Bounce Cosmology and Primordial Perturbations

Speaker: Yi-Fu Cai (IHEP)

July, 2009 in USTC

Outline

- Introduction on bounce cosmology
  - Basic idea
  - Models
- A specific realization: Lee-Wick Bounce
- Primordial perturbations in a bouncing universe
- Non-Gaussianity in bounce cosmology
- Conclusions

Introduction

Bounce Cosmology: Preliminary

Problems in hot Big Bang cosmology:

- Horizon problem
- Flatness problem
- Monopole
- Structure formation
- And singularity…

A successful model of early universe: **Inflation**

suggests an acceleration at early times

solves the first three problems

[Guth, Phys.Rev.D.23, 347 (1981)]

Bounce Cosmology: Models

Inflation suffers initial singularity problem

[Borde and Vilenkin, Phys.Rev.Lett.72,3305 (1994)]

Our universe comes from a contracting phase,

Solving singularity problem,

Viewed as an alternative to inflation.

Models of bounce cosmology:

- Pre-big-bang
- Modified gravity
- Loop quantum cosmology
- String gas cosmology
- And so on…

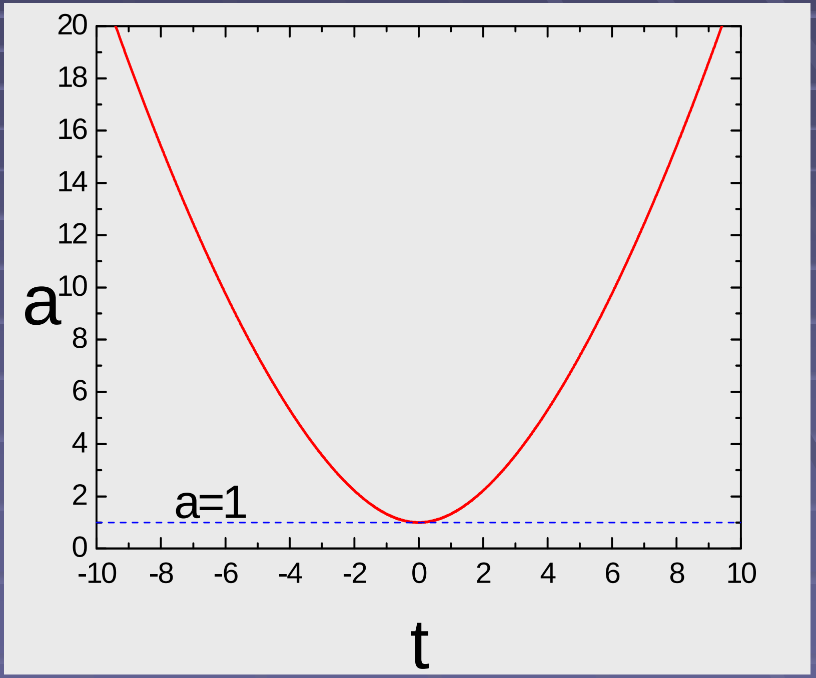
Bounce Cosmology: Basic feature

In the effective description of 4D Einstein's gravity:

Contracting: $H < 0$ Expanding: $H > 0$

Bounce point: $H = 0$ Around it: $\dot{H} > 0$

$$\dot{H} = -4\pi G\rho(1+w) \quad \Rightarrow \quad w < -1$$



Transition to the observable universe (radiation/matter/DE dominant, …) So in 4D Einstein's gravity an effective w crosses -1, and so a Quintom scenario!

This feature also holds even if $\dot{H} = 0$ and so is quite generic.

Cai, Qiu, Piao, Li, Zhang, JHEP 0710, 071 (2007), 0704.1090 [gr-qc]

A recent specific realization of bounce cosmology

Lee-Wick Bounce
(Matter Bounce)

Cai, Qiu, Brandenberger, Zhang,
PRD to be appeared,
0810.4677 [gr-qc]

Lee-Wick model

The Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 - V(\hat{\phi}),$$

A higher derivative term is involved.
Classically, a new degree of freedom is obtained.
(so-called LW partner)

Equivalent Lagrangian :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi-\tilde{\phi})^2 - V(\phi-\tilde{\phi}).$$

Regular Higgs $\phi \equiv \hat{\phi} + \tilde{\phi}$ LW partner $\tilde{\phi} \equiv \frac{\partial L}{\partial \Box \hat{\phi}}$

The mass terms can be diagonalized by rotating the field basis.
Generically, there is a coupling between the two fields.
For example: $V(\phi - \tilde{\phi}) = \frac{\lambda}{4}(\phi - \tilde{\phi})^4.$

Equations of Motion

- Metric of space-time:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \,,$$

- Einstein action coupled to Lee-Wick Model leads to the following equations for cosmological dynamics:

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2 + \frac{\lambda}{4}(\phi - \tilde{\phi})^4\right] \,,$$

$$\dot{H} = -4\pi G\left(\dot{\phi}^2 - \dot{\tilde{\phi}}^2\right) \,.$$

- In addition, there are the coupled Klein-Gordon equations for ϕ and $\tilde{\phi}$.

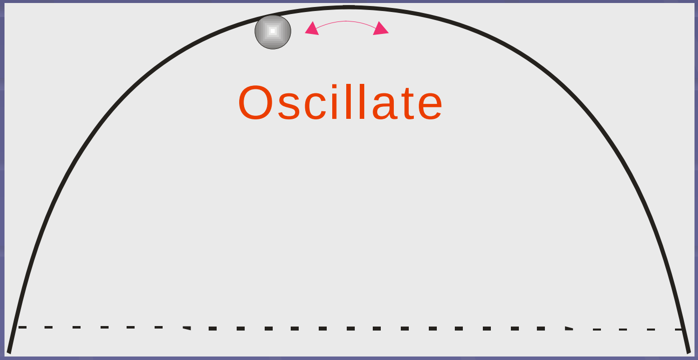
Sketch

A heavier field is much more stable than a lighter one at low energy densities and curvatures.

Contracting: ϕ dominates

Near the bounce:

$\phi_f \simeq \frac{M_{pl}}{\sqrt{12\pi}}$ ϕ freezes $\tilde{\phi}$ still oscillates



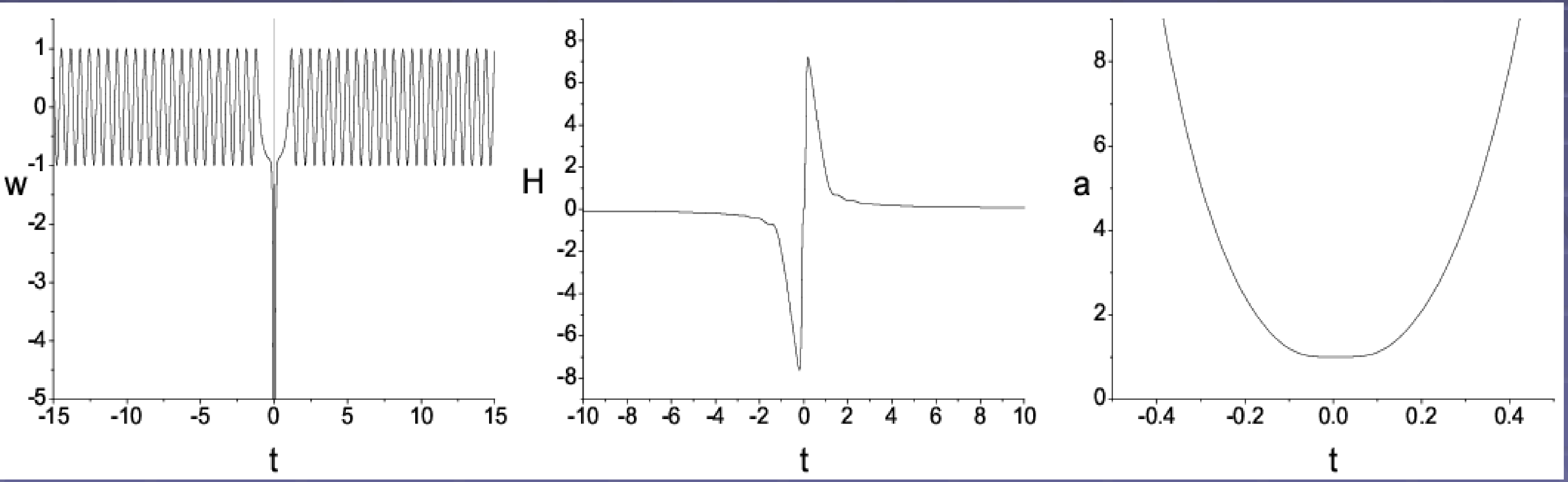
A bounce happens:

when the contribution of LW scalar to the energy density catches up to that of the normal scalar

Expanding: ϕ dominates again

Numerical Results

The plots of the equation-of-state, Hubble parameter, and scale factor in the model:

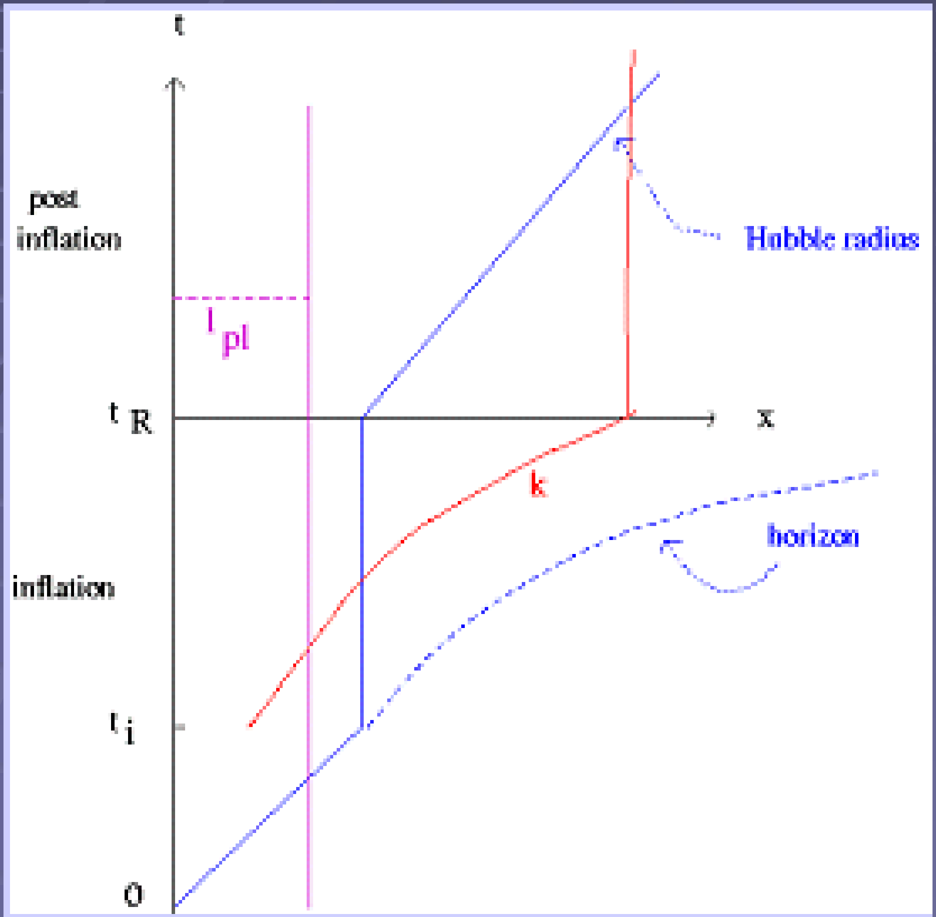


Primordial perturbations in bounce cosmology

- Preliminaries
- Formalism

Comparison with inflationary cosmology

[R.Brandenberger, hep-ph/9910410]

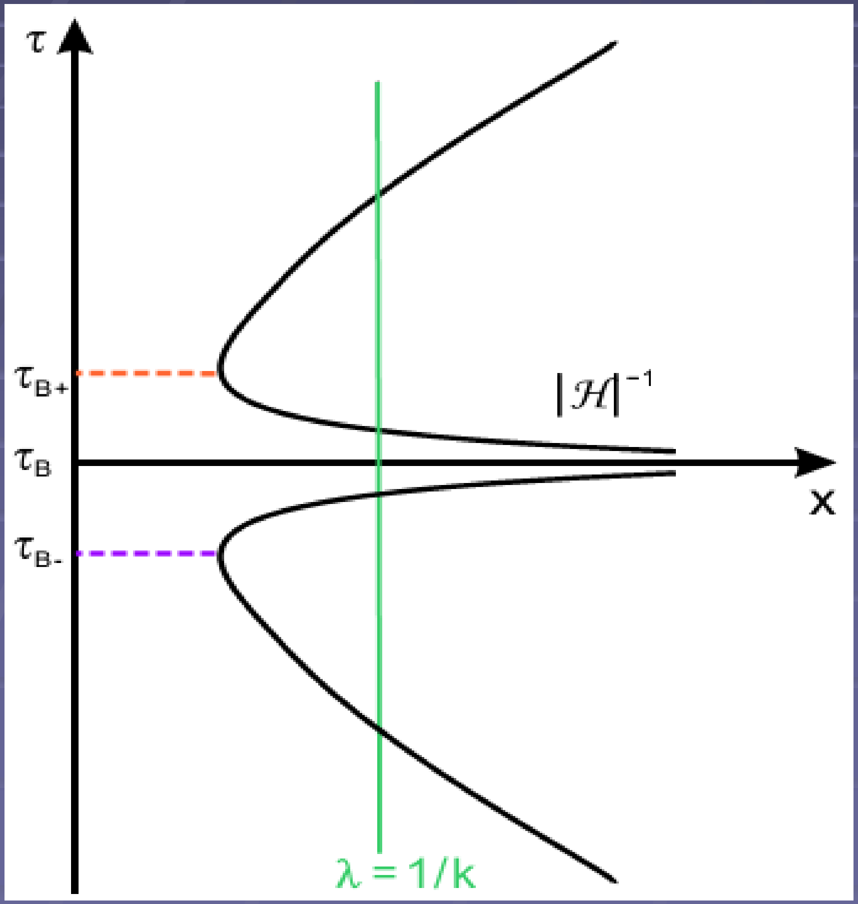


Crucial facts:

- Fluctuations originate on sub-Hubble scales
- Fluctuations propagate for a long time on super-Hubble scales
- Trans-Planckian problem

Success: a scale-invariant spectrum

Space-time sketch for a bounce model



Note:

✓Fluctuations emerge inside the Hubble radius

✓and propagate on super-Hubble scales as in inflationary cosmology.

✓No trans-Planckian problem

Can we obtain a scale-invariant spectrum as in inflation?

Primordial perturbations in bounce cosmology

- Preliminaries
- Formalism

Cai, Qiu, Brandenberger, Zhang,
PRD to be appeared,
0810.4677 [gr-qc]

Setup of Perturbations

Perturbed scalars: $\phi \to \phi + \delta\phi$

Perturbed metric: $ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)dx^i dx^i]$

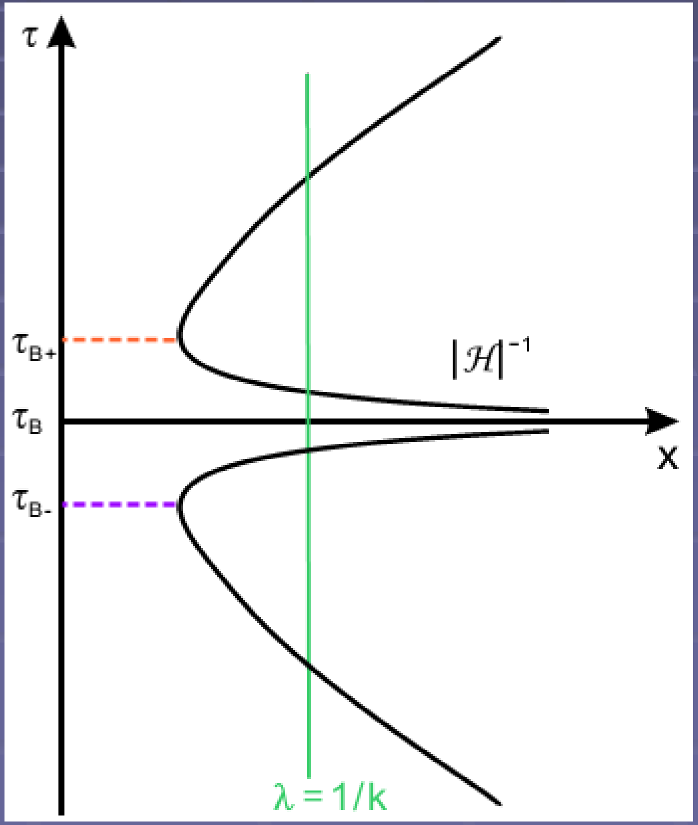
Pert. Equation: $\Phi'' + 2(\mathcal{H} - \frac{\phi''}{\phi'})\Phi' + 2(\mathcal{H}' - \mathcal{H}\frac{\phi''}{\phi'})\Phi - \nabla^2\Phi = 0$

Status：

Contracting phase

The Bounce

Expanding phase



Curvature perturbation on uniform density

Specifically, we consider a matter bounce

$$\zeta = \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}(\Phi' + \mathcal{H}\Phi)$$

Contracting: $\zeta_k = \frac{A}{\sqrt{2k^3}} X_k$

$$A = i\frac{4H_B}{\sqrt{3}\mathcal{H}_B^3}\ , \ X_k = \frac{e^{ik(\eta-\tilde{\eta}_B)}}{(\eta-\tilde{\eta}_B)^3}[1 - ik(\eta - \tilde{\eta}_B)]$$

Expanding: $\zeta_k = \zeta_k|_B \sim k^{-\frac{3}{2}}$

Scale-invariant and constant

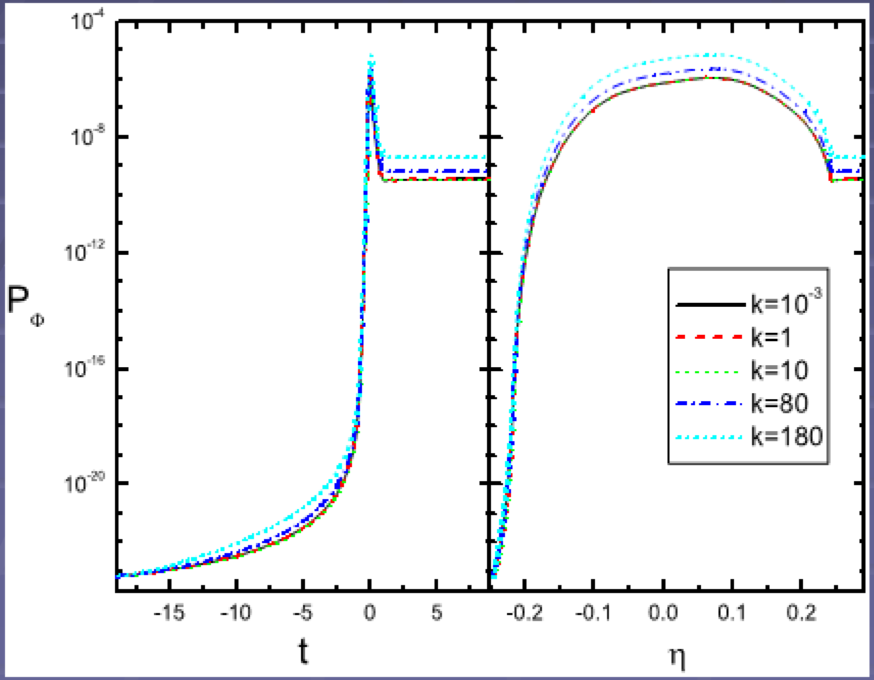
Comments:

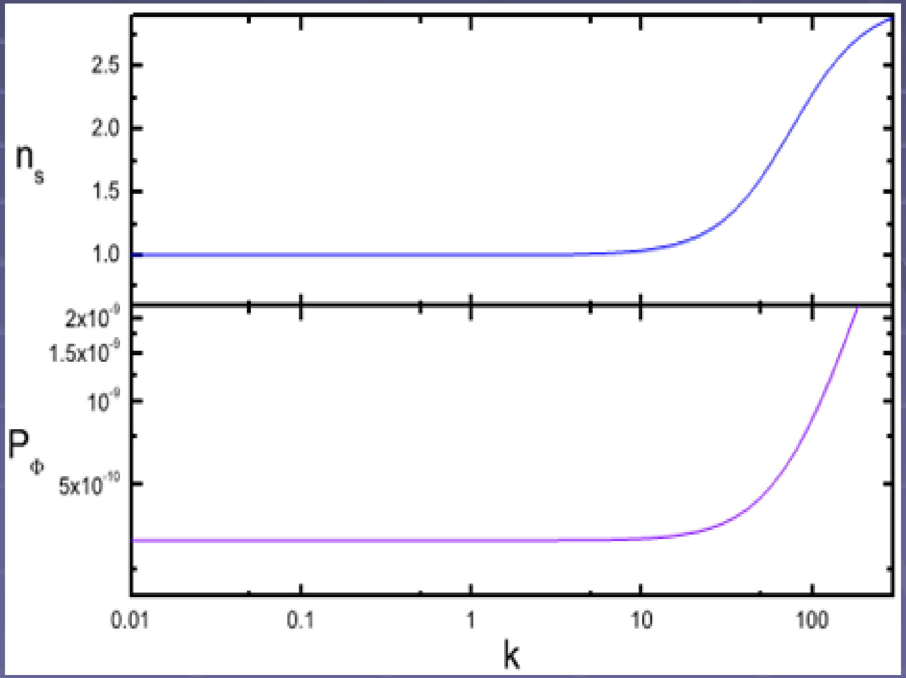
1, zeta is **no longer a conserved quantity** outside Hubble radius when the universe is contracting, but it is continuous through the bounce;

2, curv. pert. in Ekpyrotic bounce is not scale-invariant $\zeta_k \sim k^{-\frac{1}{2}}$

Numerical Results

The plots of the power spectrum and spectral index





Non-Gaussianity in Bounce Cosmology

Cai, Xue, Brandenberger, Zhang,
JCAP 0905,011 (2009)
arXiv:0903.0631 [astro-ph]

Non-Gaussianity in early universe

Non-gaussianity parameter:

$$\zeta = \zeta_g(x) + \frac{3}{5} f_{NL} \zeta_g^2$$

WMAP5 data:

$$-9 < f_{NL}^{\text{local}} < 111 \ (95\% \ \text{CL}) \qquad -151 < f_{NL}^{\text{equil}} < 253 \ (95\% \ \text{CL})$$

- The contribution from redefinition:

$$\zeta \to \zeta - \epsilon \zeta^2 + \ldots \qquad \epsilon = \frac{3}{2}(1+w)$$

So it gives

$$f_{NL} \supseteq \frac{5}{3}\epsilon$$

- Also other contributions…

Non-Gaussianity in bounce cosmology

Three-point correlation function:

$$< \zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3) > \; = \; i\int_{t_i}^{t} dt' < [\zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3), L_{int}(t')] >$$
$$= \; (2\pi)^7\delta(\sum \vec{k}_i)\frac{P_\zeta^2}{\prod k_i^3} \times \mathcal{A}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

Lagrangian in cubic order:

$$\mathcal{L}_3 = (\epsilon^2 - \frac{\epsilon^3}{2})a^3\zeta\dot{\zeta}^2 + \epsilon^2 a\zeta(\partial\zeta)^2 - 2\epsilon^2 a^3\dot{\zeta}(\partial\zeta)(\partial\chi) + \frac{\epsilon^3}{2}a^3\zeta(\partial_i\partial_j\chi)^2 + f(\zeta)\frac{\delta\mathcal{L}_2}{\delta\zeta}|_1$$

with $\chi \equiv \partial^{-2}\dot{\zeta}$ and

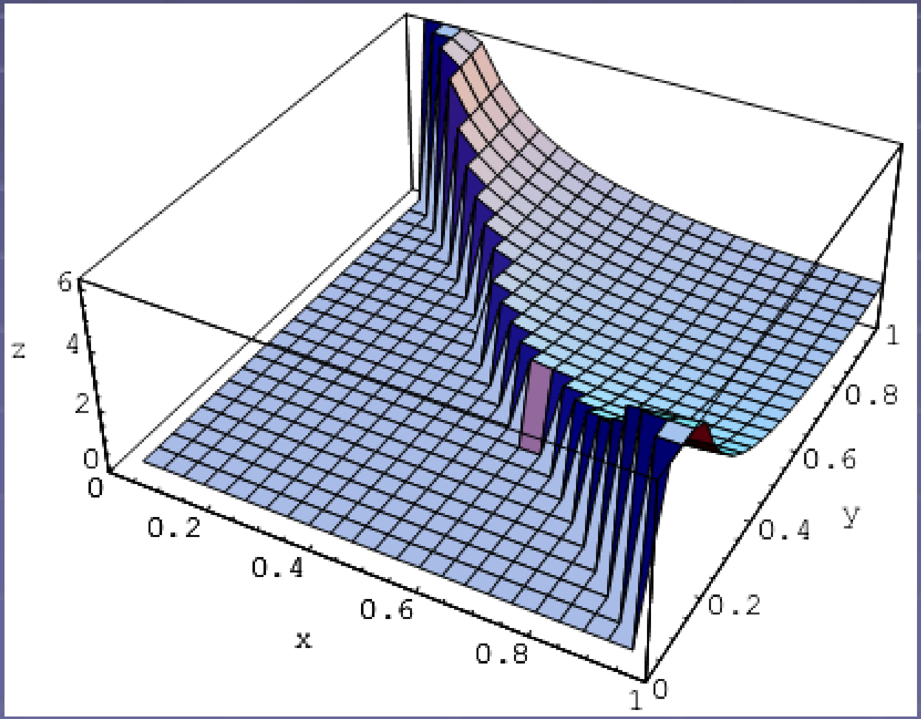
$$f(\zeta) = \frac{1}{4\mathcal{H}^2}(\partial\zeta)^2 - \frac{1}{4\mathcal{H}^2}\partial^{-2}\partial_i\partial_j(\partial_i\zeta\partial_j\zeta)$$
$$-\frac{a}{\mathcal{H}}\zeta\dot{\zeta} - \frac{\epsilon a}{2\mathcal{H}}\partial_i\zeta\partial_i\partial^{-2}\dot{\zeta}$$
$$+\frac{\epsilon a}{2\mathcal{H}}\partial^{-2}\partial_i\partial_j(\partial_i\partial^{-2}\dot{\zeta}\partial_j\zeta) \; ,$$

Non-Gaussianity in bounce cosmology

shape: $\frac{\mathcal{A}_T}{k_1 k_2 k_3}$

- of order ε

$$\mathcal{A}^{\epsilon} = -\frac{\epsilon}{2}\sum k_i^3$$

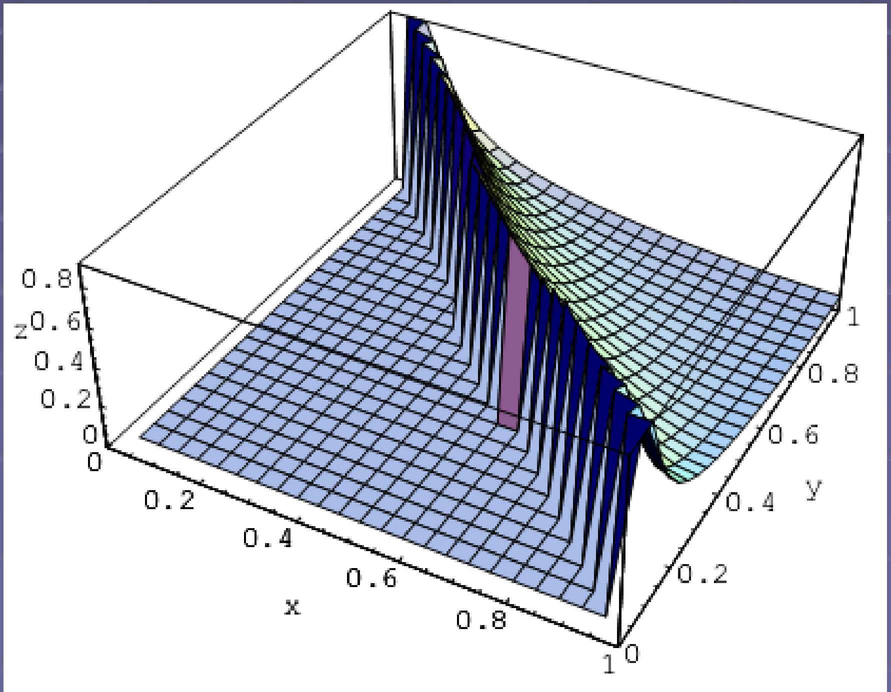


Non-Gaussianity in bounce cosmology

shape: $\frac{\mathcal{A}_T}{k_1 k_2 k_3}$

- of order ε^2

$$\mathcal{A}^{\epsilon^2} = -\frac{\epsilon^2}{24}\sum k_i^3 + \frac{\epsilon^2}{32}\sum_{i\neq j} k_i k_j^2 + \frac{\epsilon^2}{96\prod k_i^2}\left\{5\sum_{i\neq j} k_i^7 k_j^2 - 3\sum_{i\neq j} k_i^6 k_j^3 - 2\sum_{i\neq j} k_i^5 k_j^4\right\}$$

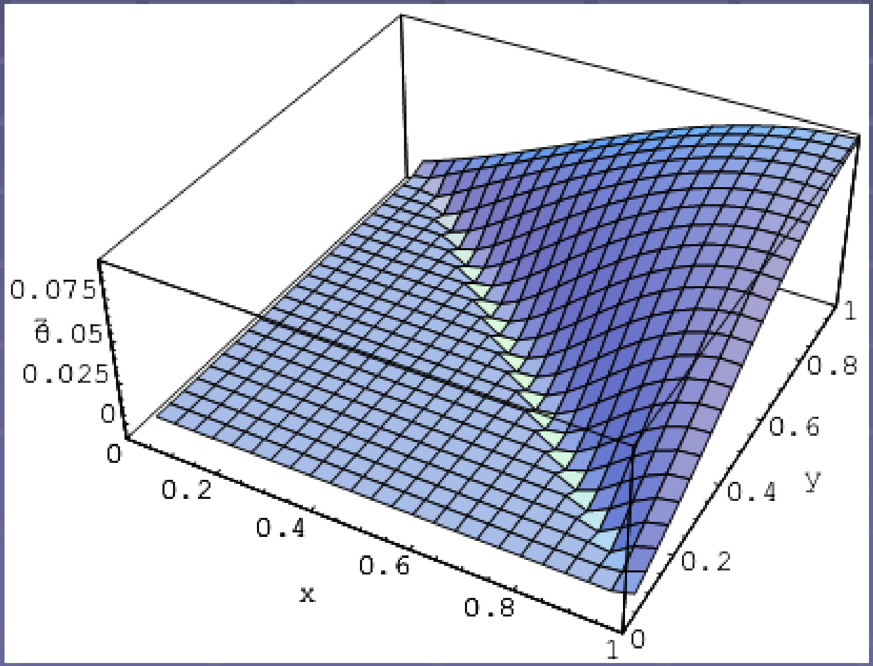


Non-Gaussianity in bounce cosmology

shape: $\dfrac{\mathcal{A}_T}{k_1 k_2 k_3}$

- of order ϵ^3

$$\mathcal{A}^{\epsilon^3} = \frac{\epsilon^3}{48}\sum k_i^3 + \frac{\epsilon^3}{96}\sum_{i\neq j} k_i k_j^2 + \frac{\epsilon^3}{96\prod k_i^2}\left\{\sum_i k_i^9 - 3\sum_{i\neq j} k_i^7 k_j^2 - \sum_{i\neq j} k_i^6 k_j^3 + 3\sum_{i\neq j} k_i^5 k_j^4\right\}$$



Non-Gaussianity in bounce cosmology

Main results:

- No slow roll —— a sizable amplitude;
- No slow roll —— new shapes;
- No conservation (zeta) —— new origins;
- Specifically, we consider a matter bounce

$$f_{NL}^{\text{local}} = -\frac{35}{8}$$

Consequence:

- Detectable in Planck?

Non-Gaussianity in bounce cosmology

- **Squeezed limit:** $k_1 = k_2 \equiv k$ **and** $k_3 \to 0$

  The leading terms of above three shape functions takes

  $$\mathcal{A} \sim \frac{k^5}{k_3^2}$$

  which looks super-local.
  However, by summing up, the super-local terms cancel each others, and finally

  $$\mathcal{A}_T|_{\text{squeezed}} = -\frac{21}{8}k^3$$

- Question: is this cancellation a coincidence or a generic feature?
- Application of our method in other bounce models.

Thermal fluctuations in Bounce Cosmology

Cai, Xue, Brandenberger, Zhang,
JCAP 0906, 037 (2009)
arXiv:0903.4838 [astro-ph]

Thermal fluctuations in bounce cosmology

Metric perturbation: gravitational potential

The solution: $\Phi = \Phi_D + \Phi_S$

Initial conditions:

Bunch-Davies (matter bounce)

Thermal initial state

Matching relations:

Contracting

S_- growing

D_- constant

Expanding

$$\Phi_D \sim \mathcal{O}(1) D_- + \mathcal{O}(1) \left(\frac{k}{k^*} \right)^2 S_-$$

Φ_S irrelevant

Observations: Scale-invariant Spectrum

$$P_\Phi = k^3 |\Phi_D|^2 \sim k^0$$

Thermal fluctuations in bounce cosmology

- Normal radiation: w_r=1/3

and heat capacity $C_V(R) = R^3 \frac{\partial \rho}{\partial T} \sim R^3 T^{\frac{1}{w_r}}$

Only when the background EoS w=7/3 a scale-invariant spectrum could be obtained;

- Holographic radiation: $C_V(R) \sim R^2 M_p^2$

Only when the background EoS w=0 (matter bounce) a scale-invariant spectrum could be obtained.

Conclusions

Summary

- An alternative to inflation
- Avoiding initial singularity
- Scale-invariant spectra in bounce models
- Large non-gaussianity

Outlook

- Unsettled issues
- A scenario of bounce inflation

  Piao et.al., Phys.Rev.D69,103520 (2004)

  Cai et.al., Phys.Rev.D79,021303 (2009)

  Cai et.al., JCAP 0906,003 (2009)
- String cosmology: a nonsingular model

  [Tolman, 1934]

  [Steinhardt et.al., Phys.Rev.D64,123522 (2001)]

Unsettled issues

- Excess of tensor to scalar ratio
- Anisotropic instability
- Causality
- …

These issues deserve future studies

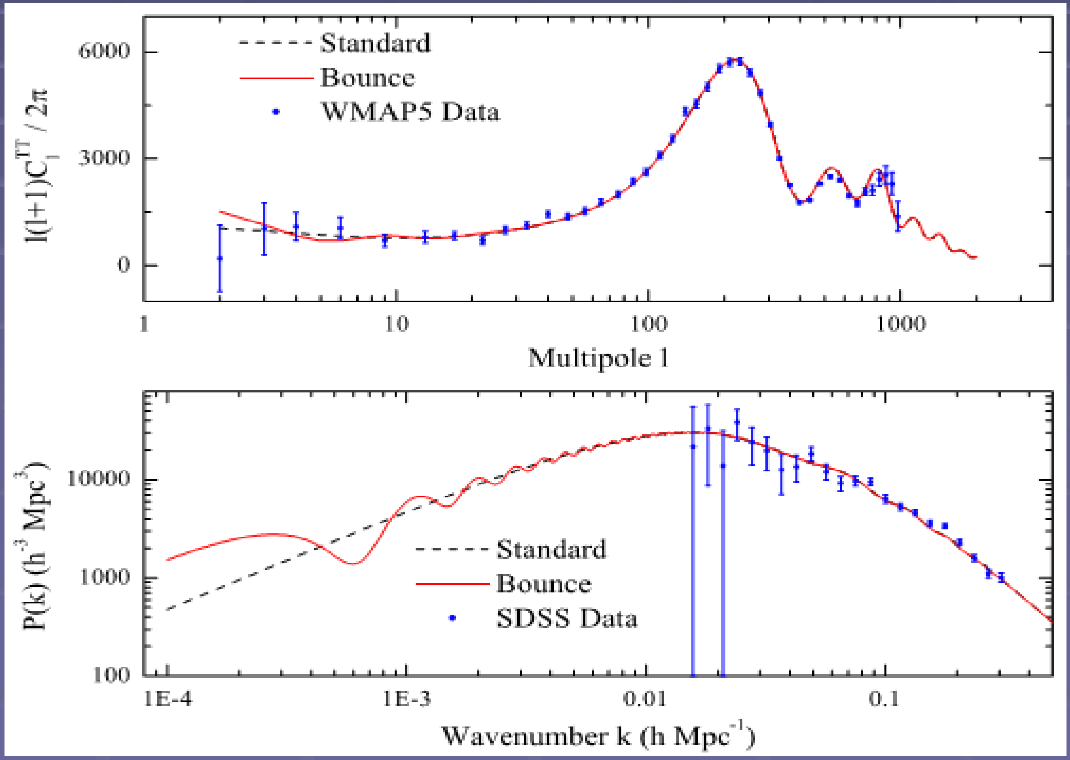
Bounce + inflation

Large field model: [Brandenberger, and Zhang, arXiv:0903.2065]

the spectrum is far away from scale-invariance

Small field model: $$V = \frac{1}{4}\lambda\phi^4\left(\ln\frac{|\phi|}{v} - \frac{1}{4}\right) + \frac{1}{16}\lambda v^4$$ Phys.Rev.D79, 021303 (2009)

A featured spectrum,

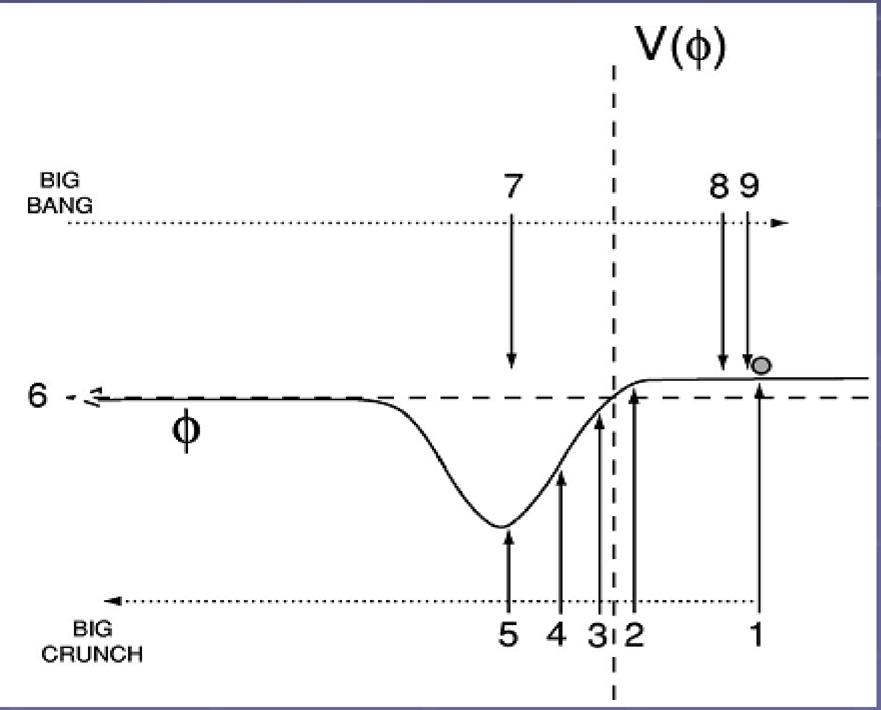


String cosmology

Ekpyrotic model:

The collision of two M branes in 5D gives rise to a cyclic universe, and 4D effective description is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a **singular** bounce
7 after big bang
8 radiation domination
9 matter domination

String cosmology

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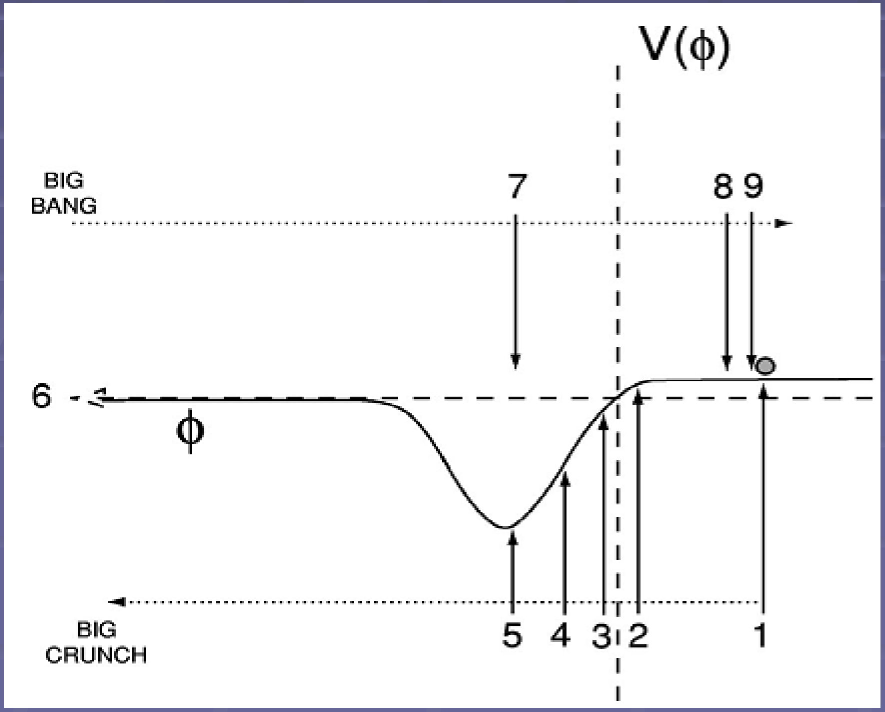

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6 a **singular** bounce
7 after big bang
8 radiation domination
9 matter domination

Problem: singularities unsolved
effective description fails

String cosmology

Quintom scenario at late time evolution:
A dark energy model inspired by string theory,
Cai, Li, Lu, Piao, Qiu, Zhang, Phys.Lett.B651,1 (2007)
Relations with bounce cosmology?

A potential solution to singularity in string cosmology:
Mirage Braneworld in heterotic M-theory
In preparation…

THANK YOU!

History of Lee-Wick I

- 1969: T. D. Lee and G. C. Wick proposed the Lee-Wick mechanism.
  - Higher derivative action → new degrees of freedom.
  - Equivalent to adding fields with opposite kinetic term.
  - → problem of ghosts.
- 1970's: debates on the consistency of the theory.
- 1970's: super-symmetry discovered → interest in Lee-Wick theory wanes.

History of Lee-Wick II

- 2007: Lee-Wick construction resurrected by Grinstein, O'Connell and Wise:

  Lee-Wick Standard Model.
  - No progress on conceptual issues related to ghosts.
  - Phenomenological studies (LHC).

Unsettled issue:

Quantization? Some attempts,

van Tonder, arXiv:0810.1928,

Shalaby, arXiv:0812.3419.

Considered as an effective description of field theory beyond SM.

Trans-Planckian Problem for Inflationary Cosmology

- Success of inflation: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- Problem: If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.
- → Planck scale physics testable with cosmological observations!

[J. Martin, and R.Brandenberger, Phys.Rev.D63:123501,2001]

No Trans-Planckian problem for Bouncing Cosmology

- Note: The physical length of fluctuation modes is always greater than Planck length
- → fluctuations are in the far IR regime throughout, and so no trans-Planckian in Matter Bounce