Emergence of Time Semicrystals in Holographic Driven-Dissipative Systems

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2026.04.03

To be submitted, in cooperation with Yu-Qi Lei, Yu Tian and Shao-Feng Wu.

Outline

1. Background and Motivation
2. Holographic Setup
3. Numerical results and Emergence of Time Semi-Crystal
4. Dynamic Phase Transition and Scaling Behaviors
5. Summary & Outlook

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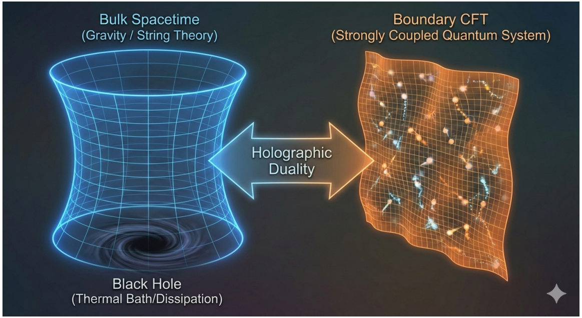
Holographic Approach (AdS/CFT)



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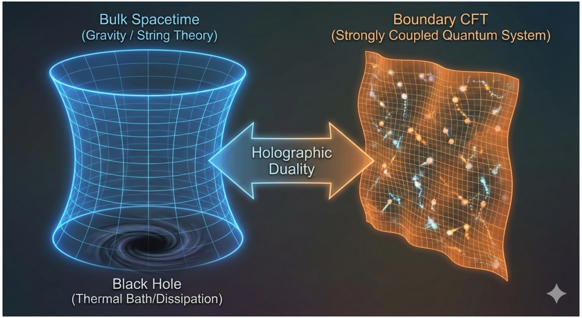
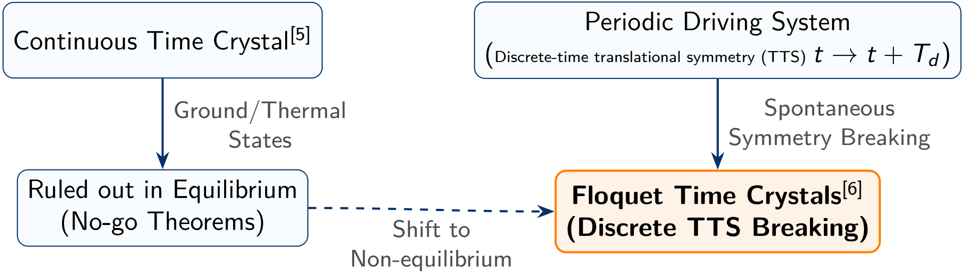


Why Holography?

- Rooted in macroscopic brane geometries[1], holographic duality[2] provides a **powerful framework** for strongly coupled systems.
- The bulk black hole naturally provides a thermal bath, introducing **dissipation** to the boundary quantum system[3].
- Ideal for exploring the long-time evolution of **non-equilibrium dynamics** and driven-dissipative systems[4].

[1] M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rept. **259**, 213-326 (1995).
[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231-252 (1998).
[3] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, JHEP **12**, 015 (2008).
[4] H. Liu and J. Sonner, Rept. Prog. Phys. **83**, no.1, 016001 (2019).

Origin of Time Crystals



- **Signature of DTC:** Subharmonic response ($T = nT_d$) $\to$ a stable temporal order.

[5] F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012).
[6] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).

Non-equilibrium Dynamic phase

The research of DTC in holography: Holographic dissipative space-time supersolids.[7]

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More non-equilibrium phases in driven quantum manybody systems:

- Higher-order and fractional driven time crystal[8,9]
- Time quasicrystal[10]
- Topological time crystals[11]
- ...

[7] P. Yang, M. Baggioli, Z. Cai, Y. Tian and H.B. Zhang. Phys. Rev. Lett. 131, 221601 (2023).
[8] A. Pizzi, J. Knolle and A. Nunnenkamp, Nature Commun. **12**, 2341 (2021).
[9] B. Liu, L. H. Zhang, Q. F. Wang, Y. Ma, T. Y. Han, J. Zhang, Z. Y. Zhang, S. Y. Shao, Q. Li and H. C. Chen, *et al.*, Nature Commun. **15**, no.1, 9730 (2024).
[10] S. Autti, V. B. Eltsov and G. E. Volovik, Phys. Rev. Lett. **120**, no.21, 215301 (2018).
[11] K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski and K. Sacha, New J. Phys. **21**, no.5, 052003 (2019).

From Order to Disorder: The Melt of Time Crystals

- Time crystals eventually melt into thermalized chaos through dynamical phase transitions[12−14].

[12] N. Y. Yao, A. C. Potter, I. D. Potirniche and A. Vishwanath, Phys. Rev. Lett. **118**, no.3, 030401 (2017).
[13] P. Frey and S. Rachel, Sci. Adv. **8**, no.9, abm7652 (2022).
[14]R. Yousefjani, K. Sacha and A. Bayat, Phys. Rev. B **111**, no.12, 125159 (2025).

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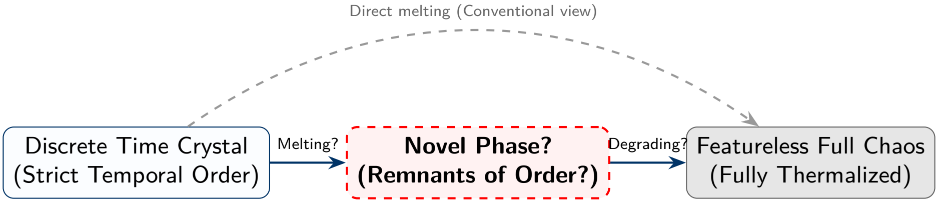

[12] N. Y. Yao, A. C. Potter, I. D. Potirniche and A. Vishwanath, Phys. Rev. Lett. **118**, no.3, 030401 (2017).
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The Time Semi-Crystal Phase

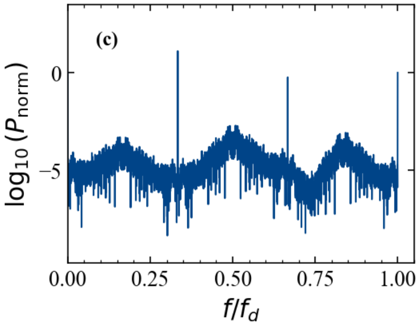

Fig. 1. 3-Time semi-crystal.

Physical Essence

- **Order & Chaos Coexistence:** Temporal order "melted" but not destroyed.
- **Residual "periodic skeleton"** embedded in a **chaotic background** ($\lambda > 0$).

Mathematical Signature (PSD):

$$P(f) = \underbrace{\sum_k A_k \delta(f - f_k)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}},$$

where $f_k = \frac{q}{n} f_d$ with coprime integers q and n.

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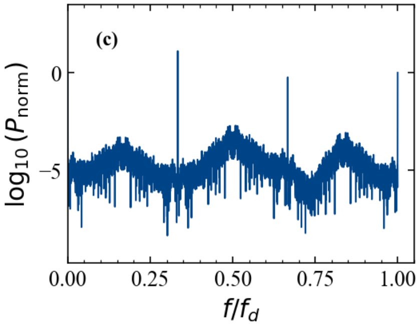

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The *n*-Time semi-crystal: Dominant subharmonic peak at $\boldsymbol{f_d/n}$, coexisting with continuous broadband spectrum.

In This Work

Periodically driven holographic system with **spontaneous Z_2 symmetry breaking**.

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- **Emergence of a New Dynamical Phase**

  Discrete Time Semi-Crystal (TSC), a novel non-equilibrium phase representing **residual temporal order within chaos**.

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- **Dynamical Phase Transitions**

  Investigating the melting processes, critical scaling behaviors, and transition routes related to the TSC.

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How temporal order changes from time crystals to full chaos?

Outline

A minimal holographic model described by the Einstein-scalar action[15]

$$S = \int d^4x \sqrt{-g}\left[R + \frac{6}{L^2} - (\nabla\phi)^2 - V(\phi)\right]. \tag{1}$$

where AdS radius $L = 1$. The real scalar field ϕ has the nonlinear potential $V(\phi) = m^2\phi^2 + \frac{\phi^4}{6}$.

[15] T. Faulkner, G. T. Horowitz and M. M. Roberts. JHEP. 2011, 51 (2011).

Holographic model

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Work in the probe limit, and fix the background as the Schwarzschild-AdS black hole

$$ds^2 = \frac{L^2}{z^2}\left(-h(z)dt^2 - 2dtdz + dx^2 + dy^2\right), \quad h(z) = 1 - \frac{z^3}{z_h^3}. \tag{2}$$

The black hole horizon $z_h = 1$, and the AdS boundary at $z = 0$. The Hawking temperature is $T_H = \frac{3}{4\pi z_h}$.

[15] T. Faulkner, G. T. Horowitz and M. M. Roberts. JHEP. 2011, 51 (2011).

The Klein-Gordon equation

$$\nabla^2\phi - \frac{1}{2}\frac{\partial V(\phi)}{\partial \phi} = 0. \tag{3}$$

Near the AdS boundary ($z \to 0$), the asymptotic behavior of scalar field

$$\phi = z\psi = z\left(\mathcal{A} + z\mathcal{B} + \cdots\right), \tag{4}$$

where $\mathcal{A}$ is the condensate response $\langle \mathcal{O} \rangle$ with the *alternative quantization*.

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With the double trace deformation, the action of the dual field can be modified by a scalar operator $\mathcal{O}$

$$S \to S - \int d^3x\, \varkappa_d\, \mathcal{O}^\dagger \mathcal{O}, \tag{5}$$

where $\varkappa_d = 2(3 - 2\Delta_-)\varkappa$ is the coupling parameter, $\varkappa$ is the rescaled coupling parameter used for convenience later. The Z_2 symmetry can spontaneous breaking.

This deformation imposes a mixed boundary condition to the scalar field near the AdS boundary

$$\mathcal{B} = \varkappa \mathcal{A}. \tag{6}$$

Under the isotropy assumption, the equation of motion can be reduced

$$2\partial_t \partial_z \psi - h\, \partial_z^2 \psi - h'\, \partial_z \psi + z\psi + \frac{1}{3}\psi^3 = 0. \tag{7}$$

With periodic driving $F_d(t)$, the boundary condition generalizes to

$$\begin{aligned} \partial_t \psi|_{z=0} &= (\partial_z \psi - \bar{\varkappa}\psi + F_d(t))|_{z=0}\,, \\ F_d(t) &= F_0 \cdot \sin(\omega_d \cdot t). \end{aligned} \tag{8}$$

Here $\bar{\varkappa} = \varkappa / z_h$, and Z_2 symmetry breaking occurs for $\bar{\varkappa} \lesssim -0.386$. For definiteness, we fix $\bar{\varkappa} = -2$ and $F_0 = 4$, focusing on the system's response to varying driving frequency ω_d.

Outline

Characterizing the Dynamical Phases

The distinct dynamical phases of the system based on their spectral and phase-space features.

The Time Semi-Crystal Signature

The Time semi-crystal phase exhibits a unique power spectrum combining order and chaos:

$$P(f) = \underbrace{\sum_k A_k \delta(f - f_k)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}},$$

where $f_k = \frac{q}{n} f_d$ with coprime integers q and n.

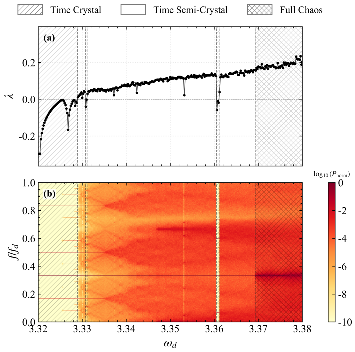


Fig. 2. Dynamical Phase Diagram

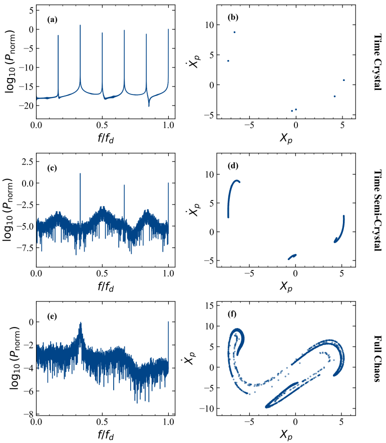
Phase Classifications:

- **Discrete Time Crystal (DTC)**
  - Sharp subharmonic peaks
  - $\lambda < 0$ (Strict temporal order)
- **Time Semi-Crystal (TSC)**
  - Subharmonic skeleton + Continuous background
  - $\lambda > 0$ (Residual order survives)
- **Full Chaos**
  - Broadband continuous spectrum
  - $\lambda > 0$ (Order vanishes)

Spectrum & Poincaré Section Features

- **DTC Phase:**
  - *Spectrum:* Sharp discrete subharmonic peaks only.
  - *Poincaré section:* Discrete points.
- **Time Semi-Crystal Phase:**
  - *Spectrum:* Subharmonic skeleton + continuous background.
  - *Poincaré section:* **Finite non-closed curves**.

  The coexistence of temporal order and chaos
- **Full Chaos:**
  - *Spectrum:* Broadband continuous spectrum.
  - *Poincaré section:* Discrete point cloud with fractal structure.



The Melting of Temporal Order

Discrete Time Crystal
Strict Periodicity
$\lambda < 0$

→ Melting →

Time Semi-Crystal (TSC)
Periodic Skeleton + Chaos
$\lambda > 0$

→ Collapse →

Full Chaos
Completely Disorder
$\lambda > 0$

The Time Semi-Crystal Phase

- The transition is **not an abrupt collapse** into full chaos.
- In the TSC phase, chaotic fluctuations ($\lambda > 0$) coexist with a long-lived **subharmonic skeleton**.

Investigate how this temporal order melts, and how the **residual order evolves** as parameters vary. —**Dynamic phase transition**

Outline

Dynamical Phase Transitions in the Holographic Model

1. The Melting Transition
(DTC $\rightarrow$ Time Semi-Crystal)
Order Parameter: Lyapunov Exponent λ

The onset of disorder

2. Internal Restructuring
(e.g., 6-TSC $\rightarrow$ 3-TSC)
Order Parameter: Subharmonic Peak Height A

Change of the discrete skeleton

What are the **scaling behaviors** governing these two distinct dynamical transitions?

How **temporal order** changes?

The transition of DTC to Time Semi-Crystal

As ω_d approaches a critical value ω_c, the discrete time crystal begins to melt, giving way to the Time semi-crystal phase.

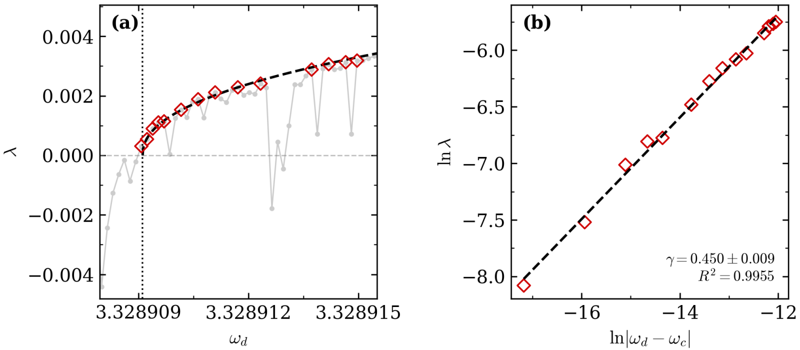
Order Parameter & Scaling Law

Maximal Lyapunov Exponent (λ): Serves as the order parameter characterizing the onset of chaos.

- $\lambda < 0$: Ordered DTC phase.
- $\lambda > 0$: Time semi-crystal with disorder.

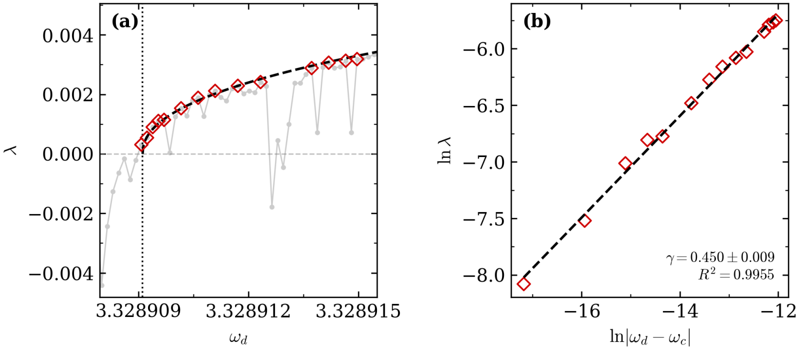
Near the critical point ω_c, λ follows a continuous power law:

$$\lambda \sim |\omega_d - \omega_c|^\gamma \tag{9}$$



Universal Scaling

The critical parameter $\omega_c \approx 3.329$. The fitted critical exponent $\boldsymbol{\gamma = 0.450 \pm 0.009}$ is in excellent agreement with the universal value for **period-doubling cascades.**



Melting & Residual Order

Melting of temporal order & **Emergence of temporal disorder** ($\lambda > 0$)

↓

Emergence of **residual order embedded in chaos** (Time semi-crystal)

Internal Restructuring: 6-Time semi-crystal to 3-Time semi-crystal

Inside the Time semi-crystal phase, the discrete "skeleton" undergoes dynamical restructuring (e.g., the melting of $f_d/6$ subharmonic peak).

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Order Parameter

To quantify this internal transition, we define the order parameter A as the normalized height of the $\boldsymbol{f_d/6}$ **subharmonic peak**:

$$A \equiv \frac{\int_{f_d/6-\Delta f}^{f_d/6+\Delta f} P_{\text{norm}}(f)\, df}{\int_0^{f_d} P_{\text{norm}}(f)\, df} \tag{10}$$

- $P_{\text{norm}}(f)$: Normalized power spectral density.
- The decay of A dynamically tracks the “melting” of the 6-TSC structure.

Scaling near Criticality:

The decay of A deviates from a simple power law, exhibiting **log-periodic oscillations**:

1. Trend:

$$A_{\text{trend}} \sim (\omega_s - \omega_d)^{\nu}$$

2. LPPL Modulation:

$$A = A_{\text{trend}} \left[1 + \alpha \cos\left(\Omega \ln(\omega_s - \omega_d) + \theta_0\right)\right]$$

- $\nu = 0.360 \pm 0.017$: Critical exponent.
- $\Omega = 16.40 \pm 0.71$: Log-periodic frequency.

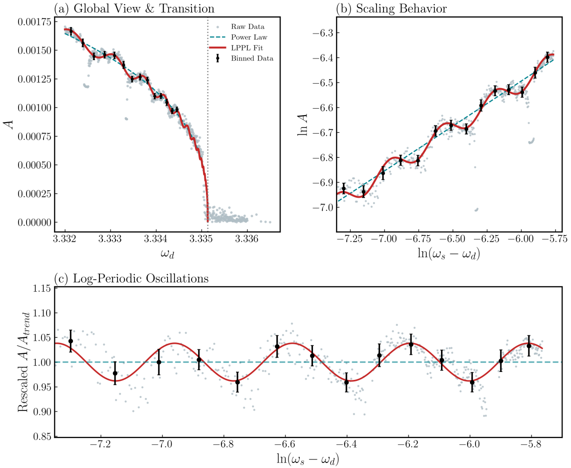

Fig. 3. Transition from 6-TSC to 3-TSC.

Log-Periodic Power Law (LPPL)

Scaling near Criticality: The decay of A deviates from a simple power law, exhibiting **log-periodic oscillations**:

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Decoding the Internal Structure

1. **Hierarchical Skeleton Phase Transition**
   - The Time semi-crystal phase is **not a featureless chaotic state**.
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This reveals a deeply **nested, hierarchical temporal structure** embedded within the chaos.

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2. **Discrete Scale Invariance (DSI)**
   - The log-periodic correction (Ω) is a direct physical signature of DSI.

Outline

Summary

1. **Phase Characterization**
   - PSD reveals a hybrid phase:

$$P(f) = \underbrace{\sum_n A_n \delta(f - f_d/n)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}}$$

Residual temporal order within chaos.

2. **Dynamical Phase Transitions**
   - **Onset:** Melting of Time Crystal → Emergence of Time semi-crystal.
   - **Internal:** Restructuring of the residual skeleton → **Log-periodic corrections**.

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2. Dynamical Phase Transitions

- **Onset:** Melting of Time Crystal → Emergence of Time semi-crystal.
- **Internal:** Restructuring of the residual skeleton → **Log-periodic corrections**.

The Time semi-crystal is **not a featureless chaotic state**. Its hierarchical residual order holds fundamental physical significance.

Outlook

1. **Theoretical Exploration**
   - Investigate the **universality classes** of these novel dynamical transitions.
   - Deepen the physical understanding of **Discrete scale invariance** in chaotic environments.

2. **Experimental Verification**
   - Connect our holographic model results with quantum many-body experiments, such as ferroelectric/ferromagnetic phase transitions and Rydberg atom simulations.

Thank you!