A brief introduction to brane inflation

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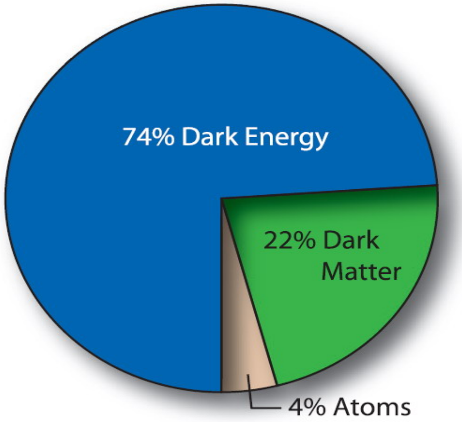
Mar. 18, 2010 at USTC

Cosmology: science based on observations



CMB: WMAP, Planck

LSS: SDSS ⇒ **hot Big Bang**

SuperNovae

Theoretical Problems:

- Horizon
- Flatness
- Monopole
- Structure formation

→ **Inflation**

- And singularity…

→ **Big Bounce**

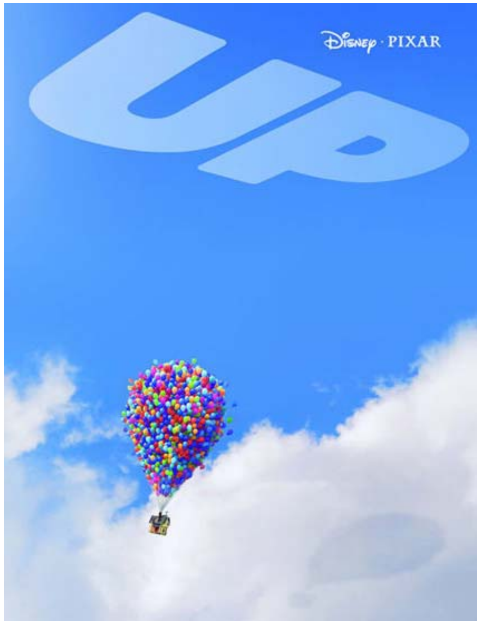
Open Questions in early universe

1. How to realize inflation at early universe?

2. What is inflaton made of?

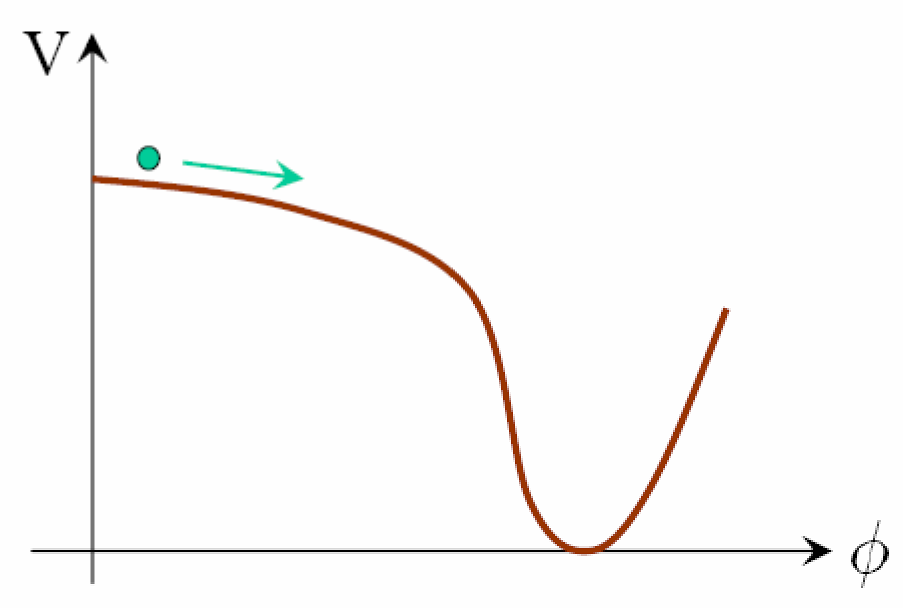
3. Does it fit to observations?

Adventure is out there!



• Preliminary of Inflationary cosmology

Guth, PRD 23:347,1981;
Linde, PLB 108:389,1982;
Albrecht & Steinhardt, PRL 48:1220,1982



Slow-roll conditions:

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 << 1$$

$$\eta_V = M_{pl}^2 \frac{V''}{V} << 1$$

Low energy description of string theory

• Preliminary of Inflationary cosmology

Type I describes both open and closed, un-oriented strings with SO(32) gauge group with low energy effective theory being N=1 SUGRA.

Type II theories are of closed strings only (not counting the open string excitations which live on the D-branes themselves), with N = 2 SUGRA effective theories.

Type IIA theories contain Dp-branes with p even, while IIB contains odd-dimensional branes.

Low energy description of string theory

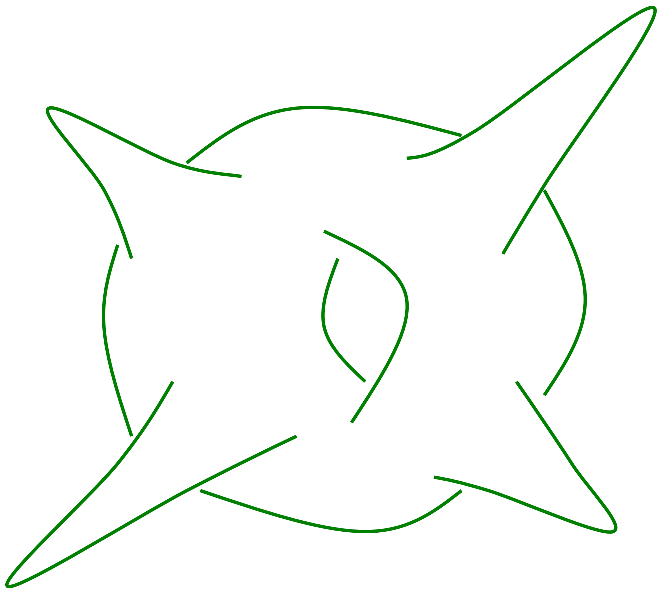
•Preliminary of Inflationary cosmology

Old conception of string theories: No 4D de-Sitter solution with stabilized moduli. Contradict with inflation!

In 2003 KKLT mechanism was proposed to illustrate the possibility of 4D dS solution.

•Warped compactification

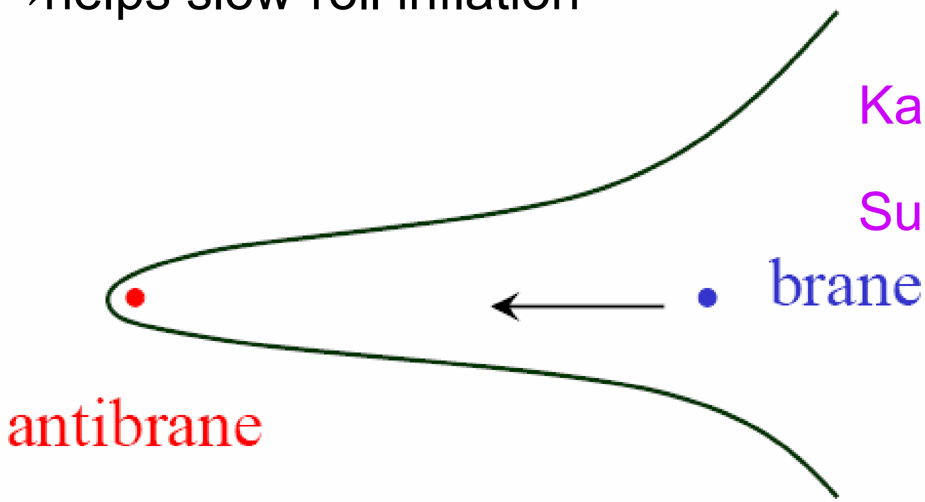
(Gidding, Kachru, Polchinski, 01;
Klebanov, Strassler, 00;
Verlinde, 99;
Randall, Sundrum, 99)



- 6D bulk
- Warped space generated by point-like (6D) sources

A first glance of KKLMMT brane inflation

•Warped space Reduce (red-shift) brane tension:
→helps slow-roll inflation



Kahler potential $K = -3\log\left(\rho + \overline{\rho} - \phi\overline{\phi}\right)$

Superpotential $W = W_0 + Ae^{-a\rho}$

KKLMMT, 03

$T_3 \to h^4 T_3$, h is the warp factor.

The inflaton gets the mass

$$m_\phi^2 = 2H^2$$

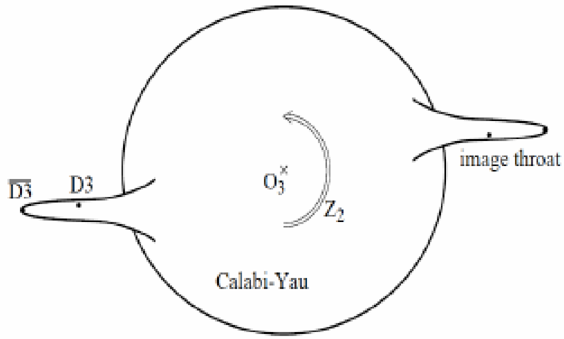
Observations: $m_\phi^2 : 10^{-2} H^2$

•The **eta**-problem

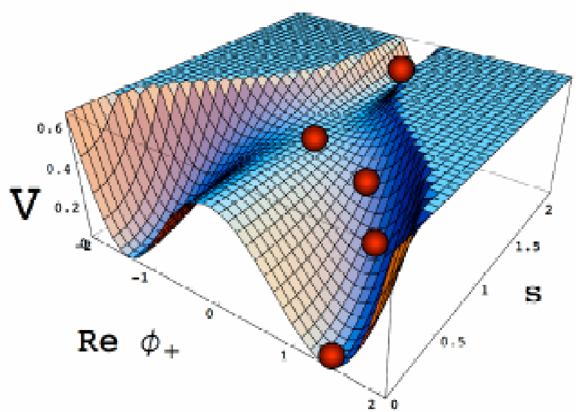
Volume stabilization;
Volume-scalar coupling;

$$V \sim \pm H^2\phi^2 \Longrightarrow \eta \sim 1$$

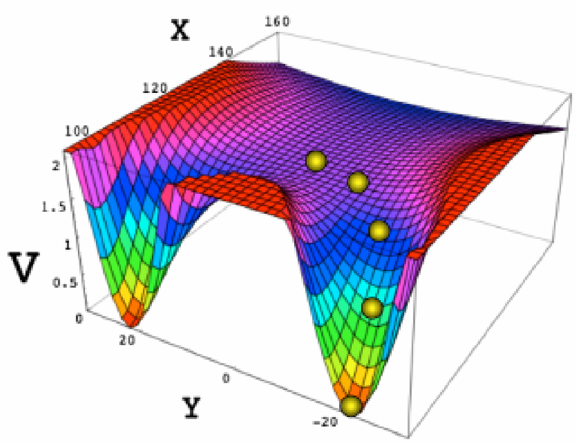
Brane inflation in II-B string theory



KKLMMT brane-anti-brane inflation



D3/D7 brane inflation



Racetrack modular inflation

DBI inflation (non-minimal kinetic terms)

DBI inflation

- A mobile D3-brane with a relativistic speed
- The effective action

$$S = \int d^4\xi \sqrt{-g^{(4)}} \left[-T(\phi)\sqrt{1 + \partial^\mu \phi \partial_\mu \phi / T(\phi)} + T(\phi) - V(\phi) \right]$$

$T(\phi) = T_3^{1/2} h^4$ ϕ: radial position of the probe brane

- Energy density and Pressure

$$\rho = T(\phi)(\gamma - 1) + V(\phi)$$

$$p = T(\phi)(1 - \gamma^{-1}) - V(\phi)$$

$$\gamma = \frac{1}{\sqrt{1 - \dot{\phi}^2 / T(\phi)}}$$

DBI inflation: Features

- A realization of k-inflation with a small sound speed **motivated by string theory!**

- No need for small mass:
  a potential solution to the η problem

- Large equilateral non-Gaussianity:

$$f_{NL} \approx \frac{1}{3}(\gamma^2 - 1)$$

signature of stringy inflation?

DBI inflation: Constraints

Baumann & Mcllister 2006; Lidsey & Huston 2007

- (UV) DBI inflation with too large non-Gaussianity seems **inconsistent with WMAP data.**

- Upper bound on r:

$$\left(\frac{\Delta\phi}{M_{\rm P}}\right)^2_* < \frac{T_3\kappa_{10}^2(\Delta\rho_*)^2}{|\Delta V_{6,*}|} \quad \Longrightarrow \quad r_* < \frac{2.5\times 10^{-6}}{(\Delta N_*)^6 {\rm Vol}(X_5)} < 10^{-7}$$

- Lower bound on r:

$$1-n_s = \frac{r}{4}\sqrt{1+3f_{NL}} + \frac{n_{NL}}{1+3f_{NL}} \mp \sqrt{\frac{r}{8}}\left(\frac{T'}{T}M_{\rm P}\right)$$

$$r_* > \frac{4(1-n_s)}{\sqrt{3f_{NL}}} > \frac{1-n_s}{8} \quad \Longrightarrow \quad r > 0.002$$

Inconsistence!

DBI inflation: Formulae

$$1 - n_s = \frac{r}{4}\sqrt{1+3f_{\rm NL}} - \frac{2\tilde{s}}{3f_{NL}} + \frac{\dot{T}}{TH}$$

$$\tilde{\epsilon} \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'}{H}\right)^2$$

$$\left(\frac{\Delta\phi}{M_p}\right)^2 \simeq \frac{r}{8}(\Delta\mathcal{N})^2$$

$$\tilde{\eta} \equiv \frac{2M_p^2 H''}{\gamma H}$$

$$\frac{\pi^2}{16} r^2 P_s \left(1 + \frac{1}{3f_{\rm NL}}\right) = \frac{T(\phi)}{M_p^4}$$

$$\tilde{s} \equiv \frac{2M_p^2\gamma' H'}{\gamma^2 H}$$

$$P_s = \frac{1}{8\pi^2 M_p^2}\frac{H^2}{c_s\tilde{\epsilon}} \qquad P_t = \frac{2}{\pi^2}\frac{H^2}{M_p^2} \qquad n_s - 1 = 2\tilde{\eta} - 4\tilde{\epsilon} - 2\tilde{s}$$

$$n_t = -2\tilde{\epsilon} \qquad r = 16 c_s\tilde{\epsilon} \qquad f_{\rm NL} = \frac{1}{3}\left(\frac{1}{c_s^2} - 1\right) \qquad c_s = \frac{1}{\gamma}$$

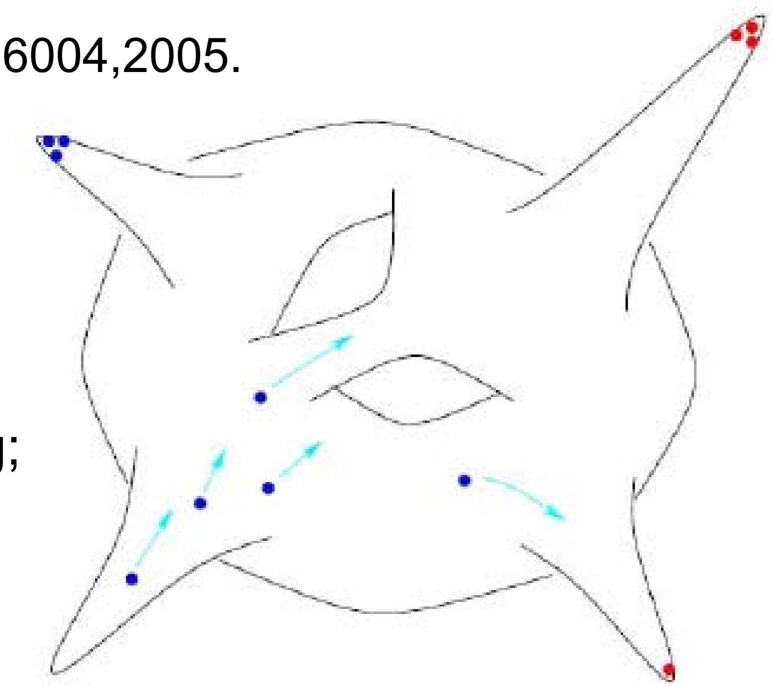
Inflationary landscape with multiple branes

➢Basic idea:

Slow-roll can be relaxed by introducing multiple degrees of freedom;
Inflation is realized by a collection of branes in warped throats

KKLT version: Cline&Stoica,Phys.Rev.D72:126004,2005.

DBI version: Cai & Xia, PLB 677:226,2009;
Cai & Xue, PLB 680:395,2009



➢Features:

Brane positions as inflatons;
Brane annihilation or collision as ending;
Warped compactifications

➢Motivations:

To solve η -problem in usual brane inflation;
To develop cosmological perturbation theory;
To find more potential testable predictions

•Dynamics of the branes:

$$S = \int d^4x \sqrt{-g}\left[\sum_I P_I(X_I, \phi_I)\right]$$

which involves N scalar fields, with

$$P_I(X_I, \phi_I) = \frac{1}{f(\phi_I)}\left[1 - \sqrt{1 - 2f(\phi_I)X_I}\right] - V_I(\phi_I)$$

•Consider the case of IR type potential: $V_I = V_{0I} - \frac{1}{2}m_I^2\phi_I^2$

We have the solution:

$$\phi_I = -\frac{\sqrt{\lambda}}{t}\left(1 - \frac{9H^2}{2m_I^4t^2} + \cdots\right)$$

and obtain a series of small sound speeds for these branes

$$c_{sI} \equiv \sqrt{1 - 2f(\phi_I)X_I} \sim 0$$

•Primordial perturbations:
(we consider double-brane inflation)

Curvature perturbations:

$$\mathcal{R} \simeq \frac{\mathcal{N}^2}{2\pi\sqrt{\lambda_I}}\left(1+\frac{27H^4}{2m_1^2m_2^2\mathcal{N}^2}\right)$$

Entropy perturbations:

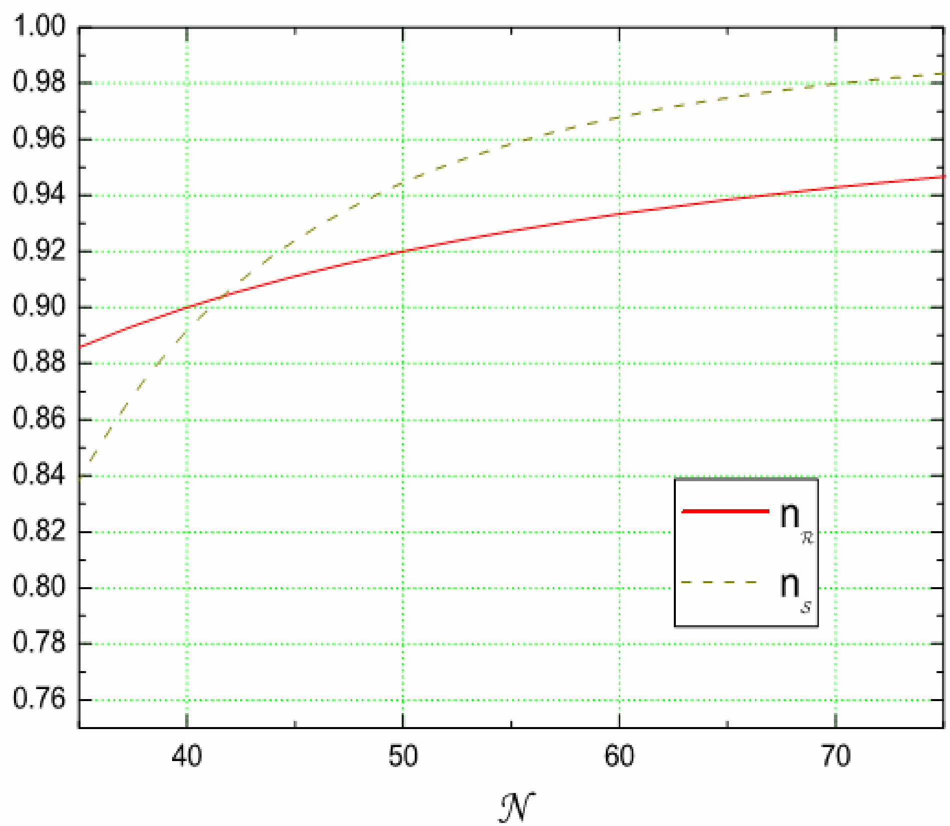
$$\mathcal{S} \simeq \frac{27H^4(m_1^2-m_2^2)}{4\pi\sqrt{\lambda_I}m_1^3m_2^3} + O(\mathcal{N}^{-2})$$

Non-Gaussianity of local type:

$$f_{\rm NL}^{\rm local} \sim \frac{q}{\mathcal{N}_1}$$

Non-Gaussianity of equilateral type:

$$f_{\rm NL} \sim 1/c_{s2}^2$$



Benefits of multiple brane inflation:

- Explaining the current observations well;
- Relaxing the eta-problem in usual brane inflation;
- Plentiful phenomena in the early universe

Main predictions:

- A homogeneous, isotropic and flat universe;
- Nearly scale-invariant CMB spectrum;
- Sizable local non-gaussianity & large equilateral one;
- Possible scale-invariant entropy perturbations;
- Existence of cosmic strings

Problems:

- Reheating process is still unclear;
- The initial singularity is still not addressed   Borde and Vilenkin, PRL 72:3305, 1994

Summary

- A brief review of brane inflation models
- Current difficulties in brane inflation
- Inflation can be obtained on multiple branes
- Detectable predictions in multi-brane inflation models


Thank You !