5D Meyers-Perry Black Holes Cannot be Over-Spun by Gedanken Experiments

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Based mainly on the work

arxiv:1711.04310 w/

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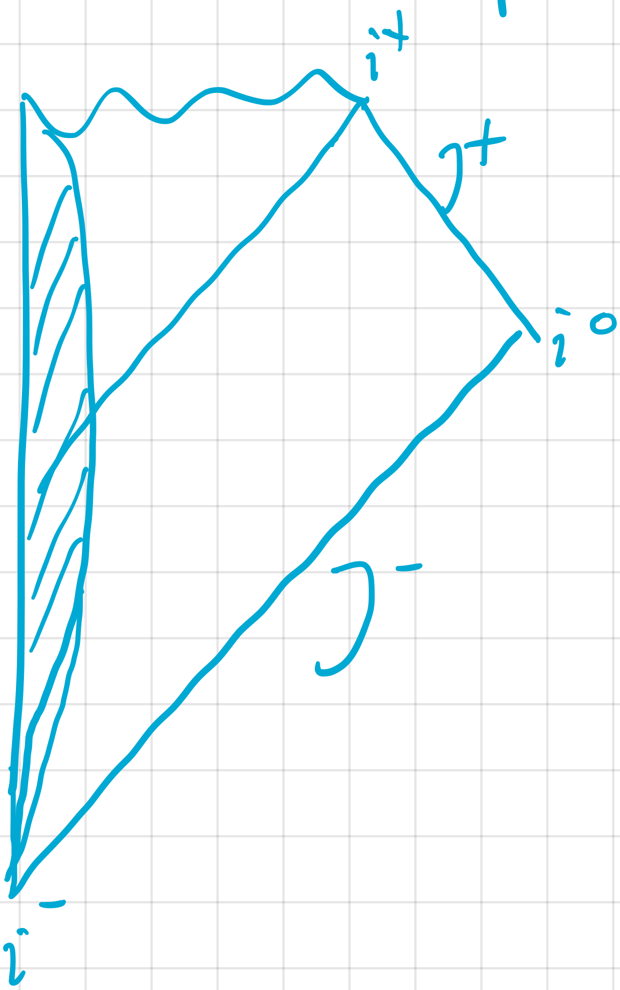
Outline of this talk

1. Cosmic Censorship Conjecture
   1. Weak
   2. Strong
2. High Dimensional Case
3. Gedanken Experiments
   1. Old
   2. New
4. Iyer-Wald Formalism
   1. First
   2. Second
5. 5D Myers-Perry BHs
6. Conclusion

Cosmic Censorship Conjecture



4D
AF

① WCC:

The gravitational collapse from generic initial condition of suitable matters, gives rise to the singularity hidden inside the BHs.

② SCC:

The singularity is spacelike or null

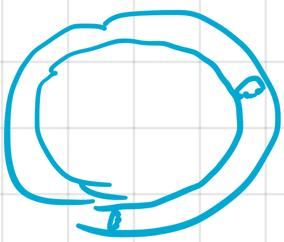
High Dimensional Case

1 5D Black strings

Lehner and Pretorius

2010 PRL

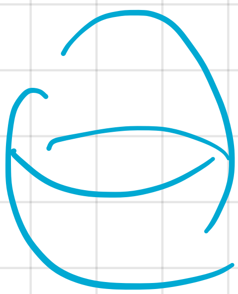
2 5D Black Rings



Figueras et al.

2016 PRL

3 6D MP Black Holes



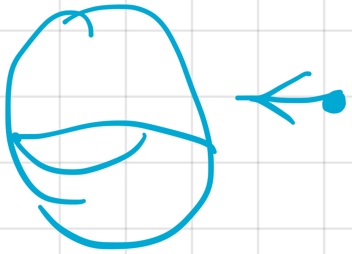
Figueras et al.

2017 PRL

What happens to 5D MP BHs?

Gedanken Experiments

① Old Version



$$E = -(m u_a + q A_a)\xi^a \geq -q A_a \xi^a = q\Phi_H = q$$

($\Phi_H = 1$ for extremal BHs)

Wald

1974 Ann. phys.

Nearly extremal BHs

WCC can be violated!

Hubeny

1999 PRD

② New Version

point particles	Generic Matters
trajectories	NEC
first order	Second order

Sorce and Wald

2017 PRD

Iyer Wald Formalism

Lagrangian n-form

Iyer and Wald 1994. 1995 PRD

$$\underline{L} = \mathcal{L}(\phi)\,\underline{\epsilon} \qquad \phi = (g_{ab}, \psi)$$

Variation of Lagrangian

$$\delta L = E\,\delta\phi + d\,\Theta(\phi, \delta\phi)$$

Symplectic potential

Symplectic Current

$$\omega(\phi, \delta\phi_1, \delta\phi_2) = \delta_1 \Theta(\phi, \delta\phi_2) - \delta_2 \Theta(\phi, \delta\phi_1)$$

Noether Current

$$J_X = \Theta(\phi, \mathcal{L}_X \phi) - X \cdot \underline{L}$$

$dJ_X = -E\,\mathcal{L}_X\phi$

$$J_X = dQ_X + C_X$$

Noether Charge

Constraint

Iyer-Wald Formalism

First Variational Identy

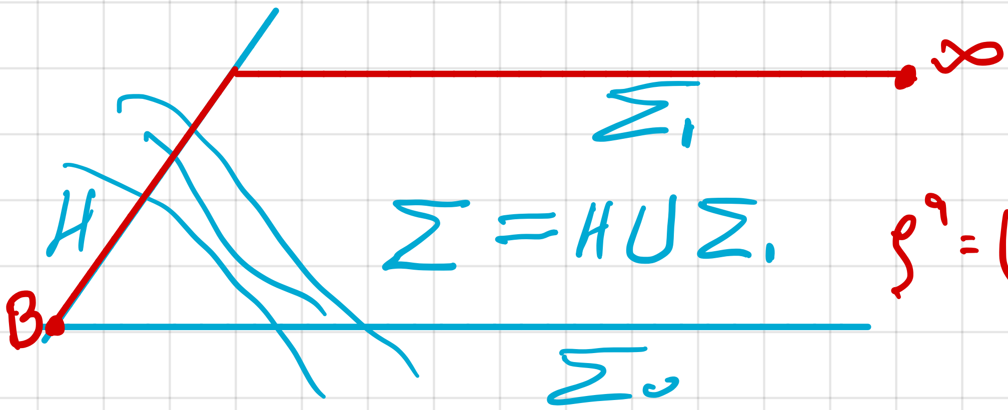
$$d\left[\delta Q_X - X\cdot\Theta(\phi,\delta\phi)\right] = \omega(\phi,\delta\phi,\mathcal{L}_X\phi) - X\cdot E\delta\phi - \delta C_X$$

Second Variational Identy

$$d\left[\delta^2 Q_\xi - \xi\cdot\delta\Theta(\phi,\delta\phi)\right] = \omega(\phi,\delta\phi,\mathcal{L}\,\delta\phi) - \xi\cdot\delta E\delta\phi - \delta^2 C_\xi$$

(ξ is a Killing field on the background with $E=0$)

Application to BHs



$$\xi^a = \left(\frac{\partial}{\partial t}\right)^a + \Omega^H\left(\frac{\partial}{\partial \varphi}\right)^a$$

Iyer-Wald Formalism

Application to BHs

$$\delta M - \Omega^1 \delta J_2 = \int_B \left[\delta Q_\xi - \xi \cdot \Theta(\phi, \delta\phi)\right] - \int_\Sigma \delta C_\xi$$

$$= -\int_H \delta C_\xi = \int_H \hat{\epsilon}\, \delta T_{ab} K^a \xi^b \geq 0$$

(null energy condition)

$$\delta^2 M - \Omega^1 \delta^2 J_1 = \mathcal{E}_\Sigma(\phi, \delta\phi) + \int_B \left[\delta^2 Q_\xi - \xi \cdot \delta\Theta(\phi, \delta\phi)\right] - \int_\Sigma \xi \cdot \delta E \delta\phi - \int_\Sigma \delta^2 C_\xi$$

Canonical Energy: $\mathcal{E}_\Sigma(\phi, \delta\phi) = \int_\Sigma \omega(\phi, \delta\phi, \mathcal{L}_\xi \delta\phi)$

$$= -\int_H \delta^2 C_\xi + \mathcal{E}_H(\phi, \delta\phi) + \mathcal{E}_{\Sigma_1}(\phi, \delta\phi)$$

$$= \int_H \hat{\epsilon}\, \delta^2 T_{ab} K^a \xi^b + \frac{1}{4\pi} \int_H \hat{\epsilon}\, \delta\sigma_{cd} \delta\sigma^{cd} \xi^a \nabla_a u + \mathcal{E}_{\Sigma_1}(\phi, \delta\phi)$$

$$\geq \mathcal{E}_\Sigma(\phi, \delta\phi) = -\frac{\kappa}{8\pi} \delta^2 A_B^{MP} \quad \text{(trick!)}$$

5D Myers-Perry BHs

$$ds^2 = -dt^2 + \frac{\mu}{\Xi}\left(dt - a_1 \sin^2\theta\, d\varphi^1 - a_2 \cos^2\theta\, d\varphi^2\right)^2$$

$$+ \frac{r^2 \Xi}{\Pi - \mu r^2} dr^2 +$$

Myers and Perry
1986 Ann. Phys.

$$(r^2 + a_1^2) \sin^2\theta (d\varphi^1)^2 + (r^2 + a_2^2) \cos^2\theta (d\varphi^2)^2$$

$$\Xi = r^2 + a_1^2 \cos^2\theta + a_2^2 \sin^2\theta$$

$$\Pi = (r^2 + a_1^2)(r^2 + a_2^2)$$

$\mu \geq (a_1 + a_2)^2$ BHs

$\mu < (a_1 + a_2)^2$ NSs

Mass: $M = \frac{3\pi\mu}{8}$, Angular Momentum: $J_I = \frac{\pi \mu a_I}{4}$

Horizon radius: $r_h = \frac{\sqrt{\mu - (a_1 + a_2)^2} + \sqrt{\mu - (a_1 - a_2)^2}}{2}$

Area: $A = 2\pi^2 \mu r_h$

Angular velocity: $\Omega^1 = \frac{a_1}{r_h^2 + a_1^2}$

Surface gravity: $\kappa = \frac{2 r_h^2 + a_1^2 + a_2^2 - \mu}{\mu r_h}$

BH condition: $\alpha = \sqrt{32 M^3 - 27\pi (J_1 + J_2)^2} \geq 0$

Conclusion

① Extremal 5D MP BHs can not be over-spun at the linear level

$$32M^3 - 27\pi (J_1 + J_2)^2 = 0$$

$$\delta M - \frac{9\pi (J_1+J_2)}{16M^2}(\delta J_1 + \delta J_2) < 0$$

impossible because

$$\Omega^1 = \Omega^2 \equiv \frac{9\pi (J_1+J_2)}{16M^2}.$$

Conclusion

② Nearly extremal MP BHs can be over-spun at the linear level but can not be over-spun at the quadratic level

$$f(\lambda) = 32 M^3(\alpha) - 27\pi \left(J_1(\alpha) + J_2(\alpha) \right)^2$$

$$f(\alpha) = \underline{\alpha^2 + \gamma_1 \lambda} + \gamma_2 \lambda^2 + O(\lambda^3, \lambda^2\alpha, \lambda\alpha^2, \alpha^3)$$

$$\gamma_1 = -6\sqrt{3\pi} \left(\sqrt{\frac{J_2}{J_1}}\,\delta J_1 + \sqrt{\frac{J_1}{J_2}}\,\delta J_2 \right) \alpha$$

$$\gamma_2 = 27\pi \left(\sqrt{\frac{J_2}{J_1}}\,\delta J_1 + \sqrt{\frac{J_1}{J_2}}\,\delta J_2 \right)^2$$

$$f(\lambda) = \underline{\left[\alpha - 3\sqrt{3\pi} \left(\sqrt{\frac{J_2}{J_1}}\,\delta J_1 + \sqrt{\frac{J_1}{J_2}}\,\delta J_2 \right) \right]^2}$$

+ high order terms

Conclusion

③ It is still possible to violate WCC around 5D MP BHs by fully non-linear dynamics, although highly unlikely.

— Pau Figueras

Thank you for your listening!