Radiative Processes in the Dynamics of Compact Binary Systems

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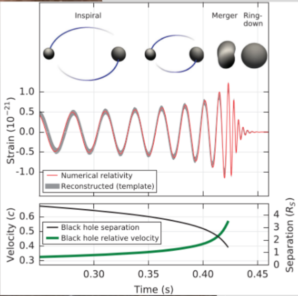
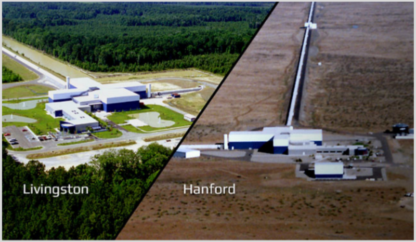
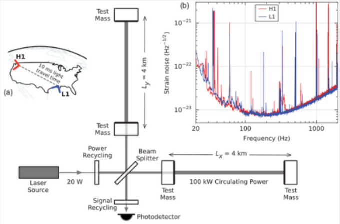
中国科学技术大学
University of Science and Technology of China

GW150914: First direct detection of gravitational waves



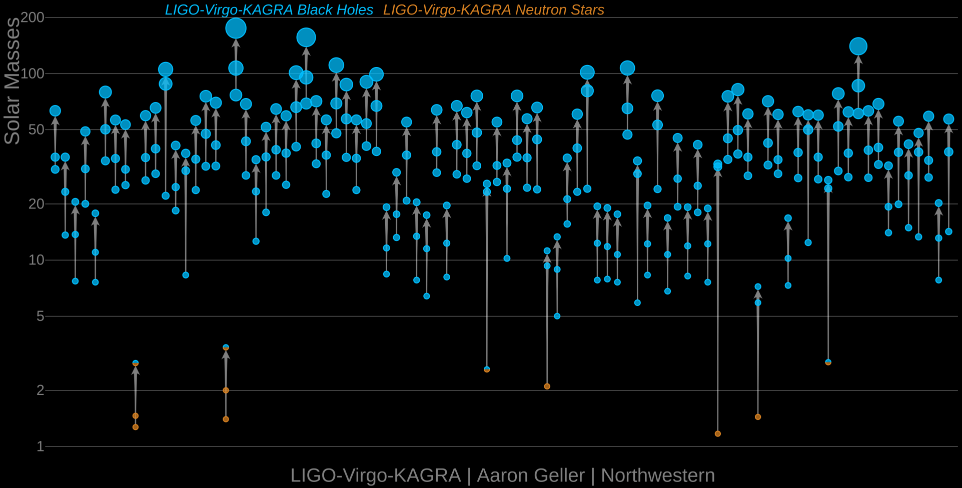




New window to observe the universe!

Today, 90 GW events have been observed by the LVK collaboration.

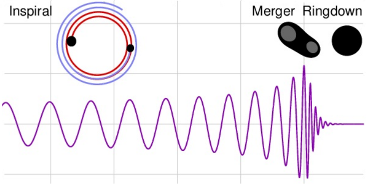
Masses in the Stellar Graveyard



Phases of the Coalescence

Different methods are used to study each of these phases.

Inspiral: *Post-Newtonian approximation.*



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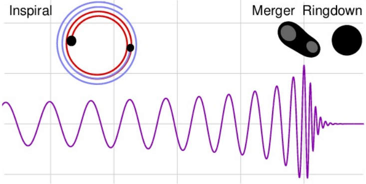


In the adiabatic approximation to the circular motion

$$\begin{cases} h_+ = \frac{4}{D}(G_N\mathcal{M}_c)^{5/3}(\pi f_{\rm gw})^{2/3}\left(\frac{1+\cos^2\iota}{2}\right)\cos\left[\Phi(t)\right]\ , \\ h_\times = \frac{4}{D}(G_N\mathcal{M}_c)^{5/3}(\pi f_{\rm gw})^{2/3}\cos\iota\sin\left[\Phi(t)\right]\ . \end{cases}$$

The **Gravitational-Wave Phase** $\Phi(t)$ can be modelled through

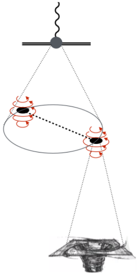
$$\Phi(t) = 2\int_{t_0}^{t} dt\,\omega(t) = -\frac{2}{G_N M}\int_{v(t_0)}^{v(t)} dv\,\frac{v^3}{P(v)}\frac{dE}{dv}\ .$$

Hierarchy of Scales

Hierarchy of scales and the **method of regions** for bound binary systems
[M. Beneke and V. A. Smirnov, Nucl. Phys. B **522**, 321-344 (1998)]

$$\text{Orbital scale:} \quad v^2 \sim \frac{G_N m}{r} \quad \Rightarrow \quad r_s \sim 2G_N m \sim r v^2$$

$$\text{GW scale:} \quad \lambda \sim \frac{r}{v}$$



$$\Rightarrow \quad r_s \sim r v^2 \sim \lambda v^3$$

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$$r_s \ll r \ll \lambda$$

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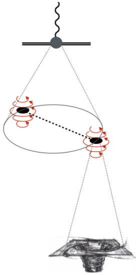

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Method of regions: $h_{\mu\nu} = \underbrace{H_{\mu\nu}}_{\text{potential modes}} + \underbrace{\bar{h}_{\mu\nu}}_{\text{radiative modes}}$

$H_{\mu\nu}$: off-shell modes scaling as $(k^0, \mathbf{k}) \sim (v/r, 1/r)$

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The EFT for the Binary System

Non-Relativistic General Relativity construction of Goldberger and Rothstein.
[W. D. Goldberger and I. Z. Rothstein, Phys. Rev. D **73** (2006) 104029]

⇒ **Idea**: We write down a general Lagrangian containing all possible types of *local* terms consistent with the symmetries of the problem.

The starting point: Theory of relativistic point particles coupled to gravity

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R[g_{\mu\nu}] + S_{\rm pp}\,,$$

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Symmetries of the theory:

- Diffeomorphism covariance
- Reparameterization invariance

$$S_{\text{pp}} = -\sum_{a}^{2} m_a \int d\tau_a \underbrace{-\frac{1}{2}\int d\tau\, S^{\mu\nu}\omega_{\mu\nu}}_{\text{spin DOFs}} + \underbrace{c_E \int d\tau\, E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau\, B_{\mu\nu}B^{\mu\nu}}_{\text{finite size effects}} + \dots$$

Wilson's ideia: by adding all these infinitely many operators (scaling as $\sim r_s/r$) allowed by the symmetries, finite size effects are systematically taken into account.

The Near Zone

The **Near Zone** is the orbital scale where the potential modes $H_{\mu\nu}$ are exchanged between the constituents of the binary system. In this case, $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}$ and

$$(k^0, \mathbf{k})_{\text{pot}} \sim (v/r, 1/r)\,.$$

$k^0/|\mathbf{k}| \sim v \Rightarrow$ Departure from instantaneity: implementation of the PN expansion:

$$\frac{1}{k_0^2 - \mathbf{k}^2} = -\frac{1}{\mathbf{k}^2}\left(1 + \frac{k_0^2}{\mathbf{k}^2} + \frac{k_0^4}{\mathbf{k}^4} + \cdots\right) \sim r^2(1 + v^2 + v^4 + \dots)\,.$$

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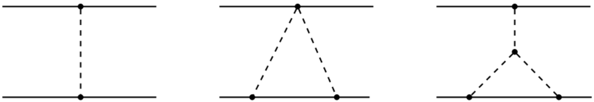
The 1PN correction, whose contributing diagrams are



gives

$$L_{1\text{PN}} = \frac{m_a v_a^4}{8} + \frac{G_N m_1 m_2}{2r}\left[3(v_1^2 + v_2^2) - 7(v_1 \cdot v_2) - \frac{(v_1 \cdot r)(v_2 \cdot r)}{r^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2r^2}\,.$$

[Einstein, Infeld, Hoffmann, Annals Math. **39**, 65-100 (1938)]

The Far Zone

Integrating out the potential modes:

$$e^{iS_{\text{eff}}[x_a,\bar{h}_{\mu\nu}]} = \int \mathcal{D}H_{\mu\nu} \exp\left\{iS_{\text{EH+GF}}[H_{\mu\nu}+\bar{h}_{\mu\nu}] + iS_{\text{pp}}[x_a(t), H_{\mu\nu}+\bar{h}_{\mu\nu}]\right\}$$

$$\Rightarrow S_{\text{eff}} = \frac{1}{2}\int d^4x\, T^{\mu\nu}\bar{h}_{\mu\nu}$$

Multipole expansion, $\lambda \gg r$, makes $S_{\text{eff}} \to S_{\text{mult}}$:
[Goldberger and Ross, Phys. Rev. D **81**, 124015 (2010)]

$$S_{\text{mult}} = -\int d\tau\, E - \frac{1}{2}\int dx^\mu\, L_{ab}\omega_\mu^{ab} + \frac{1}{2}\sum_{r=0}^{\infty}\int d\tau\, c_r^{(I)} I^{ijR}(\tau)\nabla_R E_{ij}(x)$$
$$+\frac{1}{2}\sum_{r=0}^{\infty}\int d\tau\, c_r^{(J)} J^{ijR}(\tau)\nabla_R B_{ij}(\tau)\,.$$

[Multi-index notation $R = i_1 \dots i_r$]
GW observables can be computed, e.g.:

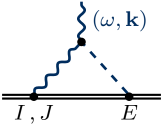
$$P = \frac{1}{2T}\sum_{\text{pol}}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,|\mathcal{A}(\omega,\mathbf{k})|^2$$

Nonlinear Effects: Emission and Self-Energy

Emission

The tail effect

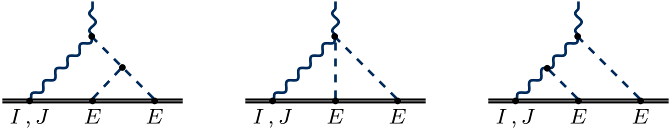
$$i\mathcal{A}_{\rm tail}(\omega, \mathbf{k}) =$$



- UV and IR divergences
- Renormalization group evolution

[G. Almeida, S. Foffa, R. Sturani, Phys. Rev. D **104**, 084095 (2021)]

The tail-of-tail effect contains three contributions:



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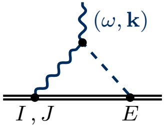

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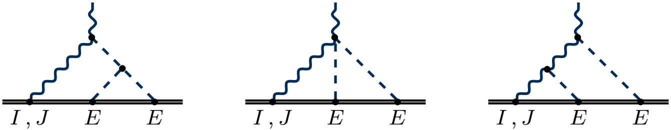
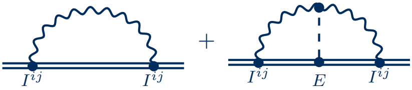


Self-energy

$iS_{\mathrm{eff}} =$



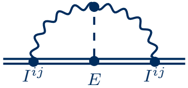
- $\mathrm{Im}(S_{\mathrm{eff}}) \Rightarrow$ Energy flux
- $\mathrm{Re}(S_{\mathrm{eff}}) \Rightarrow$ Conservative

Computed for arbitrary multipole moments

[G. Almeida, S. Foffa, R. Sturani, Phys. Rev. D **104**, 124075 (2021)]

State-of-the-art of the conservative sector (spinless case)

$\Rightarrow$ State-of-the-art on the conservative sector: 4PN (2019) (Potential + Radiation)
[S. Foffa, R. Porto, I. Rothstein and R. Sturani, Phys. Rev. D **100**, 024048 (2019)]



$$= -\frac{1}{5}\int \frac{d\omega}{2\pi}\omega^6 \left[\frac{1}{\epsilon} - \frac{41}{30} + \log\left(\frac{\omega^2 e^{\gamma}}{\pi\mu^2}\right) - i\pi\right] I^{ij}(\omega) I_{ij}(-\omega)\,.$$

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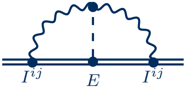

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⇒ 5PN Potential contributions: 10^6 diagrams via multi-loop integration methods.
[J. Blümlein, A. Maier, P. Marquard and G. Schäfer, Nucl. Phys. B **965** (2021) 115352]

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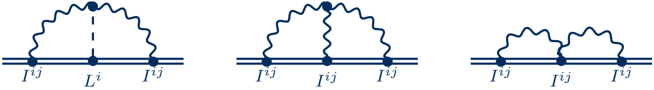
[S. Foffa and R. Sturani, Phys. Rev. D **101**, 064033 (2020)]



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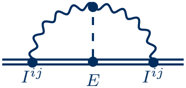

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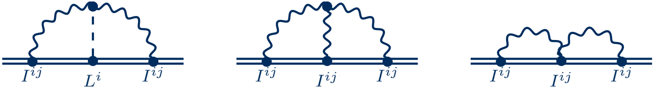

⇒ Results are inconsistent with predictions from Self-Force formalism.
[D. Bini, T. Damour, A. Geralico, Phys. Rev. D **104**, 084031 (2021)]

Finite size effects start to enter at the 5PN level, hence being the main motivation for going forward to the completion of this order.

The Gravitational Field

[G. Almeida, A. Müller, S. Foffa, R. Sturani, arXiv:2307.05327]

The **Classical Gravitational Field** at a spacetime position x is given by

$$\langle h_{\mu\nu}(x)\rangle = \int \mathcal{D}h\, e^{iS[h]} h_{\mu\nu}(x)\,.$$

The most relevant role is played by the **trace-reversed** quantity $\bar{h}_{\mu\nu}$, defined by

$$\bar{h}_{\mu\nu} = P_{\mu\nu}{}^{\alpha\beta} h_{\alpha\beta}\,, \qquad \text{with} \qquad P_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2}\left(\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} + \delta^{\beta}_{\mu}\delta^{\alpha}_{\nu} - \eta_{\mu\nu}\eta^{\alpha\beta}\right)\,.$$

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When interactions are considered, the field $h_{\mu\nu}$ will have the generic form

$$\langle h_{\mu\nu}(x)\rangle = \int_{\mathbf{k}} \frac{d\omega}{2\pi} \frac{e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}}{\mathbf{k}^2 - (\omega + i\mathsf{a})^2} \times i\mathcal{A}_{\mu\nu}(\omega, \mathbf{k})\,.$$

This equation defines the **Gravitational Scattering Amplitude** $i\mathcal{A}_{\mu\nu}$.

In particular, in real space, this takes the form

$$\langle \bar{h}_{\mu\nu}(x)\rangle = -16\pi G_N \int d^{d+1}x' G_R(t - t', \mathbf{x} - \mathbf{x}') T_{\mu\nu}(x')\,.$$

Hence, we have the identification

$$T_{\mu\nu}(x) \qquad \sim \qquad i\bar{\mathcal{A}}_{\mu\nu}(\omega, \mathbf{k})\,.$$

Gauge Condition and Ward Identity

It follows directly from the trace-reversed version of $\langle h_{\mu\nu}(x)\rangle$ that

$$\partial^\mu \langle \bar{h}_{\mu\nu}(x)\rangle = -\int_{\mathbf{k}} \frac{d\omega}{2\pi} \frac{e^{ik\cdot x}}{\mathbf{k}^2 - (\omega + i\mathsf{a})^2} \times k^\mu \bar{\mathcal{A}}_{\mu\nu}(\omega, \mathbf{k})\,.$$

Hence, we immediately see that, if the condition $k^\mu \bar{\mathcal{A}}_{\mu\nu} = 0$ is satisfied, we have

$$k^\mu \bar{\mathcal{A}}_{\mu\nu}(\omega, \mathbf{k}) = 0 \qquad \Longrightarrow \qquad \partial^\mu \langle \bar{h}_{\mu\nu}(x)\rangle = 0 \qquad \text{and} \qquad \partial^\mu T_{\mu\nu}(x) = 0\,.$$

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The harmonic gauge condition: $\Gamma^\mu = 0$.
From this, it is easy to derive the important result: $k^\mu \bar{\mathcal{A}}_{\mu\nu}(\omega, \mathbf{k}) \propto (\omega^2 - \mathbf{k}^2)$.

Thus: Physically relevant amplitudes $i\mathcal{A}_{\mu\nu}$ are such that, on-shell ($\mathbf{k} \equiv \omega\hat{\mathbf{n}}$),

$k^\mu \bar{\mathcal{A}}_{\mu\nu}(\omega, \omega\hat{\mathbf{n}}) = 0 \quad \Rightarrow \quad$ This is the statement of the **Ward identity**.

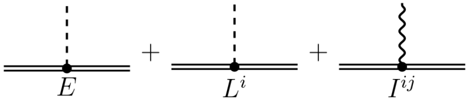
On-shell amplitudes: Useful to build h_{ij}^{TT} in the far field approximation, $D \gg r$:

$$h_{ij}^{TT}(x) \equiv \langle \bar{h}_{ij}^{TT}(x)\rangle = \frac{1}{4\pi D}\Lambda_{ijkl} \int \frac{d\omega}{2\pi}\, i\bar{\mathcal{A}}_{kl}(\omega, \omega\mathbf{n}) e^{-i\omega t_{\mathrm{ret}}}\,.$$

Processes to be investigated

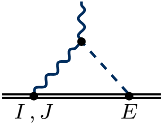
Diagrams for the emission processes that we want to investigate:

- Leading-order processes:



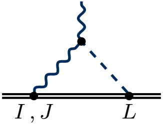
⇒ Gauge condition implies conservation of E, L_i at this perturbative order.

- Mass tails:



⇒ Ward identity fully satisfied.

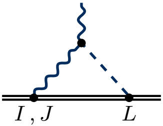
- Angular momentum (failed) tails:



⇒ Presence of a (consistent) "Gravitational-Wave" anomaly.

The Angular Momentum (Failed) Tail

Angular momentum (failed) tail:
[Here we use the representation $L_i = \frac{1}{2}\epsilon_{ijk}J^{j|k}$]



$\Rightarrow$ Computation of $i\bar{\mathcal{A}}_{\mu\nu}$.

From the full amplitude, we have

$$k^\mu \bar{\mathcal{A}}_{\mu 0}(\omega, \omega\mathbf{n}) = 0\,.$$

$$k^\mu \bar{\mathcal{A}}_{\mu l}(\omega, \omega\mathbf{n}) = (-i)^{r+1}\frac{c_r^{(I)}}{2\Lambda^4}\left(\frac{i\omega}{4}\right)\left[k_a\omega^2 J^{i|a} I^{iRl}(\omega)\int_{\mathbf{q}}\frac{q_R}{(\mathbf{q}^2-\omega^2)}\right].$$

$\Rightarrow$ Hence, we notice that, since the integral in $\mathbf{q}$ is proportional to δ_R, this result vanishes on account of the tracelessness of I^{iRl}, unless $r=0$, in which case

$$k^\mu \bar{\mathcal{A}}_{\mu l}\big|_{r=0} = 16\pi i G_N^2 k_a \omega^4 J^{i|a} I^{il}(\omega)\,.$$

Einstein's Equation in Perturbation Theory

Variation of the Einstein-Hilbert plus gauge-fixing action, with metric expanded as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, yields

$$S_{EH+GF} \sim \Lambda^2 \int d^4x \, (h\partial^2 h + h^2 \partial^2 h + \dots)$$

$$\Rightarrow \qquad \Box \bar{h}_{\mu\nu} = N_{\mu\nu}[h,h] + M_{\mu\nu}[h,h,h] + \dots .$$

Perturbative expansion in G_N, with $h^{(n)}$ denoting contributions of $\mathcal{O}(G_N^n)$, gives

$$\begin{aligned}
\Box \bar{h}^{(1)}_{\mu\nu} &= 0 \,, \\
\Box \bar{h}^{(2)}_{\mu\nu} &= N_{\mu\nu}[h^{(1)}, h^{(1)}] \,, \\
\Box \bar{h}^{(3)}_{\mu\nu} &= N_{\mu\nu}[h^{(1)}, h^{(2)}] + M_{\mu\nu}[h^{(1)}, h^{(1)}, h^{(1)}] \,, \\
&\dots .
\end{aligned}$$

$\Rightarrow$ Explicit check for $h^{(\mathrm{L-tail})}_{\mu\nu}$, for the electric quadrupole, shows that it is indeed a solution of the perturbed Einstein's equation $\Box \bar{h}^{(2)}_{\mu\nu} = N_{\mu\nu}[h^{(1)}, h^{(1)}]$.

Equations of motion for the full problem

A solution to the full problem is obtained once we solve simultaneously

$$\Box\bar{h}_{\mu\nu} = \Lambda_{\mu\nu} \qquad \text{and} \qquad \partial^\mu \bar{h}_{\mu\nu} = 0\,.$$

Once we obtain a particular solution $h^p_{\mu\nu}$ of $\Box\bar{h}_{\mu\nu} = \Lambda_{\mu\nu}$, we can always find a homogeneous solution $h^h_{\mu\nu}$ that precisely cancels the divergence in $\partial^\mu \bar{h}_{\mu\nu}$.

Solution can be given in terms of four STF tensors, say N_L, P_L, Q_L, R_L.

$\Rightarrow$ General solution to the full problem is $h_{\mu\nu} = h^p_{\mu\nu} + h^h_{\mu\nu}$.

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In terms of amplitudes

$$\bar{\mathbf{a}}_{\mu\nu} \quad \Longleftrightarrow \quad h^h_{\mu\nu} \qquad \text{and} \qquad k^\mu \bar{\mathbf{a}}_{\mu\nu} = -k^\mu \bar{\mathcal{A}}_{\mu\nu}\,.$$

For the anomalous case presented before, we derive

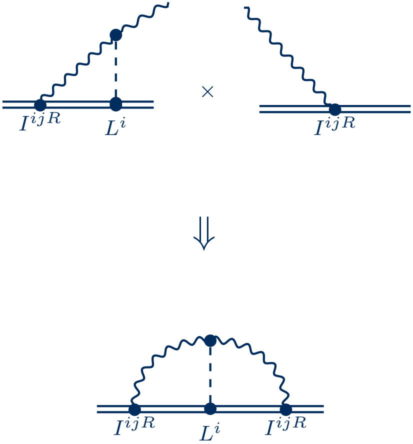
$$Q_{al} = 16\pi i G_N^2 \omega^4 J^{i|(a} I^{l)i} \qquad \text{and} \qquad R_b = -8\pi i G_N^2 \omega^4 \epsilon_{bcd} J^{i|c} I^{id}\,.$$

From which the following results are obtained

$$\bar{\mathbf{a}}_{00} = 0\,, \quad \bar{\mathbf{a}}_{0i} = -8\pi i G_N^2 \omega^3 J^{b|k}(k_j\delta_{ib} - k_b\delta_{ij}) I^{jk}\,, \quad \bar{\mathbf{a}}_{ij} = -16\pi i G_N^2 \omega^4 J^{m|(i} I^{j)m}\,.$$

Self-energy diagrams from emission amplitudes

Radiation × Conservative contributions



Self-energy diagrams from emission amplitudes

For the gluing of two amplitudes $\mathcal{A}_{\mu\nu}$ and $\mathcal{B}_{\mu\nu}$, we have

$$iS_{\text{eff}} = \frac{1}{2}\int_{\mathbf{k}} \frac{d\omega}{2\pi}\mathcal{A}_{\mu\nu}(\omega,\mathbf{k})\mathcal{D}[h_{\mu\nu},h_{\rho\sigma}]\mathcal{B}_{\rho\sigma}(-\omega,-\mathbf{k})\,.$$

No anomalies for $r>0$: the computation of the self-energy from standard EFT methods or by gluing of amplitudes result in the same expression, given by

$$iS_{\text{eff}}^{(L-\text{tail})} = G_N^2\frac{2^r(12+50r+35r^2+10r^3+r^4)}{(r+1)^3(r+2)^3(r+3)(1+2r)(3+2r)(5+2r)(2r)!} \times J^{b|a}\int\frac{d\omega}{2\pi}\,\omega^{7+2r}I^{aiR}(\omega)I^{biR}(-\omega)\,. \qquad (r>0)$$

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⇒ For the quadrupole case, $r = 0$, we must glue the anomaly-fixed amplitude $i\mathcal{M}_{\mu\nu} = i\mathcal{A}_{\mu\nu} + i\mathbf{a}_{\mu\nu}$ previously obtained. In this case, we get:

$$iS_{\text{eff}}^{(r=0,L-\text{tail})} = -\frac{1}{30} G_N^2 J^{i|k} \int \frac{d\omega}{2\pi} I^{ij}(\omega) I^{jk}(-\omega) \omega^7 \, .$$

⇒ Standard self-energy computation gives the coefficient 8/15.

Conclusions

- We have studied Gravitational Scattering Amplitudes for leading-order processes, the mass tails, and the angular momentum failed tail;
- We have identified, for the first time, a classical anomaly in the quadrupole cases of the angular momentum failed tails;
- A fixing at the level of the amplitudes could be implemented by the introduction of counter-terms, within a consistent framework.

Besides this,

- We have learned how emission amplitudes could be used to compute self-energy diagrams;
- In this case, we were able to correct previous results for the conservative dynamics stemming from the angular momentum failed tail.

⇒ The work presented here is important to correctly account for the far-zone effects of back-scattering in the conservative dynamics of compact binary systems.

⇒ Particularly important in the completion of the 5PN dynamics.

Thank you!