

# Gauge/Gravity duality and Nuclear Symmetry energy

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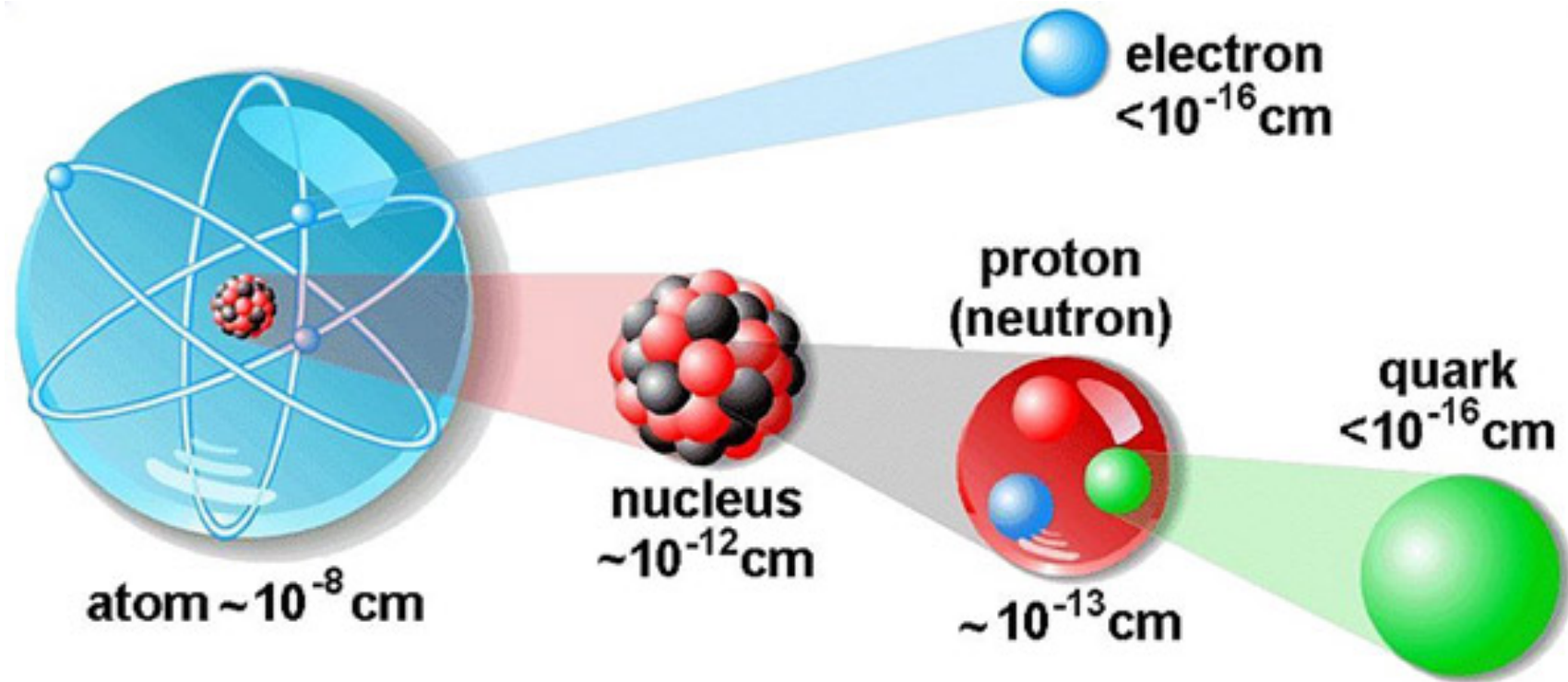
2011.07.08@USTC

Greeks called **indivisible** Building block  
of the universe **Atom**

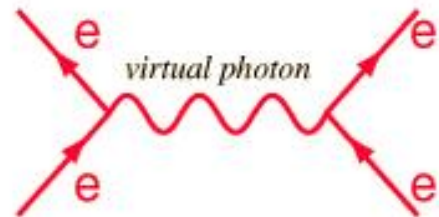
- Two most basic questions in physics are
  1. What is the Atom?
  2. What Forces glue them together to make the visible world?

# Modern Theory of Atom

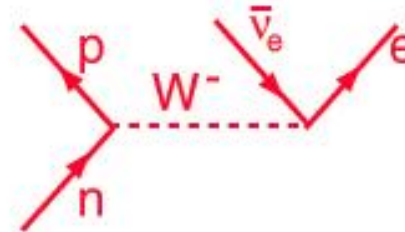
Dalton's story was not the end...



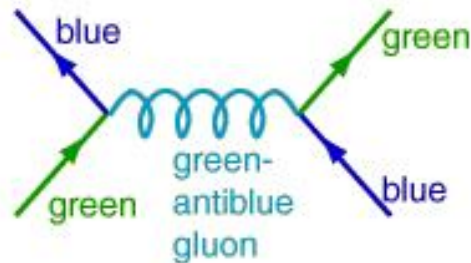
# Force=exchange of glue particles



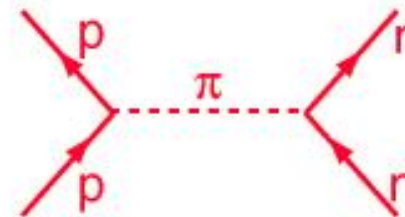
Electromagnetic



Weak



between quarks



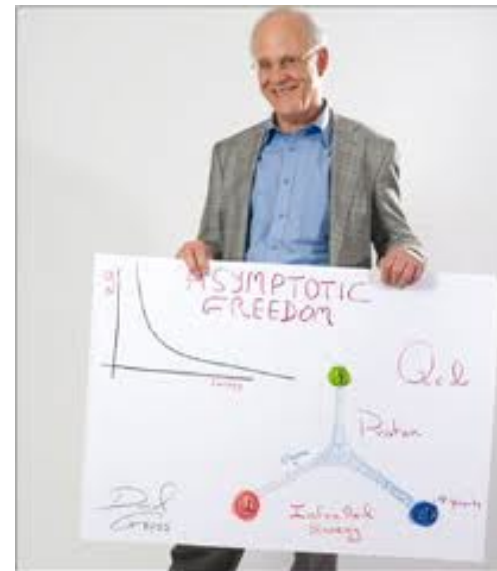
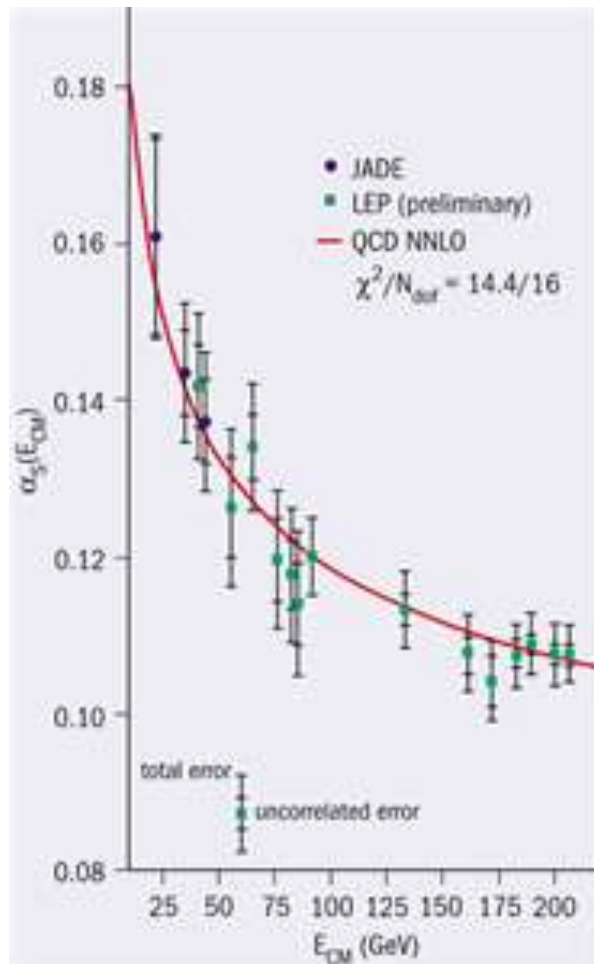
between nucleons

Strong Interaction

Two levels of strong force. (QCD,meson)  
Gravity is very special and not included here

# QCD: today's theme

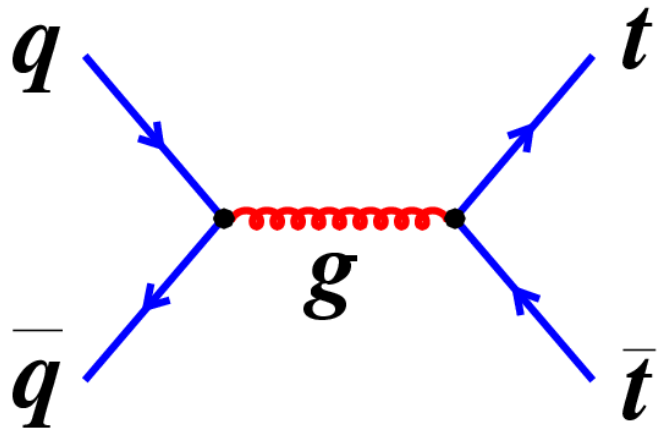
- For High E:  $g \ll 1$ : Asymptotic freedom



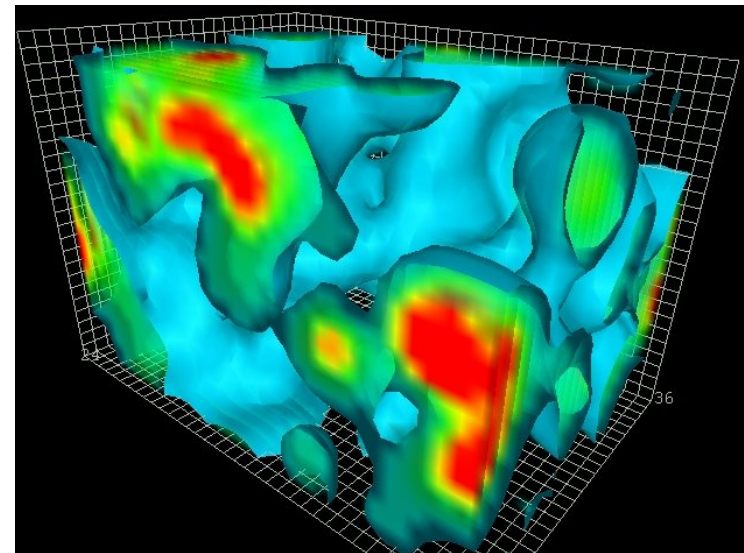
$$\beta_1(\alpha) = \frac{\alpha^2}{\pi} \left( -\frac{11N}{6} + \frac{n_f}{3} \right)$$

$$\alpha_s(k^2) \stackrel{\text{def}}{=} \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)},$$

- For Low E: QCD is still difficult  
interaction strong, fluctuation large



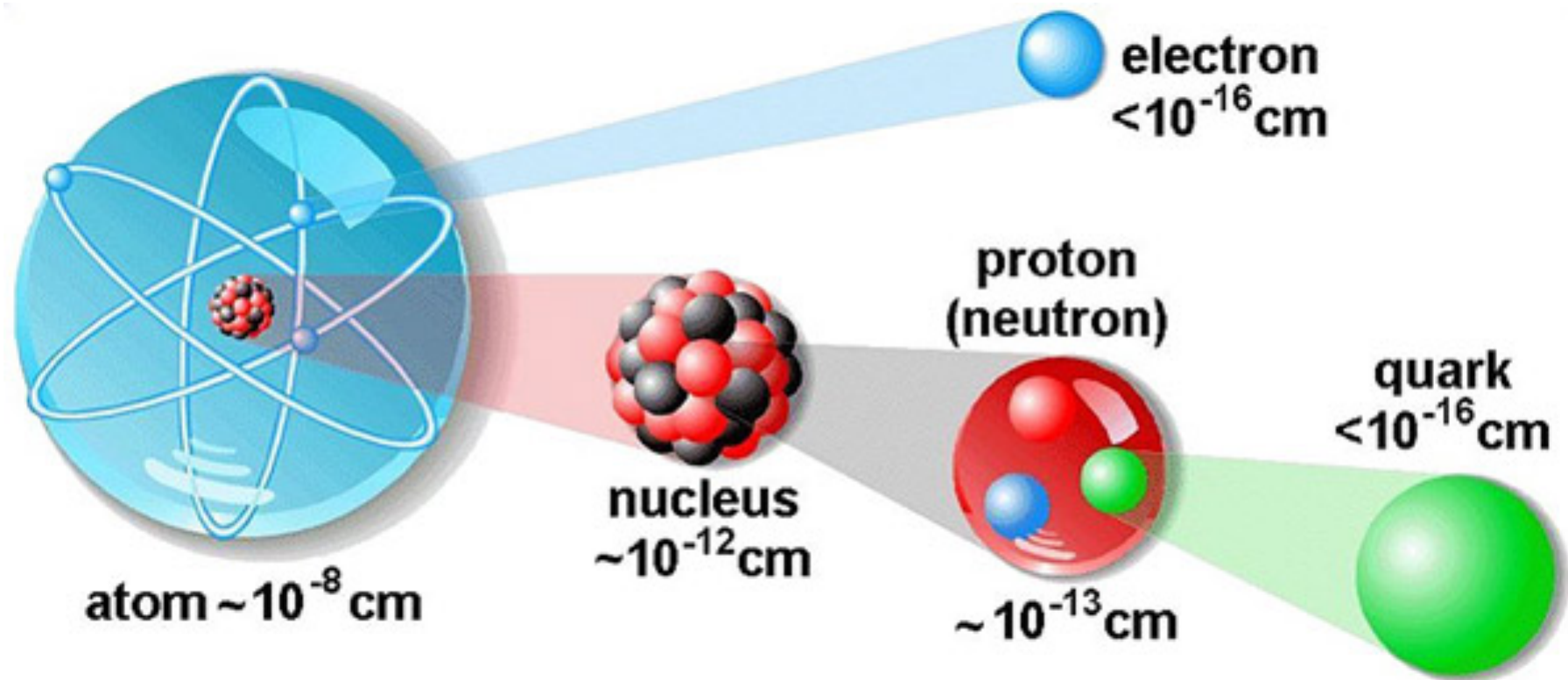
High E v.s



Low E of Strong int.

For strong int. Not clear even what is particle<sub>6</sub>

# Asymptotic freedom and confinement



# General Idea for strong int.

- Reformulate theory in terms of new degree of freedom whose interactions are weak.

## Duality

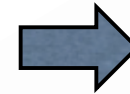
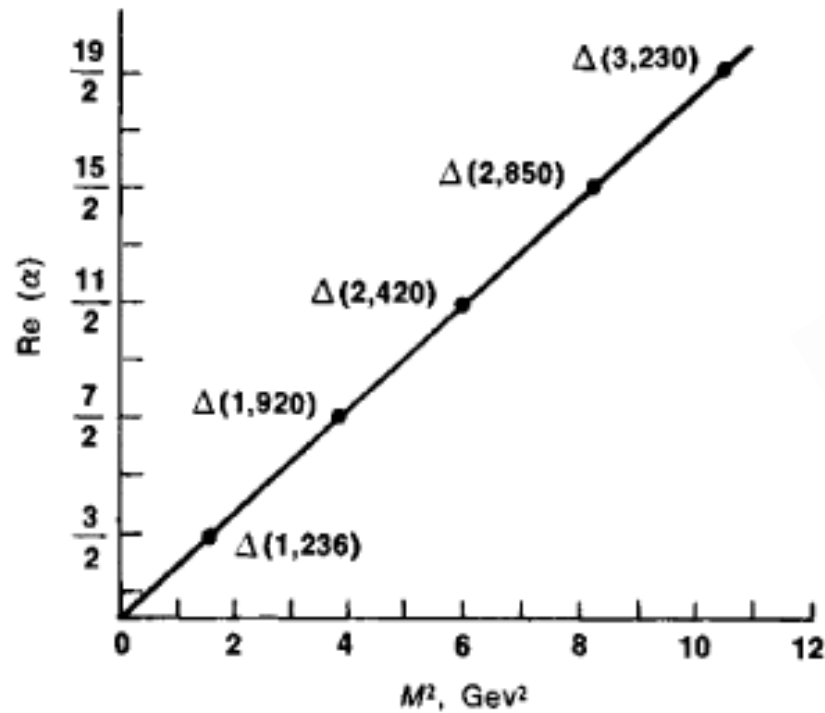


## An idea from string theory

- Replace strong nuclear force by classical gravity(geometry).
- What is the string theory?

# Old String theory

- Regge trajectory  $\rightarrow$  spectrum of string



# Difficulty of old string theory

- Hard to explain the quark's free behavior.
- abandoned and re-incarnated as a quantum gravity, which is a BIG deal!
- Why? qm gravity is difficult! puzzle

# Specialty of Quantum gravity

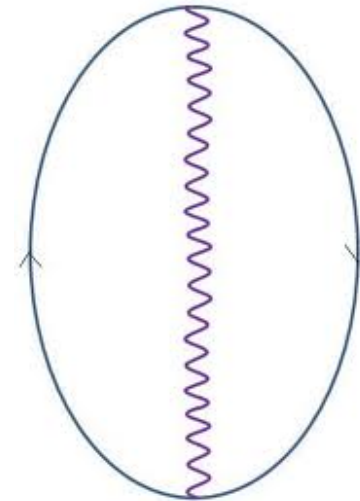
- Force = - grad (potential Energy)
- Einstein: gravity is not a force but a curvature.

$$\text{Curvature} = k (\text{Energy (mass)})$$

- Quantum fluctuation of any field generate infinite Energy → infinite gravity  
essential difficulty

# Origin of the vacuum energy

- Uncertainty principle.  $\Delta p \sim \frac{\hbar}{\Delta x}$



- in a particle theory, any large energy is possible ( $\Delta x \rightarrow 0$ )

# Finiteness of String theory

- stringy uncertainty principle

$$\Delta x \sim \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq \sqrt{\alpha'} = l_{\text{string}}$$

energy of the string per unit length is constant  $\rightarrow$  E up means length up

- Minimal length  
 $\rightarrow$  UV divergence disappear

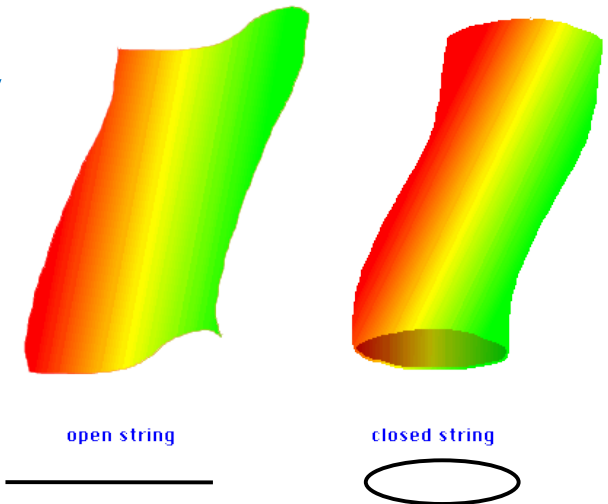
# Open – closed duality

- Open String : gauge theory

Theory of Matter

- Closed string: gravity

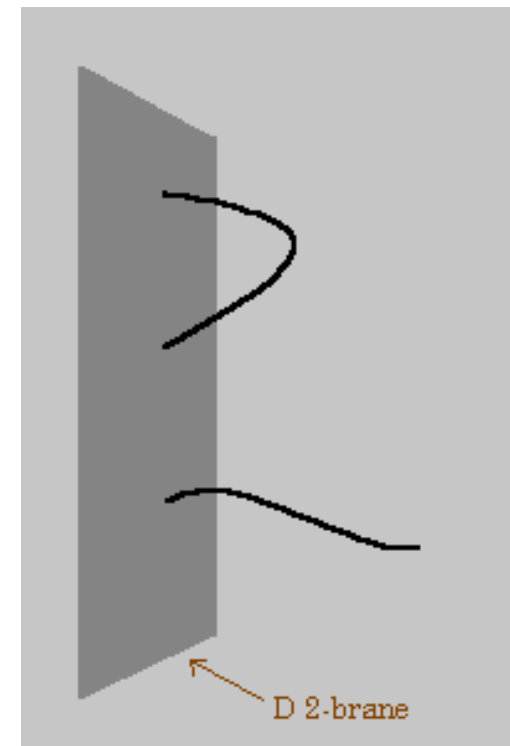
Theory of space-time



- Two are dual to each other through D-brane:

# D-branes

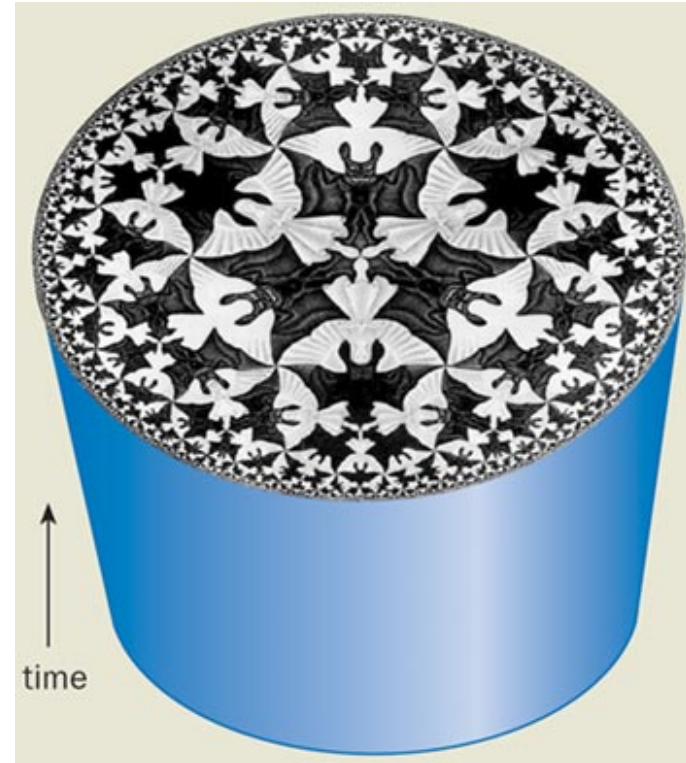
- Closed string soliton
  - That this object has dual description by dynamics of open string ending on it, is the major discovery of 90'. (Polchinski)
  - vibration of D brane
- =The low energy dynamics  $SU(N)$  SYM
- =string theory at warped geometry.





# D-brane & gravity

- Stack of D-brane  
→ space time warp  
due to the tension.  
→(AdS).



Eschers's "Angels and devils"

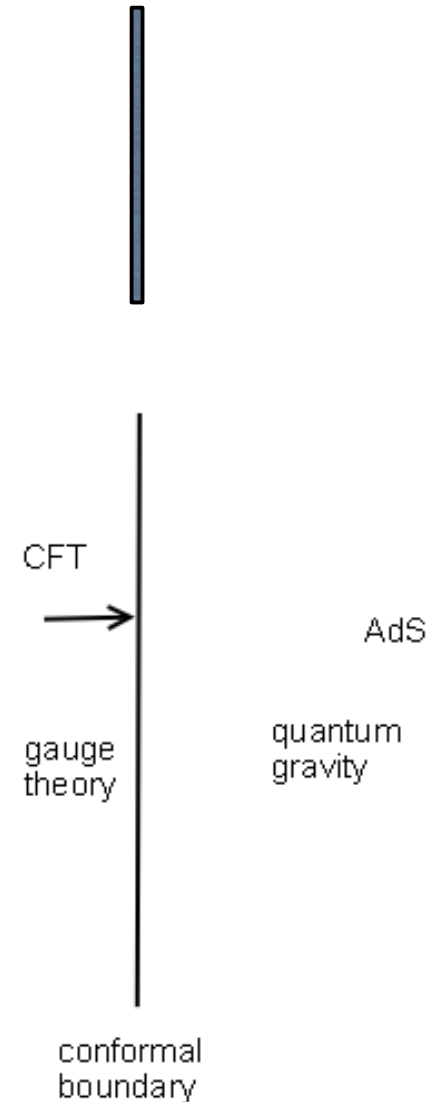
# AdS/CFT duality

–String in AdS5 is dual to SYM in 4d.

–5d v.s 4d → Holographic corr.

–For large coupling,  
**AdS/CFT → gravity/gauge duality**

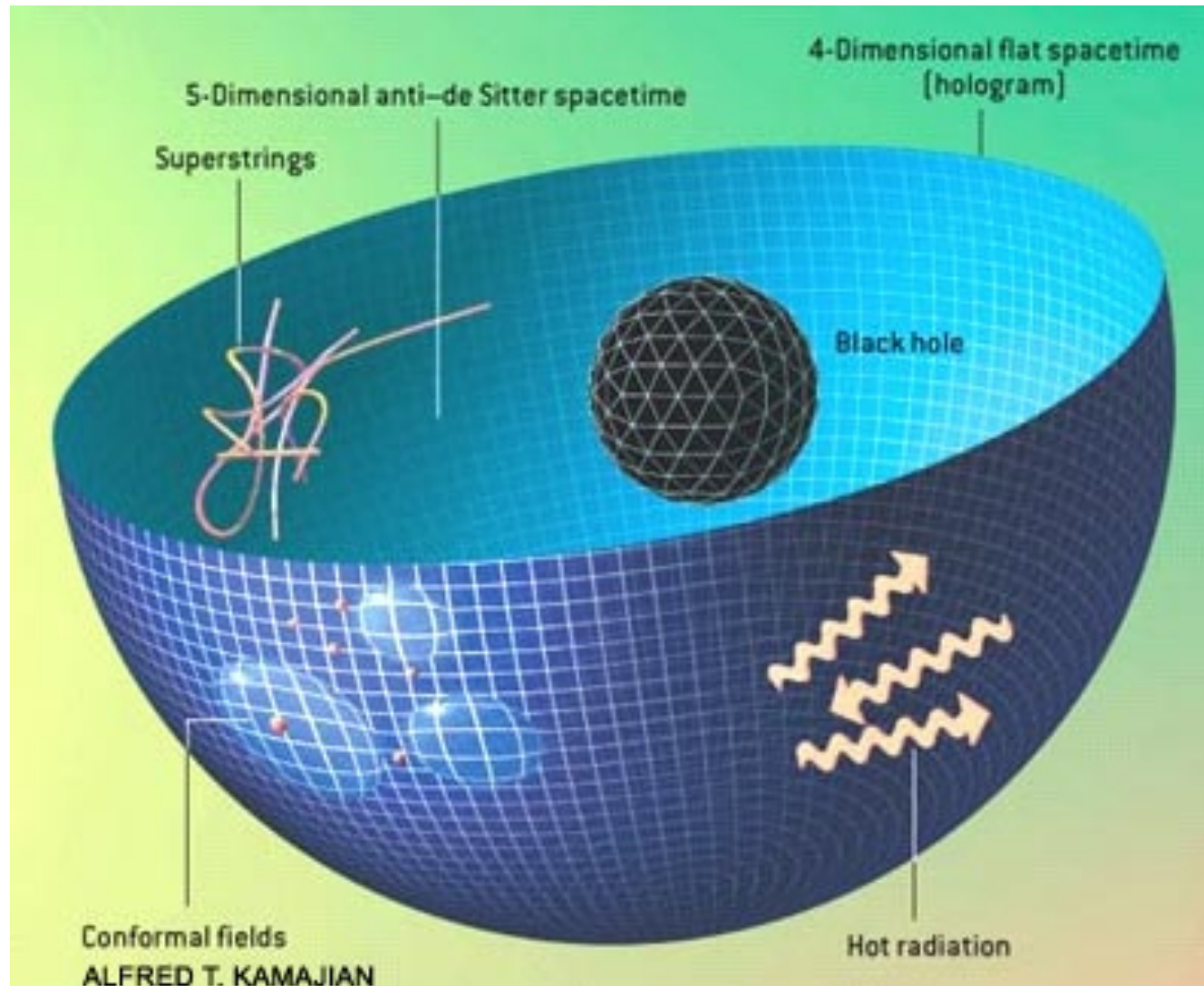
$$\lambda = g_s N_c$$



# Consequence of duality

- Gluon dynamics is replaced by ads gravity.
- For large  $N_c$ , gravity is weakly coupled.
- Correlation function in 4d can be calculated by the classical dynamics at the ads bulk.

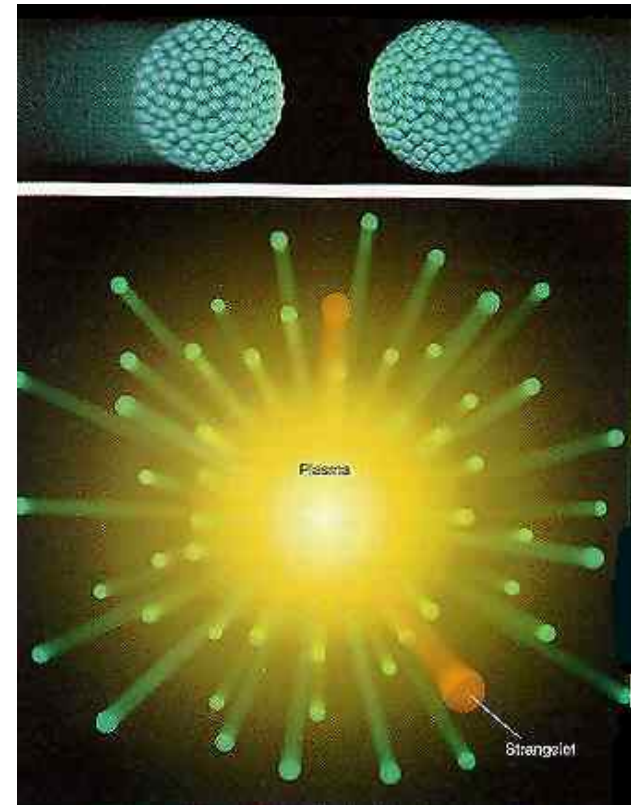
# AdS/CFT at finite T



# II. String theory in RHIC/LHC collider

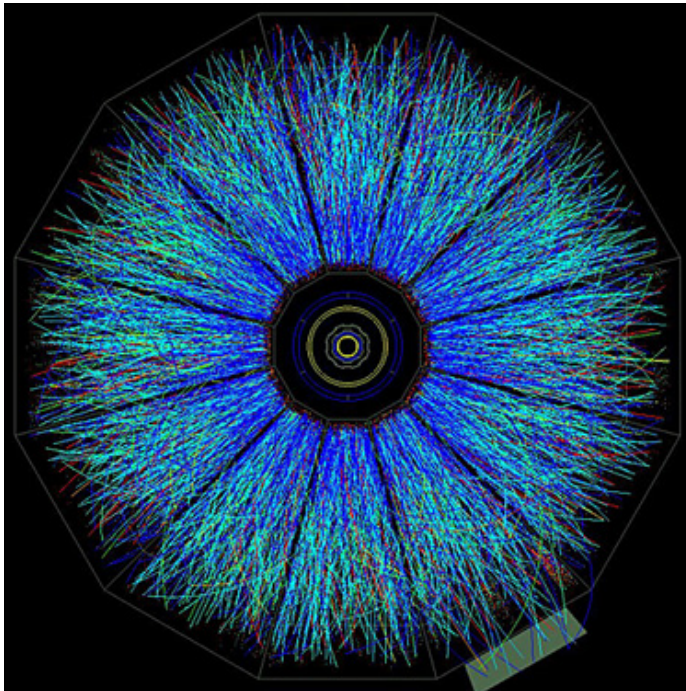
# Relativistic Heavy Ion Collider (Brookhaven N.L)

- Au-Au collision
- $E \sim 200$  GeV/nucleon
- Seek quark-gluon plasma(QGP)



# Detectors

star



phenix



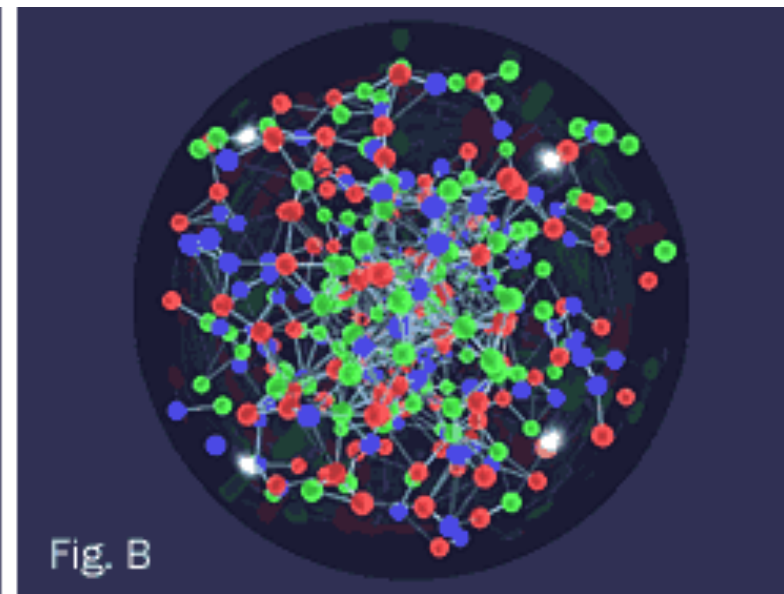
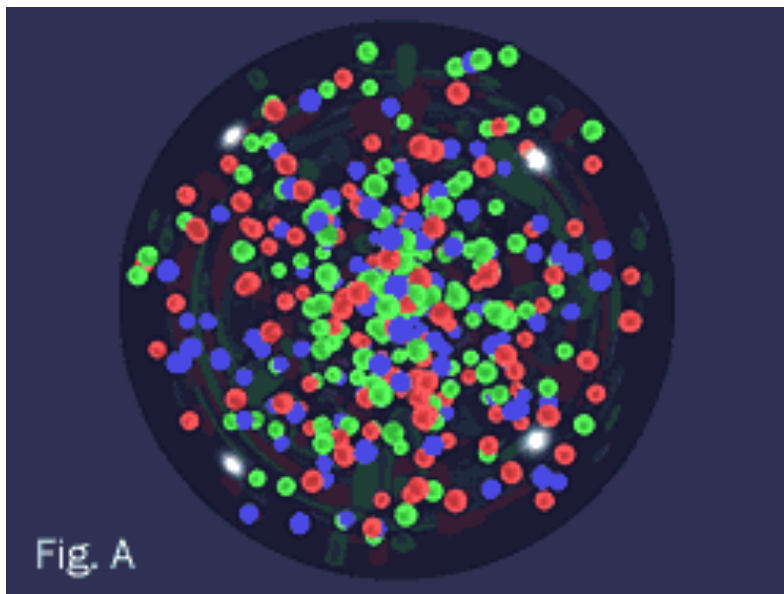
# Some Puzzles (my contributions)

- Elliptic flow (hep-th/0610113)\*
- Perfect fluid : (0806.4460 , 0901.0610 )
- Jet quenching (hep-th/0607123)
- Early thermalization (hep-th/0511199 )

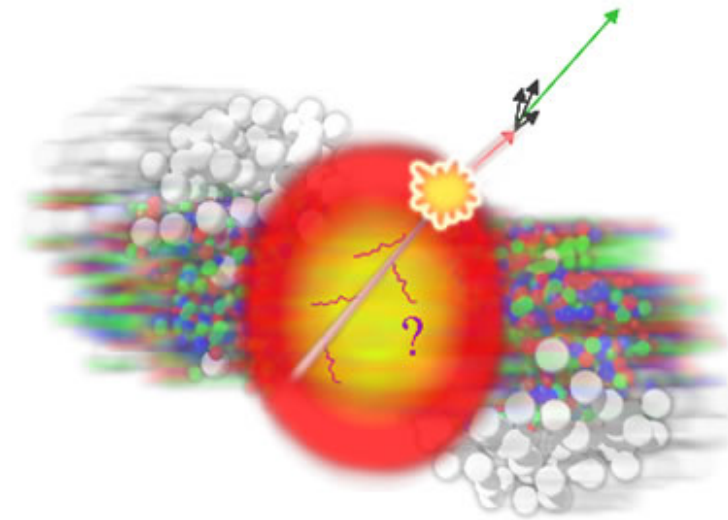
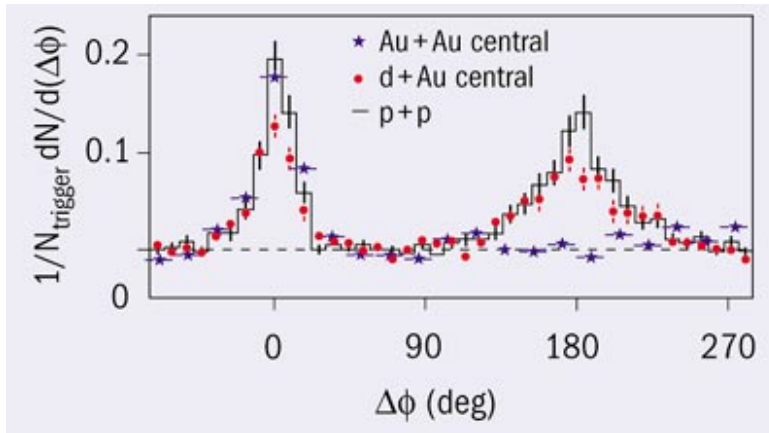
→ QGP is strongly interacting



# Liquid than Gas

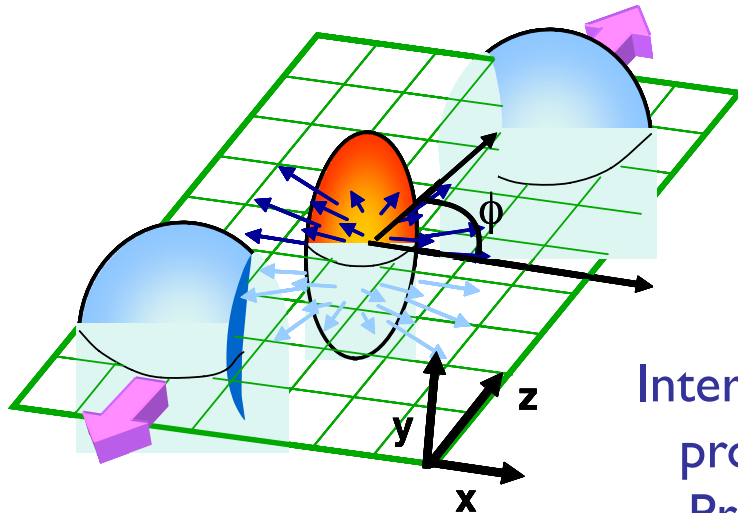


# Jet-Quenching



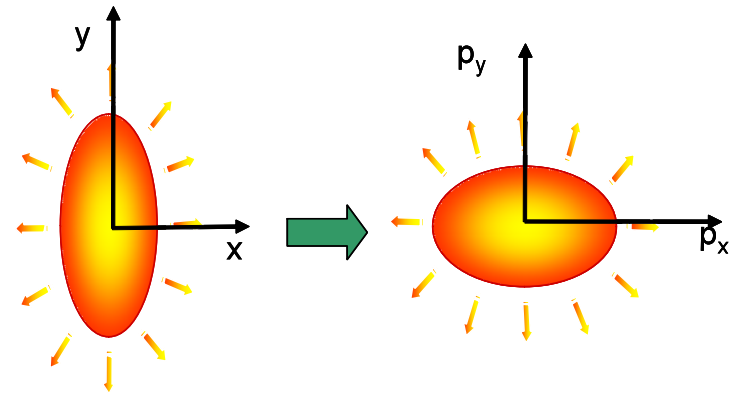
- Energy Loss Problem.

# Elliptic flow $v_2$



$$\frac{dN}{d\phi} = \frac{1}{2\pi} (1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi + \dots)$$

Interactions among the produced particles: Pressure gradients generate positive elliptic flow  $v_2$



Early observation at RHIC:

$v_2$  as large as predicted by **perfect fluid dynamics!**

# Perfect fluid

Exp:  $\eta / s < 0.1 \times \hbar / k_B$

While perturbative evaluation  $\sim 1/g^4$

ads/cft :  $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$  Universal value

# Transport Coeff.

[X.Ge, Y.Matsuo, F.Shu, SJS, Takuya Tsukioka](#) , [arXiv:0806.4460](#)

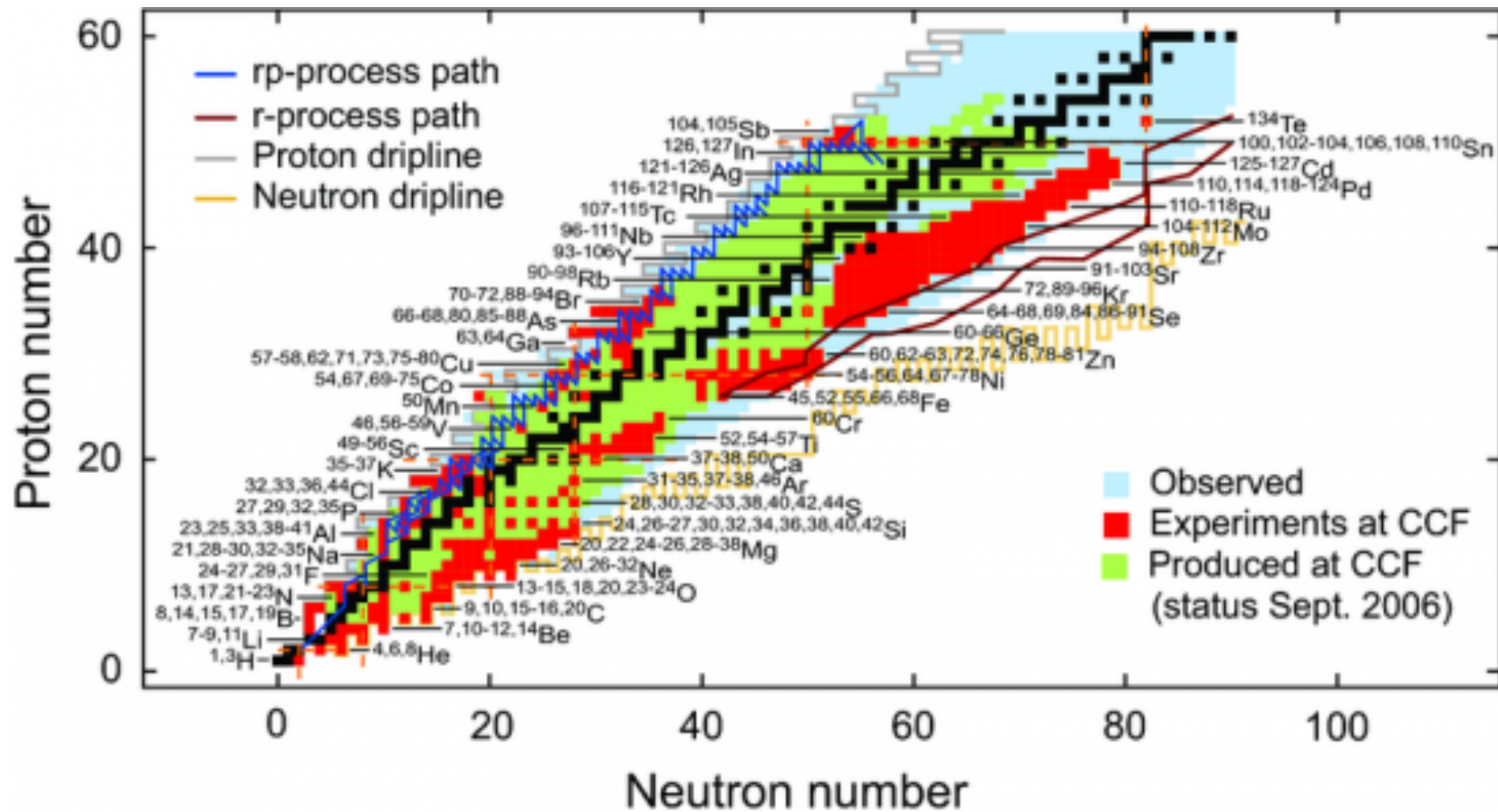
Y. Matsuo, SJS. S.Takeuchi, T.Tsukioka, C.Yoo (APCTP), [archiv:0901.0610](#)

- Linear response theory: causal Green function  $\rightarrow$  T.C
- Ads/cft can calculate  $\langle JJ \rangle$ ,  $\langle TT \rangle$  easily.

# III. String theory and Nuclear Symmetry Energy.

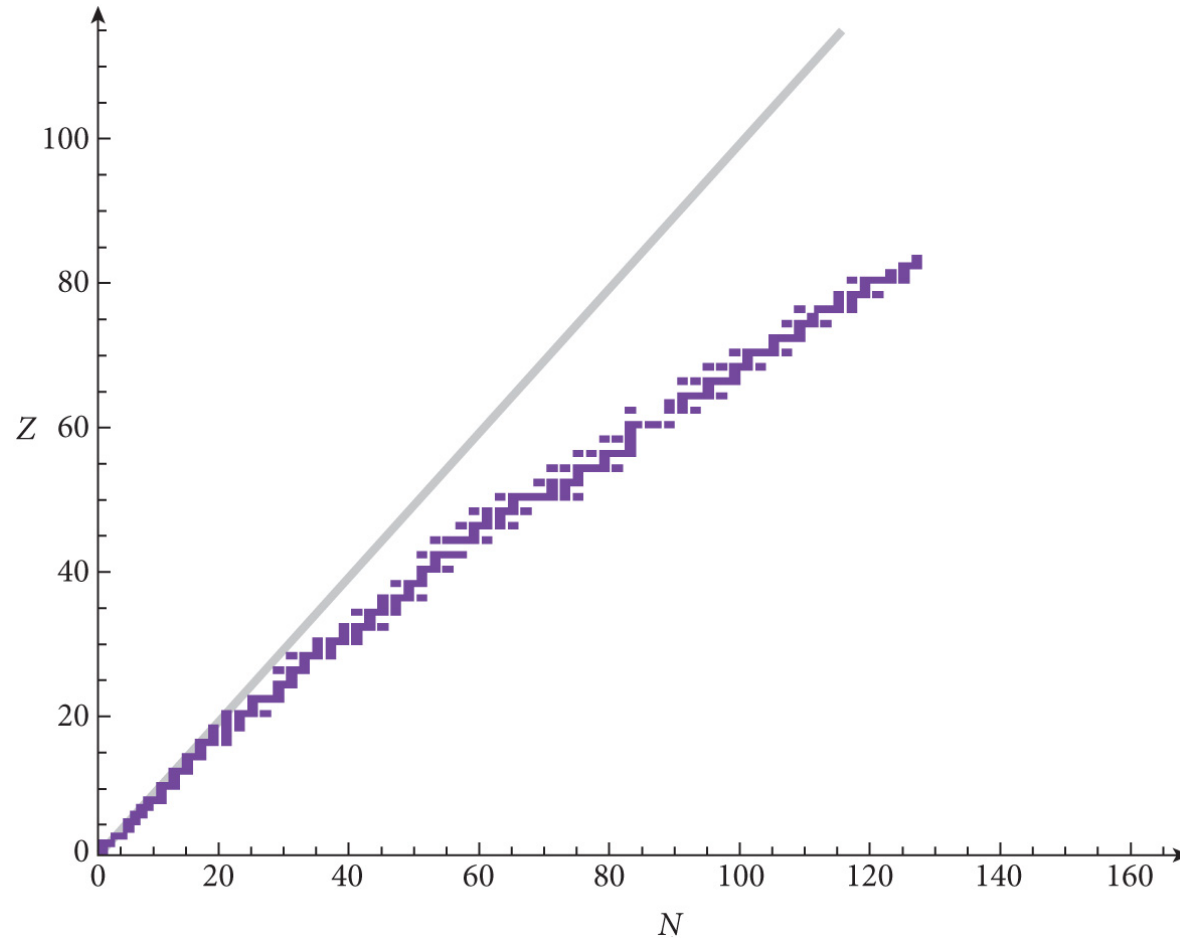
- There will be a heavy ion collider constructed in Korea in 5 year with ~billion dollar.

# KoR(are)I(sotope)A



- Valley of stability: Pauli v.s Coulomb

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# Symmetry Energy

- Liquid Drop Model  
Bethe-Weizsäcker formula (1935):

$$m = Zm_p + Nm_n - \frac{E_B}{c^2}$$

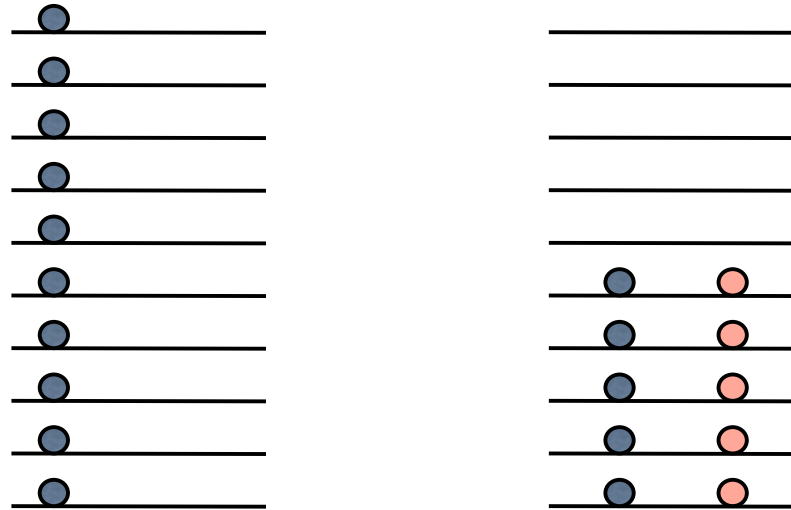
$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

$$E(\rho, \tilde{\alpha}) \simeq E(\rho, 0) + S_2(\rho) \tilde{\alpha}^2 \quad \tilde{\alpha} \equiv (N-Z)/A$$

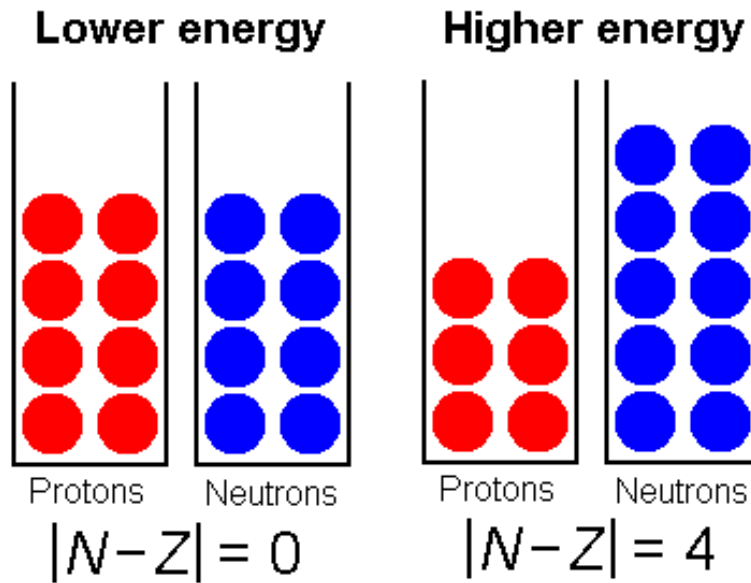
It determines the curvature of valley of Stability.

# Es and Pauli principle

Asymmetry term  $E_s(N-Z)^2$  is the consequence of Pauli principle.  $\rightarrow$  Pauli term



$$A = 16$$



$$E_s(N-Z)^2 :$$

If  $E_s \rightarrow 0$ , pure neutron star is possible.

If  $E_s \rightarrow \text{infinity}$ :  $N=P$

# Importance of Es

- **Structure of Neutron Star**  
the mass and width of neutron-star crusts.
- **Properties of Exotic Nuclei**
- **Nucleo-Synthesis during the supernova explosion.**

# non-interacting fermi gas

$$E_k = \frac{3}{5}(N_p \epsilon_{Fp} + N_n \epsilon_{Fn})$$

$$E_k = \frac{3}{5} \epsilon_F (N_p + N_n)^{2/3} + \frac{1}{3} \epsilon_F \frac{(N_n - N_p)^2}{(N_p + N_n)} + O((N_n - N_p)^4) = \frac{3}{5} \epsilon_F A^{2/3} + \frac{1}{3} \epsilon_F \frac{(A - 2Z)^2}{A} + O((A - 2Z)^4)$$

$$E_{sym} = \frac{\epsilon_F}{3} \sim \rho^{2/3}$$

**Non-relativistic**

$$\sim \rho^{1/3}$$

**Relativistic**

# Contribution of Pot.

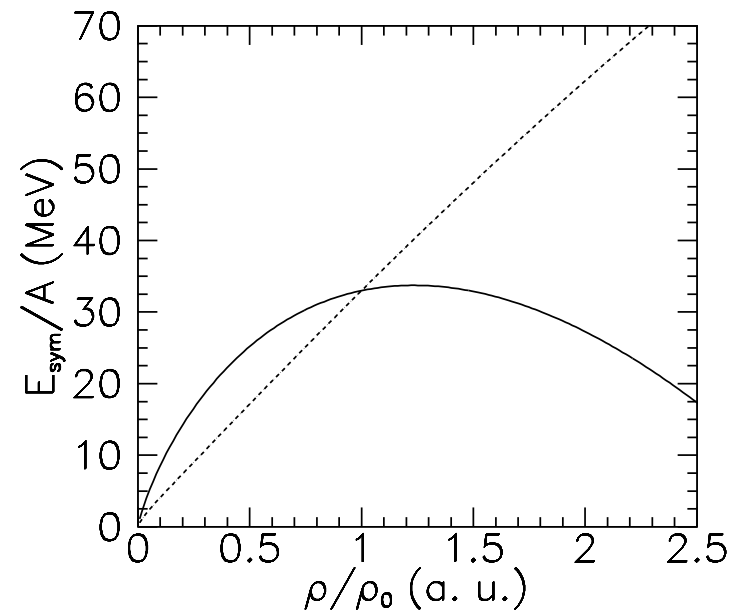
$$E_{sym} = E_{sym}(kin) + E_{sym}(pot) \equiv \frac{\epsilon_F}{3} + C_{sym}(\rho)$$

stronger density dependence,

$$E_{sym}(\rho) = a \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + b \cdot \frac{2(\rho/\rho_0)^2}{1 + (\rho/\rho_0)},$$

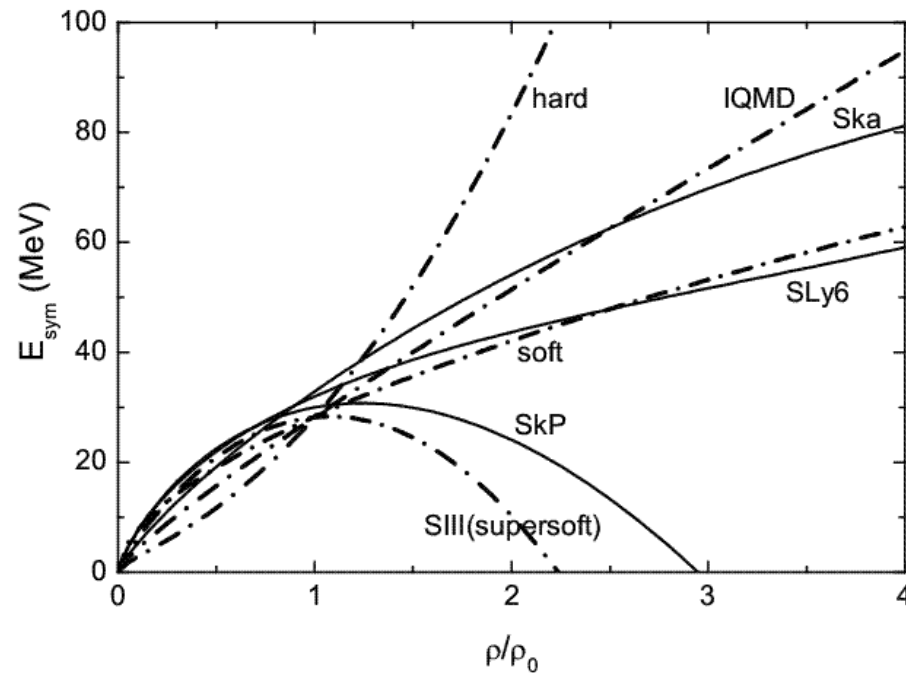
weaker density dependence

$$E_{sym}(\rho) = a \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + 240.9\rho - 819.1\rho^2,$$



# What is known for $E_s$ ?

- Little is known for high density.  
not Exp. nor theoretical.



# Even for the low density

- The separation of free part and potential part may not be valid.

So both low as well as high density regime is to be trusted.



# Why difficult?

1. Strongly interacting.  
No good calculational tool in this regime.
2. Density effect: Even lattice qcd does not help much.

# Repeat:

## String theory idea is

- Replace Nuclear force by classical gravity

Gluon dynamics  $\rightarrow$  Geometry.

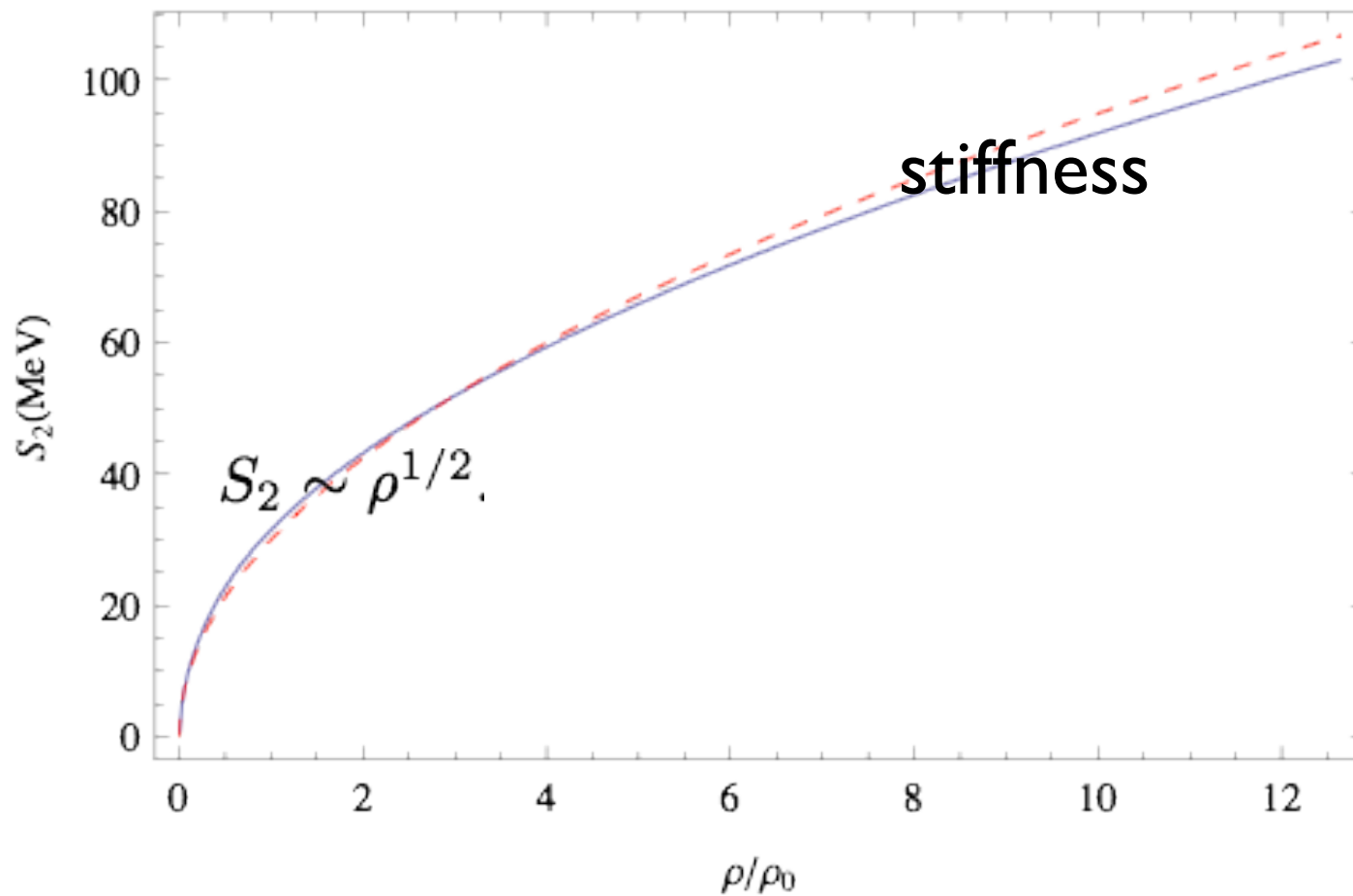
Confinement or deconfinement  
depends on geometry.

Flavor dynamics by classical fields  
in warped geometry.

# Result

arXiv:1011.0868 → to appear in JHEP

by Y.Kim, Y.Seo, I. Shin, SJS

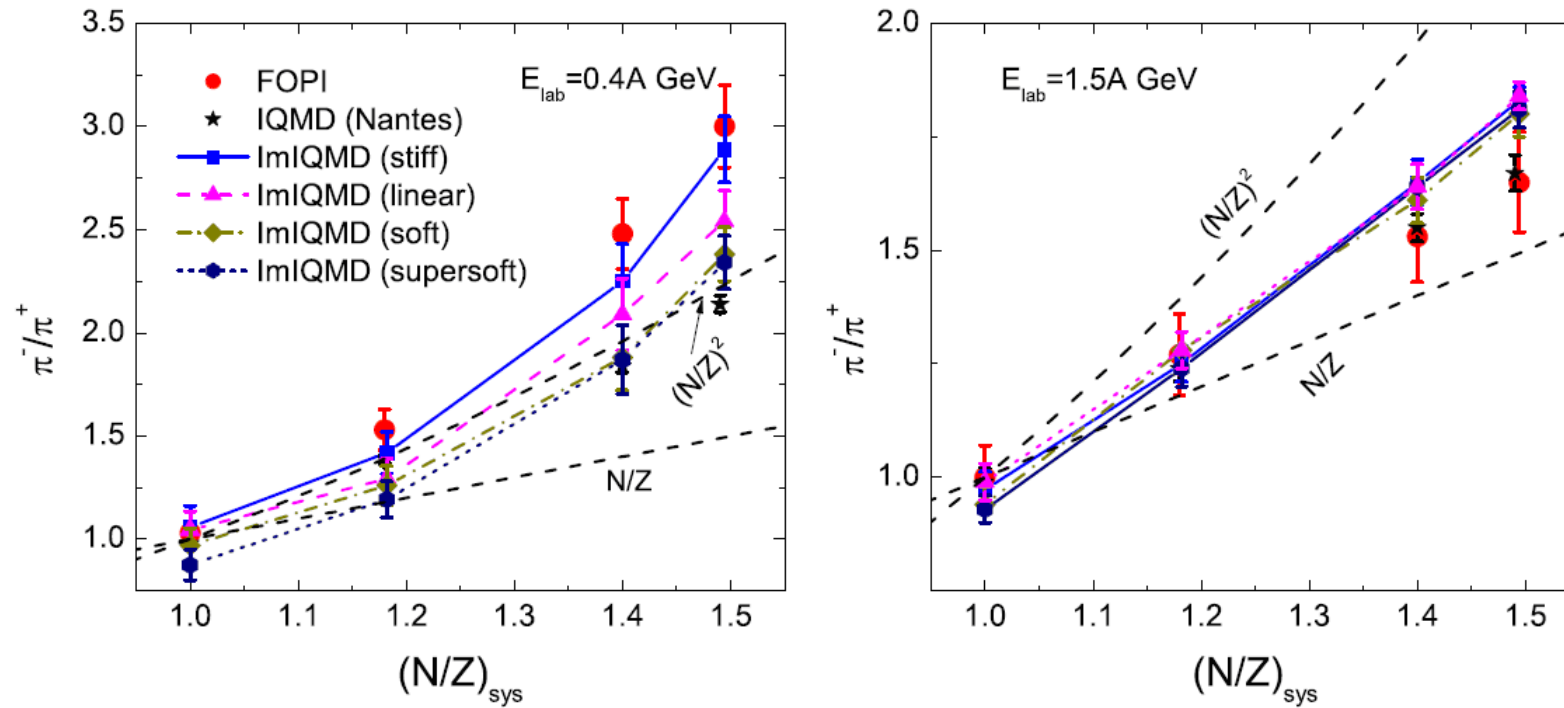


# How to detect $E_s$ ?

- Asymmetry in N-P is  
~ that in  $\pi^- \pi^+$
- $\pi^-/\pi^+$  yields are sensitive to  
the stiffness of the symmetry energy  
near threshold energy.

# Experiment → stiff

Z.-Q. Feng, G.-M. Jin / *Physics Letters B* 683 (2010) 140–144



$^{40}\text{Ca}+^{40}\text{Ca}$ ,  $^{96}\text{Ru}+^{96}\text{Ru}$ ,  $^{96}\text{Zr}+^{96}\text{Zr}$  and  $^{197}\text{Au}+^{197}\text{Au}$ , and also plotted the ratios of  $N/Z$  and  $(N/Z)^2$  as a function of  $N/Z$  at incident energy 0.4A GeV and 1.5A GeV, respectively.

# Pauli principle in hQCD.

- DBI action  $[Q_1, Q_2] = f(Q) + S^2(Q_1 - Q_2)^2 + \dots$   
Positivity of  $S^2$  is equivalent to the Pauli principle in hqcd
- Parallel to “Driving force of  $Z=N$  is Pauli principle”
- Dual of fermion number is the local  $U(1)$  in 5d.
- Coulomb repulsion of the dual E&M is responsible for the Es.

Why  $S_2 \sim \rho^{1/2}$ .

- analytic expression for symmetry energy.

$$S_2 = \frac{2\tau_6}{N_B} \int d\rho \frac{\sqrt{1 + \dot{y}^2} \tilde{Q}^2 \omega_+^{10/3} \rho^4}{(\tilde{Q}^2 + 4\omega_+^{8/3} \rho^4)^{3/2}},$$

- For the flat embedding approximation.

$$S_2 = \left(\Gamma\left(\frac{5}{4}\right)\right)^2 \sqrt{\frac{\lambda\rho_0}{2M_{KK}}} \sqrt{\frac{\rho}{\rho_0}}.$$

$$S_2 \sim \rho^{1/2}.$$

- Have confidence on



# Dispersion relation for non-fermi-Liquid

$$E_{sym} \sim \rho^{2/3} \text{ comes from } E = p^2$$

$$E_{sym} \sim \rho^{1/3} \text{ from } E = p$$

$$E_{sym} \sim \rho^{1/2} \rightarrow E = p^{3/2}$$

# Why interesting?

- The anomaly in dispersion relation is closely related to the fermi surface structure.
- For strongly interacting system, fermi surface is fuzzy and its implication is a big and interesting issue.
- Entire Thermodynamics as well as hydrodynamics of the strongly interacting Non-fermi liquid system will be a hot issue.

# Non-fermi Liquid in Nuclear system?

$$G_R(\omega, k) = \frac{h_1}{k_{\perp} - \frac{1}{v_F}\omega - h_2\omega^{2\nu_{k_F}}}$$

Liu et. al  
0903.2477  
0907.2694

if  $\nu_{k_F} < \frac{1}{2}$ ,  $\omega_{\star}(k) \sim k_{\perp}^z$ ,  $z = \frac{1}{2\nu_{k_F}} > 1$

- If we choose  $\nu_{k_F} = \frac{1}{3}$ ,

we get  $E = p^{3/2} \rightarrow E_{sym} = \rho^{1/2}$

# Conclusion

- String theory is useful as well as interesting.
- Symmetry Energy can be calculated using the holographic principle.
- Physics of fermi surface for strongly interacting system will be Interesting issue.
- It can make the KoRIA more interesting project.