de Sitter, Newton-Hooke, pp Waves, and All That

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- 宇宙学常数 (CC)

$\Lambda > 0, \quad \Lambda \sim 10^{-120} M_p^4$

$\sim 10^{-120} (10^{18+9} ev)^4$

$\sim (10^{-3} ev)^4$

解释 Λ，需要至少两个能标:

$M_p \sim 10^{18}$ Gev ; $\quad M_{SUSY} \sim 10^{3}$ Gev

仿照 Banks:

$$\Lambda \sim \frac{M_{SUSY}^{\lambda}}{M_p^{\lambda-4}}$$

- $\lambda = 0, \quad \Lambda \sim M_p^4$

  QG short distance behavior

- $\lambda = 4, \quad \Lambda \sim M_{SUSY}^4$

  SUSY breaking

- The observed value of λ is

  $\lambda \sim 8$

$$\Lambda \sim \frac{M_{SUSY}^8}{M_p^4} \sim \left(\frac{M_{SUSY}}{M_p}\right)^8 M_p^4 \sim 10^{-120} M_p^4$$

粗略地说，有两类不同的问题

(i) 构造理论模型说明 Λ 何以如此之小；（RG 的 Talk）

(ii) 给定 $\Lambda>0$，建立相应真空背景中引力量子涨落的微观描述

"de Sitter 引力"

de Sitter 空间：带正宇宙学常数的真空 Einstein 场方程的解

· 常曲率空间

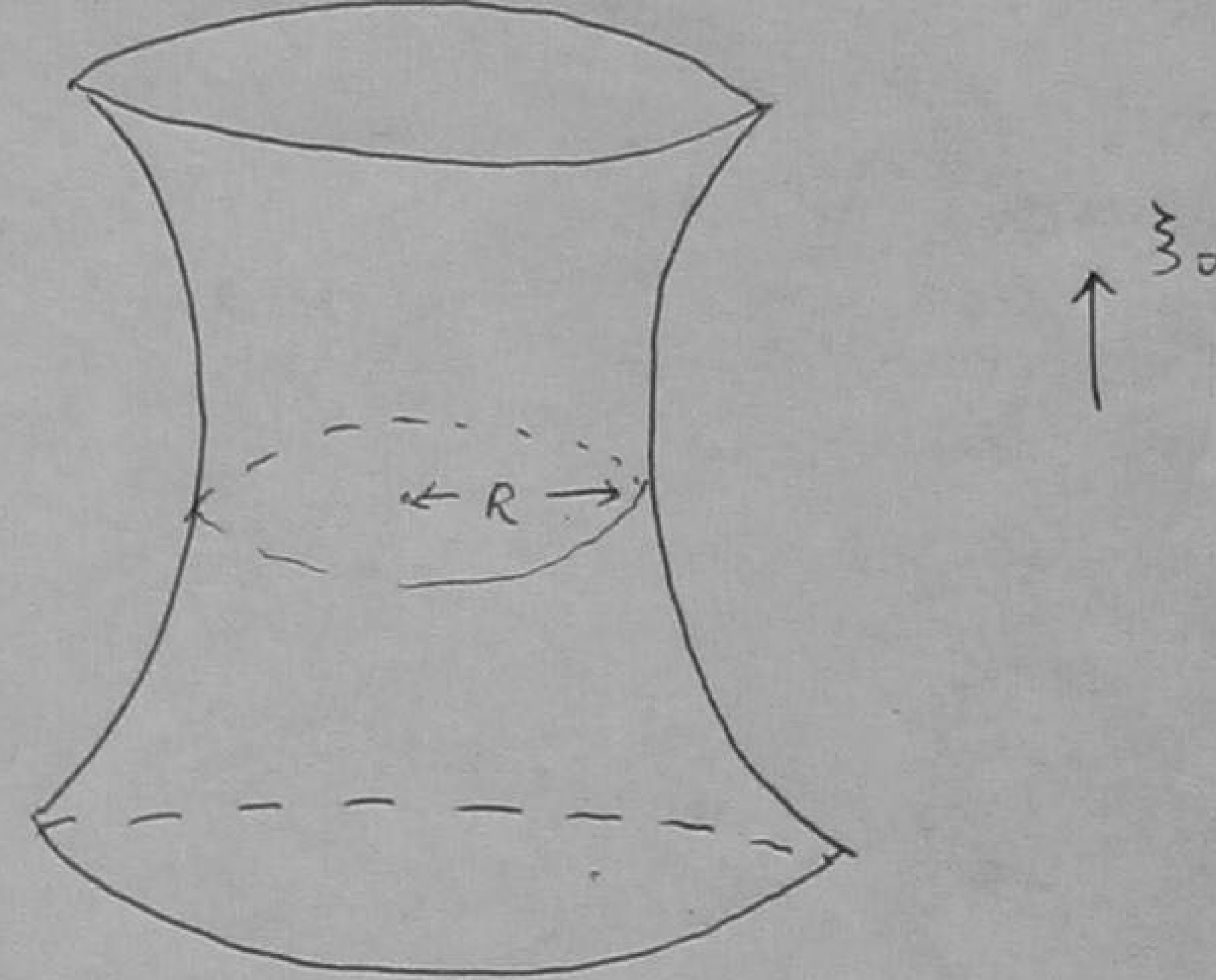
· 极大对称空间

$$dS_D \rightsquigarrow \mathbb{R}^{D,1}$$

$$-\xi_0^2 + \xi_1^2 + \cdots + \xi_D^2 = R^2$$

dS_D 的"尺度"

$$R \sim \frac{1}{\sqrt{\Lambda}}$$



dS_D 的度规是 $\mathbb{R}^{D,1}$ 上平度规的诱导

- Isometry Group: $SO(D,1)$
- Global coordinates:

$$\begin{cases} \xi_0 = R\, \mathrm{sh}\dfrac{t}{R} \\ \xi_i = R\, \mathrm{ch}\dfrac{t}{R} \cdot \omega_i \qquad (i=1,\cdots,D) \end{cases}$$

$$\omega_i \in S^{D-1}\ ; \quad \sum_{i=1}^{D} \omega_i^2 = 1\ ; \quad \sum_{i=1}^{D} \omega_i d\omega_i = 0$$

$$\sum d\omega_i^2 \equiv d\Omega_{D-1}^2$$

$$ds^2 = -d\xi_0^2 + \sum_{i=1}^{D} d\xi_i^2$$

$$= -dt^2 + R^2 \operatorname{ch}^2 \frac{t}{R} \cdot d\Omega_{D-1}^2$$

• dS 引力的若干特点

(1) $dS_D \sim \mathbb{R}^1 \times S^{D-1}$

$\downarrow$

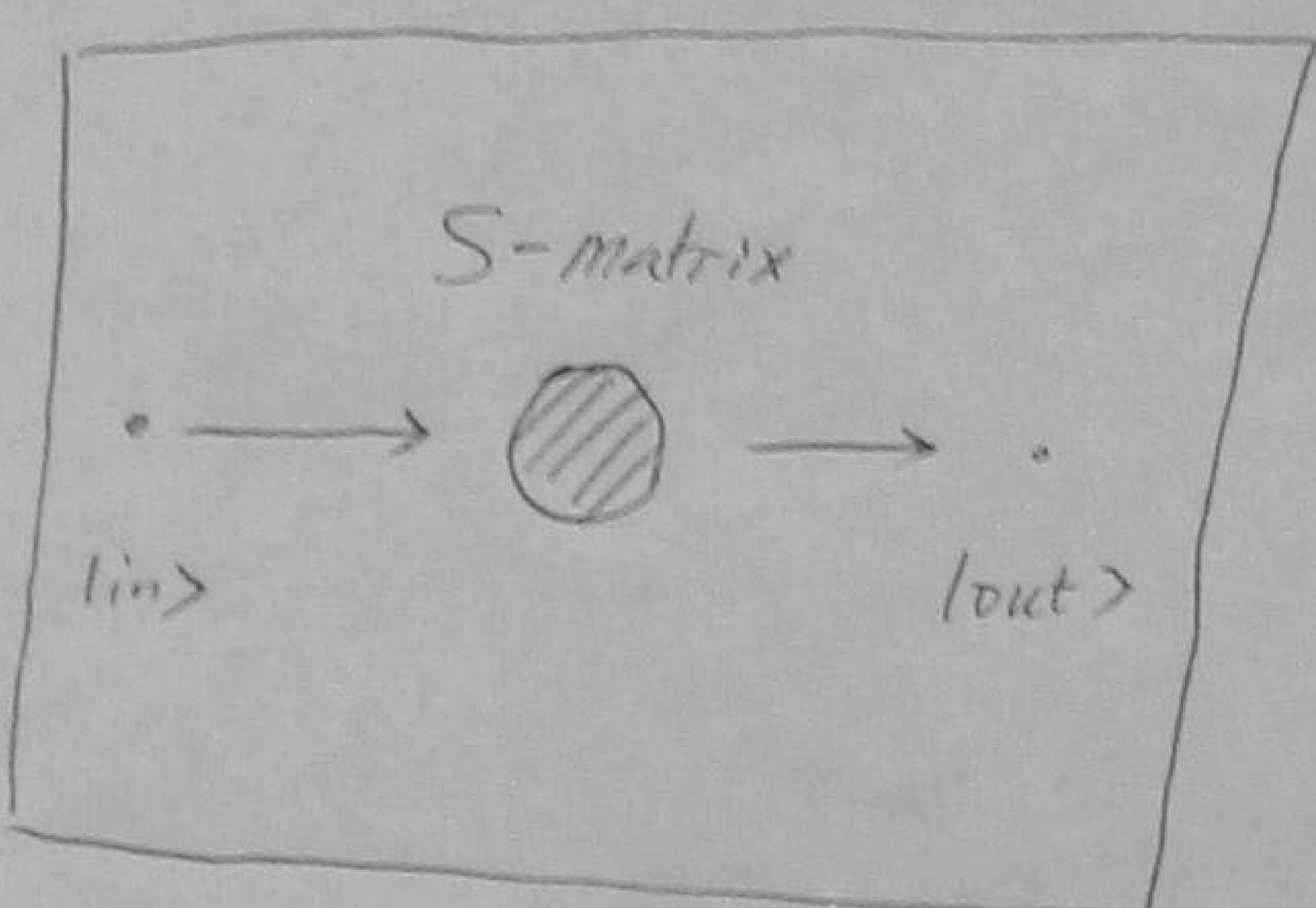
空间部分紧致

无"空间无穷远处"的概念

不能在通常意义下谈论

渐近 de Sitter 空间

(只能定义"无限过去"和"无限将来")



无法绝热地关掉相互作用，来定义渐近态，进而定义 S-matrix

(2) 无法定义正的、守恒的能量

$$E_{Killing} \geq 0 \qquad (不成立)$$

比较"物理"的 Argument:

dS_4, Euclidean formulation

$$\tilde{g}_{\mu\nu} = \Omega^2 \cdot g_{\mu\nu}^{background}$$

$\uparrow$

conformal fluctuation

$$\tilde{R} = \Omega^{-2} R - 6\Omega^{3} \Box \Omega$$

$$I = -\frac{1}{16\pi G} \int d^4x \sqrt{g} \left(\Omega^2 R + 6 \nabla\Omega \cdot \nabla\Omega - 2\Lambda \Omega^4 \right)$$

$\rightarrow$ $\lambda\phi^4$ theory, w/ wrong sign of the K.T.

$\frac{1}{2} \nabla\phi \cdot \nabla\phi$

Rapid modes of Ω $\rightarrow$ $I \ll 0$

解析延拓： $\Omega = 1 + i\varphi$

$$I_{Kinetic} = \int \frac{1}{2} \varphi (-6\Box - 8\Lambda) \varphi$$

$-\Box \geq 0$ （正定算子）

- 当 $\Lambda \leq 0$，动能项 ≥ 0

- 当 $\Lambda > 0$，动能项无下界

(3) 作为推论，dS 引力不允许有

超对称.

SUSY 保持的条件：

$$\sum_A \{\bar{Q}_A, Q_A\} = H \geq 0$$

(4) Hawking 温度、熵

Wick rotation: $t \to i\tau$

$$ds^2 = d\tau^2 + R^2 \cos^2 \frac{\tau}{R} \cdot d\Omega_{D-1}^2$$

若 $\tau \sim \tau + 2\pi R$,

$$ds^2 = R^2 \, d\Omega_D^2$$

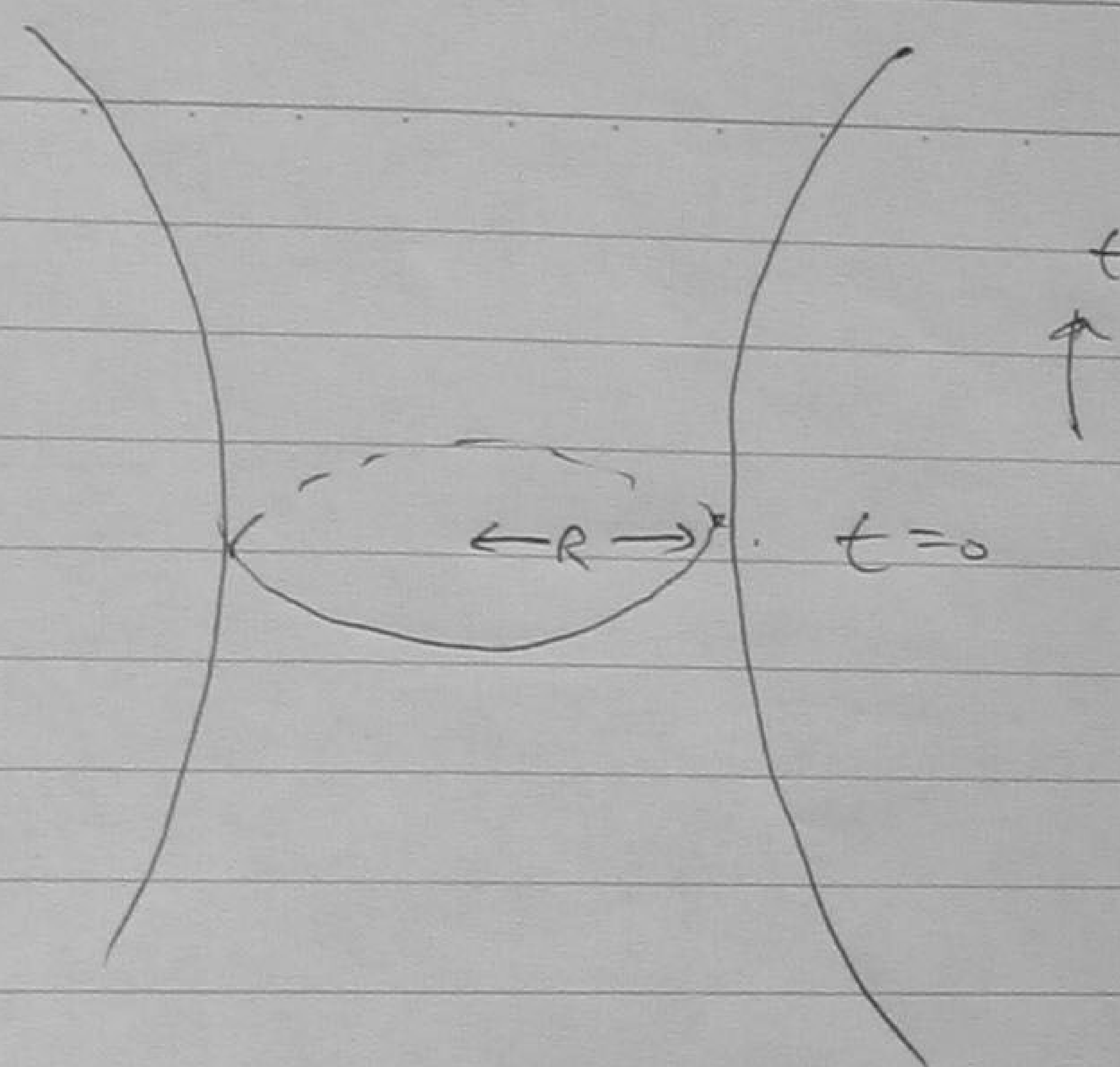
否则度量所描述的空间有 conic singularity.

$$\beta = 2\pi R, \quad T_H = \frac{1}{\beta} = \frac{1}{2\pi R}.$$

$$S = \beta \cdot \frac{\partial I}{\partial \beta} - I$$

$$= \frac{A}{4G} \sim \frac{R^{D-2}}{G} \sim \frac{1}{G \Lambda^{(D-2)/2}}$$

静态解存在的条件:



$$L \le R$$

$$\Rightarrow \quad \frac{1}{8} g^2 R^2 (N^2-1) \le R^2$$

$$\Rightarrow \quad N \le N_{max} \sim \frac{1}{g} \sim S$$

(2) $\quad \mathcal{H}_{tot} \sim \mathcal{H}^{\otimes N_{max}}$

如何保证 $\dim \mathcal{H} < \infty$?

The relation $N \sim S$ is familiar in the matrix model approach to Schwarzschild black holes

(Banks, Fischler, Klebanov, Susskind, Miao Li, Martinec, ...)

Indicating that N should be thought of as the number of effective degrees of freedom of the "boundary" system, as least for $g \cdot R$ being small.

$$\mathcal{H}_{tot} \sim \mathcal{H}^{\otimes N_{max}}$$

Problem If we keep $g \cdot R$ small, how to get a finite dimensional "1-particle" Hilbert space $\mathcal{H}$?

When $g \cdot R$ is extremally small, the non-flat degrees of freedom in the Hamiltonian are frozen

$$\hat{H} = Tr\left\{ \frac{1}{2\mu} \Pi_i^2 - \frac{\mu}{2R^3} (X^i)^2 - \frac{\mu}{2R^3} \cdot \frac{1}{2g^2R^2} [X^i, X^j]^2 \right\}$$

So if $g \cdot R \simeq 0$, the matrix model gets drastically simplified, and it becomes $(d-1)$ copies of decoupled one-matrix model, each has the Hamiltonian:

$$\hat{H}_0(X) = \mathrm{Tr}\left\{ -\frac{1}{2\mu}\left(\frac{\partial}{\partial X}\right)^2 - \frac{\mu}{2R^2}X^2 + \cdots \right\}$$

$\longrightarrow$ $c=1$ strings at finite temperature $T_H \sim \frac{1}{R}$.

- Two ways to reach the limit $gR \to 0$.

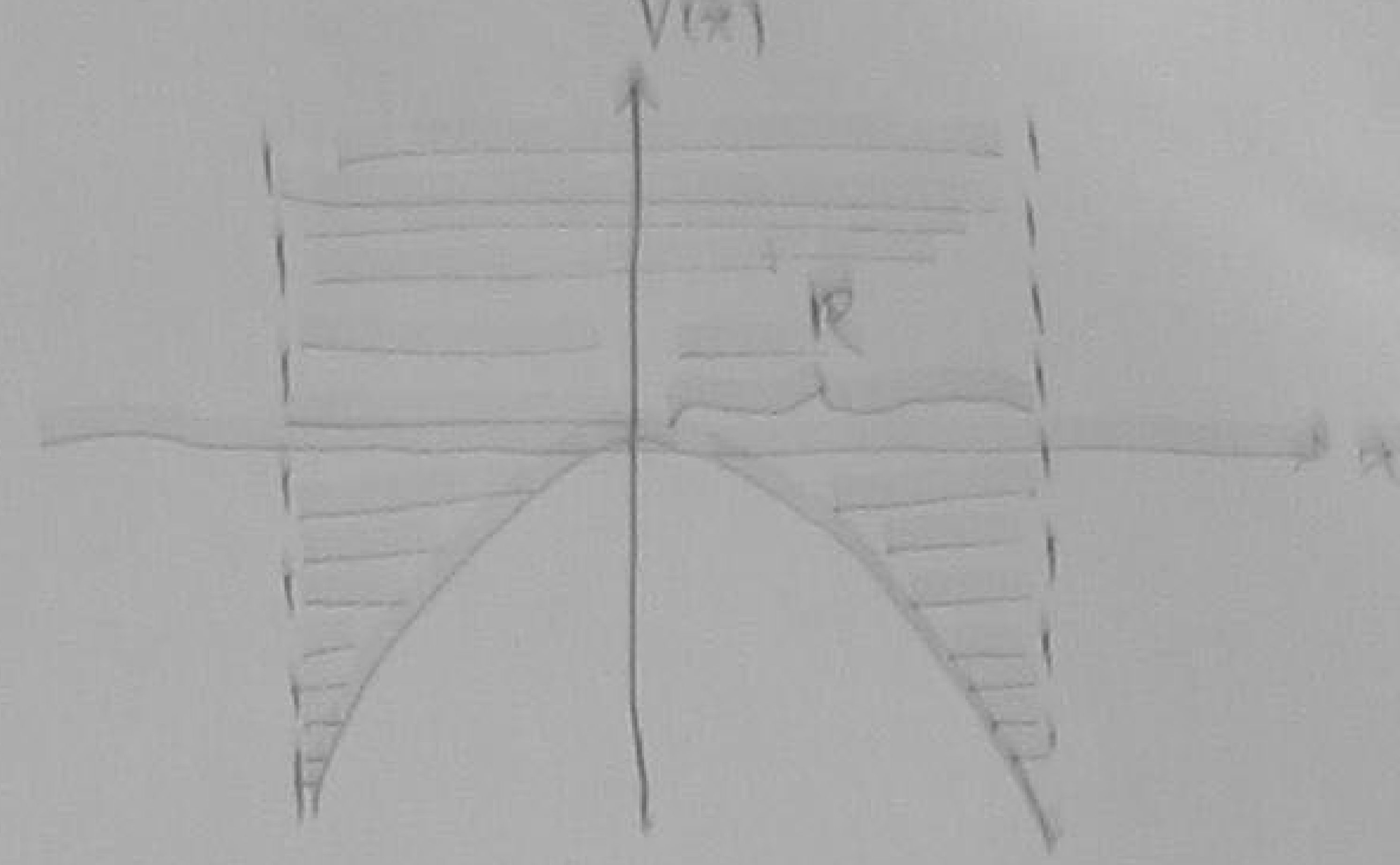
① $\boxed{R \text{ large but fixed}, \quad g \to 0}$

$\Leftrightarrow$ perturbative regime of gravity in large de Sitter space, or equivalently at low Hawking temperature $T_H \sim \frac{1}{R} \sim 0$.

$\sim$ $c=1$ strings in non-compact target space

There are N^2 degrees of freedom, $N(N-1)$ of which are angular and decoupled, they can be integrated out, thus leaving N degrees of freedom.

$$\begin{cases} \mathcal{H}_{tot} \sim \mathcal{H}^{\wedge N_{max}} \\ \dim \mathcal{H} = \infty \end{cases}$$



(2)

$$\boxed{R \to 0, \quad g \text{ finite}}$$

$\Leftrightarrow$ Non-perturbative regime of gravity in an extremely small de Sitter space, or at very high Hawking temperature $T_H \sim 1/R \sim \infty$

A kind of the Kosterlitz - Thouless phase transition may occur here (Gross, Klebanov)

$$\exists \quad T_c \sim \frac{1}{R_c}, \quad \text{when} \quad T_H \gg T_c,$$

the model goes into a phase of matrix chains describing $c < 1$ strings !

The time variable is descretized with a spacing,

$$\epsilon \sim \frac{1}{R} \sim T_H$$

UV - IR Relations

ϵ: UV - cut off in the "boundary theory"

T_H: IR - cut off in the bulk gravity

(A particle in de Sitter space w/ $E < T_H$ will be immediately thermalized.)

Dual description of this phase:

$$S \sim \sum_{a=1}^{Q} \left\{ \frac{1}{2\epsilon} \operatorname{Tr} (X_{a+1} - X_a)^2 + \epsilon \cdot \operatorname{Tr} W(X_a) \right\}$$

$$\begin{cases} \mathcal{H}_{tot} \sim \mathcal{H}^{\otimes N} \\ \dim \mathcal{H} < \infty . \end{cases}$$

Imagine we have an ordinary QM particle, but with both a UV - cut-off ϵ and an IR cut off $Q \cdot \epsilon$.

UV cut-off: $|E| \leq E_{max} \sim \frac{1}{\epsilon}$

IR cut-off: the energy spectrum discrete, w/ separation

$$\Delta E \sim \frac{1}{Q\epsilon}$$

# of Energy levels $\sim \dfrac{E_{max}}{\Delta E} \sim Q$

$$\Rightarrow \quad \dim \mathcal{H} \sim Q < \infty$$

Note There is a third way to reach the limit $g \cdot R \to 0$, namely

$$\boxed{\text{Both } g, R \to 0}$$

$\Leftrightarrow$ Perturbative regime of gravity in extremally small de Sitter space. In this case one expects

↳ high Hawking temperature

$$\dim \mathcal{H} \to \infty.$$

This would correspond to a matrix chain with $Q \to \infty$ (infinitely long chain)