

Towards a new paradigm of heavy-flavor hadron spectrum

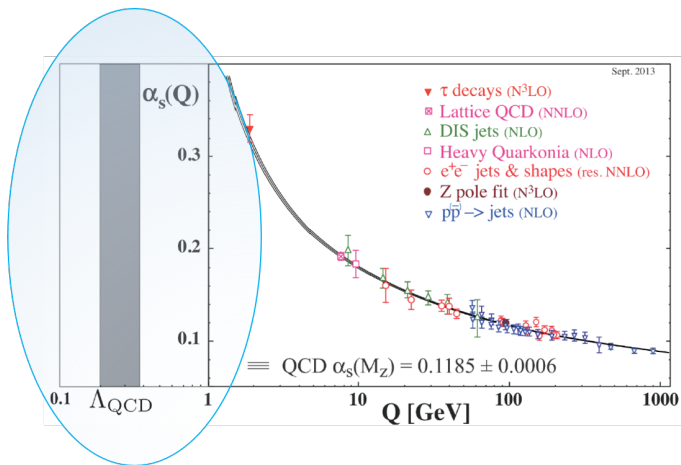
Revealing the mysteries of positive-parity charm mesons

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Institute of Theoretical Physics, Chinese Academy of Sciences

Seminar, USTC, Nov. 22, 2018

Low-energy QCD: big challenge

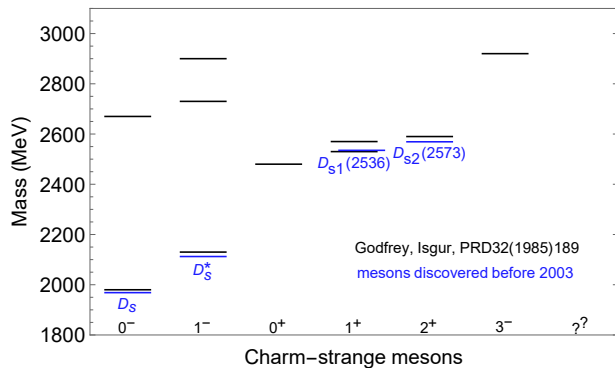


Confinement

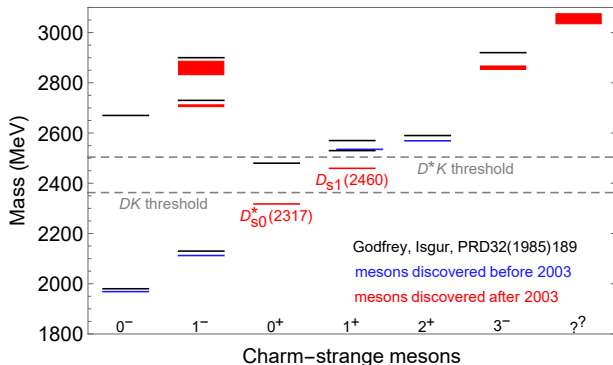
nucleon structure

mysteries in hadron
spectroscopy

Mysteries of charm mesons: Charm-strange



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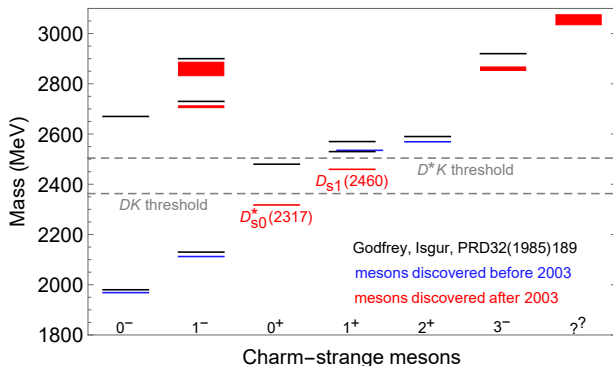


- $D_{s0}^*(2317)$: BaBar (2003)
 $J^P = 0^+$, $\Gamma < 3.8$ MeV
- $D_{s1}(2460)$: CLEO (2003)
 $J^P = 1^+$, $\Gamma < 3.5$ MeV
- no isospin partner observed, tiny widths
 $\Rightarrow I = 0$

- Mystery 1: Mass problem: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- Mystery 2: Naturalness problem:

$$\text{Why } \underbrace{M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)}}_{(141.8 \pm 0.8) \text{ MeV}} \simeq \underbrace{M_{D^{*+}} - M_{D^+}}_{(140.67 \pm 0.08) \text{ MeV}} ?$$

Mysteries of charm mesons: Charm-strange

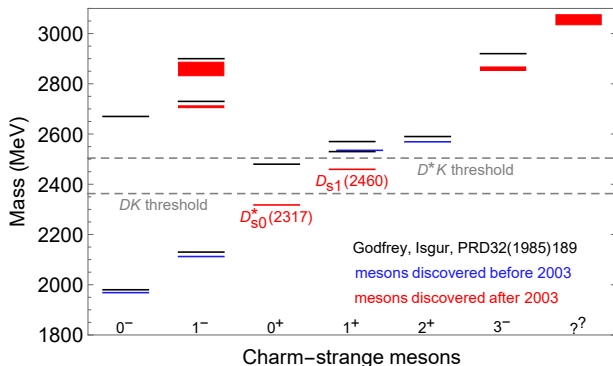


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Mysteries of charm mesons: Charm-nonstrange (1)

Observations of charm-nonstrange excited mesons in 2003

$$B^- \rightarrow D^{(*)+} \pi^- \pi^-$$

Belle, PRD69(2004)112002 [hep-ex/0307021]

- $D_0^*(2400): J^P = 0^+$

$$\Gamma = (267 \pm 40) \text{ MeV}$$

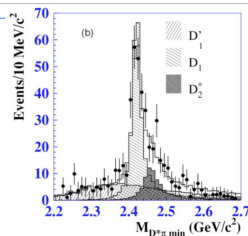
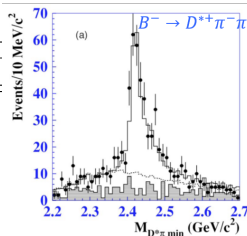
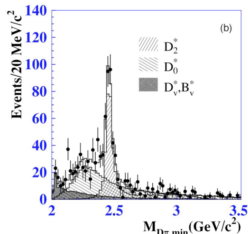
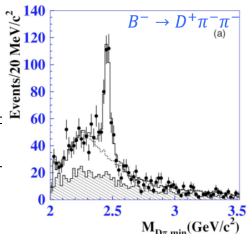
Mass (MeV):

| | | |
|---------------|-------|---------------------------------------|
| 2318 ± 29 | PDG18 | |
| 2297 ± 22 | BaBar | B decays |
| 2308 ± 36 | Belle | B decays |
| 2401 ± 41 | FOCUS | γA |
| 2360 ± 34 | LHCb | $B^0 \rightarrow \bar{D}^0 \pi^- K^+$ |

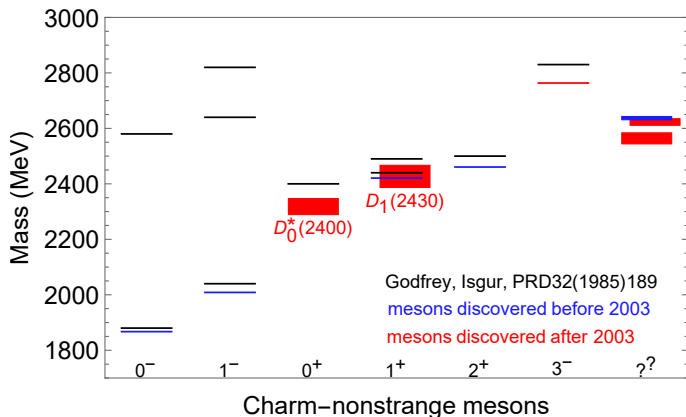
- $D_1(2430): J^P = 1^+$

$$\Gamma = 384_{-110}^{+130} \text{ MeV}$$

$$M = (2427 \pm 36) \text{ MeV}$$



Mysteries of charm mesons: Charm-nonstrange (2)

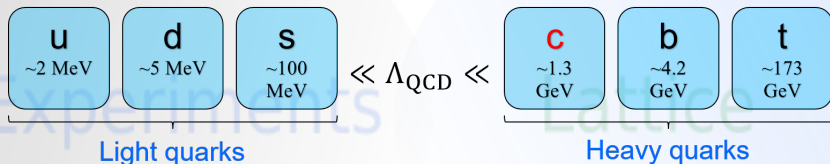


- **Mystery 3:** Hierachy problem:

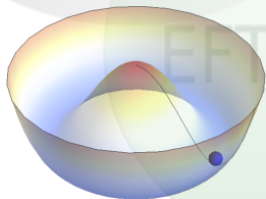
Why $M_{D_0^*(2400)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

Theoretical tools

- Lattice QCD
- Symmetries:



- ☞ Spontaneously broken **chiral symmetry**: π , K and η as the pseudo-Goldstone bosons



- ☞ Heavy quark spin symmetry
- ☞ Heavy quark flavor symmetry



- Quark model, QCD sum rules, ...

Lattice studies of the charmed scalar mesons: strange

- Early studies using **only $c\bar{s}$ -type** interpolators typically give **mass larger** than that for $D_{s0}^*(2317)$ Bali (2003); UKQCD (2003); ...

- $c\bar{s} + DK$ interpolators: \sim right mass Mohler et al., PRL111(2013)222001

$$M_{D_{s0}^*} - \frac{1}{4} (M_{D_s} + 3M_{D_s^*}):$$

| Mohler et al. | PDG2018 |
|--------------------|-----------------------|
| (266 ± 16) MeV | (241.5 ± 0.8) MeV |

- Calculation with $M_\pi = 150$ MeV Bali et al. [RQCD Col.], PRD96(2017)074501

| | Energy [MeV] | Expt [MeV] |
|----------|--------------|------------------|
| m_{0-} | 1976.9(2) | 1966.0(4) |
| m_{1-} | 2094.9(7) | 2111.3(6) |
| m_{0+} | 2348(4)(+6) | 2317.7(0.6)(2.0) |
| m_{1+} | 2451(4)(+1) | 2459.5(0.6)(2.0) |

Lattice studies of the charmed scalar mesons: nonstrange (1)

- $(S, I) = (0, \frac{1}{2}): c\bar{q} + D\pi$

interpolators:

Mohler et al., PRD87(2013)034501

$$M_\pi \approx 266 \text{ MeV},$$

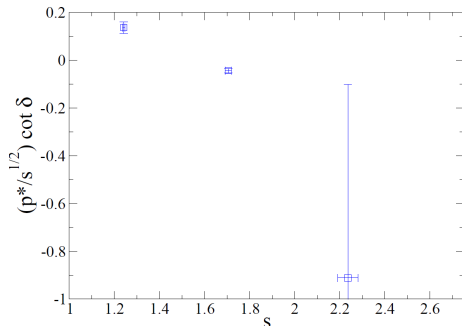
$$M_D \approx 1558 \text{ MeV},$$

$$M_{D^*} \approx 1690 \text{ MeV}$$

Lüscher's formula $\Rightarrow D\pi$ phase

shifts

\Rightarrow BW parameters of $D_0^*(2400)$ consistent with PDG values



| | Mohler et al. | PDG2018 |
|---|----------------------------|----------------------------|
| $M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$ | $(351 \pm 21) \text{ MeV}$ | $(347 \pm 29) \text{ MeV}$ |
| $M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$ | $(380 \pm 21) \text{ MeV}$ | $(456 \pm 40) \text{ MeV}$ |

Lattice studies of the charmed scalar mesons: nonstrange (2)

- $(S, I) = (0, \frac{1}{2})$: first coupled-channel lattice calculation including interpolating fields for $c\bar{q} + D\pi + D\eta + D_s\bar{K}$: Moir et al. [Hadron Spectrum Col.], JHEP1610(2016)011
- $M_\pi = 391$ MeV, $M_D = 1885$ MeV: $D\pi$ threshold (2276.4 ± 0.9) MeV
- for coupled channels:
parametrizing the T -matrix with the K -matrix formalism

$$T_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s)$$

$I_{ij}(s)$: 2-point loop function evaluated with a subtracted dispersion integral

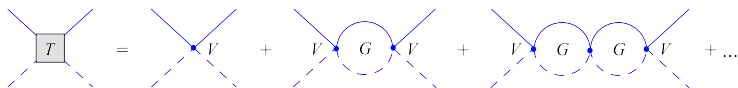
$K_{ij}(s)$: different forms of the K -matrix were used, summarized as

$$K_{ij}(s) = \left(g_i^{(0)} + g_i^{(1)} s \right) \left(g_j^{(0)} + g_j^{(1)} s \right) \frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s$$

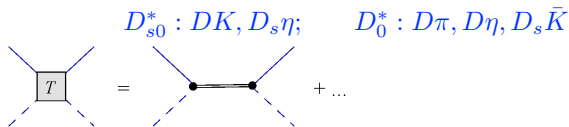
- \Rightarrow a pole below threshold (2275.9 ± 0.9) MeV. relation to $D_0^*(2400)$?

Interactions between charm mesons and light mesons

- S -wave interactions between charm mesons (D, D_s) and light pseudoscalar mesons (π, K, η)



- not far from the thresholds \Rightarrow chiral EFT for matter field
- D_{s0}^*/D_0^* should appear as poles in scattering amplitudes:



\Rightarrow needs a nonperturbative treatment: ChPT + unitarization

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

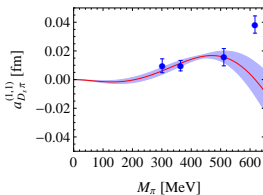
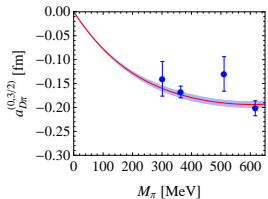
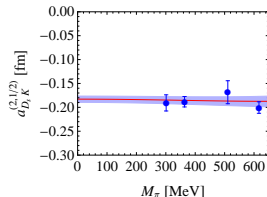
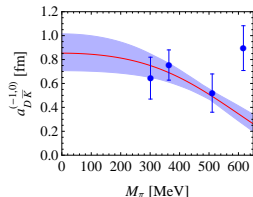
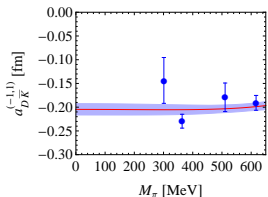
$V(s)$: to be derived from SU(3) chiral Lagrangian, 6 LECs up to NLO

$G(s)$: 2-point scalar loop functions, regularized with a subtraction constant $a(\mu)$

- Fit to lattice data on scattering lengths in 5 simple channels:

$D\bar{K}(I=1, I=0)$, $D_s K$, $D\pi(I=3/2)$, $D_s\pi$: no disconnected contribution

5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$

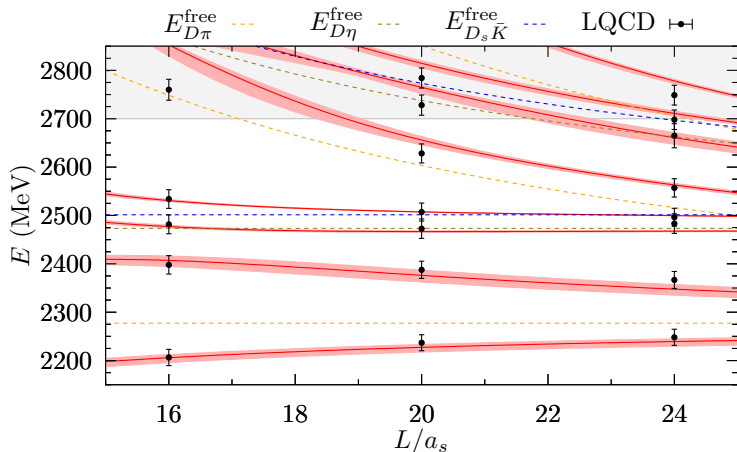


Postdictions versus recent lattice results: charm-nonstrange

- In a finite volume: $\vec{q} = \frac{2\pi}{L}\vec{n}$, $\vec{n} \in \mathbb{Z}^3$; loop integral $G(s)$: $\int d^3\vec{q} \rightarrow \frac{1}{L^3} \sum_{\vec{q}}$
- Postdicted $I = 1/2 D\pi, D\eta, D_s\bar{K}$ finite volume energy levels versus lattice QCD results by [G. Moir *et al.* [Hadron Spectrum Collaboration], JHEP1610(2016)011]

NOT a fit!

M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465



- Heavy-strange

| meson | J^P | prediction (MeV) | PDG2018 (MeV) | lattice (MeV) |
|------------|-------|--------------------|------------------|----------------------|
| D_{s0}^* | 0^+ | 2315_{-28}^{+18} | 2317.7 ± 0.6 | 2348_{-4}^{+7} [1] |

- Heavy-nonstrange, two $I = 1/2$ states ($M, \Gamma/2$):

| | Lower (MeV) | Higher (MeV) | PDG2018 (MeV) |
|---------|--------------------------------------|-------------------------------------|----------------------------------|
| D_0^* | $(2105_{-8}^{+6}, 102_{-11}^{+10})$ | $(2451_{-26}^{+36}, 134_{-8}^{+7})$ | $(2318 \pm 29, 134 \pm 20)$ |
| D_1 | $(2247_{-6}^{+5}, 107_{-10}^{+11})$ | $(2555_{-30}^{+47}, 203_{-9}^{+8})$ | $(2427 \pm 40, 192_{-55}^{+65})$ |
| B_0^* | $(5535_{-11}^{+9}, 113_{-17}^{+15})$ | $(5852_{-19}^{+16}, 36 \pm 5)$ | — |
| B_1 | $(5584_{-11}^{+9}, 119_{-17}^{+14})$ | $(5912_{-18}^{+15}, 42_{-4}^{+5})$ | — |

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

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| B_{s0}^* | 0^+ | 5720_{-23}^{+16} | — | $5711 \pm 23[2]$ |
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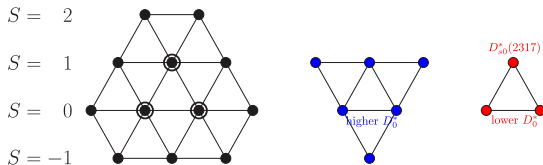
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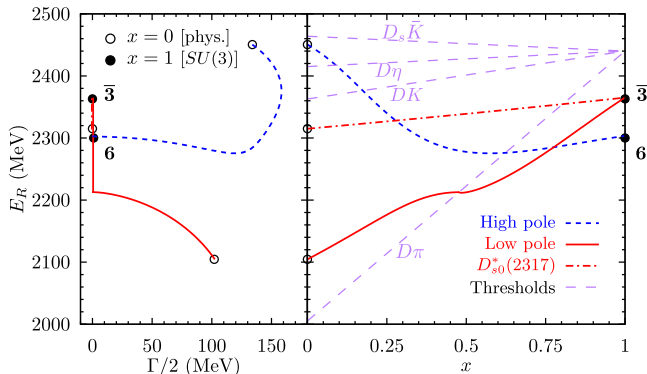
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SU(3) analysis

- In the SU(3) limit, irreps: $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$



- Evolution of the two poles from the physical to the SU(3) symmetric case



DK component from lattice QCD

- Compositeness $(1 - Z)$ related to the S -wave scattering length: Weinberg (1965)

$$a \simeq -2 \frac{1 - Z}{2 - Z} \frac{1}{\sqrt{2\mu E_B}}$$

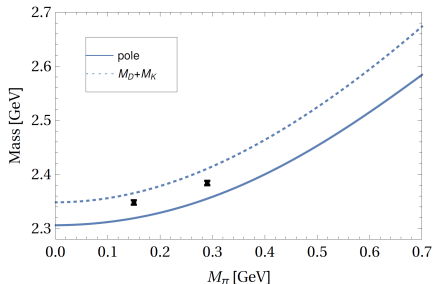
- From the lattice energy levels in C. Lang et al., PRD90(2014)034510
 $D_{s0}^*(2317)$ contains $\sim 70\%$ DK Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,053
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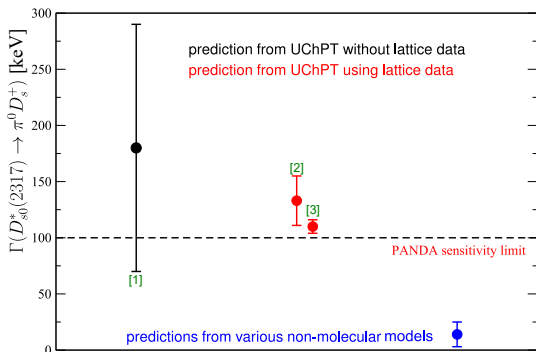
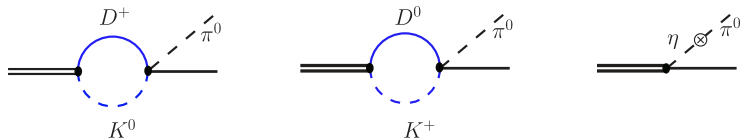
$$1 - Z = 1.04(0.08)(+0.30)$$

| | | |
|----------------------------|--------------|--------------|
| M_π [MeV] | 150 | 290 |
| $M_{D_{s0}^*(2317)}$ [MeV] | 2348 ± 4 | 2384 ± 3 |
| M_{D_s} [MeV] | 1977 ± 1 | 1980 ± 1 |

strong M_π dependence!

curves: prediction in Du et al., EPJC77(2017)728

Decay width of $D_{s0}^*(2317)$: smoking gun

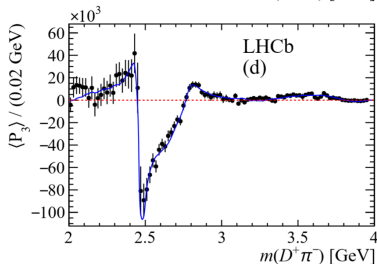
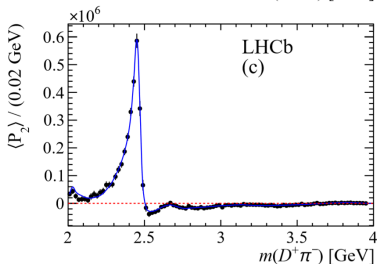
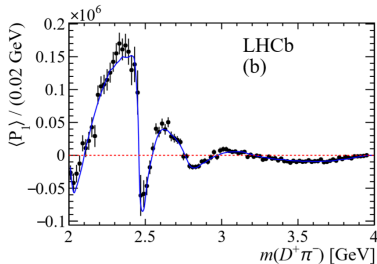
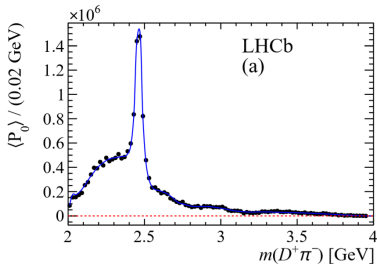


measurement planned at PANDA

[1] FKG et al., PLB666(2008)251; [2] L. Liu et al., PRD87(2013)014508; [3] X. Guo et al., PRD98(2018)014510

non-molecular: e.g., P. Colangelo and F. De Fazio, PLB570(2003)180

Angular moments: $\langle P_L \rangle \propto \int_{-1}^{+1} d \cos \theta P_L(\cos \theta) \frac{d\Gamma}{dm_{D\pi} d \cos \theta}$

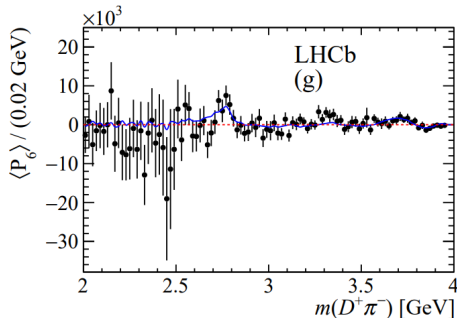


- Consider only S, P, D waves, up to around 2.5 GeV:

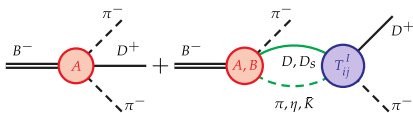
$$\mathcal{A}(B^- \rightarrow D^+ \pi^- \pi^-) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z);$$

higher partial waves negligible:

$$\langle P_6 \rangle \propto |\mathcal{A}_3|^2$$



- P -wave: D^* , $D^*(2680)$ [$M = 2681$ MeV, $\Gamma = 187$ MeV]; D -wave: $D_2(2460)$ parametrized (with the same masses and widths) as in the LHCb paper: Breit–Wigner with Blatt–Weisskopf barrier factors, one constant phase for each as free parameters



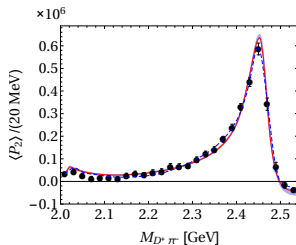
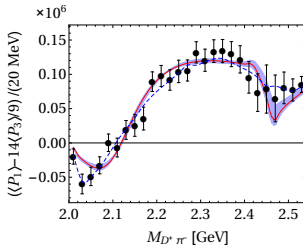
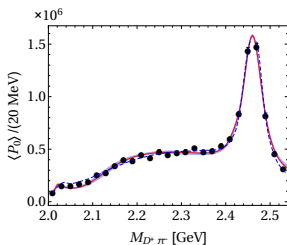
- *S*-wave: FSI, two new parameters
- *P*, *D*-wave: BWs from the LHCb fit

Angular moments:

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2, \quad \langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0),$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$

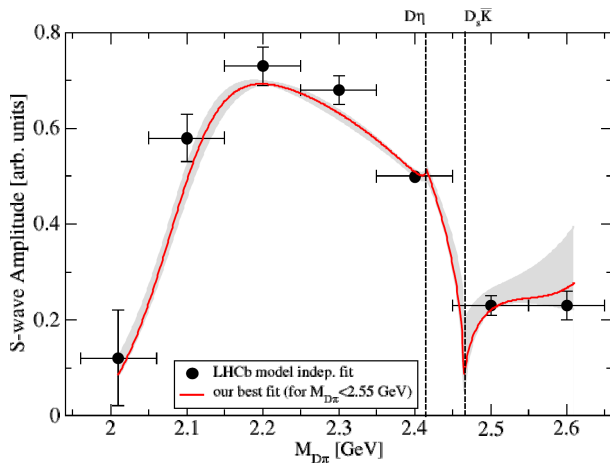
Data: LHCb, PRD94(2016)072001



- **Fast variation** in [2.4, 2.5] GeV in $\langle P_{13} \rangle$: cusps at $D\eta$ and $D_s \bar{K}$ thresholds

$$B^- \rightarrow D^+ \pi^- \pi^- \quad (4)$$

$D\pi$ S-wave amplitude: comparison with the LHCb determination



Coupled-channel thresholds, effects enhanced by the pole at $(2.45 - i0.13)$ GeV

Thanks for the recent experiment, lattice and EFT developments

⇒ likely resolution to all 3 mysteries of positive-parity charm mesons:

- Q: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ much lighter than quark model predictions for $c\bar{s}$ mesons ?

A: They are dominantly DK and D^*K molecular states, respectively.

- Why $M_{D_{s1}(2460)^\pm} - M_{D_{s0}^*(2317)^\pm} \simeq M_{D^{*\pm}} - M_{D^\pm}$?

A: Consequence of HQSS as dominantly DK and D^*K molecules.

- Q: Why $M_{D_0^*(2400)} \lesssim M_{D_{s0}^*(2317)}$ and $M_{D_1(2300)} \sim M_{D_{s1}(2460)}$?

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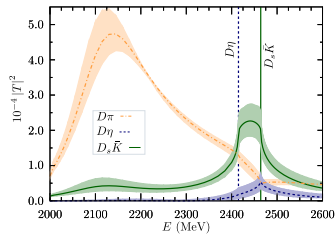
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- Q: Why $M_{D_0^*(2400)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

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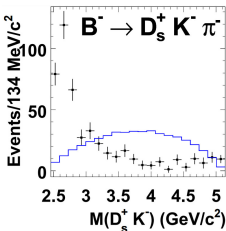
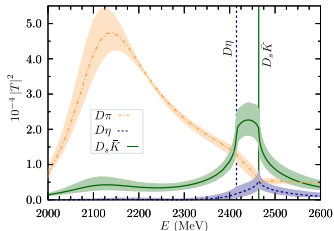
Searching for the higher nonstrange state

- Near-threshold enhancement in $D_s \bar{K}$?

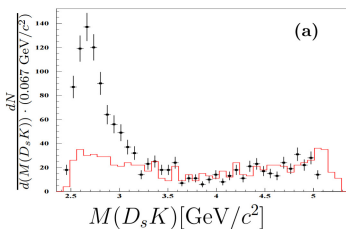


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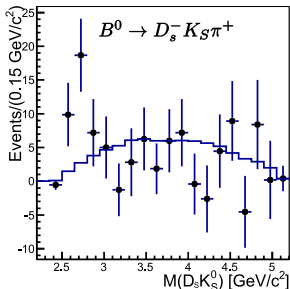
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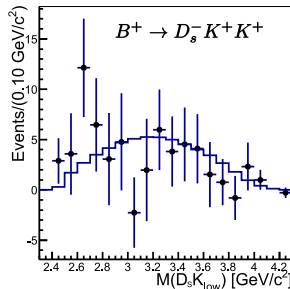
BaBar, PRL100(2008)171803;



Belle, PRD80(2009)052005

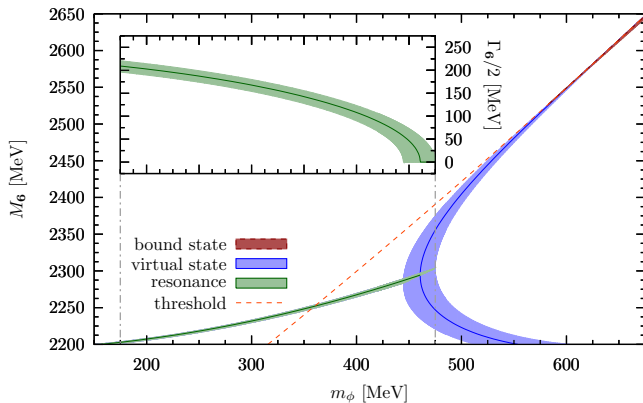


Belle, PRD91(2015)032008



Searching for the higher nonstrange state: lattice

- Lattice QCD calculation with a **SU(3) symmetric large quark mass**:



- Mysteries of positive-parity charmed mesons naturally understood
- There will be further experimental and lattice checks
- π, K, η are pseudo-Goldstone bosons, interactions for other hadrons could be more strong
 - ⇒ importance of *S-wave multi-hadron channels* should be generally expected
 - ⇒ **new paradigm** shifted from quark model (old paradigm)
- **Suggestion for PDG:**
to reconsider the D_0^* parameters

Experiments

Lattice

Thank you for your attention !

EFT, models

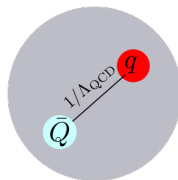
Doubly-charmed baryons

- Heavy anti-quark–diquark symmetry (HADS):

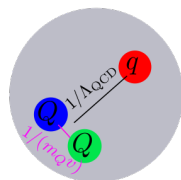
$$m_Q v \gg \Lambda_{\text{QCD}},$$

the diquark serves as a point-like color- $\bar{3}$ source, like a heavy anti-quark.

doubly-heavy baryons \Leftrightarrow anti-heavy mesons



Savage, Wise (1990)



- HADS + CHPT with virtual photons: Brodsky, FKG, Hanhart, Meißner, PLB698(2011)251

$$M_{D^+} - M_{D^0} \Rightarrow M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+} = (1.5 \pm 2.7) \text{ MeV}$$

- LHCb observation of Ξ_{cc}^{++} : $M = (3621.40 \pm 0.78) \text{ MeV}$ LHCb, PRL119(2017)112001

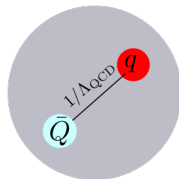
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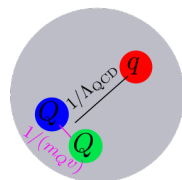
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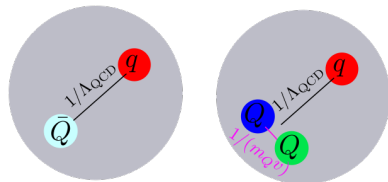
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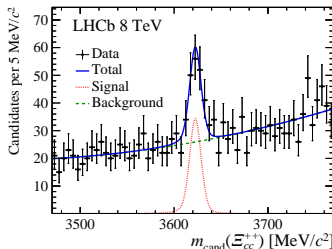
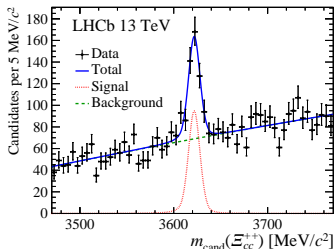


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Doubly-charmed baryons with $J^P = 1/2^-$

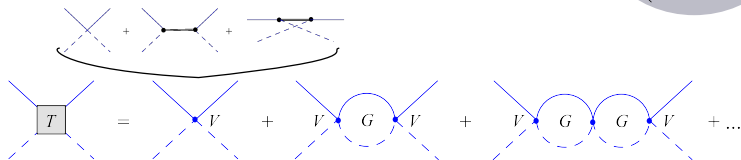
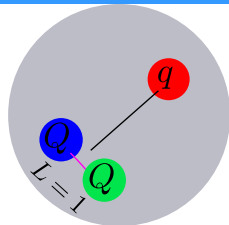
- P -wave QQ

excitation energy

Mehen, PRD96(2017)094028

$$\sim \frac{1}{2}(M_{hc} - M_{J/\psi}) = \mathcal{O}(200 \text{ MeV})$$

- $\Rightarrow \Xi_{cc}^P, \Omega_{cc}^P$ as dynamical degrees of freedom



- S -wave QQ : spin $s_{QQ} = 1$, P -wave QQ : spin $s_{QQ} = 0$

$$\Xi_{cc}^P \lambda = \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right)$$

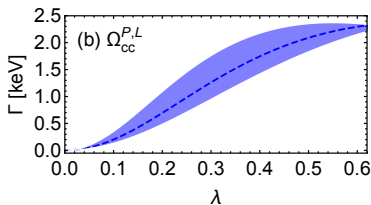
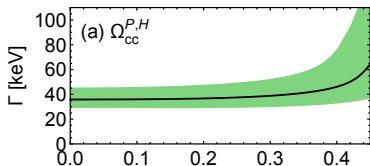
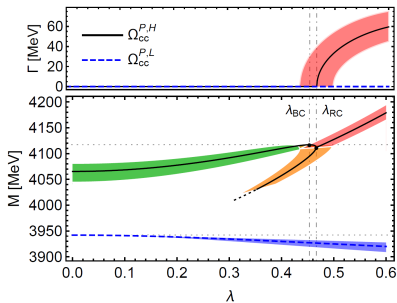
Doubly-charmed strange baryons with $J^P = 1/2^-$: Ω_{cc}^P

M.-J. Yan, X.-H. Liu et al., PRD98(2018)091502(R)

- Very likely **two states** below the $\Xi_{cc}\bar{K}$ threshold

Inputs: bare $\dot{M}_{\Xi_{cc}^P} = 3838$ MeV from quark model [D. Ebert et al., PRD96\(2002\)024008](#)

$$M_{\Omega_{cc}} - M_{\Xi_{cc}} = M_{D_s} - M_D, \quad \dot{M}_{\Omega_{cc}^P} - M_{\Omega_{cc}} = 217 \text{ MeV}$$



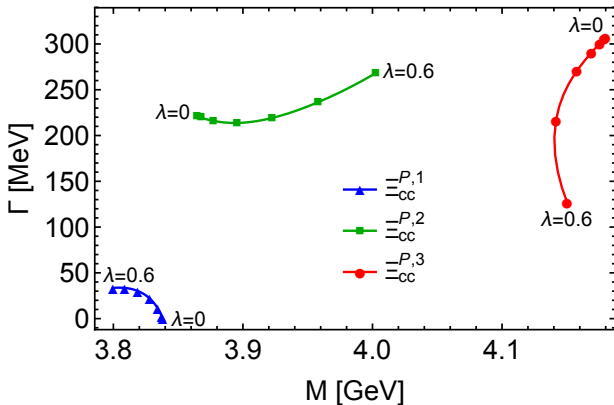
- Tiny widths due to **isospin breaking**:

$$\Omega_{cc}^P \rightarrow \Omega_{cc}\pi^0$$

Doubly-charmed nonstrange baryons with $J^P = 1/2^-$: Ξ_{cc}^P

M.-J. Yan, X.-H. Liu et al., PRD98(2018)091502(R)

- Three $\frac{1}{2}^-$ Ξ_{cc}^P states below 4.2 GeV:



- $\mathcal{B}(\Xi_{cc}^{P,1}, \Xi_{cc}^{P,2} \rightarrow \Xi_{cc}\pi) \simeq 100\%$, searching for $\Xi_{cc}^{P,1}$ in $\Xi_{cc}^{++}\pi^- \Rightarrow \lambda$

HQS for D_{s0}^* (2317) and D_{s1} (2460)

- Heavy quark flavor symmetry:

$$\text{for a singly-heavy hadron, } M_{HQ} = m_Q + A + \mathcal{O}(m_Q^{-1})$$

- rough estimates of bottom analogues **whatever the D_{sJ} states are**

$$M_{B_{s0}^*} = M_{D_{s0}^*(2317)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.65 \pm 0.15) \text{ GeV}$$

$$M_{B_{s1}} = M_{D_{s1}(2460)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.79 \pm 0.15) \text{ GeV}$$

here $\Delta_{b-c} \equiv m_b - m_c \simeq \overline{M}_{B_s} - \overline{M}_{D_s} \simeq 3.33 \text{ GeV}$, where

$\overline{M}_{B_s} = 5.403 \text{ GeV}$, $\overline{M}_{D_s} = 2.076 \text{ GeV}$: spin-averaged g.s. $Q\bar{s}$ meson masses

 both to be discovered ¹

- more precise predictions can be made in a given model, e.g. **hadronic molecules**

¹The established meson $B_{s1}(5830)$ is probably the bottom partner of $D_{s1}(2536)$.

- Heavy quark flavor symmetry (HQFS) for any hadron containing **one** heavy quark: velocity remains unchanged in the limit $m_Q \rightarrow \infty$: $\Delta v = \frac{\Delta p}{m_Q} = \frac{\Lambda_{\text{QCD}}}{m_Q}$
 \Rightarrow heavy quark is like a **static** color triplet source, m_Q is irrelevant
- Predicting the bottom-partner masses in 1 minute:

$$M_{B_{s0}^*} \simeq M_B + M_K - 45 \text{ MeV} \simeq 5.730 \text{ GeV}$$

$$M_{B_{s1}} \simeq M_{B^*} + M_K - 45 \text{ MeV} \simeq 5.776 \text{ GeV}$$

nice agreement with lattice results: [Lang, Mohler, Prelovsek, Woloshyn, PLB750\(2015\)17](#)

$$M_{B_{s0}^*}^{\text{lat.}} = (5.711 \pm 0.013 \pm 0.019) \text{ GeV}$$

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Energy levels in a finite volume

- **Goal:** predict **finite volume** (FV) energy levels for $I = 1/2$, and compare with recent lattice data by the Hadron Spectrum Col. in **JHEP1610(2016)011**
 \Rightarrow insights into $D_0^*(2400)$
- In a FV, momentum gets quantized: $\vec{q} = \frac{2\pi}{L}\vec{n}$, $\vec{n} \in \mathbb{Z}^3$
- Loop integral $G(s)$ gets modified: $\int d^3\vec{q} \rightarrow \frac{1}{L^3} \sum_{\vec{q}}$, and one gets

M. Döring, U.-G. Meißner, E. Oset, A. Rusetsky, EPJA47(2011)139

$$\tilde{G}(s, L) = G(s) + \lim_{\Lambda \rightarrow +\infty} \underbrace{\left[\frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}| < \Lambda} I(\vec{q}) - \int_0^\Lambda \frac{q^2 dq}{2\pi^2} I(\vec{q}) \right]}_{\text{finite volume effect}}$$

$I(\vec{q})$: loop integrand

- FV energy levels obtained by as poles of $\tilde{T}(s, L)$:

$$\tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{G}(s, L)$$

- The leading order Lagrangian:

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with $P = (D^0, D^+, D_s^+)$ denoting the D -mesons, and the covariant derivative being

$$D_\mu P = \partial_\mu P + P \Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu) P^\dagger,$$
$$\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger),$$

where $u_\mu = i [u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger]$, $u = e^{i\lambda_a \phi_a / (2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the [Weinberg–Tomozawa term](#) for $P\phi$ scattering

- At the next-to-leading order $\mathcal{O}(p^2)$: FKG, Hanhart, Krewald, Meißner, PLB666(2008)251

$$\mathcal{L}_{\phi P}^{(2)} = P [-h_0 \langle \chi_+ \rangle - h_1 \chi_+ + h_2 \langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu] P^\dagger \\ + D_\mu P [h_4 \langle u_\mu u^\nu \rangle - h_5 \{u^\mu, u^\nu\}] D_\nu P^\dagger,$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 \text{diag}(m_u, m_d, m_s)$$

- LECs: $h_{1,3,5} = \mathcal{O}(N_c^0)$, $h_{2,4,6} = \mathcal{O}(N_c^{-1})$

$$M_{D_s} - M_D \Rightarrow h_1 = 0.42$$

h_0 : can be fixed from lattice results of charmed meson masses

$h_{2,3,4,5}$: to be fixed from lattice results on scattering lengths

- Extensions to $\mathcal{O}(p^3)$, see Y.-R. Liu, X. Liu, S.-L. Zhu, PRD79(2009)094026; L.-S. Geng et al., PRD82(2010)054022; D.-L. Yao, M.-L. Du, FKG, U.-G. Meißner, JHEP1511(2015)058;

M.-L. Du, FKG, U.-G. Meißner, D.-L. Yao, EPJC77(2017)728

renormalization:

M.-L. Du, FKG, U.-G. Meißner, JPG44(2017)014001

PCB-term subtraction in EOMS scheme using path integral:

M.-L. Du, FKG, U.-G. Meißner, JHEP1610(2016)122

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M.-L. Du, FKG, U.-G. Meißner, D.-L. Yao, EPJC**77**(2017)728

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M.-L. Du, FKG, U.-G. Meißner, JPG**44**(2017)014001

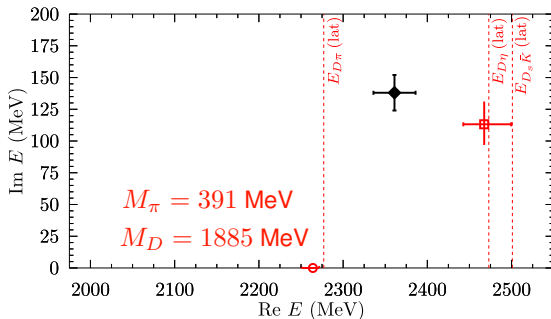
PCB-term subtraction in EOMS scheme using path integral:

M.-L. Du, FKG, U.-G. Meißner, JHEP**1610**(2016)122

Two $I = 1/2$ states !

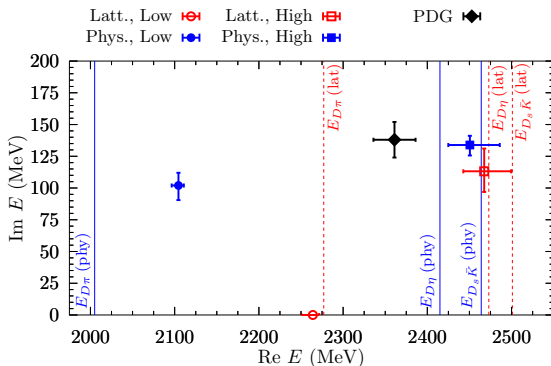
| Masses | M (MeV) | $\Gamma/2$ (MeV) | RS | $ g_{D\pi} $ | $ g_{D\eta} $ | $ g_{D_s \bar{K}} $ |
|---------|--------------------|-------------------|-------|---------------------|---------------------|----------------------|
| lattice | 2264^{+8}_{-14} | 0 | (000) | $7.7^{+1.2}_{-1.1}$ | $0.3^{+0.5}_{-0.3}$ | $4.2^{+1.1}_{-1.0}$ |
| | 2468^{+32}_{-25} | 113^{+18}_{-16} | (110) | $5.2^{+0.6}_{-0.4}$ | $6.7^{+0.6}_{-0.4}$ | $13.2^{+0.6}_{-0.5}$ |

Latt., Low \circ Latt., High \square PDG \blacklozenge



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| physical | 2105^{+6}_{-8} | 102^{+10}_{-11} | (100) | $9.4^{+0.2}_{-0.2}$ | $1.8^{+0.7}_{-0.7}$ | $4.4^{+0.5}_{-0.5}$ |
| | 2451^{+36}_{-26} | 134^{+7}_{-8} | (110) | $5.0^{+0.7}_{-0.4}$ | $6.3^{+0.8}_{-0.5}$ | $12.8^{+0.8}_{-0.6}$ |



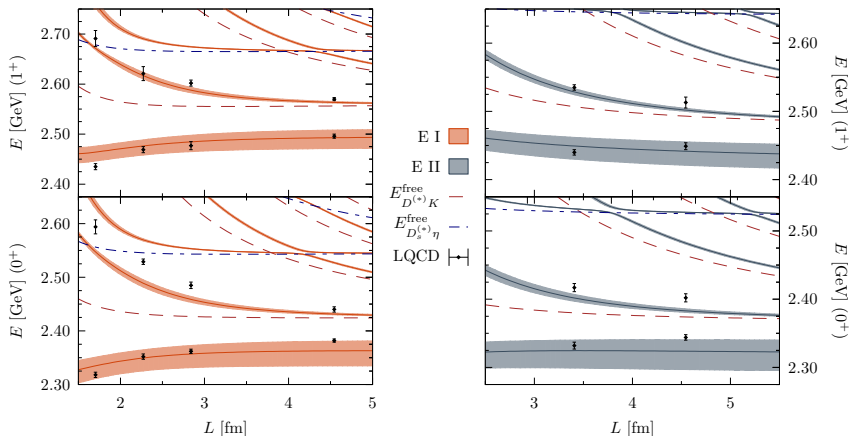
Postdictions versus recent lattice results: charm-strange

- **Postdicted** finite volume energy levels for $(S, I) = (1, 0) D^{(*)}K, J^P = 1^+ \text{ \& } 0^+$ versus lattice QCD results by [G. Bali, S. Collins, A. Cox, A. Schäfer, PRD96(2017)074501]

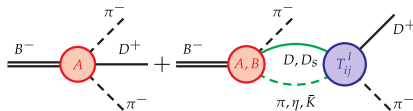
M. Albaladejo, P. Fernandez-Soler, J. Nieves, P. G. Ortega, arXiv:1805.07104

E I: $M_\pi = 290 \text{ MeV}$

E II: $M_\pi = 150 \text{ MeV}$



- $B^- \rightarrow D^+ \pi^- \pi^-$ contains **coupled-channel** $D\pi$ FSI
- consider S, P, D waves: $\mathcal{A}(B^- \rightarrow D^+ \pi^- \pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$
 - P -wave: $D^*, D^*(2680)$; D -wave: $D_2(2460)$ as in the LHCb paper
 - S -wave: use the coupled-channel (1: $D\pi$; 2: $D\eta$; 3: $D_s \bar{K}$) amplitudes with **all parameters fixed before**

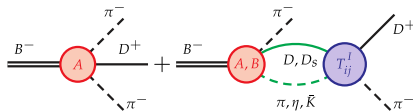


only 2 parameters in S -wave: C and a subtraction constant in $G_i(s)$

$$\text{SU}(3)+\text{chiral} \Rightarrow \mathcal{A}_0(s) \propto E_\pi \left[2 + G_{D\pi}(s) \left(\frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T^{3/2}(s) \right) \right] \\ + \frac{1}{3} E_\eta G_{D\eta}(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_{D_s \bar{K}}(s) T_{31}^{1/2}(s) \\ + C E_\eta G_{D\eta}(s) T_{21}^{1/2},$$

$$\text{Im } G_i(s) = -\rho_i(s) \Rightarrow \text{Unitarity: } \text{Im } \mathcal{A}_{0,i}(s) = -\sum_j T_{ij}^*(s) \rho_j(s) \mathcal{A}_{0,j}(s)$$

- $B^- \rightarrow D^+ \pi^- \pi^-$ contains **coupled-channel** $D\pi$ FSI
- consider S, P, D waves: $\mathcal{A}(B^- \rightarrow D^+ \pi^- \pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$
 - P -wave: $D^*, D^*(2680)$; D -wave: $D_2(2460)$ as in the LHCb paper
 - S -wave: use the coupled-channel (1: $D\pi$; 2: $D\eta$; 3: $D_s \bar{K}$) amplitudes with **all parameters fixed before**



- only 2 parameters in S -wave:** C and a subtraction constant in $G_i(s)$

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- Chiral symmetry \Rightarrow **universal Weinberg–Tomozawa term**
applicable to any hadrons with **a small width $\Gamma \ll$ inverse of force range**
- nice candidates: $D_1(2420)$ & $D_2(2460)$, $\Gamma \sim 30$ MeV
more speculative (using the same subtraction constant) predictions of
 $D_1(2420)K$ and $D_2(2460)K$ bound states

| Constituents | $D_1(2420)K$ | $D_2(2460)K$ | $\bar{B}_1(5720)K$ | $\bar{B}_2(5747)K$ |
|--------------|---------------------------|-------------------|---------------------------------------|-------------------------------|
| J^P | 1^- | 2^- | 1^- | 2^- |
| Predictions | 2870 ± 9 | 2910 ± 9 | 6151 ± 33 | 6169 ± 33 |
| Decays | $D^{(*)}K, D_s^{(*)}\eta$ | $D^*K, D_s^*\eta$ | $\bar{B}^{(*)}K, \bar{B}_s^{(*)}\eta$ | $\bar{B}^*K, \bar{B}_s^*\eta$ |

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What is $D_s^*(2860)$?

- $D_{s1}^*(2860)$: puzzling decay pattern: $\Gamma(D^*K)/\Gamma(DK) = 1.10 \pm 0.24$

Predictions from HQSS:

P.Colangelo et al., PRD77(2008)014012

| $D_{sJ}(2860)$ | $D_{sJ}(2860) \rightarrow DK$ | $\frac{\Gamma(D_{sJ} \rightarrow D^*K)}{\Gamma(D_{sJ} \rightarrow DK)}$ |
|--|-------------------------------|---|
| $s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 1$ | p -wave | 1.23 |
| $s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 1$ | s -wave | 0 |
| $s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 1$ | d -wave | 0.63 |
| $s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 0$ | p -wave | 0.06 |
| $s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 0$ | f -wave | 0.39 |

but, better candidate for $(2S, 1^-)$: $D_{s1}^*(2700)$ $\Gamma(D^*K)/\Gamma(DK) = 0.91 \pm 0.18$

$M(2P, 2^+) \sim 3.16$ GeV

M. Di Piero, E. Eichten, PRD64(2001)114004

- A natural explanation of the decay pattern:

$$: \frac{\Gamma(D_{s1}^*(2860) \rightarrow D^*K)}{DK} \simeq 2 \frac{M_{D^*}}{M_D} \left| \frac{\vec{k}_{D^*}}{\vec{k}_D} \right|^3 = 1.23$$

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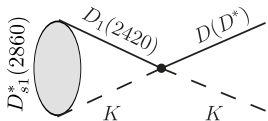
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