High Spin Gravity

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Based on many works on HS AdS gravity
and the works with Jiang Long and Jun-bao Wu, 1106.5141,
with Jiang Long, 1110.5113
with jiang Long and Jian-dong Zhang, to appear soon

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Outline

- Brief review of HS field theory
- Recent developments in HS AdS gravity
- TMG in AdS_3 and chiral gravity;
- Spin-3 TMG
- Higher spin TMG: general case;
- Conclusion and discussions

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- Interacting HS field theory only well-defined in a spacetime with cosmological constant, positive or negative, to have a gauge invariant theory;Vasiliev

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- Though it looks intractable, HS field theory has drawn much attention in the past decade, for its close relation with string theory and AdS/CFT correspondence;

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- However, the degrees of freedom on both sides in $\text{AdS}_5/\text{CFT}_4$ do not match;

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- This provides new examples of AdS/CFT correspondence;

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- However, there could be boundary d.o.f. $==>$ BTZ black hole;

AdS_3 gravity as CS theory

For pure AdS_3 gravity, it could be written as a Chern-Simons theory: Achucarro and Townsend 1986;E. Witten 1988

1. Combine the dreibein and the spin connection into two SL(2,R) gauge potentials:

$$A = (\omega_\mu^a + \frac{1}{l} e_\mu^a) J_a dx^\mu, \qquad \tilde{A} = (\omega_\mu^a - \frac{1}{l} e_\mu^a) J_a dx^\mu.$$

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2. Einstein action + C.C. term

$$S_{EH} + S_\Lambda = S_{CS}[A] - S_{CS}[\tilde{A}] \tag{2.1}$$

where with $k = \frac{l}{4G}$

$$S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A); \tag{2.2}$$

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3. In CS formulation, the asymptotic symmetry of AdS_3 gravity could be analyzed as well, leading to the same conclusion as Brown-Henneaux (1986).

Spin-3 AdS$_3$ gravity Campoleoni et.al. 1008.4744

- To account for spin-3 field, SL(2,R) $-->$ SL(3,R);
- The $SL(3,R)$ group has the generators $J_a, T_{ab}(a,b=1,2,3)$ with T_{ab} being symmetric and traceless;
- They satisfy the following commutation relations:

$$[J_a, J_b] = \epsilon_{abc}J^c, \qquad [J_a, T_{bc}] = \epsilon^d{}_{a(b}T_{c)d},$$

$$[T_{ab}, T_{cd}] = \sigma(\eta_{a(c}\epsilon_{d)be} + \eta_{b(c}\epsilon_{d)ae})J^e.$$

- We combine the vielbein-like fields and the connections of spin-2 and spin-3 into two gauge potentials $A, \tilde{A}$

$$\begin{aligned} A &= ((\omega^a_\mu + \frac{1}{l}e^a_\mu)J_a + (\omega^{ab}_\mu + \frac{1}{l}e^{ab}_\mu)T_{ab})dx^\mu, \\ \tilde{A} &= ((\omega^a_\mu - \frac{1}{l}e^a_\mu)J_a + (\omega^{ab}_\mu - \frac{1}{l}e^{ab}_\mu)T_{ab})dx^\mu; \end{aligned}$$

- Here e^{ab}_μ is the frame-like field for spin-3 field, and ω^{ab}_μ is corresponding spin-connection.

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- Very recently, the similar analysis was applied to super-HS;S.J.Rey et.al.

HS/CFT correspondence

- 3D Vasiliev theory based on high spin algebra $hs[\lambda]$ coupled to two additional complex scalars is dual to the large N 't Hooft limit of the $\mathcal{W}_N$ coset CFT $SU(N)_k \otimes SU(N)_1 / SU(N)_{k+1}$;Gaberdiel and Gopakumar, 1011.2986

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- The t' Hooft limit: $N, k \to \infty$ while keeping $\lambda = N/(k+N)$ fixed;
- The support to this conjecture: partition function, 3-pt correlation functions;Gaberdiel et.al., Xi Yin et.al. ...

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- Nevertheless, one may use gauge-invariant quantity: holonomy to characterize the classical solutions;

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- Conjecture: chiral gravity is holographically dual to a 2D chiral CFT by imposing self-consistent Brown-Henneaux B.C.;

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- It turns out that these modes are legal and with suitable B.C., the theory corresponds to another CFT, possibly the log CFT;
- However in this case the chiral nature gets lost;
- In TMG, there exist other vacua: warped AdS_3 spacetimes, leading to warped AdS/CFT correspondence;

Motivation

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- Some kind of generalization of chiral gravity conjecture?
- In our work, we mainly tried to answer the first two questions.

1st order formulation of TMG

In first order formalism, TMG with a negative cosmological constant $\Lambda = -l^{-2}$ is described by the action Deser et.al.(1991),S.Carlip (1991)

$$S_{\text{TMG}} = \frac{1}{8\pi G}\int \left(e^a \wedge R_a + \frac{1}{6l^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c\right) - \frac{1}{16\pi G\mu}\int \left(\mathcal{L}_{\text{CS}} + \beta^a \wedge T_a\right),$$

where

$$\mathcal{L}_{\text{CS}} = \omega^a \wedge d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a \wedge \omega^b \wedge \omega^c. \quad (4.1)$$

The field β^a is just a Lagrangian multiplier, imposing the torsion free condition such that the above action is equivalent to the action in terms of Christoffel symbol. It would be illuminating to rewrite the above action in a form relating to Chern-Simons gravity with gauge group $SL(2,R) \times SL(2,R)$:

$$S_{\text{TMG}} = \left(1 - \frac{1}{\mu l}\right) S_{CS}[A] - \left(1 + \frac{1}{\mu l}\right) S_{CS}[\tilde{A}] - \frac{k}{4\pi\mu l}\int \left(\tilde{\beta}^a \wedge T_a\right).$$

Action of spin-3 TMG BC, J. Long and J.b Wu 1106.5141

To study the topological massive gravity coupled to the spin-3 fields, we now propose the following action:

$$S = \left(1 - \frac{1}{\mu l}\right) S_{CS}[A] - \left(1 + \frac{1}{\mu l}\right) S_{CS}[\tilde{A}] - \frac{k}{4\pi\mu l}\int \left(\tilde{\beta}^a \wedge T_a - 2\sigma\tilde{\beta}^{ab} \wedge T_{ab}\right). \tag{4.2}$$

Here the last two terms are introduced to impose the torsion free conditions. Now the torsions are defined as

$$T^a = de^a + \epsilon^{abc}\omega_b \wedge e_c - 4\sigma\epsilon^{abc}e_{bd} \wedge \omega_c{}^d, \tag{4.3}$$

$$T^{ab} = de^{ab} + \epsilon^{cd(a|}\omega_c \wedge e_d{}^{|b)} + \epsilon^{cd(a|}e_c \wedge \omega_d{}^{|b)}. \tag{4.4}$$

Note that the torsion T^a for veilbein gets modified by the spin-3 field and the torsion T^{ab} is for spin-3 field.

Action in terms of frame-like field

In terms of the frame-like field and connection, the action (4.2) could be written in a more familiar form:

$$\begin{aligned} S &= \frac{1}{8\pi G}\int \Big(e^a\wedge d\omega_a+\frac{1}{2}\epsilon_{abc}e^a\wedge\omega^b\wedge\omega^c+\frac{1}{6l^2}\epsilon_{abc}e^a\wedge e^b\wedge e^c \\ &\quad -2\sigma e^{ab}\wedge d\omega_{ab}-2\sigma\epsilon_{abc}e^a\wedge\omega^{bd}\wedge\omega^c{}_d-2\sigma e^{ab}\wedge\epsilon_{(a|cd}\omega^c\wedge\omega_{|b)}{}^d \\ &\quad -\frac{2\sigma}{l^2}\epsilon_{abc}e^a\wedge e^{bd}\wedge e^c{}_d\Big)-\frac{1}{16\pi G\mu}\int\Big(\omega^a\wedge d\omega_a+\frac{1}{3}\epsilon_{abc}\omega^a\wedge\omega^b\wedge\omega^c \\ &\quad -2\sigma\omega^{ab}\wedge d\omega_{ab}-4\sigma\epsilon_{abc}\omega^a\wedge\omega^{bd}\wedge\omega^c{}_d+\beta^a\wedge T_a-2\sigma\beta^{ab}\wedge T_{ab}\Big), \end{aligned}$$

where

$$\beta^{ab}=\tilde{\beta}^{ab}+\frac{e^{ab}}{l^2}$$

could be taken as an independent field.

Remarks

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- Massive spin-3 modes: degenerate with massless left-moving modes at the chiral point, both for traceless and trace partBC, J. Long and J.b Wu 1106.5141, A. Bagchi et.al. 1107.0915;
- The theory could be still chiral at the critical point $\mu l = 1$;

High spin TMG: general case BC and J. Long, 1110.5113

We start from the following action

$$S_{TMG} = (1-\frac{1}{\mu l})S_{CS}[A]-(1+\frac{1}{\mu l})S_{CS}[\bar{A}]-\frac{k}{4\pi\mu}\int tr(\beta\wedge(F-\bar{F})). \quad (5.1)$$

where we have the gauge curvature

$$F = dA + A\wedge A, \qquad \bar{F} = d\bar{A} + \bar{A}\wedge\bar{A}. \quad (5.2)$$

and one-form Lagrangian multiplier β. The gauge field A, $\bar{A}$ and the Lagrangian multiplier β are in the adjoint representation of the corresponding group, which is chosen to be $SL(n,R)\times SL(n,R)$ to describe the high spin fields from spin 2 to n.

Remarks

- when $\beta = 0$ and $\mu \to \infty$, it reduces to the action of the well-known high spin AdS_3 gravity which describes a tower of higher spin fields from spin 3 to spin n coupled to gravity;

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- The action describes all the high spin fields coupled to topological massive gravity: there are not only topologically Chern-Simons term for graviton, but also the similar parity-breaking Chern-Simons terms for higher spin fields.
- The equations of motion in a concise form:

$$(1 - \frac{1}{\mu})F - \frac{1}{2\mu}(d\beta + \beta \wedge A + A \wedge \beta) = 0 \tag{5.3}$$

$$(1 + \frac{1}{\mu})\bar{F} - \frac{1}{2\mu}(d\beta + \beta \wedge \bar{A} + \bar{A} \wedge \beta) = 0 \tag{5.4}$$

$$F = \bar{F}. \tag{5.5}$$

Symmetry

- $\beta = 0$ case:

$$\delta_\Lambda A = d\Lambda + [A, \Lambda], \qquad \delta_\Lambda \bar{A} = d\Lambda + [\bar{A}, \Lambda]$$

$$\delta_{\tilde{\Lambda}} A = d\tilde{\Lambda} + [A, \tilde{\Lambda}], \qquad \delta_{\tilde{\Lambda}} \bar{A} = -d\tilde{\Lambda} - [\bar{A}, \tilde{\Lambda}].$$

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- However, the gauge symmetry is preserved on-shell;
- More importantly, in the AdS$_3$ vacuum, the gauge symmetry is recovered at linearized level;
- The re-emergence of gauge symmetry allows us to study the fluctuations of various spin systematically.

Physical fluctuations

- Spin-s gauge fields:

$$h_{\nu,\nu_1\cdots\nu_{s-1}} = \bar{e}_{\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}} e_{\nu a_1\cdots a_{s-1}} \tag{5.6}$$

- After a Lorentz transformation, it could be changed into a symmetric field:

$$\Phi_{\nu\nu_1\cdots\nu_{s-1}} = \frac{1}{s}\bar{e}_{(\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}} e_{\nu)a_1\cdots a_{s-1}} \tag{5.7}$$

- Φ has a gauge symmetry

$$\delta_\xi\Phi_{\nu\nu_1\cdots\nu_{s-1}} = \nabla_{(\nu}\xi_{\nu_1\cdots\nu_{s-1})}. \tag{5.8}$$

where $\xi_{\nu_1\cdots\nu_{s-1}} = \bar{e}_{\nu_1}^{\ a_1}\cdots\bar{e}_{\nu_{s-1}}^{\ a_{s-1}}\tilde{\Lambda}_{a_1\cdots a_{s-1}}$ is symmetric and traceless;

- More precisely, we have

$$\begin{aligned} h_{\nu,\nu_1\cdots\nu_{s-1}} &= \Phi_{\nu\nu_1\cdots\nu_{s-1}} + \Theta_{\nu,\nu_1\cdots\nu_{s-1}} + \frac{s-2}{2(s-1)}g_{\nu(\nu_1}\Phi'_{\nu_2\cdots\nu_{s-1})} \\ &\quad -\frac{1}{s-1}g_{(\nu_1\nu_2}\Phi'_{|\nu|\nu_3\cdots\nu_{s-1})}. \end{aligned} \tag{5.9}$$

Fluctuations: I

1. In AdS_3 background, the equation of fluctuations

$$\begin{aligned} G(da + a\wedge A + A\wedge a) &= d\beta + \beta\wedge A + A\wedge\beta \\ \bar{G}(d\bar{a} + \bar{a}\wedge\bar{A} + \bar{A}\wedge\bar{a}) &= d\beta + \beta\wedge\bar{A} + \bar{A}\wedge\beta \\ da + a\wedge A + A\wedge a &= d\bar{a} + \bar{a}\wedge\bar{A} + \bar{A}\wedge\bar{a} \end{aligned}$$

   where we have defined $G = 2(\mu - 1), \bar{G} = 2(\mu + 1)$.
2. This linearized equations describe free fluctuations of spin 2 to spin n;
3. When $\beta = 0$, we use the previous decomposition of $h_{\nu\nu_1\cdots\nu_{s-1}}$ and choose a gauge that $\Theta_{\nu\nu_1\cdots\nu_{s-1}} = 0$, then we can derive the Fronsdal equation

$$\begin{aligned} \mathcal{F}_{\nu_1\cdots\nu_s} \equiv \Box\Phi_{\nu_1\cdots\nu_s} - \nabla_{(\nu_1|}\nabla^{\sigma}\Phi_{\sigma|\nu_2\cdots\nu_s)} + \frac{1}{2}\nabla_{(\nu_1}\nabla_{\nu_2}\Phi'_{\nu_3\cdots\nu_s)} \\ -(s^2 - 3s)\Phi_{\nu_1\cdots\nu_s} - 2g_{(\nu_1\nu_2}\Phi'_{\nu_3\cdots\nu_s)} = 0 \end{aligned}$$

Fluctuations: II

1. When $\beta \neq 0$, in principle we will get a third order differential equation for each spin s field;
2. The calculation is quite tedious;
3. For arbitrary spin $s \geq 2$, we finally obtain the equations of the physical fields

$$\mathcal{F}_{a_1 \cdots a_s} + \frac{1}{\mu s(s-1)} \epsilon_{(a_1|}^{\ \ bc} \nabla_b \mathcal{F}_{c|a_2 \cdots a_s)} = 0. \tag{5.10}$$

4. To discuss the equations of motion of the free fluctuations, we only need to know the commutation relation between SL(2,R) generators and high spin generators. This allows us to obtain the equations of motion for arbitrary spin up to n in our formulation.

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- Is the theory chiral at the critical point?

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  1. AdS-pp wave: its high spin cousin has been found by us;
  2. Warped AdS spacetime: we managed to find these solutions in the first order formulation of TMG;

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Thank you!