THE BEAUTIFUL STANDARD MODEL

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USTC, Hefei, May 18, 2007

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.

Jules H. Poincare

If the SM were not beautiful, it would not be worth studying.

Tianjun Li

I. Introduction

The Standard Model (SM) is a model that describes the elementary particles in the nature and the fundamental interactions between them.

Fundamental Interactions

Interactions	Invariant	Symmetry	Fields	Spin
Gravity	Diffeomorphism		Graviton	2
Strong	Gauge	$SU(3)_C$	Gluon	1
Weak	Gauge	$SU(2)_L$	$W^{\pm}, W^0$	1
Hypercharge	Gauge	$U(1)_Y$	B^0	1

Properties for the theories:

Gauge theory is renormalizable, and described by quantum field theory which is consistent with both quantum mechanics and special relativity. However, gravity theory is non-renormalizable, and we do not have a correct quantum gravity theory.

Elementary Particles

- Three families of SM fermions:

$$\textbf{Quarks}: \quad Q_1 = \begin{pmatrix} U & U & U \\ D & D & D \end{pmatrix}_{\mathrm{L}}, \quad (U \;\; U \;\; U)_R\,, \quad (D \;\; D \;\; D)_R\,.$$

$$\textbf{Leptons}: \quad L_1 = \begin{pmatrix} \nu \\ E \end{pmatrix}_{\mathrm{L}}, \quad E_R\,.$$

- One Higgs doublet

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}.$$

$$
\begin{aligned}
\mathcal{L}_{MSM} = & -\frac{1}{2g_s^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2g^2}\mathrm{Tr}W_{\mu\nu}W^{\mu\nu} \\
& -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} + i\frac{\theta}{16\pi^2}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} + M_{Pl}^2 R \\
& +|D_\mu H|^2 + \bar{Q}_i i\not{D}Q_i + \bar{U}_i i\not{D}U_i + \bar{D}_i i\not{D}D_i \\
& +\bar{L}_i i\not{D}L_i + \bar{E}_i i\not{D}E_i - \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2 \\
& -\left(h_u^{ij}Q_i U_j \tilde{H} + h_d^{ij}Q_i D_j H + h_l^{ij}L_i E_j H + c.c.\right) ,
\end{aligned}
$$

where $\tilde{H} \equiv i\sigma_2 H^*$.

The SM has 20 parameters (19 without gravity).

The Higgs potential is

$$V_{\rm Higgs} = \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2 ,$$

At minimum, Higgs field has a non-zero VEV

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}} .$$

All the gauge symmetries, under which H^0 is charged, are broken after Higgs mechanism.

Symmetry Breaking

- $SU(2)_L \times U(1)_Y$ is broken down to the $U(1)_{\rm em}$ symmetry.
- $W^{\pm}$ and Z^0 become massive, and γ is massless

$$Z^0 \;=\; \cos\theta_W W^0 - \sin\theta_W B^0 \;, \quad \gamma \;=\; \sin\theta_W W^0 + \cos\theta_W B^0 \;.$$

- The SM quarks and leptons obtain masses via Yukawa couplings, except the neutrinos.
- Unknown: Higgs boson and its mass.

The SM explains existing experimental data very well, including electroweak precision tests.

The convincing evidence for physics beyond the SM:

- Dark matter
- Dark energy
- Neutrino masses and mixings
- Baryon asymmetry
- Inflation

The SM is incomplete!

Major Problems in the SM

- Fine-Tuning Problems
- Aesthetic Problems

Fine-Tuning Problems:

- Cosmological constant problem

$$\Lambda_{\mathrm{CC}} \sim 10^{-122} M_{\mathrm{Pl}}^4 \, .$$

- Gauge hierarchy problem

$$M_{\mathrm{EW}} \sim 10^{-16} M_{\mathrm{Pl}} \, .$$

- Strong CP probelm

$$\theta < 10^{-9} \, .$$

- The SM fermion masses and mixings

$$m_{\mathrm{electron}} \sim 10^{-5} m_{\mathrm{top}} \, .$$

Aesthetic Problems:

- Interaction unification
- Fermion unification
- Gauge coupling unification
- Charge quantization

The first two prolems can be solved when we embed the SM into the Grand Unified Theories (GUTs) and string models.

II. Major Fine-Tuning Problem

String Theory

- String theory is the only known theory which might correctly describe quantum gravity
- Boson string theory: 26 dimensions
- Superstring theory: 10 dimensions
- The observed world is 4-dimensional
- Calabi-Yau compactifications for extra 6 dimensions
- Preserving the 4-dimensional $N = 1$ supersymmetry

String Landscape

- An enormous "landscape" for long-lived metastable string/M theory vacua due to flux compactifications [a].
- Weak anthropic principle [b].
- The first concrete explanation of the very tiny value of the cosmological constant, which can take only discrete values.
- Solution to gauge hierarchy problem.

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[a] Giddings, Kachru and Polchinski; Kachru, Kallosh, Linde and Trivedi; Susskind; Denef and Douglas.
[b] Weinberg.

Although the tiny cosmological constant and light Higgs mass are not technically natural in QFT, they can indeed be natural in the string landscape if the vacua with tiny cosmological constant and light Higgs mass are populated in the string landscape!

Adjustable scenario:

- The Hartle-Hawking wavefunction strongly favors the smallest positive value of cosmological constant.
- The de Sitter entropy suggests that the Hartle-Hawking wavefunction has some statistical interpretation in terms of the system exploring all possible states.
- The Coleman-de Lucchia amplitude for tunneling from positive to negative cosmological constant vanishes for some parameter range, so the universe would be stuck in the state with the smallest positive energy density.

Strong CP Problem

- $\overline{\theta} = \theta + \theta_{\rm q}$ parameter is a dimensionless coupling constant which is infinitely renormalized by radiative corrections.
- No theoretical reason for $\overline{\theta}$ as small as 10^{-9} required by the experimental bound on the EDM of the neutron.
- $\overline{\theta}$ may be a random variable with a roughly uniform distribution in the string landscape [a].

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[a] Donoghue.

Peccei–Quinn Mechanism

- $\overline{\theta} = \theta + \theta_{\rm q} + a/f_a$

$$V_{\rm Instanton} \simeq \Lambda_{QCD}^4 \left(1 - \cos\overline{\theta}\right) \ .$$

- Weak axion is ruled out by $K \to \pi a$ and $J/\Psi \to a\gamma$ experiments: $f_a \sim 300$ GeV and $m_a \sim 30$ keV

- Invisible DFSZ or KSVZ axions: 10^{10} GeV $< f_a < 10^{12}$ GeV

- Axion can be a cold dark matter candidate

$$f_a \sim 10^{11}{\rm GeV} \ , \ \ m_a \sim 10^{-5}{\rm eV} \ .$$

- The axion solution can be stabilized by the gauged discrete PQ symmetry from the breaking of an anomalous gauged $U(1)$ symmetry in string models [a].

- Universal high-scale supersymmetry breaking.

- Canonical gauge coupling unification can be achieved due to additional vector-like particles.

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[a] Barger, Chiang, Jiang, and TL

Major Prediction in String Landscape

The supersymmetry breaking scale can be high if there exist many supersymmetry breaking parameters or many hidden sectors [a].

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[a] Giryavets, Kachru and Tripathy; Susskind; Douglas; Dine, Gorbatov and Thomas; Arkani-Hamed and Dimopoulos.

Supersymmetry Breaking Scale [a].

- String landscape is based on the Type II orientifolds with flux compactifications.
- The supersymmetry breaking soft masses are universal and roughly M_S^2/M_{Pl}.
- M_S is about 10^{17} GeV, so, $M_{soft} \sim 10^{16}$ GeV.
- Universal GUT-scale supersymmetry breaking.

The SM from weak scale to GUT scale.

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[a] Barger, Chiang, Jiang, and TL

III. Gauge Coupling Unification and Charge Quantization

Charge quantization can easily be realized by embedding the SM into GUTs or string models.

Key problem: gauge coupling unification.

- No gauge coupling unification in the SM
- Implicit assumption: canonical normalization of the $U(1)_Y$ hypercharge interaction

Gauge coupling unification in the SM [a]:

- The gauge couplings for $SU(3)_C$ and $SU(2)_L$ are unified at about 10^{16-17} GeV.
- The gauge coupling for the $U(1)_Y$ depends on its normalization.
- With a suitable normalization of the $U(1)_Y$, the three gauge couplings for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ can in fact be unified at about 10^{16-17} GeV.

Question: *is the canonical normalization for $U(1)_Y$ unique?*

For a 4-dimensional GUT with a simple group, the canonical $U(1)_Y$ normalization is the only possibility, assuming that the SM fermions form complete multiplets under the GUT group.

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[a] V. Barger, J. Jiang, P. Langacker and TL.

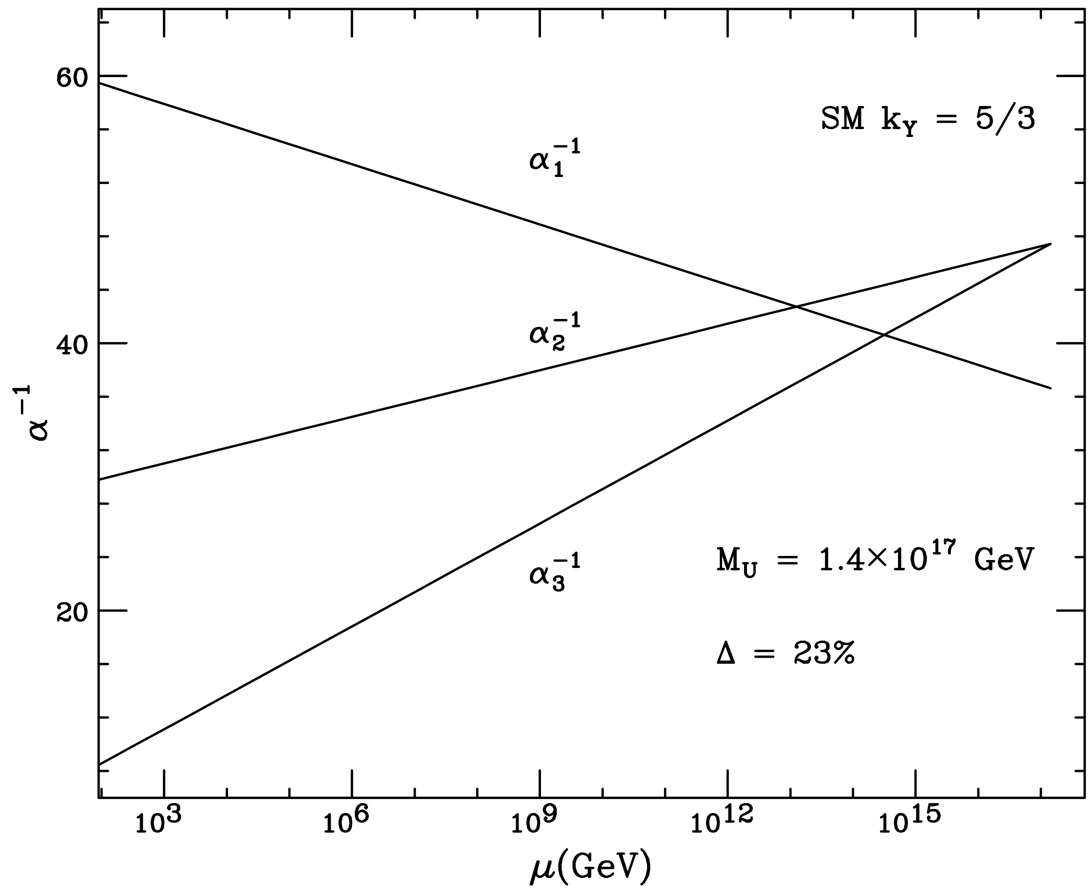

Figure 1: One-loop gauge coupling unification for the SM with $k_Y = 5/3$ where $\alpha_i \equiv g_i^2/4\pi$.

Non-canonical $U(1)_Y$ normalization:

- In weakly coupled heterotic string theory, the gauge and gravitational couplings always automatically unify [a]

$$k_Y g_Y^2 = k_2 g_2^2 = k_3 g_3^2 = 8\pi \frac{G_N}{\alpha'} = g_{\rm string}^2 ~.$$

- In intersecting D-brane model building on Type II orientifolds, the normalization for the $U(1)_Y$ (and also other gauge factors) is not canonical in general [b].

- In orbifold GUTs [c] and their deconstruction [d], and the 4D GUTs with product gauge groups.

We assume $k_2 = k_3 = 1$. And $k_Y = 5/3$ for canonical $U(1)_Y$ normalization.

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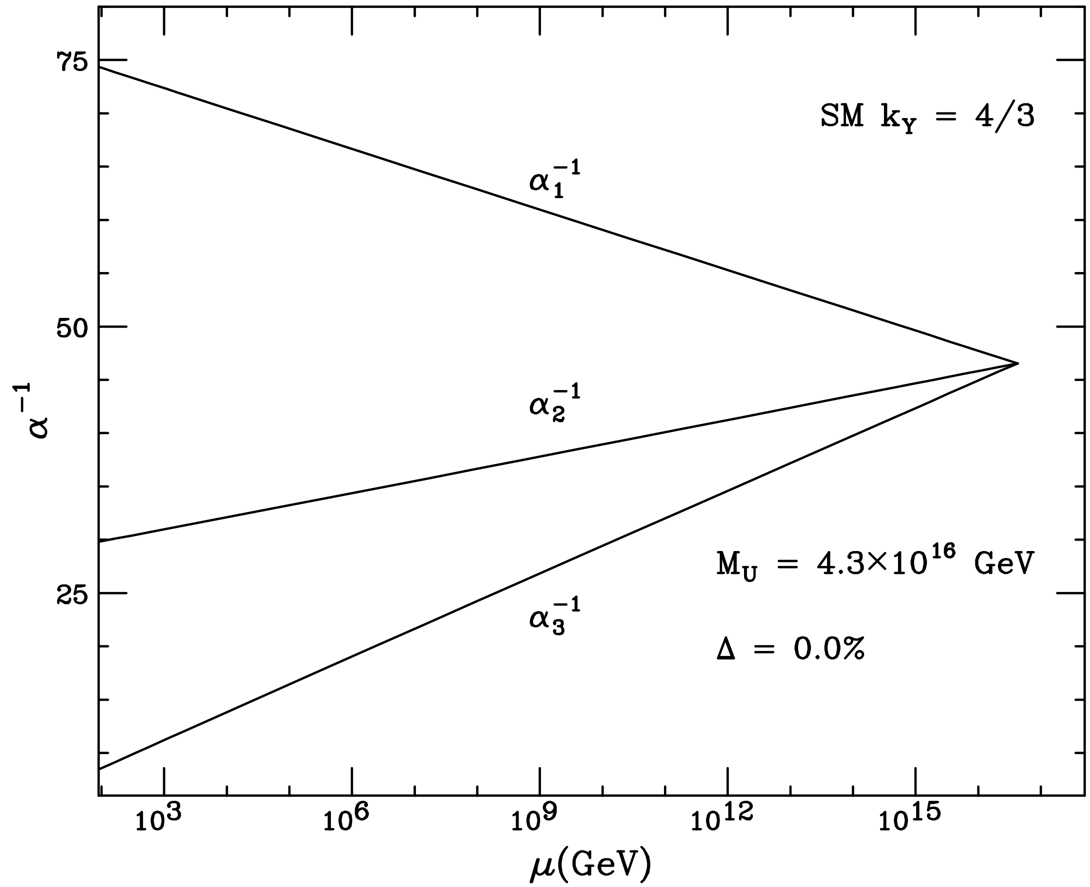
[a] Dienes.

[b] Blumenhagen, Cvetic, Langacker and Shiu.

[c] Kawamura; Altarelli and Feruglio; Hall and Nomura; Hebecker and March-Russell; TL; Asaka, Buchmuller and Covi; Gogoladze, Mimura and Nandi; Babu, Barr and Kyae; Kyae and Shafi.

[d] Arkani-Hamed, Cohen and Georgi; Hill, Pokorski and Wang; Csaki, Kribs and Terning; Cheng, Matchev and Wang; TL and Liu; Huang, Jiang and TL.

Two-loop gauge coupling unification for the SM with $k_Y = 4/3$ [a].



[a] V. Barger, J. Jiang, P. Langacker and TL.

7D $\mathcal{N} = 1$ Supersymmetric $SU(8)$ Models with $k_Y = 4/3$ [a]:

Background:

The $\mathcal{N} = 1$ supersymmetry in 7D has 16 supercharges corresponding to $\mathcal{N} = 4$ supersymmetry in 4D, and only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under 4D $\mathcal{N} = 1$ supersymmetry into a gauge vector multiplet V and three chiral multiplets Σ_1, Σ_2, and Σ_3 in the adjoint representation.

[a] I. Gogoladze, TL, V. N. Senoguz and Q. Shafi.

The space-time is $M^4 \times T^2/Z_6 \times S^1/Z_2$ [a]

Representation for Z_6 and Z_2

$$R_{\Gamma_T} = \mathrm{diag}\left(+1, +1, +1, \omega^{n_1}, \omega^{n_1}, \omega^{n_1}, +1, \omega^{n_2}\right) ,$$

$$R_{\Gamma_S} = \mathrm{diag}\left(+1, +1, +1, +1, +1, -1, -1, +1\right) ,$$

where $n_1 \neq n_2 \neq 0$. We choose $n_1 = 5,\ n_2 = 2$ or 3.

Gauge symmetry breaking

$$SU(8)/R_{\Gamma_T} = SU(4) \times SU(3) \times U(1)^2 ,$$

$$SU(8)/R_{\Gamma_S} = SU(6) \times SU(2) \times U(1) ,$$

$$SU(8)/\{R_{\Gamma_T} \cup R_{\Gamma_S}\} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^3 .$$

[a] I. Gogoladze, TL, Y. Mimura and S. Nandi

Properties [a]

- Gauge interaction unification.
- All the SM fermions in the third family and Higgs fields arise from the chiral multiplets Σ_i: Gauge-Fermion-Higgs Unification.
- Unification of the gauge couplings and Yukawa couplings.
- Charge quantization–Hypercharges are determined from the construction.
- $SU(8)$ is broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^3$ via orbifold projections.
- The supersymmetry can be broken at the GUT scale by the Scherk–Schwarz mechanism.

[a] I. Gogoladze, TL, V. N. Senoguz and Q. Shafi.

$$\mathbf{63} = \begin{pmatrix} 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & H_u & H_d & 0 \\ t^c & L_3 & 0 & 0 & 0 \\ d^c & 0 & \tau^c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} .$$

$\Sigma_1 \supset \{Q_3,\ \tau^c\}$; $\Sigma_2 \supset \{H_u,\ L_3,\ b^c\}$; $\Sigma_3 \supset \{H_d,\ t^c\}$.

Comments: The minimal orbifold GUT model with gauge-fermion-Higgs unification is 6D $\mathcal{N} = (1,1)$ supersymmetric $SU(7)$ model on the space-time $M^4 \times T^2/Z_6$ with $k_Y = 23/21$ [a].

[a] I. Gogoladze, TL, V. N. Senoguz and Q. Shafi.

IV. The Rest Problems and New Physics

Neutrino Masses and Mixings

- $(L\tilde{H})(L\tilde{H})/M_{\rm Pl}$ gives $m_\nu \lesssim 10^{-5}$ eV
- Seesaw mechanism
- Type I, Type II, and Type III
- Minimal Type I seesaw: two right-handed neutrinos

See-Saw Mechanism:

$$-\mathcal{L}_{\rm neutrino} = h_\nu L N \tilde{H} + m_N N N \ .$$

$$M_\nu \ = \ \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \ .$$

If $m_N >> m_D$**, two eigenvalues:** m_D^2/m_N**,** m_N

$$m_N \sim 10^{11-14}\ {\rm GeV} \ .$$

Baryon Asymmetry in the SM:

- Electroweak baryogenesis
- Leptogenesis

Electroweak baryogenesis does not work

- No strong first order phase transition for $m_{H^0} > 114$ GeV.
- The CP violation in CKM matrix is too small.

Leptogenesis:

- N_i decays generate net lepton numbers due to CP violation
- Sphaleron process preserve the $B - L$ while violate $B + L$
- Some of net lepton numbers transform into net baryon numbers
- Minimal leptogenesis scenario: two right-handed neutrinos
- $T_R \gtrsim 10^{10}$ GeV

SM fermion masses and mixings: Froggatt-Nielsen mechanism [a]

- Introducing a global $U(1)_{FN}$ symmetry, and a SM singlet ϕ with $U(1)_{FN}$ charge -1. Assigning suitable charges to the SM fermions.

- ϕ obtains a VEV, and $\epsilon = \langle \phi \rangle / M_{\rm Pl}$ is a small parameter about 0.22.

$$
\begin{aligned}
M_{ij}^U &\sim \mathcal{O}(1)\ \sin\beta\, \epsilon^{nQi+nUj}\ v\ , \\
M_{ij}^D &\sim \mathcal{O}(1)\ \cos\beta\, \epsilon^{nQi+nDj}\ v\ , \\
M_{ij}^E &\sim \mathcal{O}(1)\ \cos\beta\, \epsilon^{nLi+nEj}\ v\ .
\end{aligned}
$$

- Stabilization: anomalous gauged $U(1)$ symmetry in the string models.

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[a] I. Gogoladze, C. A. Lee T. Li, and Q. Shafi.

$$
\begin{aligned}
-\mathcal{L} \;=\;& h_u^{ij} \sin\beta \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{nQi+nUj} Q_i U_j \tilde{H} \\
&+ h_d^{ij} \cos\beta \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{nQi+nDj} Q_i D_j H \\
&+ h_l^{ij} \cos\beta \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{nLi+nEj} L_i E_j H \\
=\;& h_u^{ij} \sin\beta \epsilon^{nQi+nUj} Q_i U_j \tilde{H} + h_d^{ij} \cos\beta \epsilon^{nQi+nDj} Q_i D_j H \\
&+ h_l^{ij} \cos\beta \epsilon^{nLi+nEj} L_i E_j H \; .
\end{aligned}
$$

Inflation [a]

- Inflaton: a real SM singlet field φ
- Quadratic term drives inflation–Chaotic Inflation
- $\varphi \simeq 3.718 \times 10^{19}$ GeV and $m \simeq 1.5 \times 10^{13}$ GeV for $N = 60$
- Stabilization: $\mu \lesssim 10^{6}$ GeV and $\kappa \lesssim 10^{-14}$.

Lagrangian

$$\mathcal{L}_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\mu}{3!}\varphi^3 - \frac{\kappa}{4!}\varphi^4 \, .$$

Other possible inflaton candidates: right-handed sneutrinos, etc [b].

[a] H. Davoudiasl, R. Kitano, TL and H. Murayama

[b] J. Jiang, TL and Y. H. Wang, in preparation.

V. Prediction: Higgs Boson Mass

If the Higgs particle is the only new physics discovered at the LHC and the SM is thus confirmed as the low energy effective theory, the most interesting parameter is the Higgs mass [a].

Can we predict the Higgs boson mass in a narrow range?

[a] V. Barger, J. Jiang, P. Langacker and TL.

Procedure:

- Calculating the Higgs boson quartic coupling λ at the GUT scale due to supersymmetry:

$$\lambda(M_U) = \frac{k_Y g_2^2(M_U) + g_1^2(M_U)}{4k_Y} \cos^2 2\beta \ .$$

- Evolving λ down to the weak scale via RGEs.
- Minimizing the one-loop effective Higgs potential with top quark radiative corrections.
- Calculating the Higgs boson mass.

The one-loop effective Higgs potential

$$V_{eff} = m_h^2 H^\dagger H - \frac{\lambda}{2!}(H^\dagger H)^2 - \frac{3}{16\pi^2} h_t^4 (H^\dagger H)^2 \left[\log \frac{h_t^2 (H^\dagger H)}{Q^2} - \frac{3}{2}\right] .$$

For the $\overline{MS}$ top quark Yukawa coupling, we use the one-loop corrected value:

$$m_t = h_t v \left(1 + \frac{16}{3}\frac{g_3^2}{16\pi^2} - 2\frac{h_t^2}{16\pi^2}\right) .$$

Results [a]**:**

- If we vary α_3 within its 1σ range, m_t within its 1σ range $(171.4 \pm 2.1$ GeV), and $\tan\beta$ from 1.5 to 50, the predicted Higgs boson mass will range from 130.8 GeV to 148.5 GeV.
- The top quark mass can be measured to about 1 GeV accuracy at the LHC.
- Assuming this accuracy and a central value of 171.4 GeV, the Higgs boson mass is predicted to be between 132.7 GeV and 146.9 GeV.

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[a] V. Barger, J. Jiang, P. Langacker and TL.

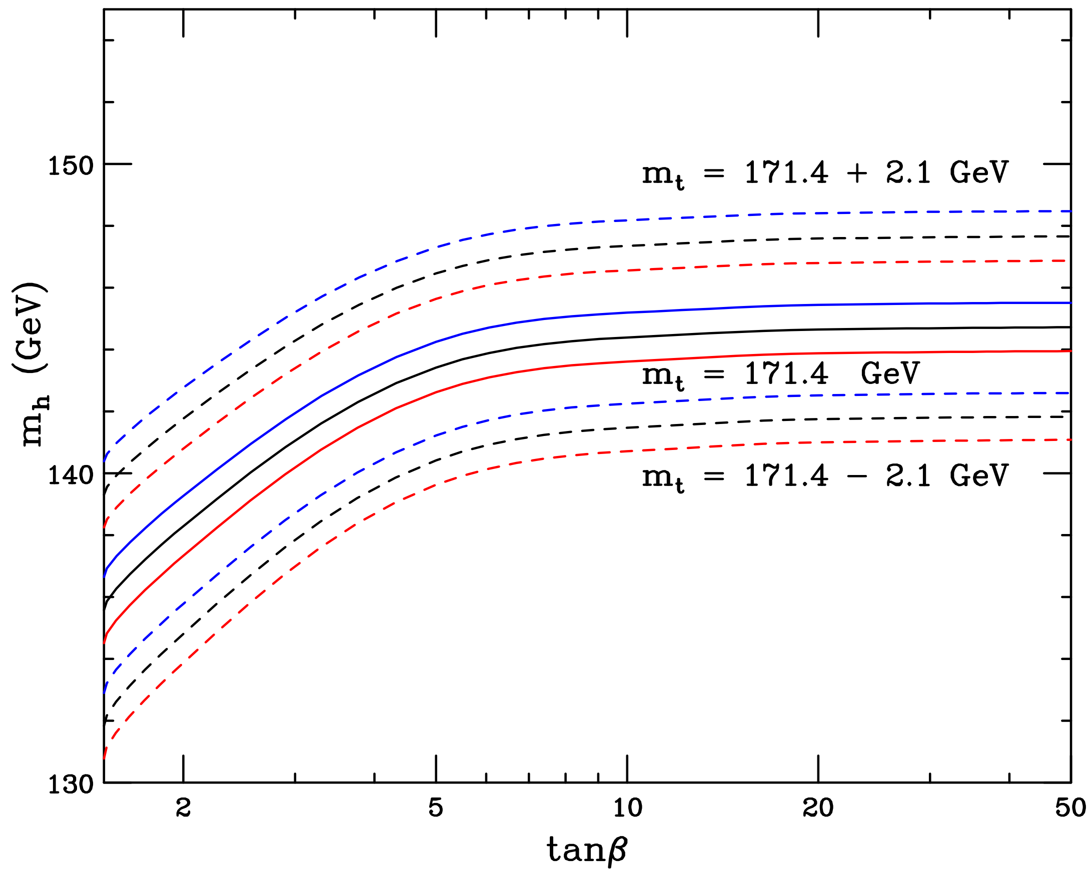

Figure 2: The predicted Higgs mass for the SM with $k_Y = 4/3$. The red (lower) curves are for $\alpha_3 + \delta\alpha_3$, the blue (upper) for $\alpha_3 - \delta\alpha_3$, and the black for α_3. The dash ones for $m_t \pm \delta m_t$, and the solid ones for m_t.

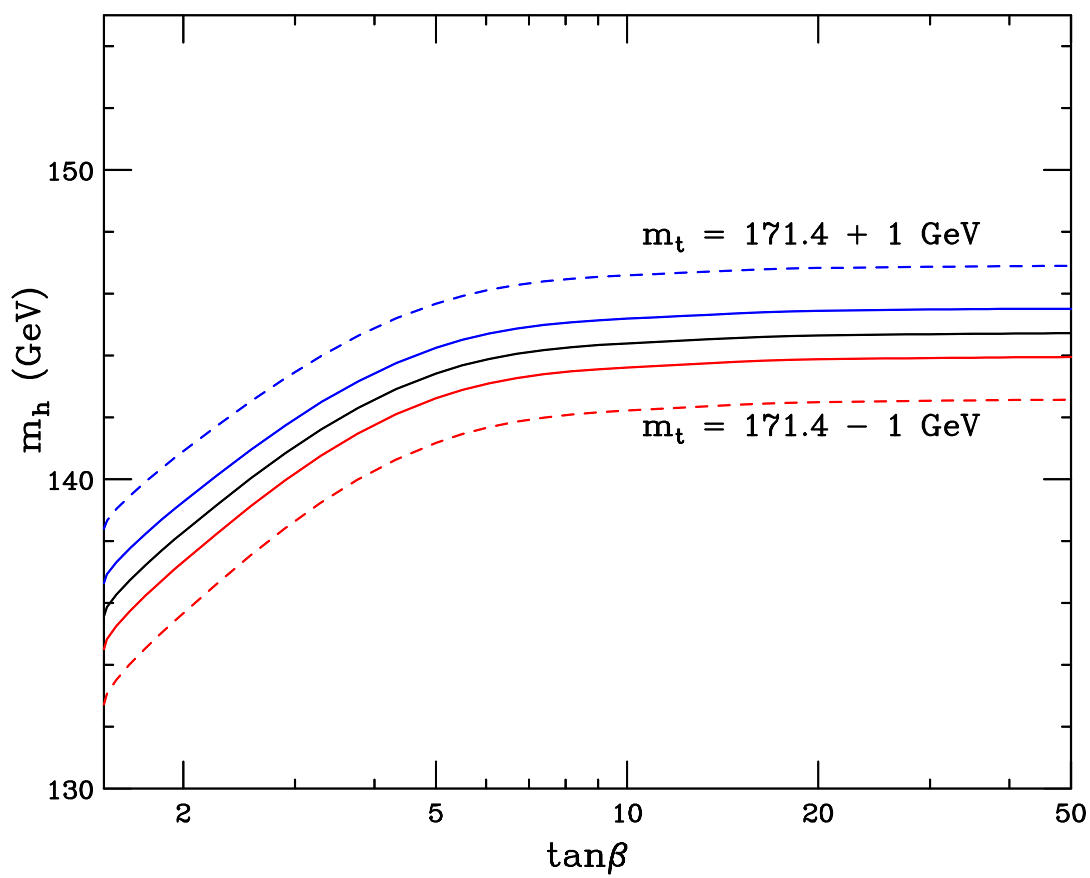

Figure 3: The predicted Higgs mass for the SM with $k_Y = 4/3$. The red (lower) curves are for $\alpha_3 + \delta\alpha_3$, the blue (upper) for $\alpha_3 - \delta\alpha_3$, and the black for α_3. The dash ones for $m_t \pm \delta m_t$, and the solid ones for m_t.

Higgs Physics at the LHC [a]:

- For $m_H < 135$ GeV, the Higgs dominant decay mode is $b\bar{b}$, and for $135 \text{ GeV} < m_H < 2m_W$, the Higgs dominant decay mode is $WW^{(*)}$.

- For $120 \text{ GeV} < m_H < 400$ GeV, the accurate Higgs boson mass can be determined up to 0.1% uncertainty via the channel $H \to ZZ^{(*)} \to l^+l^-l^+l^-$, assuming an integrated luminosity of 300 fb^{-1}.

[a] M. Carena and H. E. Haber

- For $110 \text{ GeV} < m_H < 150 \text{ GeV}$, the Higgs boson can be detected (with 100 fb^{-1} of data) via the chanels $gg \to H \to \gamma\gamma$, and $qq \to qqV^{(*)}V^{(*)} \to qqH$ and then $H \to \gamma\gamma,\ \tau^+\tau^-$.
- For $m_H > 130 \text{ GeV}$, the Higgs boson can also be observed in the gluon-gluon fusion through its decay to $WW^{(*)}$ and $ZZ^{(*)}$ with both final gauge bosons decaying leptonically.

Our model can be tested at the LHC!

V. SUMMARY

The beautiful Standard Model in string landscape

- Cosmological constant problem and gauge hierarchy problem could be solved due to weak anthropic principle.
- Axion–Strong CP problem/Dark matter.
- Gauge coupling unification can be achieved and charge quantization can be realized.
- Gauge interaction unification and fermion unification.
- Two right-handed neurtrinos–Neutrino masses and mixings/Baryon asymmetry.

- Froggatt-Nielsen mechanism–SM fermion masses and mixings.
- Inflation: A real SM singlet scalar/sneutrinos, etc.
- The Higgs mass can be predicted in a narrow range, and can be tested at the LHC and ILC.