2016年04月 · 中国科技大学交叉学科理论研究中心

宇宙学常数导致的狭义相对论改动

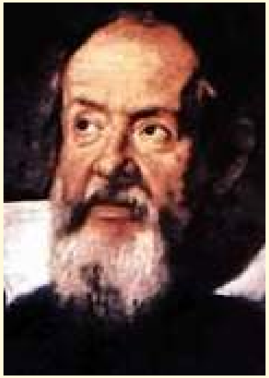
———-(de-Sitter 不变狭义相对论)

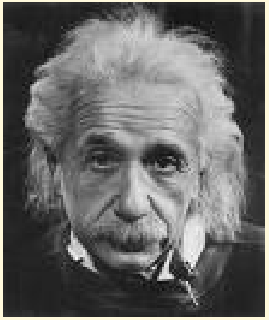
闫沐霖(USTC)


狭义相对论
还能改动吗？
E=mc2

目录

0, 一本书
1, 相对论的基本精神
2, 一个问题
3, 伽利略相对论
4, Einstein 狭义相对论
5, de Sitter 狭义相对论
6, 对称性产生动力学
7, 对爱氏狭义相对论的挑战
8, 结论与讨论





0，一本2015出版的介绍dS-SR的书：

封面上的简介:

De Sitter Invariant Special Relativity:

Einstein's Special Relativity (E-SR) is one of the cornerstones of the modern physics. When the Einstein's cosmological constant Λ is non-zero in spacetime, E-SR will naturally become De Sitter Invariant SR (dS/AdS-SR). Hence it is essentially related to the foundation of physics. This book provides a description of dS/AdS-SR in terms of Lagrangian-Hamiltonian formulation with spacetime metric of inertial reference frames.

One of the outstanding features of the book is that all descriptions of SR are in the inertial reference frames. This is a requirement due to the first principle of SR theory. The descrip- tions of dS/AdS-SR in this book satisfy this principle. It is highly non-trivial for the curved spacetime in dS/AdS-SR theory.

封面上的简介:

De Sitter Invariant Special Relativity:

Einstein's Special Relativity (E-SR) is one of the cornerstones of the modern physics. When the Einstein's cosmological constant Λ is non-zero in spacetime, E-SR will naturally become De Sitter Invariant SR (dS/AdS-SR). Hence it is essentially related to the foundation of physics. This book provides a description of dS/AdS-SR in terms of Lagrangian-Hamiltonian formulation with spacetime metric of inertial reference frames.

One of the outstanding features of the book is that all descriptions of SR are in the inertial reference frames. This is a requirement due to the first principle of SR theory. The descrip- tions of dS/AdS-SR in this book satisfy this principle. It is highly non-trivial for the curved spacetime in dS/AdS-SR theory.

封面上的简介:

De Sitter Invariant Special Relativity:

Einstein's Special Relativity (E-SR) is one of the cornerstones of the modern physics. When the Einstein's cosmological constant Λ is non-zero in spacetime, E-SR will naturally become De Sitter Invariant SR (dS/AdS-SR). Hence it is essentially related to the foundation of physics. This book provides a description of dS/AdS-SR in terms of Lagrangian-Hamiltonian formulation with spacetime metric of inertial reference frames.

One of the outstanding features of the book is that all descriptions of SR are in the inertial reference frames. This is a requirement due to the first principle of SR theory. The descrip- tions of dS/AdS-SR in this book satisfy this principle. It is highly non-trivial for the curved spacetime in dS/AdS-SR theory.

封面上的简介:

De Sitter Invariant Special Relativity:

Einstein's Special Relativity (E-SR) is one of the cornerstones of the modern physics. When the Einstein's cosmological constant Λ is non-zero in spacetime, E-SR will naturally become De Sitter Invariant SR (dS/AdS-SR). Hence it is essentially related to the foundation of physics. This book provides a description of dS/AdS-SR in terms of Lagrangian-Hamiltonian formulation with spacetime metric of inertial reference frames.

One of the outstanding features of the book is that all descriptions of SR are in the inertial reference frames. This is a requirement due to the first principle of SR theory. The descrip- tions of dS/AdS-SR in this book satisfy this principle. It is highly non-trivial for the curved spacetime in dS/AdS-SR theory.

作者写这本书的动机:

de Sitter 不变狭义相对论是Einstein宇宙学常数Λ不为零时空中的狭义相对论。当今，尽管这样一个普适性常数Λ的存在的可能性已广泛接受，然而与之相应的，在40年前完成的de Sitter 不变狭义相对论却至今在物理界知知甚少，在这种情况下，一种教科书式地介绍de Sitter狭义相对论的读物看来是需要的，这就是作者写这本书的动机。主要参考文献:

1，P.A.M. Dirac, *The electron wave equation in de-Sitter space*, Annals of Mathematics, **35**, 657 (1935)

2，K.H. Look (Q.K. Lu), C.L. Tsou (Z.L. Zou) and H.Y. Kuo, (H.Y. Guo), 1974, *“ Motion effects and cosmologic red-shift phenomenons in classical domain space-time”*, Acta Physica Sinica, **23** (1974) 225 (in Chinese)

3，A.G. Riess, *et al.*, *“ Observational evidence from supernovae for an accelerating universe and a cosmological constant”*, Astro. J. **116** 1009 (1998); S. Perlmutter , *et al.*, *“Measurements of Omega and Lambda from 42 high redshift supernovae”*, Astro. J. **517**, 565 (1999) [astro-ph/9812133]

4，Yan, M.L., Xiao,N.C., Huang,W., Li,S., 2005, *“ Hamiltonian Formalism of de-Sitter Invariant Special Relativity”* Commun. Theor. Phys. (Beijing, China) **48**, 27 (2007), hep-th/0512319.

1，相对论理论的基本精神:

1，狭义相对论：时空几何确定物质运动。（度规是固定的；该度规满足真空Einstein 方程；自由粒子作惯性运动；时间-空间具有最大对称性。）

2，广义相对论：物质运动确定时空几何（Einstein 方程），时空几何影响物质运动（短程线方程、弯曲空间场论等）。

2，一个问题：

i，当Einstein 宇宙学常数$\Lambda = 0$ 时，固定狭义相对论时空几何的Minkowski 度规

$$\{\eta^{\mu\nu}\} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{1}$$

是满足**真空**Einstein 方程的：（且满足惯性运动定律；满足最大对称性时空要求）

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \Lambda g_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 0. \tag{2}$$

ii，但是，当Einstein 宇宙学常数$\Lambda \neq 0$ 时，上述结论不对！这提供了一个可能性：通常的Einstein 狭义相对论（or Poincaré 不变狭义相对论）可能不对，要接受一个来自$\Lambda \neq 0$ 的修改！

2，一个问题:

i，当Einstein 宇宙学常数$\Lambda = 0$ 时，固定狭义相对论时空几何的Minkowski 度规

$$\{\eta^{\mu\nu}\} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{3}$$

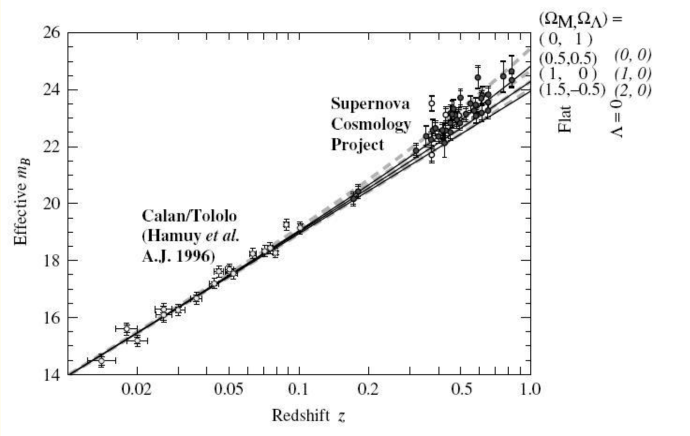
是满足真空Einstein 方程的：（且满足惯性运动定律；满足最大对称性时空要求）

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 0. \tag{4}$$

ii，但是，当Einstein 宇宙学常数$\Lambda \neq 0$ 时，上述结论不对！这提供了一个可能性：通常的Einstein 狭义相对论（or Poincaré 不变狭义相对论）可能不对，要接受一个来自$\Lambda \neq 0$ 的修改！

iii，宇宙加速膨胀的实验观测： 减速因子：

$$q_0 \equiv -a_0\ddot{a}_0/\dot{a}_0^2 \simeq \frac{1}{2}(\Omega_{\rm M} - 2\Omega_\Lambda) \approx -0.64^{+0.14}_{-0.12} < 0, \tag{5}$$



$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu} = -8\pi G T^{\rm M}_{\mu\nu} - 8\pi G\rho_{\text{dark energy}}g_{\mu\nu}, \tag{6}$$

$$\Lambda_{\rm eff} = \Lambda + 8\pi G\rho_{\text{dark energy}} = \Lambda + \Lambda_{\text{dark energy}} \simeq 1.26\times 10^{-56}{\rm cm}^{-2}. \tag{7}$$

注意：实验测量的是$\Lambda_{\rm eff} = \Lambda + 8\pi G\rho_{\text{dark energy}} = \Lambda + \Lambda_{\text{dark energy}}$，而非$\Lambda$. 作为普适常数的$\Lambda$，可正可负，还有待测定。$\Lambda_{\text{dark energy}}$是模型相关的，是时间函数，人为假定$\Lambda = 0$不可取。（Yan,Hu,Huang, Mod Phys Lett, **A27** 1250041 (2012)）

A.G. Riess et al, Astro J **116**, 1009(1998); S. Perlmutter et al Astro J, **517** 565 (1999)

iv, 狭义相对论的基本度规$g_{\mu\nu}$ 由下述条件确定:

a，真空Einstein 方程:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \Lambda g_{\mu\nu} = 0. \tag{8}$$

b，惯性定律（牛顿第一定律）:

$$\delta S = -mc\,\delta \int ds = -mc\,\delta \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu} = 0, \quad \Rightarrow \ddot{x}^i = 0, \tag{9}$$

c，最大对称空间(存在10个保度规$g_{\mu\nu}$映射生成元(Killing 矢量) $\xi^\mu(1), \cdots \xi^\mu(10)$):

$$\pounds_{\xi(i)} g_{\mu\nu} = 0, \;\; \xi^\mu(i) : i = \{1,\ 2,\ \cdots \frac{N(N+1)}{2} = 10.\} \tag{10}$$

满足这3个条件的原来只知是Minkowski度规$\eta_{\mu\nu}$, with $\Lambda = 0$。**1970- 1974** **陆启铿、邹振隆、郭汉英发现了$\Lambda \neq 0$解: $g_{\mu\nu}$ =Beltrami度规$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{\eta_{\mu\lambda}x^\lambda \eta_{\nu\rho}x^\rho}{R^2\sigma(x)^2}$ with $\sigma = 1 - \eta_{\mu\nu}x^\mu x^\nu / R^2$ and $R^2 = 3/\Lambda$也满足这3个条件.**

这就导致了De Sitter不变狭义相对论的发现!

iv, 狭义相对论的基本度规$g_{\mu\nu}$ 由下述条件确定:

a，真空Einstein 方程:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \Lambda g_{\mu\nu} = 0. \tag{11}$$

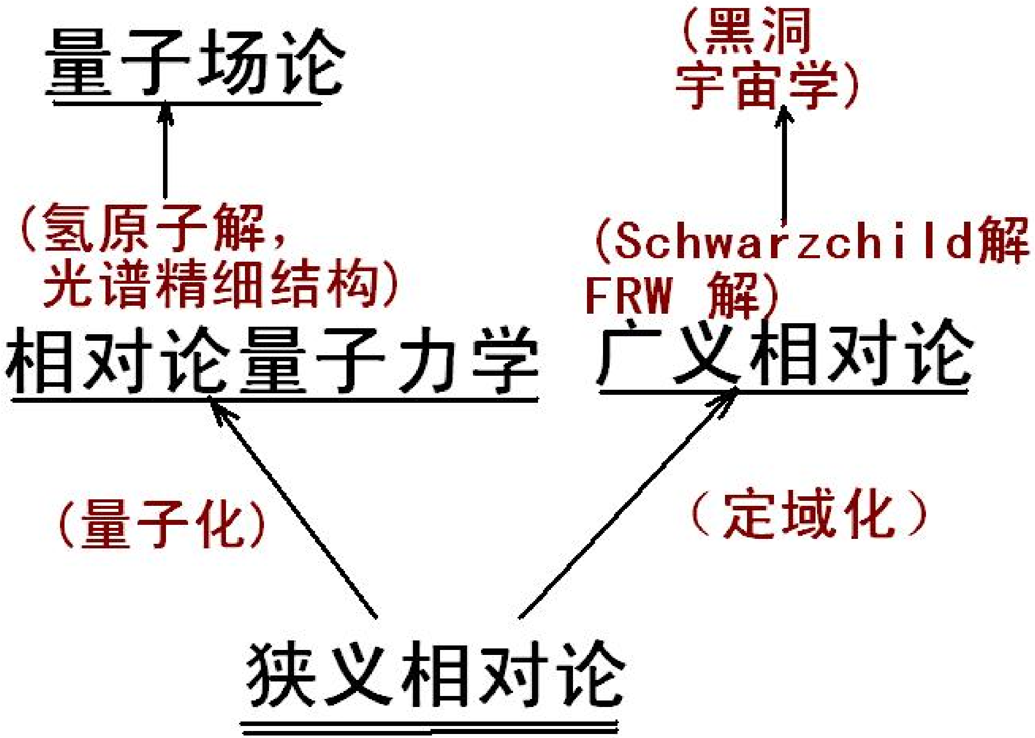
b，惯性定律（牛顿第一定律）:

$$\delta S = -mc\, \delta \int ds = -mc\, \delta \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu} = 0, \quad \Rightarrow \ddot{x}^i = 0, \tag{12}$$

c，最大对称空间(存在10个保度规$g_{\mu\nu}$映射生成元(Killing 矢量) $\xi^\mu(1), \cdots \xi^\mu(10)$):

$$\pounds_{\xi(i)} g_{\mu\nu} = 0, \;\; \xi^\mu(i) : i = \{1,\ 2,\ \cdots \frac{N(N+1)}{2} = 10.\} \tag{13}$$

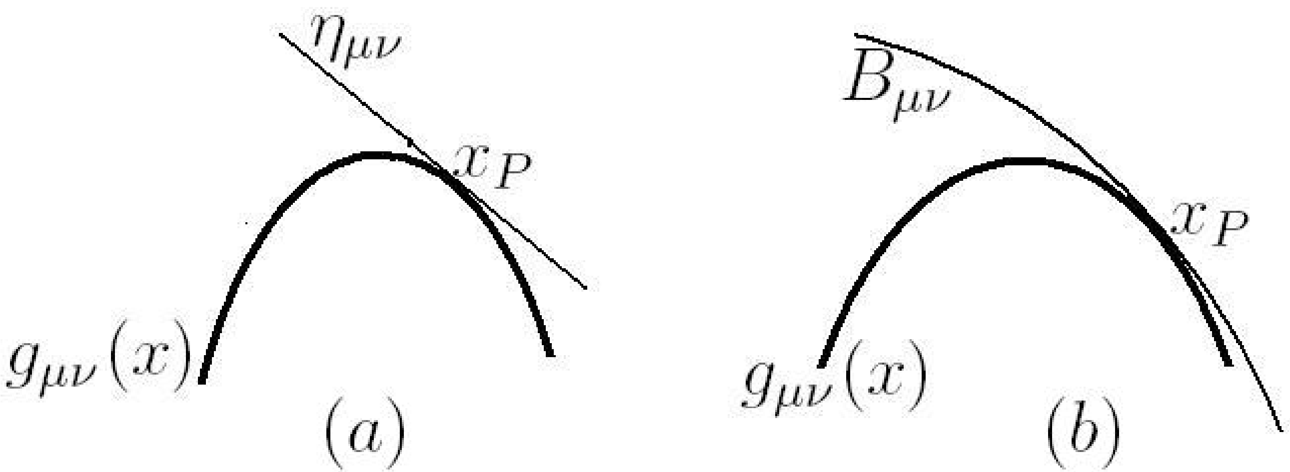
满足这3个条件的原来只知是Minkowski度规$\eta_{\mu\nu}$, with $\Lambda = 0$。**1970- 1974 陆启铿、邹振隆、郭汉英发现了$\Lambda \neq 0$解: $g_{\mu\nu}$ =Beltrami度规$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{\eta_{\mu\lambda} x^\lambda \eta_{\nu\rho} x^\rho}{R^2 \sigma(x)^2}$ with $\sigma = 1 - \eta_{\mu\nu} x^\mu x^\nu / R^2$ and $R^2 = 3/\Lambda$也满足这3个条件. 这就导致了De Sitter不变狭义相对论的发现!**


量子场论
（黑洞
宇宙学）
（氢原子解，
光谱精细结构）
（Schwarzchild解
FRW 解）
相对论量子力学
广义相对论
（量子化）
（定域化）
狭义相对论

$$\mathbf{E-GR}: \quad \Lambda = 0 \qquad\qquad \mathbf{dS-GR}: \quad \Lambda \neq 0$$

$$g_{\mu\nu}(x): \quad G_{\mu\nu} = \kappa T_{\mu\nu} \qquad\qquad g_{\mu\nu}(x): \quad G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\eta_{\mu\nu} \quad : \quad G_{\mu\nu} = 0 \qquad\qquad B_{\mu\nu}(x): \quad G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$



3, 伽利略相对论:

i) 自由粒子拉氏量: $L_{\text{Newton}} = \frac{1}{2}m_0 v^2$, 拉氏方程: $\frac{d}{dt}\frac{\partial L}{\partial \dot{\vec{x}}} - \frac{\partial L}{\partial \vec{x}} = 0$

ii) 惯性定律: $\vec{v}$ =常数⇒ 惯性系

iii)伽利略变换+平移变换+转动变换(惯性系间的变换):

$$\begin{aligned} x^1 \to \tilde{x}^1 &= x^1 - v^1 t + a^1 \\ x^2 \to \tilde{x}^2 &= x^2 + a^2 \\ x^3 \to \tilde{x}^3 &= x^3 + a^3 \\ t \to \tilde{t} &= t + t_0 \end{aligned}$$

iv) 无普适常数.

v) 在伽利略变换+平移变换+转动变换下,L_{Newton}不变,即$L_{\text{Newton}} \to \widetilde{L}_{\text{Newton}} = L_{\text{Newton}}$+常数, 因此有能量守恒($E$),动量守恒($\mathbf{p}$),角动量守恒(Neother charge).

vi) 哈氏量(正则能量): $H = \dot{\mathbf{x}} \cdot \overrightarrow{\pi} - L$, 其中正则动量$\overrightarrow{\pi} = \frac{\partial L}{\partial \dot{\mathbf{x}}}$.

vii) 正则方程:

$$\begin{aligned} \dot{x}^i &= \frac{\partial H}{\partial \pi_i} = \{H, x^i\}_{PB} \\ \dot{\pi}_i &= -\frac{\partial H}{\partial x^i} = \{H, \pi_i\}_{PB}, \end{aligned}$$

where the Poisson bracket $\{x^i, \pi_j\}_{PB} = \delta^i_j$, $\{x^i, x^j\}_{PB} = 0$, $\{\pi_i, \pi_j\}_{PB} = 0$ are as usual. It is straightforward to check $\dot{x}^i = constant$ by using $L_{\rm Newton}$.

viii) 量子化: $\{\ ,\ \}_{PB} \Rightarrow \frac{1}{i\hbar}[\ ,\ ]$

ix) 正则量=Neother charge: 哈氏量=物理测量能量; 正则动量=物理测量动量. (循环坐标, 拉氏量与时间无关)

4, Einstein 狭义相对论:

i) 自由粒子拉氏量:

$$L_c = -m_0 c \frac{ds}{dt} = -m_0 c \frac{\sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}}{dt} = -m_0 c^2 \sqrt{1 + \frac{\eta_{ij} \dot{x}^i \dot{x}^j}{c^2}} = -m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}},$$

where Lorentz metric $\eta_{\mu\nu} = \mathrm{diag}\{+,-,-,-\}$, $dx^\mu = \{d(ct), dx^1, dx^2, dx^3\}$ and $i, j = 1, 2, 3$. 拉氏方程: $\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{x}}} - \frac{\partial L}{\partial \vec{x}} = 0$

ii) 惯性定律: $\vec{v}$ =常数⇒ 惯性系

iii)Lorentz变换+平移变换+转动变换(惯性系间的变换, Poincare' 变换):

$$\begin{aligned} x^1 \to \tilde{x}^1 &= \gamma \left[x^1 - v^1 t - a^1 + \beta a^0 \right] \\ x^2 \to \tilde{x}^2 &= x^2 - a^2 \\ x^3 \to \tilde{x}^3 &= x^3 - a^3 \\ t \to \tilde{t} &= \gamma \left[t - \beta x^1 / c - a^0 + \beta a^1 \right] \end{aligned}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v^1/c$.

iv) 普适常数c: 任何惯性系中测量到的光速相等.

v) 在Lorentz变换+平移变换+转动变换下,L_c不变,即$L_c \to \widetilde{L}_c = L_c +$, 因此有能量守恒($E$),动量守恒(**p**),角动量守恒, 及Boost荷守恒(Neother charge).

vi) 哈氏量(正则能量): $H = \dot{\mathbf{x}} \cdot \vec{\pi} - L = c\sqrt{\vec{\pi}^2 + m_0^2 c^2}$,

其中正则动量$\pi_i = \frac{\partial L}{\partial \dot{x}^i} = \frac{-m_0 \dot{x}^j \eta_{ij}}{\sqrt{1-\beta^2}}$.

vii) 正则方程:

$$\dot{x}^i = \frac{\partial H}{\partial \pi_i} = \{H, x^i\}_{PB}$$

$$\dot{\pi}_i = -\frac{\partial H}{\partial x^i} = \{H, \pi_i\}_{PB}.$$

Poisson bracket $\{x^i, \pi_j\}_{PB} = \delta^i_j$, $\{x^i, x^j\}_{PB} = 0$, $\{\pi_i, \pi_j\}_{PB} = 0$.

viii) 量子化: $\{\ ,\ \}_{PB} \Rightarrow \frac{1}{i\hbar}[\ ,\ ]$. 这样, Poisson bracket 就变成了Heisenberg 代数:

$$[x^i, \pi_j] = i\hbar\delta^i_j,\ \ [x^i, x^j] = 0,\ \ [\pi_i, \pi_j] = 0. \tag{14}$$

注意$\pi_\mu = \eta_{\mu\nu} p^\nu$, $[\pi_i, \pi_j] = 0 \Rightarrow [p^\mu, p^\nu] = 0$, 与Poincaré群相容, 可见狭义相对论和量子理论相容,自恰.

ix) Heisenberg 代数的解与Poincaré algebra: *the 4-dimensional invariant commutation relations* for x^μ and $\hat{\pi}_\nu$ (i.e., Heisenberg algebra):

$$[x^\mu, \pi_\nu] = i\hbar\delta^\mu_\nu, \quad [x^\mu, x^\nu] = 0, \quad [\pi_\mu, \pi_\nu] = 0. \tag{15}$$

The solution is

$$\pi_\mu = -i\hbar\partial_\mu, \qquad \text{or} \quad p^\mu = -i\hbar\eta^{\mu\nu}\partial_\nu. \tag{16}$$

p^μ together with operator $L^{\mu\nu} = (x^\mu p^\nu - x^\nu p^\mu)/i\hbar$ form an algebra as follows

$$\begin{aligned} [p^\mu, p^\nu] &= 0 \\ [L^{\mu\nu}, p^\rho] &= \eta^{\nu\rho}p^\mu - \eta^{\mu\rho}p^\nu \\ [L^{\mu\nu}, L^{\rho\sigma}] &= \eta^{\nu\rho}L^{\mu\sigma} - \eta^{\nu\sigma}L^{\mu\rho} + \eta^{\mu\sigma}L^{\nu\rho} - \eta^{\mu\rho}L^{\nu\sigma} \end{aligned} \tag{17}$$

which is just the Poincaré algebra, (i.e.,Poincaré Invariant 狭义相对论的时空对称性).**说明在正则量子化下的通常狭义相对论量子力学自洽!**

ix) 正则量=Neother charge: 哈氏量=物理测量能量; 正则动量=物理测量动量. (循环坐标, 拉氏量与时间无关):即$E = H$, $\overrightarrow{p} = \overrightarrow{\pi}$.

x) 注意: 牛顿力学: $E = \frac{1}{2}mv^2$, $p = mv$, 所以$m \to 0$时, 必有$E \to 0$, $p \to 0$, 即真空. 而对于狭义相对论,在$m \to 0$以及$v \to c$极限下,除$E = p = 0$的真空解外, 还有$E \neq 0$, $p \neq 0$的光量子解:

$$E = \lim_{m\to 0,\, v\to c} \frac{mc^2}{\sqrt{1 - v^2/c^2}} \neq 0, \quad p = \lim_{m\to 0,\, v\to c} \frac{mv}{\sqrt{1 - v^2/c^2}} \neq 0.$$

而且$E/p = c$. 与$\lambda\nu = c$ 比较, 可有$E = h\nu$和$p = h/\lambda$, 其中h为Plank常数. 因此量子论"几乎"可以从狭义相对论推导出来.

xi) 几何: Minkowski 度规: $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$. 在Poincaré变换下: $x^\mu \to \widetilde{x}^\mu$, $\eta_{\mu\nu} \to \widetilde{\eta}_{\mu\nu} = \frac{\partial x^\lambda}{\partial \widetilde{x}^\mu} \frac{\partial x^\rho}{\partial \widetilde{x}^\nu} \eta_{\lambda\rho} = \eta_{\mu\nu}$

5, de Sitter 不变狭义相对论:

o)问题的由来:

1, Einstein 1917年提出宇宙学常数Λ, (量纲为cm^{-1});

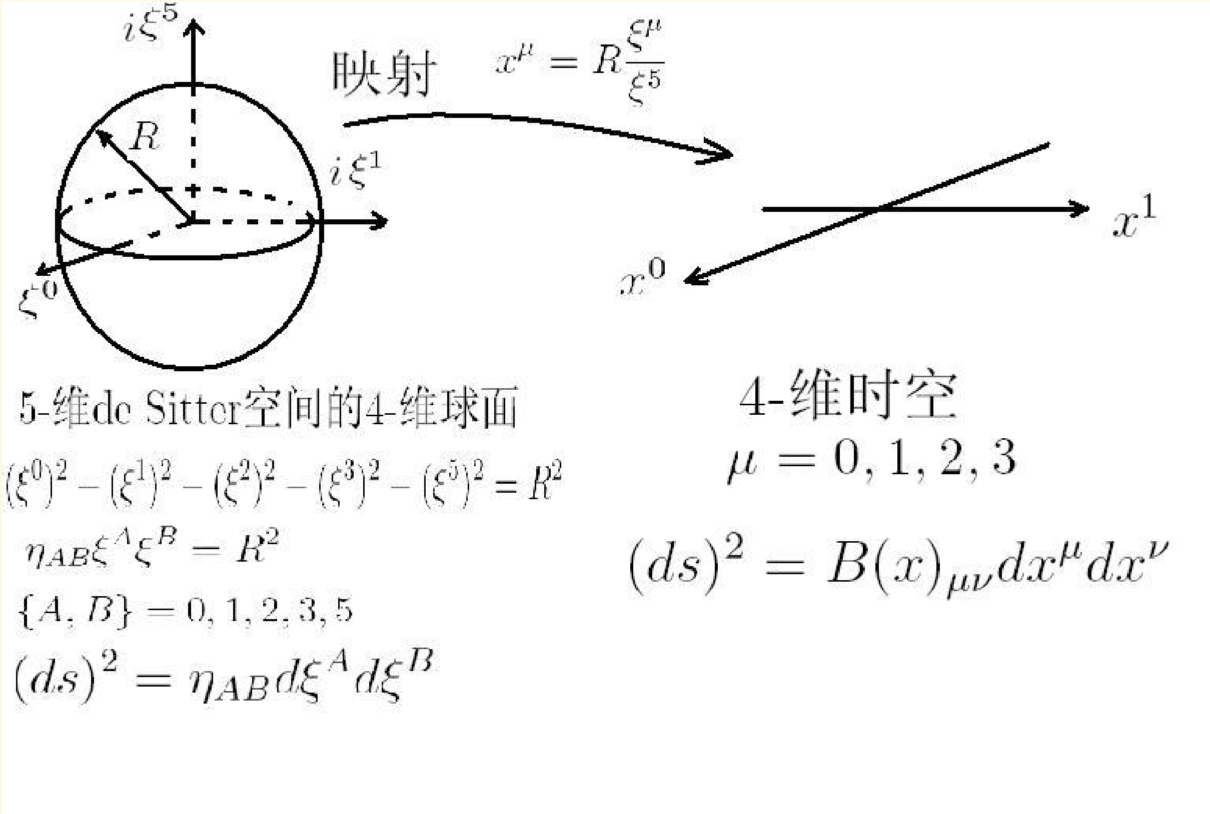
2,Dirac 1935年指出可以存在$\Lambda \neq 0$的狭义相对论（但他本人没有构造出来），和讨论相关的原子物理;

3, 1974年陆启铿,邹振隆,郭汉英, 构造出了"典型时空"理论，并讨论了宇宙学; "典型时空"理论就是$\Lambda \neq 0$ 的狭义相对论，即De Sitter 不变狭义相对论;

4, 1998年发现宇宙加速膨胀现象，表明$\Lambda_{eff} = \Lambda + \Lambda_{dark\ matter} \neq 0$;（这说明$\Lambda$可不为0，可按物理要求任意取值）

5, 这以后，讨论陆、邹、郭的典型时空"理论（即De Sitter 不变狭义相对论）的物理就有必要了。2004年提出了De Sitter 不变狭义相对论力学的Lagrange-Hamiltonian理论，实现了De Sitter 不变狭义相对论量子力学、De Sitter 不变广义相对论的构造，基于新理论求解了氢原子问题和黑洞问题等(Yan et al, 2005, 2011, 2012, 2014, 2015).

i), de Sitter 时空对称性：**5-维dS伪球面的$(ds)^2$在SO(4,1)旋转变换下不变。**



其中$\eta_{AB} = diag\{+,-,-,-,-\}$, $\eta_{\mu\nu} = diag\{+,-,-,-\}$.

由$(ds)^2 = \eta_{AB}d\xi^A d\xi^B|_{(\xi^{A,B}\in\{R\})} = B_{\mu\nu}(x)dx^\mu dx^\nu$ 可求得

$$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{1}{R^2\sigma(x)^2}\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho, \quad \text{with} \quad \sigma(x) \equiv 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu,$$

ii) Beltrami metric and Lagrangian L_{dS} for free particle:

$$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{1}{R^2\sigma(x)^2}\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho, \quad \text{with} \quad \sigma(x) \equiv 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu, \qquad (18)$$

where the constant R is the radius of the pseudo-sphere in dS-space, and it can be related to cosmological constant via $R = \sqrt{3/\Lambda}$. Seting up the time $t = x^0/c$, $B_{\mu\nu}(x)$ can be rewritten as follows

$$\begin{aligned} ds^2 &= B_{\mu\nu}(x)dx^\mu dx^\nu = \widetilde{g}_{00}d(ct)^2 + \widetilde{g}_{ij}\left[(dx^i + N^i d(ct))(dx^j + N^j d(ct))\right] \qquad (19) \\ &= c^2(dt)^2\left[\widetilde{g}_{00} + \widetilde{g}_{ij}(\frac{1}{c}\dot{x}^i + N^i)(\frac{1}{c}\dot{x}^j + N^j)\right], \end{aligned}$$

The free particle in $\mathcal{B}_\Lambda$ is described by the corresponding Landau-Lifshitz (L-L) action:

$$S_{dS} = -mc\int ds = -mc\int\sqrt{B_{\mu\nu}(x)dx^\mu dx^\nu} \equiv \int dt L_{dS}(t, x^i, \dot{x}^i), \qquad (20)$$

$$L_{dS} = -mc\frac{ds}{dt} = -mc\frac{\sqrt{B_{\mu\nu}(x)dx^\mu dx^\nu}}{dt} = -mc\sqrt{B_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu}, \qquad (21)$$

And then 自由粒子拉氏量:(含时并无循环坐标L氏系统)

$$L_{dS} = -mc^2 R \sqrt{\frac{R^2(c^2 - \dot{\mathbf{x}}^2) - \mathbf{x}^2\dot{\mathbf{x}}^2 + (\mathbf{x}\cdot\dot{\mathbf{x}})^2 + c^2(\mathbf{x} - \dot{\mathbf{x}}t)^2}{c^2(R^2 + \mathbf{x}^2 - c^2t^2)^2}}.$$

When $R \to \infty$,

$$L_{dS} \to L_{Eins} = -mc^2 \sqrt{1 - \frac{\dot{\mathbf{x}}^2}{c^2}}.$$

iii) 由拉氏方程: $\frac{d}{dt}\frac{\partial L}{\partial \dot{\vec{x}}} - \frac{\partial L}{\partial \vec{x}} = 0$, $\Rightarrow \ddot{x}^i = 0$,惯性定律成立!: $\vec{v}$ =常数$\Rightarrow$ 惯性系.

iv)保Beltrami度规的时空变换(de Sitter变换):

$$t \to \tilde{t} = \frac{\sqrt{\sigma(a)}}{c\sigma(a,x)}\gamma\left[ct - \beta x^1 - a^0 + \beta a^1 + \frac{a^0 - \beta a^1}{R^2}\frac{a^0 ct - a^1 x^1 - (a^0)^2 + (a^1)^2}{\sigma(a) + \sqrt{\sigma(a)}}\right]$$

.

$$x^1 \to \tilde{x}^1 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\gamma\left[x^1 - \beta ct + \beta a^0 - a^1 + \frac{a^1 - \beta a^0}{R^2}\frac{a^0 ct - a^1 x^1 - (a^0)^2 + (a^1)^2}{\sigma(a) + \sqrt{\sigma(a)}}\right]$$

$$x^2 \to \tilde{x}^2 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^2$$

$$x^3 \to \tilde{x}^3 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^3$$

where $\sigma(a,x) = 1 - \eta_{\mu\nu}a^\mu x^\nu/R^2$. 在该变换下$B_{\mu\nu}(x)$不变,即:

$$B_{\mu\nu}(x) \to \widetilde{B}_{\mu\nu}(\widetilde{x}) \equiv \frac{\partial x^\lambda}{\partial \widetilde{x}^\mu}\frac{\partial x^\rho}{\partial \widetilde{x}^\nu}B_{\lambda\rho}(x) = B_{\mu\nu}(\widetilde{x}).$$

v) 正则动量和正则能量(即Hamiltonian): 引进变量:

$$\Gamma \equiv \frac{cdt}{ds} = \frac{1}{\sqrt{1 - \frac{\dot{\mathbf{x}}^2}{c^2} + \frac{(\mathbf{x}\cdot\dot{\mathbf{x}})^2 - \mathbf{x}^2\dot{\mathbf{x}}^2}{c^2 R^2} + \frac{(\mathbf{x} - \dot{\mathbf{x}}t)^2}{R^2}}}, \qquad \Gamma|_{R\to\infty} = \gamma.$$

由理论力学得到:

$$\begin{aligned} \pi_i &= \frac{\partial L_{cR}}{\partial \dot{x}^i} = -m_0 B_{i\mu}\dot{x}^\mu \Gamma, \\ H_{cR} &= \sum_{i=1}^{3} \frac{\partial L_{cR}}{\partial \dot{x}^i}\dot{x}^i - L_{cR} = m_0 c B_{0\mu}\dot{x}^\mu \Gamma. \end{aligned}$$

vi) 4-正则动量, 正则色散关系, 正则方程, Poisson括号:

$$\pi_\mu \equiv (\pi_0,\ \pi_i) = (-\frac{H_{cR}}{c}, \pi_i) = -m_0 c B_{\mu\nu}\frac{dx^\nu}{ds},$$

$$B^{\mu\nu}\pi_\mu\pi_\nu \quad = \quad m_0^2 c^2. \tag{22}$$

$$H_{dS} \equiv -c\pi_0 = \frac{1}{2B^{00}}\left\{2cB^{0i}\pi_i \pm c\sqrt{4(B^{0i}\pi_i)^2 - 4B^{00}(B^{ij}\pi_i\pi_j - m^2c^2)}\right\}, \tag{23}$$

where the metric $B^{\mu\nu}$ has be given in above and hence

$$B^{00} = \sigma(x)\left(1 - \frac{c^2t^2}{R^2}\right),\ \ B^{0i} = -\sigma(x)\frac{ctx^i}{R^2},\ \ B^{ij} = \sigma(x)\left(\eta^{ij} - \frac{x^i x^j}{R^2}\right),$$

It is straightforward to to get the following canonical equations

$$\dot{x}^i \quad = \quad \frac{\partial H_{cR}}{\partial \pi_i} = \{H_{cR}, x^i\}_{PB} \tag{24}$$

$$\dot{\pi}_i \quad = \quad -\frac{\partial H_{cR}}{\partial x^i} = \{H_{cR}, \pi_i\}_{PB},$$

where the Poisson bracket $\{x^i, \pi_j\}_{PB} = \delta^i_j$, $\{x^i, x^j\}_{PB} = 0$, $\{\pi_i, \pi_j\}_{PB} = 0$ are as usual. It is also straightforward to check $\dot{x}^i = constant$ by eq.(24).

vii) 在$R \to \infty$ 时

$$\begin{aligned} H_{dS} &= \frac{1}{2B^{00}}\left\{2cB^{0i}\pi_i \pm c\sqrt{4(B^{0i}\pi_i)^2 - 4B^{00}(B^{ij}\pi_i\pi_j - m^2c^2)}\right\} \\ &\longrightarrow H_{Eins} = \pm\sqrt{c^2\mathbf{p}^2 + m^2c^4} \end{aligned} \tag{25}$$

可见，在H_{Eins}下，正反粒子的动力学是对称的；而对于H_{dS}，正反粒子的动力学是不对称的。这是重要启示！(沙哈诺夫准则(1960's)、丁肇中实验(2013))

vii)Beltrami-度规下的(Lorentz变换+平移变换+转动变换):惯性系间的变换, de Sitter 变换下, 度规不变, 作用量不变:

由前: 在$x^\mu \to \tilde{x}^\mu$下, $B_{\mu\nu}(x) \to \tilde{B}_{\mu\nu}(\tilde{x}) = B_{\mu\nu}(\tilde{x})$, 这导至

$$\begin{aligned} A = -m_0 c \int \sqrt{B_{\mu\nu}(x) dx^\mu dx^\nu} \quad &\to \quad \tilde{A} = -m_0 c \int \sqrt{\tilde{B}_{\mu\nu}(\tilde{x}) d\tilde{x}^\mu d\tilde{x}^\nu} \\ &= \quad -m_0 c \int \sqrt{B_{\mu\nu}(\tilde{x}) d\tilde{x}^\mu d\tilde{x}^\nu} = A. \end{aligned}$$

于是有10个不变量(Noether 定理). 用运动方程$\ddot{x}^i = 0$, 可证

$$\frac{d}{dt}(\Gamma) = 0$$

因此进一步可证:

$$\pounds_{\xi(i)} B_{\mu\nu} = 0, \;\; \xi^\mu(i) : i = \{1,\ 2,\ \cdots \frac{N(N+1)}{2} = 10.\} \tag{26}$$

满足最大对称空间条件。

1)3个boost charges:

$$G_{\beta_i} = m_0 c(x^i - t\dot{x}^i)\Gamma.$$

2) 动量:

$$p^i = G_{a^i} = m_0 \dot{x}^i \Gamma.$$

3)能量:

$$E \equiv cp^0 = cG_{a^0} = m_0 c^2 \Gamma.$$

4-动量:

$$p^\mu \equiv \{p^0,\ p^i\} = m_0 \dot{x}^\mu \Gamma = \frac{m_0 c}{\sigma(x)} \frac{dx^\mu}{ds} = -\frac{1}{\sigma(x)} B^{\mu\nu} \pi_\nu,$$

4) 角动量:

$$G_{\omega^i} = -m_0 \Gamma \epsilon^{ijk} x^j \dot{x}^k.$$

可验证:

$$\frac{d}{dt} G_{a=(1,\cdots,10)} = 0.$$

viii) 量子化: $\{\ ,\ \}_{PB} \Rightarrow \frac{1}{i\hbar}[\ ,\ ]$. **由于对de Sitter狭义相对论正则方程成立，由Heisenberg 代数，我们有:**

$$H_{cR} = i\hbar B^{-1/4}\frac{\partial}{\partial t}B^{1/4}, \qquad \pi_i = -i\hbar B^{-1/4}\frac{\partial}{\partial x^i}B^{1/4}$$

其中$B = \det(B_{\mu\nu})$. 代入上面的能量-动量表达式，可得4-动量算符

$$p^\mu \;=\; i\hbar[(\eta^{\mu\nu} - \frac{x^\mu x^\nu}{R^2})\partial_\nu + \frac{5x^\mu}{2R^2}]. \tag{27}$$

p^μ together with operator $L^{\mu\nu} = (x^\mu p^\nu - x^\nu p^\mu)/i\hbar$ form an algebra as follows

$$\begin{aligned} [p^\mu, p^\nu] &= \frac{\hbar^2}{R^2}L^{\mu\nu} \\ [L^{\mu\nu}, p^\rho] &= \eta^{\nu\rho}p^\mu - \eta^{\mu\rho}p^\nu \\ [L^{\mu\nu}, L^{\rho\sigma}] &= \eta^{\nu\rho}L^{\mu\sigma} - \eta^{\nu\sigma}L^{\mu\rho} + \eta^{\mu\sigma}L^{\nu\rho} - \eta^{\mu\rho}L^{\nu\sigma} \end{aligned} \tag{28}$$

which is just the de-Sitter algebra SO(1,4).

结论：对dS-SR QM 的Heisenberg 代数与dS时空的dS对称性代数相自洽！

由$B^{\mu\nu}\pi_\mu\pi_\nu = m_0^2c^2$, 单粒子运动的波方程:

$$(\eta^{\mu\nu} - \frac{x^\mu x^\nu}{R^2})\partial_\mu\partial_\nu\psi - 2\frac{x^\mu}{R^2}\partial_\mu\psi + \frac{m_0^2c^2}{\hbar^2\sigma(x)}\psi = 0$$

当$R \to \infty$, 上述方程成为平面波方程: $(\Box - \frac{m_0^2c^2}{\hbar^2})\psi = 0.$

ix) 色散关系: $p^\mu p^\nu B_{\mu\nu} = (m_0c/\sigma(x))^2$

以上2点可能会导至区别Einstein狭义相对论与de Sitter不变狭义相对论的天体观测实验!

---

dS-时空对称性的定域化会导致dS-GR

6，对称性产生动力学：（总结）

	Einstein狭义相对论(E-SR)	de Sitter狭义相对论(dS-SR)
Global对称性	Poincaré	dS or anti dS
惯性系度规	$\eta_{\mu\nu} = diag(+,-,-,-)$	Beltrami metric $B_{\mu\nu} = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{1}{R^2\sigma(x)^2}\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho$, 其中$\sigma(x) \equiv 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu$
作用量	$S = -m_0c^2\int\sqrt{\eta_{\mu\nu}dx^\mu x^\nu}$ $\equiv \int dtL_E$ $L_E = -m_0c^2\gamma^{-1}$ $\gamma^{-1} = \sqrt{1+\frac{\eta_{ij}\dot{x}^i\dot{x}^j}{c^2}}$	$S = -m_0c^2\int\sqrt{B_{\mu\nu}dx^\mu x^\nu} \equiv \int dtL_{dS}$, $L_{dS} = -m_0c^2\sigma^{-1}\Gamma^{-1}$, $\Gamma^{-1} \hat{=} \frac{1}{R}\sqrt{(R^2-\eta_{ij}x^ix^j)(1+\frac{a}{c^2})+2tb-at^2+\frac{b^2}{c^2}}$ 其中：$a \equiv \eta_{ij}\dot{x}^i\dot{x}^j$, $b \equiv \eta_{ij}x^i\dot{x}^j$
惯性定律	$\delta S = 0, \Rightarrow \ddot{x}^i = 0$	$\delta S = 0, \Rightarrow \ddot{x}^i = 0$
保惯性系 度规的变换	$x^\mu \to \tilde{x}^\mu = L^\mu_\nu x^\nu + a^\mu$ 注意L^μ_ν 和a^μ 是与x 无关 的常数	$x^\mu \to \tilde{x}^\mu = \pm\sigma(a)^{1/2}\sigma(a,x)^{-1}(x^\nu - a^\nu)D^\mu_\nu$ $D^\mu_\nu := L^\mu_\nu + R^{-2}\eta_{\nu\rho}a^\rho a^\lambda(\sigma(a)+\sigma^{1/2}(a))^{-1}L^\mu_\lambda$, $L := (L^\mu_\nu) \in SO(1,3)$, 注意和E-SR一样$L^\mu_\nu$ 和a^μ 是与x 无关的常数

定域化	$L^\mu_\nu \Rightarrow L^\mu_\nu(x)$, $a^\mu \Rightarrow a^\mu(x)$ 这时惯性系之间的坐标变换成为广义坐标变换： $x^\mu \to \tilde{x}^\mu = f^\mu(x)$	和E-SR一样,$L^\mu_\nu \Rightarrow L^\mu_\nu(x)$, $a^\mu \Rightarrow a^\mu(x)$ 导致惯性系之间的坐标变换同样成为广义坐标变换：$x^\mu \to \tilde{x}^\mu = f^\mu(x)$
联络和动力学	广义坐标变换的无绕联络是Christoffel联络：$\{{}^{\ \lambda}_{\mu\ \nu}\}$, 对应的度规是$g_{\mu\nu}(x) \neq \eta_{\mu\nu}$, 动力学方程： $\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = -\frac{8\pi G}{c^4}T_{\mu\nu}$ (see: Utiyama, PR 101(1956)1597; Kibble, J. Math Phys 2 (1961) 212 Cho, PRD14, (1976) 2521, etc)	和E-SR一样,广义坐标变换的无绕联络也只能是$\{{}^{\ \lambda}_{\mu\ \nu}\}$, 对应的度规是$g_{\mu\nu}(x) \neq B_{\mu\nu}$, 动力学方程： $\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} - \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$ 要求$B_{\mu\nu}(x)$是动力学方程的真空解， 则$\Lambda = \frac{3}{R^2}$(see: Yan, arXiv: 1004.3023).
均匀和各向同性时空(FRW度规)	$ds^2 = c^2dt^2 - a(t)^2(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2)$ $a(t)$满足不含Λ的Friedmann方程	和E-SR一样:$ds^2 = c^2dt^2 - a(t)^2(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2)$ $a(t)$满足含Λ的Friedmann方程
切空间	用$\eta_{\mu\nu}$描写的平空间	用$B_{\mu\nu}$描写的Betrami空间
“纤维丛”	底空间度规：$g_{\mu\nu}$ 切丛度规：$\eta_{\mu\nu}$	底空间度规：$g_{\mu\nu}$ 切丛度规：$B_{\mu\nu}$

7, 对爱因斯坦狭义相对论(E-SR)的一个挑战:

I, 爱因斯坦狭义相对论: The spacetime metric of the local inertial system is Minkowski metric:

$$\{\eta_{\mu\nu}\} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{29}$$

which is spacetime independent.

氢原子光谱: E **satisfies Dirac spectrum equation:**

$$E\psi = \left(-i\hbar c\boldsymbol{\alpha}\cdot\nabla - \frac{e^2}{r} + m_e c^2\beta\right)\psi, \tag{30}$$

E 的精确解:

$$\begin{aligned} E \equiv W_{n,\kappa} &= m_e c^2\left(1+\frac{\alpha^2}{(n-|\kappa|+s)^2}\right)^{-1/2} \\ &\alpha \equiv \frac{e^2}{\hbar c}, \quad |\kappa| = (j+1/2) = 1,\ 2,\ 3\ \cdots \\ &s = \sqrt{\kappa^2-\alpha^2}, \quad n = 1,\ 2,\ 3\ \cdots. \end{aligned} \tag{31}$$

（注意: e^2 是**Dirac** 方程中势能$1/r$ 前的系数，$\hbar c$是**Dirac** 方程中动能$i\boldsymbol{\alpha}\cdot\nabla$ 前的系数，$\alpha \equiv e^2/(\hbar c) \simeq 1/137$ 是这**2**个系数之比）

这样，由于在宇宙各处的定域惯性系的度规都是与坐标无关（即与宇宙红移z无关）的Mikowski 度规$\eta_{\mu\nu}$，爱因斯坦狭义相对论预言：

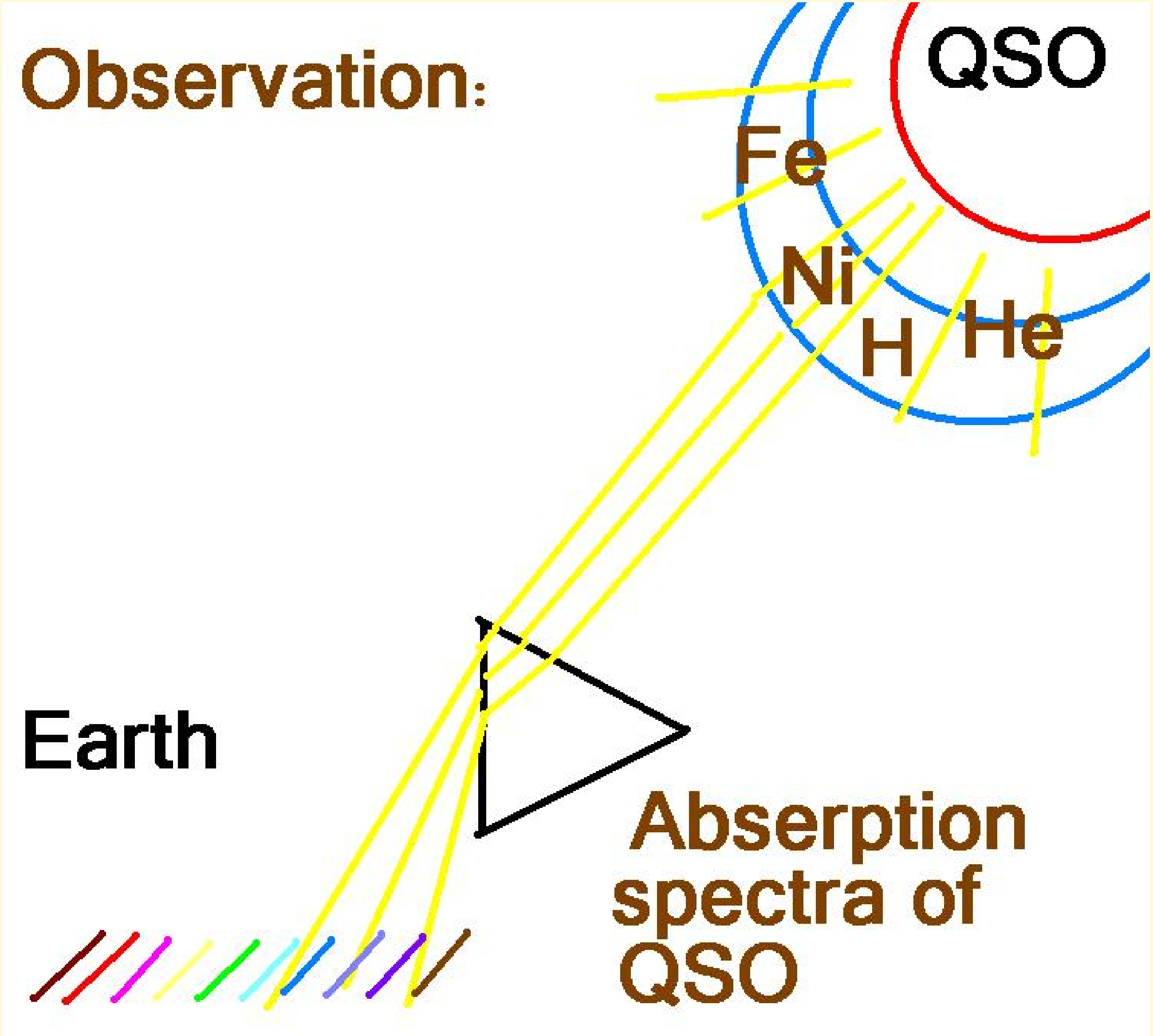
$$\alpha = \alpha_z \tag{32}$$

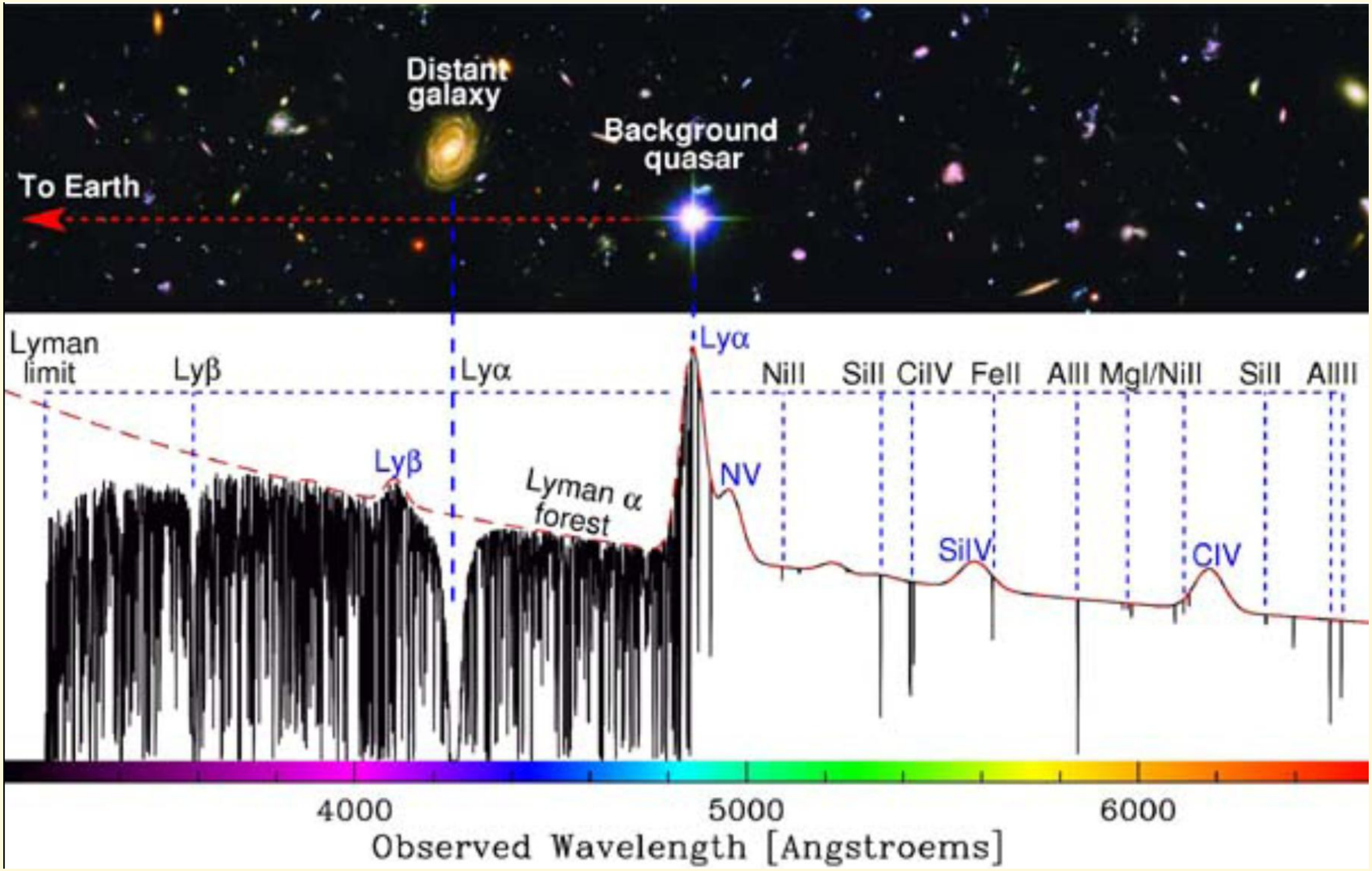
其中z 是原子所在处的红移。

- 这个E-SR的理论预言对吗？实验：

美国夏威夷keck天文台

南美智利VLT天文台


Observation:
QSO
Fe
Ni
H
He
Earth
Absorption spectra of QSO


Distant galaxy
Background quasar
To Earth
Lyman limit
Lyβ
Lyα
Lyα
NII
SiII
CII
CIV
FeII
AlII
MgI/NiII
SiII
AlIII
Lyβ
Lyman α forest
NV
SiIV
CIV
4000
5000
6000
Observed Wavelength [Angstroems]

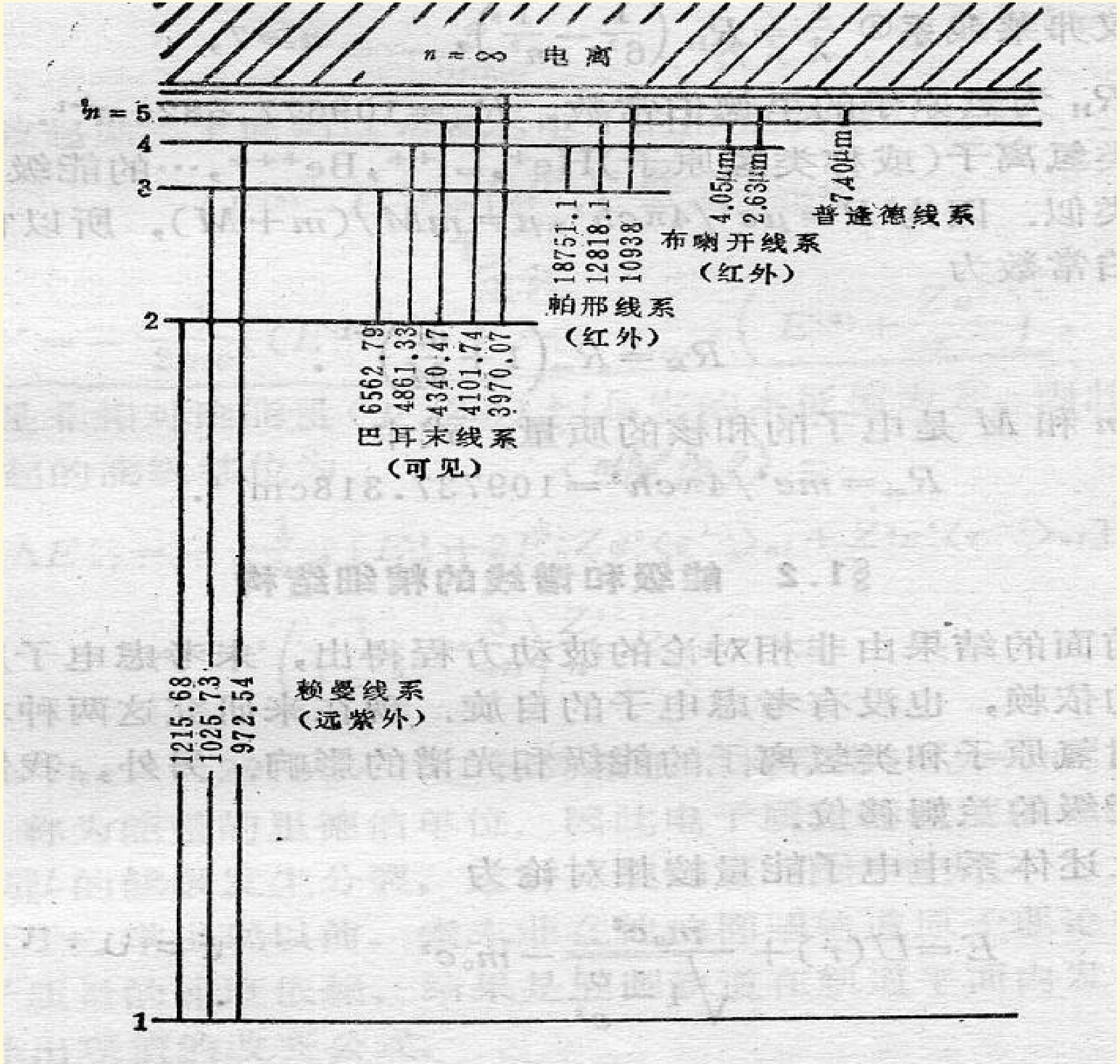
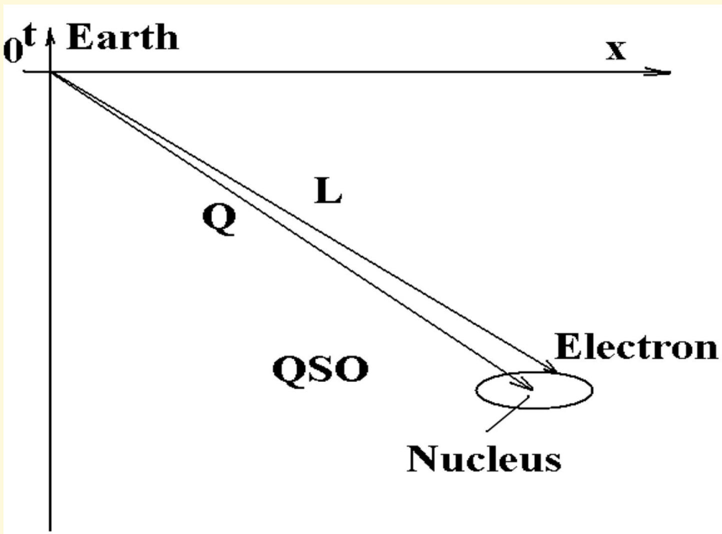


图1.1 氢原子的能级和观察的光谱（左边三组数字的单位为Å）

♣用dS-SR量子力学求解氢原子:

$2s^{1/2} - 2p^{1/2}$-splitting in dS-SR

Dirac equation:



$$(ie_{\mu a}\gamma^a \mathcal{D}_L^\mu - \frac{\mu c}{\hbar})\psi = 0, \quad (33)$$

Yan, arXiv:1004.3023 [physics.gen-ph]

$$\mathcal{D}_L^\mu \equiv D_L^\mu - ie/(c\hbar)A^\mu, \qquad A^\mu = \{\phi_B,\ \mathbf{A}\}$$

地球上的氢原子能谱:

n	l	j	κ	d	E (eV)
5	4	9/2	-5	10	- 4618
5	3,4	7/2	-4,4	16	- 4639
5	2,3	5/2	-3,3	12	- 4676
5	1,2	3/2	-2,2	8	- 4751
5	0,1	1/2	-1,1	4	- 5036
4	3	7/2	-4	8	- 7235
4	2,3	5/2	-3,3	12	- 7304
4	1,2	3/2	-2,2	8	- 7453
4	0,1	1/2	-1,1	4	- 8017
3	2	5/2	-3	6	- 12935
3	1,2	3/2	-2,2	8	- 13281
3	0,1	1/2	-1,1	4	- 14637
2	1	3/2	-2	4	- 29591
2	0,1	1/2	-1,1	4	- 34135
1	0	1/2	-1	2	- 131982

n	l	d
5	0,1,2,3,4	25
4	0,1,2,3	16
3	0,1,2	9
2	0,1	4
1	0	1

- **测红移z处的α_z:**

- 用i 代表光学跃迁**Mg I** $(3s^2\ ^1S_0 \to 3s3p\ ^1P_1)$（满足$\triangle\ell = \pm 1$），所观测到的跃迁频率可用下式表达:

$$\begin{aligned} \omega_i \equiv \omega_{i\ \mathrm{obs}} &= \omega_{i\ \mathrm{lab}} + q_i x + q_i' x^2 + \ldots \\ &= 35051.277(1) + 106x - 10y, \end{aligned} \tag{34}$$

其中$x \equiv (\alpha_z/\alpha)^2$, $y \equiv (\alpha_z/\alpha)^4 - 1$, $\omega_{i\ \mathrm{lab}}$ 是地球实验室的ω_i的测量值，式中3项前的系数是基于爱因斯坦狭义相对论，用原子物理中的量子多电子(E-SR)-Dirac-Hartree-Fock数值计算方法算出的。这样原则上只要通过天文观测测得$\omega_{i\ \mathrm{obs}}$，就得到了α_z. (如果发现: $\alpha_z = \alpha$就肯定了E-SR, 如果发现$\alpha_z \neq \alpha$,就否定了E-SR.)

- **部分(E-SR)-Dirac-Hartree-Fock 计算结果:**

(PRL 82(1999) 884)

$$
\begin{aligned}
\text{Mg II }^2P\ J=1/2: &\quad \omega = 35669.286(2)+119.6x,\\
J=3/2: &\quad \omega = 35760.835(2)+211.2x,\\
\text{Fe II }^6D\ J=9/2: &\quad \omega = 38458.9871(20)+1394x+38y,\\
J=7/2: &\quad \omega = 38660.0494(20)11632x10y,\\
{}^6FJ=11/2: &\quad \omega = 41968.0642(20)+1622x+3y,\\
J=9/2: &\quad \omega = 42114.8329(20)+1772x+0y,\\
{}^6P\ J=7/2: &\quad \omega = 42658.2404(20)+1398x-13y.
\end{aligned}
$$

- **2011结果：通过测遥远天体原子光谱的动力学移动，测**

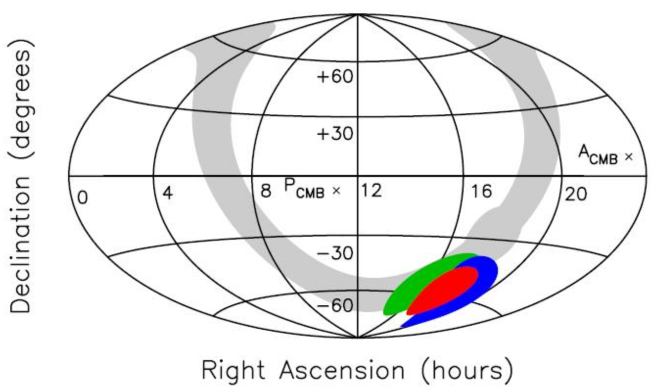
$$\alpha \equiv \frac{e^2}{\hbar c}:$$

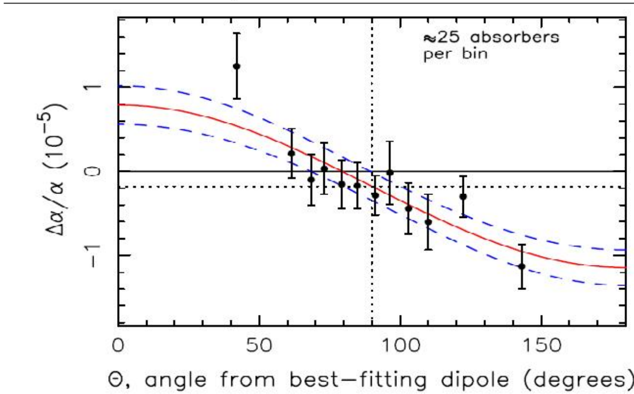
（Keck 用所发现的140 absorption systems，VLT 用了所发现的153 个吸收系统）

PRL **107,** 191101 (2011) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2011

Indications of a Spatial Variation of the Fine Structure Constant

J. K. Webb,[1] J. A. King,[1] M. T. Murphy,[2] V. V. Flambaum,[1] R. F. Carswell,[3] and M. B. Bainbridge[1]
[1]*School of Physics, University of New South Wales, Sydney, New South Wales 2052, Australia*
[2]*Centre for Astrophysics and Supercomputing, Swinburne University of Technology, Mail H30, PO Box 218, Victoria 3122, Australia*
[3]*Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, United Kingdom*
(Received 23 August 2010; published 31 October 2011)





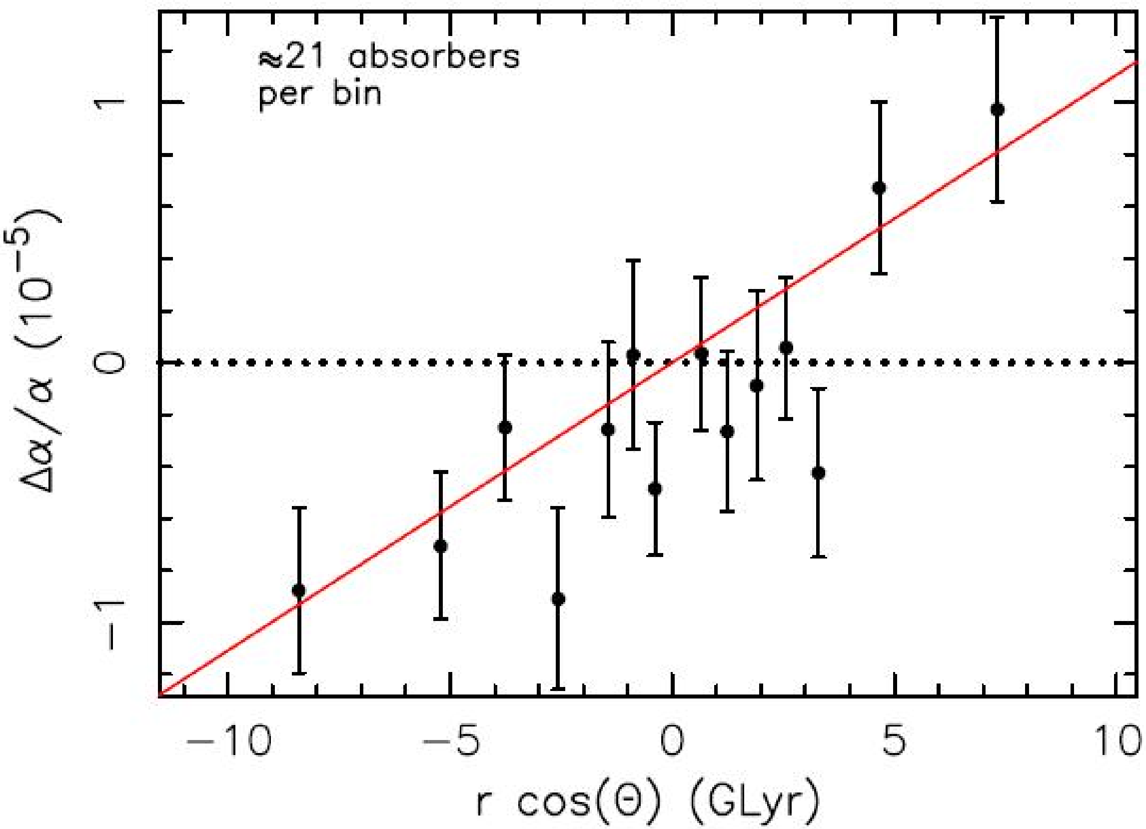


FIG. 3 (color online). $\Delta\alpha/\alpha$ vs $Ar\cos\Theta$ showing an apparent gradient in α along the best-fit dipole. The best-fit direction is at right ascension 17.5 ± 0.9 h, declination -58 ± 9 deg, for which $A = (1.1 \pm 0.25) \times 10^{-6}$ GLyr^{-1}. A spatial gradient is statistically preferred over a monopole-only model at the 4.2σ level. A cosmology with parameters $(H_0, \Omega_M, \Omega_\Lambda) =$ (70.5, 0.2736, 0.726) was used [18].

结果发现:

$$\alpha_z \neq \alpha$$

$$\alpha_z = \alpha_z(\mathbf{\Omega})$$

表明α是变化的，而且其变化是各向异性的。这与爱因斯坦狭义相对论以及宇宙学原理相冲突！是对爱因斯坦狭义相对论的一个挑战！

II, 理论考虑:

(a) 在原子物理中精细结构常数定义是氢原子Direc 谱方程的势能项系数与动能项系数之比: $\alpha \equiv e^2/(\hbar c)$

$$E\psi = \left[(\hbar c)(-i)\vec{\alpha}\cdot\nabla + \beta mc^2 - e^2\frac{1}{r}\right]\psi. \tag{35}$$

(b)在dS-SR 中，对于红移为z的远处氢原子Dirac 方程会被修正，一般地总可有

$$E\psi = \left[f(z)(-i)\vec{\alpha}\cdot\nabla + \beta m(z)c^2 - g(z)\frac{1}{r}\right]\psi. \tag{36}$$

于是，求得: $\alpha_z = g(z)/f(z) \neq \alpha$**. 这是原子物理的结果，可与实验比较。**

(M.L.Yan, Commu Theor Phys, 57(2012) 930; 62(2014)189)

(c) 上述关于α_z 的天文观测，是基于原子光谱的物理实验，不涉及Planck 能标关于新物理的理论：如string、额外维、“第5种力”$\cdots$ 等。

(d)在用天文观测测得$\omega_{i\ \mathrm{obs}}$的值来定α_z时，要用基于爱因斯坦狭义相对论的量子多电子(E-SR)-Dirac-Hartree-Fock数值计算。在实验结果与E-SR 相违背时，这样处理实验结果是不正确的，包含了来自宇宙学常数的系统误差！

III, 一个建议:

由于基于爱因斯坦狭义相对论不能自洽处理实验观察数据$\omega_{i\ \mathrm{obs}}$ 来测定α_z，建议使用基于de Sitter 不变狭义相对论的(dS-SR)-Dirac-Hartree-Fock多电子计算结果，来处理实验数据。这样爱因斯坦宇宙学常数的效应将被考虑。原则上多了可调参数，期望解决宇宙学原理问题，消弭实验中的系统误差:

（1）推导(dS-SR)-Dirac-Hartree-Fock公式;

（2）用上述公式和多电子量子理论计算Mg和Fe的原子和离子能谱;

（3）变动计算中α的输入值，导出$\omega_{i\ \mathrm{obs}}$展开式中x和y 前的系数;

（4）要求：(i) α_z各向同性；(ii)用上面的数字计算处理后，Keck和VLT的α_z果相一致;

（5）用dS-SR计算氢原子光谱，导出α_z的表达式，比较实验与理论。

S.S.Feng and M.L.Yan, IJTP, 55 (2016) 1049.

上面的工作将导致普适常数Λ的数值测定，这对物理学有基础性的重要性！

8, 结论和讨论:

1) 存在自恰的de-Sitter不变狭义相对论的Lagrangian和Hamiltonian形式;

2)惯性运动定律成立;包含2个普适常数:$c,\ R,\ R\to\infty$时回到狭义相对论;

3)理论的L与H与时间相关,且没有循环坐标,但有de Sitter不变性;

4)因此理论的正则量和守恒量不相等: $\vec{\pi}\neq\vec{p},\quad H\neq E$

5)量子化来自正则形式的Poisson括号,所以应是:$[x^i,\pi_j]=i\hbar\delta^i_j$, 和$[t,H]=i\hbar$, 而$[x^i,p^j]\neq i\hbar\delta^{ij}$, 和$[t,E]\neq i\hbar$

6)dS-SR的定域化可导致dS-GR.

7)位于遥远宇宙空间的氢原子可用dS-SR QM + dS-GR求出非平凡解，导致超出哈勃红移的原子光谱动力学移动.

8)力学是物理学的基础:

$$(\mathrm{dS-SR}\ mechanics)|_{R\to\infty}\Rightarrow(\mathrm{E-SR}\ mechanics)$$

$$(\mathrm{E-SR}\ mechanics)|_{c\to\infty}\Rightarrow(\mathrm{Newton}\ mechanics).$$

谢谢

In this case the frequency of each transition can be expanded in a series in α^2 :

$$\omega_i = \omega_i^{(0)} + \omega_i^{(2)}\alpha^2 + \cdots, \tag{37}$$

$$= \omega_{i\ \mathrm{lab}} + q_i x + \cdots, \quad x \equiv (\alpha/\alpha_0)^2 - 1, \tag{38}$$

where α_0 stands for the laboratory value of the fine structure constant. Note that Eq.(37) corresponds to the expansion at $\alpha = 0$, while Eq.(38) at $\alpha = \alpha_0$. In both cases, parameters $\omega_i^{(2)}$ and q_i appear due to relativistic corrections. We note that both $\omega_{i\ \mathrm{lab}}$ and q_i are calculable by the Dirac-Hartree-Fock method and the quantum many body perturbation theories (and $\omega_{i\ \mathrm{lab}}$ are of course measurable in laboratories).

For a fine structure transition the first coefficient on the right hand side of (37) turns to zero, while for the optical transitions it does not. Thus, for the case of a fine structure and an optical transition one can write:

$$\frac{\omega_{\mathrm{fs}}}{\omega_{\mathrm{op}}} = \frac{\omega_{\mathrm{fs}}^{(2)}}{\omega_{\mathrm{op}}^{(0)}}\alpha^2 + \mathcal{O}(\alpha^4) \tag{39}$$

while for two optical transitions i and k the ratio is:

$$\frac{\omega_i}{\omega_k} = \frac{\omega_i^{(0)}}{\omega_k^{(0)}} + \left(\frac{\omega_i^{(2)} - \omega_k^{(2)}}{\omega_k^{(0)}} \right) \alpha^2 + \mathcal{O}(\alpha^4). \tag{40}$$

More often than not, the coefficients $\omega_i^{(2)}$ for optical transitions are about an order of magnitude larger than corresponding coefficients for the fine structure transitions $\omega_{\rm fs}^{(2)}$ (this is because the relativistic correction to a ground state electron energy is substantially larger than the spin-orbit splitting in an excited state [?] [?]). Therefore, the ratio (40) is, in general, more sensitive to the variation of α than the ratio (39).