

QCD Aspects of Radiative Leptonic B Decays (with Subleading Power Corrections)

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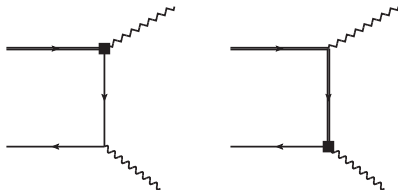
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Why power corrections?

- Understanding the general properties power expansion in EFTs (HQET, SCET, NRQCD).
- Interesting to understand the strong interaction dynamics of heavy quark decays.
 - ▶ Factorization properties of the subleading-power amplitudes.
 - ▶ Renormalization and asymptotic properties of higher-twist B -meson DAs.
 - ▶ Interplay of different QCD techniques.
- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
Power corrections, QED corrections, BSM physics.
- Crucial to understand the CP violation in B -meson decays.
Strong phase of $\mathcal{A}(B \rightarrow M_1 M_2)$ @ m_b scale in the leading power.
- Indispensable for understanding the flavour puzzles.
 - ▶ P'_5 anomaly in $B \rightarrow K^* \ell^+ \ell^-$.
 - ▶ Color suppressed hadronic B -meson decays.
 - ▶ Polarization fractions of penguin dominated $B_{(s)} \rightarrow VV$ decays.

General aspects of $B \rightarrow \gamma \ell \nu$

- Tree diagrams:



Kinematics:

$$p_B \equiv p + q = m_B v, \quad p = \frac{n \cdot p}{2} \bar{n}, \quad q = \frac{n \cdot q}{2} \bar{n} + \frac{\bar{n} \cdot q}{2} n.$$

- Decay amplitude:

$$\mathcal{M}(B^- \rightarrow \gamma \ell \nu) = \frac{G_F V_{ub}}{\sqrt{2}} (i g_{em} \boldsymbol{\varepsilon}_\nu^*) \left\{ T^{\nu\mu}(p, q) \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu + Q_\ell f_B \bar{\ell} \gamma^\nu (1 - \gamma_5) \nu \right\}.$$

Hadronic tensor:

$$\begin{aligned} T_{\nu\mu}(p, q) &\equiv \int d^4x e^{ip \cdot x} \langle 0 | T \{ j_{\nu,em}(x), [\bar{u} \gamma_\mu (1 - \gamma_5) b](0) \} | B^-(p+q) \rangle, \\ &= v \cdot p \left[-i \varepsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V(n \cdot p) + g_{\mu\nu} \hat{F}_A(n \cdot p) \right] + v_\nu p_\mu F_1(n \cdot p) \\ &\quad + v_\mu p_\nu F_2(n \cdot p) + v \cdot p v_\mu v_\nu F_3(n \cdot p) + \frac{P_\mu P_\nu}{v \cdot p} F_4(n \cdot p). \end{aligned}$$

General aspects of $B \rightarrow \gamma \ell \nu$

- Ward identity [Grinstein and Pirjol, 2000; Khodjamirian and Wyler, 2001]:

$$p_\nu T^{\nu\mu}(p, q) = -(Q_b - Q_u) f_B p_B^\mu.$$

↓

$$\hat{F}_A(v \cdot p) = -F_1(v \cdot p), \quad F_3(v \cdot p) = -\frac{(Q_b - Q_u) f_B m_B}{(v \cdot p)^2}.$$

- Reduced parametrization:

$$T_{\nu\mu}(p, q) = -i v \cdot p \varepsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V(n \cdot p) + [g_{\mu\nu} v \cdot p - v_\nu p_\mu] \hat{F}_A(n \cdot p) - \underbrace{\frac{(Q_b - Q_u) f_B m_B}{v \cdot p} v_\mu v_\nu}_{\text{contact term}}.$$

- Absorb the photon emission off the lepton [Beneke and Rohrwild, 2011]:

$$[g_{\mu\nu} v \cdot p - v_\nu p_\mu] \hat{F}_A(n \cdot p) = -Q_\ell f_B g_{\mu\nu} + [g_{\mu\nu} v \cdot p - v_\nu p_\mu] \underbrace{\left[\hat{F}_A(n \cdot p) + \frac{Q_\ell f_B}{v \cdot p} \right]}_{F_A(n \cdot p)} + \underbrace{\frac{v_\nu p_\mu}{v \cdot p} Q_\ell f_B}_{\text{irrelevant after the contraction with } \varepsilon_\nu^*}.$$

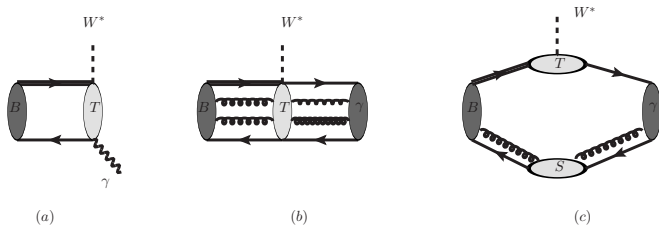
irrelevant after the contraction with ε_ν^*

Current status of $B \rightarrow \gamma \ell \nu$

- **Factorization properties at leading power** [Korchemsky, Pirjol and Yan, 2000; Descotes-Genon and Sachrajda, 2002; Lunghi, Pirjol and Wyler, 2003; Bosch, Hill, Lange and Neubert, 2003].
- Leading power contributions at NLL and **(partial)-subleading power corrections at tree level** [Beneke and Rohrwild, 2011].
- **Subleading power corrections from the dispersion technique:**
 - ▶ Soft two-particle correction **at tree level** [Braun and Khodjamirian, 2013].
 - ▶ Soft two-particle correction **at one loop** [Wang, 2016].
 - ▶ **Three-particle B -meson DA's contribution** at tree level [Wang, 2016; Beneke et al, 2018].
 - ▶ Subleading effective current and twist-5 and 6 corrections at tree level. [Beneke et al, 2018].
- **Subleading power corrections from the direct QCD approach:**
 - ▶ Hadronic photon corrections **at tree level** up to the twist-4 accuracy [Khodjamirian, Stoll, Wyler, 1995; Ali, Braun, 1995; Eilam, Halperin, Mendel, 1995].
 - ▶ Hadronic photon corrections of **twist-two at one loop** and of **higher-twist at tree level** [Ball and Kou, 2003; Wang and Shen, 2018].

The general picture

- Schematic structure of the distinct mechanisms:



A: hard subgraph that includes both photon and W^* vertices

B: real photon emission at large distances

C: Feynman mechanism: soft quark spectator

$$\left(\frac{\Lambda}{m_b}\right)^{1/2} + \left(\frac{\Lambda}{m_b}\right)^{3/2} + \dots$$

$$\left(\frac{\Lambda}{m_b}\right)^{3/2} + \dots$$

$$\left(\frac{\Lambda}{m_b}\right)^{3/2} + \dots$$

- Operator definitions of different terms needed for an unambiguous classification.

Method I: Dispersion technique

- Basic idea [Khodjamirian, 1999]:

$$\begin{aligned}\tilde{T}_{\nu\mu}(p, q) &\equiv \int d^4x e^{ip \cdot x} \langle 0 | T \{ j_{\nu,em}(x), [\bar{u}\gamma_\mu(1-\gamma_5)b](0) \} | B^-(p+q) \rangle \Big|_{p^2 < 0}, \\ &= v \cdot p \left[-i \varepsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p) + g_{\mu\nu}^\perp \hat{F}_A^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p) \right] + \dots\end{aligned}$$

- Power counting: $n \cdot p \sim \mathcal{O}(m_b)$, $\bar{n} \cdot p \sim \mathcal{O}(\Lambda)$.
- Dispersion relations [Braun and Khodjamirian, 2013]:

$$\begin{aligned}F_V^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p) &= \frac{2}{3} \frac{f_\rho m_\rho}{m_\rho^2 - p^2 - i0} \frac{2m_B}{m_B + m_\rho} V(q^2) + \frac{1}{\pi} \int_{\omega_s}^{\infty} d\omega' \frac{\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega')}{\omega' - \bar{n} \cdot p - i0}, \\ \hat{F}_A^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p) &= \frac{2}{3} \frac{f_\rho m_\rho}{m_\rho^2 - p^2 - i0} \frac{2(m_B + m_\rho)}{n \cdot p} A_1(q^2) + \frac{1}{\pi} \int_{\omega_s}^{\infty} d\omega' \frac{\text{Im}_{\omega'} \hat{F}_A^{B \rightarrow \gamma^*}(n \cdot p, \omega')}{\omega' - \bar{n} \cdot p - i0}.\end{aligned}$$

- LCSR for the $B \rightarrow \rho$ form factors:

$$\begin{aligned}\frac{2}{3} \frac{f_\rho m_\rho}{n \cdot p} \text{Exp} \left[-\frac{m_\rho^2}{n \cdot p \omega_M} \right] \frac{2m_B}{m_B + m_\rho} V(q^2) &= \frac{1}{\pi} \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \left[\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right], \\ \frac{2}{3} \frac{f_\rho m_\rho}{n \cdot p} \text{Exp} \left[-\frac{m_\rho^2}{n \cdot p \omega_M} \right] \frac{2(m_B + m_\rho)}{n \cdot p} A_1(q^2) &= \frac{1}{\pi} \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \left[\text{Im}_{\omega'} \hat{F}_A^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right].\end{aligned}$$

Method I: Dispersion technique

- Improved dispersion relations (setting $\bar{n} \cdot p = 0$) [Master formula I]:

$$F_V(n \cdot p) = \underbrace{\frac{1}{\pi} \int_0^{\omega_s} d\omega' \frac{n \cdot p}{m_p^2} \text{Exp} \left[\frac{m_p^2 - \omega' n \cdot p}{n \cdot p \omega_M} \right]}_{\text{nonperturbative modification}} \left[\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right],$$

$$+ \frac{1}{\pi} \int_{\omega_s}^{\infty} d\omega' \frac{1}{\omega'} \left[\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right].$$

- Comparison with the HQE result [Master formula II]:

$$F_V(n \cdot p) = \underbrace{\frac{1}{\pi} \int_0^{\infty} d\omega' \frac{1}{\omega'} \left[\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right]}_{\text{HQE expression, not always well defined}}$$

$$+ \underbrace{\frac{1}{\pi} \int_0^{\omega_s} d\omega' \left\{ \frac{n \cdot p}{m_p^2} \text{Exp} \left[\frac{m_p^2 - \omega' n \cdot p}{n \cdot p \omega_M} \right] - \frac{1}{\omega'} \right\}}_{\text{nonperturbative modification}} \left[\text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') \right].$$

- Spectral density at tree level:

$$\frac{1}{\pi} \text{Im}_{\omega'} F_V^{B \rightarrow \gamma^*}(n \cdot p, \omega') = \frac{Q_u f_B m_B}{n \cdot p} \underbrace{\phi_B^+(\omega', \mu)}_{\mathcal{O}(1)} + \mathcal{O}(\alpha_s, \Lambda/m_b).$$

of $\mathcal{O}(1/\Lambda)$ [$\mathcal{O}(1/m_b)$] for $\omega' \sim \mathcal{O}(\Lambda)$ [$\omega' \sim \mathcal{O}(\Lambda^2/m_b)$]

Power suppressed soft contribution!

Method I: Dispersion technique

- New Improvement I [Wang, 2016]:
 - ▶ Soft two-particle correction at NLO:

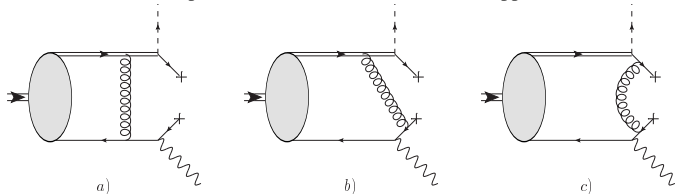
$$F_{V,2P}^{\text{soft,NLL}}(n \cdot p) \propto \int_0^{\omega_s} d\omega' \left\{ \frac{n \cdot p}{m_p^2} \text{Exp} \left[\frac{m_p^2 - \omega' n \cdot p}{n \cdot p \omega_M} \right] - \frac{1}{\omega'} \right\} \phi_{B,\text{eff}}^+(\omega', \mu),$$

$$\phi_{B,\text{eff}}^+(\omega', \mu) = \phi_B^+(\omega', \mu) + \frac{\alpha_s(\mu) C_F}{4\pi} [\text{New}].$$

- ▶ Three-particle correction with **KKQT parametrization** at tree level.

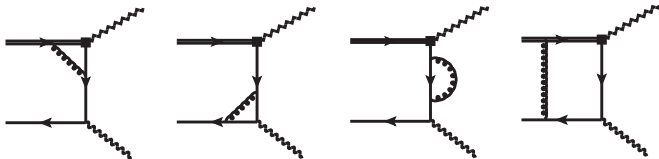
- New Improvement II [Beneke et al, 2018]:

- ▶ Compute the matrix element of the subleading effective current $\bar{q} \gamma_\mu \frac{i \overrightarrow{D}}{2m_b} h_v$ at LO.
- ▶ Three-particle correction with **BJM parametrization** at tree level.
- ▶ Twist-five and -six four-particle corrections in factorization approximation.



Method II: Direct QCD approach

- Leading power contribution at NLO:



- Perturbative QCD factorization formula:

$$F_{V,LP}(n \cdot p) = F_{A,LP}(n \cdot p) = \frac{Q_u m_B}{n \cdot p} \tilde{f}_B(\mu) C_{\perp}(n \cdot p, \mu) \int_0^{\infty} d\omega \frac{\phi_B^+(\omega, \mu)}{\omega} J_{\perp}(n \cdot p, \omega, \mu).$$

- Both the hard function and the jet function can be determined with two different methods.

$$C_{\perp} = 1 - \frac{\alpha_s C_F}{4\pi} \left[2 \ln^2 \frac{\mu}{n \cdot p} + 5 \ln \frac{\mu}{m_b} - 2 \text{Li}_2 \left(1 - \frac{1}{r} \right) - \ln^2 r + \frac{3r-2}{1-r} \ln r + \frac{\pi^2}{12} + 6 \right],$$

$$J_{\perp} = 1 + \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{\mu^2}{n \cdot p (\omega - \bar{n} \cdot p)} - \frac{\pi^2}{6} - 1 \right] + \mathcal{O}(\alpha_s^2).$$

QCD resummation for the leading power contribution

- RG evolution of the hard functions at NLL:

$$\frac{d}{d \ln \mu} C_{\perp}(n \cdot p, \mu) = \left[\underbrace{-\Gamma_{\text{cusp}}(\mu)}_{\text{three loops}} \ln \frac{\mu}{n \cdot p} + \underbrace{\gamma_h(\mu)}_{\text{two loops}} \right] C_{\perp}(n \cdot p, \mu),$$

$$\frac{d}{d \ln \mu} \tilde{f}_B(\mu) = \underbrace{\tilde{\gamma}(\mu)}_{\text{two loops}} \tilde{f}_B(\mu).$$

- RG evolution of the twist-two B -meson LCDA at one loop [Lange and Neubert, 2003]:

$$\frac{d\phi_B^+(\omega, \mu)}{d \ln \mu} = - \left[\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu}{\omega} + \gamma_+(\alpha_s) \right] \phi_B^+(\omega, \mu) - \omega \int_0^{\infty} d\omega' \Gamma_+(\omega, \omega', \mu) \phi_B^+(\omega', \mu).$$

- ▶ Renormalization of $[\bar{q}_s(t\bar{n}) \not{n} \Gamma b_v(0)]$ does not commute with the short-distance expansion [Braun, Ivanov and Korchemsky, 2004].

$$[(\bar{q}_s Y_s)(t\bar{n}) \not{n} \Gamma (Y_s^\dagger b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!} \left[\bar{q}_s(0) (n \cdot \overleftarrow{D})^p \not{n} \Gamma b_v(0) \right]_R.$$

- ▶ Eigenfunctions of the Lange-Neubert renormalization kernel [Bell, Feldmann, YMW and Yip, 2013].
- ▶ **Two-loop evolution of $\phi_B^+(\omega, \mu)$ essential to the NLL resummation.**

QCD resummation for the leading power contribution

- (Partial)-NLL resummation improved factorization formula:

$$\begin{aligned}
 F_{V,LP}(n \cdot p) &= F_{A,LP}(n \cdot p) \\
 &= \frac{Q_u m_B}{n \cdot p} [U_2(n \cdot p, \mu_{h2}, \mu) \tilde{f}_B(\mu_{h2})] [U_1(n \cdot p, \mu_{h1}, \mu) C_\perp(n \cdot p, \mu_{h1})] \\
 &\quad \times \underbrace{\int_0^\infty d\omega \frac{\phi_B^+(\omega, \mu)}{\omega} J_\perp(n \cdot p, \omega, \mu)} \\
 &= \lambda_B^{-1}(\mu) \left\{ 1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left[\ln^2 \frac{\mu^2}{n \cdot p \mu_0} + 2 \ln \frac{\mu^2}{n \cdot p \mu_0} \sigma_1(\mu) + \sigma_2(\mu) - \frac{\pi^2}{6} - 1 \right] \right\}.
 \end{aligned}$$

- Inverse-logarithmic moments [Beneke and Rohrwild, 2011]:

$$\begin{aligned}
 \lambda_B^{-1}(\mu) &= \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu), \quad \sigma_B^{(n)}(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\mu_0}{\omega} \phi_B^+(\omega, \mu). \\
 \frac{\lambda_B(\mu_0)}{\lambda_B(\mu)} &= 1 + \frac{\alpha_s(\mu_0) C_F}{4\pi} \ln \frac{\mu}{\mu_0} \left[2 - 2 \ln \frac{\mu}{\mu_0} - 4 \sigma_B^{(1)}(\mu_0) \right] + \mathcal{O}(\alpha_s^2). \\
 \frac{d}{d \ln \mu} \sigma_B^{(n)}(\mu) &= \mathcal{O}(\alpha_s) \Rightarrow \text{No } [\alpha_s(\mu)]^0 \ln(\mu/\mu_0) \text{ due to the evolution.}
 \end{aligned}$$

- Not aiming at resummation of $\ln^k(\mu/\mu_0)$ given the fact that $\mu \sim 1.5\text{GeV}$ and $\mu_0 \sim 1.0\text{GeV}$.
Resummation in the “dual” momentum space [Bell, Feldmann, YMW and Yip, 2013].

Sub-leading power corrections in QCD

- Subleading-power local contributions [Beneke and Rohrwild, 2011]:

$$F_{V,NLP}^{LC}(n \cdot p) = -F_{A,NLP}^{LC}(n \cdot p) = \frac{Q_u f_B m_B}{(n \cdot p)^2} + \frac{Q_b f_B m_B}{n \cdot p m_b}.$$

- ▶ Q_u term: Subleading correction from the hard-collinear u -quark propagator.
- ▶ Q_b term: Subleading correction from the photon radiation off the b -quark.
- ▶ Power suppressed local contributions violate the large-recoil symmetry relation.

- Subleading-power non-local contribution:

$$F_{V,NLP}^{NLC}(n \cdot p) = \xi(v \cdot p) \stackrel{?}{=} -\frac{iQ_u}{(n \cdot p)^2} \bar{C}_{\perp,NLP}(n \cdot p, \mu) \times \int ds \left\langle 0 \left| [\bar{u} Y_s](s \bar{n}) \frac{i n \cdot \overleftarrow{\partial}}{i \bar{n} \cdot \overleftarrow{\partial}} \not{n} (1 - \gamma_5) [Y_s^\dagger b_v](0) \right| B^-(v) \right\rangle \tilde{J}_{NLP} \left(\frac{\mu^2 s}{v \cdot p} \right).$$

A systematic SCET treatment of power corrections to $B \rightarrow \gamma \ell \nu$ in demand.

- $B \rightarrow \gamma \ell \nu$ with scalar fields [Beneke and Feldmann, 2003]:

$$\langle \gamma | J | \bar{q} b \rangle = (C_0 + C_1) \star \phi_{q\bar{b}} + \left[C_2 \star \phi_{q\bar{b}g} \right]_{\mathbf{v}} + \left[\phi_{q\bar{q}}^\gamma \star C_3 \right]_{\mathbf{v}} \star \phi_{q\bar{b}}.$$

Communication between the soft and collinear systems.

Twist-two hadronic photon correction from LCSR

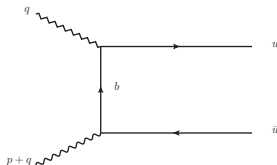
- Main task: QCD factorization for the vacuum-to-photon correlation function.

$$\Pi_\mu(p, q) = \int d^4x e^{iq \cdot x} \langle \gamma(p) | T \{ \underbrace{\bar{u}(x) \gamma_{\mu\perp} (1 - \gamma_5) b(x)}_{\text{weak current}}, \bar{b}(0) \gamma_5 u(0) \} | 0 \rangle.$$

- Power counting scheme:

$$n \cdot p \sim \mathcal{O}(m_b),$$

$$|(p+q)^2 - m_b^2| \sim \mathcal{O}(m_b^2).$$



- QCD matrix element at LO:

$$F_\mu(p, q) = \int d^4x e^{iq \cdot x} \langle q(zp) \bar{q}(\bar{z}p) | T \{ \bar{u}(x) \gamma_{\mu\perp} (1 - \gamma_5) b(x), \bar{b}(0) \gamma_5 u(0) \} | 0 \rangle,$$

$$= \frac{i}{2} \frac{\bar{n} \cdot q}{z(p+q)^2 + \bar{z}q^2 - m_b^2 + i0} \bar{u}(zp) \gamma_{\mu\perp} \not{n} (1 + \gamma_5) v(\bar{z}p).$$

Hard-collinear factorization for the considered correlation function (pion-photon form factor).

Twist-two factorization at tree level

- Operator matching automatically:

$$F_{\mu}^{(0)}(p, q) = \frac{i}{2} \frac{\bar{n} \cdot q}{z'(p+q)^2 + \bar{z}' q^2 - m_b^2 + i0} * \langle O_{A, \mu}(z, z') \rangle^{(0)}.$$

- Collinear operator in the momentum space:

$$\begin{aligned} O_{j, \mu}(z') &= \frac{n \cdot p}{2\pi} \int d\tau e^{-iz' \tau n \cdot p} \bar{\xi}(\tau n) W_c(\tau n, 0) \Gamma_j \xi(0), \\ \Gamma_A &= \gamma_{\mu\perp} \not{n} (1 + \gamma_5). \end{aligned}$$

- Matrix element of the collinear operator:

$$\langle O_{A, \mu}(z, z') \rangle = \langle q(zp) \bar{q}(\bar{z}p) | O_{A, \mu}(z') | 0 \rangle = \bar{\xi}(zp) \gamma_{\mu\perp} \not{n} (1 + \gamma_5) \xi(zp) \delta(z - z') + \mathcal{O}(\alpha_s).$$

- Operator matching with the collinear operator defining the standard photon DAs:

$$\begin{aligned} O_{A, \mu} &= O_{1, \mu} + O_{2, \mu} + O_{E, \mu}, \\ \Gamma_1 &= \gamma_{\mu\perp} \not{n}, \quad \Gamma_2 = \frac{n^\nu}{2} \varepsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}, \quad \underbrace{\Gamma_E}_{\text{evanescent operator}} = \gamma_{\mu\perp} \not{n} \gamma_5 - \frac{n^\nu}{2} \varepsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}. \end{aligned}$$

Twist-two factorization at tree level

- Operator matching with the evanescent operator:

$$F_\mu(p, q) = \sum_i C_i(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z, z') \rangle.$$

Expansion \Downarrow at tree level

$$C_1^{(0)} = C_2^{(0)} = C_E^{(0)} = \frac{i}{2} \frac{\bar{n} \cdot q}{z'(p+q)^2 + \bar{z}'q^2 - m_b^2 + i0}.$$

- Hard-collinear factorization at tree level:

$$\begin{aligned} \Pi_\mu(p, q) &= \frac{i}{2} g_{\text{em}} Q_u \chi(\mu) \langle \bar{q}q \rangle(\mu) \varepsilon^{*\alpha}(p) \left[g_{\mu\alpha}^\perp - i \varepsilon_{\mu\alpha\nu\beta} n^\nu v^\beta \right] \\ &\times \int_0^1 dz \phi_\gamma(z, \mu) \frac{n \cdot p \bar{n} \cdot q}{z(p+q)^2 + \bar{z}q^2 - m_b^2 + i0} + \mathcal{O}(\alpha_s). \end{aligned}$$

Evanescent operator does not mix into the physical operators at LO.

- Twist-two photon DA:

$$\langle \gamma(p) | \bar{\xi}(x) W_c(x, 0) \sigma_{\alpha\beta} \xi(0) | 0 \rangle = i g_{\text{em}} Q_q \chi(\mu) \langle \bar{q}q \rangle(\mu) (p_\beta \varepsilon_\alpha^* - p_\alpha \varepsilon_\beta^*) \int_0^1 dz e^{izp \cdot x} \phi_\gamma(z, \mu).$$

Twist-two LCSR at tree level

- Hadronic dispersion relation:

$$\begin{aligned} \Pi_\mu(p, q) &= \frac{i}{2} g_{\text{em}} \frac{f_B m_B^2}{m_b + m_u} \varepsilon^{*\alpha}(p) \left[g_{\mu\alpha}^\perp F_{A, \text{photon}}^{2\text{PLT}}(n \cdot p) - i \varepsilon_{\mu\alpha\nu\beta} n^\nu v^\beta F_{V, \text{photon}}^{2\text{PLT}}(n \cdot p) \right] \\ &\times \frac{n \cdot p}{(p+q)^2 - m_B^2 + i0} + \int_{s_0}^\infty ds \frac{\rho_\mu^h(s, q^2)}{s - (p+q)^2 - i0}. \end{aligned}$$

- Tree-level sum rules:

$$\frac{f_B m_B}{m_b + m_u} F_{V(A), \text{photon}}^{2\text{PLT}}(n \cdot p) = Q_u \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_{z_0}^1 \frac{dz}{z} \exp \left[-\frac{m_b^2 - \bar{z} q^2}{z M^2} + \frac{m_B^2}{M^2} \right] \phi_\gamma(z, \mu) + \mathcal{O}(\alpha_s).$$

- Power counting scheme:

$$(s_0 - m_b^2) \sim M^2 \sim \mathcal{O}(m_b \Lambda), \quad \bar{z}_0 = \frac{s_0 - m_b^2}{s_0 - q^2} \sim \Lambda/m_b.$$

↓

$$F_{V, \text{photon}}^{2\text{PLT}} \sim F_{A, \text{photon}}^{2\text{PLT}} \sim \mathcal{O} \left(\frac{\Lambda}{m_b} \right)^{3/2}.$$

Hadronic photon correction is power suppressed in the heavy quark expansion.

Twist-two factorization at NLO

- QCD matrix element at NLO:



- Extracting the hard contribution with the method of regions:

$$F_{\mu}^{(1)}(p, q) = \sum_{i=1,2} T_{i,\text{hard}}^{(1)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z, z') \rangle^{(0)} + \dots$$

One-loop QCD amplitude in **NDR scheme**:

$$\begin{aligned} T_{i,\text{hard}}^{(1)}|_{\text{NDR}} = & \frac{\alpha_s C_F}{4\pi} \left\{ (-2) \left[\frac{1-r_2}{r_1-r_2} \ln \frac{1-r_1}{1-r_2} + \frac{1-r_3}{r_1-r_3} \ln \frac{1-r_1}{1-r_3} + \frac{3}{1-r_1} \right] \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{m_b^2} \right) \right. \\ & + \frac{2(1-r_2)}{r_1-r_2} \text{Li}_2 \left(1 - \frac{1-r_1}{1-r_2} \right) + \frac{2(1-r_3)}{r_1-r_3} \text{Li}_2 \left(1 - \frac{1-r_1}{1-r_3} \right) \\ & + \left(\frac{1-r_2}{r_1-r_2} + \frac{1-r_3}{r_1-r_3} \right) \ln^2(1-r_1) - \frac{1-r_2}{r_1-r_2} \ln^2(1-r_2) - \frac{1-r_3}{r_1-r_3} \ln^2(1-r_3) \\ & \left. + \left[\frac{2}{r_1(r_3-r_1)} + \frac{2(r_3-2)}{r_3-r_1} + \frac{6}{1-r_1} - \frac{2-r_2}{r_1-r_2} + \frac{4}{r_1} - 4 \right] \ln(1-r_1) + \dots \right\} C_{i,\text{hard}}^{(0)}. \end{aligned}$$

Twist-two factorization at NLO

- Expanding the matching equation at NLO:

$$\begin{aligned} & \sum_i T_i^{(1)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z, z') \rangle^{(0)} \\ = & \sum_i \left[C_i^{(1)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z, z') \rangle^{(0)} + C_i^{(0)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z, z') \rangle^{(1)} \right]. \end{aligned}$$

- One-loop renormalized matrix elements of collinear operators:

$$\langle O_{i,\mu} \rangle^{(1)} = \sum_j \left[M_{ij, \text{bare}}^{(1),R} + Z_{ij}^{(1)} \right] \langle O_{j,\mu} \rangle^{(0)}.$$

Vanishing bare matrix element $M_{ij, \text{bare}}^{(1),R}$ in dimensional regularization \Rightarrow

$$C_i^{(1)} = T_i^{(1)} - \sum_{j=1,2,E} C_j^{(0)} * Z_{ji}^{(1)} = \underbrace{T_i^{(1)} - C_i^{(0)} * Z_{ii}^{(1)}}_{T_{i, \text{hard}}^{(1), \text{reg}}} - C_E^{(0)} * \underbrace{Z_{Ei}^{(1)}}_{\text{operator mixing}}.$$

- The IR finite matrix element of the evanescent operator vanishes (Dugan and Grinstein, 1991).

$$Z_{Ei}^{(1)} = -M_{Ei, \text{bare}}^{(1), \text{off}}.$$

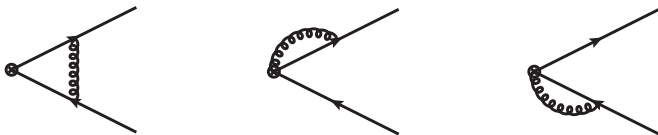
IR singularities regularized by any parameter other than the dimensions of spacetime.

Twist-two factorization at NLO

- Master formula for the hard function:

$$C_i^{(1)} = T_i^{(1)} - C_i^{(0)} * Z_{ii}^{(1)} + C_E^{(0)} * M_{Ei, \text{bare}}^{(1), \text{off}} = T_{i, \text{hard}}^{(1), \text{reg}} + C_E^{(0)} * M_{Ei, \text{bare}}^{(1), \text{off}}.$$

- The IR subtraction:



Only the first diagram can potentially generate the operator mixing.

$$C_E^{(0)} * M_{Ei, \text{bare}}^{(1), \text{off}} = 0, \text{ with } i = 1, 2.$$

γ_5 -scheme independent subtraction term!

$$C_i^{(1)} = T_{i, \text{hard}}^{(1), \text{reg}}.$$

Twist-two factorization at NLO

- The NLO factorization formula:

$$\begin{aligned} \Pi_\mu(p, q) &= g_{\text{em}} Q_u n \cdot p \chi(\mu) \langle \bar{q}q \rangle(\mu) \varepsilon^{*\alpha}(p) \left[g_{\mu\alpha}^\perp - i \varepsilon_{\mu\alpha\nu\beta} n^\nu v^\beta \right] \\ &\int_0^1 dz \phi_\gamma(z, \mu) \left[C_i^{(0)}(z, (p+q)^2, q^2) + C_i^{(1)}(z, (p+q)^2, q^2) \right] + \mathcal{O}(\alpha_s^2). \end{aligned}$$

- RG evolution of the twist-two photon LCDA [Lepage and Brodsky, 1979]:

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} \left[\chi(\mu) \langle \bar{q}q \rangle(\mu) \phi_\gamma(z, \mu) \right] &= \int_0^1 dz' \tilde{V}(z, z') \left[\chi(\mu) \langle \bar{q}q \rangle(\mu) \phi_\gamma(z', \mu) \right], \\ \tilde{V}_0(z, z') &= 2C_F \left[\frac{\bar{z}}{\bar{z}'} \frac{1}{z-z'} \theta(z-z') + \frac{z}{z'} \frac{1}{z'-z} \theta(z'-z) \right]_+ \underbrace{-C_F \delta(z-z')}_{\text{New term!}}. \end{aligned}$$

Multiplicative renormalization at LO:

$$\chi(\mu) \langle \bar{q}q \rangle(\mu) \phi_\gamma(z, \mu) = 6z\bar{z} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_{T,n}^{(0)}/(2\beta_0)} \chi(\mu_0) \langle \bar{q}q \rangle(\mu_0) a_n(\mu_0) C_n^{3/2}(2z-1).$$

Twist-two factorization at NLL

- NLL resummation needs two-loop evolution kernel [Belitsky, Müller and Freund, 1999; etc]:

$$\tilde{V}_1(z, z') = \frac{N_f}{2} C_F \tilde{V}_N(z, z') + C_F C_A \tilde{V}_G(z, z') + C_F^2 \tilde{V}_F(z, z').$$

Two-loop evolution of the Gegenbauer moment [Müller, 1994/1995]:

$$\begin{aligned} \chi(\mu) \langle \bar{q}q \rangle(\mu) a_n(\mu) &= E_{T,n}^{\text{NLO}}(\mu, \mu_0) \chi(\mu_0) \langle \bar{q}q \rangle(\mu_0) a_n(\mu_0) \\ &+ \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{n-2} E_{T,n}^{\text{LO}}(\mu, \mu_0) d_{T,n}^k(\mu, \mu_0) \chi(\mu_0) \langle \bar{q}q \rangle(\mu_0) a_n(\mu_0). \end{aligned}$$

Construction from the forward anomalous dimensions and the special conformal anomaly matrix.

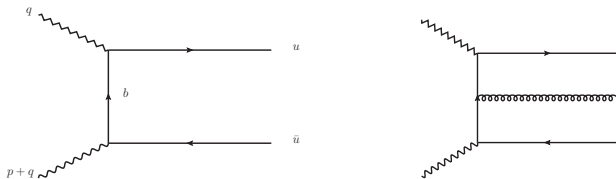
- NLL resummation improved factorization formula:

$$\begin{aligned} \Pi_\mu(p, q) &= g_{\text{em}} Q_u n \cdot p \chi(\mu) \langle \bar{q}q \rangle(\mu) \varepsilon^{*\alpha}(p) \left[g_{\mu\alpha}^\perp - i \varepsilon_{\mu\alpha\nu\beta} n^\nu v^\beta \right] \\ &\times \underbrace{\sum_{n=0} a_n(\mu) K_n((p+q)^2, q^2)}_{\text{dispersion form}} + \mathcal{O}(\alpha_s^2), \\ &\int_0^\infty \frac{ds}{s - (p+q)^2 - i0} \left[\rho^{(0)}(s, q^2) + \frac{\alpha_s C_F}{4\pi} \rho^{(1)}(s, q^2) \right]. \end{aligned}$$

- NLL twist-two LCSR of $F_{V,\text{photon}}^{2\text{PLT}}(n \cdot p)$ can be constructed immediately.

Higher-twist hadronic photon corrections

- General picture:



Higher twist corrections from both two-particle and three-particle LCDAs.

- Systematic studies of the photon LCDAs [Ball, Braun and Kivel, 2002]:

$$\begin{aligned}
 & \langle \gamma(p) | \bar{q}(x) W_c(x, 0) \sigma_{\alpha\beta} q(0) | 0 \rangle \\
 &= i g_{\text{em}} Q_q \langle \bar{q}q \rangle(\mu) (p_\beta \epsilon_\alpha^* - p_\alpha \epsilon_\beta^*) \int_0^1 dz e^{izp \cdot x} \left[\chi(\mu) \overbrace{\phi_\gamma(z, \mu)}^{\text{twist 2}} + \frac{x^2}{16} \overbrace{\mathbb{A}(z, \mu)}^{\text{twist 4}} \right] \\
 &+ \frac{i}{2} g_{\text{em}} Q_q \frac{\langle \bar{q}q \rangle(\mu)}{q \cdot x} (x_\beta \epsilon_\alpha^* - x_\alpha \epsilon_\beta^*) \int_0^1 dz e^{izp \cdot x} \underbrace{h_\gamma(z, \mu)}_{\text{twist 4}}. \\
 & \langle \gamma(p) | \bar{q}(x) W_c(x, 0) g_s G_{\alpha\beta}(vx) i \gamma_\rho q(0) | 0 \rangle \quad \text{twist 3} \\
 &= g_{\text{em}} Q_q f_3 \gamma(\mu) p_\rho (p_\beta \epsilon_\alpha^* - p_\alpha \epsilon_\beta^*) \int [\mathcal{D}\alpha_i] e^{i(\alpha_q + v\alpha_g)p \cdot x} \overbrace{V(\alpha_i, \mu)}^{\text{twist 3}}.
 \end{aligned}$$

Can be rewritten as collinear matrix elements in SCET [Hardmeier et al, 2006].

Two-particle higher-twist hadronic photon corrections

- Two-particle higher twist corrections:

$$\begin{aligned} \Pi_\mu(p, q) \supset & \frac{i}{4} g_{\text{em}} Q_q (p \cdot q) \int_0^1 dz \left\{ \varepsilon_\mu^* \left[\frac{\rho_{A,2}^{2\text{PHT}}((p+q)^2, q^2, z)}{[(zp+q)^2 - m_b^2 + i0]^2} + \frac{\rho_{A,3}^{2\text{PHT}}((p+q)^2, q^2, z)}{[(zp+q)^2 - m_b^2 + i0]^3} \right] \right. \\ & \left. - i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} n^\alpha v^\beta \left[\frac{\rho_{V,2}^{2\text{PHT}}((p+q)^2, q^2, z)}{[(zp+q)^2 - m_b^2 + i0]^2} + \frac{\rho_{V,3}^{2\text{PHT}}((p+q)^2, q^2, z)}{[(zp+q)^2 - m_b^2 + i0]^3} \right] \right\}. \end{aligned}$$

- Two-particle higher twist corrections violate the symmetry relations of $B \rightarrow \gamma$ form factors.

$$\rho_{A,2} \neq \rho_{V,2}, \quad \rho_{A,3} \neq \rho_{V,3}.$$

- Heavy-quark scaling of the two-particle corrections:

$$F_{V,\text{photon}}^{2\text{PHT,LL}}(n \cdot p) \sim F_{A,\text{photon}}^{2\text{PHT,LL}}(n \cdot p) \sim \left(\frac{\Lambda}{m_b} \right)^{3/2}.$$

- Only suppressed by one factor of Λ/m_b compared with the “direct” photon contribution.
- No correspondence between the heavy-quark expansion and the twist expansion [see also $B \rightarrow \pi$ form factors, pion-photon form factor].

Three-particle higher-twist hadronic photon corrections

- Quark propagator in the background gluon field [Balitsky and Braun, 1988]:

$$\langle 0|T\{\bar{b}(x), b(0)\}|0\rangle \\ \supset i g_s \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot x} \int_0^1 dv \left[\frac{vx_\mu}{k^2 - m_b^2} G^{\mu\nu}(vx) \gamma_\nu - \frac{\not{k} + m_b}{2(k^2 - m_b^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} \right] + \dots$$

- Three particle corrections at tree level:

$$\Pi_\mu(p, q) \supset i g_{\text{em}} Q_q(p \cdot q) \int_0^1 dv \int [\mathcal{D}\alpha_i] \left\{ \varepsilon_\mu^* \left[\frac{\rho_{A,2}^{3P}((p+q)^2, q^2, \alpha_i, v)}{[(\alpha_q + v\alpha_g)p + q]^2 - m_b^2 + i0]^2} \right. \right. \\ \left. \left. + \frac{\rho_{A,3}^{3P}((p+q)^2, q^2, \alpha_i, v)}{[(\alpha_q + v\alpha_g)p + q]^2 - m_b^2 + i0]^3} \right] - i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} n^\alpha v^\beta \right. \\ \left. \times \left[\frac{\rho_{V,2}^{3P}((p+q)^2, q^2, \alpha_i, v)}{[(\alpha_q + v\alpha_g)p + q]^2 - m_b^2 + i0]^2} + \frac{\rho_{V,3}^{3P}((p+q)^2, q^2, \alpha_i, v)}{[(\alpha_q + v\alpha_g)p + q]^2 - m_b^2 + i0]^3} \right] \right\}.$$

- Heavy-quark scaling of the three-particle corrections:

$$F_{V,\text{photon}}^{3P,LL}(n \cdot p) \sim F_{A,\text{photon}}^{3P,LL}(n \cdot p) \sim \left(\frac{\Lambda}{m_b} \right)^{5/2}.$$

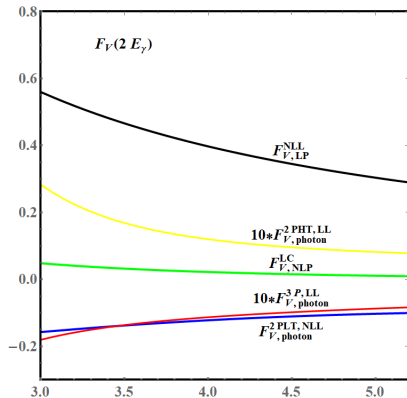
Double suppression compared with the “direct” photon contribution.

Final expressions of the $B \rightarrow \gamma$ form factors

- Adding up different pieces together:

$$\begin{aligned}
 F_V(n \cdot p) &= F_{V,LP}(n \cdot p) + F_{V,NLP}^{LC}(n \cdot p) + F_{V,photon}^{2PLT}(n \cdot p) + F_{V,photon}^{2PHT,LL}(n \cdot p) + F_{V,photon}^{3P,LL}(n \cdot p), \\
 F_A(n \cdot p) &= F_{A,LP}(n \cdot p) + F_{A,NLP}^{LC}(n \cdot p) + F_{A,photon}^{2PLT}(n \cdot p) + F_{A,photon}^{2PHT,LL}(n \cdot p) + F_{A,photon}^{3P,LL}(n \cdot p) \\
 &\quad + \frac{Q\ell f_B}{v \cdot p}.
 \end{aligned}$$

- Breakdown of various contributions [$\lambda_B = 354 \text{ MeV}$]:



Numerics with central inputs:

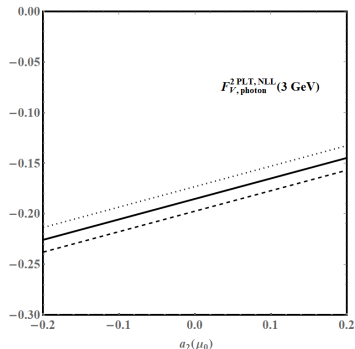
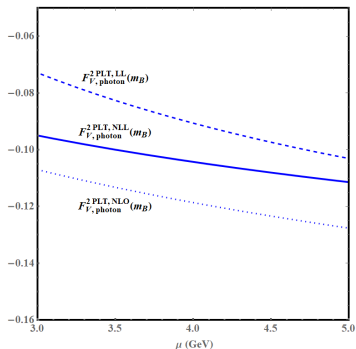
$$\begin{aligned}
 F_{V,2P}^{LP,NLL}(m_B) &= 0.28, \\
 F_{V,photon}^{2PLT,NLL}(m_B) &= -0.10, \\
 F_{V,photon}^{2PHT,LL}(m_B) &= 0.008, \\
 F_{V,photon}^{3P,LL}(m_B) &= -0.008, \\
 F_{V,NLP}^{LC}(m_B) &= 0.009, \\
 F_{A,NLP}^{LC}(m_B) &= 0.048.
 \end{aligned}$$

Twist-two hadronic photon effect yields $\mathcal{O}(30\%)$ correction.

Strong cancellation between $F_{V,photon}^{2PHT,LL}$ and $F_{V,photon}^{3P,LL}$.

Numerical impacts of higher-order corrections

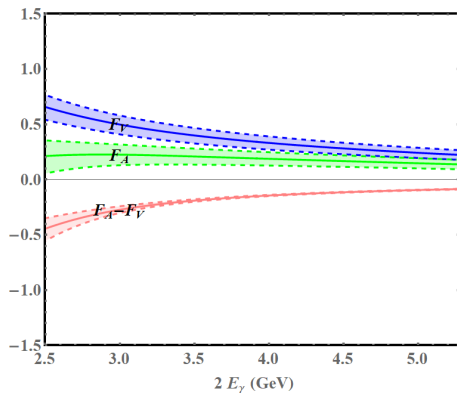
- Radiative correction and a_2 dependence of the twist-two hadronic photon correction:



- ▶ NLO QCD correction shifts the LO twist-two hadronic photon correction $\mathcal{O}(20 - 40)\%$.
- ▶ NLL resummation yields $\mathcal{O}(10 - 20)\%$ correction to the NLO prediction.
- ▶ Variation of $a_2(\mu_0) \in [-0.2, 0.2]$ leads to $\mathcal{O}(35\%)$ uncertainty at $n \cdot p = 3$ GeV.
⇒ More information about $a_2(\mu_0)$ necessary.
- ▶ Variation of $a_4(\mu_0) \in [-0.2, 0.2]$ yields minor uncertainty.
⇒ Yet higher conformal spin contributions might not be sizeable numerically.

Photon-energy dependence of the $B \rightarrow \gamma$ form factors

Including power suppressed hadronic photon corrections and the local contribution:



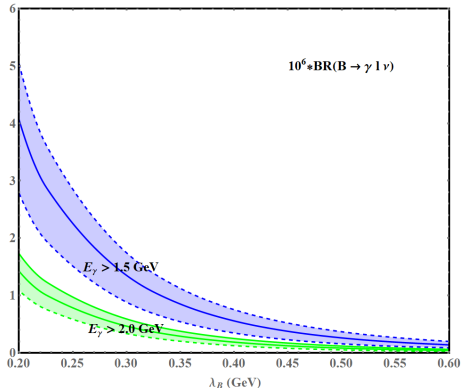
- Dominant uncertainties from $\lambda_B(\mu_0)$, $a_2(\mu_0)$ and μ .
- $F_A - F_V$ only comes from the subleading power contributions:
 - (a) Power suppressed local corrections.
 - (b) Higher-twist hadronic photon corrections.
- Faster growing F_V than F_A with the decrease of E_γ .

Partial branching fractions of $B \rightarrow \gamma \ell \nu$

- Integrated decay rate $\Delta BR(E_{\text{cut}})$:

$$\Delta BR(E_{\text{cut}}) = \tau_B \int_{E_{\text{cut}}}^{m_B/2} dE_\gamma \frac{d\Gamma}{dE_\gamma} (B \rightarrow \gamma \ell \nu).$$

- $\lambda_B(\mu_0)$ dependence of $\Delta BR(E_{\text{cut}})$:



- ▶ Belle 2015 data:
 $\Delta BR(1 \text{ GeV}) < 3.5 \times 10^{-6}$.
- ▶ Belle-II 2018 data [Talk by Bernlochner]: $\Delta BR(1 \text{ GeV}) = (1.4 \pm 1.0 \pm 0.4) \times 10^{-6}$.
- ▶ Expected statistical error for $\Delta BR(1 \text{ GeV})$ with 50 ab^{-1} of Belle-II data: ${}^{+0.18}_{-0.17} \times 10^{-6}$.
- ▶ **The photon-energy cut not sufficiently large.**
Power corrections numerically important for $E_\gamma < 1.5 \text{ GeV}$.

Concluding Remarks

- Understanding power corrections in $B \rightarrow \gamma \ell \nu$ important for precision flavour physics.
- Subleading power contributions from hadronic photon corrections.
 - ▶ NLL twist-two hadronic photon effect yields $\mathcal{O}(30\%)$ correction at $\lambda_B = 354 \text{ MeV}$.
 - ▶ Strong cancellation between $F_{V,\text{photon}}^{2\text{PHT,LL}}$ and $F_{V,\text{photon}}^{3\text{P,LL}}$.
 - ▶ Large-recoil symmetry violation due to $F_{V(A),\text{NLP}}^{\text{LC}}$, $F_{V,\text{photon}}^{2\text{PHT,LL}}$ and $F_{V,\text{photon}}^{3\text{P,LL}}$.
- The inverse moment $\lambda_B(\mu_0)$ not sufficient to describe $B \rightarrow \gamma \ell \nu$ in general.
- Connection between the dispersion technique and the direct QCD approach?
Can be also investigated in a simpler process $\gamma^* \gamma \rightarrow \pi$.
- A systematic treatment of subleading power corrections in SCET in demand.
 - ▶ SCET representation of the QCD $b \rightarrow u$ current at NLP including QED interaction.
 - ▶ QCD factorization for the subleading power SCET matrix elements.
 - ▶ QCD resummation for the parametrically large logarithms in the heavy quark expansion.