Spin-3 Topological Massive Gravity

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Outline

- Brief review of HS field theory
- TMG in AdS_3 and chiral gravity;
- Spin-3 TMG
- Spin-3 fluctuations about AdS_3;
- Spin-3 chiral gravity;
- Asym. symmetry of spin-3 chiral gravity
- Conclusion and discussions

High spin field

- Free HS theory well-defined in flat and curved spacetime; Frondal,
- Interacting HS field theory only well-defined in a spacetime with cosmological constant, both positive and negative; Vasiliev
- Remarkable feature: in $D \geq 4$, once we include one massless field with spin higher than two, we must include an infinite number of massless fields with various higher spins and also other compensator fields;
- Though it looks intractable, HS field theory has drawn much attention in the past decade, for its close relation with string theory and AdS/CFT correspondence;

HS and string theory

- It has been known for a long time string theory has a rich symmetry;
- The massless HS fields appear as the excitations of tensionless string;
- In AdS/CFT correspondence, what's the dual to the free SYM theory?
- Naively, one may expect that it's the tensionless string in AdS;
- However, the degrees of freedom on both sides in AdS_5/CFT_4 do not match;
- Nevertheless, it was conjectured by Polyakov and Klebanov in 2002 that three dim. O(N) model in the large N limit is dual to HS theory in AdS_4;
- It is very interesting, but will not be the topic I am going to talk about;

HS in AdS$_3$

- Extra compensator fields vanish;
- Finite truncation to a spin n is possible;
- Especially, the action of HS in AdS$_3$ could be rewritten as a Chern-Simons gravity;
- For pure AdS$_3$ gravity, it could be written as a Chern-Simons theory: Achucarro and Townsend 1986;E. Witten 1988
  1. Combine the frame-like fields and the spin connections into two SL(2,R) gauge fields:
  $$A = (\omega^a_\mu + \frac{1}{l} e^a_\mu) J_a dx^\mu, \qquad \tilde{A} = (\omega^a_\mu - \frac{1}{l} e^a_\mu) J_a dx^\mu.$$
  2. Einstein action + C.C. term
  $$S_{EH} + S_\Lambda = S_{CS}[A] - S_{CS}[\tilde{A}] \qquad (1.1)$$
  where with $k = \frac{l}{4G}$
  $$S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A); \qquad (1.2)$$

Spin-3 AdS$_3$ gravity Campoleoni et.al. 1008.4744

- To account for spin-3 field, SL(2,R) $--->$ SL(3,C);
- The $SL(3,C)$ group has the generators $J_a, T_{ab}(a,b=1,2,3)$ with T_{ab} being symmetric and traceless;
- They satisfy the following commutation relations:

$$[J_a, J_b] = \epsilon_{abc} J^c, \qquad [J_a, T_{bc}] = \epsilon^d{}_{a(b} T_{c)d},$$

$$[T_{ab}, T_{cd}] = \sigma(\eta_{a(c}\epsilon_{d)be} + \eta_{b(c}\epsilon_{d)ae}) J^e.$$

- We combine the vielbein-like fields and the connections of spin-2 and spin-3 into two gauge potentials $A, \tilde{A}$

$$A = ((\omega^a_\mu + \frac{1}{l} e^a_\mu) J_a + (\omega^{ab}_\mu + \frac{1}{l} e^{ab}_\mu) T_{ab}) dx^\mu,$$
$$\tilde{A} = ((\omega^a_\mu - \frac{1}{l} e^a_\mu) J_a + (\omega^{ab}_\mu - \frac{1}{l} e^{ab}_\mu) T_{ab}) dx^\mu;$$

Spin-3 AdS_3 gravity II

- Then the CS action gives the correct theory for spin-3 field coupled to gravity with a negative cosmological constant.
- Starting from CS theory, the asymptotic symmetry has been studied;
- It was found that with Brown-Henneaux b.c., spin-3 gravity in AdS_3 has W_3 asym. symmetry algebra, with the same central charge $c_L = c_R = 3l/2G$.
- It has been conjectured that for spin-n HS gravity in AdS_3, its asymp. symmetry algebra is W_n algebra with the same central charges;
- In the $n \to \infty$ limit, there is another approach starting from HS algebra directly; M. Henneaux and S.J. Rey 1008.4579

3D gravity

- No local physical d.o.f. in 3D gravity;
- This is also true for high spin fluctuations in 3-dim;
- However, there could be boundary d.o.f. ==> BTZ black hole;
- Nevertheless, one may add higher-derivative terms to have local d.o.f. (but also ghost usually);
- A simple choice is to add a gravitational Chern-Simons term, which is parity breaking and topological: S.Deser et.al. 1982

$$I_{CS} = \frac{1}{2\mu} \int d^3x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^{\rho}_{\lambda\sigma} \left(\partial_\mu \Gamma^{\sigma}_{\rho\nu} + \frac{2}{3} \Gamma^{\sigma}_{\mu\tau} \Gamma^{\tau}_{\nu\rho} \right) \tag{2.1}$$

- It leads to a new massive, propagating d.o.f;
- However, 3D TMG in AdS_3 is not well-defined for generic value μl, either because of the instability or because of negative energy for BTZ black hole;
- It may allowed other vacua: warped spacetime; D.Anninos et.al. 0807.3040

Chiral gravity conjecture W.Li et.al. 0801.4566

- Li, Song and Strominger in 2008 found that at the critical point $\mu l = \pm 1$, 3D TMG in AdS_3 is well-defined;
- In this case, the local massive mode becomes massless and degenerates with the massless graviton in the left-mover;
- Both local mode and left-moving graviton are just pure gauge;
- The only physical d.o.f. is the right-moving boundary graviton;
- That's why it's called chiral gravity;
- Moreover, by imposing self-consistent Brown-Henneaux B.C., it was found that the ASG is one copy of Virasoso algebra with central charge $c_R = 3l/G$;
- Conjecture: chiral gravity is holographically dual to a 2D CFT with c_R;
- The partition function is holomorphic and modular invariant, supporting this conjecture. A. Maloney et.al. 0903.4573

Remarks on chiral gravity conjecture

- There is actually a log mode, as the equation of fluctuation is a third order differential equation;
- Its existence once brought much debate on stability and chiral nature of the chiral gravity; Carlip et.al. (2008);Grumiller et.al.0805.2610
- It boils down that one may impose Brown-Heanneaux B.C. and get a chiral CFT dual;
- Instead, one may also impose a relaxed B.C. to allow the log mode, which could break chiral nature of the theory, but is still well-defined; Maloney et.al. 0903.4573
- In the latter case, the theory is conjectured to be dual to a logarithmic CFT;
- The graviton 1-loop determinant in TMG breaks the chiral nature and is consistent with Log CFT conjecture;

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- What's the asym. symmetry?
- Some kind of generalization of chiral gravity conjecture?

1st order formulation of TMG

In first order formalism, TMG with a negative cosmological constant $\Lambda = -l^{-2}$ is described by the action

$$S_{\mathsf{TMG}} = \frac{1}{8\pi G}\int \left(e^a \wedge R_a + \frac{1}{6l^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c\right) - \frac{1}{16\pi G\mu}\int \left(\mathcal{L}_{\mathsf{CS}} + \beta^a \wedge T_a\right), \tag{3.1}$$

where

$$\begin{aligned} R_a &= d\omega_a + \frac{1}{2}\epsilon_{abc}\left(\omega^b \wedge \omega^c + \frac{e^b \wedge e^c}{l^2}\right) \\ T^a &= de^a + \epsilon^{abc}\omega_b \wedge e_c, \\ \mathcal{L}_{\mathsf{CS}} &= \omega^a \wedge d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a \wedge \omega^b \wedge \omega^c. \end{aligned} \tag{3.2}$$

The field β^a is just a Lagrangian multiplier, imposing the torsion free condition such that the above action is equivalent to the action in terms of Christoffel symbol.

Relation with CS gravity

It would be illuminating to rewrite the above action in a form relating to Chern-Simons gravity with gauge group $SL(2,R) \times SL(2,R)$:

$$S_{\mathsf{TMG}} = \left(1 - \frac{1}{\mu l}\right) S_{CS}[A] - \left(1 + \frac{1}{\mu l}\right) S_{CS}[\tilde{A}] - \frac{k}{4\pi\mu l} \int \left(\tilde{\beta}^a \wedge T_a\right).$$

- The solutions of AdS$_3$ gravity are automatically the solutions of TMG;
- In these cases, $\tilde{\beta}^a = 0$;
- An important subtlety: it does not mean AdS$_3$ TMG is equivalent to CS gravity,
  1. In TMG, there is a local massive mode at generic μl;
  2. CS gravity is just topological, no local d.o.f.;
- A different story at chiral point:
  1. No more local d.o.f.;
  2. Central charge $c_L = 0, c_R = \frac{3l}{G}$ <== CS gravity;
  3. Holographic nature of partition function;

Action of spin-3 TMG

To study the topological massive gravity coupled with spin-3 fields, we now propose the following action:

$$S = \left(1-\frac{1}{\mu l}\right) S_{CS}[A] - \left(1+\frac{1}{\mu l}\right) S_{CS}[\tilde{A}] - \frac{k}{4\pi\mu l}\int \left(\tilde{\beta}^a \wedge T_a - 2\sigma \tilde{\beta}^{ab} \wedge T_{ab}\right). \quad (3.3)$$

Here the last two terms are introduced to impose the torsion free conditions. Now the torsions are defined as

$$T^a = de^a + \epsilon^{abc}\omega_b \wedge e_c - 4\sigma\epsilon^{abc} e_{bd} \wedge \omega_c{}^d, \quad (3.4)$$

$$T^{ab} = de^{ab} + \epsilon^{cd(a|}\omega_c \wedge e_d{}^{|b)} + \epsilon^{cd(a|} e_c \wedge \omega_d{}^{|b)}. \quad (3.5)$$

Note that the torsion T^a for veilbein gets modified by the spin-3 field and the torsion T^{ab} is for spin-3 field.

Action in terms of frame-like field

In terms of the frame-like field and connection, the action (3.3) could be written in a more familiar form:

$$\begin{aligned} S \;=\; & \frac{1}{8\pi G}\int \Big(e^a\wedge d\omega_a+\frac{1}{2}\epsilon_{abc}e^a\wedge\omega^b\wedge\omega^c+\frac{1}{6l^2}\epsilon_{abc}e^a\wedge e^b\wedge e^c \\ & -2\sigma e^{ab}\wedge d\omega_{ab}-2\sigma\epsilon_{abc}e^a\wedge\omega^{bd}\wedge\omega^c{}_d-2\sigma e^{ab}\wedge\epsilon_{(a|cd}\omega^c\wedge\omega_{|b)}{}^d \\ & -\frac{2\sigma}{l^2}\epsilon_{abc}e^a\wedge e^{bd}\wedge e^c{}_d\Big)-\frac{1}{16\pi G\mu}\int\Big(\omega^a\wedge d\omega_a+\frac{1}{3}\epsilon_{abc}\omega^a\wedge\omega^b\wedge\omega^c \\ & -2\sigma\omega^{ab}\wedge d\omega_{ab}-4\sigma\epsilon_{abc}\omega^a\wedge\omega^{bd}\wedge\omega^c{}_d+\beta^a\wedge T_a-2\sigma\beta^{ab}\wedge T_{ab}\Big), \end{aligned} \tag{3.6}$$

where

$$\beta^{ab}=\tilde\beta^{ab}+\frac{e^{ab}}{l^2}$$

could be taken as an independent field.

Remarks

- The first part of the action, proportional to $1/8\pi G$, is the same as the one of pure spin-3 AdS_3 gravity;
- The two terms proportional to T_a, T_{ab} are just to impose the torsion free conditions:

$$\begin{aligned} T^a &= 0, \\ T^{ab} &= 0, \end{aligned}$$

- The remaining parts are just the spin-3 generalization of gravitational Chern-Simons term, which has been discussed by Damour and Deser in 1987;
- Such gravitational CS terms are parity breaking.

Equations of motion

$$R_a - \frac{1}{2\mu}(d\beta_a + \epsilon_{abc}\beta^b \wedge \omega^c - 2\sigma\epsilon_{(c|da}\beta^{bc} \wedge \omega^d{}_{|b)}) = 0,$$

$$R_a + \frac{1}{2}\epsilon_{abc}\left[\beta^b \wedge e^c - \frac{e^b \wedge e^c}{l^2} + 4\sigma\left(\frac{e^{bd} \wedge e^c{}_d}{l^2} - e^{bd} \wedge \beta^c{}_d\right)\right] = 0,$$

$$R_{ab} - \frac{1}{2\mu}\left(d\beta_{ab} + \epsilon_{cd(a|}\beta^c \wedge \omega^d{}_{|b)} + \epsilon_{cd(a|}\omega^c \wedge \beta^d{}_{|b)}\right) = 0,$$

$$R_{ab} + \frac{1}{2}\left(\epsilon_{cd(a|}\beta^c \wedge e^d{}_{|b)} + \epsilon_{cd(a|}e^c \wedge \beta^d{}_{|b)}\right) - \frac{1}{l^2}\epsilon_{cd(a|}e^c \wedge e^d{}_{|b)} = 0,$$

where

$$R_a = d\omega_a + \frac{1}{2}\epsilon_{abc}(\omega^b \wedge \omega^c + \frac{e^b \wedge e^c}{l^2}) - 2\sigma\epsilon_{abc}(\omega_{bd} \wedge \omega_c{}^d + \frac{e_{bd} \wedge e_c{}^d}{l^2}$$

$$R_{ab} = d\omega_{ab} + \epsilon_{cd(a|}\omega^c \wedge \omega^d{}_{|b)} + \frac{1}{l^2}\epsilon_{cd(a|}e^c \wedge e^d{}_{|b)}.$$

Remarks

- The equations are different from the pure gravity case, with contributions from β^a, β^{ab} terms;
- However, notice that when

$$\beta^a = \frac{e^a}{l^2}, \qquad \beta^{ab} = \frac{e^{ab}}{l^2}, \tag{3.7}$$

the extra terms vanish due to the torsion free conditions.
- This suggests that the solutions of pure gravity coupled to spin-3 field theory proposed in 1008.4744 remain the solutions of above equations of motion;
- But it could be possible that there exist more solutions: for example, the warped spacetime with vanishing spin-3 fields;
- In our work, we just focus on AdS$_3$ solution;

Equations of fluctuation

To the leading order, we have the following equations of motion of the fluctuations:

$$de^a + \epsilon^{abc}\bar{\omega}_b \wedge e_c + \epsilon^{abc}\omega_b \wedge \bar{e}_c = 0,$$
$$d\omega_a + \epsilon_{abc}(\bar{\omega}^b \wedge \omega^c + \frac{\bar{e}^b \wedge e^c}{l^2}) - \frac{1}{2\mu}(d\beta_a + \epsilon_{abc}\bar{\beta}^b \wedge \omega^c + \epsilon_{abc}\beta^b \wedge \bar{\omega}^c) = 0,$$
$$d\omega_a + \epsilon_{abc}(\bar{\omega}^b \wedge \omega^c + \frac{\bar{e}^b \wedge e^c}{l^2}) + \frac{1}{2}\epsilon_{abc}\left[\bar{\beta}^b \wedge e^c + \beta^b \wedge \bar{e}^c - \frac{2}{l^2}\bar{e}^b \wedge e^c\right] = 0,$$
$$de^{ab} + \epsilon^{cd(a|}\bar{\omega}_c \wedge e_d{}^{|b)} + \epsilon^{cd(a|}\bar{e}_c \wedge \omega_d{}^{|b)} = 0,$$
$$R_{ab} - \frac{1}{2\mu}\left(d\beta_{ab} + \epsilon_{cd(a|}\bar{\beta}^c \wedge \omega^d{}_{|b)} + \epsilon_{cd(a|}\bar{\omega}^c \wedge \beta^d{}_{|b)}\right) = 0,$$
$$R_{ab} + \frac{1}{2}\left(\epsilon_{cd(a|}\bar{\beta}^c \wedge e^d{}_{|b)} + \epsilon_{cd(a|}\bar{e}^c \wedge \beta^d{}_{|b)}\right) - \frac{1}{l^2}\epsilon_{cd(a|}\bar{e}^c \wedge e^d{}_{|b)} = 0,$$

where

$$R_{ab} = d\omega_{ab} + \epsilon_{cd(a|}\bar{\omega}^c \wedge \omega^d{}_{|b)} + \frac{1}{l^2}\epsilon_{cd(a|}\bar{e}^c \wedge e^d{}_{|b)}.$$

Graviton fluctuation

- To the leading order, the fluctuations of the spin-3 fields decouple from the gravitons;
- TT gauge: $\nabla^\mu h_{\mu\nu} = 0$ and $h^\mu_\mu = 0$;
- In our formalism, the fluctuations of gravitons $h_{\mu\nu}$ satisfy the following equation

$$(\Box + \frac{2}{l^2})h^\rho_\sigma + \frac{1}{\mu}\epsilon^{\rho\mu\nu}\nabla_\mu(\Box + \frac{2}{l^2})h_{\nu\sigma} = 0, \qquad (4.1)$$

- It is a third order differential equation, the same as the one got by Andy's group in 2008;

Spin-3 fluctuation I

- It is much more involved;
- 1st step: solve the torsion free condition;
- 2nd step: get $\tilde{\beta}^{\mu\alpha\beta} = \frac{1}{2}\Box\Phi^{\mu\alpha\beta}$;
- Physical field: $\Phi_{\mu\nu\lambda} = e_{\mu ab}\bar{e}^a_\nu \bar{e}^b_\lambda$;
- Gauge condition:

$$\Phi_\mu{}^{\mu\rho} = 0, \qquad \nabla^\mu \Phi_{\mu\nu\rho} = 0, \tag{4.2}$$

- Finally, we obtain:

$$\Box\Phi^{\rho\alpha\beta} + \frac{1}{2\mu}\epsilon^{\rho\mu\nu}\nabla_\mu \Box\Phi_\nu{}^{\alpha\beta} = 0. \tag{4.3}$$

- The spin-3 fluctuations satisfy a third order differential equation as well;
- The above gauge condition is a little bit too strong, it eliminate the trace part of spin-3 fluctuation; A. Bagchi et.al. 1107.0915
- In our work, we focused on the traceless spin-3 fluctuations;

Spin-3 fluctuation II

1. In AdS_3 background, the equation could be rewritten as

$$\Box(\Phi^{\rho\alpha\beta} + \frac{1}{2\mu}\epsilon^{\rho\mu\nu}\nabla_\mu\Phi_\nu{}^{\alpha\beta}) = 0; \tag{4.4}$$

2. This third order differential equation could be decomposed into three first-order differential equations, each corresponding to different degrees of freedom.
3. The massive degree of freedom satisfies a first order equation

$$\epsilon^{\rho\mu\nu}\nabla_\mu\Phi_\nu^{(M)\ \alpha\beta} = -2\mu\Phi^{(M)\rho\alpha\beta}. \tag{4.5}$$

4. Both the left mover and right mover are massless and satisfy respectively

$$\epsilon^{\rho\mu\nu}\nabla_\mu\Phi_\nu^{(L)\ \alpha\beta} = -\frac{2}{l}\Phi^{(L)\rho\alpha\beta}, \qquad \epsilon^{\rho\mu\nu}\nabla_\mu\Phi_\nu^{(R)\ \alpha\beta} = \frac{2}{l}\Phi^{(R)\rho\alpha\beta}; \tag{4.6}$$

Spin-3 fluctuation II

1. The three equations share the same structure, which could be denoted simply as

$$\epsilon^{\rho\mu\nu}\nabla_\mu\Phi_\nu^{(A)\ \alpha\beta} = m_A\Phi^{(A)\rho\alpha\beta}, \tag{4.7}$$

   Where A can be M, L, R and $m_A = -2\mu, -\frac{2}{l}, \frac{2}{l}$ correspondingly.

2. One can derive the second order equation it satisfies as

$$\Box\Phi_{\rho\mu\nu}^{(A)} = (m_A^2 - \frac{4}{l^2})\Phi_{\rho\mu\nu}^{(A)}. \tag{4.8}$$

Conformal weights I

In the global coordinates the metric of AdS_3 is:

$$ds^2 = l^2(-\cosh^2\rho d\tau^2 + \sinh^2\rho d\phi^2 + d\rho^2).$$

It has the isometry group $SL(2,\mathbf{R})_L \times SL(2,\mathbf{R})_R$. By defining $u \equiv \tau + \phi, v \equiv \tau - \phi$, the generators of $SL(2,\mathbf{R})_L$ can be written as:

$$\begin{aligned} V_0 &= i\partial_u, \\ V_{-1} &= ie^{-iu}\left(\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v + \frac{i}{2}\partial_\rho\right), \\ V_1 &= ie^{iu}\left(\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v - \frac{i}{2}\partial_\rho\right), \end{aligned}$$

satisfying

$$[V_i, V_j] = (i-j)V_{i+j}.$$

By exchanging u and v in the above equations, we can get the generators $\bar{V}_0, \bar{V}_{-1}, \bar{V}_1$ of $SL(2,\mathbf{R})_R$.

Conformal weights II

1. Define Lie-induced Casimir
$$\mathcal{L}^2=\mathcal{L}_{V_0}\mathcal{L}_{V_0}-\frac{1}{2}(\mathcal{L}_{V_1}\mathcal{L}_{V_{-1}}+\mathcal{L}_{V_{-1}}\mathcal{L}_{V_1}), \qquad (4.9)$$
and similarly for $\bar{\mathcal{L}}^2$.
2. Then the quadratic equation could be written as
$$\begin{aligned}\mathcal{L}^2\Phi^{(A)}_{\rho\mu\nu} &= \frac{(m_A l)^2-6(m_A l)+8}{4}\Phi^{(A)}_{\rho\mu\nu},\\ \bar{\mathcal{L}}^2\Phi^{(A)}_{\rho\mu\nu} &= \frac{(m_A l)^2+6(m_A l)+8}{4}\Phi^{(A)}_{\rho\mu\nu}.\end{aligned}$$
3. Considering the highest weight state with conformal weight $(h^{(A)},\bar{h}^{(A)})$,
$$\begin{aligned}\mathcal{L}_{V_1}\Phi^{(A)}_{\mu\nu\lambda} &= \mathcal{L}_{\bar{V}_1}\Phi^{(A)}_{\mu\nu\lambda}=0,\\ \mathcal{L}_{V_0}\Phi^{(A)}_{\mu\nu\lambda} &= h^{(A)}\Phi^{(A)}_{\mu\nu\lambda}, \qquad \mathcal{L}_{\bar{V}_0}\Phi^{(A)}_{\mu\nu\lambda}=\bar{h}^{(A)}\Phi^{(A)}_{\mu\nu\lambda},\end{aligned}$$
then we have
$$\mathcal{L}^2\Phi^{(A)}_{\rho\mu\nu}=(h^{(A)2}-h^{(A)})\Phi^{(A)}_{\rho\mu\nu},$$

Conformal weights III

- The conformal weights of spin-3 fluctuations are:

$$h^{(M)} = \mu l + 2, \qquad \bar{h}^{(M)} = \mu l - 1; \tag{4.10}$$
$$h^{(L)} = 3, \qquad \bar{h}^{(L)} = 0; \tag{4.11}$$
$$h^{(R)} = 0, \qquad \bar{h}^{(R)} = 3. \tag{4.12}$$

where we have assumed $\mu l \geq 1$.

- Note that for the chiral gravity at $\mu l = 1$, we have $h^{(M)} = 3$ and $\bar{h}^{(M)} = 0$, which degenerates with the left-moving massless spin-3 fluctuations.
- In the similar way, we may discuss the spin-2 fluctuations, which have conformal weights

$$h^{(M)} = \frac{\mu l + 3}{2}, \qquad \bar{h}^{(M)} = \frac{\mu l - 1}{2}; \tag{4.13}$$
$$h^{(L)} = 2, \qquad \bar{h}^{(L)} = 0; \tag{4.14}$$
$$h^{(R)} = 0, \qquad \bar{h}^{(R)} = 2. \tag{4.15}$$

Explicit solution of spin-3 fluctuations

- From the Killing symmetry of the background, we may make ansatz

$$\Phi_{\mu\nu\lambda} = e^{-ihu-i\bar{h}v} F_{\mu\nu\lambda},$$

- All the components of $F_{\mu\nu\lambda}$ could be characterized by one undetermined function φ

$$\begin{aligned} F_{\tau\tau\tau} &= \pm\varphi, \\ F_{\phi\phi\phi} &= \varphi, \\ F_{\rho\rho\rho} &= \mp\frac{i\varphi}{\sinh^3\rho\cosh^3\rho}, \\ F_{\tau\tau\phi} &= \varphi, \cdots \end{aligned}$$

where

$$\varphi = C(\cosh\rho)^{-(h+\bar{h})}\sinh^3\rho \tag{4.16}$$

with a constant C.

Log mode

- This is feasible because the equation of fluctuations is a third order differential equation.
- The log mode can be constructed as

$$\Phi^{(Log)}_{\mu\nu\sigma} \equiv \lim_{\mu l \to 1} \frac{\Phi^{(M)}_{\mu\nu\sigma} - \Phi^{(L)}_{\mu\nu\sigma}}{\mu l - 1}. \quad (4.17)$$

- We define a function $A(\tau,\rho)$ as $A(\tau,\rho) \equiv -2(i\tau + \ln\cosh\rho)$, then the log mode can be written as $\Phi^{(Log)}_{\mu\nu\sigma} = A(\tau,\rho)\Phi^{(L)}_{\mu\nu\sigma}$.
- This new mode grows linearly in time, and grows logarithmically in the radial coordinate ρ.
- By using the fact that

$$\mathcal{L}_{V_0} A = \bar{\mathcal{L}}_{\bar{V}_0} A = 1; \qquad \mathcal{L}_{V_1} A = \bar{\mathcal{L}}_{\bar{V}_1} A = 0,$$

  One can show that

$$\Box \Phi^{(Log)}_{\mu\nu\sigma} \propto \Phi^{(L)}_{\mu\nu\sigma}.$$

- Hence the log mode satisfies the classical equations of motion.

Trouble with log modes

- The existence of the log graviton mode in TMG brought much debates on the stability and the chiral nature of the theory;
- One has to impose appropriate boundary conditions on the metric fluctuations;
- If one impose the standard Brown-Henneaux boundary conditions, such mode disappears and the only physical modes are right-moving boundary gravitons, then the theory is chiral;
- One may relax the boundary condition to allow the log mode, whose presence break the chiral nature of the theory;
- Such relaxed boundary condition is well-defined, leading to finite conserved charge. It was conjectured that the log gravity could be dual to a logarithmic CFT;
- We just focus on the Brown-Henneaux boundary conditions.

Energies of spin 3 fluctuations

- The free action of the spin-3 fluctuations is of the form

$$S_2 = \frac{1}{64\pi G}\int d^3x\sqrt{-g}\left\{-\bar{\nabla}^\lambda \Phi^{\mu_1\mu_2\mu_3}\bar{\nabla}_\lambda \Phi_{\mu_1\mu_2\mu_3} - \frac{1}{2\mu}\bar{\nabla}_\alpha \Phi_{\mu_1\mu_2\mu_3}\epsilon^{\mu_1\alpha\beta}\square\right.$$

- One subtle point: as there are three time derivatives in the action, we need to use the Ostrogradsky method to define the Hamiltonian;
- The energies of different spin-3 fluctuations are

$$\begin{aligned}
E_M &= \frac{2}{T\mu}(\mu^2 - \frac{1}{l^2})\int d^3x\frac{\sqrt{-g}}{64\pi G}\epsilon_\beta{}^{0\mu_1}\Phi_M^{\beta\mu_2\mu_3}\dot{\Phi}_{M\mu_1\mu_2\mu_3},\\
E_L &= -\frac{1}{T}(1 - \frac{1}{\mu l})\int d^3x\frac{\sqrt{-g}}{32\pi G}\bar{\nabla}^0\Phi_L^{\mu_1\mu_2\mu_3}\dot{\Phi}_{L\mu_1\mu_2\mu_3},\\
E_R &= -\frac{1}{T}(1 + \frac{1}{\mu l})\int d^3x\frac{\sqrt{-g}}{32\pi G}\bar{\nabla}^0\Phi_R^{\mu_1\mu_2\mu_3}\dot{\Phi}_{R\mu_1\mu_2\mu_3}.
\end{aligned}$$

- The above three integrals are negative for primary fields;
- Then $\mu l > 1$ gives $E_M < 0$, while $\mu l < 1$ gives $E_L < 0$;

Spin-3 chiral gravity

- So only at the critical point $\mu l = 1$, there are no modes with negative energy, and in this case, we have $E_M = E_L = 0$, suggesting that they are pure gauge;
- At the chiral point $\mu l = 1$, the local massive spin-3 mode becomes degenerate with the left-moving massless mode;
- Both of them are just pure gauge;
- This is the same as the spin-2 fluctuations;
- There are only right-moving spin-2 and spin-3 fluctuation at the chiral point;
- Even with spin-3 field, the chiral nature of the theory at $\mu l = 1$ keep intact;
- What is the asyp. symmetry of spin-3 chiral gravity in AdS_3?
- Is there a CFT dual?

Our speculation

- The chiral high spin gravity is equivalent to a holomorphic Chern-simons gravity with gauge group $SL(3,C)$;
- More precisely, the action of the holomorphic Chern-Simons takes the form

$$S_{CS}[\tilde{A}] = -\frac{k}{2\pi}\int \mathrm{Tr}(\tilde{A}\wedge d\tilde{A} + \frac{2}{3}\tilde{A}\wedge\tilde{A}\wedge\tilde{A}), \qquad (5.1)$$

  where $\tilde{A}$ takes values in $SL(3,C)$ and the level is enhanced to $2k$;
- Comparing with the action (3.3) at the chiral point $\mu l = 1$, we find that the main difference resides at the last two terms relating to the torsions;
- They play a key role in studying the fluctuations around the vacuum and induce higher derivative terms.
- Once the fluctuations become pure gauge at the chiral point, it is safe to ignore these two terms;

Chiral gravity = holomorphic CS?

- The only physical degrees of freedom are the right-moving boundary gravitons, in accordance with the ones of a holomorphic Chern-Simons gravity;
- From the Chern-Simons gravity, the central charge could be read easily and reproduce the result from the analysis of asymptotic symmetry with Brown-Henneaux boundary condition;
- The partition function of chiral gravity is holomorphic and modular invariant, in accordance with the expectation of holomorphic factorization of Chern-Simons gravity
- Therefore, the chiral gravity is expected to be equivalent to a holomorphic Chern-Simons gravity.
- We expect that the same equivalence could be extended to the case with the spin-3 fields, though we are short of evidence from the partition function.

Asymptotic symmetry

- The asym. symm. could be read straightforwardly from CS gravity;
- One needs to introduce the boundary term for Chern-Simons action, and impose appropriate boundary conditions

$$\tilde{A}_+ = 0, \tag{5.2}$$

- The gauge choice $\tilde{A}_\rho = b(\rho)\partial_\rho b^{-1}(\rho)$;
- Here it has been assumed that the space $\mathcal{M} = R \times \Sigma$, where Σ is a two-dimensional manifold with boundary $\partial\Sigma \sim S^1$ and R is the radial direction. And we have defined $x^\pm = t \pm \phi$;
- In the case with spin-3 field such that the gauge group is $SL(3)$, one needs to require the solution is asymptotically AdS,

$$(\tilde{A} - \tilde{A}_{\text{AdS}})|_{\text{boundary}} = \mathcal{O}(1). \tag{5.3}$$

- The above boundary conditions are equivalent to the Brown-Henneaux boundary conditions on metric fields
- The asymptotic symmetry algebra turns out to be a classical W_3-algebra with central charge

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- The asymp. symmetry of spin-3 chiral gravity is one copy of W_3 symmetry algebra with central charge $c_R = 3l/G$;
- Namely the chiral spin-3 gravity could be described holographically by a two-dimensional chiral CFT with W_3 algebra and central charge c_R.

Discussions

- Other spin case? especially the large N limit?
- Partition function?
- Other B.C.?
- Large N duality;
- Black hole solution?
- SUSY case?
- HS fields in warped spacetime?

Thank you!