QCD and String Theory

Chuan-Jie Zhu

Institute of Theoretical Physics, Chinese Academy of Sciences

CITS, USTC, November 19, 2004

Contents

1. Quantum Chromodynamics (QCD)

What is QCD? The answer is quite simple in the short distance (high energy).

The Lagrange is:

$$\mathcal{L} = -\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a} + \bar{\psi}_I[i\gamma^\mu(\partial_\mu - igT^a A^a_\mu) - m_I]\psi_I, \quad (1)$$

$$[T^a, T^b] = if^{abc}T^c. \quad (2)$$

Non-abelian gauge theory (Yang and Mills, 1954). Perturbative expansion is OK because of Asymptotic Freedom (1973).

2004 Nobel price in physics

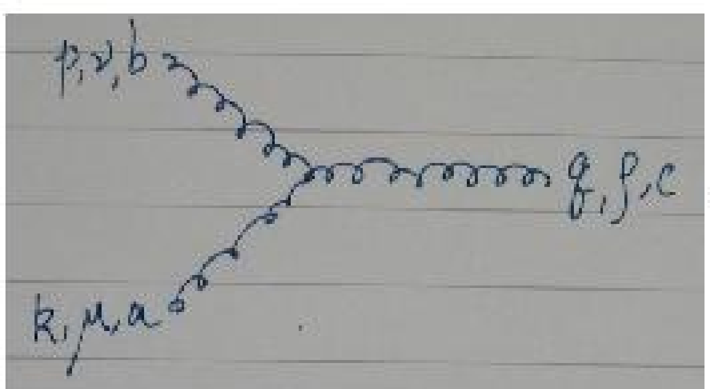
David J. Gross

H. David Politzer

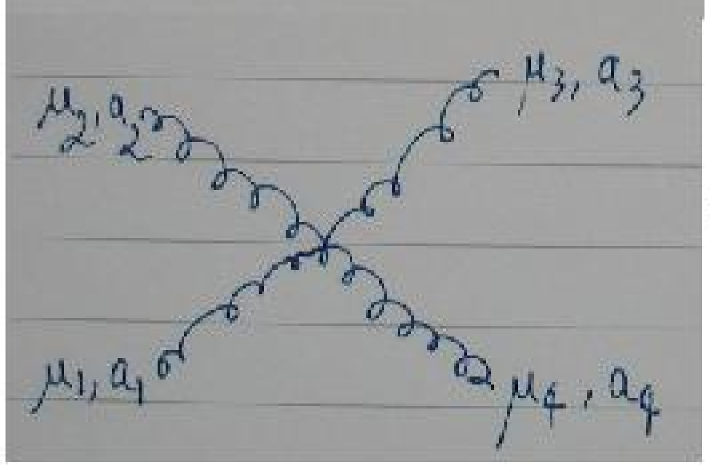
Frank Wilcezk

But the perturbative computation by using Feynman diagrams is very difficult, especially for multi-particle ($n \geq 6$) and higher loops ($L \geq 1$ and $n \geq 4$). The reasons are as follows:

1) The vertices are quite complicated: the tri-vertex and the quartic-vertex are given as follows:



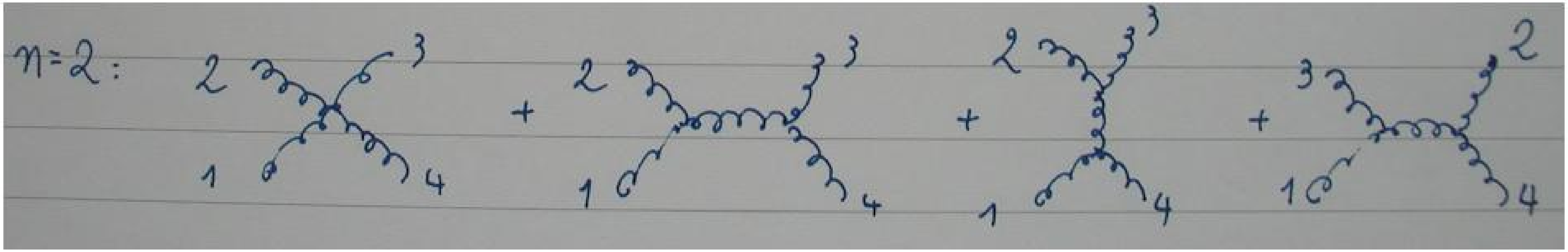
$$= gf^{abc}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{rho\mu}(q-k)^{\nu}\right],$$



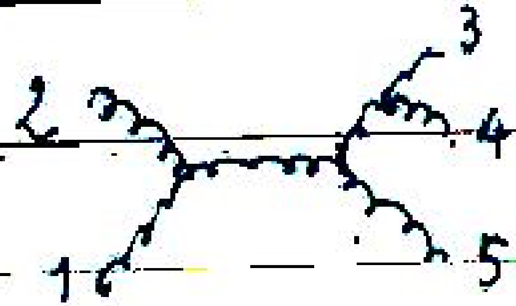
$$= -ig^2\left[(f^{a_1a_2a}f^{a_3a_4a} - f^{a_1a_4a})g^{\mu_1\mu_3}g^{\mu_2\mu_4} + (2\leftrightarrow 3) + (1\leftrightarrow 3)\right].$$

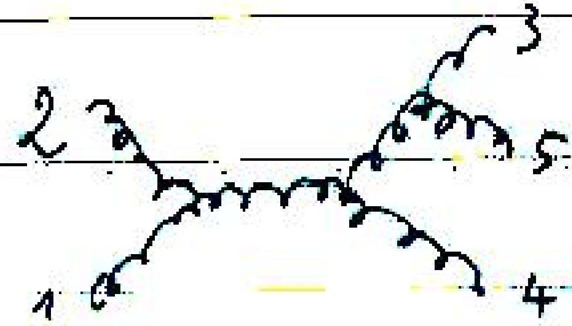
2) There are a lot of different Feynman diagrams. The different diagrams for $g + g \to n\ g$ process is as follows (M. L. Mangano and S. J. Parke, *Multi-Parton Amplitudes in Gauge Theories*, Phys. Repts. 200 (1991) 301–367):

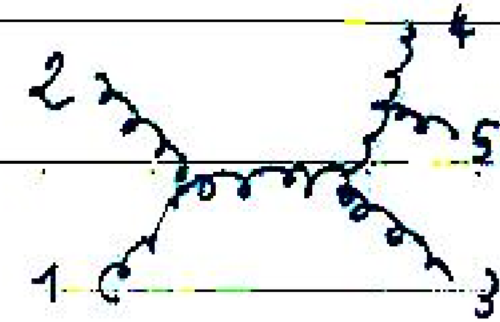
n	2	3	4	5	6	7	8
no. of diagrams	4	25	220	2,485	34,300	559,405	10,525,900

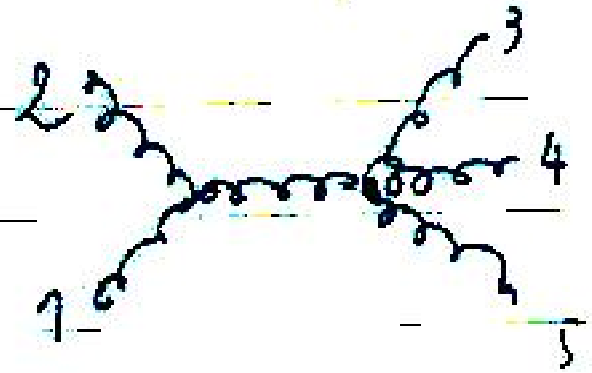


n=3时:

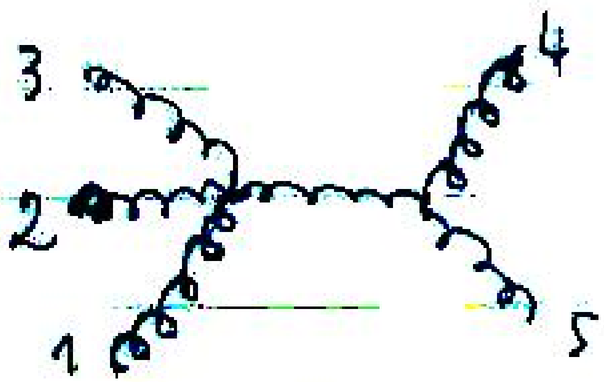








+ (13) + (14) + (15)　　4×4 = 16个图



+ (124) + (125) + (134) + (135) + (145)　　6个图

+ (24)(35) + (25)(34)　　3个图: 16+6+3=25个图

3) The intermediate expressions are quite complicated but the final result is comparatively simple. This is quite important for numerical calculations. (Instability at higher energy if you do numerical calculation by hand.)

Expressions like

$$\sum_i \frac{D_i}{N_i}, \tag{5}$$

with very big D_i and N_i must be simplified, especially at high energy (where D_i and N_i are big).

And also the final results are not so simple:

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

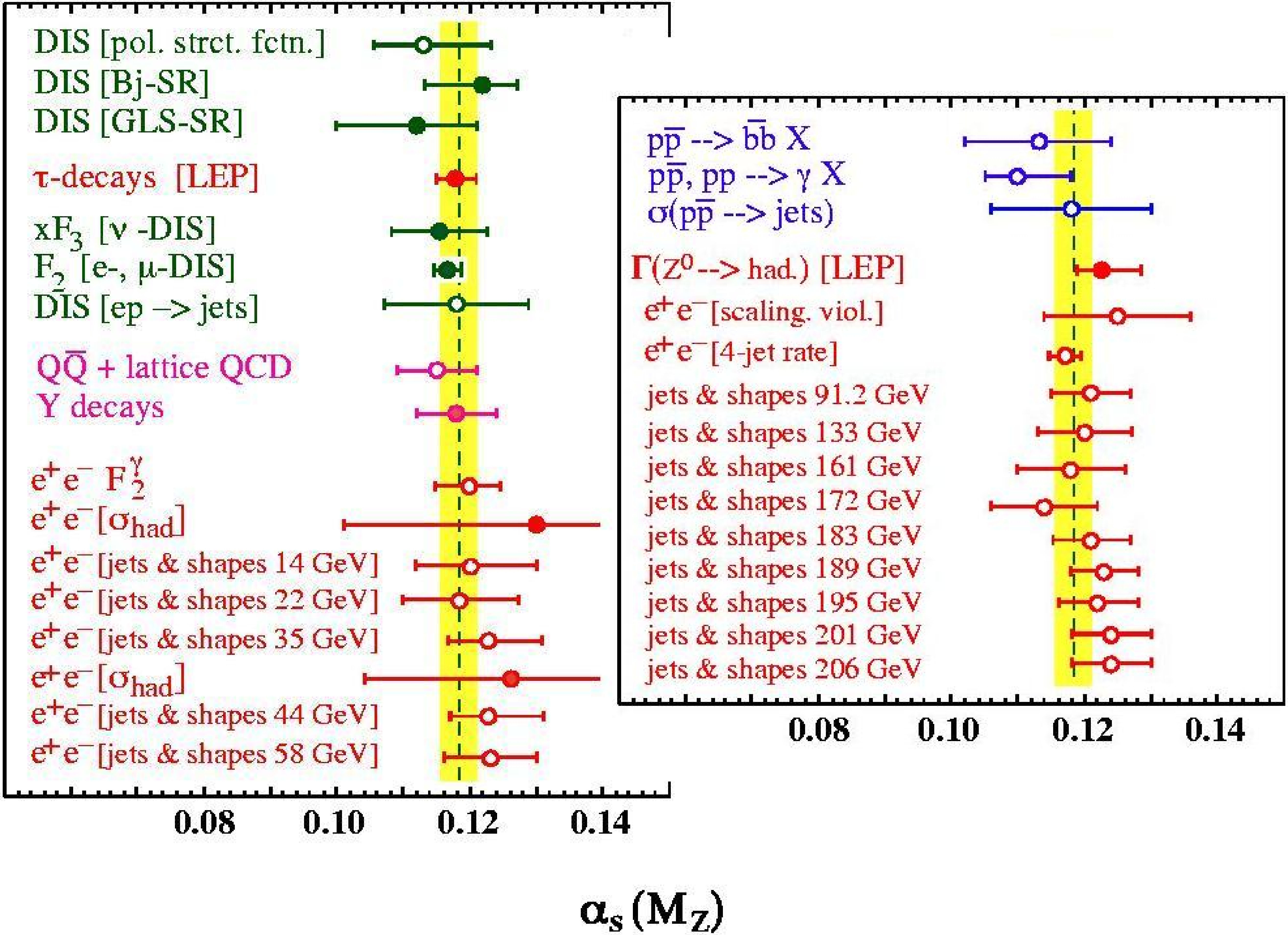
The era of "testing QCD" is finished. Now we can do the following 2 things:

1) precision test of QCD;

2) discovering new physics.

Either way, we need to understand QCD better. Now the theory is behind the experiment. As an example, CDF find from their Run I data:

$$\alpha_s(M_z) = 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys})^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf}). \quad (6)$$



DIS [pol. strct. fctn.]
DIS [Bj-SR]
DIS [GLS-SR]
τ-decays [LEP]
xF3 [ν -DIS]
F2 [e-, μ-DIS]
DIS [ep -> jets]
QQ̄ + lattice QCD
Υ decays
e+e− F2γ
e+e−[σhad]
e+e−[jets & shapes 14 GeV]
e+e−[jets & shapes 22 GeV]
e+e−[jets & shapes 35 GeV]
e+e−[σhad]
e+e−[jets & shapes 44 GeV]
e+e−[jets & shapes 58 GeV]
0.08
0.10
0.12
0.14
pp̄ --> bb̄ X
pp̄, pp --> γ X
σ(pp̄ --> jets)
Γ(Z0 --> had.) [LEP]
e+e−[scaling. viol.]
e+e−[4-jet rate]
jets & shapes 91.2 GeV
jets & shapes 133 GeV
jets & shapes 161 GeV
jets & shapes 172 GeV
jets & shapes 183 GeV
jets & shapes 189 GeV
jets & shapes 195 GeV
jets & shapes 201 GeV
jets & shapes 206 GeV
0.08
0.10
0.12
0.14
αs (MZ)

Because of the coming experiments (LHC in 2007 or 500GeV NLC thereafter), we do need more precise theoretical calculations. So it is mandatory to understand perturbative QCD better.

NNLO (next-to-next-to-leading) order or 2-loop calculations are needed.

The promise

1) Reduced renormalization scale dependence;

2) Event has more partons in the final state hence closer to real word;

3) Better description of p_T of final state due to double radiation off initial state;

4) Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$;

5) Full NNLO global fits of PDF's should also reduce the factorization scale uncertainty.

The various methods used:

1) Color decomposition;

2) Spinor helicity;

3) Recursive relations;

4) Supersymmetric identities;

5) String theory techniques (one-loop);

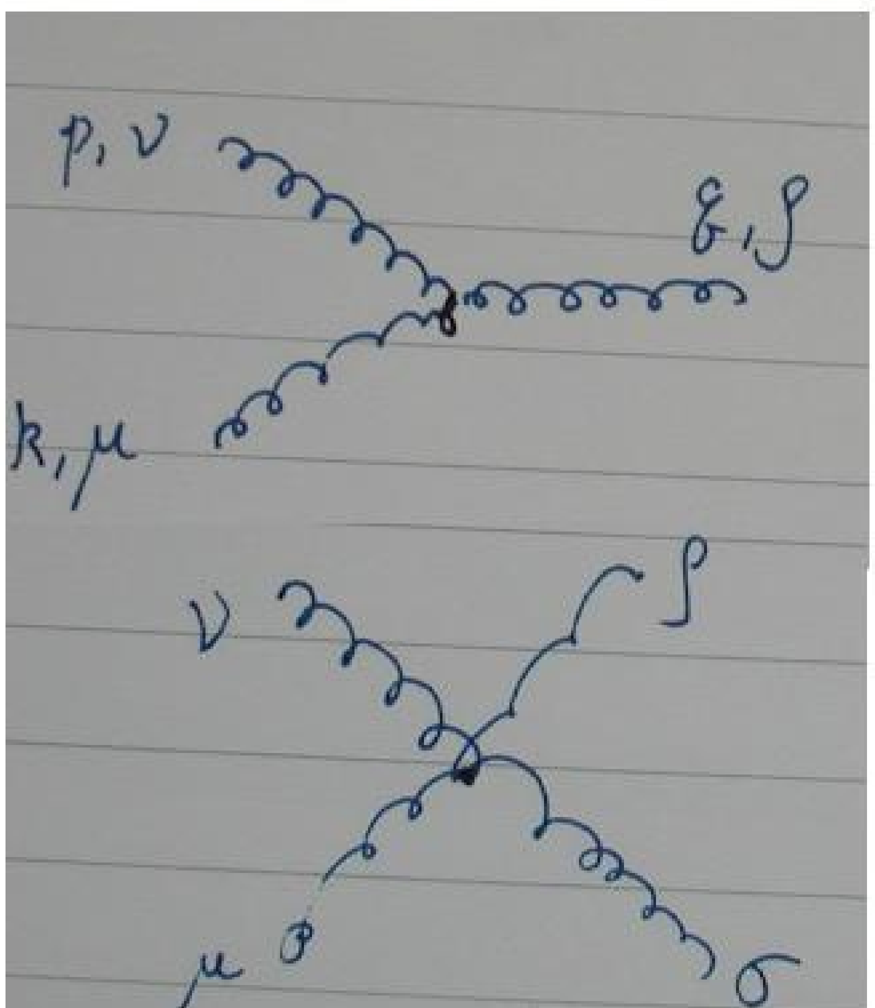
6) Unitarity (one- and two-loops) and educated guess, $\cdots$.

Color decomposition

$$\mathcal{A}_n = \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}})\, A_n(k_{\sigma(1)}), \cdots k_{\sigma(n)}), \tag{7}$$

and only planar diagrams keeping the cyclic ordering contribute to $A_n(k_1, \cdots, k_n)$.

The Feynman rules are simpler:



$$= \frac{i}{\sqrt{2}} \left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{rho\mu}(q-k)^{\nu} \right],$$

$$= \frac{i}{2} \left[2g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho} \right].$$

Spinor helicity: the Chinese magic

A clever choice for the (external) polarization vector in helicity basis:

$$\begin{aligned} \epsilon^{+}_{\mu}(k,p) &= \frac{\mu_\alpha \bar{\lambda}_{\dot{\alpha}}}{\langle \lambda, \mu \rangle}, \\ \epsilon^{-}_{\mu}(k,p) &= \frac{\lambda_\alpha \bar{\mu}_{\dot{\alpha}}}{[\bar{\lambda}, \bar{\mu}]}, \end{aligned} \tag{10}$$

for the particle momentum $k_\mu = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}$ and the reference momentum $p_\mu = \mu_\alpha \bar{\mu}_{\dot{\alpha}}$.

The helicity amplitude is defined as $A_n(k_i^{h_i}) = A_n(k_i, \epsilon_i^{h_i})$.

By choosing this helicity basis, we have:

$$A_5(1^+, 2^+, 3^+, 4^+, 5^+) = 0, \tag{11}$$

$$A_5(1^-, 2^+, 3^+, 4^+, 5^+) = 0, \tag{12}$$

$$A_5(1^+, \cdots, j^-, \cdots, k^-, 5^+) = \frac{\langle j, k\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}, \tag{13}$$

where $\langle ij\rangle \equiv \langle \lambda_i, \lambda_j\rangle = \lambda_i^\alpha \lambda_{j\alpha} = \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta$.

The Parke-Taylor MHV (maximally helicity violating) amplitudes:

$$A_n(1^+, \cdots, j^-, \cdots, k^-, n^+) = \frac{\langle j, k\rangle^4}{\prod_{i=1}^n \langle i, i+1\rangle}\,. \tag{14}$$

2-loop superstring amplitude is also simple:

$$A_{II} = c_{II}\, K(k_i,\epsilon_i) \int \frac{\prod_{i=1}^{6} \mathrm{d}^2 a_i / \mathrm{d}V_{pr}}{T^5 \prod_{i<j} |a_i - a_j|^2} \prod_{i=1}^{4} \frac{\mathrm{d}^2 z_i}{|y(z_i)|^2} \prod_{i<j} \exp\left[-k_i \cdot k_j\ G(z_i, z_j)\right]$$
$$\times |s(z_1 z_2 + z_3 z_4) + t(z_1 z_4 + z_2 z_3) + u(z_1 z_3 + z_2 z_4)|^2, \qquad (15)$$

where

$$\mathrm{d}V_{pr} = \frac{\mathrm{d}^2 a_i \mathrm{d}^2 a_j \mathrm{d}^2 a_k}{|a_{ij} a_{jk} a_{ki}|^2}, \qquad (16)$$

$$G(z,w) = -\ln |E(z,w)|^2 + 2\pi\, \mathrm{Im} \int_z^w \omega_I\, (\mathrm{Im}\Omega)^{-1}_{IJ}\, \mathrm{Im} \int_z^w \omega_J, \qquad (17)$$

$$y^2(z) = \prod_{i=1}^{6} (z - a_i), \qquad (18)$$

$$T = \int \mathrm{d}^2 z_1 \mathrm{d}^2 z_2 \frac{|z_1 - z_2|^2}{|y(z_1) y(z_2)|^2}. \qquad (19)$$

2. String Theory

You need to learn from the following 4 books:

Green-Schwarz-Witten, *Superstring Theory*, Cambridge University Press, 1987.

Vol. 1: Introduction, 469 pages;

Vol. 2: Loop amplitides, anomalies and phenomenology, 596 pages.

J. Polchinski, *String Theory*, Cambridge University Press, 1998.

Vol. 1: An Introduction to the Bosonic String, 402 pages;

Vol. 2: Superstring and Beyond, 531 pages.

(All are in the Cambridge Monographs on Mathematical Physics. Other series include Particle Physics, Nuclear Physics and Cosmology.)

Why Strings?

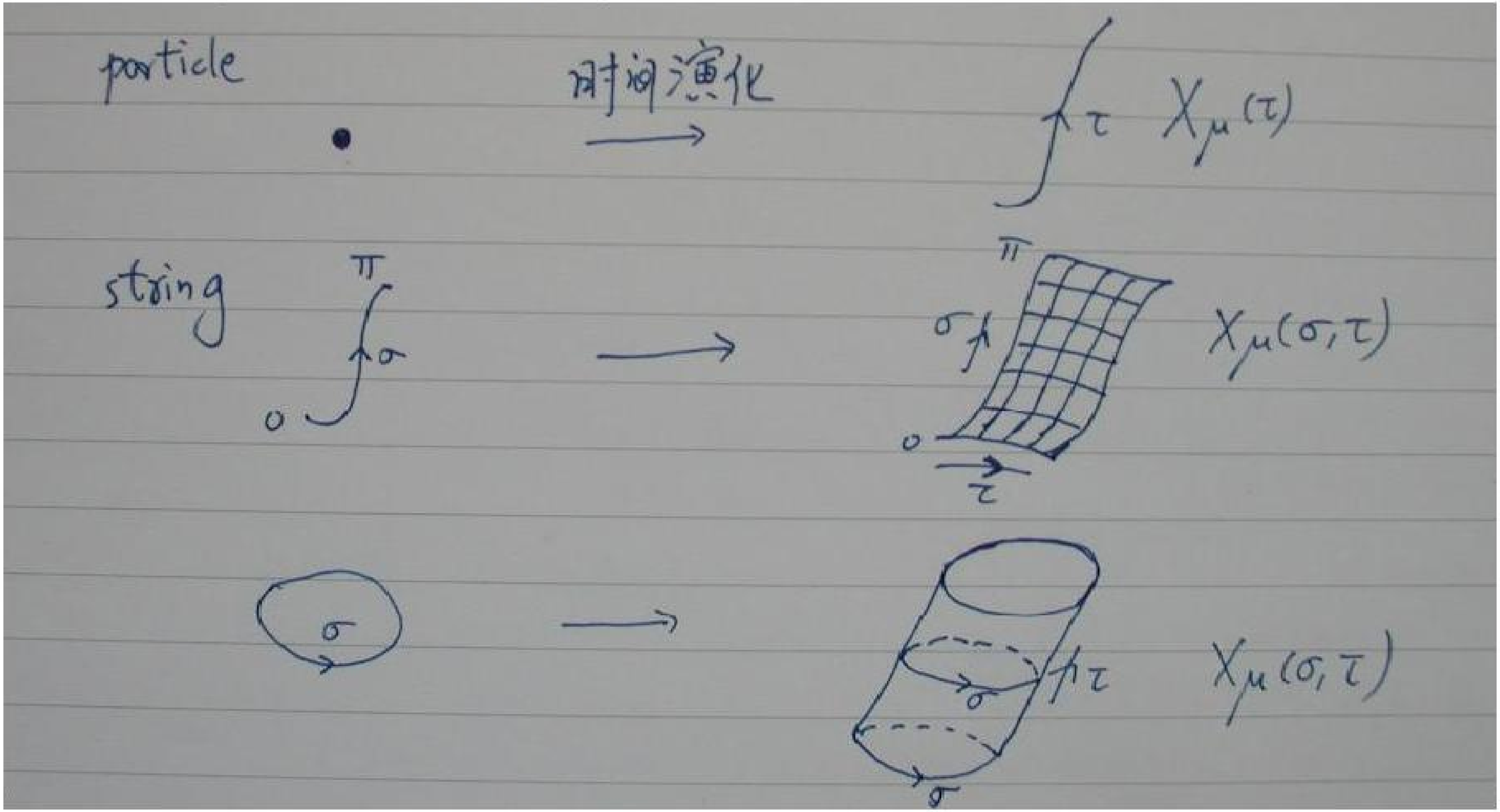
1. Gravity
2. A consistent theory of quantum gravity, at least in perturbation theory
3. Grand unification
4. Extra dimensions
5. supersymmetry
6. chiral gauge couplings
7. no free parameters
8. uniqueness

It's the only know generalization of standard QFT that makes any sense.

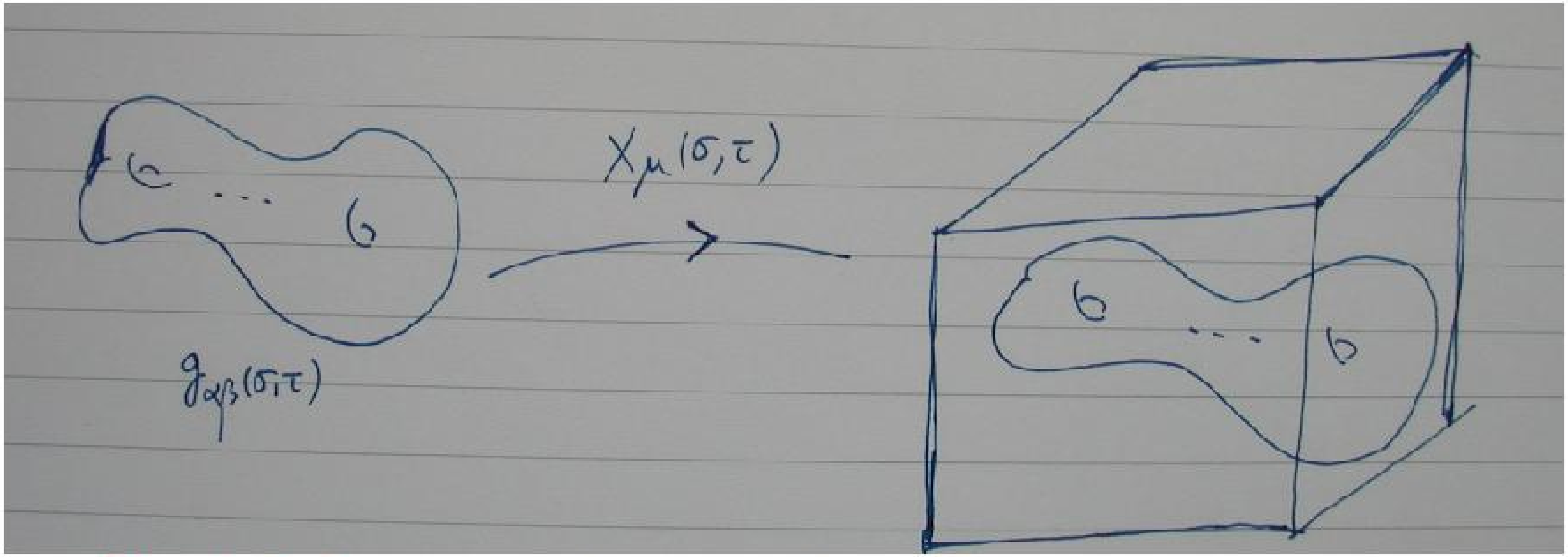
The basics of string theory

The time evolution of a "point" particle (0 dimensional object)is a worldline.

A string (1 dimensional object) sweeps out a worldsheet:



Or more abstractly, we study string theory as a mapping of an abstract 2 dimensional surface imbedding in the target spacetime:



Bosonic string: the target space is a 26 dimensional spacetime.

Superstring: the target space is a 10 dimensional spacetime.

Topologcal string: the target is a (super) Calabi-Yau space.

String theory is:

1) a new framework for physics that goes beyond QFT;

2) a new framework for unifying forces of nature (gravity + quantum theory in a consistent way)

3) based on a new kind of geometry, going beyond standard differential geometry, that we do not yet understand well. There is no analogue of Einstein's equivalence principle.

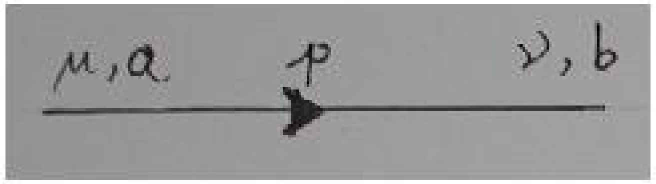
4) yielding many new insights about physical theories of an established kind. Gauge/Gravity correspondence.

Various dualities

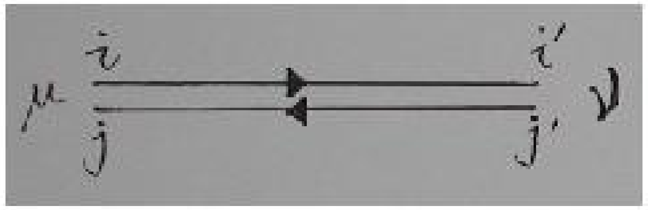
3. Gauge/String Duality

't Hooft's large N limit

The Feynman rules for the propagator:

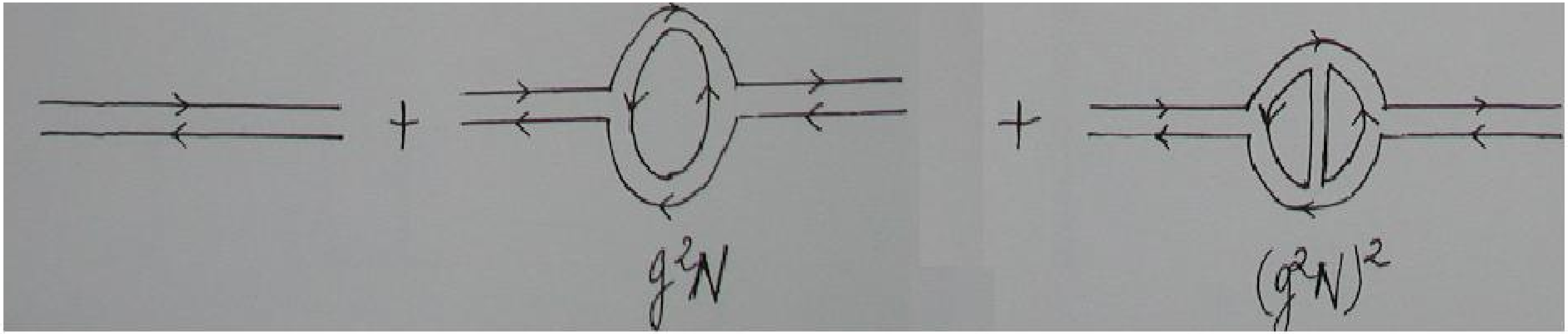


$$= \frac{-i\delta_{ab}g_{\mu\nu}}{p^2+i\epsilon} = \langle A_\mu^a A_\nu^b \rangle,$$



$$= \frac{-i\delta_{ii'}\delta_{jj'}g_{\mu\nu}}{p^2+i\epsilon} = \langle A_{\mu ij} A_{\nu j'i'} \rangle,$$

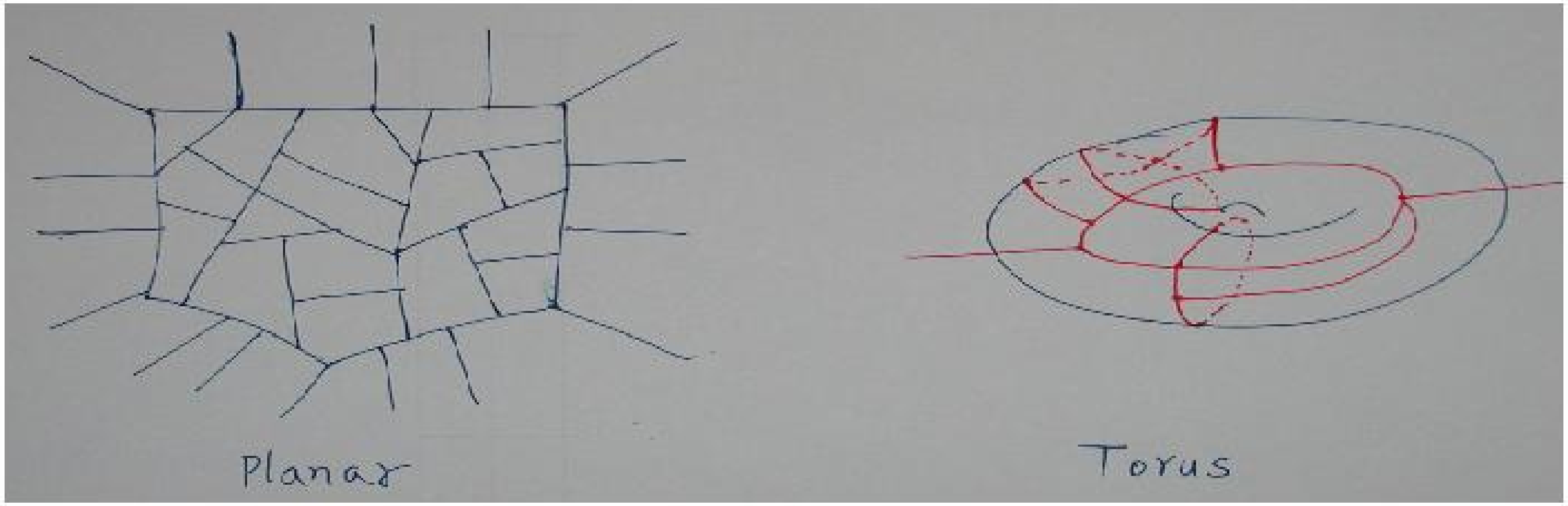
with $A_{\mu ij} = \sum_a T^a_{ij} A^a_\mu$.



Planar diagrams is an expansion in g^2N and the complete expansion is:

$$F(g,N) = \sum_{n=0}^{\infty} \frac{1}{N^{2n}} f_n(g^2N). \tag{22}$$

This suggests that the nonperturbative effects fill in the holes.



If we consider the above as an expansion of string theory, the string coupling constant is: $g_s = \frac{1}{N}$. So we have:

QCD = A String Theory with $\frac{1}{N}$ as the String Coupling Constant.

Maldacena conjecture: the AdS/CFT correspondence (1997) and its extensions (holographic principle) make it clear that gauge/string duality is correct and is useful at least for large g^2N.

This is the wrong limit for asymptotically free QCD, where $g^2N << 1$ at small distances. To make gauge/string duality useful for QCD, we need to understand it for all g^2N.

The goal: Find a string theory construction that is relevant to 4-dimensional gauge theory at small g^2N, i.e.

interpret perturbative gauge theory in 4 dimensions as a string theory.

weak/weak duality

E. Witten, *Perturbative Gauge Theory as a String Theory in Twistor Space*, **hep-th/0312171.**

studied the topological B model on supertwistor space $CP^{3|4}$ and conjectured that:

the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory is equivalent to the D-instanton expansion of a certain string theory, namely the topological B model whose target space is the Calabe-Yau supermanifold $CP^{3|4}$.

can't promise that it is useful in a string description of QCD, but at least that there appears some interesting things about perturbative gauge theory.

The evidence:

a) Helicity amplitudes and twistor space

$$\hat{A}_n(\lambda_i, \tilde{\lambda}_i, h_i) = i(2\pi)^4 \delta^{(4)} A_n(\lambda_i, \tilde{\lambda}_i, h_i), \tag{23}$$

and for MHV:

$$A_n(r^-, s^-) = g^{n-2} \frac{\langle j, k \rangle^4}{\prod_{i=1}^n \langle i, i+1 \rangle} \,. \tag{24}$$

b) Scattering amplitudes in twistor space: (λ_i, μ_i)

$$A_n(\lambda_i, \tilde{\lambda}_i) = i g^{n-2} \int \mathrm{d}^4 x \, \mathrm{e}^{i x_{\alpha\dot{\alpha}} \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}}} f(\lambda_i), \tag{25}$$

for MHV amplitudes.

By doing a Fourier transformation to twistor space:

$$\tilde{f}(\mu) = \int \frac{\mathrm{d}^2\tilde{\lambda}}{(2\pi)^2} \exp(\mathrm{i}[\mu, \tilde{\lambda}])\mathrm{f}(\tilde{\lambda}), \tag{26}$$

$$\tilde{A}(\lambda_i, \mu_i) = ig^{n-2} \int \mathrm{d}^4x \prod_{i=1}^{n} \delta^{(2)}(\mu_{i\dot{\alpha}} + x_{\alpha\dot{\alpha}}\lambda_i^{\alpha}) f(\lambda_i). \tag{27}$$

So all the points (λ_i, μ_i) are on the same straight line in the twistor space. The moduli of the line is parametrized by $x_{\alpha\dot{\alpha}}$.

And Witten proved:

the twistor version of the n particle scattering amplitude $\hat{A}(\lambda_i, \mu_i)$ is nonzero only if the points $P_i = (\lambda_i, \mu_i)$ are all supported on an algebric curve (of degree $d = q - 1 + l$) in twistor space.

c) Interpretation as a string theory

$$\hat{A}(\lambda,\mu,\psi) = \int \frac{\mathrm{d}^{4d+4}a\,\mathrm{d}^{4d+4}\beta\,\mathrm{d}^n\sigma}{\mathrm{vol.}(GL(2))}\,\mathcal{J}\prod_{i=1}^{n}\delta^{(3)}\left(\frac{z_i^I}{z_i^J}-\frac{P^I(\sigma-i)}{P^J(\sigma_i)}\right) \times\prod_{i=1}^{n}\delta^{(4)}\left(\frac{\psi_A^I}{z_i^J}-\frac{G^A(\sigma-i)}{P^J(\sigma_i)}\right), \tag{28}$$

where

$$\mathcal{J} = \prod_{i=1}^{n}\frac{1}{\sigma_i-\sigma_{i+1}}, \tag{29}$$

$$P^I(\sigma) = \sum_{k=0}^{d} a_k^I\sigma^k, \qquad I=0,1,2,3, \tag{30}$$

$$G^A(\sigma) = \sum_{k=0}^{d} \beta_A^A\sigma^k, \qquad A=1,2,3,4. \tag{31}$$

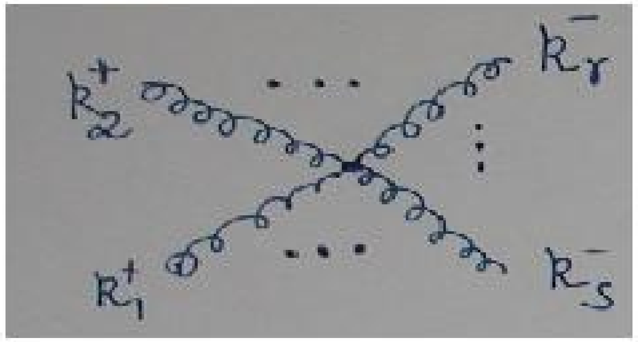
The P^I and G^A represent a degree d curve in the supertwistor space $CP^{3|4}$.

What given in the above is a formula for a multi-instanton, but recent results indicated that it can also be obtained by a summation over 1-instanton.

Beacuse MHV vertex = 1-instanton, so

F. Cachazo, P. Svrcek and E. Witten, *MHV Vertices and Tree Amplitudes in Gauge Theory*, **hep-th/0403047,** proposed a new set of rules for computing the scattering amplitudes. The rules are as follows:

The propagator is the same as before, $\frac{-i}{p^2}$, and the MHV vertices are:



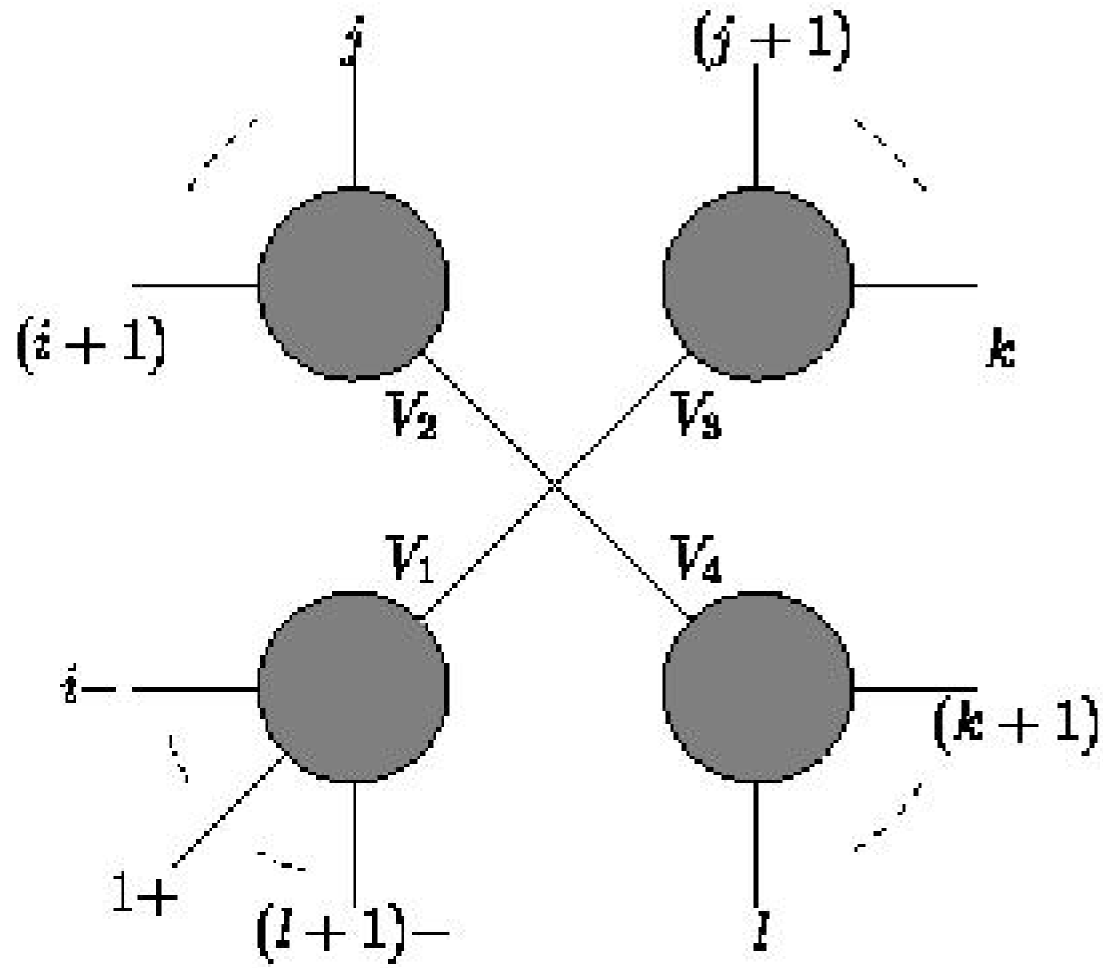
$$= V_n(k_r^-, k_s^-) = \frac{\langle \lambda_r, \lambda_s \rangle^4}{\prod_{i=1}^n \langle \lambda_i, \lambda_{i+1} \rangle},$$

where λ_i is defined as $\lambda_{i\alpha} = k_{i\alpha\dot{\alpha}} \eta^{\dot{\alpha}} = k_i^\mu (\sigma_\mu)_{\alpha\dot{\alpha}} \eta^{\dot{\alpha}}$.

These rules can be used to compute all the googly amplitudes:

$$A_n(k_r^+, k_s^+) = \frac{[\tilde{\lambda}_r, \tilde{\lambda}_s]^4}{\prod_{i=1}^{n}[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]}. \quad (33)$$

This is achieved by using the following decomposition:



In the above there is only a single quartic vertex. The rests consists of only tri-vertex and are given as follows:

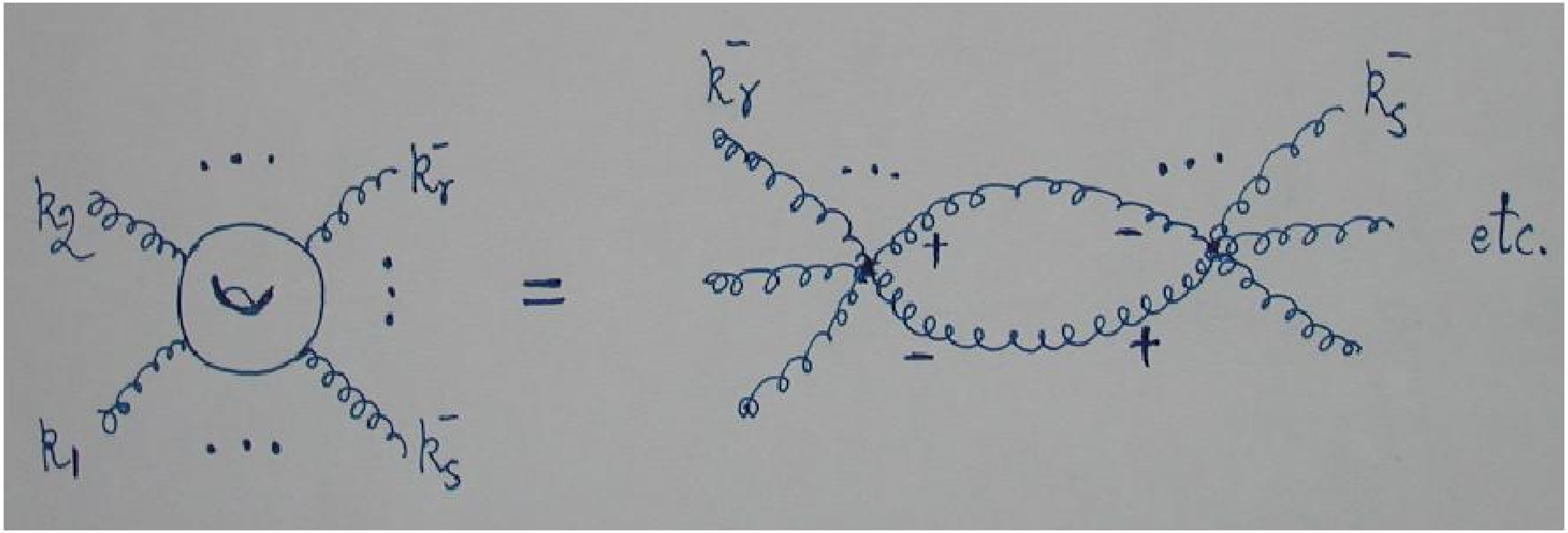
$$V_{n+1} = \frac{p_1^2}{\phi_2 \phi_{n+1}} \frac{1}{[2,3] \cdots [n, n+1]}, \tag{34}$$

where p_1 is the off-shell momentum (all the rest momenta are on-shell).

The above rules can also be used to derive a closed formula for NMHV (with 3 negative helicities) amplitude. It can also be used to derive a set of new recursive methods to compute the more general NMHV amplitudes.

It can be extended to include fermions.

It can also be used to compute one-loop MHV and NMHV amplitudes for supersymmetric theory (by using the method of unitarity).



Down of a new calculation era.

The CSW approach may be the long-waited breakthrough in perturbative QCD. (Z. Bern)