Spontaneous Chiral Symmetry Breaking in QCD

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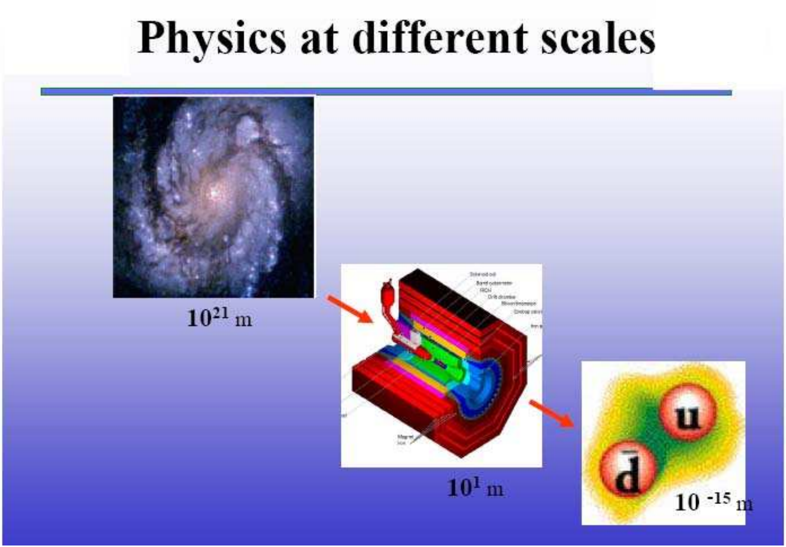
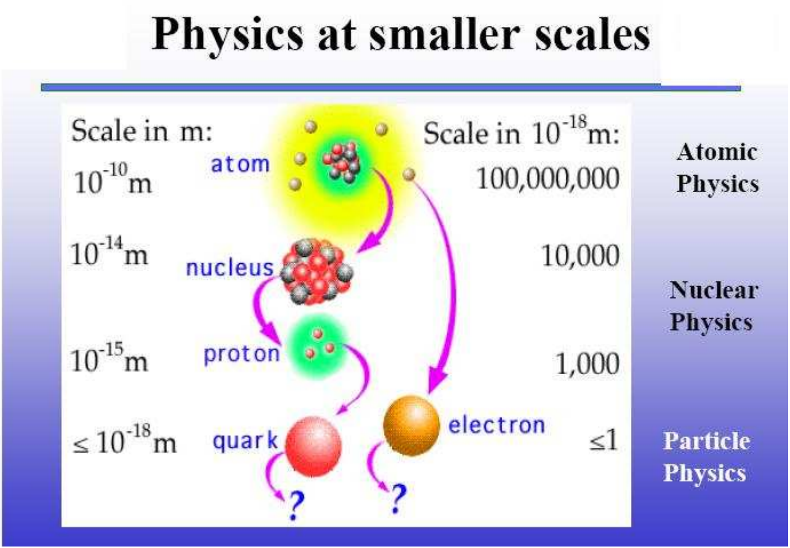
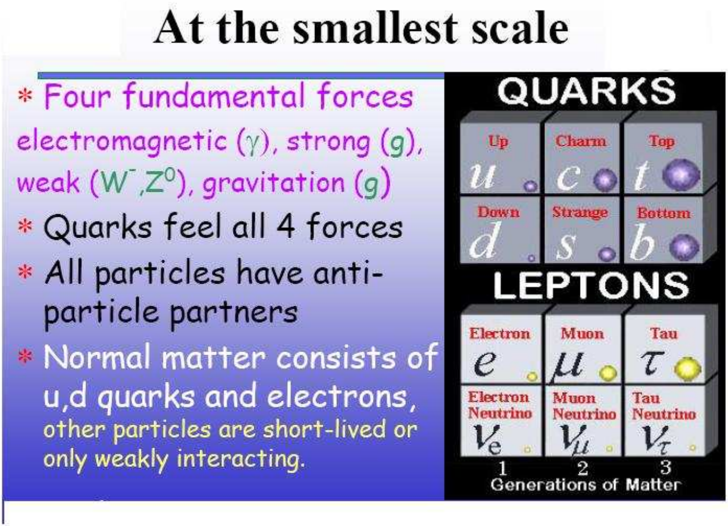
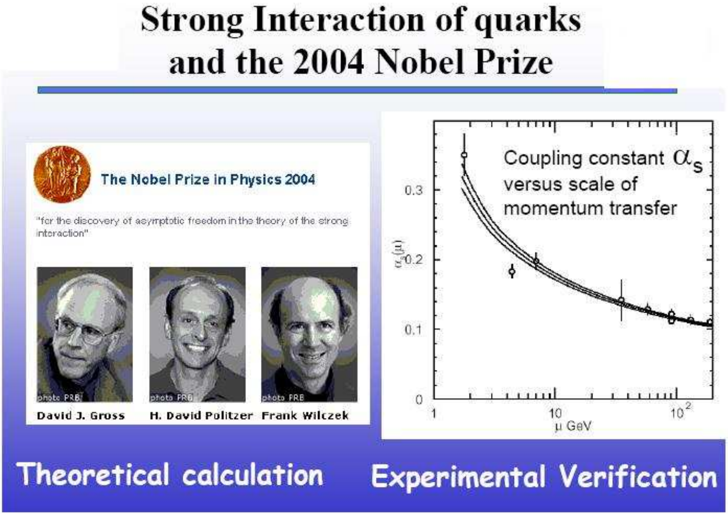

Physics at different scales
10^{21} m
10^{1} m
u
$\bar{d}$
10^{-15} m


Physics at smaller scales
Scale in m:
atom
Scale in 10^-18 m:
10^-10 m
100,000,000
Atomic Physics
10^-14 m
nucleus
10,000
Nuclear Physics
10^-15 m
proton
1,000
≤ 10^-18 m
quark
electron
≤1
Particle Physics
?
?


At the smallest scale
* Four fundamental forces
electromagnetic (γ), strong (g),
weak (W, Z0), gravitation (g)
* Quarks feel all 4 forces
* All particles have anti-particle partners
* Normal matter consists of u,d quarks and electrons,
other particles are short-lived or only weakly interacting.
QUARKS
Up
u
Charm
c
Top
t
Down
d
Strange
s
Bottom
b
LEPTONS
Electron
e
Muon
μ
Tau
τ
Electron Neutrino
νe
Muon Neutrino
νμ
Tau Neutrino
ντ
1
2
3
Generations of Matter


Strong Interaction of quarks
and the 2004 Nobel Prize
The Nobel Prize in Physics 2004
"for the discovery of asymptotic freedom in the theory of the strong interaction"
photo PRB
photo PRB
photo PRB
David J. Gross
H. David Politzer
Frank Wilczek
Coupling constant α_s versus scale of momentum transfer
$\alpha_s(\mu)$
0.3
0.2
0.1
0
1
10
10^2
μ GeV
Theoretical calculation
Experimental Verification

The QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d,s,c,b,t} \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

with $q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$, where r=red, g=green, and b=blue,

and the gauge-covariant derivative

$$D_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} - ig\sum_{a=1}^{8} \frac{\lambda_a^C}{2} \mathcal{A}_{\mu,a} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

and the gluon field tensor

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c}$$

Symmetries of the QCD Lagrangian

Quark masses

$$\begin{pmatrix} m_u = 0.005\,\mathrm{GeV} \\ m_d = 0.009\,\mathrm{GeV} \\ m_s = 0.175\,\mathrm{GeV} \end{pmatrix} \ll 1\,\mathrm{GeV} \leq \begin{pmatrix} m_c = (1.15 - 1.35)\,\mathrm{GeV} \\ m_b = (4.0 - 4.4)\,\mathrm{GeV} \\ m_t = 174\,\mathrm{GeV} \end{pmatrix}$$

Assuming $m_u,\, m_d \approx 0,$

the **QCD Lagrangian for just up and down quarks** reads

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{l=u,d} \bar{q}_l i \not{D}\, q_l - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a .$$

Symmetries of the QCD Lagrangian, cont'd

Introduce projection operators

"Right-handed" $$P_R = \frac{1}{2}(1+\gamma_5) = P_R^\dagger$$

"Left-handed" $$P_L = \frac{1}{2}(1-\gamma_5) = P_L^\dagger$$

Properties

Complete $$P_R + P_L = 1$$

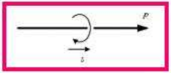
Idempotent $$P_R^2 = P_R, \quad P_L^2 = P_L$$

Orthogonal $$P_R P_L = P_L P_R = 0$$

Symmetries of the QCD Lagrangian, cont'd

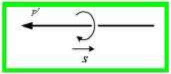
For zero-mass quarks,

quark field of **right-handed chirality**



$q_R = P_R q,$

quark field of **left-handed chirality**:



$q_L = P_L q.$

The QCD Lagrangian can be written

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{l=u,d} \left(\bar{q}_{R,l} i \not{D} q_{R,l} + \bar{q}_{L,l} i \not{D} q_{L,l} \right) - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a .$$

$\Rightarrow \quad SU(2)_L \times SU(2)_R$ symmetry **= "Chiral Symmetry"**

(Because the mass term is absent; mass term destroys chiral symmetry.)

Explicit symmetry breaking due to non-zero quark masses

The mass term that we neglected breaks chiral symmetry explicitly:

$$-\sum_{l=u,d}\bar{q}_l m_l q_l = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_R)$$

with
$$\begin{aligned} M &= \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \\ &= \frac{1}{2}(m_u+m_d)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}(m_u-m_d)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \frac{1}{2}(m_u+m_d)\, I + \frac{1}{2}(m_u-m_d)\,\tau_3 \end{aligned}$$

Both terms break $SU(2)_A$, but the 1st term is invariant under $SU(2)_V$.

⇨ if $m_u = m_d$, then there is $SU(2)_V$ symmetry (=Isospin symmetry).

Spontaneous symmetry breaking

If the ground state of QCD (=hadron spectrum) has the same symmetry as the Lagrangian (namely, chiral symmetry), then "parity doublets" exist; because:

Let $|i,+\rangle$ denote an eigenstate of $H^0_{\rm QCD}$ with eigenvalue E_i,

$$H^0_{\rm QCD}|i,+\rangle = E_i|i,+\rangle,$$

having positive parity,

$$P|i,+\rangle = +|i,+\rangle,$$

such as, e.g., a member of the ground state baryon octet (in the chiral limit). Defining $|\phi\rangle = Q^a_A|i,+\rangle$, because of $[H^0_{\rm QCD}, Q^a_A] = 0$, we have

$$H^0_{\rm QCD}|\phi\rangle = H^0_{\rm QCD}Q^a_A|i,+\rangle = Q^a_A H^0_{\rm QCD}|i,+\rangle = E_i Q^a_A|i,+\rangle = E_i|\phi\rangle,$$

i.e, the new state $|\phi\rangle$ is also an eigenstate of $H^0_{\rm QCD}$ with the same eigenvalue E_i but of opposite parity:

$$P|\phi\rangle = PQ^a_A P^{-1}P|i,+\rangle = -Q^a_A(+|i,+\rangle) = -|\phi\rangle.$$

Spontaneous symmetry breaking, cont'd

What would parity doublets look like?

Nucleons of positive parity: p(1/2+,938.3), n(1/2+,939.6), I=1/2;

nucleons of negative parity: N(1/2-,1535), I=1/2.

But, the masses are very different: NOT a parity doublet!

A meson of negative parity: $\rho(1-,770), I=1.$

the "same" with positive parity: $a_1(1+,1260), I=1.$

But again, the masses are very different: NOT a parity doublet!

Conclusion: Parity doublets are not observed in the low-energy hadron spectrum.

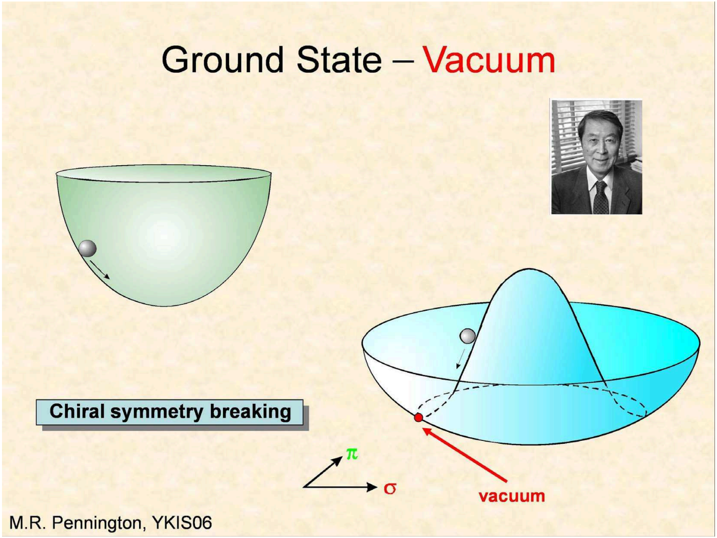
→ **Chiral symmetry is spontaneously broken.**

The linear sigma model

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_s + c\sigma \ , \\ \mathcal{L}_s &= \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\pi)^2] + \frac{m^2}{2}[\sigma^2 + \pi^2] - \frac{\lambda}{4}[\sigma^2 + \pi^2]^2 \ , \quad (1) \end{aligned}$$

when $c \to 0$, the lagrangian is invariant under $SU_L(2) \times SU_R(2)$ chiral rotations

$$\begin{aligned} \vec{\pi} &\to \vec{\pi} + \vec{\alpha} \times \vec{\pi} - \vec{\beta}\sigma \ , \\ \sigma &\to \sigma + \vec{\beta} \cdot \vec{\pi} \quad (2) \end{aligned}$$


Ground State – Vacuum
Chiral symmetry breaking
π
σ
vacuum
M.R. Pennington, YKIS06

To be or not to be, this is a question

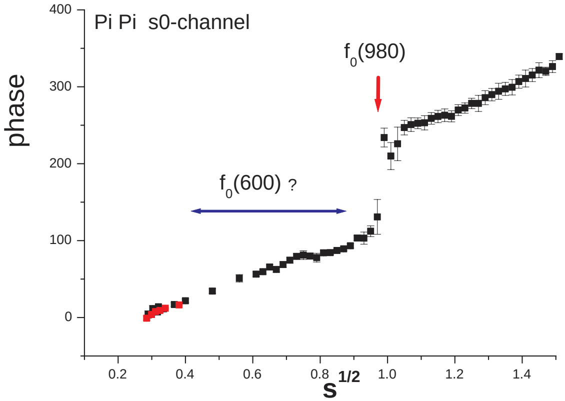


Figure: IJ=00 channel $\pi\pi$ scattering phase shift data from CERN-Munich and E865.

To be or not to be, this is a question

- One of the most important concept in strong interaction physics – spontaneous breaking of chiral symmetry.
- The σ particle was introduced in association with the linear σ model at hadron level, for SχSB.
- Early studies of nuclear physics requires σ.
- Non-linear realization of chiral symmetry was later discovered. Coleman, Wess, Zumino, rejects naturally the sigma meson
- Highly non-perturbative, strong interaction nature —- a source of debate over a few decades.

Goldstone's Theorem

When a continuous symmetry is broken, then there exists a (massless) boson with the quantum numbers of the broken generator.

Here: The 3 axial generators $Q_a^A \ (a = 1,2,3)$ are broken, therefore, 3 pseudoscalar bosons exist: the 3 pions π^+, π^0, π^-

This explains the small mass of the pion. The pion mass is not exactly zero, because the u and d quark masses are no exactly zero either.

Weinberg's "Folk Theorem"

If one writes down the most general possible Langrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Langrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles.

Physica **96A**, *327 (1979)*

Non-linear realization of chiral symmetry & χPT

$$\Sigma = \sigma + i\vec{\pi}\cdot\vec{\tau} \ ,$$
$$\mathcal{L}_s = \frac{1}{4}\mathrm{Tr}[\partial_\mu \Sigma \partial_\mu \Sigma^+] - \frac{m^2}{4}\mathrm{Tr}[\Sigma\Sigma^+] - \frac{\lambda}{16}\mathrm{Tr}[\Sigma\Sigma^+]^2 .$$
$$\Sigma \equiv \sigma U \ ,$$
$$\mathcal{L}_2 = \frac{1}{4} f_\pi^2 \mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger + 2Bm(U + U^\dagger))$$
$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 ...$$

Gasser, Leutwyler

$$\begin{aligned}
\mathcal{L}_4 \ = \ & L_1 < \partial_\mu U^+ \partial^\mu U >^2 + L_2 < \partial_\mu U^+ \partial_\nu U >< \partial^\mu U^+ \partial^\nu U > \\
& + L_3 < \partial_\mu U^+ \partial^\mu U \partial_\nu U^+ \partial^\nu U > \\
& + L_4 < \partial_\mu U^+ \partial^\mu U >< \chi^+ U + \chi U^+ > \\
& + L_5 < \partial_\mu U^+ \partial^\mu U (\chi^+ U + \chi U^+) > + \cdots \qquad (3)
\end{aligned}$$

The σ meson is CRUCIAL to adjust chiral perturbation theory to experiments

Z. G. Xiao and H.Z., Nucl. Phys. A695(2001)273
Analytic continuation of $\sin(2\delta_\pi)$ on the complex s plane:

$$F(s) \equiv \frac{1}{2i\rho}(S(s) - \frac{1}{S(s)}) \ , \ (\sin(2\delta_\pi) = \rho F, F(s) = 2\mathrm{Re}\, T(s)) \ .$$

$$\begin{aligned} F(s) &= \alpha + \sum_i \frac{\beta_i}{2i\rho(s_i)(s-s_i)} - \sum_j \frac{1/2i\rho(z_j^{II})}{S'(z_j^{II})(s-z_j^{II})} \\ &+ \frac{1}{\pi}\int_L \frac{\mathrm{Im}_L F(s')}{s'-s} ds' + \frac{1}{\pi}\int_R \frac{\mathrm{Im}_R F(s')}{s'-s} ds' \ . \end{aligned} \quad (4)$$

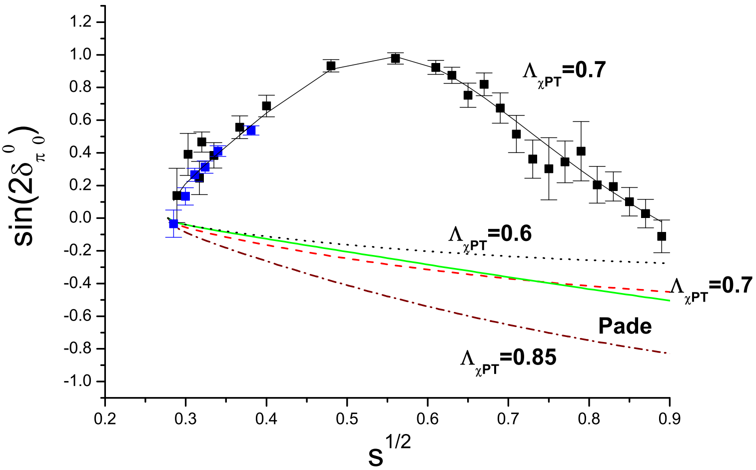


Figure: $\mathrm{Im_L} F$ is estimated using 1–loop χPT.

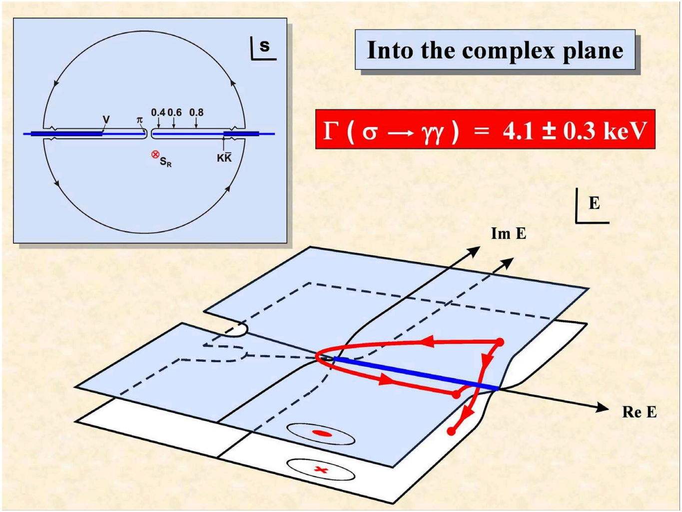
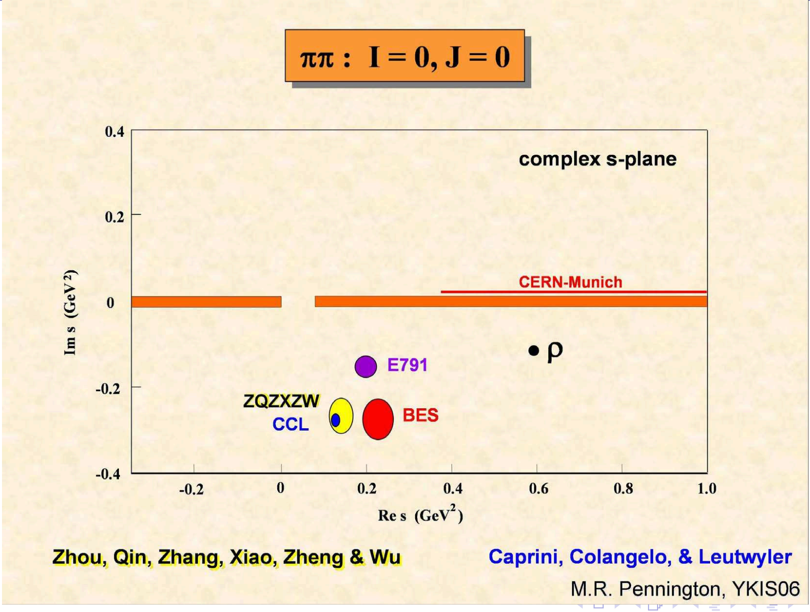


Figure: From Mike Pennington YKIS06


ππ : I = 0, J = 0
complex s-plane
CERN-Munich
ρ
E791
ZQZXZW
CCL
BES
Im s (GeV²)
Re s (GeV²)
0.4
0.2
0
-0.2
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1.0
Zhou, Qin, Zhang, Xiao, Zheng & Wu
Caprini, Colangelo, & Leutwyler
M.R. Pennington, YKIS06

Evidence for the existence of σ from BES experiments:
$J/\psi \to \omega\pi^+\pi^-$ (Phys. Lett. B598: 149-158,2004.) (also E791)

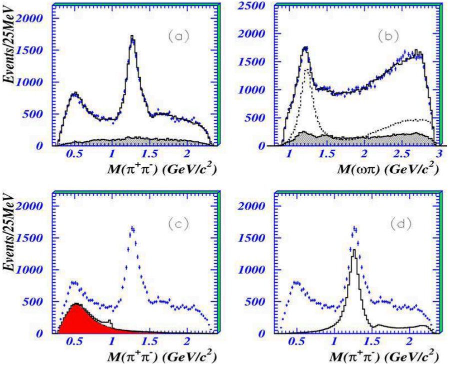


Figure: Dalitz plot and $\pi^+\pi^-$ invariant mass spectrum.

$$M_\sigma - i\Gamma_\sigma/2 = (541 \pm 39) - i(252 \pm 42)\text{MeV}$$

The PKU parametrization form

The factorized S matrix and the separable singularities:

$$S^{phy.} = \prod_i S^{R_i} \cdot S^{cut} \; . \tag{5}$$

S^{cut}: no longer contains any pole:

$$\begin{aligned} S^{cut} &= e^{2i\rho f(s)} \; , \\ f(s) &= \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi}\int_R \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} \; . \end{aligned} \tag{6}$$

Subtraction constant can be determined!

Mandelstam Analyticity(Polynomial boundedness of scattering amplitudes) (Z. Y. Zhou and H.Z., NPA775(2006)212) ⇒

$$f(0) = 0 \; . \tag{7}$$

The background phase contribution

The phase is additive, $\delta(s) = \sum_i \delta_{R_i} + \delta_{b.g.}$.

$$\delta_{b.g.}(s) = \rho(s) f(s) \ . \tag{8}$$

$$\mathrm{Im}_{L,R} f(s) = -\frac{1}{2\rho(s)} \log |S^{phy}(s)| \ . \tag{9}$$

The approximation scheme:

$$\mathrm{disc} f_{\mathcal{L}} = \mathrm{disc}\{\frac{1}{2i\rho(s)} \log \left[S^{\chi\mathrm{PT}}(s)\right]\} \ . \tag{10}$$

Background contributes negatively to scattering phase shift and scattering lengths. Crucial in stabilizing the pole position!

Quantum Scattering Theory Correspondence

For a finite range potential:

$$S(k) = e^{-2ikR} \prod_{1}^{\infty} \frac{k_n + k}{k_n - k} , \tag{11}$$

where k is the (single) channel momentum and k_n pole locations on the complex k plane. The Eq. (11) automatically predicts a negative background contribution!

Eq. (11) written down by Ning Hu, Phys. Rev. 74(1948)131.

Rigorous prove by T. Regge in 1958.

Prof. Ning Hu (1916–1997)

- 1943: PhD in Caltech.
- 1942–1945: IAS, Princeton.
- 1945–1947: IAS, Ireland.
- 1947 – 1948: Nils Bohr Institute.
- 1948 – 1950, Cornell U., Wisconsin U..
- 1951 – 1997: Peking University

Theory for meson nucleon interaction;
General relativity and gravitational wave;
S matrix theory;
Quantum electro dynamics;
Quark model.

Figure:

Fig. 9.4 Pauli with, from left to right, Ning Hu, José Leite Lopez, and Josef Maria Jauch, November 1945 (CERN)

$\kappa(700)$

The κ resonance also has a rather long history and the status is more controversial. Data also contain some problems.

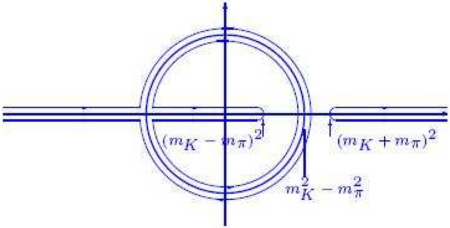
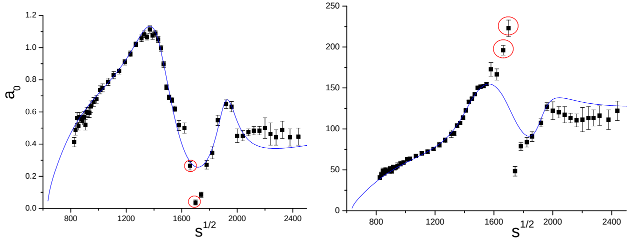


Figure 5: The left, circular and right hand cut of πK scatterings.

κ exists if the scattering length is not far from the value obtained from χPT. Conclusions (almost) model independent.
(H.Q. Zheng, et. al., Nucl.Phys.A733:235-261,2004)
Taking $f(0) = 0$ into account:
Z. Y. Zhou and H. Q. Zheng, Nucl. Phys. **A755** (2006) 212.



Large N_c expansion of PKU parametrization form

$$S = S^{\rm cut} \cdot \prod_{\rm R} S^{\rm R} ,$$

$$\chi\text{PT expansion} \leftrightarrows \text{resonance expansion} \tag{12}$$

Matching: at threshold and at leading order of $\frac{1}{N_c}$ expansion.

$$\prod_{\rm R} S^{\rm R}(s) = \prod_{\rm R} \left(1 + 2\, i\, \rho(s)\, T^{\rm sR}(s) \right) , \tag{13}$$

with

$$T^{\rm sR}(s) = \frac{s\, G_{\rm R}[z_0]}{M^2_{\rm R}[z_0] - s - i\rho(s) s\, G_{\rm R}[z_0]} , \tag{14}$$

$G_{\rm R}[z_0]/(M^2_{\rm R}[z_0] - 4m^2_\pi)$ always positive definite for any second sheet pole $\Rightarrow$ when $N_C \to \infty$, poles can only reside on the real axis or at infinity [Z.G. Xiao & HZ, MPLA 2007].

[Z.H. Guo, J.J. Sanz-Cillero, HZ 07]

$$f(s) = f_L(s) + f_R(s) \equiv \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} ds' + \frac{s}{\pi}\int_R \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} ds' ,$$

$$f(s)_{L,\,\pi\pi} = -|T(0)| + \mathcal{O}(1/N_C^2) ,$$

$$f_L(s) = -|T(0)| + \sum_R T^{\mathrm{tR}}(s) + \mathcal{O}\left(\frac{1}{N_C^2}\right) ,$$

$$T^{\mathrm{tR}}(s) = \frac{s}{\pi}\int_{-\infty}^{-M_{\mathrm{R}}^2+4m_\pi^2} \frac{\mathrm{Im}\, T^{\mathrm{tR}}(s')}{s'(s'-s)} ds' .$$

Calculable at $O(N_c^{-1})$ using Froissart–Gribov projection formula.

[Z.H. Guo, J.J. Sanz-Cillero, HZ 07]

$$T(s)_{N_C\to\infty} = T(0) + \sum_R T^{\rm tR}(s) + \sum_R T^{\rm sR}(s)\,. \qquad (15)$$

An alternative way to reach this relation is through the partial T–matrix dispersive relation

$$T(s) = T(0) + \frac{s}{\pi}\int_{-\infty}^{0} \frac{ds'\,{\rm Im}\,T(s')}{s'(s'-s)} + \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'\,{\rm Im}\,T(s')}{s'(s'-s)}\,. \qquad (16)$$

$\Rightarrow$ The PKU parametrization form is reduced to a T–matrix partial wave dispersion relation under narrow width approximation (and in the large N_c limit). The former is hence simply a combination of partial wave dispersion relation and single channel unitarity.

Matching at different orders:

$O(p^2)$:

$$\frac{1}{16\pi f^2} = \frac{9\Gamma_V^{(0)}}{M_{\rm V}^{(0)\,3}} + \frac{2\Gamma_S^{(0)}}{3M_{\rm S}^{(0)\,3}},$$

Three different channels produce the same results!

[Hikasa and Igi 1993] N/D

KSRF relation:

$$\frac{1}{16\pi f^2} = \frac{6\Gamma_V^{(0)}}{M_{\rm V}^{(0)\,3}}$$

In the IJ=11 channel, this is equivalent to neglecting crossed channel vector and scalar exchanges!

[Basdevant, Zinn-Justin 1971]

Careful chiral power counting not considered in previous studies

	$T(0)$	$t_0^{\rm tR}$	$t_0^{\rm sR}$	$t_0^{\chi PT} = m_\pi a_J^I$
$IJ = 11$	$-\frac{m_\pi^2}{24\pi f^2}$	$\frac{4\Gamma_S}{9M_S^3} + \frac{2\Gamma_V}{M_V^3}$	$\frac{4\Gamma_V}{M_V^3}$	0
$IJ = 00$	$-\frac{m_\pi^2}{32\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} + \frac{36\Gamma_V}{M_V^3}$	$\frac{4\Gamma_S}{M_S^3}$	$\frac{7m_\pi^2}{32\pi f^2}$
$IJ = 20$	$\frac{m_\pi^2}{16\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} - \frac{18\Gamma_V}{M_V^3}$	0	$-\frac{m_\pi^2}{16\pi f^2}$

Table: Summary of the different contributions $T(0)$, $t_0^{\rm tR}$, $t_0^{\rm SR}$ to the scattering lengths at leading order in the m_π^2 expansion. The generalized KSRF-relation derives from the matching of the sum of the first three columns to the χPT prediction, $t_0^{\chi PT}$. In the last line, $T(0)$ contains the sum of $-|T(0)|$ and the $IJ = 20$ virtual pole contribution.

Partial wave amplitudes remember crossing symmetry!

Matching at different orders:

$O(p^4)$:

$$L_2 = 12\pi f^4 \frac{\Gamma_{\rm V}^{(0)}}{M_{\rm V}^{(0)\,5}}\,,$$

$$L_3 = 4\pi f^4 \left(\frac{2\Gamma_{\rm S}^{(0)}}{3M_{\rm S}^{(0)\,5}} - \frac{9\Gamma_{\rm V}^{(0)}}{M_{\rm V}^{(0)\,5}} \right)\,.$$

[G. Ecker *et al.*, 1989]
Positivity:

$$L_2 > 0\ ,\ \ 3L_2 + L_3 > 0$$

[Pham and Truong 1985]
Numerically scalar effects easy to be overlooked in low energy chiral expansions.

Matching at different orders:

$O(p^2 m_\pi^2)$:
A novel relation any lagrangian model has to obey: [GSZ]

$$0 = \frac{2}{3}\,\frac{\Gamma_{\rm S}^{(0)}}{M_{\rm S}^{(0)\,5}}\,[\alpha_S+6] + \frac{9\,\Gamma_{\rm V}^{(0)}}{M_{\rm V}^{(0)\,5}}\,[\alpha_V+6]\,. \qquad (17)$$

The physical widths and masses, $\Gamma_{\rm R}$ and $M_{\rm R}$, carry an implicit dependence on m_π^2, which can be expressed in the form

$$\frac{\Gamma_{\rm R}}{M_{\rm R}^3} = \frac{\Gamma_{\rm R}^{(0)}}{M_{\rm R}^{(0)\,3}}\left[1+\alpha_R\,\frac{m_\pi^2}{M_{\rm R}^{(0)\,2}} + \mathcal{O}\left(m_\pi^4\right)\right]. \qquad (18)$$

$O(p^6)$: $r_2, \cdots, r_6$.

On the nature of $f_0(600)$?

Linear sigma models at hadron level:
N. A. Tornqvist, Z. Phys. **C68**(1995)647; N. N. Achasov and G. N. Shestakov, PRD 49(1994)5779; R. Kaminski, et al., PRD 50(1994)3154; M. Ishida et al., PTP **99**(1998)1031.

Linear sigma model at quark level:
M. D. Scadron et al., Nucl. Phys. **A724**(2003)391.

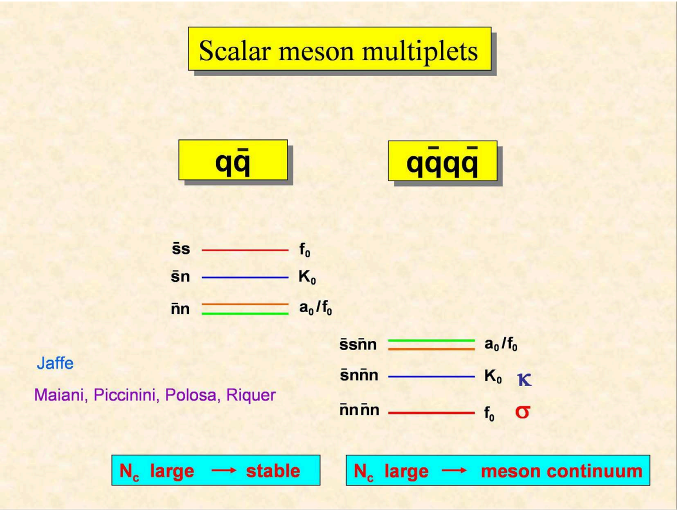
Tetra quark model: R. L. Jaffe, Phys. Rev. **D15** (1977) 267.

ENJL model:
T. Hatsuda and T. Kunihiro Phys. Rep. (1994); A. A. Osipov et al., NPA745(2004)81; V. Dmitrasinovic, PRC 53 (1996) 1383.

Dynamically generated resonance from χPT:
A. Gomez Nicola, J. R. Pelaez, PRD 65(2002)054009; J. A. Oller et al., PRD 59(1999)074001.

Dynamical resonance in $3P_0$ model: E. van Beveren et al., Z. Phys. **C30**(1986)615

The scalar puzzle



Mass relations?

Pole mass vs. line shape mass

$$m_\sigma < m_\kappa < m_{a_0} \ ? \tag{19}$$

A resonance located at z_0: The S matrix can be expressed as:

$$S(s) = \frac{M^2(z_0) - s + i\rho(s)sG[z_0]}{M^2(z_0) - s - i\rho(s)sG[z_0]} \ , \tag{20}$$

$M^2(z_0) >> \mathrm{Re}[z_0]$ for a light and broad resonance $\Rightarrow$ suggests the correct mass relation:
[L. Y. Xiao, HZ & Z. Y. Zhou, talk given at QCD06, Montpellier]

$$1GeV \simeq M_\sigma \leq M_{a_0} < M_\kappa \ ! \tag{21}$$

$$M_\sigma \sim 930MeV \ , \ \ M_\kappa \sim 1380MeV \ . \tag{22}$$

Hard to imagine how to develop a large width from a small bare mass $M_\sigma \sim 500\mathrm{MeV}$!

Scalars in NJL and E-NJL models

To explain the large widths of σ and κ:

- NJL model, i.e., linear realization of chiral symmetry (scalars as chiral partners of the pseudo-goldstones) can provide a consistent description to light and broad σ and κ, and the narrow $a_0(980)$, but un-natural.
- The E-NJL model has no chance to describe the vector sector and the scalar sector simultaneously well.
- Problem with $f_0(980)$
- light and spurious poles may occur in K-matrix unitarization

[M.X. Su, L.Y. Xiao, HZ, NPA 2007]

Unitarization approximations

- σ appear as dynamical pole in unitarized χPT amplitudes.
- Attention to spurious poles
- Method should be examined, results should be understood.

Using PKU parametrization form, one demonstrates [Z. X. Sun, et al, 2007] in the large N_C and chiral limit, one gets the σ pole position:

$$s_\sigma = \frac{16 i\pi f_\pi^2}{1+16 i\pi f_\pi^2 \triangle} \,,$$
$$\triangle = \frac{2}{3f_\pi^2}(22L_1+14L_2+11L_3) \propto O(N_C^0) \,, \qquad (23)$$

the same as [1,1] Padé prediction

under two assumptions:

- s-channel (σ) pole dominance
- neglecting dynamical (crossed channel) cuts

It helps to understand why

1. The [1,1] Padé works good in the I,J=1,1 channel
2. not so good in the IJ=00 channel
3. runs into disaster in the I,J=2,0 channel

Gives a guide on when Padé approximation works, when fails!

Dynamically generated resonance?

Different N_c behavior⇒
[Pelaez 04, Pelaez, Rios 06] ⇒ dynamically generated resonance.

O(N) linear sigma model (QCD)
The [n,n] Padé amplitudes: exact sigma pole location. The K matrix good approximation. [Willenbroch 1990]

Testing methods: Non-linearize the O(N) model lagrangian, integrate out the σ (at tree level) or forget it, studying various unitarized amplitudes and studying the N_c trajectories of the ‘dynamical’ pole.

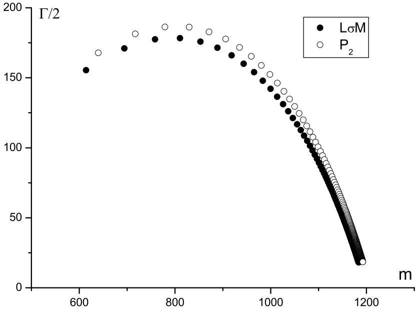


Figure: σ pole trajectory with respect to N_c, in Padé approximant and in the linear (solvable) model.

Lessons: The bent structure of the 'dynamical σ' pole trajectory does not demonstrate in any sense that the σ is 'dynamical'. Rather it is very much like the property of the true σ meson a la Nambu, Gellman and Levy [L.Y. Xiao, Z.H. Guo, H.Z., IJMPA 08]

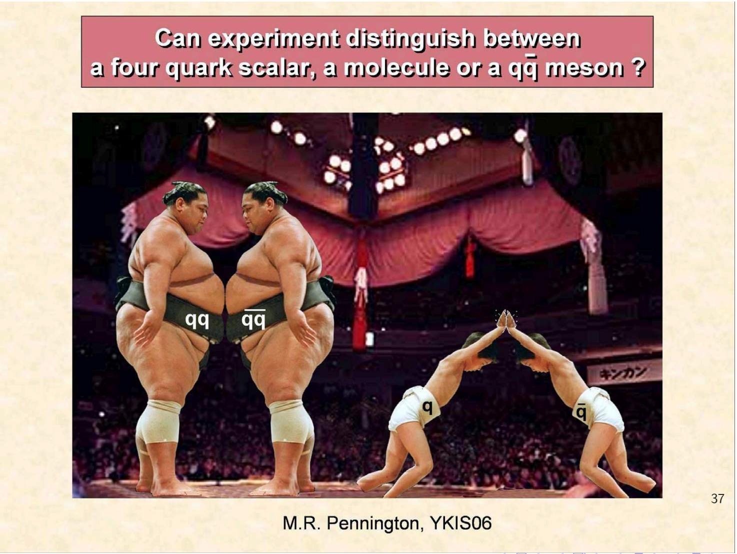
Exotica?

Here exotic pole $\equiv$ a pole with a N_c dependence different from $M \sim O(1)$, $\Gamma \sim O(\frac{1}{N_c})$.

All exotic poles so far encountered in various unitarization approximations are either spurious or accompanied by spurious poles (Conclusions obtained by analyticity+ N_c counting rules).

One exception: the virtual pole in IJ=20 channel of $\pi\pi$ scattering.

All evidences collected so far are consistent with a $M_\sigma \sim O(1)$, $\Gamma_\sigma \sim O(\frac{1}{N_c})$ behavior in the large N_c limit


Can experiment distinguish between
a four quark scalar, a molecule or a $q\bar{q}$ meson ?
qq
$\overline{qq}$
キンカン
q
$\bar{q}$
37
M.R. Pennington, YKIS06

Is σ a tetra quark state?

as suggested by Lattice QCD or QCD sum rule calculations?

Large width of σ means it strongly couples to (or renormalized by) $\pi\pi$ continuum, as already suggested by toy linear σ model.

quark quadrulinear currents $= \pi\pi$ continuum
[S. Coleman, *aspects of symmetry*]

Only unambiguous manifestation of 'tetra quark state' comes from state with exotic quantum numbers

Nevertheless see [Hosaka, Hyodo, Jido 07], in disfavor of the existence of such states!

If one has to understand σ using quark language:

VERY ROUGHLY speaking σ is a $q\bar{q}$ state (as a seed to generate such a σ) heavily renormalized by $\pi\pi$ continuum, or contain a large $q^2\bar{q}^2$ (even $q^4\bar{q}^4$) component. A $q\bar{q}$ component is however crucial to maintain the correct large N_c behavior.

A combination of $q\bar{q}$ and $q^2\bar{q}^2$ in the calculation is welcome! [Sugiyama et al., 2007]

After all, the σ (and the pions) collective mode of QCD!

Summary

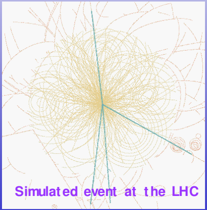
- There exists a broad $f_0(600)$.
- the $f_0(600)$ is most likely the σ responsible for the breaking of a linearly realized chiral symmetry in QCD.
- DRs crucial for a model independent study of σ.
- A broad $\kappa(700)$ exists, similar to σ according to dispersive analysis
- chiral symmetry + analyticity + unitarity + crossing symmetry + N_c counting rule very useful in studying physics related to these light and broad states.

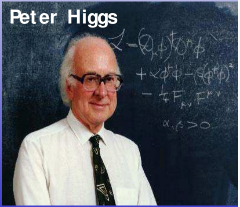
Dynamic Origin of Mass

* Most of the mass of the proton is of dynamical nature (field energy and quark-kinetic energy)
- There is a dynamical mechanism to account for masses of elementary particles (leptons, quarks, force carriers) -> Higgs mechanism

Remnant of the Mechanism:

The Higgs-Boson (searched after for almost 35 years)





Oct. 7th 2004 Eckhard von Toerne, Colloquium Benedictine College 12


Colliding-Beam Experiments
K-State
Colliders
Tevatron at Fermilab (Chicago)
P+P̄ @ 2 TeV
Started ~1980
Large Hadron Collider (LHC) at CERN, Switzerland
P+P @ 14 TeV, Start 2007
International Linear Collider at (?)
e+ e- @ 1 TeV, Start 2017(?)
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Thanks!

Backups

The simplest S matrices in PKU form

A virtual/bound state pole at s_0 (s_0 real). $s_L < s_0 < s_R$, $s_L = (m_K - m_\pi)^2$, $s_R = (m_K + m_\pi)^2$. The scattering length:

$$a(s_0) = \pm \frac{2\sqrt{s_R}}{s_R - s_L}\sqrt{\frac{s_0 - s_L}{s_R - s_0}} \ . \qquad (24)$$

The S matrix can be expressed as,

$$S(s) = \frac{1 \pm i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}{1 \mp i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}} \ . \qquad (25)$$

A resonance located at z_0 ($\mathrm{Im}[z_0] > 0$) and z_0^*: The S matrix can be expressed as:

$$S(s) = \frac{M^2(z_0) - s + i\rho(s)sG[z_0]}{M^2(z_0) - s - i\rho(s)sG[z_0]} \,, \tag{26}$$

where

$$M^2(z_0) = \mathrm{Re}[z_0] + \frac{\mathrm{Im}[z_0]\,\mathrm{Im}[z_0\,\rho(z_0)]}{\mathrm{Re}[z_0\,\rho(z_0)]} \,,\ G[z_0] = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[z_0\,\rho(z_0)]} \,, \tag{27}$$

and the scattering length is,

$$a(z_0) = \frac{\mathrm{Im}[z_0]\mathrm{Re}[z_0\,\rho(z_0)]}{\mathrm{Im}[z_0]^2 + \mathrm{Re}[z_0\rho(z_0)]^2}\,\frac{2\sqrt{s_R}\,\left(M^2(z_0) - s_R\right)}{(s_R - z_0)\,(s_R - z_0^*)} \,. \tag{28}$$

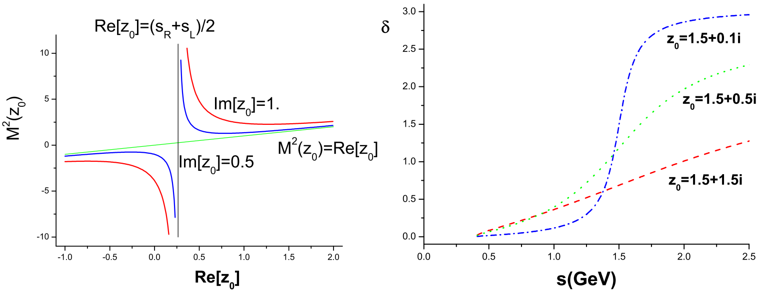

Figure: Left: $M^2(z_0)$ as a function of $\mathrm{Re}[z_0]$, for fixed $\mathrm{Im}[z_0]$. Right: some examples of resonance contribution to the phase shift.

$M(z_0)$ is the place where the resonance contribution to the phase shift passes $\pi/2$. However, a light and broad resonance can have a very large $M(z_0)^2$. [the red dragon].