Proton mass and spin from supercomputer

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Lattice 19
Wuhan·China | June 16-22

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http://lattice2019.ccnu.edu.cn, June 16-22, Wuhan

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The US National Academy of Science assessment of U.S.-based **Electron-Ion Collider** Science

https://www.nap.edu/read/25171/chapter/9#92

……

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

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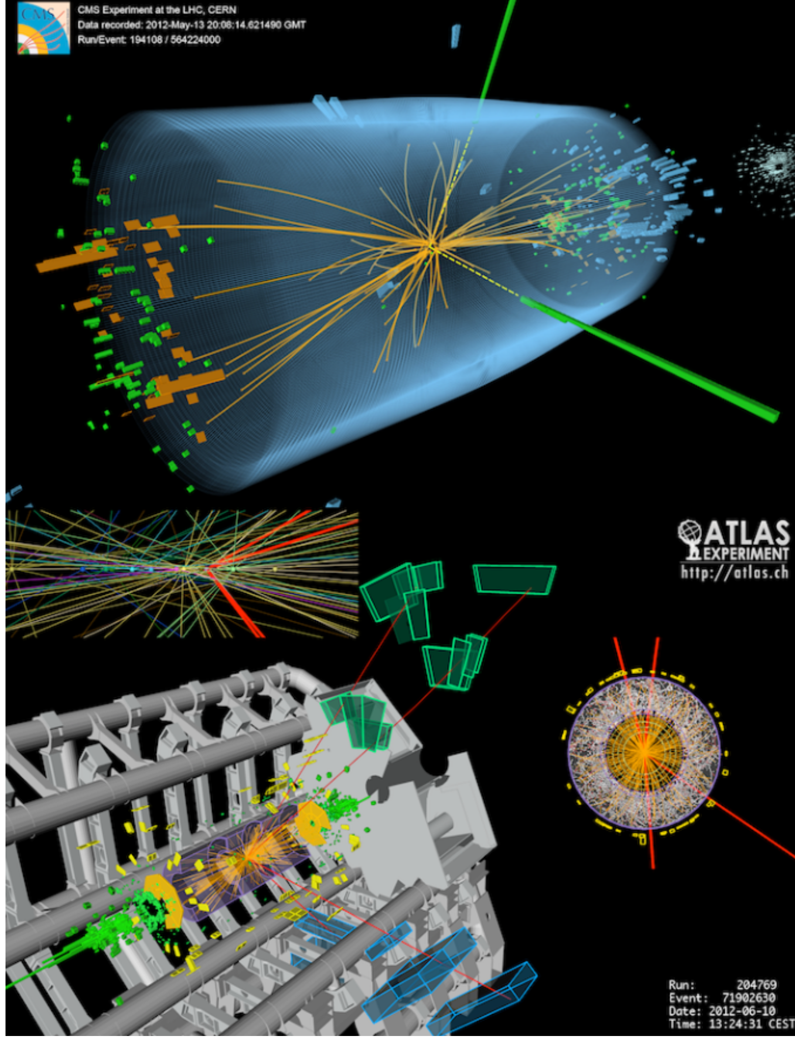
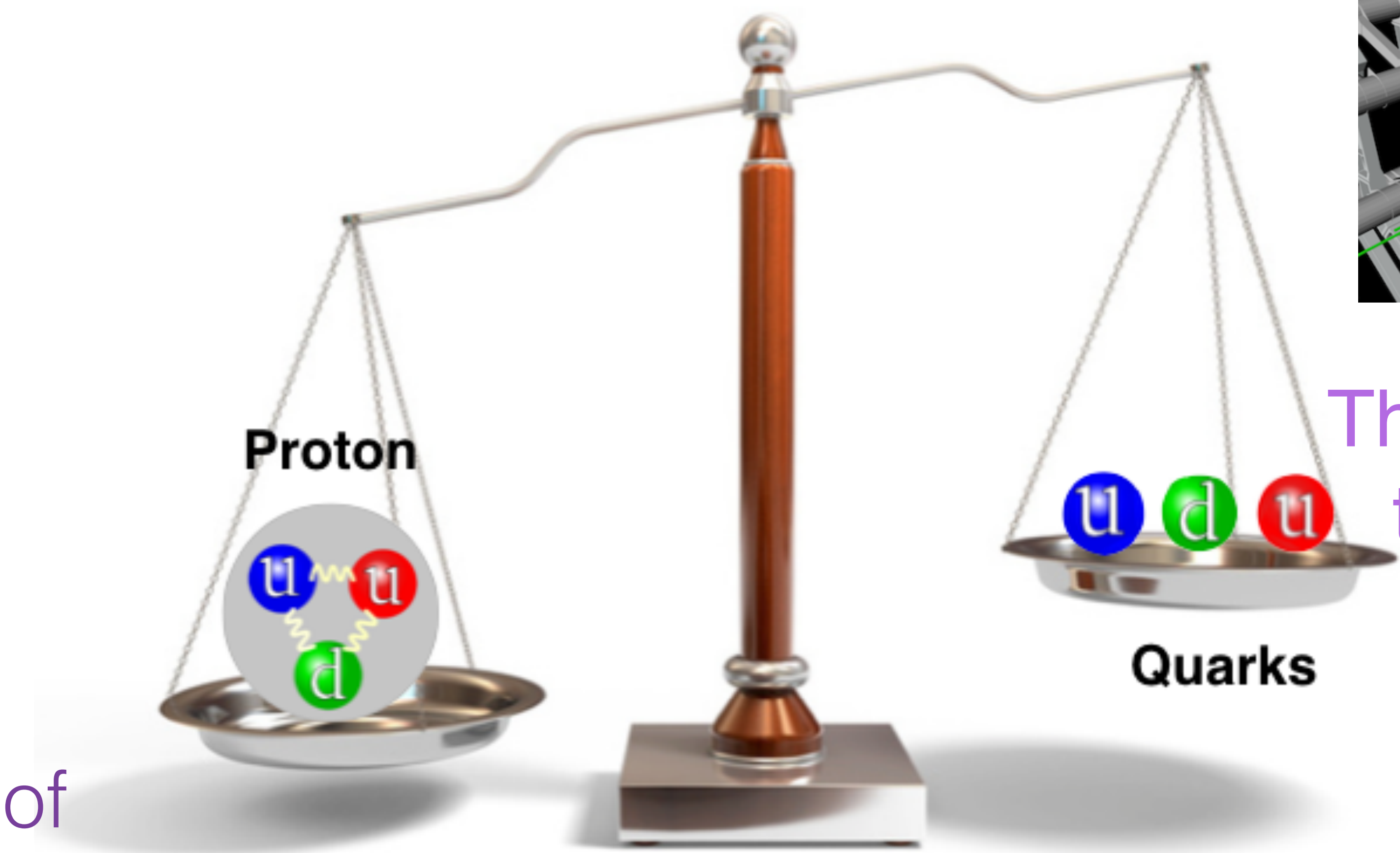
How does the mass of nucleon arise?





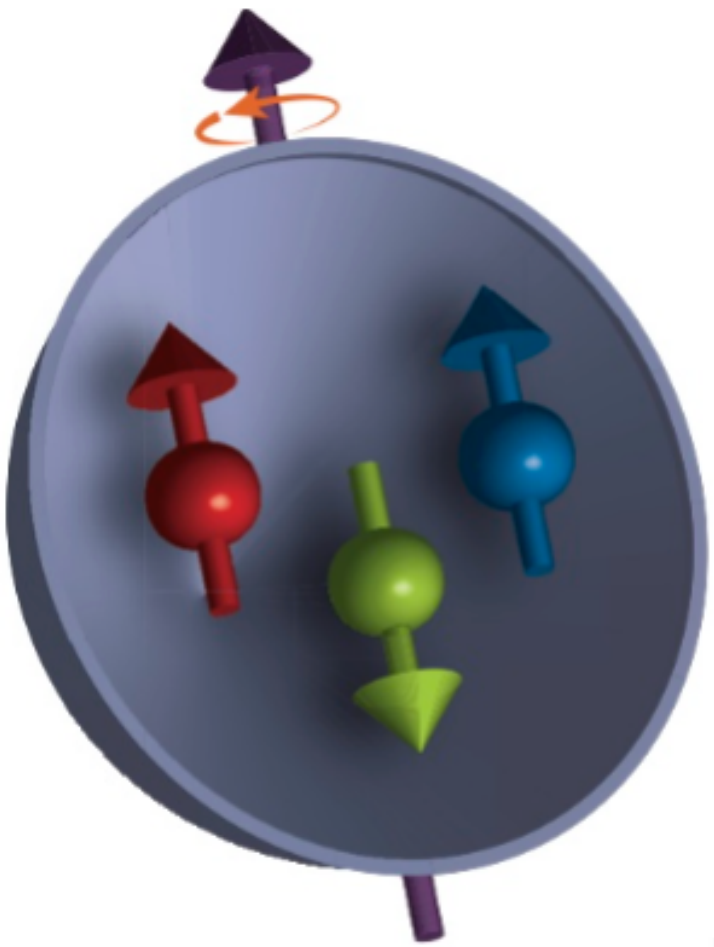
The Higgs boson make the u/d quark having masses (2GeV MS-bar):

m_u = *2.08(09) MeV*
m_d = *4.73(12) MeV*

http://www.latticeaverages.org

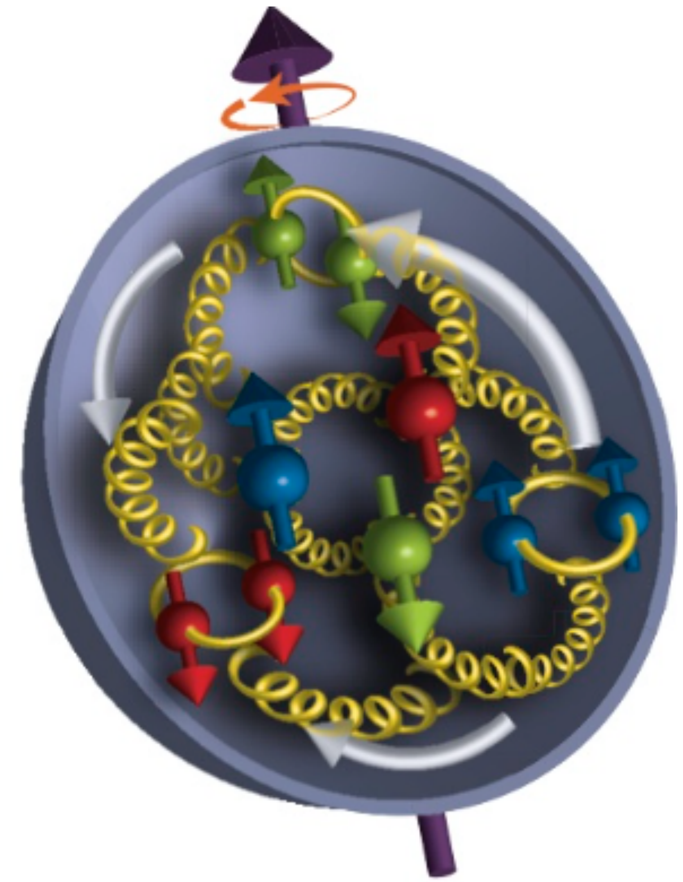
But the mass of the proton is
938.272046(21) MeV.
~100 times of the sum of the quark masses!

How does the spin of nucleon arise?



1980s

Only 30% of the proton spin comes from the quark spin, based on the experiments

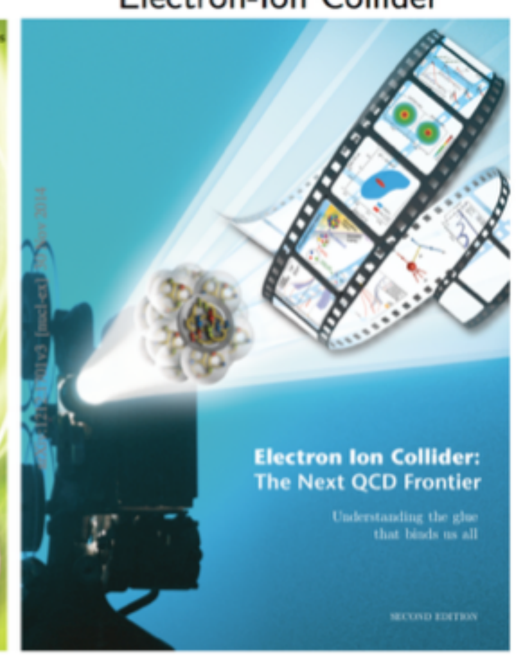


now

Jefferson Laboratory

Relativistic
Heavy Ion Collider

A future
Electron-Ion Collider



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Finding 7: To realize fully the scientific opportunities an EIC would enable, a theory program will be required to predict and interpret the experimental results within the context of QCD, and furthermore, to glean the fundamental insights into QCD that an EIC can reveal.

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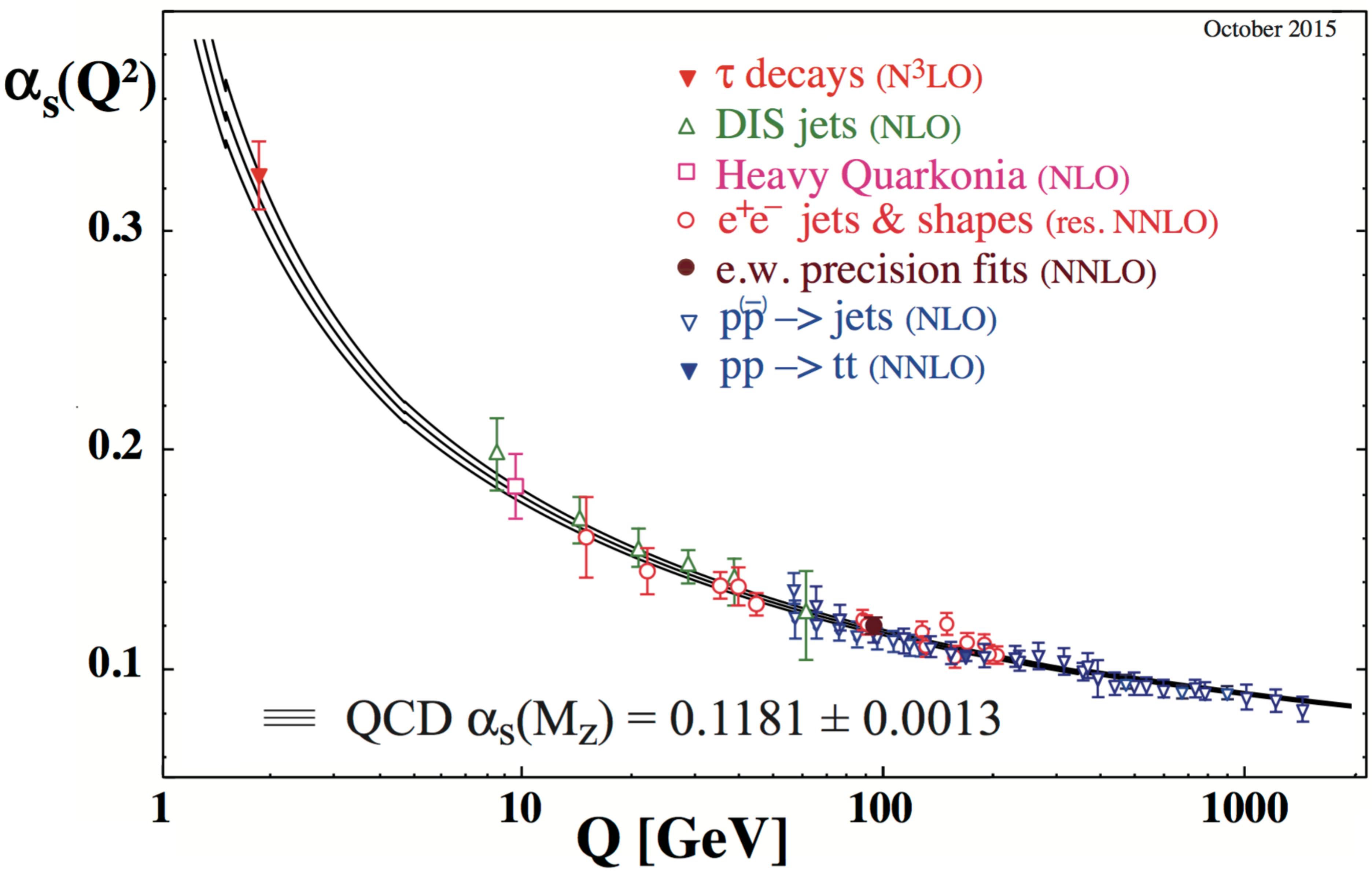
In order to take advantage of the full potential of the EIC, a theory program to match its scope is essential, comprising both continuum and lattice QCD.

……

Why Lattice QCD?

Quantum Chromodynamics

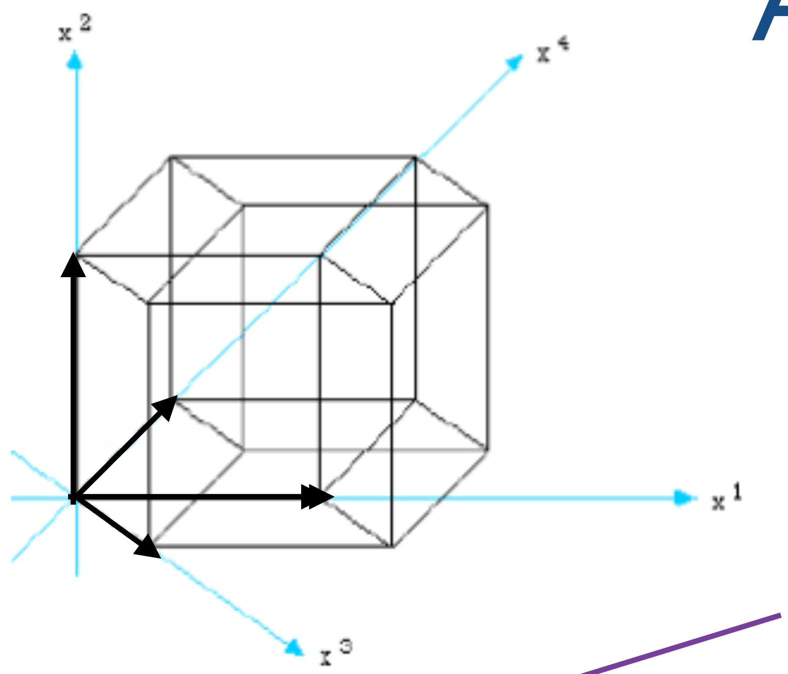
The perturbative calculation in the continuum



- **Very precise** at Q~100 GeV, the experiment confirmed the predictions;
- Limited application for nucleon properties due to **bad** convergence at Q≤2 GeV.
- The other possibilities?

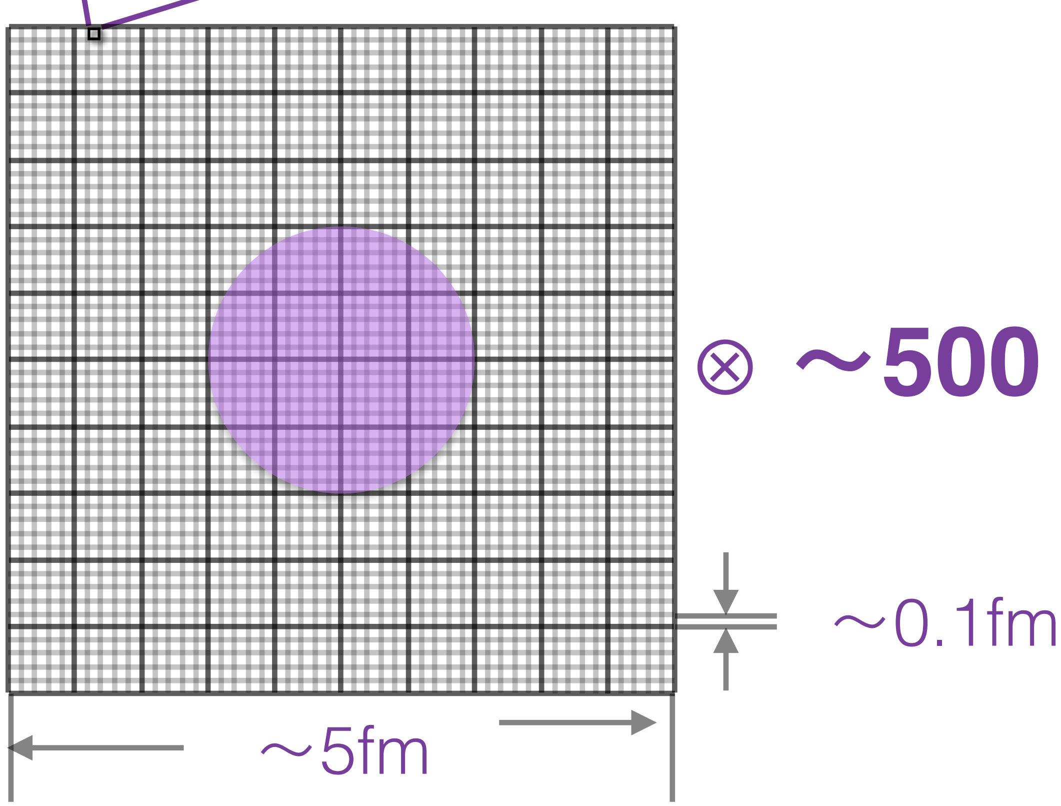
Lattice QCD

Application of the statistics in QCD



- Discretization on the 4-D integral of the space-time
- Monte Carlo sampling in the integral of the gauge phase space

- A state-of-the-art calculation requires：



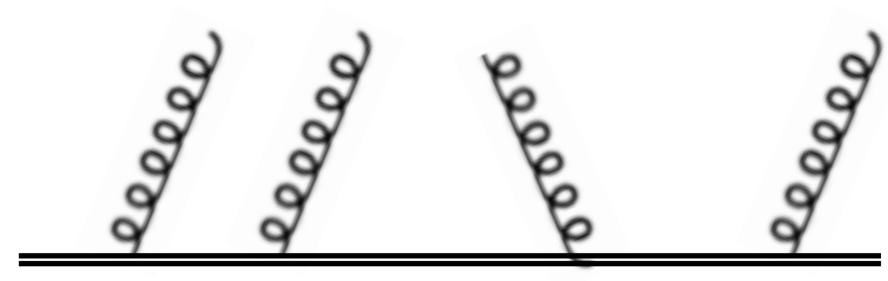
- 4-D lattice with ~5x proton size;
- With a lattice spacing ~1/10 proton size;
- ~500 samples with 1000 measurement each.
- Use 4k-8k cores per sample and handle different samples in parallel.

Lattice QCD:

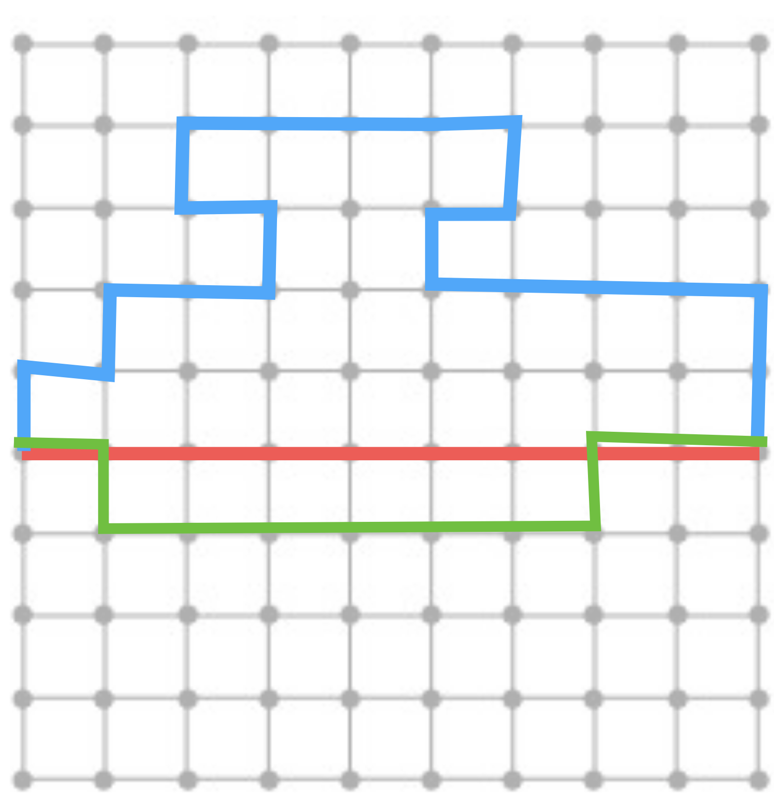
Gluon and quarks in term of the wilson lines

- The logarithm of the wilson line at very short distance corresponds to the gauge field in the continuum;

- **Multi-gluon effects** are automatically included in the longer wilson line.



$$L(x,y) = e^{-ig\int_x^y A(z)\cdot \mathrm{d}z}$$



- The quark propagation has to be through the lattice and then will be the combination of the wilson lines with the same start and end;

- The quark mass will suppress the contribution from the longer wilson lines .

- But **all the wilson lines** can contribute to the light quark propagation.

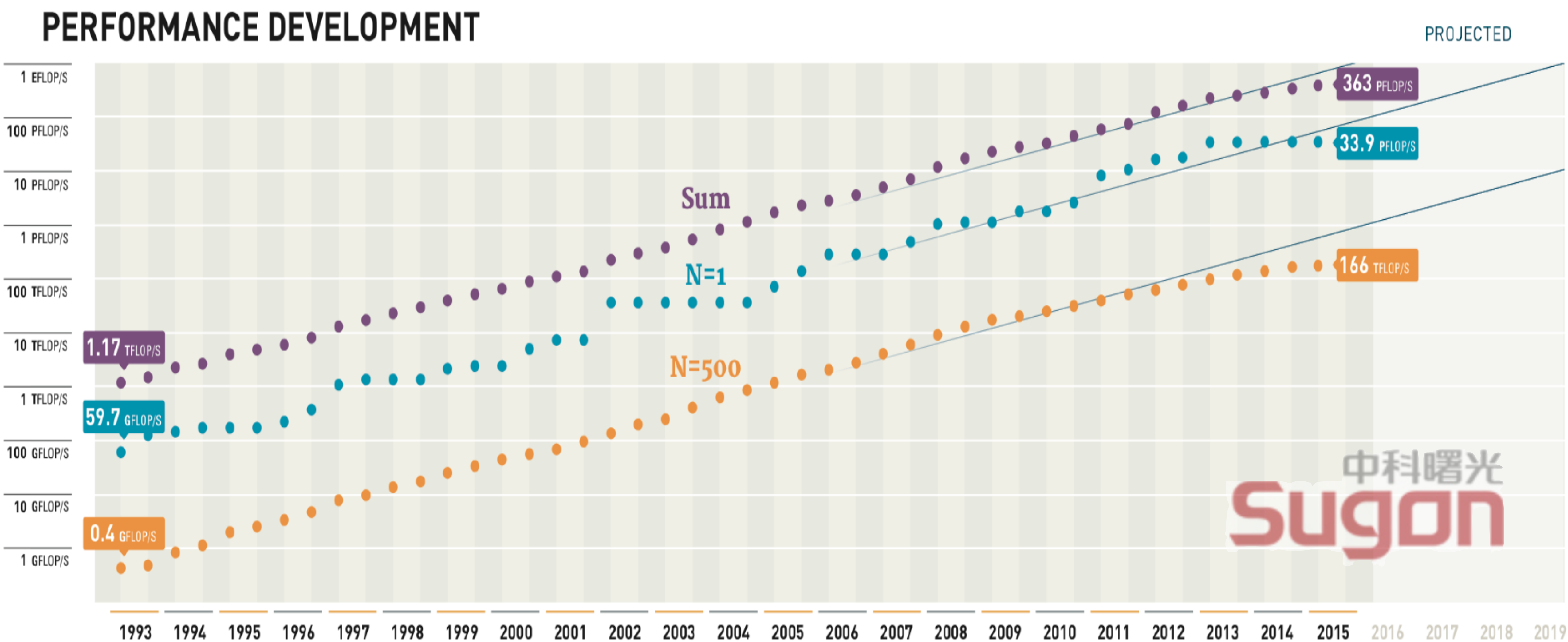
Joint efforts of

the brute force

and physics insights

Super computer:

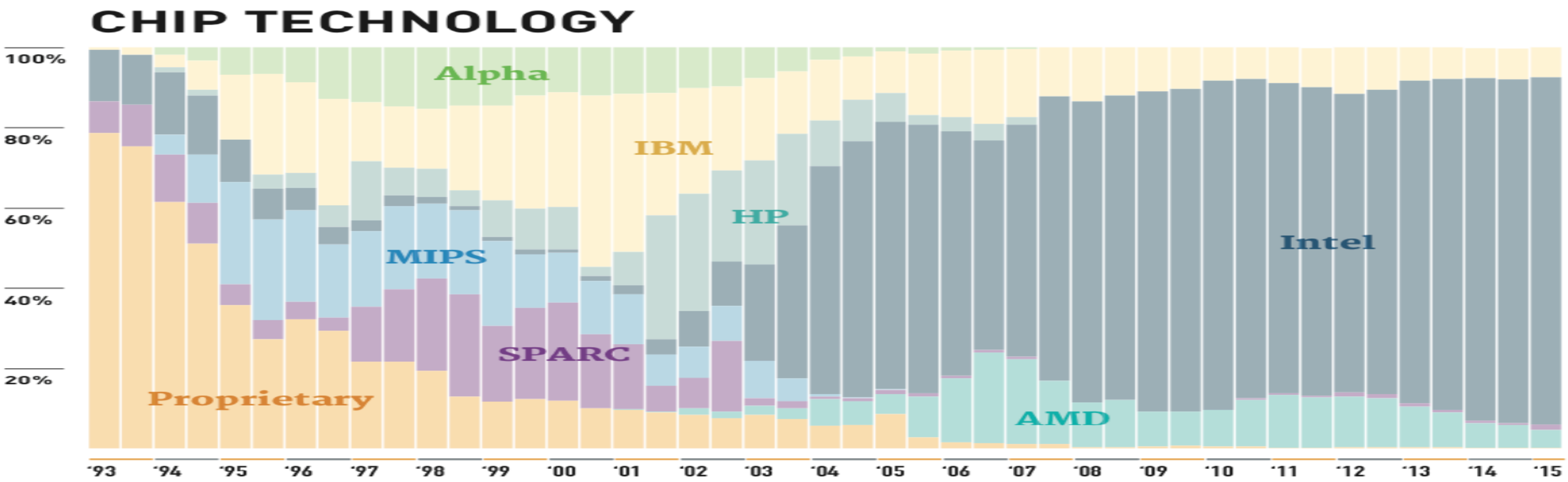
The exponential increasing performance

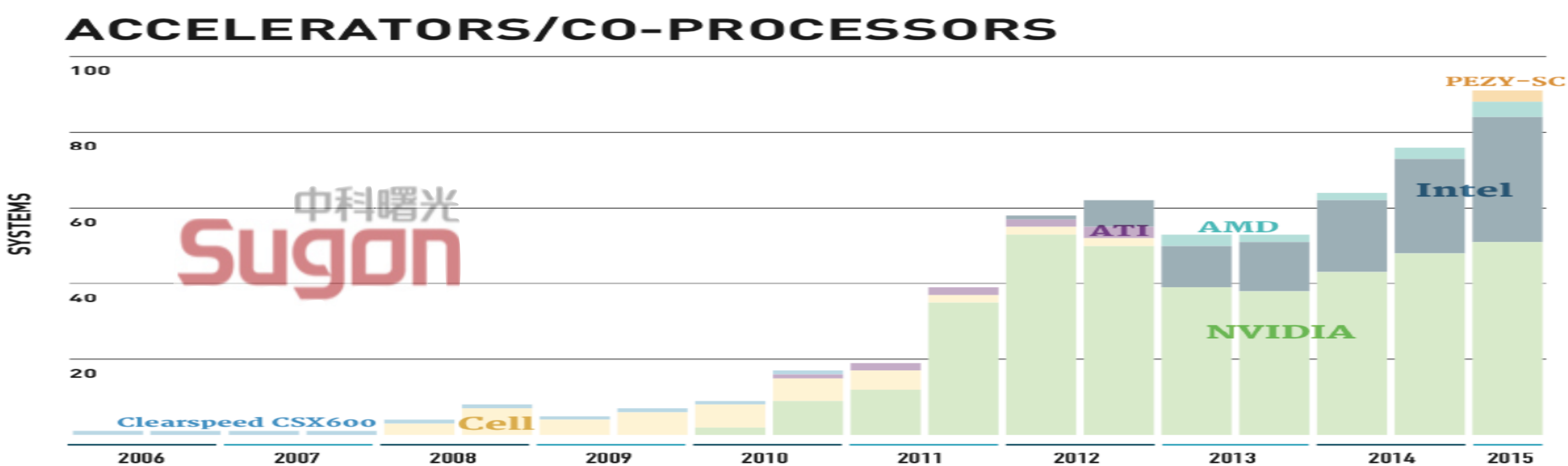


	US in Top 10	China in Top 10
2011	4.3 PFlops	3.8 PFlops
2016	51.4 PFlops	126.9 PFlops

Super computer:

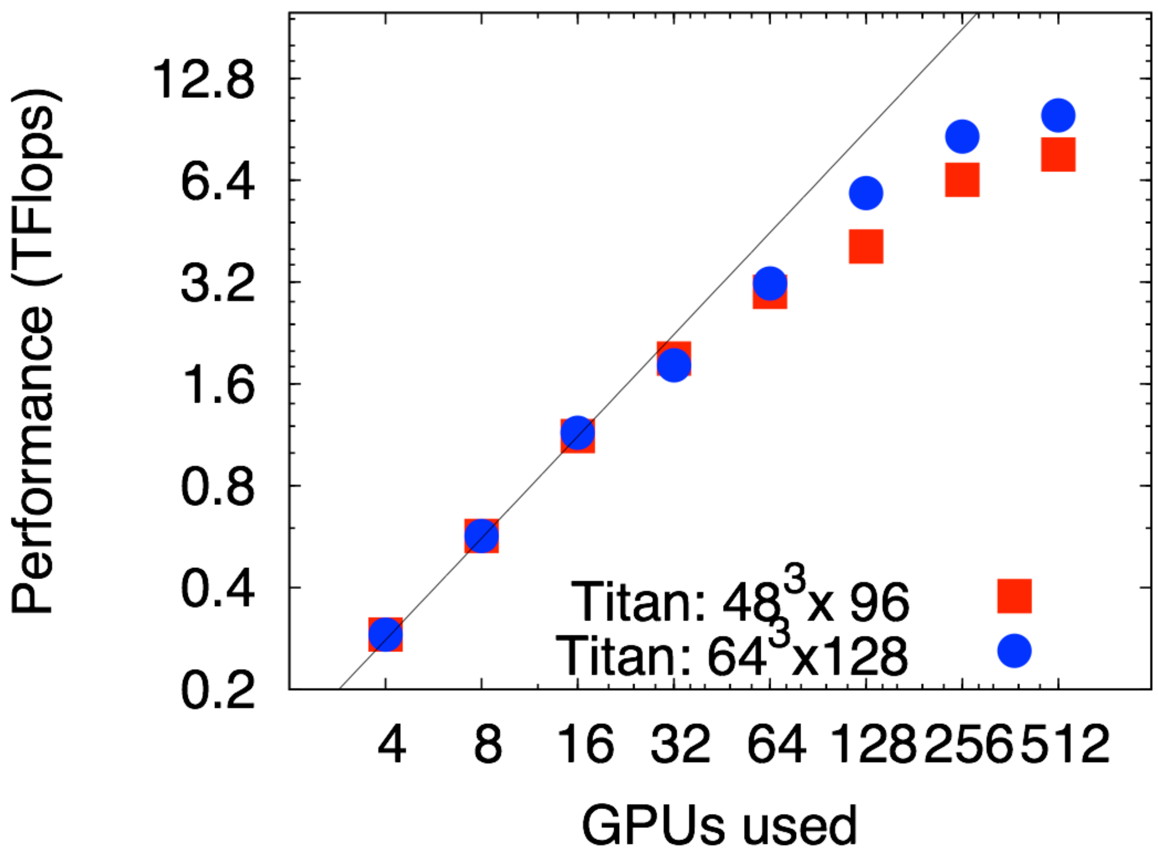
The frameworks



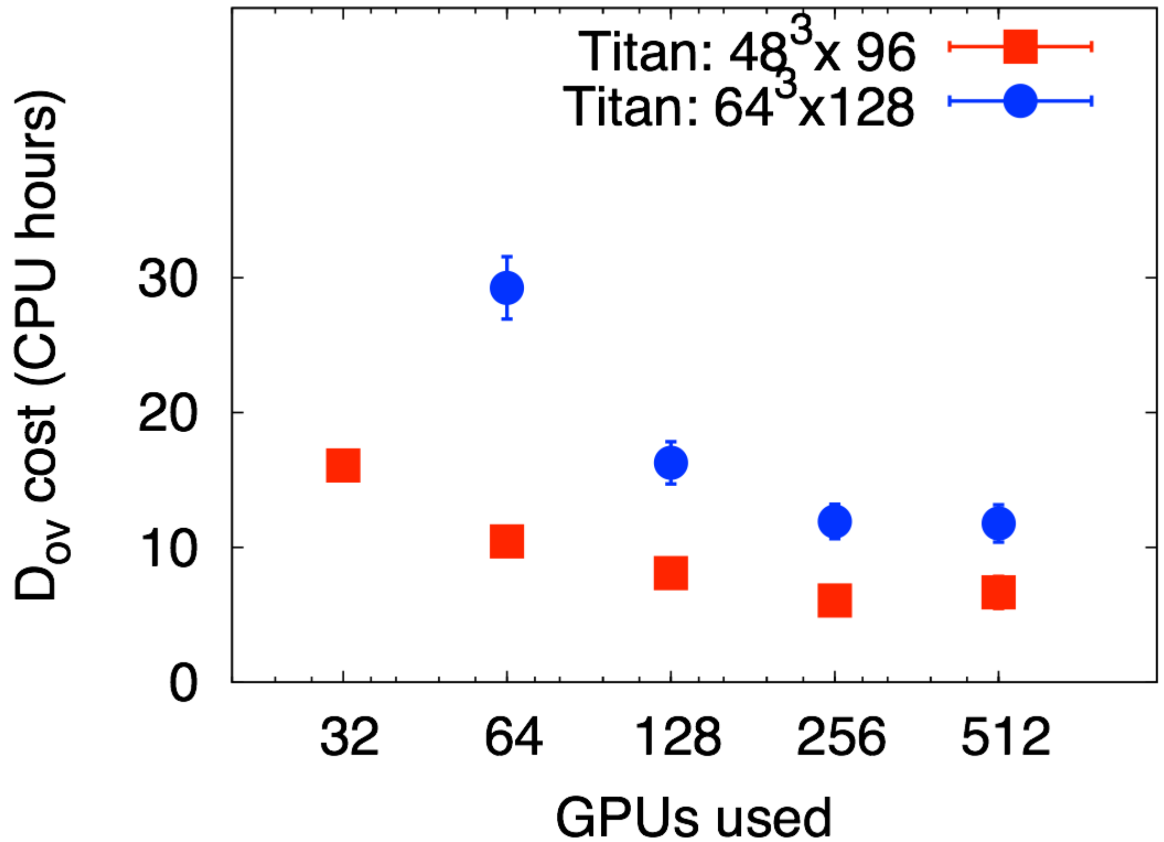


How to

benefit from the super computer?



- Parallel computation
  - 1GPUx1year ~ 365GPUx1day ~ 8760GPU x 1hour, **in the perfect case**.
- The upper bound of the speed-up
  - If 1% of the code **can not** be parallelized, then the speed-up rate **≤ 100**.



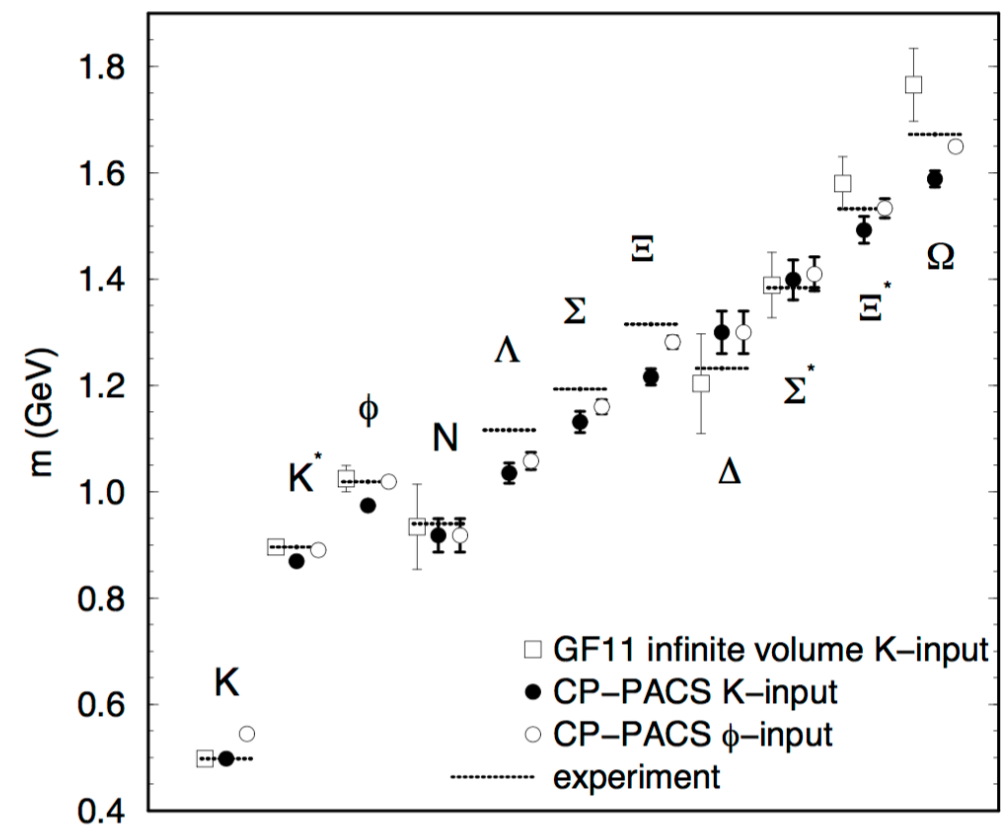
- **Super speed-up**
  - 1GPUx1year ~ 1000GPUx1hour when the data can be fully cached.

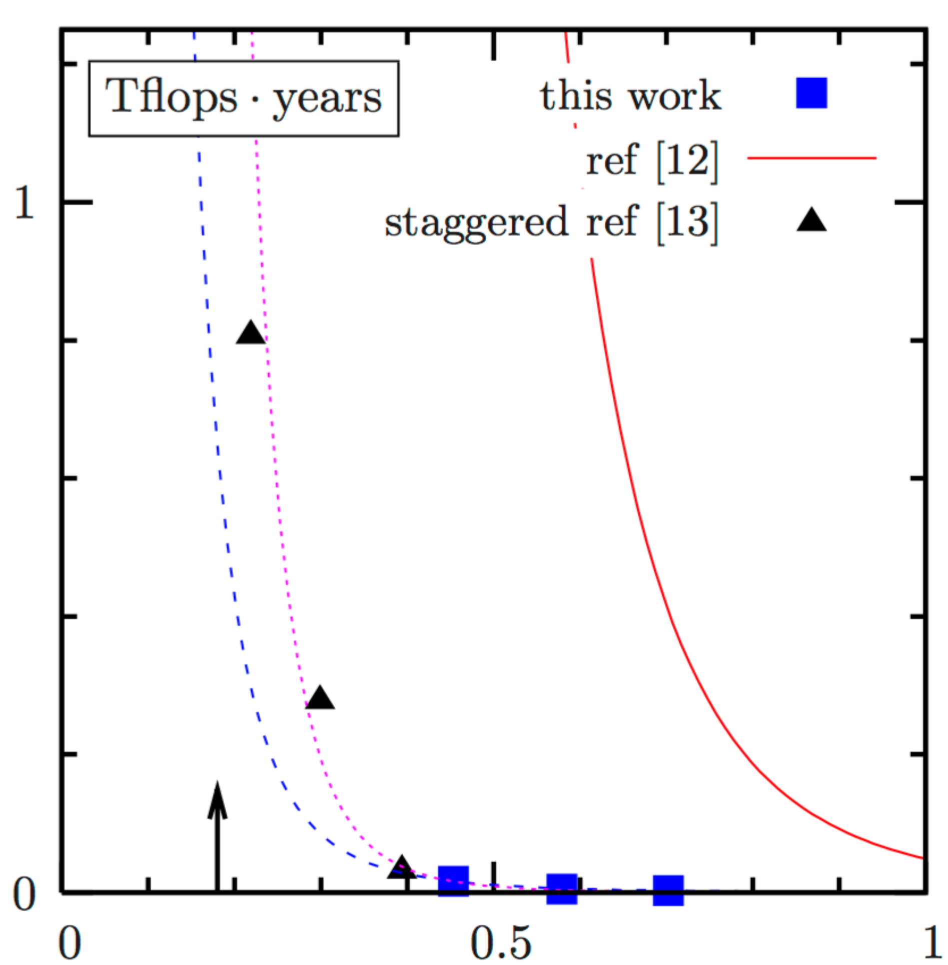
How to

have a glimpse of the Lattice QCD result of 10 years later?

- 1990's: quenching approximation
  - No quark-anti-quark pair in the background;
  - Reduce the simulation cost by **O(100)**;
  - only ~10% systematic uncertainty in some cases.

T.Yoshié, Plenary talk of Lattice97, Nucl.Phys.Proc.Suppl. 63 (1998) 3





- 2000's: deflation/multigrid technique
  - Separate the long/short distance correlation of the quark fields;
  - Reduce the simulation cost of the light quark by **O(1/m_q)**;
  - Make the direct simulation with **physical light quark mass** to be possible.

Karl Jansen, Plenary talk of Lattice08, PoS Lattice2008:010,2008

How to

have a glimpse of the Lattice QCD result of 10 years later?

- 2010's: Cluster decomposition ?

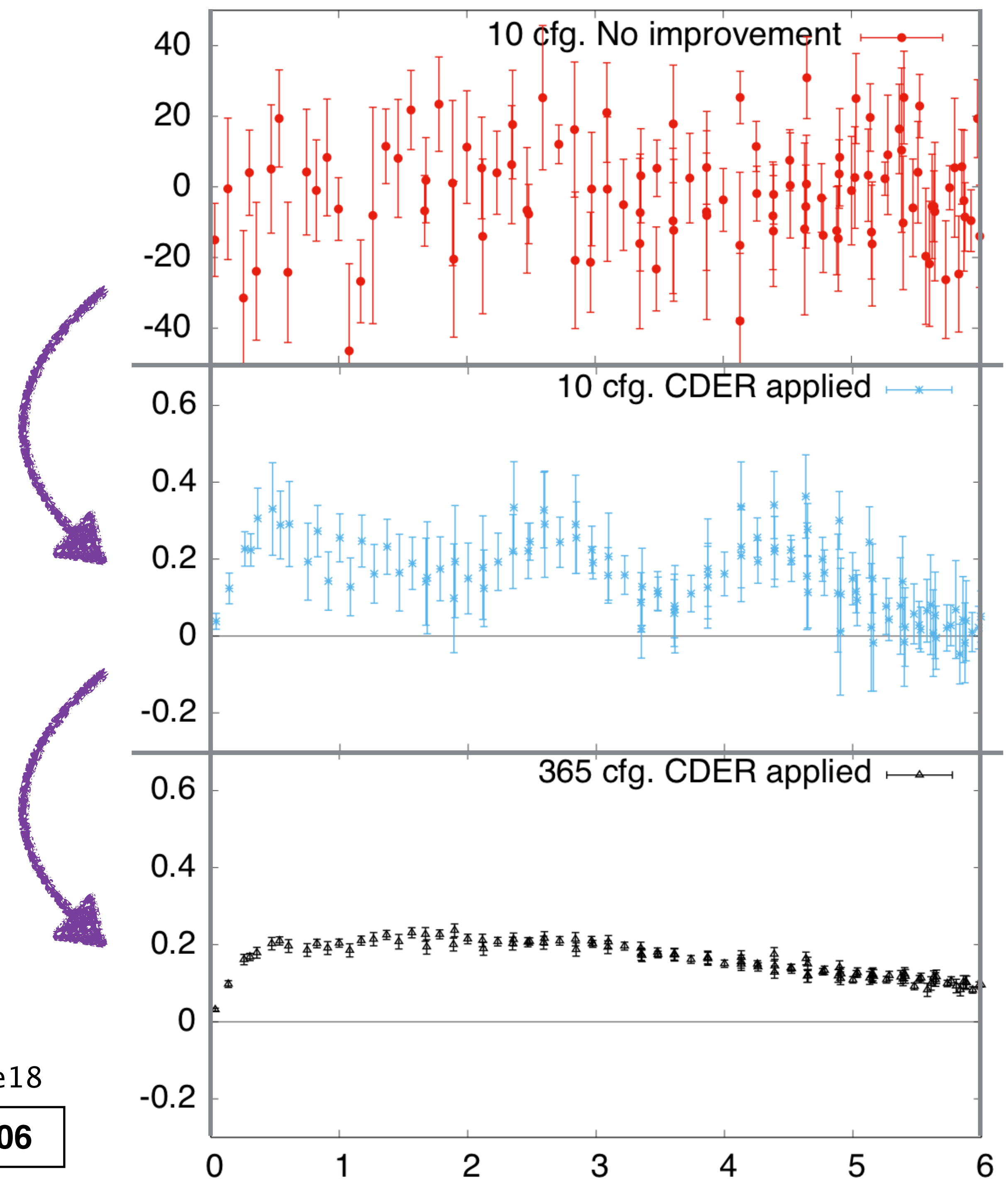
W. Sun, et.al, χQCD collaboration, CPC42, 063102(2018), 1507.02541
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

Taking our recent work as example:

- Calculate the renormalization factor of the glue EMT non-perturbatively on a ~5 fm box will require ~30,000,000 configurations to make the uncertainty to be ~0.01;

- Taking the localization of the correlations between the glue fields/operators into account, the uncertainty can be reduced by a factor ~200;

- Use reasonable computer resource (~1M CPU hours) to increase the statistics, the ~0.01 uncertainty goal can be obtained with 365 configurations.



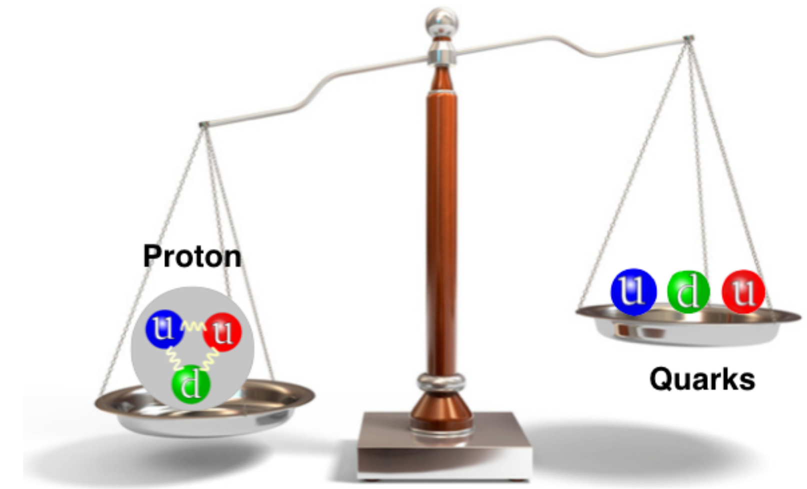
YBY, Plenary talk of Lattice18

YBY, et. al., χQCD collaboration, PRD98(2018) 074506

What can Lattice QCD says for the profound questions to be addressed by EIC/EicC

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?

How does the mass of nucleon arise?



$$M = -\langle T_{44}\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \frac{1}{4}\langle H_a\rangle$$

Xiangdong Ji, PRL 74, 1071-1074 (1995)

With

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\overline{\psi}\psi,$$ *The quark mass*

Gauge Invariant and scale independent combinations.

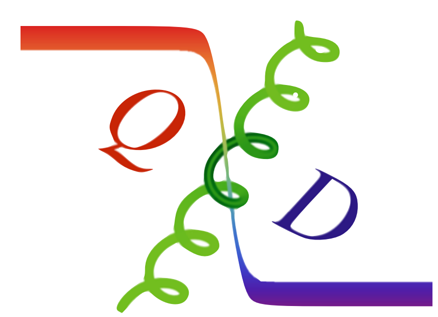
The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$ ***The glue anomaly***

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots}\int d^3x\, \gamma_m m\overline{\psi}\psi.$$

The quark mass anomaly



The total energy

$$H_E = \sum_{u,d,s\ldots}\int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

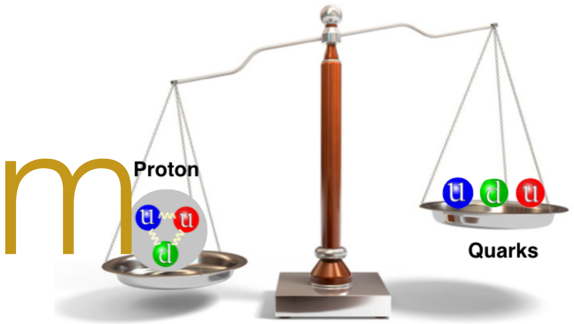
$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

YBY, et. al., χQCD collaboration, Phys. Rev. D 91, 074516 (2015)

Proton mass decomposition

The quark mass term



Then we have

$$M = -\langle T_{44} \rangle = \langle H_q \rangle + \langle H_g \rangle + \langle H_g^a \rangle + \langle H_m^\gamma \rangle$$
$$= \langle H_E \rangle + \langle H_m \rangle + \langle H_g \rangle + \langle H_a \rangle,$$
$$\frac{1}{4} M = -\langle \hat{T}_{44} \rangle = \frac{1}{4} \langle H_m \rangle + \langle H_a \rangle,$$

in the rest frame.

$$H_m = \sum_{u,d,s\cdots} \int d^3x \, m \bar{\psi} \psi,$$

The quark mass

- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.

$\sigma_{\pi N} = \langle H_m(u) + H_m(d) \rangle = 45.9(7.4)(2.8)$ MeV

$f_s^N M_N = \langle H_m(s) \rangle = 40.2(11.7)(3.5)$ MeV

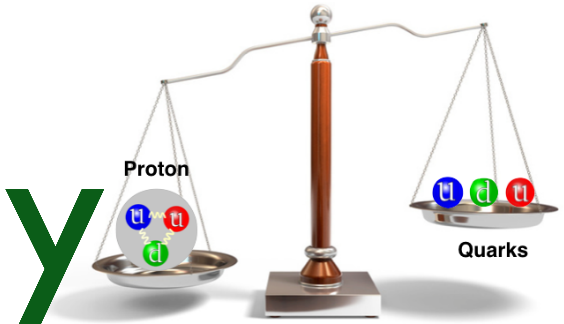
$\langle H_m(u,d,s) \rangle / M_N = 9(2)\%$

The best lattice result free of the systematic uncertainty from the explicit chiral symmetry breaking

YBY, et. al., χQCD collaboration, Phys. Rev. D 94, 054503 (2016)

Proton mass decomposition

The QCD anomaly



Then we have

$$\begin{aligned} M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\ &= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\ \frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle, \end{aligned}$$

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**
- The total QCD anomaly is renormalization scheme/scale independent.
- ***H_a / M_N = 23(1)%***

The QCD anomaly

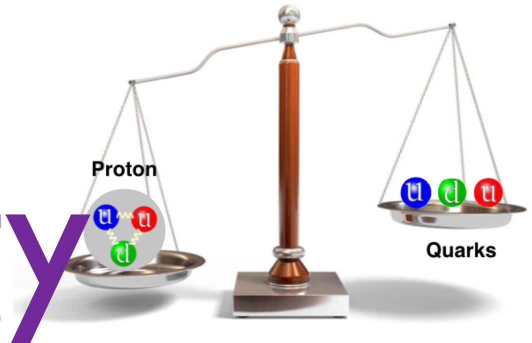
$$\begin{aligned} H_a &= H_g^a + H_m^\gamma, \\ H_g^a &= \int d^3x \, \frac{-\beta(g)}{4g}(E^2 + B^2), \\ H_m^\gamma &= \sum_{u,d,s\cdots} \int d^3x \, \frac{1}{4}\gamma_m m \bar{\psi}\psi. \end{aligned}$$

The glue anomaly

The quark mass anomaly

Proton mass decomposition

The quark/gluon energy



Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle \qquad \langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$
$$\langle H_q\rangle = \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

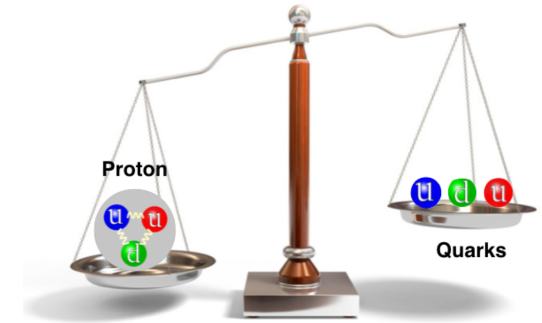
$$H_E = \sum_{u,d,s\ldots}\int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

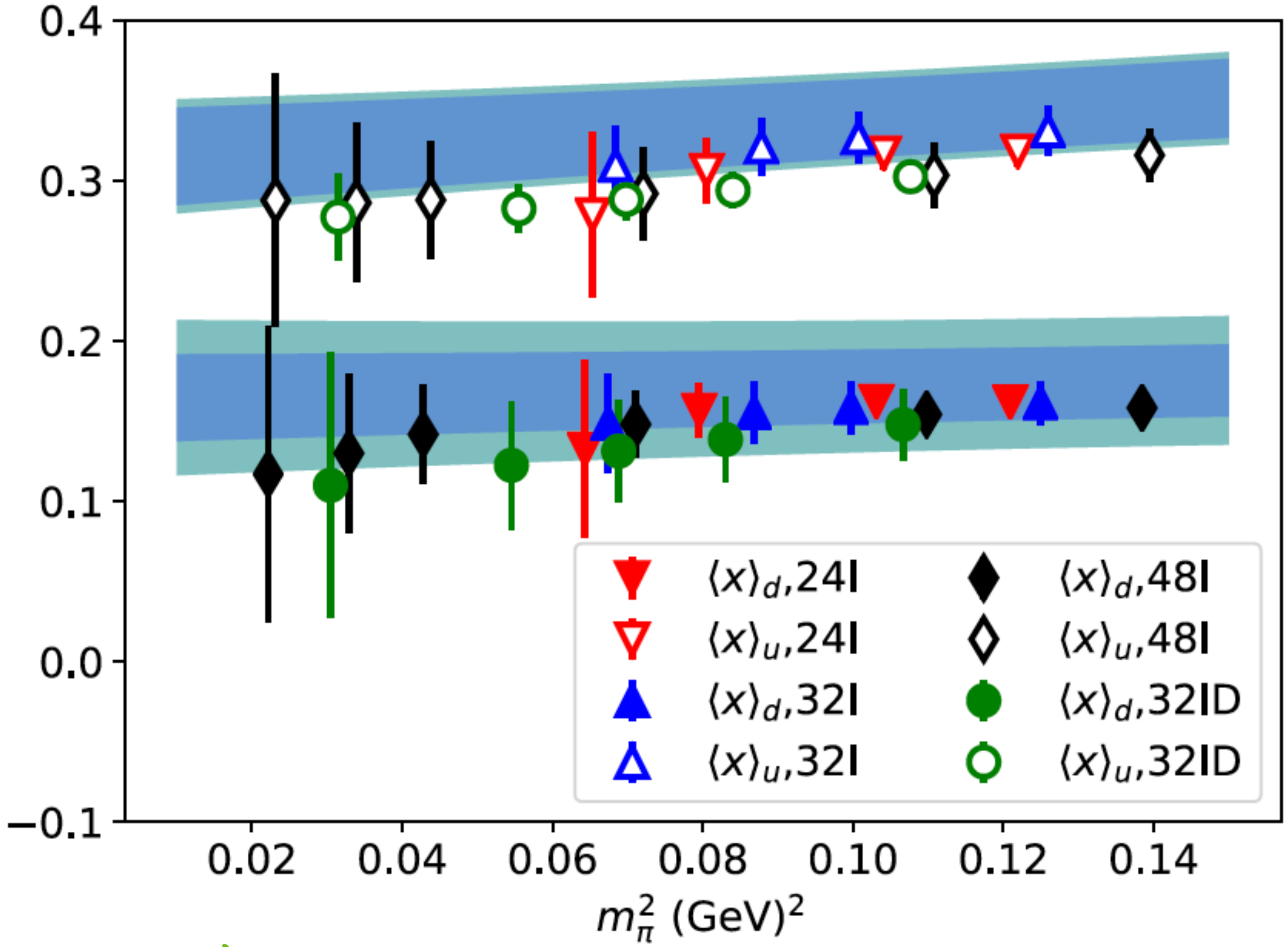
$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

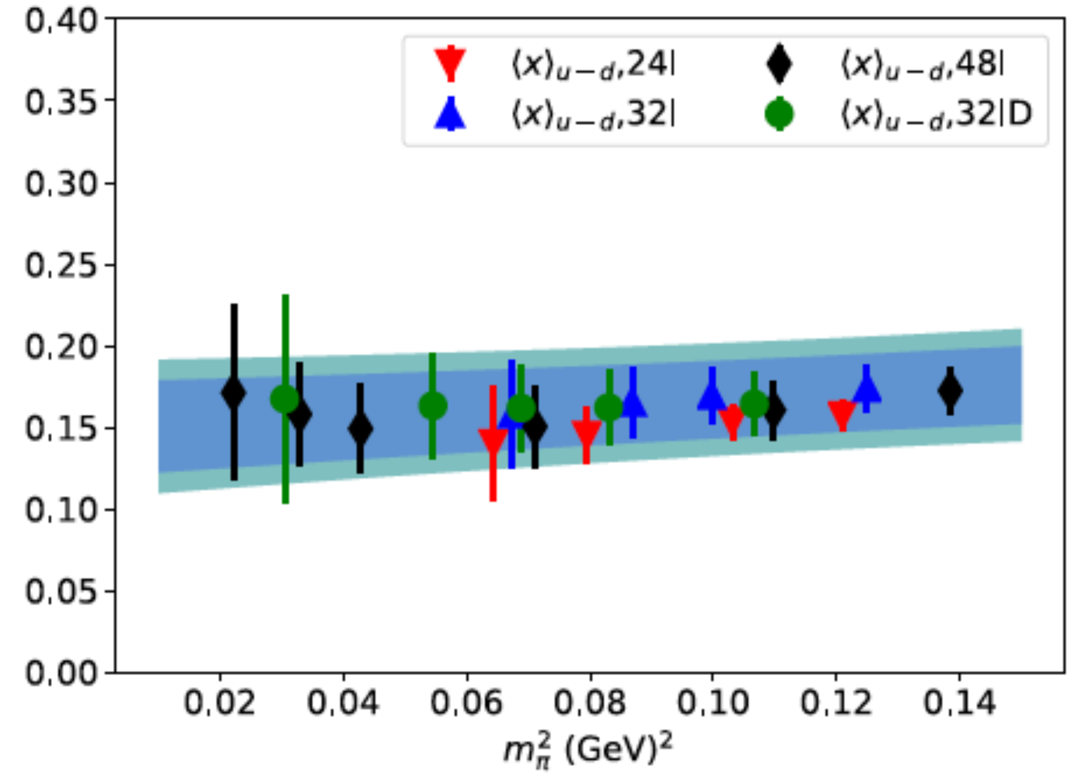
The glue field energy

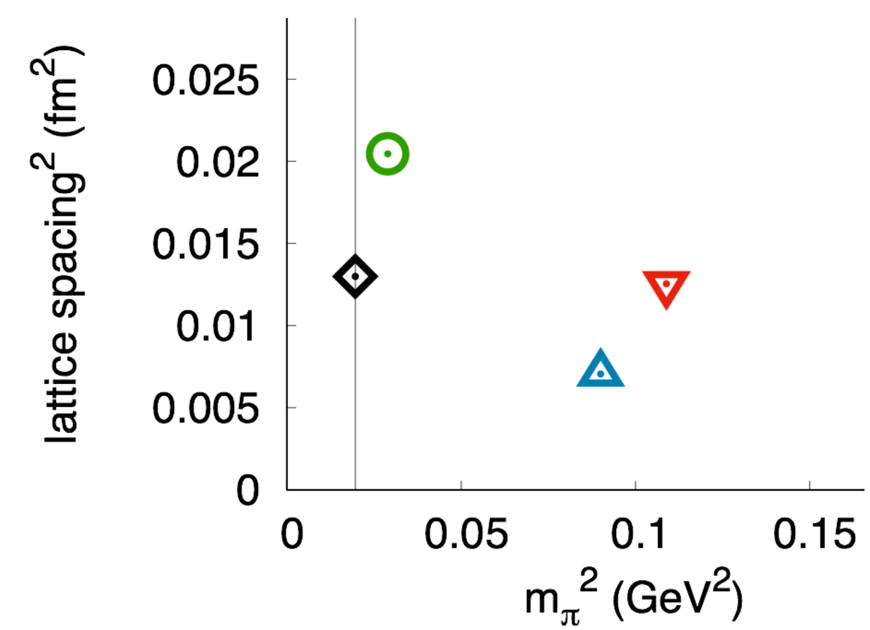
Proton mass decomposition



Momentum fractions of u and d quarks





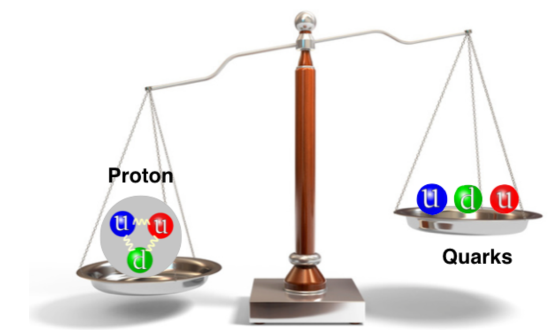


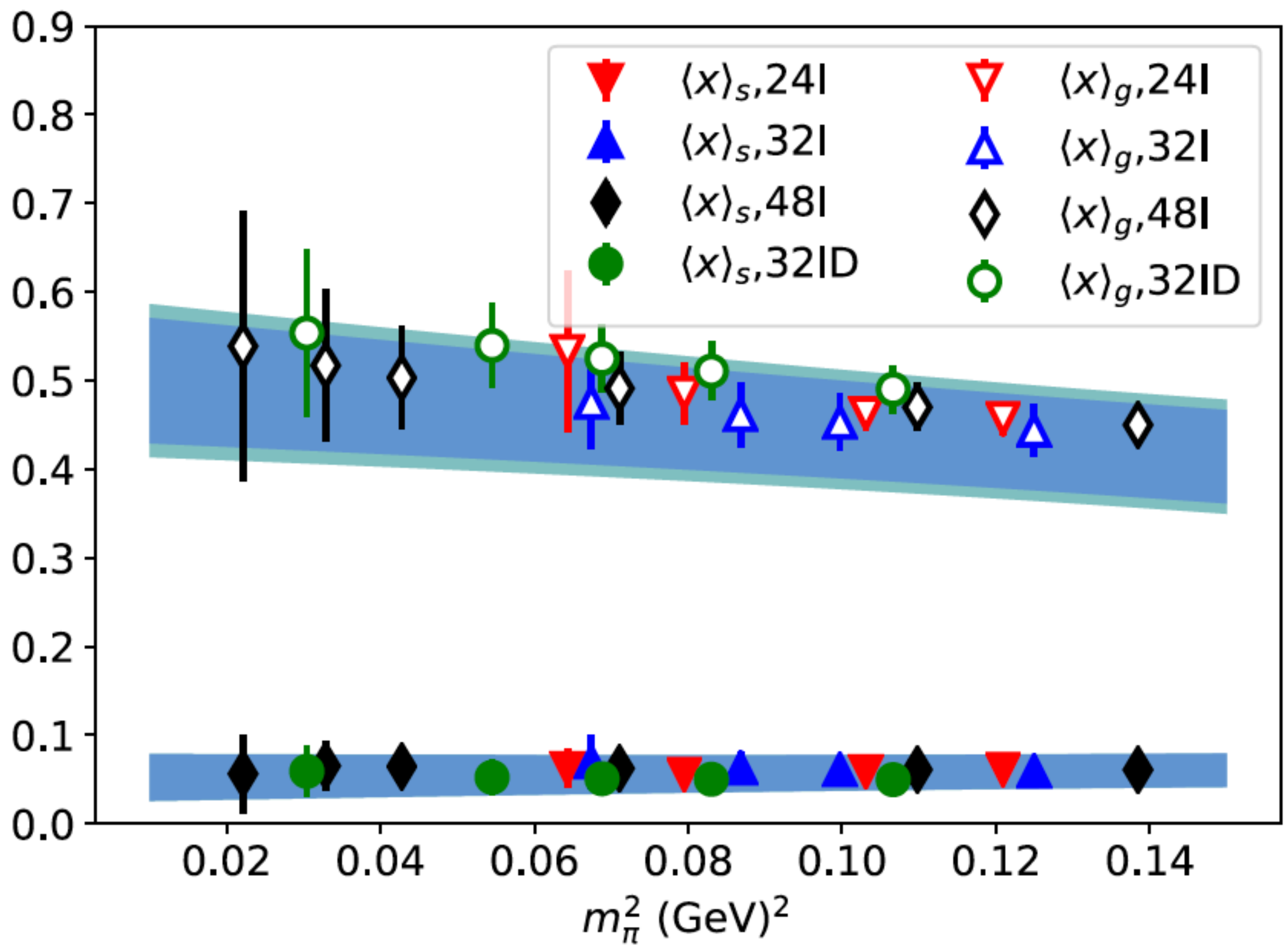
YBY, J. Liang, et. al., χQCD collaboration,
PRL121, 212001(2018), 1808.08677
ViewPoint and Editor's suggestion

Proton mass decomposition

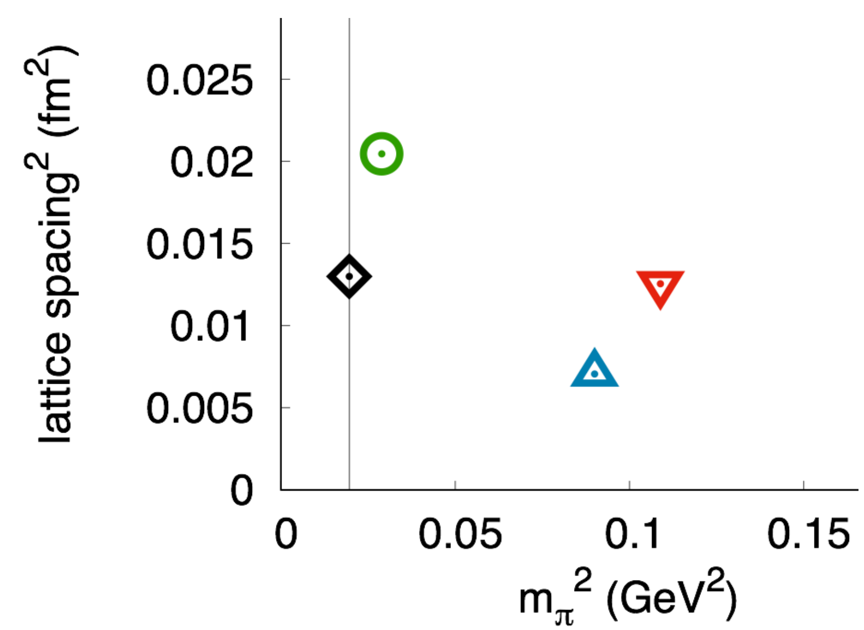
Pure DI momentum fractions:

strange quark and glue ones





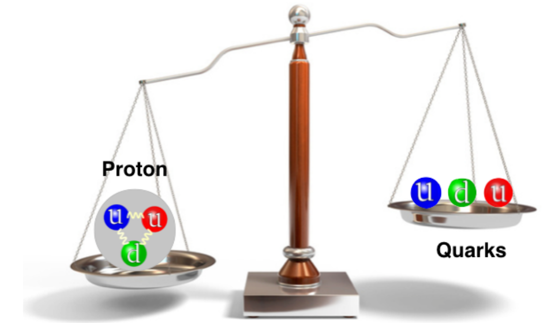
- The glue momentum fraction become **larger** when the quark mass is **lighter**;
- The strange one is small as expected.



YBY, J. Liang, et. al., χQCD collaboration, PRL121, 212001(2018), 1808.08677
ViewPoint and Editor's suggestion

Proton mass decomposition

Comparing the momentum fractions



from the experiment

YBY, J. Liang, et. al., χQCD collaboration,
PRL121, 212001(2018), 1808.08677
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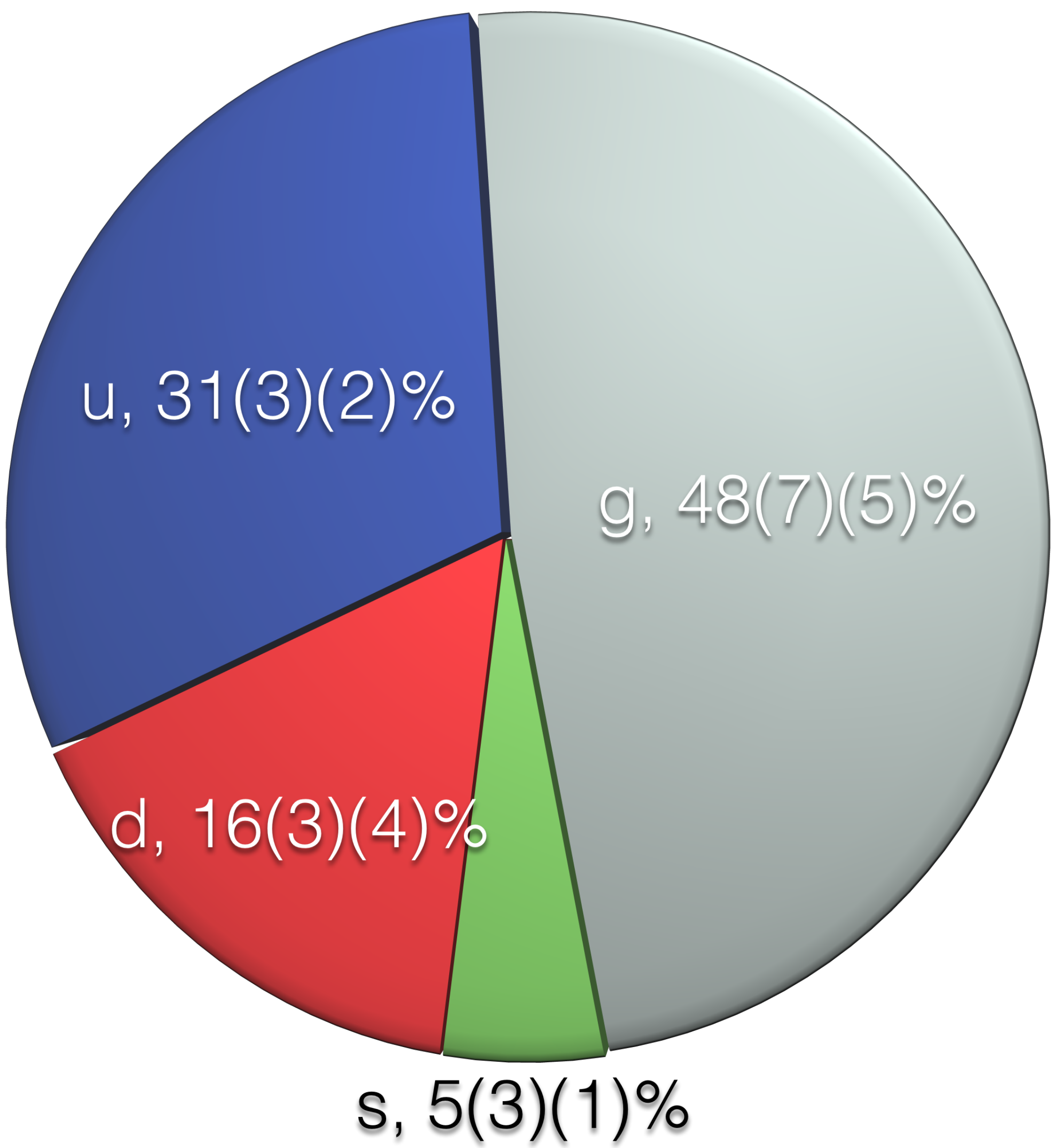
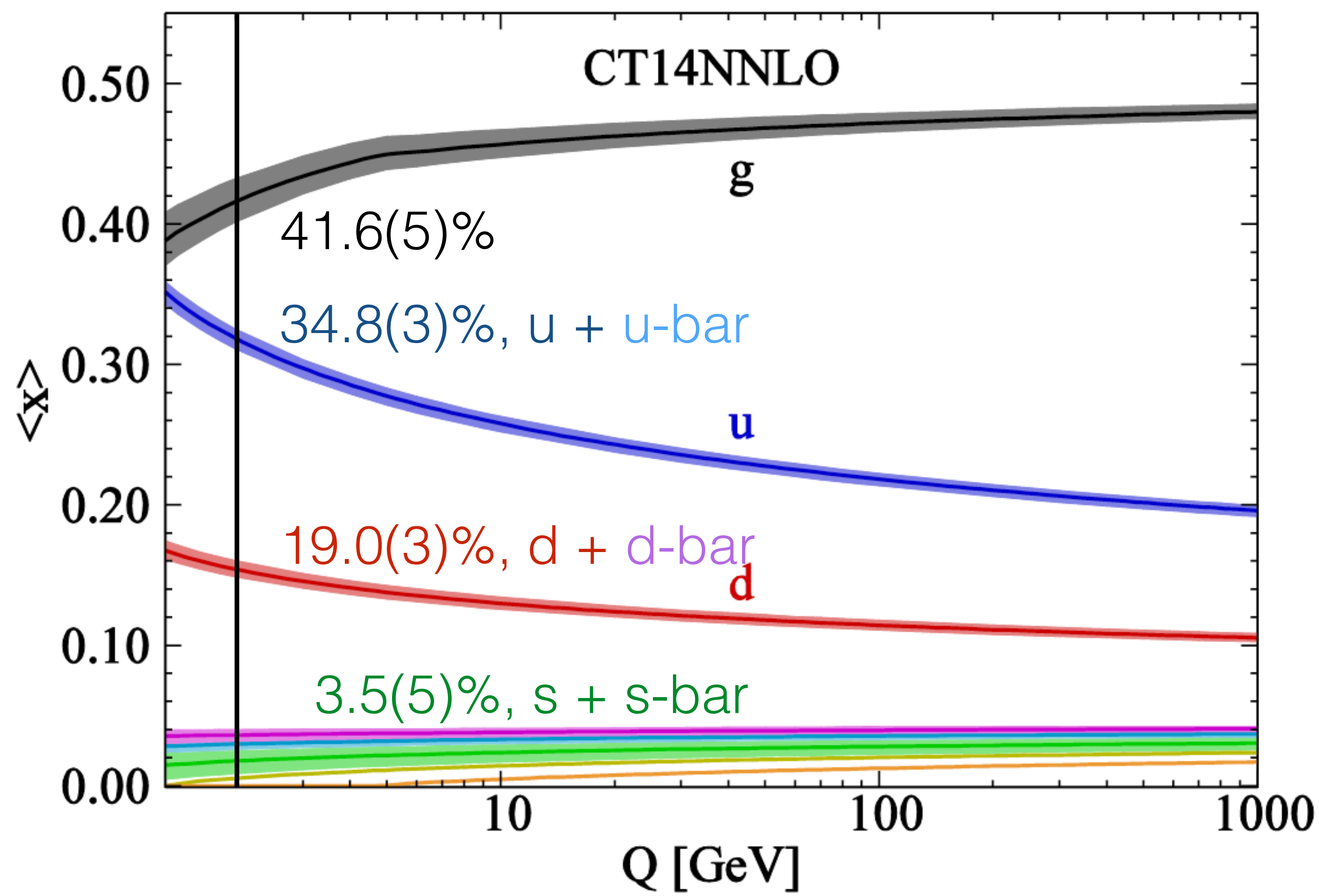
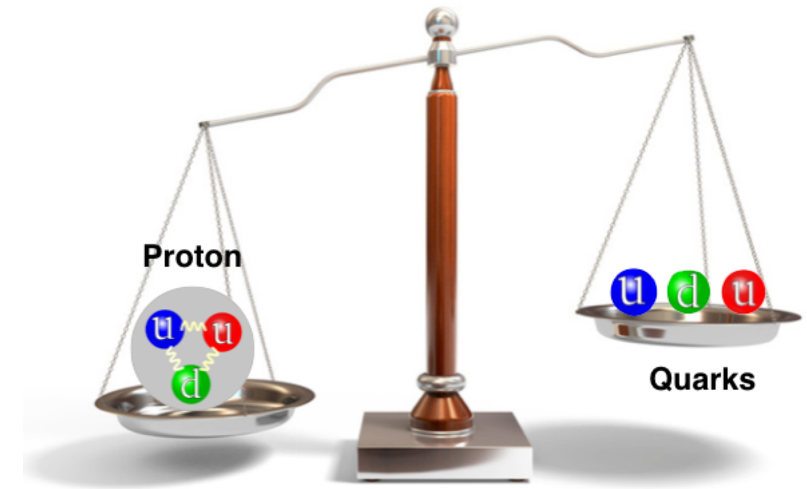




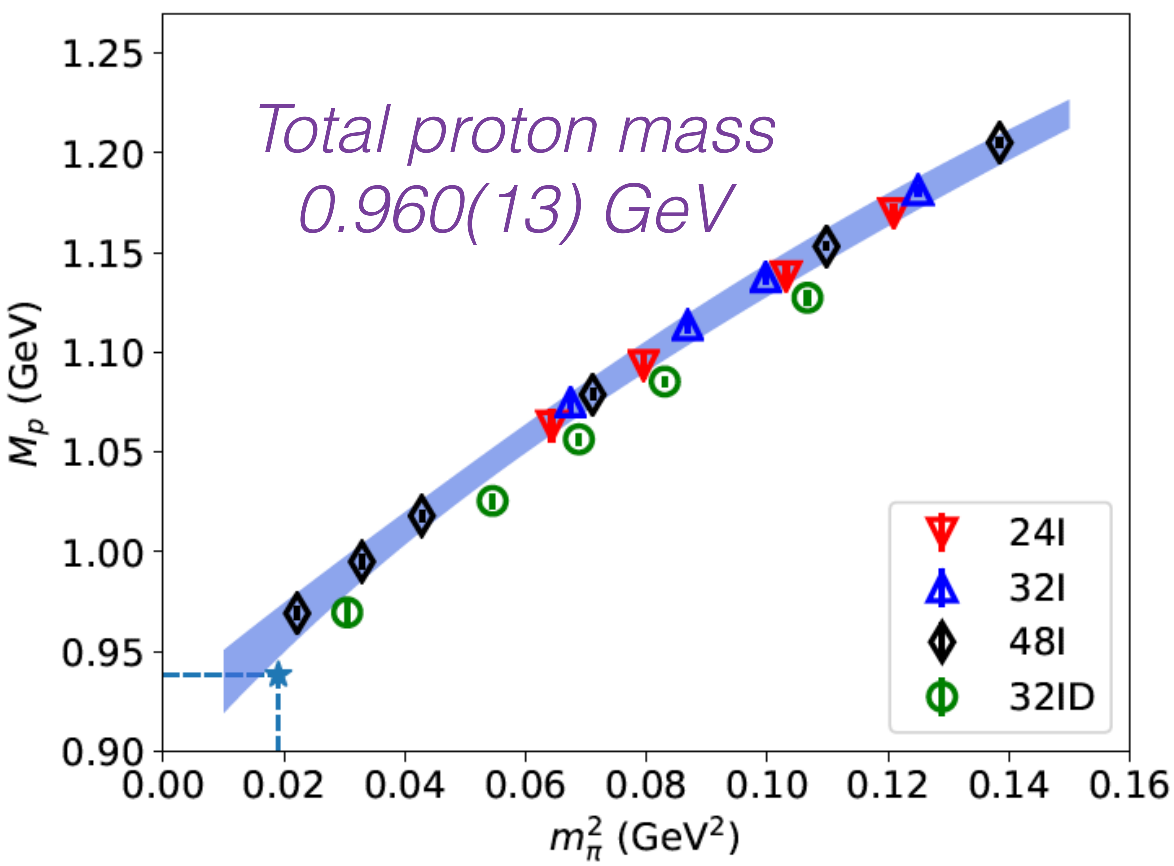
S. Dulat et al, Phys. Rev. D 93 (2016), 033006

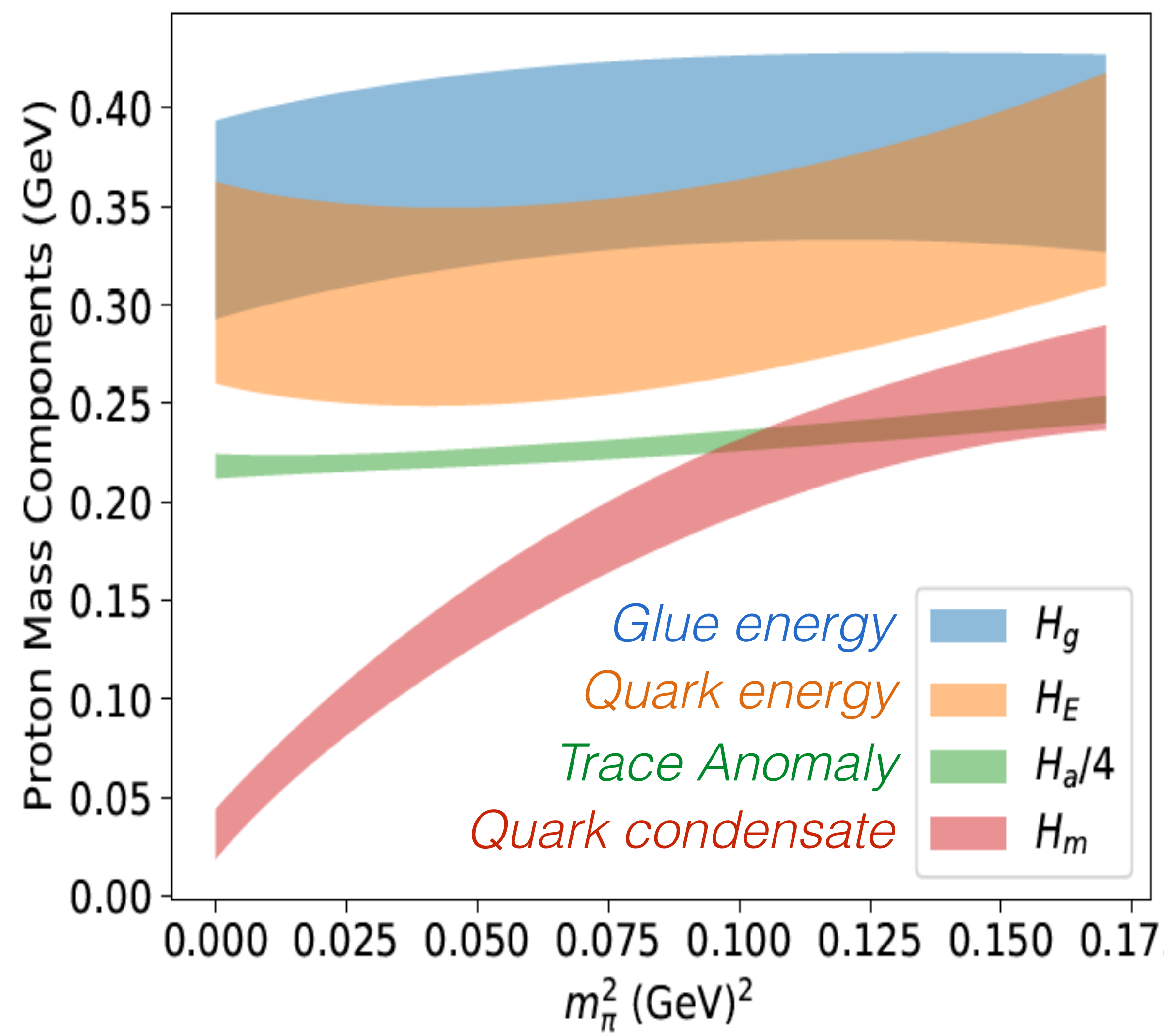
- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.



- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule

$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \frac{1}{4}\langle H_a\rangle$$





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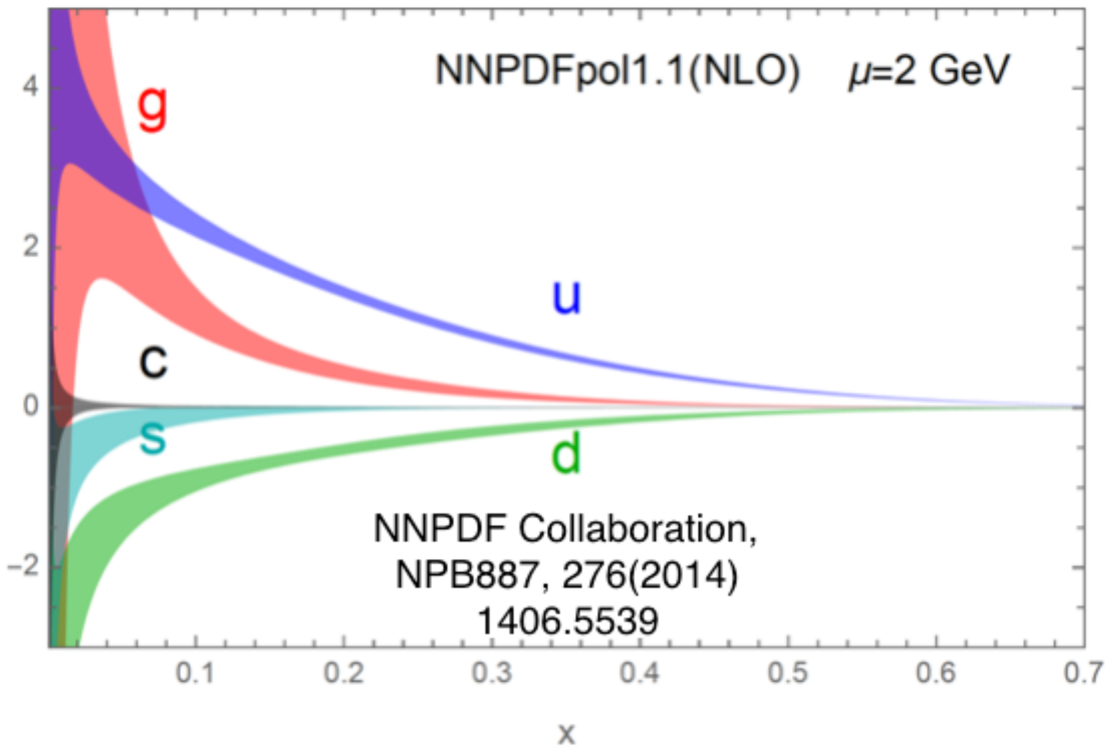
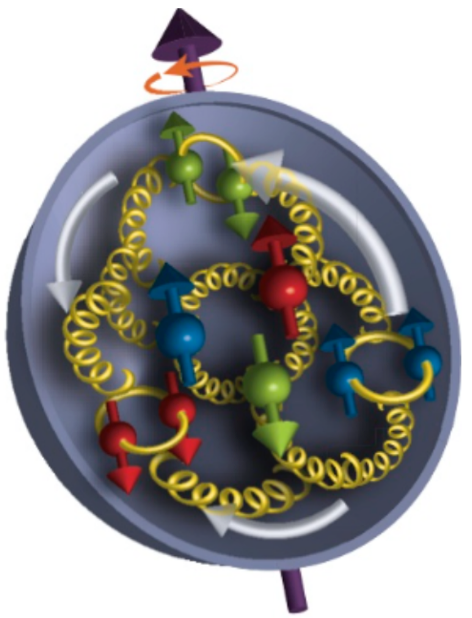
How does the spin of nucleon arise?





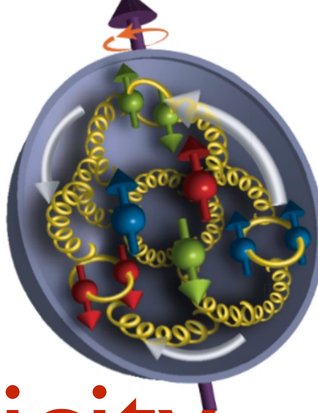
*Spin/helicity (**u**,**d**,**s**…,g): the **integration** of the polarized parton distribution function (PDF)*

$$\Delta q = \int_0^1 dx \Delta q(x)$$

$$\Delta G = \int_0^1 dx \Delta g(x)$$

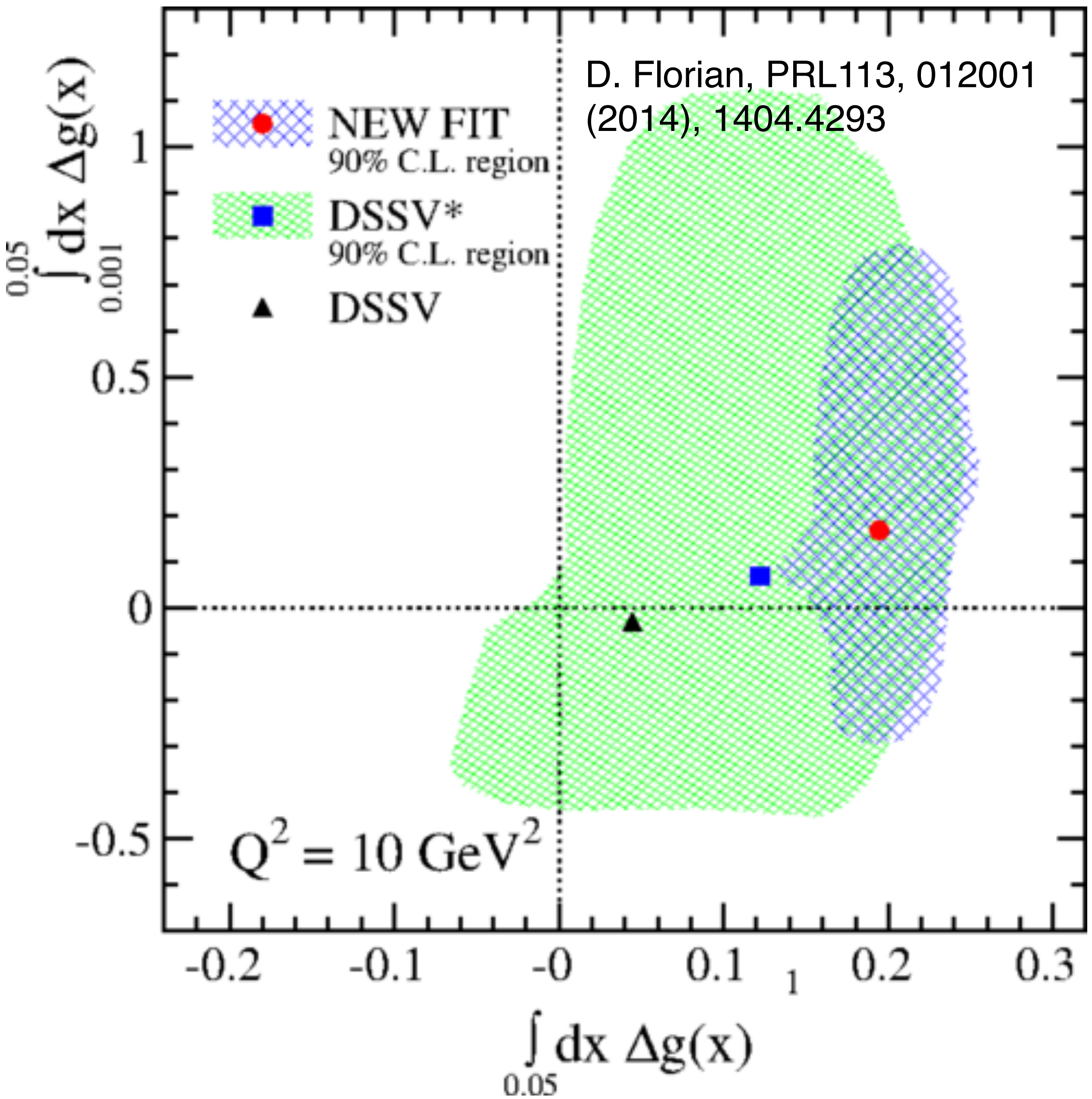
- The quark model (agrees with the lattice simulation at heavy quark limit): $\Delta u \rightarrow 4/3$, $\Delta d \rightarrow -1/3$, $\Delta s \rightarrow 0$, $\Delta g \rightarrow 0$;
- **The phenomenology fit of quark distribution based on Exp.:** $\Delta u \sim 0.8$, $\Delta d \sim -0.4$, $\Delta s \sim -0.1$, $\Delta g \sim 0.4$;
- **The experiments** are quite **different** from the **naive theoretical understanding,** just becomes the **quark masses** in the real world are actually **light!**

Proton spin



Glue helicity

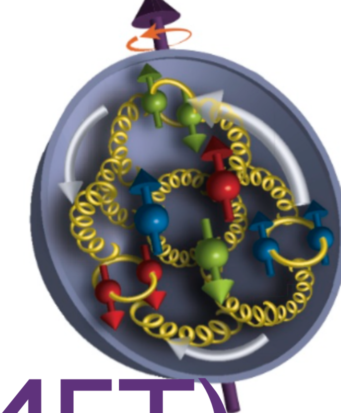
Glue helicity $\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi}$
$$e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$



has been shown to be ~0.2 or 40% of the proton spin with large uncertainty, based the global analysis with experimental results from:

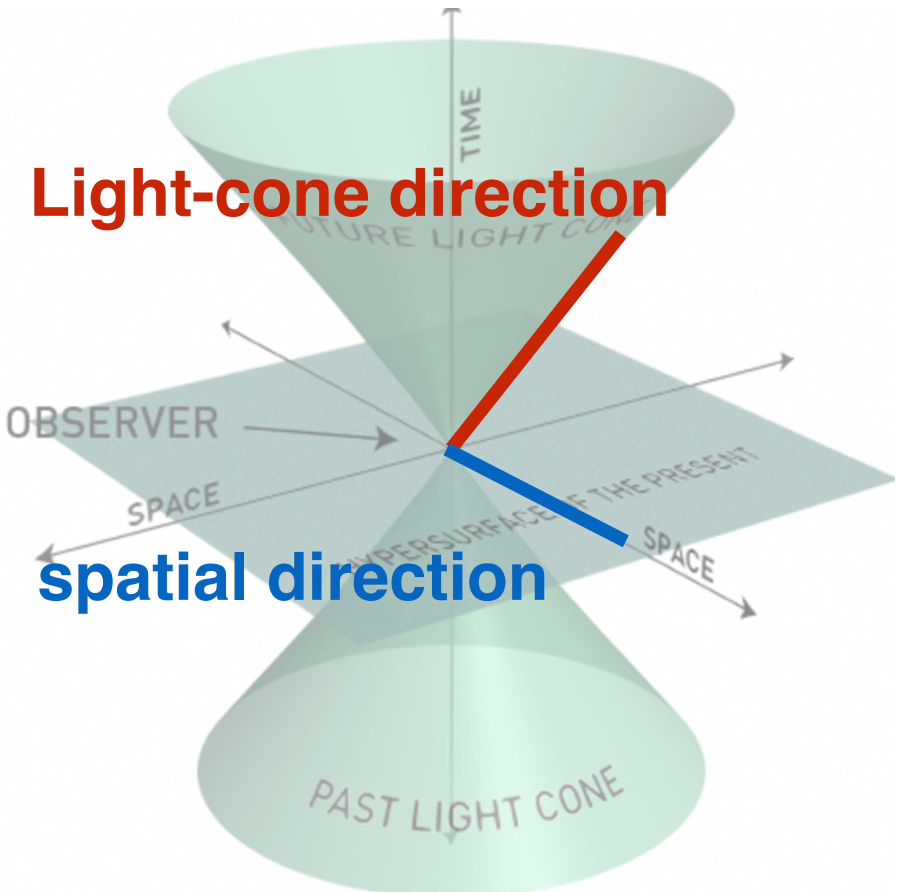
STAR NPA932, 500(2014),1404.5134
PHENIX PRD90, 012007(2014), 1402.6296
COMPASS PLB690, 466(2010), 1001.4654

Glue spin



Large momentum effective theory (LaMET)

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-,0)\right)\right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$



When nucleon is boosted:

- The Coulomb and Temporal gauge conditions become the light-cone one.
- **Glue spin** below becomes **glue helicity,** the integration of the glue polarized PDF, **at tree level**.

$$O_{S_G^c} = \vec{E}^c \times \vec{A}^c,\ \partial_i A_i^c = 0$$ ***Coulomb gauge***

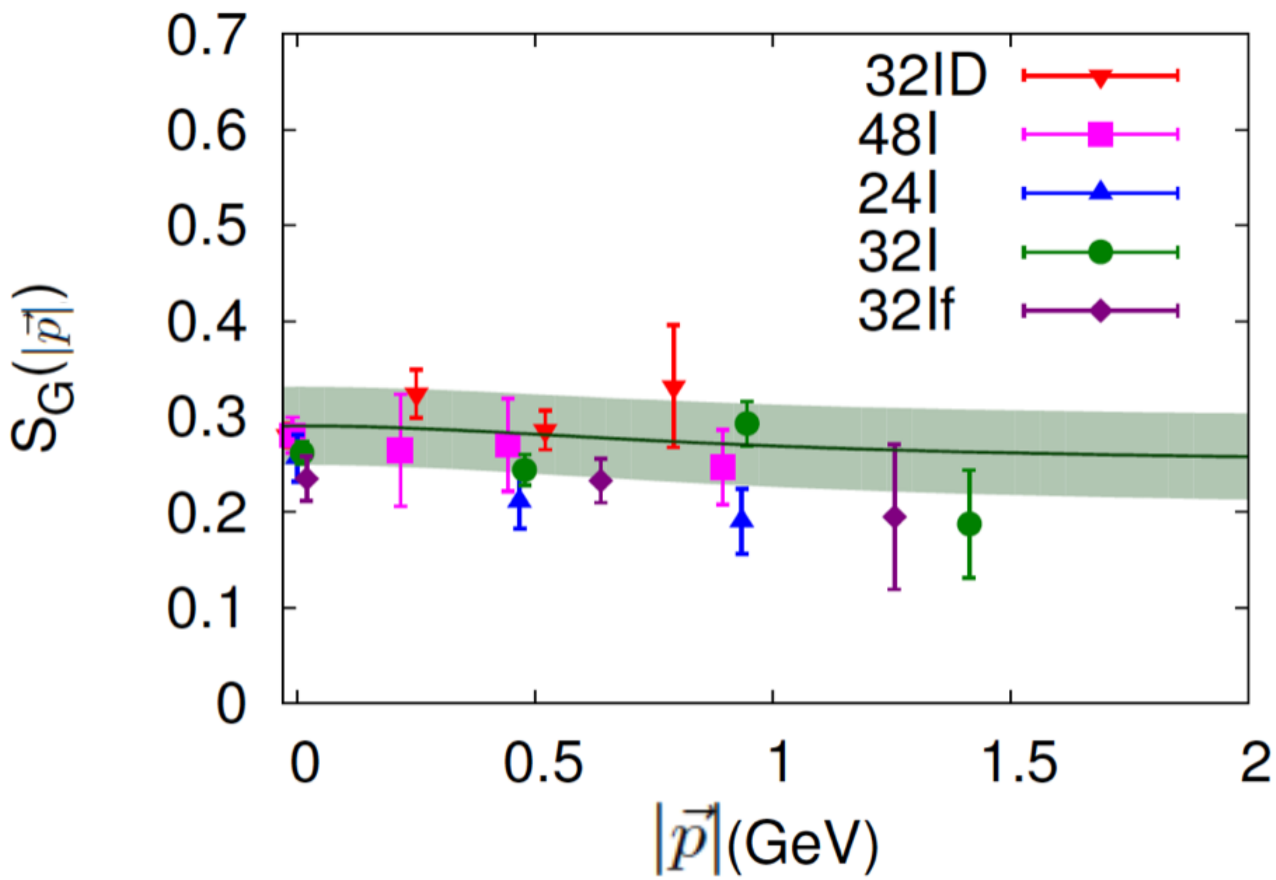
or $$O_{S_G^t} = \vec{E}^t \times \vec{A}^t,\ A_0^t = 0$$ ***Temporal gauge***

X. Ji, J. Zhang, and Y. Zhao, PRL111 112002 (2013), 1304.6708

Glue spin

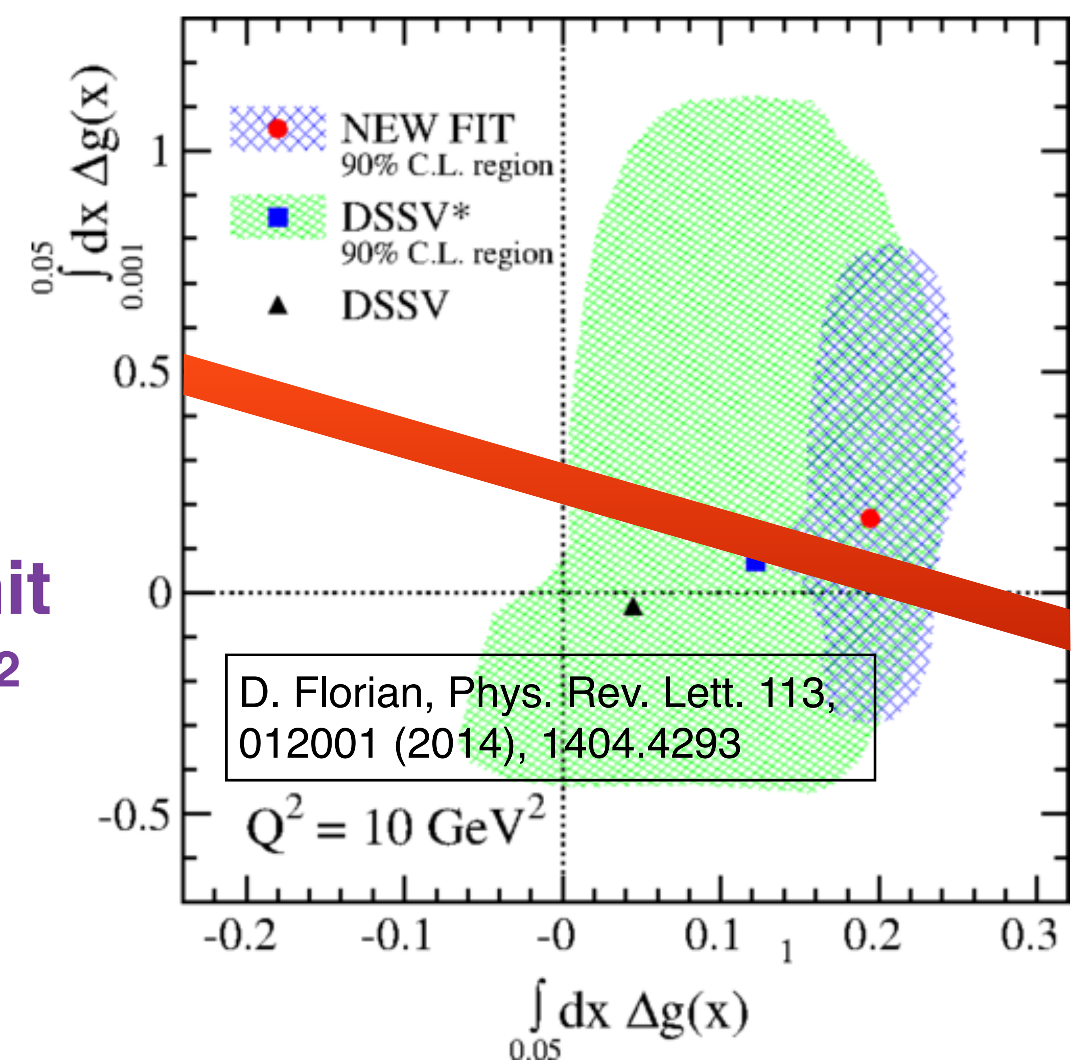
YBY, R. Sufian, et. al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion

Results



Neglect the matching and apply an empirical form to fit the data,

$$\int_{0.001}^{0.05} \mathrm{d}x \Delta g(x) + \int_{0.05}^{1} \mathrm{d}x \Delta g(x) \simeq S_g$$

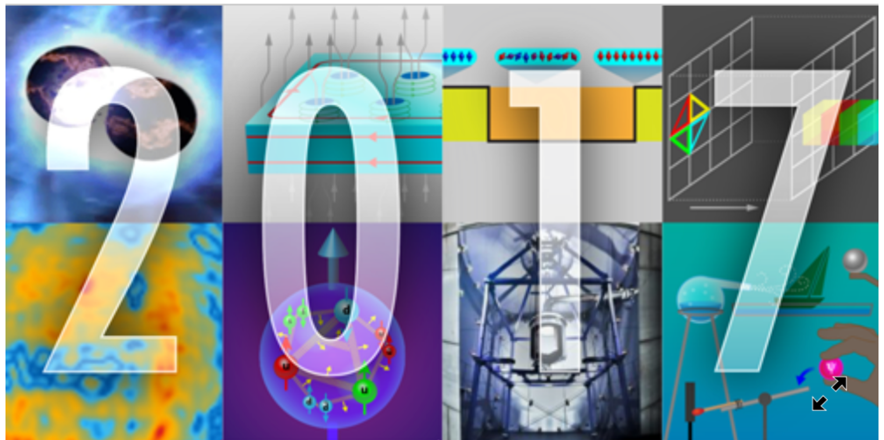


the glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2 will be

S_G=0.251(47)(16).

One of **eight**

YBY, R. Sufian, et. al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion



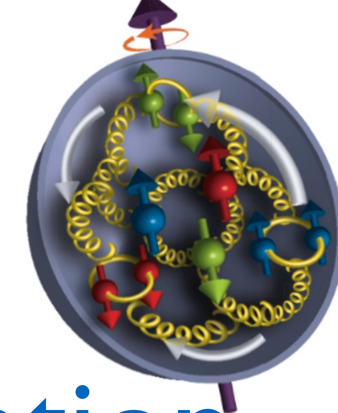
APS Highlights of 2017

https://physics.aps.org/articles/v10/137

Gluons Provide Half of the Proton's Spin

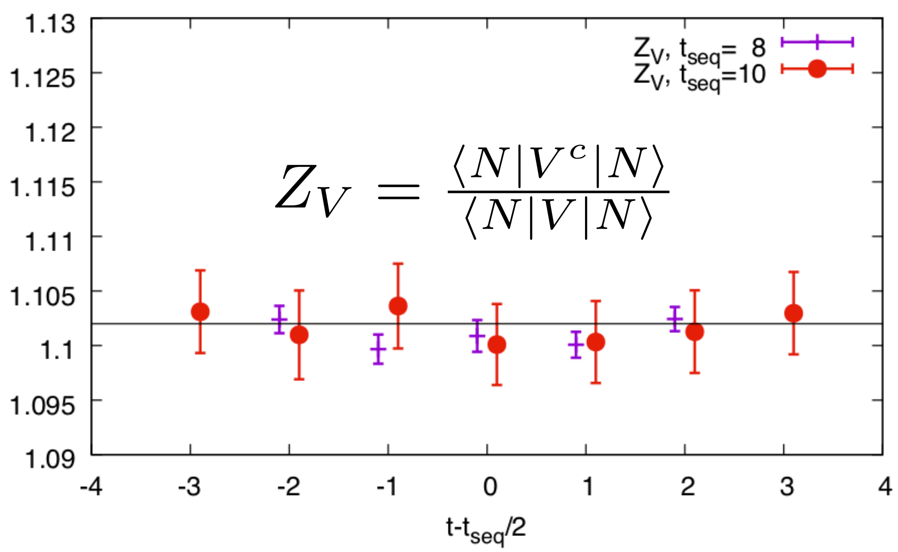
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

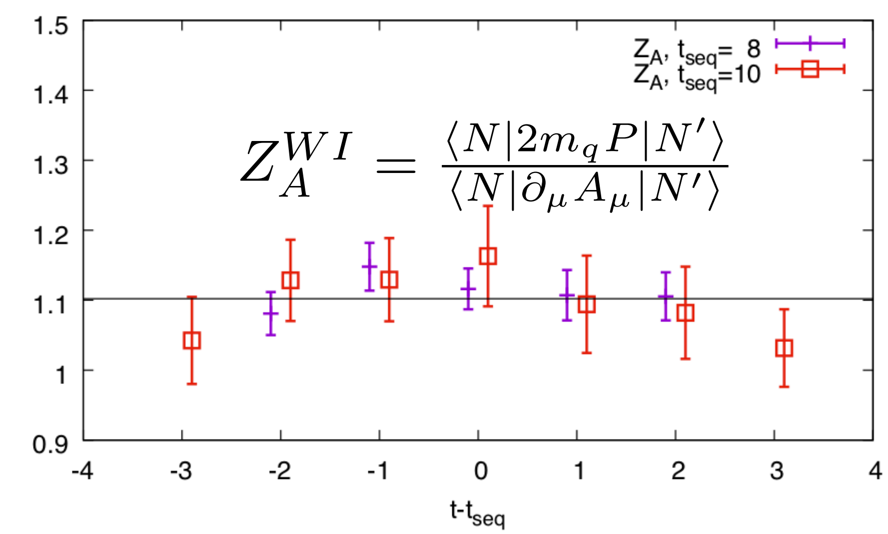
Quark spin



Normalization

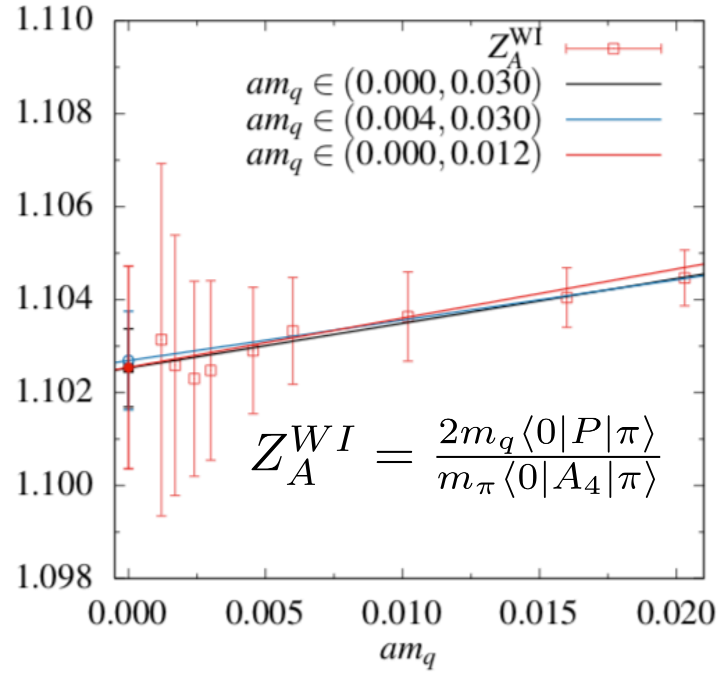
- *The normalization is the ratio between the **local** current and **conserved/ chiral** current;*

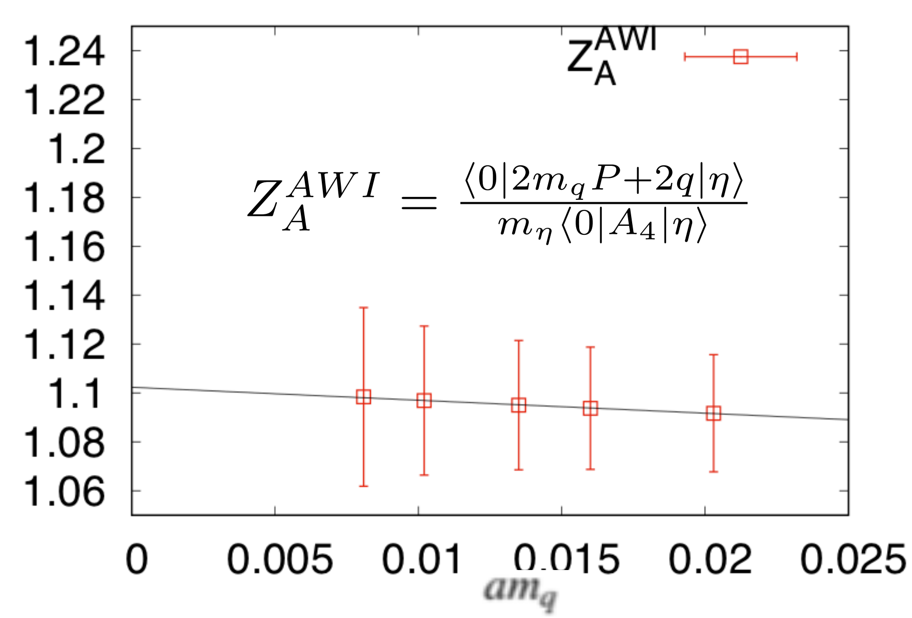




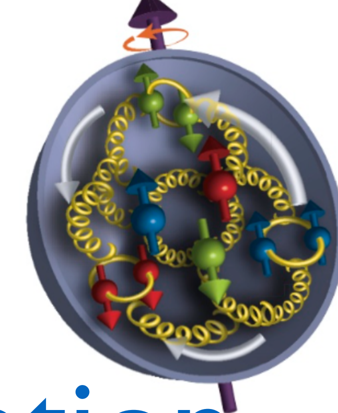
With chiral fermion, all the cases provide the same normalization $Z_V=Z_A$.

- *Can also be obtained with the (anomalous) Ward Identity.*



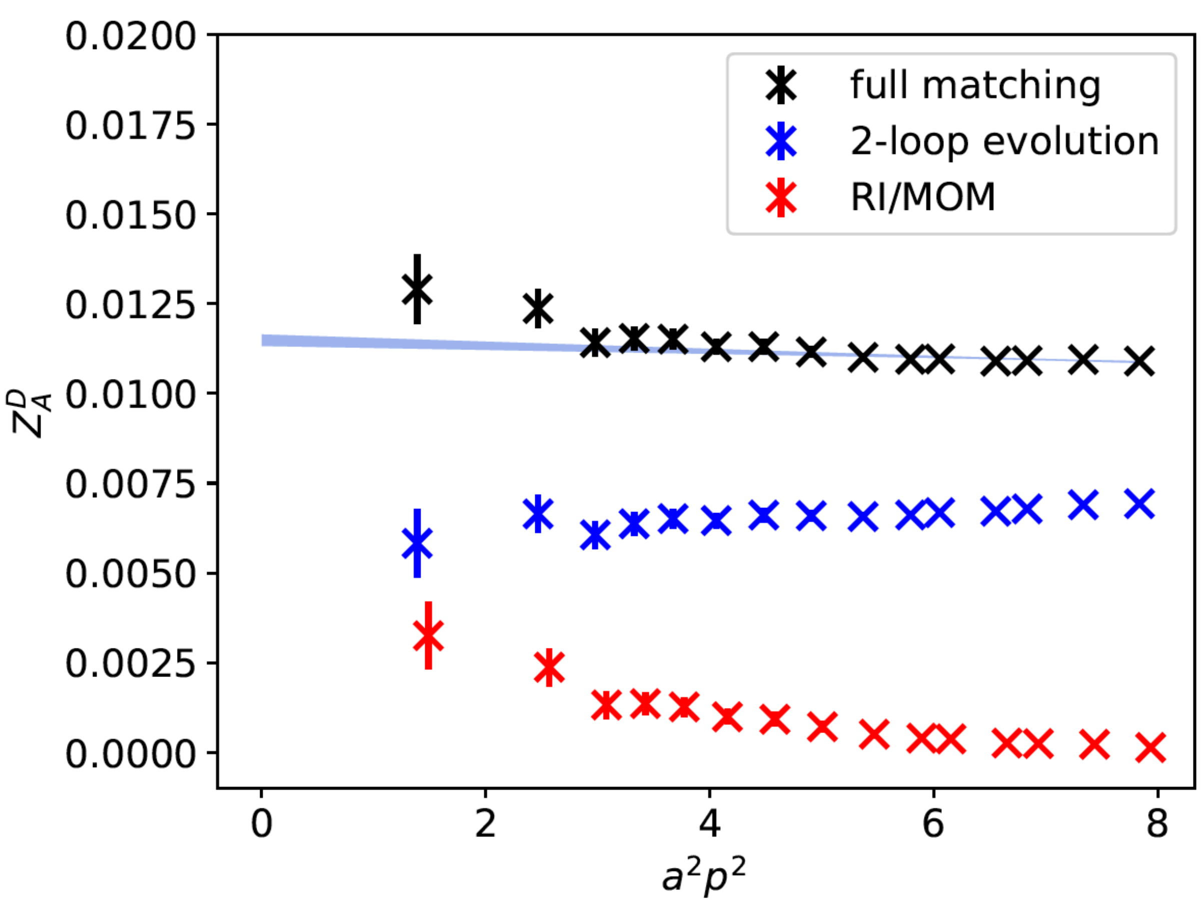


Quark spin



Renormalization

$$\begin{pmatrix} \Delta u^{\overline{\rm MS}}(\mu) \\ \Delta d^{\overline{\rm MS}}(\mu) \\ \Delta s^{\overline{\rm MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) \\ Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MS}}(\mu) \\ Z_A^{{\rm D},\overline{\rm MS}}(\mu) & Z_A^{{\rm D},\overline{\rm MSI}}(\mu) & Z_A + Z_A^{{\rm D},\overline{\rm MS}}(\mu) \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}$$



J. Liang, YBY, et al., χQCD collaboration, 1806.08366

- *An accurate renormalization of the singlet axial vector current requires the **RI/MOM** calculation of the **quark loop** and also **the 2-loop matching.***
- *The only complete renormalization calculation with 2-loop finite piece so far.*

Quark spin

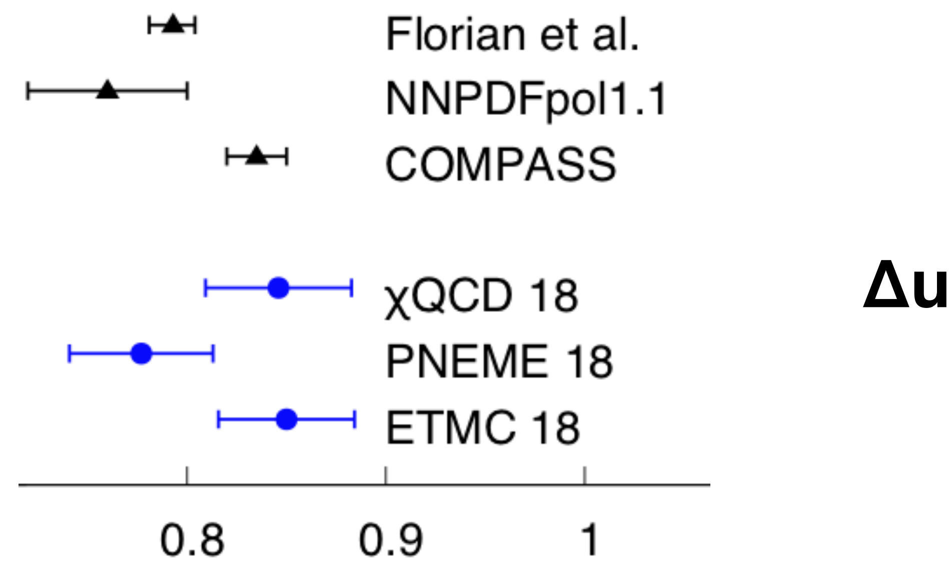
Summary of the present results

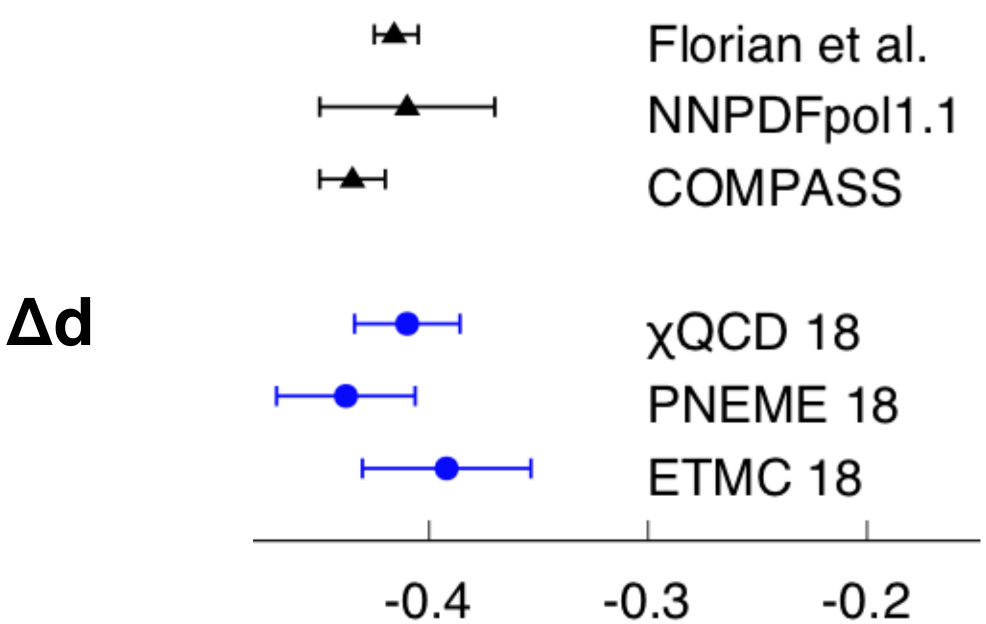
	N_f	Disc	m_π	FV	Ren	ESC
χQCD 18	2+1	o	o	★	★	★
PNDME 18	2+1+1	o	★	★	■	★
ETMC 18	2+1+1	■	★	★	★	★

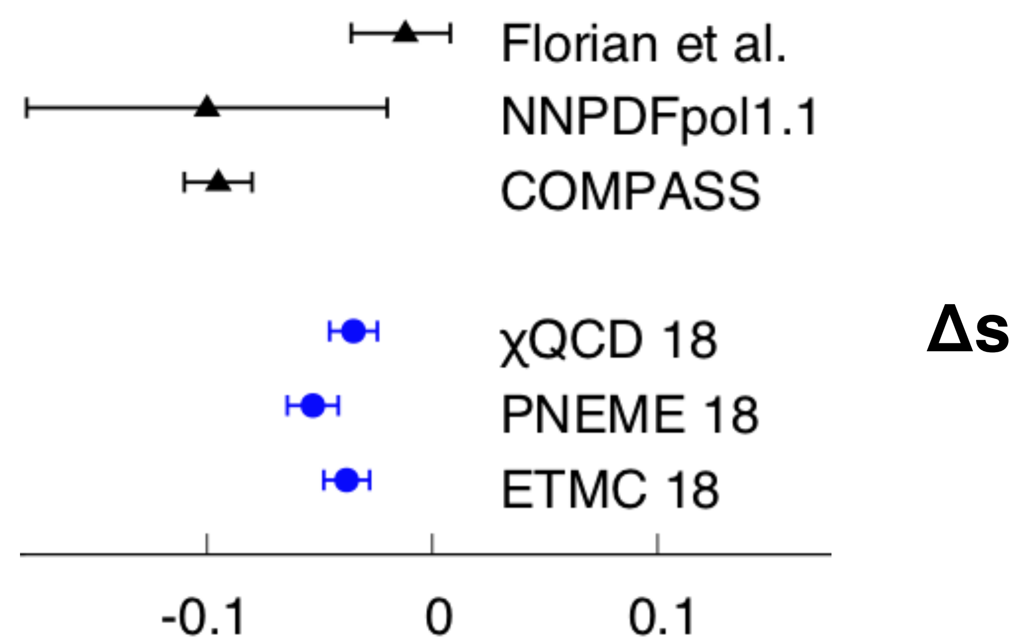
■ As the result at just one lattice spacing

■ (Ren) As the non-perturbative normalization only

★ for NP renormalization + (A)WI normalization





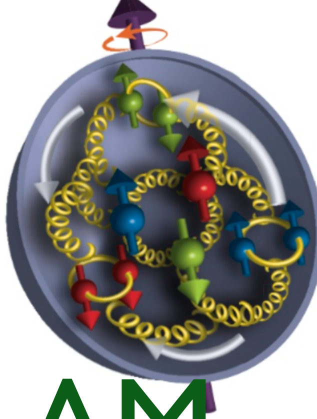


χQCD 18: 1806.08366

PNEME 18: 1806.10604

ETMC 18: in preparation

Proton spin

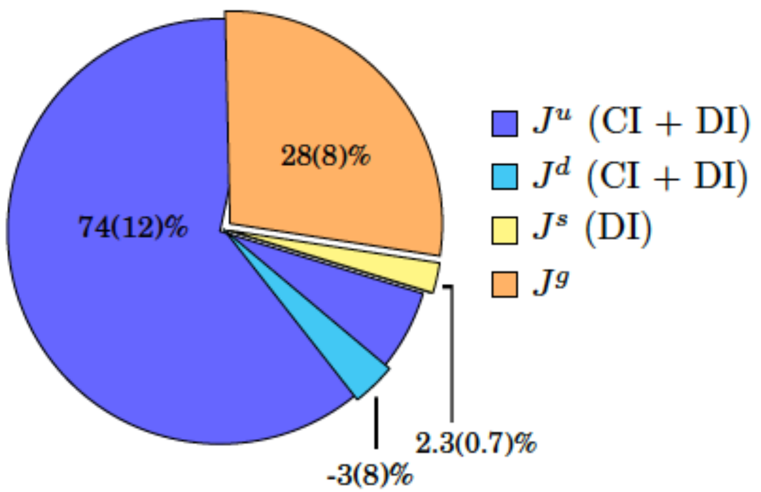


Lattice result of Ji AM

*Quark **AM*** *Glue **AM***

$$\vec{J} = \underbrace{\int d^3x \, \frac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^{\dagger} \{\vec{x} \times (i\vec{D})\} \psi}_{\text{Quark AM}} + \underbrace{\int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}}_{\text{Glue AM}}$$

Quenched result

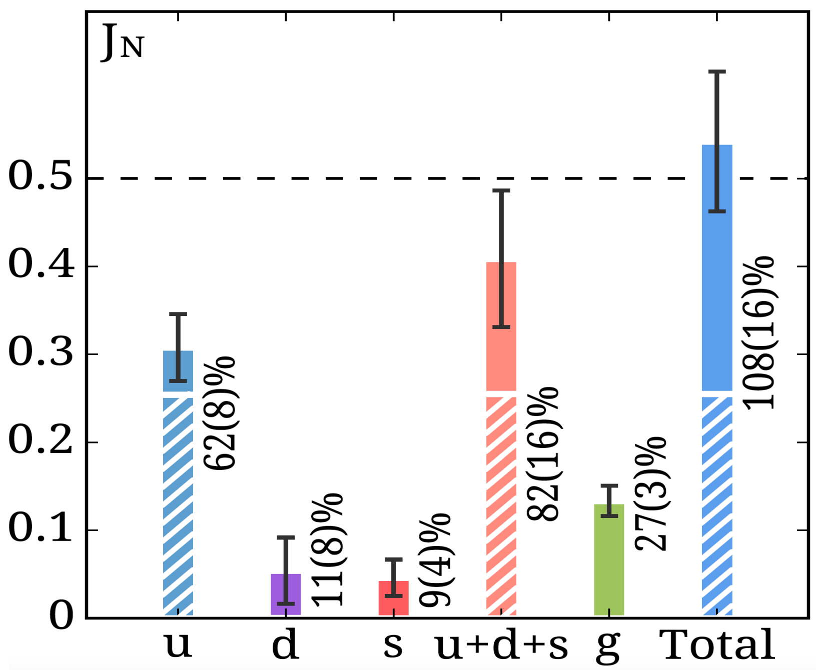


M. Deka, T. Doi, **YBY**, et. al., χQCD collaboration, PRD91, 014505 (2015), 1312.4816

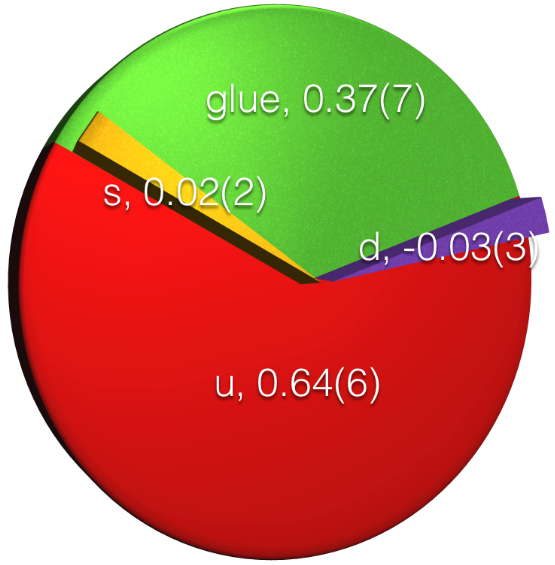
2-flavor result

(neglecting the difference between the glue momentum and AM fractions)

EMTC 17: PRL119(2017), 1706.02973



Preliminary *2+1 flavors result*



YBY, et. al., χQCD collaboration, in preparation

*All of them are based on the **perturbative** renormalization;*
***Non-perturbative** renormalization should be applied on them to get the state-of-the-art result.*

Summary

- With the support from the super computer, Lattice QCD provides a systematic way to investigate the origin of the nucleon mass and spin;

- Both the brute force and physics insights are necessary to reach the goal with reasonable cost;

- It will be more fruitful in China with the local computer resources.