Neutrino Oscillation
The 2015 Nobel Prize in Physics

Xiao-Gang He

USTC Colloquium Talk 25-3-2016

The Nobel Prize in Physics 2015

Ill. N. Elmehed. © Nobel Media AB 2015.

Takaaki Kajita

Prize share: 1/2

Ill. N. Elmehed. © Nobel Media AB 2015.

Arthur B. McDonald

Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

Four Nobel prizes so far directly related to neutrino physics

Neutrinos and different types

December 4, 1930

β decay

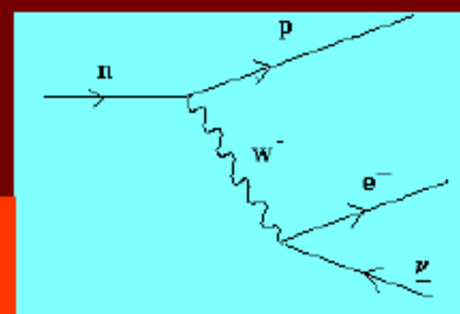
Dear radioactive ladies and gentlemen,

...I have hit upon a ‘desperate remedy’ to save...the law of conservation of energy. Namely the possibility that there exists in the nuclei electrically neutral particles, that I call neutrons...I agree that my remedy could seem incredible...but only the one who dare can win...

Unfortunately I cannot appear in person, since I am indispensable at a ball here in Zurich.

Your humble servant
W. Pauli

Note: this was before the discovery of the real neutron



December 1933: Enrico Fermi

submits a paper to Nature: “Tentativo di Una Teoria Della Emissione di raggi Beta”

It was rejected:”speculations too remote from reality to be of interest to the readers”.

Eventually published in Nuovo Cimento This paper laid out the essential theory of beta decay that has survived almost unchanged until now, with “small” modifications. It predicted the spectrum, obtained the correct value for the coupling etc………

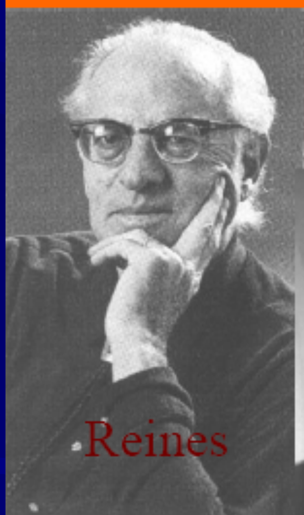
Three Types of Neutrinos

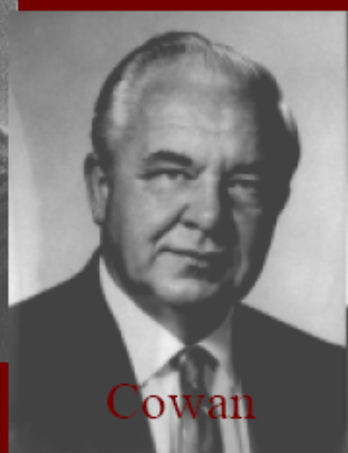
Project Poltergeist 1956

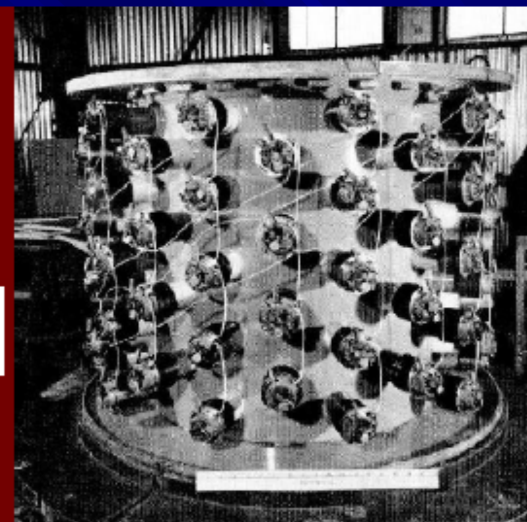
$$\nu + p^+ \to n^o + e^+$$

$$e^+ + e^- \to 2\gamma$$

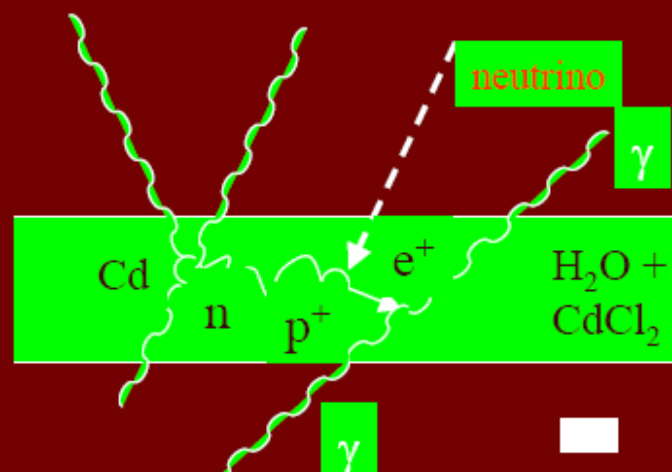
$$n^o + \mathrm{Cd} \to (\mathrm{several})\,\gamma$$





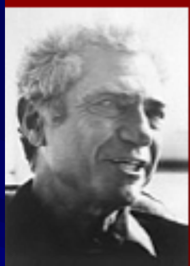


Signal 2γ, then several γ ~few μs later



Experiment attempted at Hanford in 1953, too much background. Repeated at Savannah River in 1955. [Flux: 10^{13} neutrinos/(cm^2 s)]

More than one kind of neutrino?



Date: 1962

Intent: Measure weak force at high energies

Expectation: Since neutrinos are created with muons and electrons, the neutrino beam should create both electrons and muons in the detector.

Result: No electrons produced, only muons

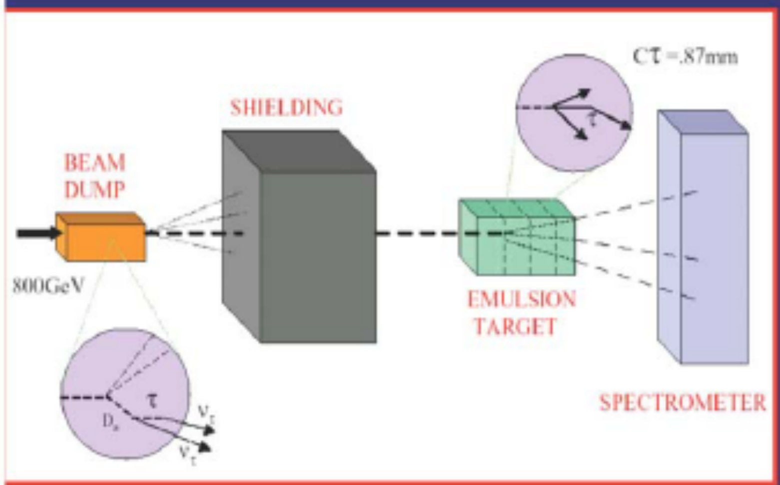
Conclusion: There must be two kinds of neutrinos.

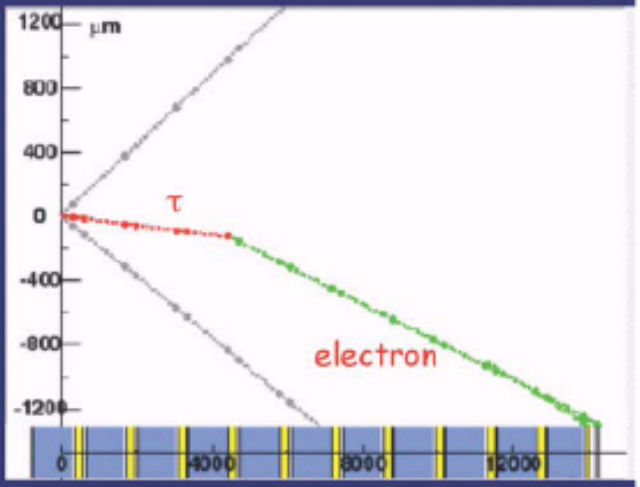
Observation of the third neutrino FermiLab 2000

ν_τ discovery: year 2000

DONUT experiment at Fermilab

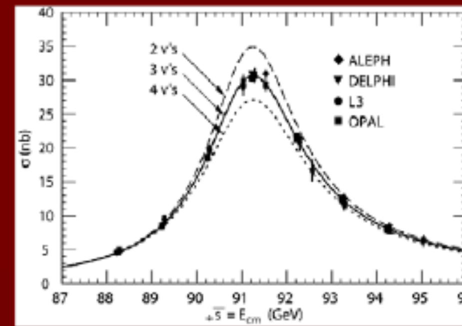
$$\text{Protons} \rightarrow \text{target} \rightarrow X + D_s$$
$$\hookrightarrow \tau + \nu_\tau \rightarrow \text{detector} \rightarrow \tau$$
$$\hookrightarrow X + \nu_\tau \rightarrow \text{detector} \rightarrow \tau$$



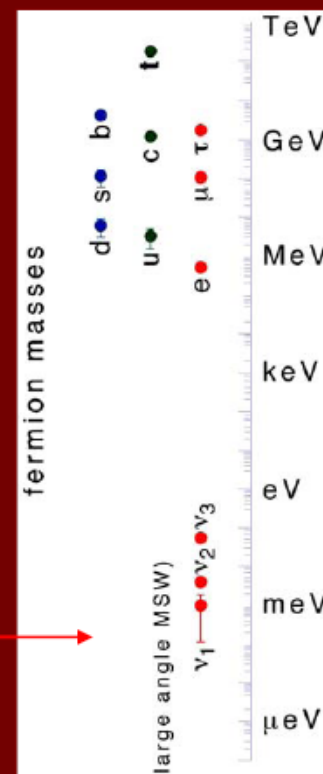


Three types of active light neutrinos

Nature's building blocks



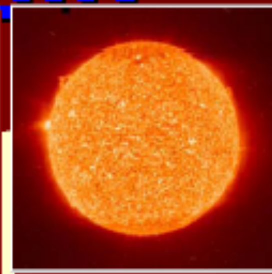
Three flavors or generations, and no more, and we do not know why.



Some mass, but curiously little.

Where do Neutrinos come from?

✓ Nuclear Reactors (power stations, ships)

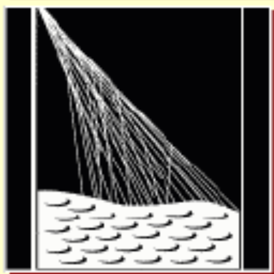


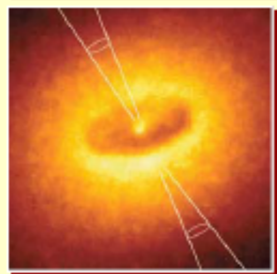
Sun ✓

✓ Particle Accelerator

Supernovae (star collapse)
SN 1987A ✓

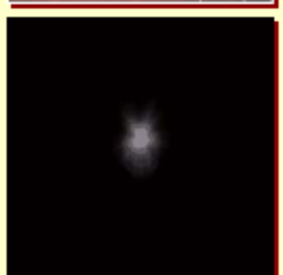
✓ Earth's Atmosphere (Cosmic Rays)





Astrophysical Accelerators
Soon ?

✓ Earth's Crust (Natural Radioactivity)



Big Bang
(here 330 ν/cm^3)
Indirect Evidence

The Nobel Prize in Physics 2015

Ill. N. Elmehed. © Nobel Media AB 2015.

Takaaki Kajita

Prize share: 1/2

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Arthur B. McDonald

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Mixing and non-zero neutrino mass

$$\begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

At t=0, neutrino $\nu_1(0)$ produced, at t, the state becomes

$$|\nu_1(0)> = \quad \cos\theta e^{-i(Et-p_1 L)}|\nu_{m1}> \; -\sin\theta e^{-i(Et-p_2 L)}|\nu_{m2}>.$$

Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_1^2/2E$,
the probability amplitude of find $\nu_2(0)$ at time t is given by

$$< \nu_2(0)|\nu_1(t) > = \cos\theta \sin\theta (e^{-im_1^2 L/2E} - e^{-im_2^2 L/2E})$$

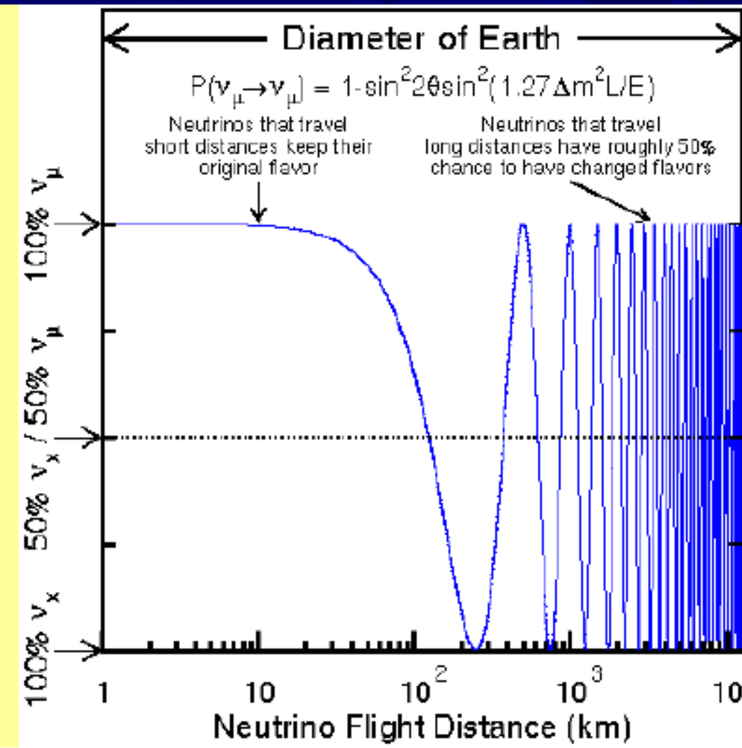
which leads to the probability of finding $\nu_2(0)$ is

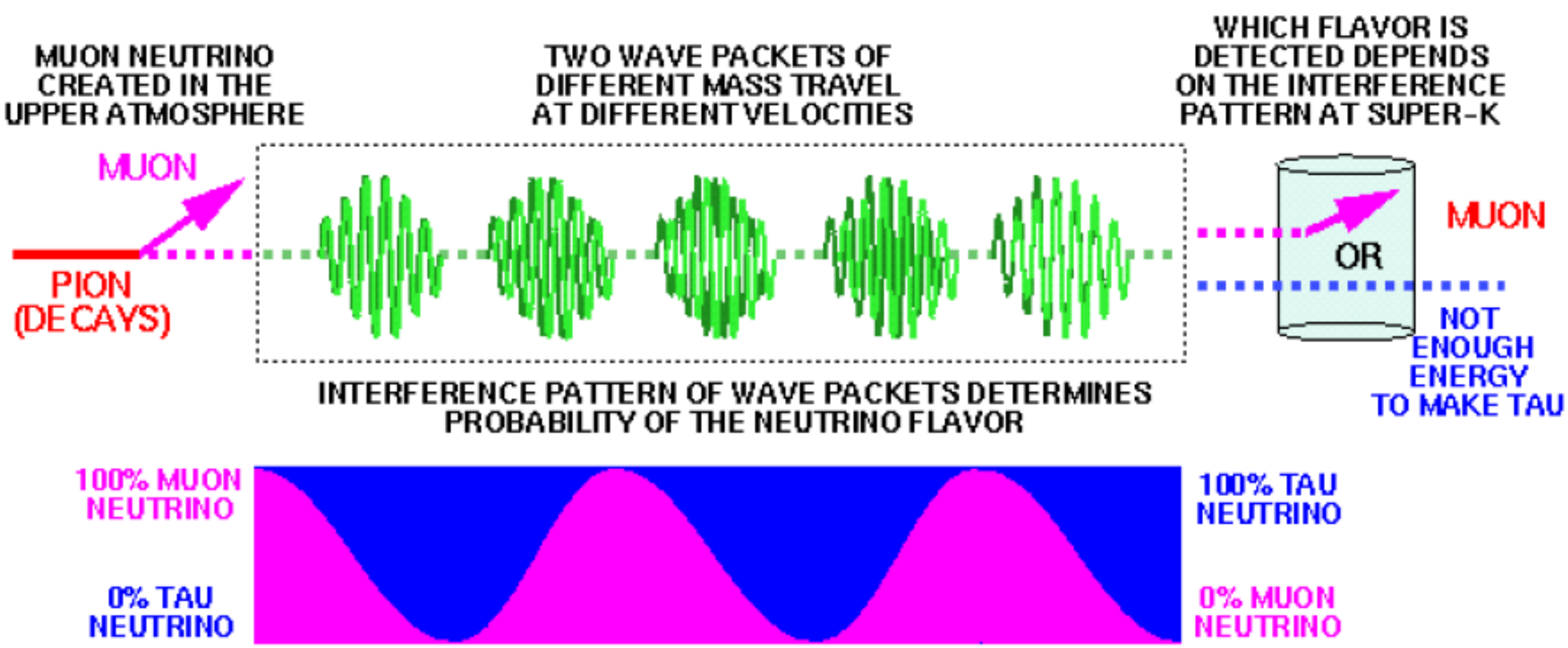
$$P(\nu_1 \to \nu_2) = | < \nu_1(0)|\nu_2(t) > |^2 = \sin^2(2\theta)\sin^2(\Delta m_{21}^2 L/4E),$$

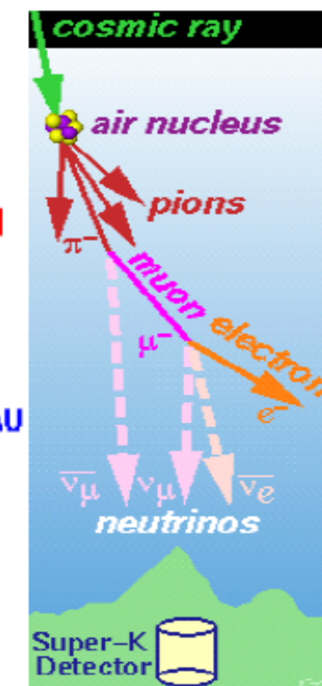
$$\Delta m_{21}^2 = m_2^2 - m_1^2.$$

$$P(\nu_e \rightarrow \nu_\mu) \cong \sin^2(2\theta)\sin^2[\frac{\Delta m^2}{4}\frac{L}{E}]$$

$$m_1 \neq m_2$$







- Active neutrinos have small mass, they mix with each other
- There may be additional light sterile neutrinos
- Electrically neutral, has the possibility of being its own anti-particle, Majorana particle. Dirac or Majorana particle?

Theoretical Models for Neutrinos

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ ,$$

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} (3,2)(1/6)\ ,\ \ U_R\ (3,1)(2/3)\ ,\ \ D_R\ (3,1)(-1/3)\ ,$$

$$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (1,2)(-1/2)\ ,\ \ E_R\ (1,1)(-1)\ ,$$

$$H = \begin{pmatrix} h^+ \\ (v+h^0)/\sqrt{2} \end{pmatrix} (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .$$

Quark and charged lepton masses are from the following Yukawa coupolings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

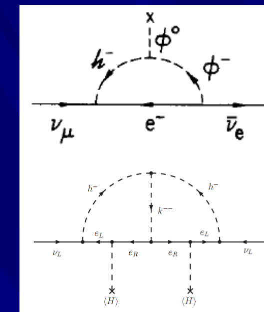
To have Dirac mass, need to introduce right handed neutrinos ν_R: (1,1)(0)

Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C\ , \to\ \ -\bar{\nu}_L m_\nu \nu_R\ , \to m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$m_{\nu_e} < 0.3$ eV, $\to\ \ Y_{\nu_e}/Y_e < 10^{-5}$, very much fine tuned!

Some theoretical models for neutrino masses



Loop generated neutrino masses:

The Zee Model(1980); Zee-Babu Model (zee 1980; Babu, 1988)

Other loop models: Babu&He; E. Ma; Mohapatra et al; Geng et al

Seesaw Models:

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v/\sqrt{2} \\ Y_\nu^T v/\sqrt{2} & M_R \end{pmatrix}$$

$$m_\nu \approx Y_\nu^2 v^2/M_R\ , \qquad M_N \approx M_R$$

Type I Introduce singlet neutrinos

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

$$L = \bar{\nu}_L (Y_\nu v/\sqrt{2})\nu_R + \bar{\nu}_R^c M_R \nu_R/2$$

Type II: Introduce triplet Higgs representation $\Delta : (1,3,1)$, (W. Konetschny, and W. Kummer, 1977; L.-F. Li and T.-P. Cheng, 1980; Gelmini and Roncadelli, 1981)

Type-III: Introduce triplet lepton representations Σ: (1,3,0)) (Foot, Lew, He and Joshi, 1989).

Neutrino Mixing

B. Pontecorvo (1957). "Mesonium and anti-mesonium". *Zh. Eksp. Teor. Fiz.* 33: 549–551.
B. Pontecorvo (1967). "Neutrino Experiments and the Problem of Conservation of Leptonic Charge". *Zh. Eksp. Teor. Fiz.* 53: 1717

Z. Maki, M. Nakagawa, and S. Sakata (1962). "Remarks on the Unified Model of Elementary Particles". *Progress of Theoretical Physics* 28 (5): 870.

Courtesy of Sakata Memorial Archival Library

S. Sakata Z. Maki M. Nakagawa

Three generation mixing in quarks and leptons

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$

$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\ ,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \left(\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{array}\right),$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Where are we now? PDG2014

Table 14.7: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters, derived from a global fit of the current neutrino oscillation data (from [174]) . The values (values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The definition of Δm^2 used is: $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m^2_{31} - \Delta m^2_{21}/2 > 0$, if $m_1 < m_2 < m_3$, and $\Delta m^2 = \Delta m^2_{32} + \Delta m^2_{21}/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit ($\pm 1\sigma$)	3σ
Δm^2_{21} $[10^{-5}$ eV $^2]$	$7.54^{+0.26}_{-0.22}$	$6.99 - 8.18$
$\lvert\Delta m^2\rvert$ $[10^{-3}$ eV $^2]$	2.43 ± 0.06 (2.38 ± 0.06)	$2.23 - 2.61$ $(2.19 - 2.56)$
$\sin^2\theta_{12}$	0.308 ± 0.017	$0.259 - 0.359$
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	$0.437^{+0.033}_{-0.023}$	$0.374 - 0.628$
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	$0.455^{+0.039}_{-0.031}$,	$0.380 - 0.641$
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	$0.0234^{+0.0020}_{-0.0019}$	$0.0176 - 0.0295$
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	$0.0240^{+0.0019}_{-0.0022}$	$0.0178 - 0.0298$
δ/π (2σ range quoted)	$1.39^{+0.38}_{-0.27}$ $(1.31^{+0.29}_{-0.33})$	$(0.00 - 0.16) \oplus (0.86 - 2.00)$
		$((0.00 - 0.02) \oplus (0.70 - 2.00))$

Where have we come from and where will we go?

Very different quark and lepton mixing patterns

Mixing pattern in quark sector

$$V_{CKM} \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}.$$

$$\theta^Q_{12} = 13.021^\circ \pm 0.039^\circ, \quad \theta^Q_{23} = 2.350^\circ \pm 0.052^\circ,$$

$$\theta^Q_{13} = 0.199^\circ \pm 0.008^\circ. \quad \delta^Q = 68.9^\circ$$

Mixing pattern in lepton sector

$$|U| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix},$$

Experimental discovery of neutrino mixing

Homestake Gold Mine

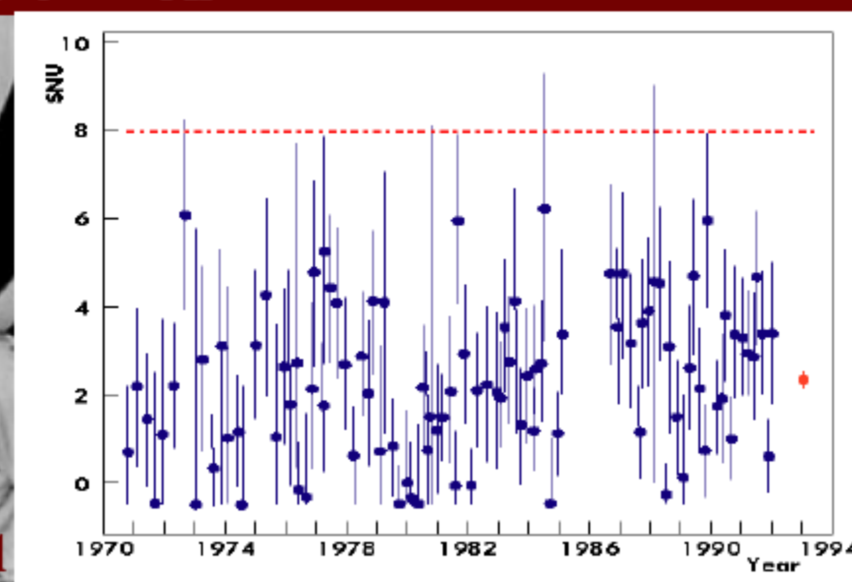
100,000 gallons of cleaning fluid C_2Cl_4

Expected 1.5 interactions per day
Measured 0.5 interactions per day

1968!

Sensitive to ^8B solar neutrinos only

$\nu_e + {}^{37}Cl \to e^- + {}^{37}Ar$



Abundant data show that neutrinos have non-zero masses and mix with each other.

Solar neutrino oscillation: Homestake, Sage+Gallex/GNO, Super-K, SNO,Borexino …

Atmospherical neutrino oscillation: Super-Kamokande, …

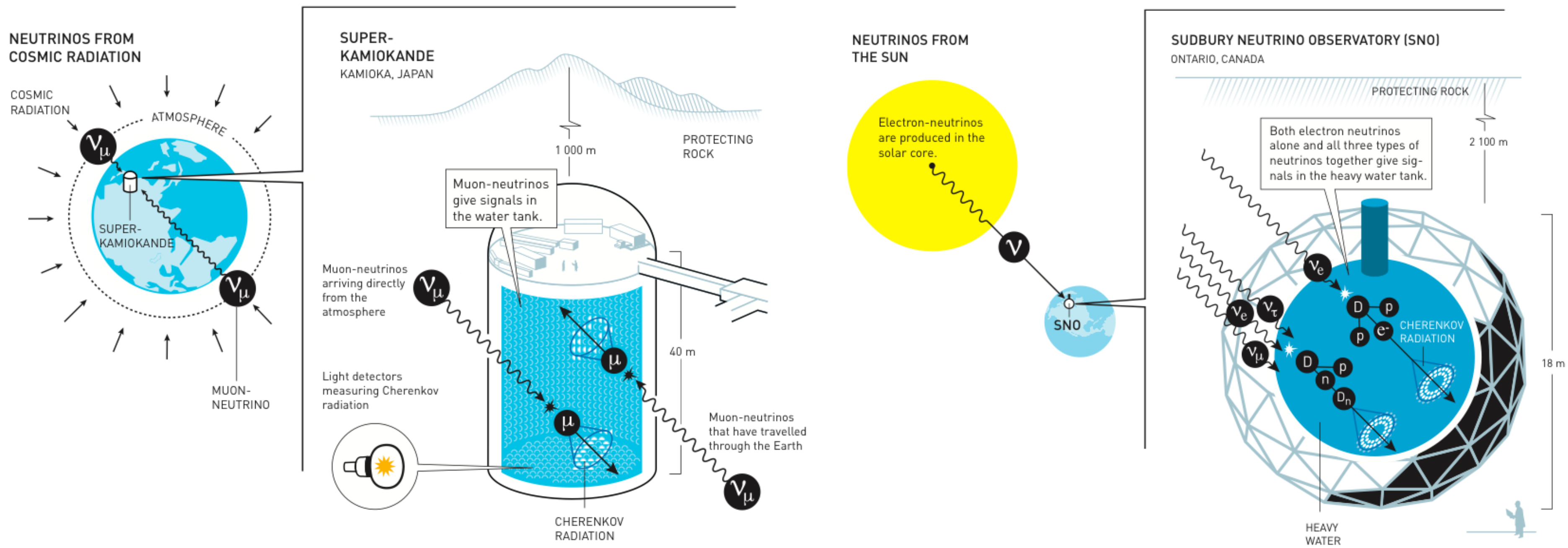
Accelerator neutrino source: K2K, Minos , Nova …

Reactor neutrino source: Kamland, T2K, Chooz, Daya-Bay, Reno…

have observed neutrino oscillation phenomenon.

LSND and Miniboon…?

Super-K and SNO



Evidence for Oscillation of Atmospheric Neutrinos

Y. Fukuda *et al.* (Super-Kamiokande Collaboration)
Phys. Rev. Lett. **81**, 1562 – Published 24 August 1998

ABSTRACT

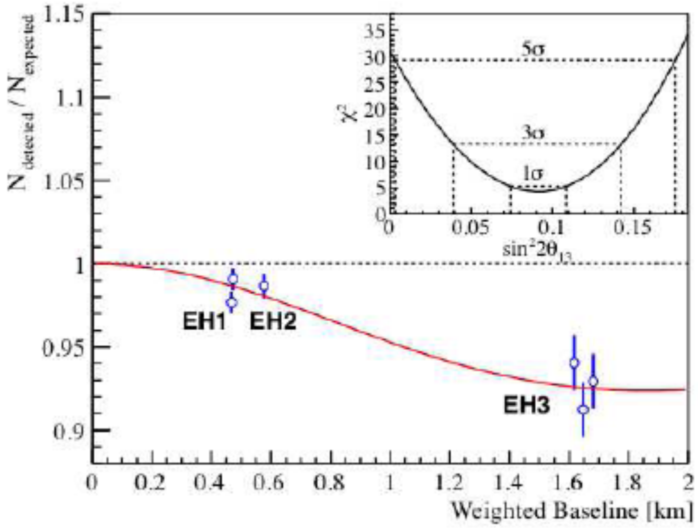
We present an analysis of atmospheric neutrino data from a 33.0 kton yr (535-day) exposure of the Super-Kamiokande detector. The data exhibit a zenith angle dependent deficit of muon neutrinos which is inconsistent with expectations based on calculations of the atmospheric neutrino flux. Experimental biases and uncertainties in the prediction of neutrino fluxes and cross sections are unable to explain our observation. The data are consistent, however, with two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with $\sin^2 2\theta > 0.82$ and $5 \times 10^{-4} < \Delta m^2 < 6 \times 10^{-3}\ \mathrm{eV}^2$ at 90% confidence level.

Received 6 July 1998

Measurement of the Rate of $\nu_e + d \rightarrow p + p + e^-$ Interactions Produced by ^8B Solar Neutrinos at the Sudbury Neutrino Observatory

Q. R. Ahmad *et al.* (SNO Collaboration)
Phys. Rev. Lett. **87**, 071301 – Published 25 July 2001

Solar neutrinos from ^{8}B decay have been detected at the Sudbury Neutrino Observatory via the charged current (CC) reaction on deuterium and the elastic scattering (ES) of electrons. The flux of ν_e's is measured by the CC reaction rate to be $\phi^{\mathrm{CC}}(\nu_e) = 1.75 \pm 0.07(\mathrm{stat})^{+0.12}_{-0.11}(\mathrm{syst}) \pm 0.05(\mathrm{theor}) \times 10^6\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Comparison of $\phi^{\mathrm{CC}}(\nu_e)$ to the Super-Kamiokande Collaboration's precision value of the flux inferred from the ES reaction yields a 3.3σ difference, assuming the systematic uncertainties are normally distributed, providing evidence of an active non-ν_e component in the solar flux. The total flux of active ^{8}B neutrinos is determined to be $5.44 \pm 0.99 \times 10^6\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.



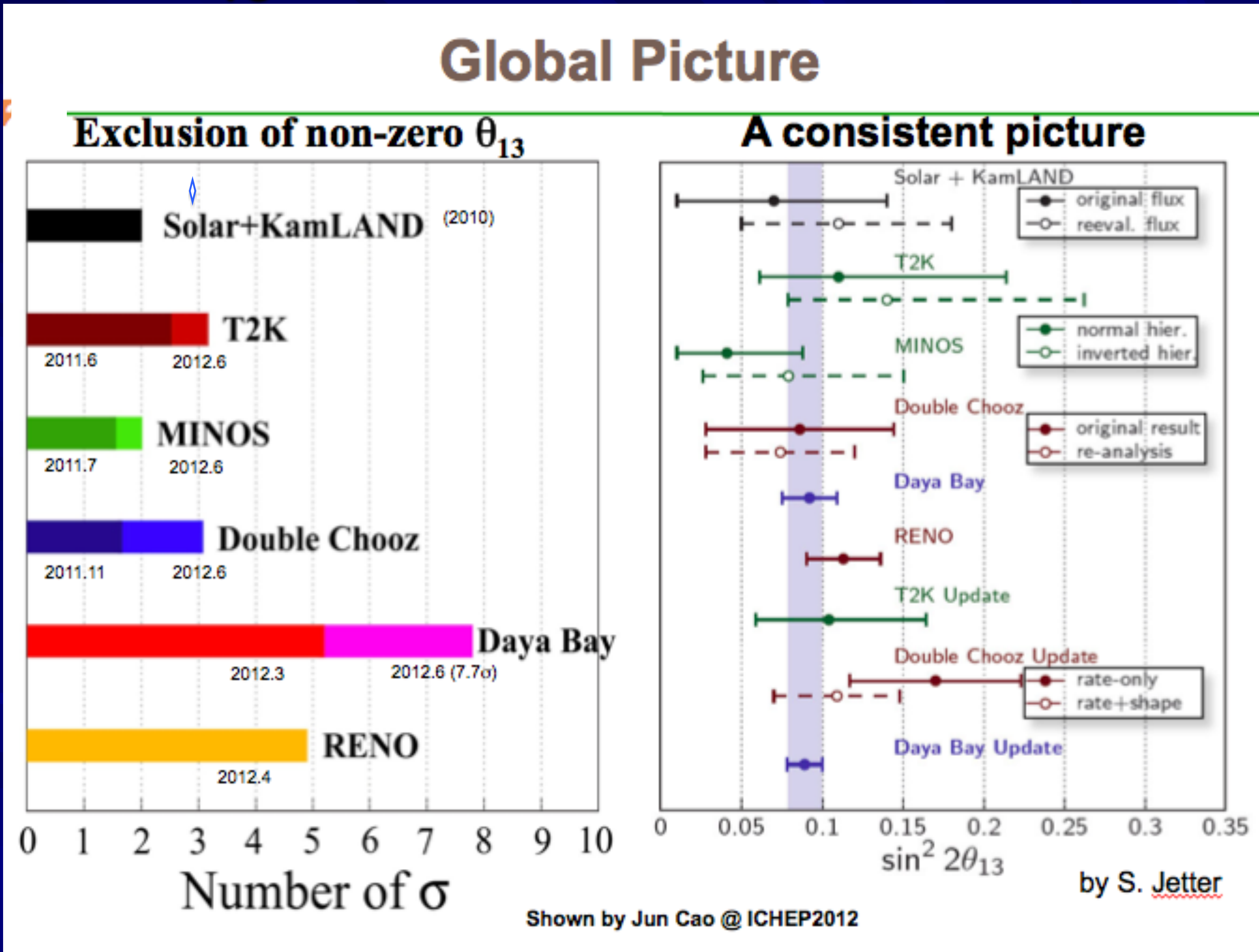
ArXiv:1203.1669 (hep-ex)

$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\mathrm{stat}) \pm 0.005(\mathrm{syst})$

Read as $\theta_{13} = 8.8^\circ$ with significance 5.2σ

Reno and others also confirmed Daya Bay result!

θ_{13} has been measured



2016 Breakthrough Prize

Theory for neutrino mixing

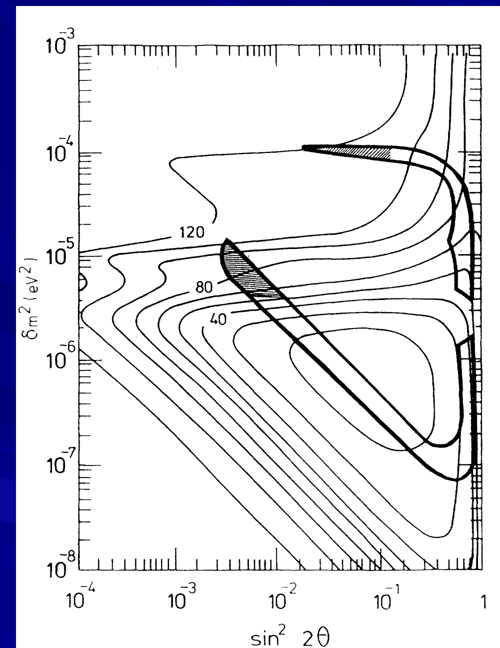
Early days, expecting neutrino mixing might be following a similar pattern as quarks, mixing angles are small.

For example, 1992 people are

trying to produce mixing on the right

Simultaneous solutions to the solar and atmospheric neutrino problems via Fritzsch-type lepton mass matrices

A. J. Davies* and Xiao-Gang He†



(Davies and He, PRD46, 3208)

But both solar and atmospheric show large mixing angles after 1998!

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing

Harrison, Perkins, Scott (2002) , Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ, with the tri-bimaximal mixing

$$V_{tri-bi} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, $\Delta(27, 96)$, $PSL_2(7)$ … discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthanpa&M-C. Chen; Frampton&Kephart; Y-L Wu, ….

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A_4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix} .$$

Tri-bimximal at higher scales and generate none zero θ_{13} at low energies?

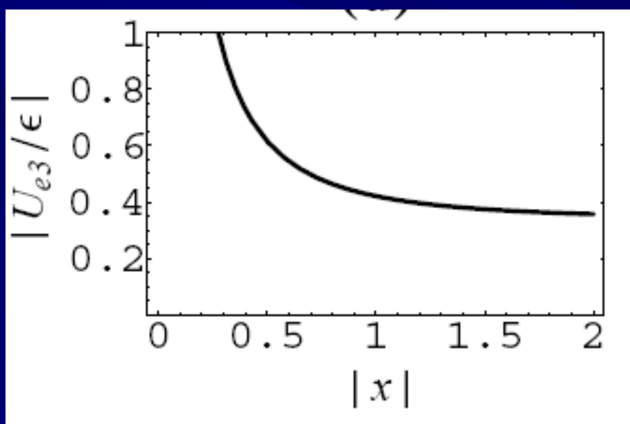
Baub and He, arxiv:0507217(hep-ph): A susy A4 model

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \, .$$

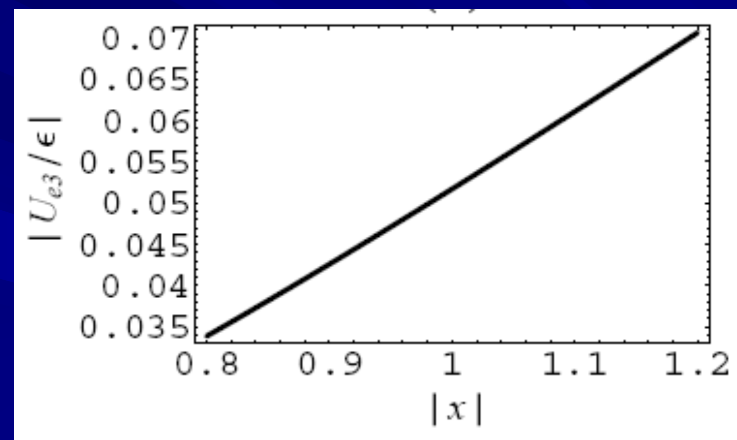
one–loop RGE $\quad \dfrac{dM_\nu^e}{d\ln t} = \dfrac{1}{32\pi^2}[M_\nu^e Y_e^\dagger Y_e + (Y_e^\dagger Y_e)^T M_\nu^e] + \ldots$

leading to the entries $M_{13,23}(1-\epsilon)$ and $M_{33}(1-2\epsilon)$

$\epsilon \simeq Y_\tau^2 \ln(M_{\rm GUT}/M_{EW})/32\pi^2.$



Inverted hierarchy



Normal hierarchy

Susy model, Y_t ~O(1), U_{e3} for inverted hierarchy, can be as large a 0.1, with RG effects!

Many other matrix form come back

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

An interesting coincidence:

$\theta_{13} = \theta_C/\sqrt{2}$

$V_{PMNS} = UV_{KM}$; $V_{Tr}V_{KM}$

B-Q. Ma et al: Ramond et al.; King et al....

Bi-Maximal Mixing

(Barger, Pakvasa, Weiler, Whisnant)

The CP violating phase gradually becomes the attention of theoretical studies.

Group theoretical studies of neutrino mass matrix, correlations between mixing angle and CP violation.

Ishimori&Kobayashi, Henandez & Smirnov, G-J Ding&King and etc..

SO(10)Grand Unification

SO(10) Yukawa couplings: $16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$

Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

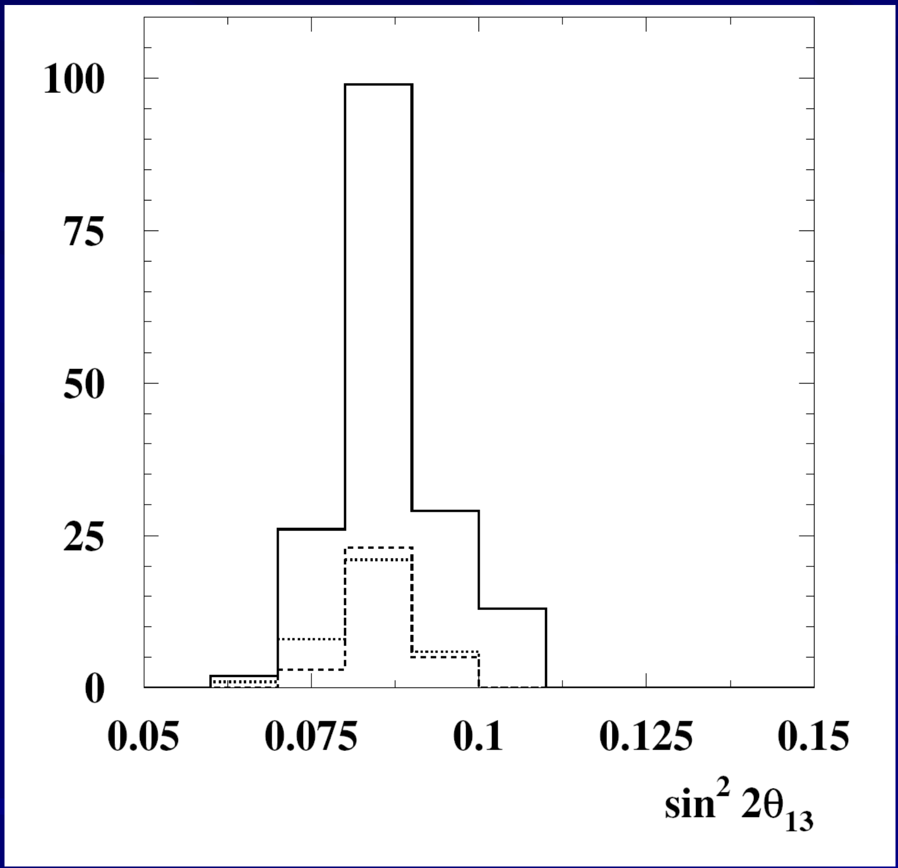
$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

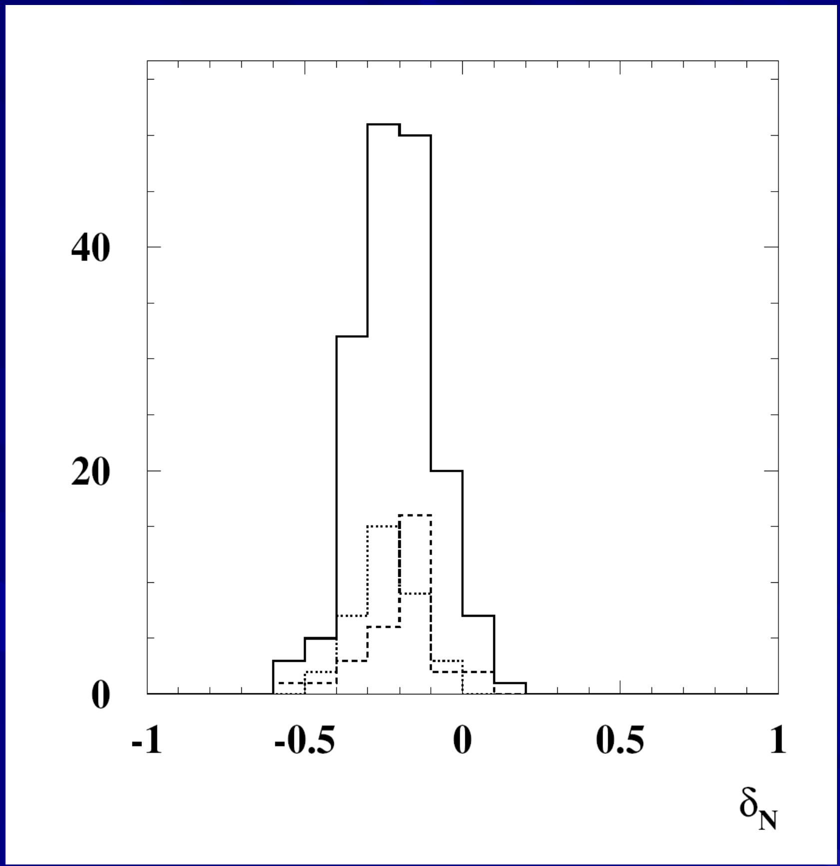
Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)

Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek
Jushipura, Patel (2011)

θ_{13} in Minimal SO(10)

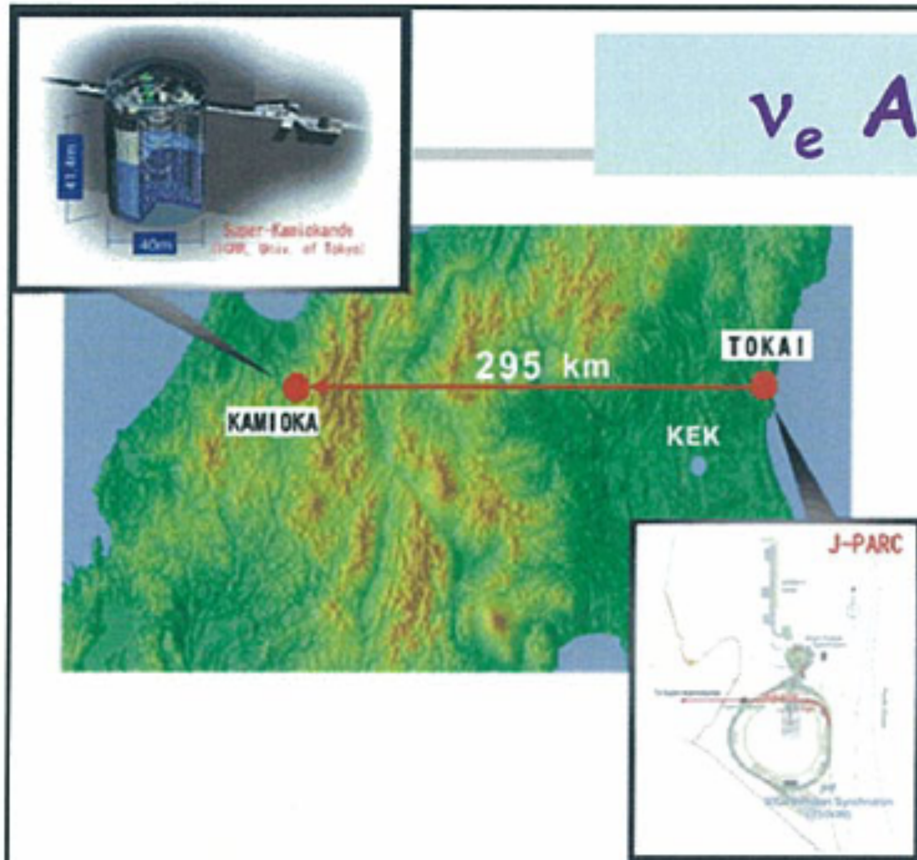




$\sin^2 2\theta_{13}$ and CP violating phase δ_N

K.S. Babu and C. Macesanu (2005)

ν_e Appearance



T2K- From Tokai To Kamioka

Mass hierarchy (+/-)

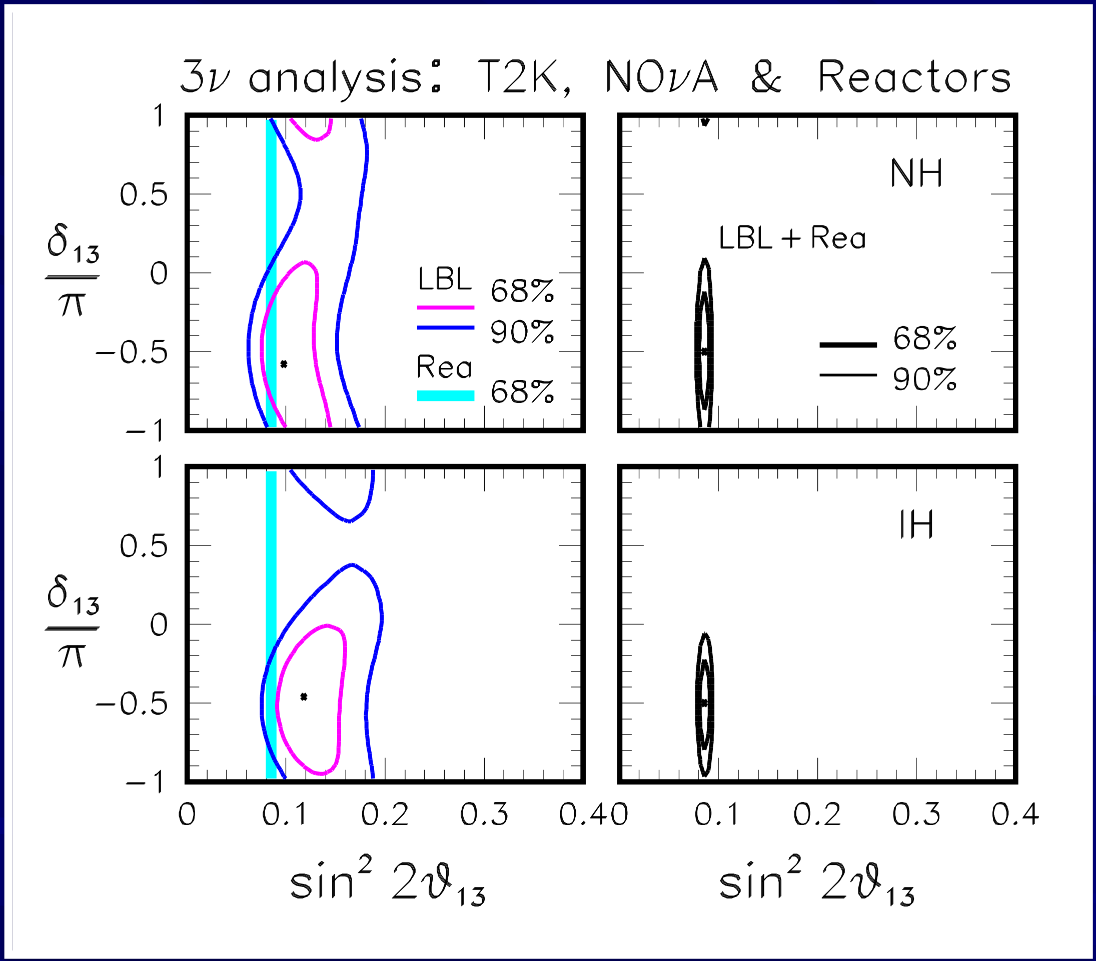
CP violation

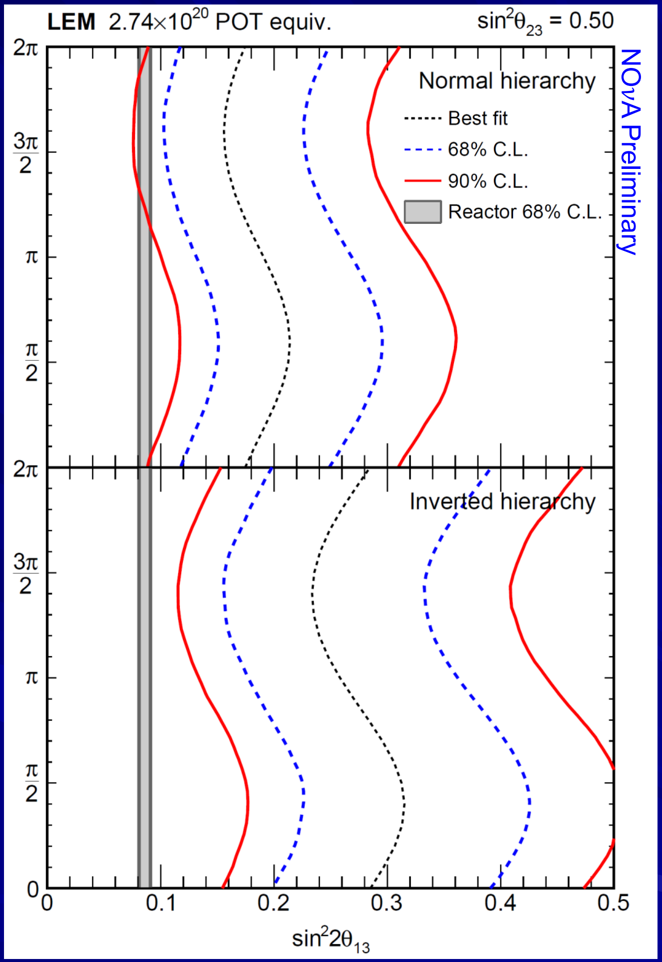
$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = {} & 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \\
& + 8c_{13}^2 s_{13} s_{23} c_{23} s_{12} c_{12} \sin \Delta_{31} \left[\cos \Delta_{32} \cos \delta - \sin \Delta_{32} \sin \delta\right] \sin \Delta_{21} \\
& - 8c_{13}^2 s_{13}^2 s_{23}^2 s_{12}^2 \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \\
& + 4c_{13}^2 s_{12}^2 \left[c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2 c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta\right] \sin^2 \Delta_{21} \\
& - 8c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2) \frac{aL}{4E_\nu} \sin \Delta_{31} \left[\cos \Delta_{32} - \frac{\sin \Delta_{31}}{\Delta_{31}}\right] .
\end{aligned}
$$

matter

Possible way of measuring mass hierarchy and CP phase

T2K, Nova new data fitting





arXiv:1509.03148
A. Palazzo

Fermilab seminar, August 6 2015
R. Patterson

Model Building with $\theta_{23}=\pi/4$ and $\delta_{CP} = 3\pi/2(-\pi/2)$

Structure of the mass matrix (charged lepton is already diagonal)

Assuming neutrinos are Majorana particles,

$$L = -\frac{1}{2}\bar{\nu}_L m_\nu \nu_L^C$$
$$m_\nu = V_{PMNS}\hat{m}_\nu V_{PMNS}^T \ , \qquad .$$

$\hat{m}_\nu = diag(m_1, m_2, m_3)$ with $m_i = |m_i| exp(i\alpha_i)$.

With $\delta = -\pi/2$ and $\theta_{23} = \pi/4$,
m_ν has the following form

$$m_\nu = \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} ,$$

$$a = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2 \ ,$$
$$\tilde{b} = -\frac{1}{2}\left(m_1(s_{12}^2 + c_{12}^2 s_{13}^2) + m_2(c_{12}^2 + s_{12}^2 s_{13}^2) - m_3 c_{13}^2\right) \ ,$$
$$c = -\frac{1}{\sqrt{2}}(m_1 - m_2) s_{12} c_{12} c_{13} \ ,$$
$$d = \frac{1}{2}\left(m_1(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2(c_{12}^2 - s_{12}^2 s_{13}^2) + m_3 c_{13}^2\right) \ ,$$
$$\beta = \frac{1}{\sqrt{2}} s_{13} c_{13}\left(m_1 c_{12}^2 + m_2 s_{12}^2 + m_3\right) \ ,$$
$$\gamma = -(m_1 - m_2) s_{12} c_{12} s_{13} \ .$$

Note that in the most general case, because non-zero Majorana phases, the parameters a, $\tilde{b}$, c, d, β and γ are all complex.

Equavilent forms

$$m_\nu = \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix} \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix},$$

where the phases p_i are arbitrary.

For $p_1 = p_2 = 0$ and $p_3 = \pi$,

$$m_\nu = \begin{pmatrix} a & c+i\beta & (c-i\beta) \\ c+i\beta & d+i\gamma & b \\ (c-i\beta) & b & d-i\gamma \end{pmatrix},$$

eq A

where $b = -\tilde{b}$.

If neutrinos do not have non-trivial Majorana phases, α, β, γ are real

The mass matrix can be written asand

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}.$$

eq B

Mass matrix in forms eq A and eq B

always predict $\theta_{23}=\pi/4$ and $\delta_{CP}=3\pi/2$?

The answer is no!

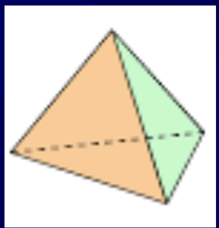
For both is change $\delta_{CP}=\pi/2$, the mass matrix has the same form.

If α, β, γ are complex, θ_{23} and δ_{CP} may not be $\pi/4$ and $\pm\pi/2$.

This also points out a method to modify the solutions of $\theta_{23}=\pi/4$ and $\delta_{CP}=3\pi/2$

Need to be careful!

Realization with A$_4$ symmetry



How to achieve in A$_4$ models?

A_4 group is defined as the set of all twelve even permutations of four object.
It has three one-dimensional representations 1, $1'$ and $1''$
and one three-dimensional irreduciblerepresentation 3.
Multiplication rules

$$3 \times 3 = 3_s + 3_a + 1 + 1' + 1''$$

$$1 \times 1_i = 1_i,\ 1' \times 1' = 1'',\ 1'' \times 1'' = 1',\ 1' \times 1'' = 1$$

$$
\begin{aligned}
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_s} &= (x_2y_3 + x_3y_2\,,\ x_3y_1 + x_1y_3\,,\ x_1y_2 + x_2y_1) \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} &= (x_2y_3 - x_3y_2\,,\ x_3y_1 - x_1y_3\,,\ x_1y_2 - x_2y_1) \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} &= x_1y_1 + x_2y_2 + x_3y_3, \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} &= x_1y_1 + \omega\, x_2y_2 + \omega^2\, x_3y_3, \\
(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} &= x_1y_1 + \omega^2\, x_2y_2 + \omega\, x_3y_3,
\end{aligned}
$$

Some general properties

(X-G He, Chin. J. Phys 53, 100101(2015); X-G He and G-N Li, Phys. Lett. B750,620(2015); E Ma, Phys. Rev. D92, 051301(2015).

Assuming that the charged lepton mass matrix M_l is diagonalized from left by U_l ,

$$M_l = U_l \hat{m}_l U_r \ , \quad U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \ ,$$

where $\omega = exp(i2\pi/3)$ and $\omega^2 = exp(i4\pi/3)$.

A_4 models usually have the above characteristic U_i.

U_r is a unitary matrix, but does not play a role in determining V_{PMNS}.

If neutrinos are Majorana particles, the most general mass matrix is

$$M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} \ ,$$

In the basis where charged lepton is digonalized,
the neutrino mass matrix is of the form given by $U_l^\dagger M_\nu U_l^*$ with

$$A_{11} = \frac{1}{3}(w_1 + w_2 + w_3 + 2(x + y + z)) ,$$
$$A_{22} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 + 2(\omega^2 x + \omega y + z)) ,$$
$$A_{33} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 + 2(\omega x + \omega^2 y + z)) ,$$
$$A_{12} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 - \omega x - \omega^2 y - z) ,$$
$$A_{13} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 - \omega^2 x - \omega y - z) ,$$
$$A_{23} = \frac{1}{3}(w_1 + w_2 + w_3 - (x + y + z)) .$$

If the parameter set $P = (w_1, w_2, w_3, x, y, z)$ is real, then

$$A_{11} = A_{11}^* \;\; A_{23} = A_{23}^*$$

$$A_{22} = A_{33}^* \;\; A_{12} = A_{13}^*$$

The mass matrix is of the form given by eq B, $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$

X-G He, Chin. J Phys. 53, 100101(2015)

Diagonalizing the mass matrices, we have

$$V_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\rho} & 1 & ce^{i\rho} - s \\ c + \omega se^{i\rho} & \omega^2 & \omega ce^{i\rho} - s \\ c + \omega^2 se^{i\rho} & \omega & \omega^2 ce^{i\rho} - s \end{pmatrix} ,$$

$\tan\rho = Im(yw_1^* + y^* w_3)/Re(yw_1^* + y^* w_3)$,
$s = \sin\theta$ and $c = \cos\theta$,

$$\tan 2\theta = \frac{2|yw_1^* + w_3 y^*|}{|w_1|^2 - |w_3|^2} .$$

Majorana phases α_i of m_i

$$\alpha_{1,3} = Arg(w_i(1 \pm \cos 2\theta) + w_2 e^{-i2\rho}(1 \mp \cos 2\theta) \pm 2\sin 2\theta y e^{-i\rho} , \quad \alpha_2 = Arg(w_2)$$

Translate into standard parameterization

$$s_{12} = \frac{1}{\sqrt{2}(1 + cs\cos\rho)^{1/2}} , \; s_{23} = \frac{(1 + cs\cos\rho + \sqrt{3}cs\sin\rho)^{1/2}}{\sqrt{2}(1 + cs\cos\rho)^{\frac{1}{2}}} ,$$

$$s_{13} = \frac{(1 - 2cs\cos\rho)^{1/2}}{\sqrt{3}} .$$

and

$$\sin\delta = \left(1 + \frac{4c^2 s^2 \sin^2\rho}{(c^2 - s^2)^2}\right)^{-1/2} \left(1 - \frac{3c^2 s^2 \sin^2\rho}{(1 + cs\cos\rho)^2}\right)^{-1/2} \times \begin{cases} -1 , & \text{if } c^2 > s^2 , \\ +1 , & \text{if } s^2 > c^2 . \end{cases}$$

It is clearly that if $\sin\rho$ is not zero,
$|\delta|$ and θ_{23} deviate from $\pi/2$ and $\pi/4$, respectively.
In the limit ρ goes to zero, that real parameter set P
$\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$.

Predictions: $|V_{i2}| = 1/\sqrt{3}$

$J = Im(V_{11}V^*_{12}V^*_{21}V_{22}) = (s^2 - c^2)/6\sqrt{3}$

independent of ρ

New model building guideline 2015

First order: θ_{23} =$\pi/4$ and δ_{CP}=3π /2, A non-zero θ_{13}.

A_4 family symmetry model provide an example to fully realize such mixing pattern. This class of A_4 models also provide direction for modifying the pattern, with complex Yukawa coefficients.

New experimental data will provide more clue about what the mixing pattern is and how theoretical model should be constructed.

A lot more for theoretical neutrino models

Neutrino mass hierarchy, (JUNO….)

Dirac or Majorana (Neutrinoless double beta decay, INPAC…)

Sterile neutrinos LSND&MiniBoon, seem still alive

Lepton number violating FCNC connection

Cosmological and Astrophysical connection

Dark matter connection,

Leptogenesis connection.

…