Hadron Properties in a class of holographic models

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Main results can be found in:
Holography, chiral Lagrangian and form factor relations, JHEP 1211 (2012) 012;
Low-energy photon and pion scattering in holographic QCD, JHEP 1306 (2013) 020;
Large-distance properties of holographic baryons, Nucl.Phys. B875 (2013) 351-367.

Extension of Hidden local symmetry

(Son & Stephanov '03)

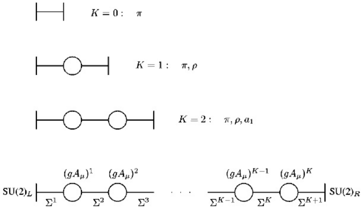
$$\mathcal{L} = \sum_{k=1}^{K+1} f_k^2 \mathrm{Tr}|D_\mu \Sigma^k|^2 - \sum_{k=1}^{K} \frac{1}{2g_k^2} \mathrm{Tr}\left(F_{\mu\nu}^k\right)^2 .$$

$$\Sigma^k = \exp\left\{ i\Pi \frac{f_\pi}{2f_k^2} \right\}, \quad \Pi = \pi^a \frac{\tau^a}{2},$$

$$U = \Sigma^1 \Sigma^2 \cdots \Sigma^{K+1}$$

$$U \to LUR^\dagger .$$

$$U = \exp\left\{ \frac{2i\Pi}{f_\pi} \right\}, \quad \frac{4}{f_\pi^2} = \sum_{k=1}^{K+1} \frac{1}{f_k^2} .$$



$K = 0$ Weinberg '79;
$K = 1$ Bando et. al, '85;
$K = 2$ Bando et. al, '88.

Large N_C and continuum limit

- When $K \to \infty$,

$$U(x^\mu) = \mathrm{P}\exp\left\{ i\int_{-z_0}^{+z_0} \mathcal{A}_z(x^\mu, z')dz' \right\},$$

  The relation is first proposed by Atiyah and Manton ('89), to connect Skyrmions and instanton solutions.
- 5D Yang-Mills action:

$$S_{\rm YM} = -\int d^5x \mathrm{tr}\left[-f^2(z)\mathcal{F}_{z\mu}^2 + \frac{1}{2g^2(z)}\mathcal{F}_{\mu\nu}^2 \right].$$

- The topological 5d Chern-Simons term provides a natural mechanism for $\mathrm{U}(1)_A$ symmetry broken and other anomalous processes:

$$S_{\rm CS} = -\kappa \int \mathrm{tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5 \right], \quad \kappa = \frac{N_c}{24\pi^2}.$$

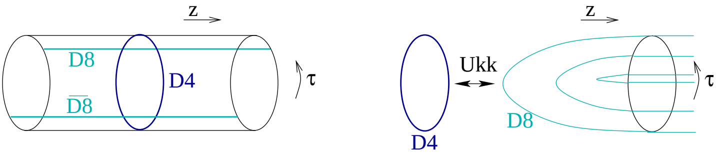
- Baryon number

$$\begin{aligned} Q &= \frac{1}{32\pi^2}\int dz\, d^3x\, \epsilon^{0\mu\nu\lambda\rho}\mathrm{Tr}\mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\rho} \\ &= \frac{i}{24\pi^2}\int d^3x\, \epsilon^{ijk}\mathrm{Tr}[(U^{-1}\partial_i U)(U^{-1}\partial_j U)(U^{-1}\partial_k U)]. \end{aligned}$$

String construction: Sakai-Sugimoto model

D-branes configuration (Sakai & Sugimoto '04):

	0	1	2	3	(4)	5	6	7	8	9
D4	X	X	X	X	X					
D8-$\overline{\text{D8}}$	X	X	X	X		X	X	X	X	X



- YM+CS action from the low energy action on the *D*8 branes.
- Baryons from D4 branes wrapping around S^4, and can be realized as the instanton configuration on the D8 brane.

Order $O(p^2)$ and $O(p^4)$ chiral Lagrangian

- Even parity (Sakai & Sugimoto '04, Hirn & Sanz '05):

$$
\begin{aligned}
S_{\rm YM} \quad \supset \quad \int d^4x \Bigg[& \frac{f_\pi^2}{4} < u_\mu u^\mu > \\
& + L_1 < u_\mu u^\mu >^2 + L_2 < u_\mu u_\nu >< u^\mu u^\nu > + L_3 < u_\mu u^\mu u_\nu u_\nu > \\
& - i L_9 < f_{+\mu\nu} u^\mu u^\nu > + \frac{L_{10}}{4} < f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} > \\
& + \frac{H_1}{2} < f_{+\mu\nu} f_+^{\mu\nu} + f_{-\mu\nu} f_-^{\mu\nu} > \Bigg],
\end{aligned}
$$

$$
U = u^2, \quad u_\mu = i \left\{ u^\dagger (\partial_\mu - i r_\mu)\, u - u\, (\partial_\mu - i l_\mu)\, u^\dagger \right\} .
$$

- Odd parity (Sakai & Sugimoto '05):

$$
S_{\rm CS} \supset S_{\rm WZW} = -\frac{iN_C}{48\pi^2} \int_{M^4} \mathcal{Z}(u, l_\mu, r_\mu) - \frac{iN_C}{240\pi^2} \int_{M^4 \times \mathbb{R}} {\rm tr}(g {\rm d} g^{-1})^5 .
$$

$$
g^{-1}(x,z) = P \exp \left\{ i \int_0^z dz'\, \mathcal{A}_z(x,z') \right\} .
$$

$$
\xi_{R(L)}(x) \equiv g^{-1}(x, \pm\infty), \quad \xi_R(x) = \xi_L(x)^\dagger = u .
$$

Alternative derivation with chirally delocalized quarks (Hill '06)

Low energy constants up to $O(p^4)$

- $$S_{\rm YM} \supset S_2^{\chi\rm PT}[\pi] + S_4^{\chi\rm PT}[\pi],$$
$$f_\pi^2 = 4\left(\int_{-z_0}^{z_0} \frac{\mathrm{d}z}{f^2(z)}\right)^{-1}, \qquad \psi_0(z) = \frac{f_\pi^2}{2}\int_0^z \frac{1}{f^2(z)}\mathrm{d}z\,,$$
$$L_1 = \frac{1}{2}L_2 = -\frac{1}{6}L_3 = \frac{1}{32}\int_{-z_0}^{z_0} \frac{(1-\psi_0^2)^2}{g^2(z)}\,\mathrm{d}z\,,$$
$$L_9 = -L_{10} = \frac{1}{4}\int_{-z_0}^{z_0} \frac{1-\psi_0^2}{g^2(z)}\,\mathrm{d}z\,, \qquad H_1 = -\frac{1}{8}\int_{-z_0}^{z_0} \frac{1+\psi_0^2}{g^2(z)}\,\mathrm{d}z\,.$$

-

	flat	Cosh	hard-wall	Sakai-Sugimoto	χPT
$10^3\ L_1$	0.5	0.5	0.5	0.5	0.9 ± 0.3
$10^3\ L_2$	1.0	1.0	1.0	1.0	1.7 ± 0.7
$10^3\ L_3$	-3.1	-3.2	-3.1	-3.1	-4.4 ± 2.5
$10^3\ L_9$	5.2	6.3	6.8	7.7	7.4 ± 0.7
$10^3\ L_{10}$	-5.2	-6.3	-6.8	-7.7	-6.0 ± 0.7

Skyrme model and Skyrme parameter

- Skyrme model (Skyrme '61 & '62)

$$S = \int d^4x \left(-\frac{f_\pi^2}{4} \mathrm{tr} \left(U^{-1}\partial_\mu U \right)^2 + \frac{1}{32e_S^2} \mathrm{tr} \left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U \right]^2 \right) ,$$

$$e_S^{-2} \quad = \quad 32\, L_1 = 16 L_2 = -\frac{16}{3} L_3 = \int \mathrm{d}z \frac{(1-\psi_0^2(z))^2}{g^2(z)} .$$

- Skyrme parameter：

$$\sum_{n=1}^{\infty} \frac{g_{v^n\pi\pi}^2}{m_{v^n}^2} = \frac{1}{3f_\pi^2}, \qquad \sum_{n=1}^{\infty} \frac{g_{v^n\pi\pi}^2}{m_{v^n}^2} = \frac{1}{4e_S^2 f_\pi^4} ,$$

ρ-dominiance valid at about 2% level，so

$$e_S^2 \sim 3m_\rho^2/4f_\pi^2 \sim 7.7^2$$

to compare with $\pi\pi$ scattering results($\sim 7.4^2$), and the value used in Skyrme model calculation (5.45^2, Adkins, Nappi and Witten '83).

Anomaly matching

- VVA correlator (one vector current carries a soft momentum)

$$i\int d^4x\, e^{iqx}\langle j^a_\mu(x) j^{5b}_\nu(0)\rangle_{\hat F} = \frac{Q^2}{8\pi^2} d^{ab} P^{\alpha\perp}_\mu \left[P^{\beta\perp}_\nu w_T(q^2) + P^{\beta\parallel}_\nu w_L(q^2)\right] \epsilon_{\alpha\beta\sigma\rho}\hat F^{\sigma\rho}.$$

$d^{ab} = (1/2)\mathrm{tr}(\mathcal{Q}\{t^a, t^b\})$, $P^{\alpha\perp}_\mu = \eta^\alpha_\mu - q_\mu q^\alpha/q^2$, $P^{\alpha\parallel}_\mu = q_\mu q^\alpha/q^2$.

- $m_q = 0$ (Adler, Bardeen '69; 't Hooft '80; Witten '83):

$$w_L(Q^2) = \frac{2N_c}{Q^2}.$$

- $m_q \neq 0$, OPE gives

$$w_L(Q^2) = \frac{2N_c}{Q^2} - \frac{16\pi^2\chi m_q\langle\bar q q\rangle}{Q^4} + \mathcal{O}(\frac{1}{Q^6}),$$

magnetic susceptibility $\langle\bar q\sigma_{\mu\nu}q\rangle_{\hat F} = \chi\langle\bar q q\rangle\hat F_{\mu\nu}$.

- Extrapolation of pion pole term (Vainshtein '02):

$$w_L(Q^2) \approx \frac{2N_c}{Q^2+m_\pi^2} = \frac{2N_c}{Q^2} - \frac{2N_c m_\pi^2}{Q^4} + \mathcal{O}(\frac{1}{Q^6}),$$

$$\Rightarrow \quad \chi = -\frac{N_c}{4\pi^2 f_\pi^2} \sim -9\ \mathrm{GeV}^{-2}$$

- SUM RULE, VMD (P. Ball '02) $\quad \chi \sim -3\ \mathrm{GeV}^{-2}$.

Anomaly matching

- $m_q = 0$, to all orders in α_s (Vainshtein '02, Knecht, Peris, Perrottet, de Rafael '03)

$$w_T(Q^2) = \frac{w_L(Q^2)}{2} = \frac{N_c}{Q^2}$$

Non-perturbative power corrections at large Q^2:

$$w_T(Q^2) = \frac{w_L(Q^2)}{2} + \frac{N_C^2 - 1}{N_C^2} \frac{16\pi^3\alpha_s}{Q^6} \chi \langle \bar{q}q \rangle^2 + \mathcal{O}(\frac{1}{Q^8})$$

-

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} \log\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{24\pi} \frac{\alpha_s \langle G_{\mu\nu} G^{\mu\nu} \rangle}{Q^4} - \frac{14}{9} \frac{N_C^2 - 1}{N_C^2} \frac{\pi\alpha_s \langle \bar{q}q \rangle^2}{Q^6} + \ldots$$

$$\Pi_A(Q^2) = -\frac{N_c}{24\pi^2} \log\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{24\pi} \frac{\alpha_s \langle G_{\mu\nu} G^{\mu\nu} \rangle}{Q^4} + \frac{22}{9} \frac{N_C^2 - 1}{N_C^2} \frac{\pi\alpha_s \langle \bar{q}q \rangle^2}{Q^6} + \ldots$$

-

$$\Pi_V(Q^2) - \Pi_A(Q^2) = -\frac{N_C^2 - 1}{N_C^2} \frac{4\pi\alpha_s}{Q^6} \langle \bar{q}q \rangle^2 + \mathcal{O}(\frac{1}{Q^8}).$$

Anomaly matching

- Holographic relation (Son & Yamamoto '10):

$$\begin{aligned} w_T(Q^2) &= \frac{w_L(Q^2)}{2} + \frac{N_C}{f_\pi^2}\left[\Pi_V(Q^2) - \Pi_A(Q^2)\right] \\ w_T(Q^2) &= \frac{N_C}{Q^2}\int_{-z_0}^{z_0} \mathrm{d}z\, A(Q,z)\partial_z V(Q,z), \quad w_L(Q^2) = \frac{2N_c}{Q^2}, \\ \Pi_V(Q^2) &= \frac{1}{Q^2} f^2(z) V(Q,z)\partial_z V(Q,z)\Big|_{z=-z_0}^{z=+z_0}, \\ \Pi_A(Q^2) &= \frac{1}{Q^2} f^2(z) A(Q,z)\partial_z A(Q,z)\Big|_{z=-z_0}^{z=+z_0}. \end{aligned}$$

- Extrapolating to large Q^2 $\Rightarrow$ $\chi = -\frac{N_c}{4\pi^2 f_\pi^2}$.
- At low energy, it gives (Knecht et. al '11)

$$C_{22}^W = -\frac{N_C}{32\pi^2 f_\pi^2} L_{10}.$$

$$O_{22}^W \equiv \epsilon^{\mu\nu\alpha\beta}\langle u_\mu \left\{\nabla_\gamma f_{+\gamma\nu}, f_{+\alpha\beta}\right\}\rangle, \quad O_{10} =< f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} > .$$

- $$C_{22}^W \sim (6-8)\cdot 10^{-3}\ \mathrm{GeV}^{-2}, \quad -\frac{N_C}{32\pi^2 f_\pi^2} L_{10} \sim 6.7\cdot 10^{-3}\ \mathrm{GeV}^{-2}.$$

Resonance Lagrangian

- $$\begin{aligned}
S_{\rm YM}\Big|_{\rm Kin.} &= \int d^4x \langle -\frac{1}{2}(\nabla_\mu v^n_\nu - \nabla_\nu v^n_\mu)^2 + m^2_{v^n} v^{n\,2}_\mu - \frac{1}{2}(\nabla_\mu a^n_\nu - \nabla_\nu a^n_\mu)^2 + m^2_{a^n} a^{n\,2}_\mu \rangle, \\
S_{\rm YM}\Big|_{1-{\rm res.}} &= \int d^4x \Big\{ -\langle \frac{f^{\mu\nu}_+}{2}\Big[(\nabla_\mu v^n_\nu - \nabla_\nu v^n_\mu)a_{Vv^n} - \frac{i}{2}([u_\mu, a^n_\nu] - [u_\nu, a^n_\mu])a_{Aa^n}\Big]\rangle \\
&\quad - \langle \frac{i}{4}[u^\mu, u^\nu]\Big[(\nabla_\mu v^n_\nu - \nabla_\nu v^n_\mu)b_{v^n\pi\pi} - \frac{i}{2}([u_\mu, a^n_\nu] - [u_\nu, a^n_\mu])b_{a^n\pi^3}]\rangle \\
&\quad + \langle \frac{f^{\mu\nu}_-}{2}\Big[(\nabla_\mu a^n_\nu - \nabla_\nu a^n_\mu)a_{Aa^n} - \frac{i}{2}([u_\mu, v^n_\nu] - [u_\nu, v^n_\mu])(a_{Vv^n} - b_{v^n\pi\pi})\Big]\rangle \Big\}, \\
S_{\rm CS}\Big|_{1-{\rm res.}} &= \int d^4x \Big\{ -\frac{N_C}{32\pi^2} c_{v^n}\epsilon^{\mu\nu\alpha\beta}\langle u_\mu\{v^n_\nu, f_{+\alpha\beta}\}\rangle + \frac{N_C}{64\pi^2} c_{a^n}\epsilon^{\mu\nu\alpha\beta}\langle u_\mu\{a^n_\nu, f_{-\alpha\beta}\}\rangle \\
&\quad + \frac{iN_C}{16\pi^2}(c_{v^n} - d_{v^n})\epsilon^{\mu\nu\alpha\beta}\langle v^n_\mu u_\nu u_\alpha u_\beta\rangle \Big\}.
\end{aligned}$$

- Anomaly matching for resonance couplings:

$$\begin{aligned}
c_{v^n} &= g_{v^n\pi\pi} \equiv \frac{m^2_{v^n}}{2f^2_\pi} b_{v^n\pi\pi}, \qquad c_{a^n} = g_{a^n\pi^3} \equiv \frac{m^2_{a^n}}{3f^2_\pi} b_{a^n\pi^3}, \\
d_{v^n} &= \frac{m^2_{v^n}}{12f^2_\pi}\int_{-z_0}^{+z_0} \frac{\psi_{2n-1}(1-\psi_0^4)}{g^2(z)} dz.
\end{aligned}$$

$$c_\omega|_{\rm exp} \sim 5.80, \qquad g_{\rho\pi\pi}|_{\rm exp} \sim 5.99.$$

Vector meson dominance

	Flat	Cosh	Hard-wall	Sakai-Sugimoto
VV-AA correlator				
$1-\frac{a_{V\rho}^2 m_\rho^2 - a_{Aa_1}^2 m_{a_1}^2}{\Sigma_n(a_{Vv^n}^2 m_{v^n}^2 - a_{Aa^n}^2 m_{a^n}^2)}$	99 %	300 %	350 %	840 %
$1-\frac{a_{V\rho}^2 - a_{Aa_1}^2}{\sum_n(a_{Vv^n}^2 - a_{Aa^n}^2)}$	8 %	34 %	41 %	110 %
$\gamma^* \to \pi\pi$				
$1-\frac{a_{V\rho} g_{\rho\pi\pi}}{\Sigma_n a_{Vv^n} g_{v^n\pi\pi}}$	18 %	0	-11 %	-30 %
$1-\frac{a_{V\rho} g_{\rho\pi\pi}/m_\rho^2}{\Sigma_n a_{Vv^n} g_{v^n\pi\pi}/m_{v^n}^2}$	0.8 %	0	-2 %	-6 %
$\gamma^*\pi \to \pi\pi\pi$				
$1-\frac{c_\rho(c_\rho - d_\rho)/m_\rho^2}{\Sigma_n c_{v^n}(c_{v^n} - d_{v^n})/m_{v^n}^2}$	2 %	0	1.5 %	4 %
$1-\frac{c_\rho(c_\rho - d_\rho)/m_\rho^4}{\Sigma_n c_{v^n}(c_{v^n} - d_{v^n})/m_{v^n}^4}$	0.3 %	0	0.3 %	1.4 %
$\gamma^* \to \pi\pi\pi$				
$1-\frac{a_{V\rho}(c_\rho - d_\rho)}{\Sigma_n a_{Vv^n}(c_{v^n} - d_{v^n})}$	- 30 %	- 20 %	-30 %	-50 %
$1-\frac{a_{V\rho}(c_\rho - d_\rho)/m_\rho^2}{\Sigma_n a_{Vv^n}(c_{v^n} - d_{v^n})/m_{v^n}^2}$	- 5 %	- 3 %	-5 %	-10 %

$\mathcal{O}(p^6)$ low energy constants: derivation

- $$C_{22}^W = \frac{N_C}{64\pi^2}\sum_{n=1}^{\infty}\frac{a_{V_{V^n}}\cdot c_{V^n}}{m_{V^n}^2} = \frac{N_C}{128\pi^2 f_\pi^2}\sum_{n=1}^{\infty} a_{V_{V^n}}\cdot b_{V^n\pi\pi}$$
$$= \frac{N_C}{128\pi^2 f_\pi^2}\sum_{n=1}^{\infty}\int_{-z_0}^{z_0}\frac{\psi_{2n-1}}{g^2(z)}\mathrm{d}z\cdot\int_{-z_0}^{z_0}\frac{\psi_{2n-1}(1-\psi_0^2)}{g^2(z')}\mathrm{d}z'$$
$$= \frac{N_C}{128\pi^2 f_\pi^2}\int_{-z_0}^{z_0}\frac{1-\psi_0^2}{g^2(z)}\mathrm{d}z = -\frac{N_C}{32\pi^2 f_\pi^2}L_{10}$$
$$\sum_{n=1}^{\infty}\frac{1}{g^2(z)}\psi_n(z)\psi_n(z') = \delta(z-z')\,.$$

- $$C_{52} = -\frac{1}{8}S_{V\pi\pi} - 2\,(\frac{N_c}{32\pi^2})^2\,S_{V\pi^4}\,,$$
$$S_{V\pi\pi} = \sum_{n=1}^{\infty}\frac{a_{V_{V^n}}\cdot b_{V^n\pi\pi}}{m_{V^n}^2} = \int_{-z_0}^{z_0}\int_{-z_0}^{z_0} G(0;z,z')(1-\psi_0^2)\mathrm{d}z\mathrm{d}z'\,,$$
$$S_{V\pi^4} = \sum_{n=1}^{\infty}\frac{c_{V^n}\cdot(c_{V^n}-d_{V^n})}{m_{V^n}^2} = \frac{1}{2f_\pi^2}\sum_{n=1}^{\infty} b_{V^n\pi\pi}\cdot(c_{V^n}-d_{V^n}) = \frac{4}{15 f_\pi^2}\,.$$
$$G(0;z,z') = \sum_{n=1}^{\infty}\frac{\psi_n(z)\psi_n(z')}{m_n^2} = \frac{1}{f_\pi^2}\left[(1-\psi_0(z))(1+\psi_0(z'))\theta(z-z') + (z\leftrightarrow z')\right]\,.$$

$\mathcal{O}(p^6)$ low energy constants: a few results

- $\pi\pi$ scattering

$$L_1 = \frac{1}{2}L_2 = -\frac{1}{6}L_3, \qquad C_1 + 4C_3 = 3C_3 + C_4.$$

- $\pi \to l\nu\gamma$ decay: $G_A(q^2) = 0 \quad \Rightarrow$

$$L_9 + L_{10} = 0, \qquad 2C_{78} - 4C_{87} + C_{88} = 0.$$

- $\gamma\gamma \to \pi\pi$ scattering

$$\begin{aligned}
a_2^{00} &= 256\pi^4\ f_\pi^2\ (8C_{53} + 8C_{55} + C_{56} + C_{57} + 2C_{59})\,, \\
b^{00} &= -128\pi^4\ f_\pi^2\ (C_{56} + C_{57} + 2C_{59})\,, \\
a_2^{+-} &= 256\pi^4\ f_\pi^2\ (8C_{53} - 8C_{55} + C_{56} + C_{57} - 2C_{59} + 4C_{78} + 8C_{87} - 4C_{88})\,, \\
b^{+-} &= -128\pi^4\ f_\pi^2\ (C_{56} + C_{57} - 2C_{59} - 4C_{78})\,.
\end{aligned}$$

	Holo.	DSE	Reso. Lagr.	ENJL
a_2^{00}	N_C^2	3.79	13 ± 3.3	14.0
b^{00}	$N_C^2/6$	1.66	3 ± 1	1.66
a_2^{+-}	0	-0.98	0.75 ± 0.65	6.7
b^{+-}	0	-0.23	0.45 ± 0.15	0.38

$\gamma^* \to \pi\pi$ vs. $\pi^0 \to \gamma\gamma^*$ form factor

-

$$\mathcal{F}_\pi(Q^2) = \frac{1}{f_\pi^2}\int_{-z_0}^{z_0} f^2(z) V(Q,z)\psi_0'(z)^2 \mathrm{d}z.$$

$$\mathcal{F}_{\gamma\pi}(Q^2) = \frac{N_C}{24\pi^2 f_\pi}\int_{-z_0}^{z_0} V(Q,z)\psi_0'(z)\mathrm{d}z.$$

$$\mathcal{F}_{\gamma\pi}(Q^2) = \frac{N_C}{12\pi^2 f_\pi}\,\mathcal{F}_\pi(Q^2).$$

- Previously found in hard wall model (Grigoryan & Radyushkin '08) and Sakai-Sugimoto model (Stoffers & Zahed '11).
- Low energy limit:

$$C_{22}^W = \frac{N_C}{32\pi^2 f_\pi^2} L_9.$$

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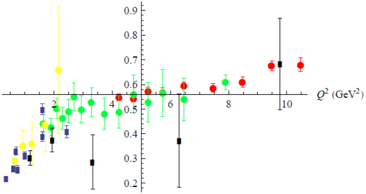
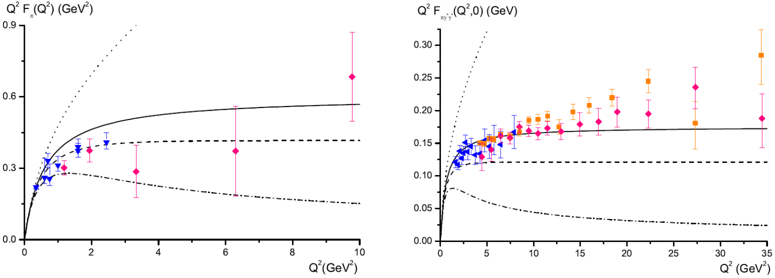


- $\frac{12\pi^2 f_\pi Q^2}{N_C}\mathcal{F}_{\gamma\pi}(Q^2)$ (circles) and $Q^2\ \mathcal{F}_\pi(Q^2)$ (squares) versus Q^2, figure taken from (Stoffers & Zahed '11) .

$$C_{22}^W \sim (6{-}8)\cdot 10^{-3}\ \mathrm{GeV}^{-2}, \quad \frac{N_C}{32\pi^2 f_\pi^2} L_9 \sim 8.3\cdot 10^{-3}\ \mathrm{GeV}^{-2}.$$

$\gamma^* \to \pi\pi$ vs. $\pi^0 \to \gamma\gamma^*$ form factor



Ultraviolet classification and constraint

- Special coordinate:

$$\begin{aligned} y &= \psi_0(z) = \frac{f_\pi^2}{2}\int_0^z \frac{1}{f^2(z)}\mathrm{d}z \ , \\ \tilde{f}^2(y) &= \frac{f_\pi^2}{2}, \\ \tilde{g}^2(y) &= \frac{f_\pi^2}{2}\frac{g^2(z(y))}{f^2(z(y))}, \end{aligned}$$

- Characteristic index in the ultraviolet ($y \to \pm 1$) :

$$\tilde{g}^2(y) \sim (1-y^2)^\alpha$$

  - flat model: $\alpha = 0$
  - asymptotic anti-de Sitter background (“cosh” and hard-wall model): $\alpha = 1$
  - Sakai-Sugimoto model: $\alpha = 4/3$
- Ultraviolet constraint ($L_1, \cdots, L_{10} < \infty$, $H_1 \to \infty$)

$$1 \le \alpha < 2.$$

Low energy π and photon scattering

- $\pi^+\pi^- \to \pi^0\pi^0$ process

$$A(s,t,u) \;=\; \frac{(t+2s)}{4}\, h(-t) \;+\; (t \leftrightarrow u)\,.$$

- $\gamma\gamma \to \pi^0\pi^0$ process

$$A(s,t,u)^{\gamma\gamma\to\pi^0\pi^0} \;=\; \frac{a_2}{(4\pi f_\pi)^4}\left[\frac{(s-4t)}{8}\, f_\pi^2 h(-t) \;+\; (t \leftrightarrow u)\right],$$

$$B(s,t,u)^{\gamma\gamma\to\pi^0\pi^0} \;=\; \frac{b}{(4\pi f_\pi)^4}\left[\frac{3}{8}\, f_\pi^2 h(-t) \;+\; (t \leftrightarrow u)\right].$$

- $\gamma \to \pi^+\pi^-\pi^0$ process

$$\mathcal{F}^{3\pi}(0,s,t) \;=\; \mathcal{F}_0^{3\pi} \times \frac{f_\pi^2}{4}\left[h(-s) + h(-t) + h(-u)\right].$$

-

$$h(Q^2) \;=\; \int_{-z_0}^{z_0} dz \int_{-z_0}^{z_0} dz'\, \psi_0'(z)\psi_0'(z')\, G(Q^2;z,z')\,,$$

$$\;=\; \sum_n \frac{4g^2_{v^n\pi\pi}}{m^2_{v^n}+Q^2} \quad \xrightarrow{Q^2\to\infty} \quad \frac{4\mathcal{H}}{Q^2}\,.$$

Asymptotic instanton solutions at large distance

- Skyrmion (hedgehog):

$$U(\vec{x}) = \exp(i\vec{\tau}\cdot\hat{r}F(r)), \quad F(r) \sim \frac{\beta}{r^2} - \frac{\beta^3}{21r^6}, \quad r \to \infty.$$

- Holographic instanton:
  - Cylindrical ansatz (Witten '79):

$$\bar{A}^a_j = \frac{\phi_2 + 1}{r^2}\epsilon_{jak}x_k + \frac{\phi_1}{r^3}[\delta_{ja}r^2 - x_j x_a] + A_r \frac{x_j x_a}{r^2},$$

$$\bar{A}^a_z = A_z \frac{x^a}{r}, \quad \hat{\bar{A}}_0 = s.$$

  - Asymptotic solutions with special coordinate y $(\tilde{f}^2(y) = f_\pi^2/2)$:

$$\phi_1 \sim \frac{\beta^3 y(y^2-1)}{3r^6}, \qquad \phi_2 \sim -1 + \frac{\beta^2(1-y^2)}{2r^4},$$

$$A_z \sim \frac{\beta}{r^2} - \frac{\beta^3}{21r^6}, \qquad A_r \sim \mathcal{O}(r^{-9}),$$

$$s \sim \frac{\beta^3}{f_\pi^2}\frac{(y^2-1)(y^2-5)}{r^9}.$$

Large-distance behavior of baryon form factors

- Electromagnetic form factors：

$$\tilde{G}_E^{I=0}(r) \to \frac{3^3}{2^{10}\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{1}{r^9}, \quad \tilde{G}_M^{I=0}(r) \to \frac{3\Delta}{2^{10}\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{1}{r^7},$$

$$\tilde{G}_E^{I=1}(r) \to \frac{\Delta}{2^5\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{1}{r^4}, \qquad \tilde{G}_M^{I=1}(r) \to \frac{1}{2^6\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{1}{r^4}.$$

$$\lim_{r\to\infty} r^2 \frac{\tilde{G}_E^{I=0}\tilde{G}_E^{I=1}}{\tilde{G}_M^{I=0}\tilde{G}_M^{I=1}} = 18.$$

- Goldberger-Treiman relation:

$$\frac{g_{\pi NN}}{M_N} = \frac{g_A}{f_\pi}.$$

- Axial form factor：

$$\tilde{G}_A(r) \to \frac{3}{7\cdot 2^6\pi^3}\frac{1}{f_\pi}\left(\frac{g_A}{f_\pi}\right)^3\frac{1}{r^7}.$$

- $\langle r^2\rangle_{E,I=1}, \langle r^2\rangle_{M,I=1}$ linearly divergent, $\langle r^2\rangle_{E,I=0}, \langle r^2\rangle_{M,I=0}$ finite, $\langle r^2\rangle_A$ quadratically divergent.

Instanton size: suppressed in Sakai-Sugimoto model

- Sakai-Sugimoto model

$$S_{\mathrm{YM}} = -\kappa \int d^5x \mathrm{tr}\left[\frac{h(z)}{2}\mathcal{F}_{\mu\nu}^2 - k(z)\mathcal{F}_{z\mu}^2\right],$$

$$\kappa = \frac{\lambda N_C}{216\pi^3}, \quad h(z) = (1+z^2)^{-1/3}, \quad k(z) = 1+z^2,$$

$$S_{\mathrm{CS}} = -\frac{N_c}{24\pi^2}\int \mathrm{tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right].$$

- (Hashimoto, Sakai & Sugimoto '08)

$$U(\rho) = 8\pi^2\kappa(1+\frac{\rho^2}{6}) + \frac{N_c^2}{40\pi^2\kappa}\frac{1}{\rho^2} + \ldots$$

- Insanton size

$$\rho_{\mathrm{cl}}^2 = \frac{N_C}{8\pi^2\kappa}\sqrt{\frac{6}{5}}$$

$$M_{I=3} - M_{I=1} \simeq 569 \text{ MeV}, \quad g_A \simeq 0.73$$

$$(M_\Delta - M_N)|_{\mathrm{exp}} \simeq 290 \text{ MeV}, \quad g_A|_{\mathrm{exp}} \simeq 1.27$$

Instanton size: an estimate using the Skyrme model

Quantity	$E_{\rm cl}$	g_A	M_N	M_Δ	$g_{\pi NN}$	$\langle r^2\rangle^{1/2}_{I=0}$	$\langle r^2\rangle^{1/2}_{M,I=0}$	μ_p	μ_n	$g_{\pi N\Delta}$	$\mu_{N\Delta}$
Skyrmion	0.83	0.31	1.11	2.23	3.88	0.31	0.48	0.58	−0.41	5.82	0.70
Instanton-1	0.84	0.47	1.05	1.89	5.62	0.32	0.57	0.87	−0.36	8.43	0.87
Instanton-2	0.94	input	1.00	1.23	14.5	0.52	0.94	2.32	−1.96	21.8	3.0
Experiment	—	1.27	0.939	1.23	13.5	0.81	0.84	2.79	−1.91	20.3	3.3

- $f_\pi = 87\,\text{MeV}$, $e = 7.70$.
- Skyrmion: numerical Skyrmion solution (Adkins, Nappi & Witten ’83)
- “Instanton-1": (Atiyah and Manton ’89) small instanton configuration with $\rho^2 = 2.11/(e^2 f_\pi^2)$;
- “Instanton-2": large instanton configuration with $\rho^2 = 5.72/(e^2 f_\pi^2)$.

Summary

- Previous holographic derivation of the χPT lagrangian is extended from $\mathcal{O}(p^4)$ to $\mathcal{O}(p^6)$. Numerical predictions for the LECs in different models are given. Model-independent relations among LECs are found.
- Some of these LEC relations are shown to follow from relations between different correlation functions, form factors or scattering amplitudes.
- Different models are classified according to the different behavior of form factors at large momentum region.
- The asymptotic instanton solutions at large distance are derived, from which the asymptotic from of various baryonic form factors are given.
- Using the Skyrme model as an approximation, large-size instanton configuration is suggested to describe the baryon properties.

Open questions

- The mass pattern is wrong, $m_n \propto n$ instead of $m_n \propto \sqrt{n}$.
- While leading power behavior in the ultraviolet is obtained in asymptotic AdS backgrounds, high power terms need to be included.
- Introduce quark mass.
- Construction of instanton solutions in curved spacetime and in presence of the Chern-Simons term with arbitrary instanton number.

Skyrmions for $1 \leq B \leq 8$