# TMD evolution at small x

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Based on the papers:
arXiv:1603.07426 JZ, 2016
arXiv:1703.06163 Bowen Xiao, Feng Yuan, JZ, 2017
arXiv:1807.00506 JZ 2018

USTC, April 2019, Hefei

# Outline:

- Background:  EIC physics
- Joint TMD and small x resummation
- Summary

# **Medium energy QCD** (12GeV-500GeV)

RHIC, Jlab, EIC, EicC

# The internal structure of nucleon and nucleus

- How are the sea quarks and gluons distributed in space and momentum inside the nucleon?

- How is proton spin decomposed into different pieces? Spin-Orbit correlation?

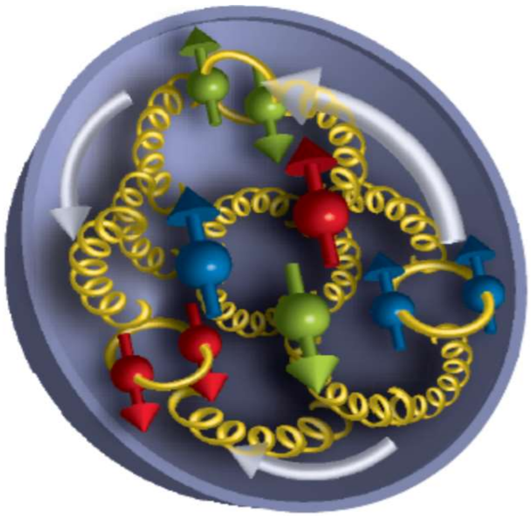
- Where does the saturation of gluon densities set in?

- Stimulate new progress of pQCD theory
- Ultimately, shed new light on QCD confinement ?

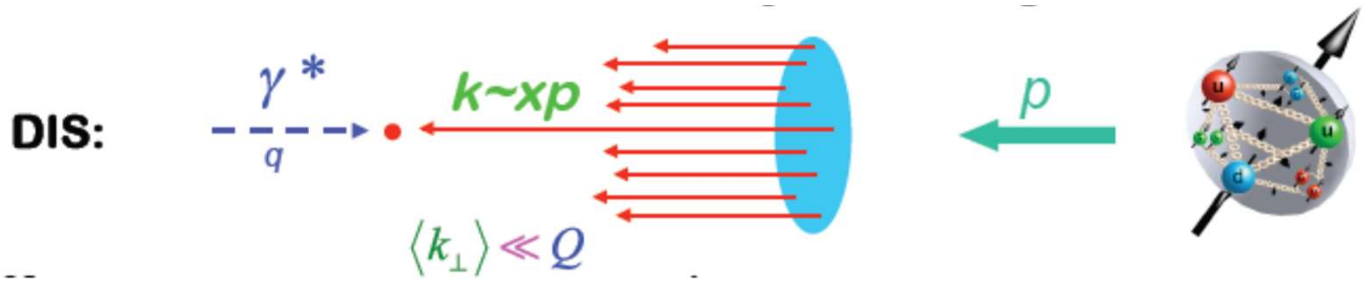
# PDFs

**The internal structure of nucleon/nuclei in terms of quark and gluon degree of freedom.**

**"Initial conditions" in ep/eA/pp/pA/AA collisions.**



## Collinear parton distributions.




**The simpliest ones: unpolarized integrated parton distributions: f(x), G(x)**

**QCD factorization theorem** tells us how to link measurable cross sections to parton distributions, in which important information on nucleon/nucleus is encoded.

# The extensions of PDFs

## 3-dimensional imaging of nucleon:

**xp, kt: TMDs SIDIS**

xp, bt: GPDs DVCS…

## 5-dimensional imaging of nucleon:

Parton Wigner distributions, GTMDs.

Measurable in High energy scatterings as well

## Nucleon spin structure:

Spin dependent PDFs, spin dependent TMDs &GPDs

## Multiple parton re-scattering:

High twist correlation functions

The study of various type parton distributions and the associated QCD factorization properties are one of main focus in the EIC physics (TMD/Spin/Small x) community.

Parton distributions not only encode information on nucleon internal structure, but also useful design for summing large logarithms in perturbation series.

$$\alpha_s \ln^2 \frac{Q^2}{k_T^2} \quad >> \quad 1$$

Parton distributions in general are not perturbative calculable.

Their change with varying x, scale, are predictable in pQCD.

$$G(x, k_\perp, \zeta_c^2 = \mu_F^2 = Q^2)$$

# QCD evolution(resummation)

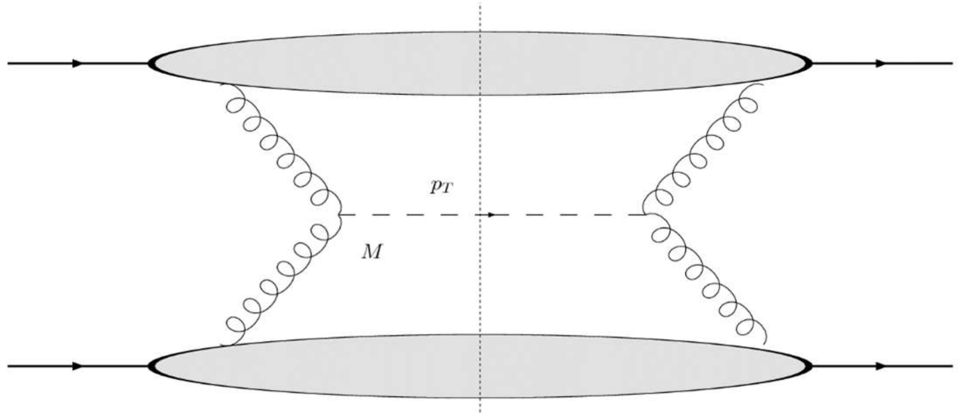
Desinged for summing large logarithms:

- DGLAP: $\ln\frac{Q^2}{\mu^2}$
- Collins-Soper: $\ln^2\frac{Q^2}{p_T^2}$
- BFKL(MQ, BK,JIMWLK): $\ln\frac{S}{Q^2}$
- Threshold resummation: $\left(\frac{\ln(1-x)}{1-x}\right)_+$
- Infrared evolution equation $\ln^2\frac{S}{Q^2}$
- …

To improve the convergence of perturbation series.

# Overlap kinematical region

# Gluon initiated Drell-Yan process




- $M^2 >> p_T^2$, TMD factorization, $\ln \frac{M^2}{p_T^2}$ resummed by Collins-Soper equation

  **1982-1983, Collins, Soper**

- $S >> M^2$, Kt factorization, $\ln \frac{S}{M^2}$ resummed by BFKL equation

  **1991, Catani, Ciafaloni and Hautmann**
  **1991, Collins and R. K. Ellis**

The overlap region $S >> M^2 >> p_T^2$

An explicit NLO cross section calculation shows that both the large logarithm appear.

**2013, Mueller, Xiao, Yuan**

Such joint resummation has been also disscussed in other literatures.

**2015, Balitsky and Tarasov; 2015 Marzani**

# Our starting point

$$xG(x,l_\perp,x\zeta)=\int\frac{dy^-d^2y_\perp}{(2\pi)^3P^+}e^{-ixP^+y^-+il_\perp\cdot y_\perp}\langle P|F_\mu^+(y^-,y_\perp)\mathcal{L}_{\tilde{n}}^\dagger(y^-,y_\perp)\mathcal{L}_{\tilde{n}}(0,0_\perp)F^{\mu+}(0)|P\rangle$$

**Gloun---->gloun splitting kernel;**

$$\mathcal{P}_{gg}(z)=2C_A\frac{(z^2-z+1)^2}{z(1-z)}$$

When z(or x) --->0, large logarithm $\ln\frac{1}{x}$ summed by BFKL

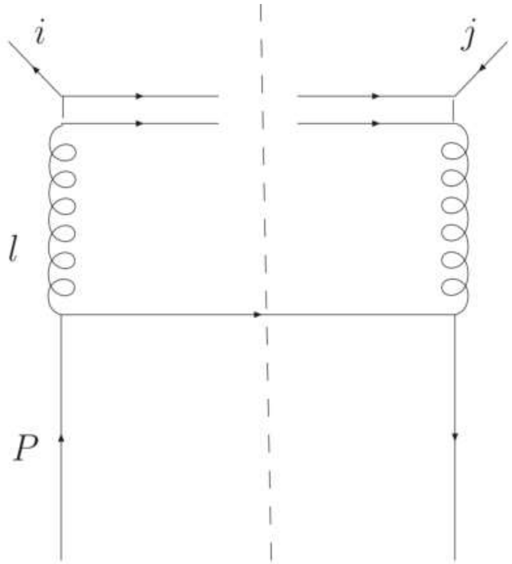
When z--->1, light cone divergence, introduce ζ to regularize

large logarithm $\ln\frac{x^2\varsigma^2}{k_T^2}$ summed by CS

$$\ln\frac{1}{x}\longrightarrow\ln\frac{S}{M^2}\qquad\qquad\ln\frac{x^2\varsigma^2}{k_T^2}\longrightarrow\ln\frac{M^2}{p_T^2}$$

# small x gluon TMD at LO

In a simple quark model, at tree level:

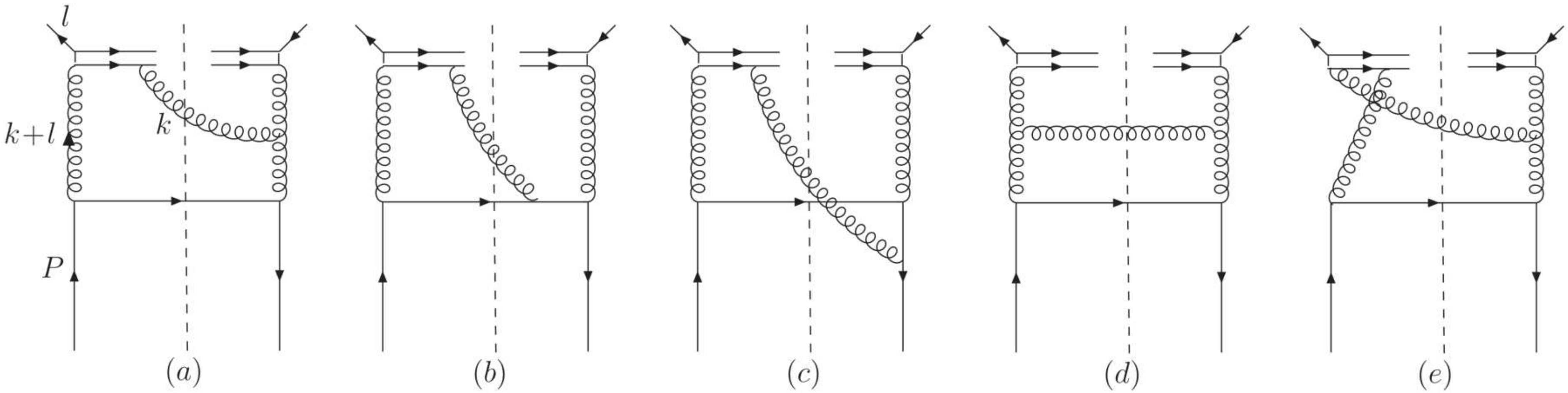



$$xG(x, l_\perp, x\zeta)_{LO}|_{x\to 0} = \frac{\alpha_s C_F}{\pi^2}\frac{1}{l_\perp^2}$$

- both large logarithms are absent at LO;
  how is it dressed by quantum corrections at NLO?

# Real graphs

Sample diagrams:




- Fig.a is the only diagram contributing to both the CS and the BFKL evolution kernel

- Calculation formulated in the Ji-Ma-Yuan scheme.

$$\int_0^\infty \frac{dk^+}{k^+} = \int_{l^+}^\infty \frac{dk^+}{k^+} + \int_0^{l^+} \frac{dk^+}{k^+}$$

# Virtual graphs

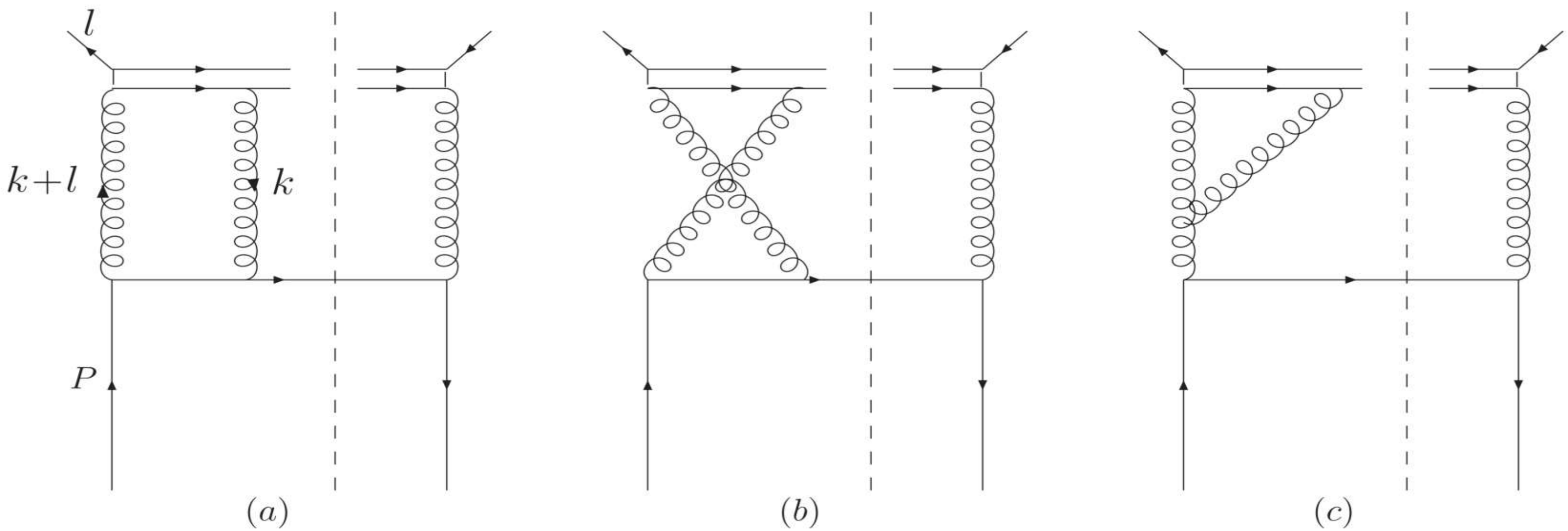



Fig.a & Fig.b ⟹ BFKL kernel

Fig.c ⟹ CS kernel

In the leading logarithm approximations,

Small x gluon TMD at NLO reads,

$$
\begin{aligned}
xG(x,l_\perp,x\zeta)_{NLO} &= xG(x,l_\perp,x\zeta)_{LO} \\
&+ \frac{\alpha_s N_c}{\pi^2}\ln\frac{1}{x}\int\frac{d^2k_\perp}{k_\perp^2}\left[xG_{LO}(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG_{LO}(x,l_\perp,x\zeta)\right] \\
&+ \frac{\alpha_s N_c}{2\pi}\left[\ln\frac{x^2\zeta^2}{l_\perp^2}-\frac{1}{2}\ln\frac{x^2\zeta^2}{\mu^2}-\left(\ln\frac{x^2\zeta^2}{l_\perp^2}\right)^2\right]xG_{LO}(x,l_\perp,x\zeta) \\
&+ \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}xG_{LO}(x,k_\perp+l_\perp,x\zeta)
\end{aligned}
$$

**2016, ZJ**

$$\ln\frac{1}{x} \Longrightarrow \ln\frac{S}{M^2} \qquad\qquad \ln\frac{x^2\varsigma^2}{k_T^2} \Longrightarrow \ln\frac{M^2}{p_T^2}$$

The resulting gluon TMD indeed simultaneously satisfies the both

BFKL equation:

$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial\ln(1/x)}=\frac{\alpha_sN_c}{\pi^2}\int\frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG(x,l_\perp,x\zeta)\right\}$$

CS equation in the Ji-Ma-Yuan scheme:

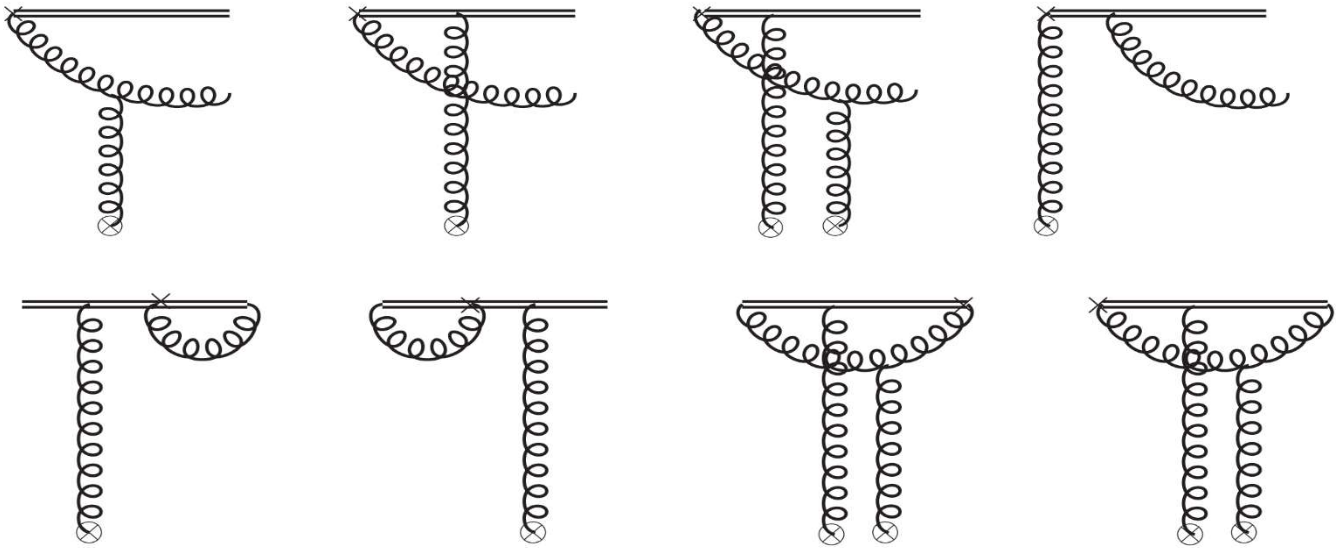
$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial\ln\zeta}=-\frac{\alpha_sN_c}{\pi}\ln\left[\frac{x^2\zeta^2b_\perp^2}{4}e^{2\gamma_E-\frac{1}{2}}\right]G(x,b_\perp,x\zeta)$$

# More formal treatment

**Computing small x gluon TMDs in CGC with Collins 2011 scheme**

# Small x TMDs in CGC at NLO

Sample diagrams



Xiao, Yuan and ZJ, 2017.

The basic nonperturbative ingredient:

$$U(x_\perp) = \langle \mathcal{P} e^{-ig\int_{-\infty}^{+\infty} dx^- A^+(x^-,\, x_\perp)} \rangle$$

Multiple gluon rescattering is treated in an equal way.

# Collinear approach vs CGC I

- **TMDs in collinear approach**
  collinear divergence DGLAP
- **TMDs in CGC,**
  rapidity divergence BK or JIMWLK $\int_{l^+}^{\infty} \frac{dk^+}{k^+}$

Collins-Soper light cone divergence appears in both collinear approach and CGC calculation

Match small x TMDs onto two point functions instead of PDFs.

$$\left\langle \mathrm{Tr} U(R_\perp + r_\perp) U^\dagger(R_\perp) \right\rangle$$

# Collinear approach vs CGC II

$$\tilde{f}_g^{(sub.)}(x, r_\perp, \zeta_c{}') = e^{-S^g_{pert}(Q,r_\perp)} \sum_i C_{g/i}(\mu_r/\mu) \otimes f_i(x,\mu)$$

**Sudakov factor** | **Hard coefficient** | **Colliner PDF**

$$xG^{(1)}(x,k_\perp,\zeta_c) = -\frac{2}{\alpha_S}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))\, e^{-\mathcal{S}_{sud}(Q^2,r_\perp^2)} \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp)$$

**Hard coefficient** | **Sudakov factor** | **Two point function**

Two step evolution: $S \rightarrow M^2 \rightarrow k_T^2$

The leading double logs can be properly taken care  by the Collins-Soper equations in saturation.

**Not the end of story!**

# Remaining large single logarithm

The Collins-Soper equation in the Collins-2011 scheme:

$$\frac{\partial \ \ln G(x, b_\perp, \mu^2, \zeta_c^2)}{\partial \ \ln \zeta_c} = K(b_\perp, \mu) = -\frac{\alpha_s C_A}{\pi} \ln \frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$$

**Evolution kernel**

# Renormalization group equation in saturation regime

$$\frac{d\ \ln G(x,b_\perp,\mu^2,\zeta_c^2)}{d\ \ln\mu} = \gamma_G\left(g(\mu),\zeta_c^2/\mu^2\right)$$

Does it hold in saturation limit?

$$\frac{\partial\ \ln G(x,b_\perp,\mu^2,\zeta_c^2)}{\partial\ \ln\zeta_c} = K(b_\perp,\mu) = -\frac{\alpha_s C_A}{\pi}\ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$$

Note: two coupled equations.

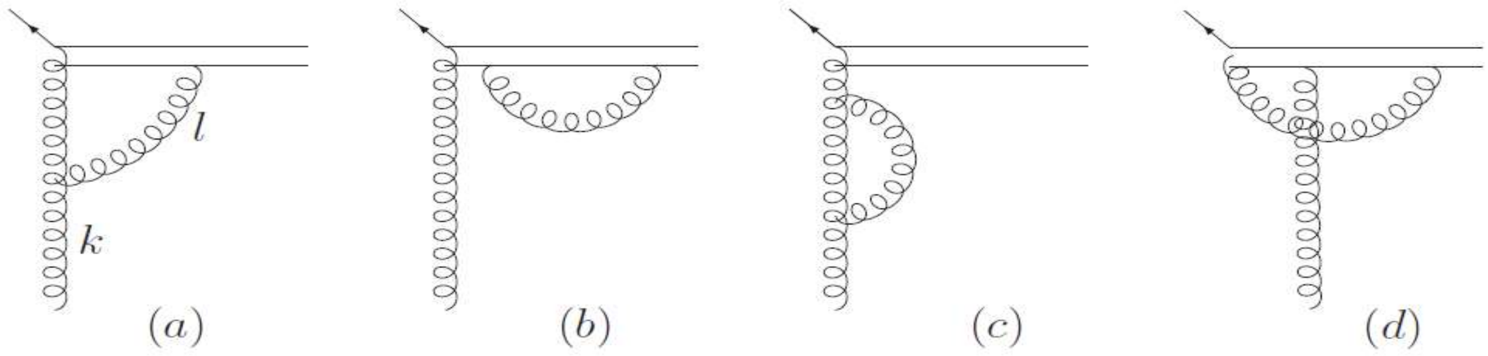
$$G(x,b_\perp,Q^2,Q^2) = \mathrm{Exp}\left\{\int_{\mu_b}^{Q}\frac{d\mu}{\mu}\left(\gamma_G\left(g(\mu),1\right) - \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\right)\right\}G(x,b_\perp,\mu_b^2,\mu_b^2)$$

# Main challenge

The large single log is subleading power contribution in $\ln \frac{1}{x}$

The systematical small x approximation can be applied
until these logs have been resummed.

# Dilute limit



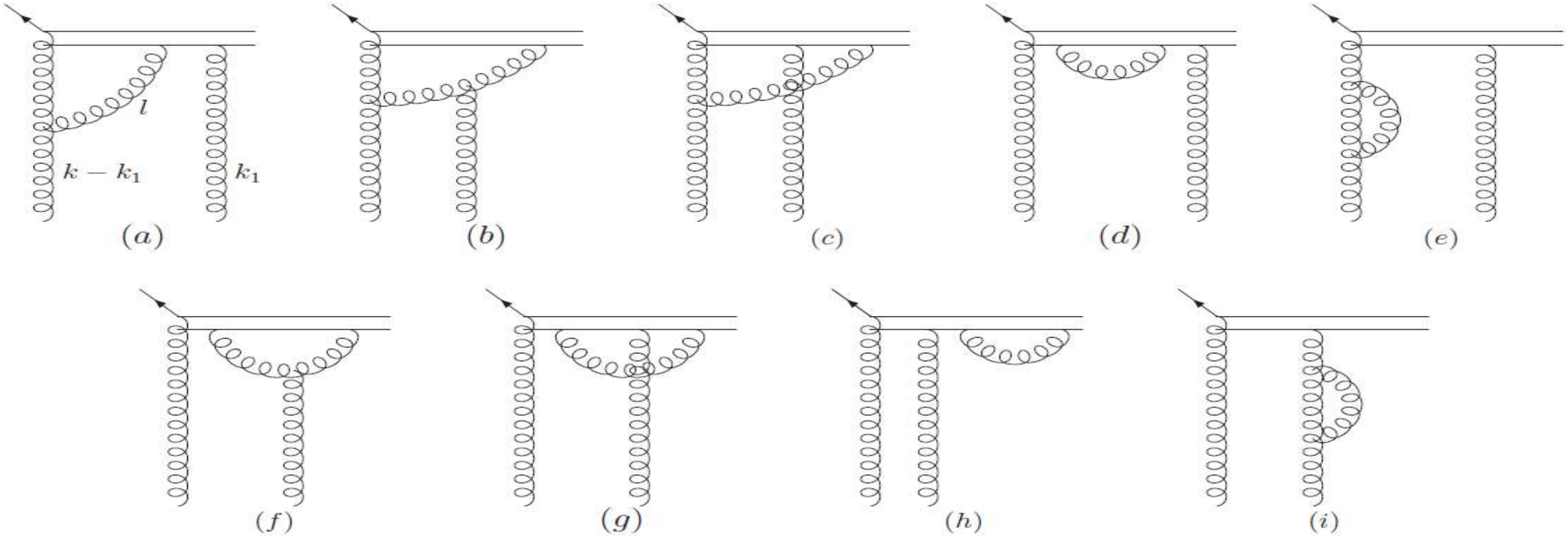

UV part:

$$xG(x,k_\perp,\mu^2,\zeta_c^2)\approx\frac{\alpha_s C_A}{2\pi^2}(2\pi\mu)^{2\epsilon}\int\frac{d^{2-2\epsilon}l_\perp}{l_\perp^2}\left\{\frac{11}{6}-\ln\frac{\zeta_c^2}{l_\perp^2}\right\}xG_0(x,k_\perp)+\text{UV c.t.}$$

Anomalous dimension:

$$\gamma_G\left(g(\mu),\zeta_c^2/\mu^2\right)=\frac{\mathrm{d}\left(-\frac{\alpha_s C_A}{2\pi}S_\epsilon\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{11}{6}-\ln\frac{\zeta_c^2}{\mu^2}\right\}\right]\right)}{\mathrm{d}\ln\mu}=\frac{\alpha_s C_A}{\pi}\left(\frac{11}{6}-\ln\frac{\zeta_c^2}{\mu^2}\right)$$

# One gluon re-scattering

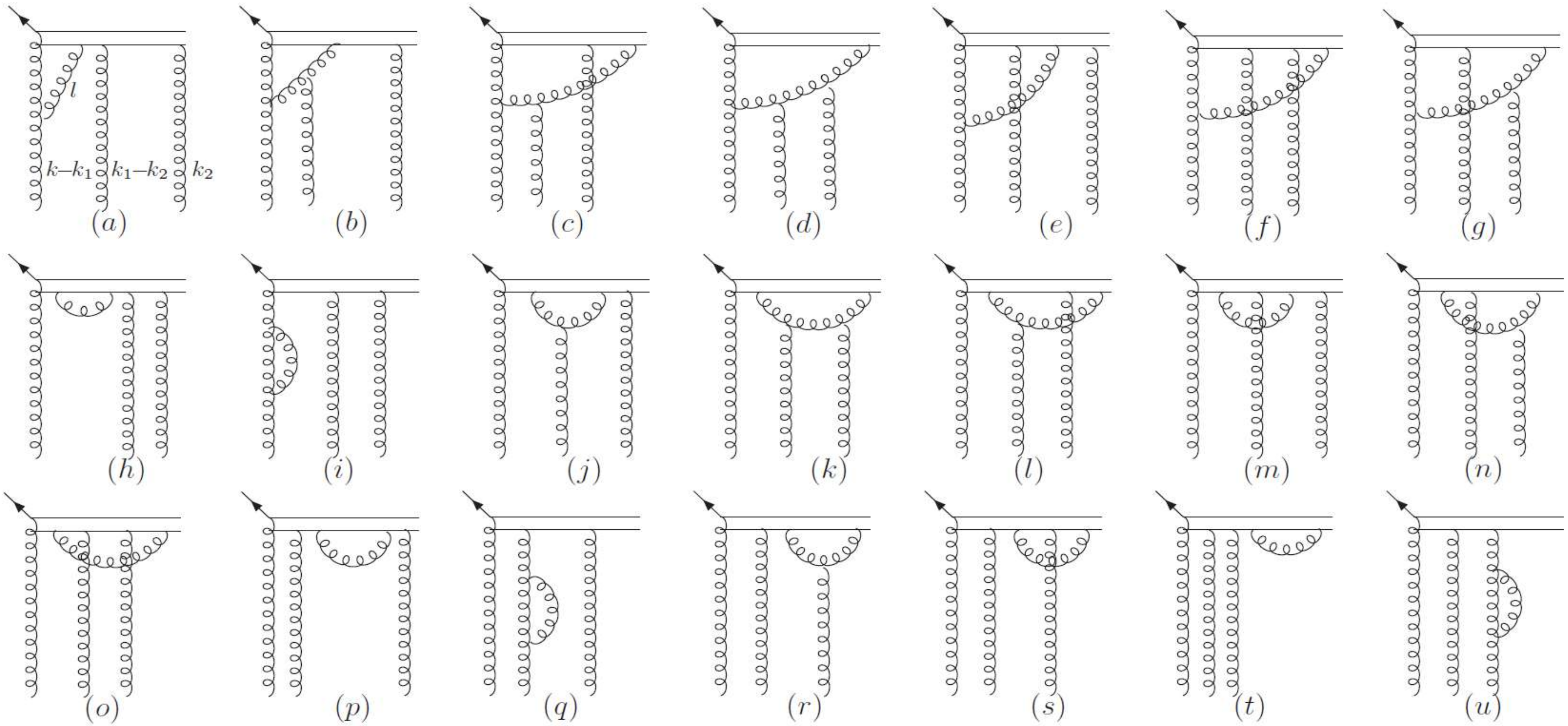



UV part: $$\text{Fig.3}(a+b+c+d+e+f+g+h+i) \approx$$
$$\frac{\alpha_s C_A}{2\pi^2}(2\pi\mu)^{2\epsilon}\int \frac{d^{2-2\epsilon}l_\perp}{l_\perp^2}\left\{\frac{11}{12}+\frac{11}{6}-\ln\frac{\zeta_c^2}{l_\perp^2}\right\} xG_1(x,k_\perp) + \text{UV c.t.}$$

Extra term is removed via:

$$-\mathbf{g}_0\int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp) \Longrightarrow -\mathbf{g}\int_{y^-}^{-\infty} dz^- f^{bac} A_b^+(z^-, y_\perp)$$

# Two gluon scattering case




JZ 2018

The similar pattern has been found.

A recursive method allows us to extend to all order gluon re-scattering case.

## Summary

Large logs: $\ln\frac{1}{x}$ $\ln\frac{Q^2}{\mu^2}$ $\ln^2\frac{Q^2}{k_T^2}$

can be resumed in a consistent and unified formalism.

**Thank you for your attention.**