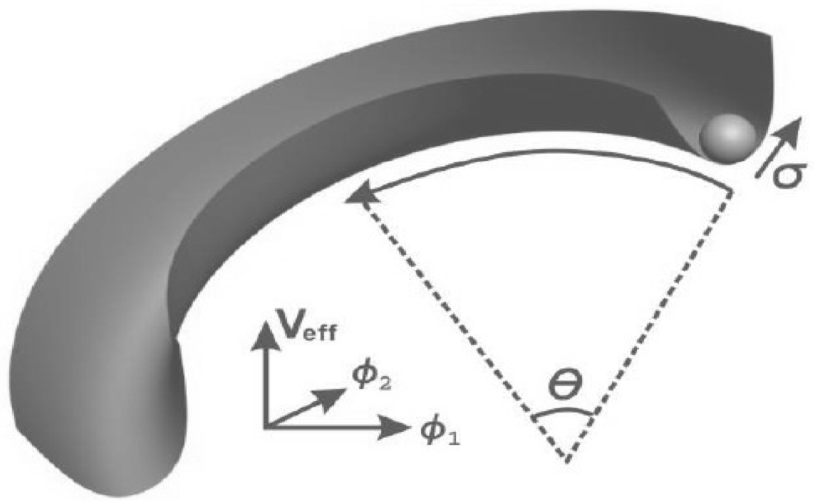
Quasi-Single Field Inflation

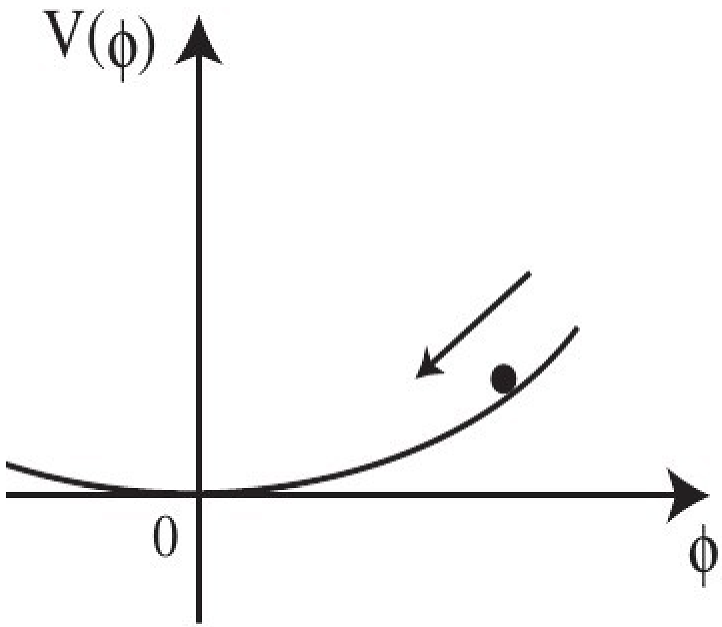


0909.0496
0911.3380
in collaboration with
Xingang Chen

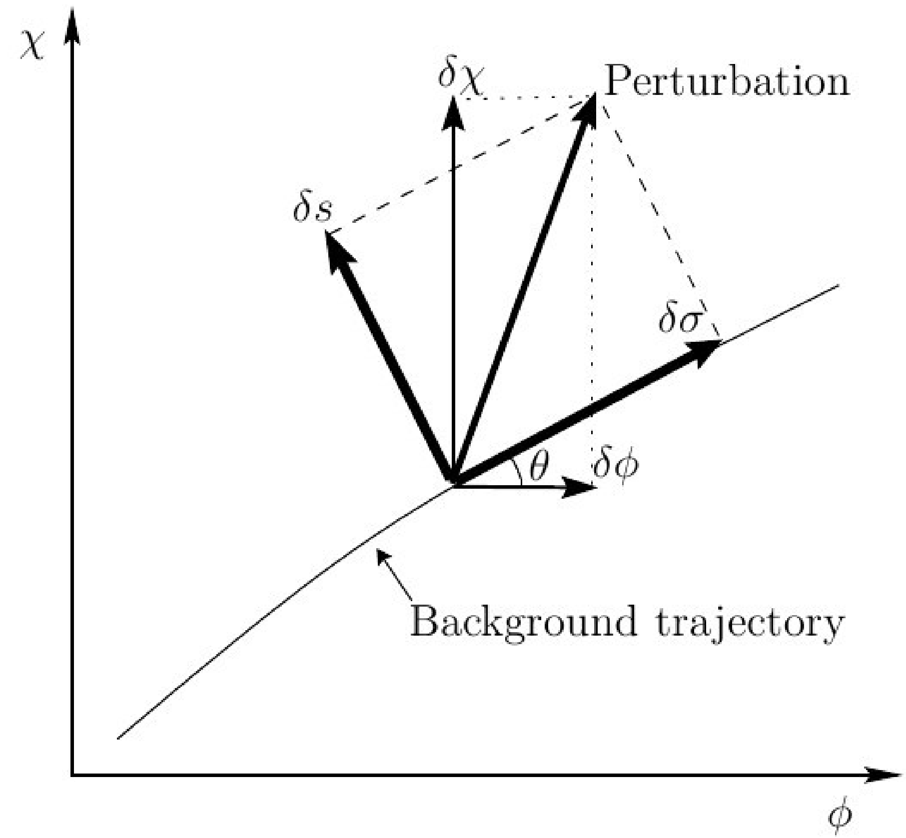
Yi Wang, McGill University
12/30/10 @ USTC

Introduction: Inflation

- Single field



- Multiple field



Gordon, Wands, Bassett, Maartens

Plan

- Motivation
- Preview
- Bkgd dynamics
- Power spectrum
- Bispectrum
- Higher order

Quasi-single field inflation

- Inflaton: slow roll as usual
- Isocurvaton: has mass m~H
- These fields are coupled

Why another inflation model?

- And without penalty?
- Here are two motivations ...

The theoretical motivation:

- Fine tuning problem
- Multiple light fields
- Have inflation anyway

Naturally, one $m \ll H$, all others $m \sim H$

The phenon. motivation:

What is $\begin{cases} \text{“light” in bkgd level?} \\ \text{“light” in pert. level?} \end{cases}$

Gap: one $m \ll H$, all others $m \sim H$

Naturally, one $m \ll H$, all others $m \sim H$

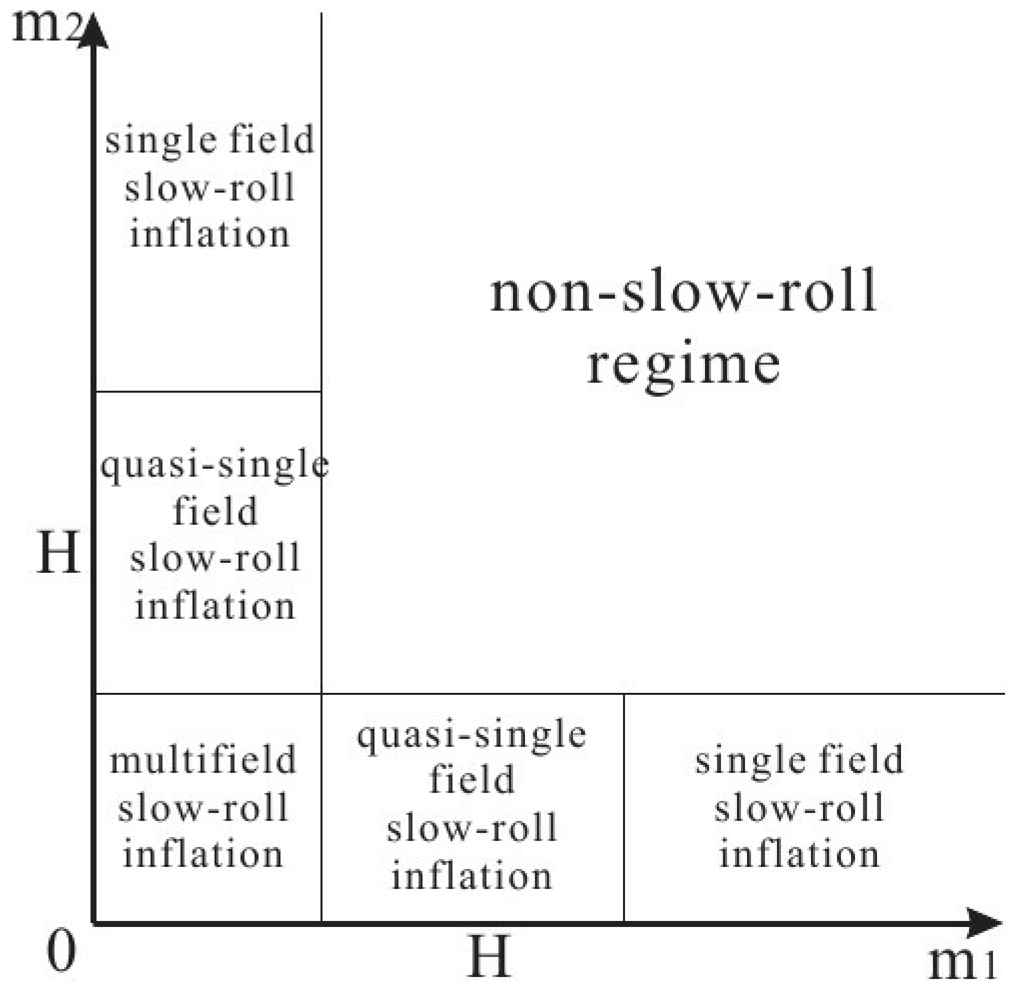
Surprisingly not noticed before!

The m~H fields:

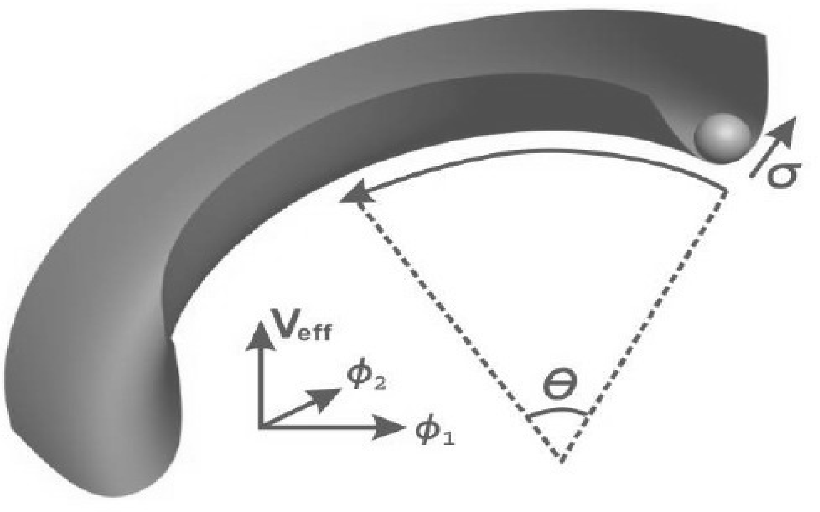
- Vanish at bkgd level
- Show up in perturbations

Thus the model has

Quasi-single field


m2
single field
slow-roll
inflation
non-slow-roll
regime
quasi-single
field
slow-roll
inflation
H
multifield
slow-roll
inflation
quasi-single
field
slow-roll
inflation
single field
slow-roll
inflation
0
H
m1

A simple model:



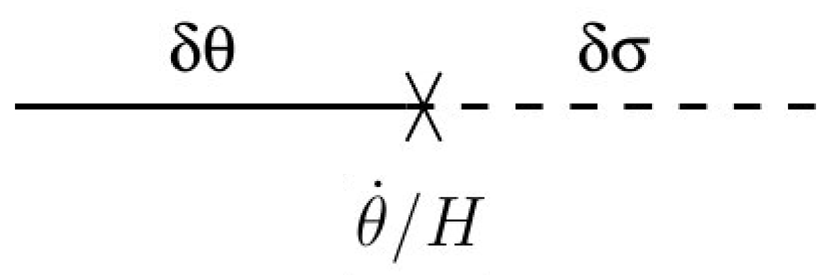
$$S_m = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma) \right]$$

Preview: transfer vertex

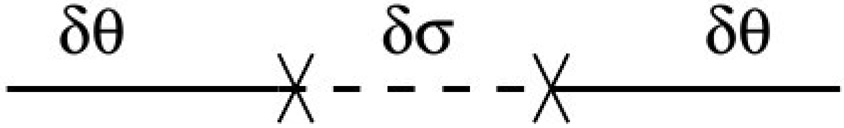
$$S_m = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma) \right]$$

2pt coupling from turning trajectory

$$\delta\mathcal{L}_2 \;=\; 2a^3 R\dot{\theta}_0\ \delta\sigma\delta\dot{\theta}$$

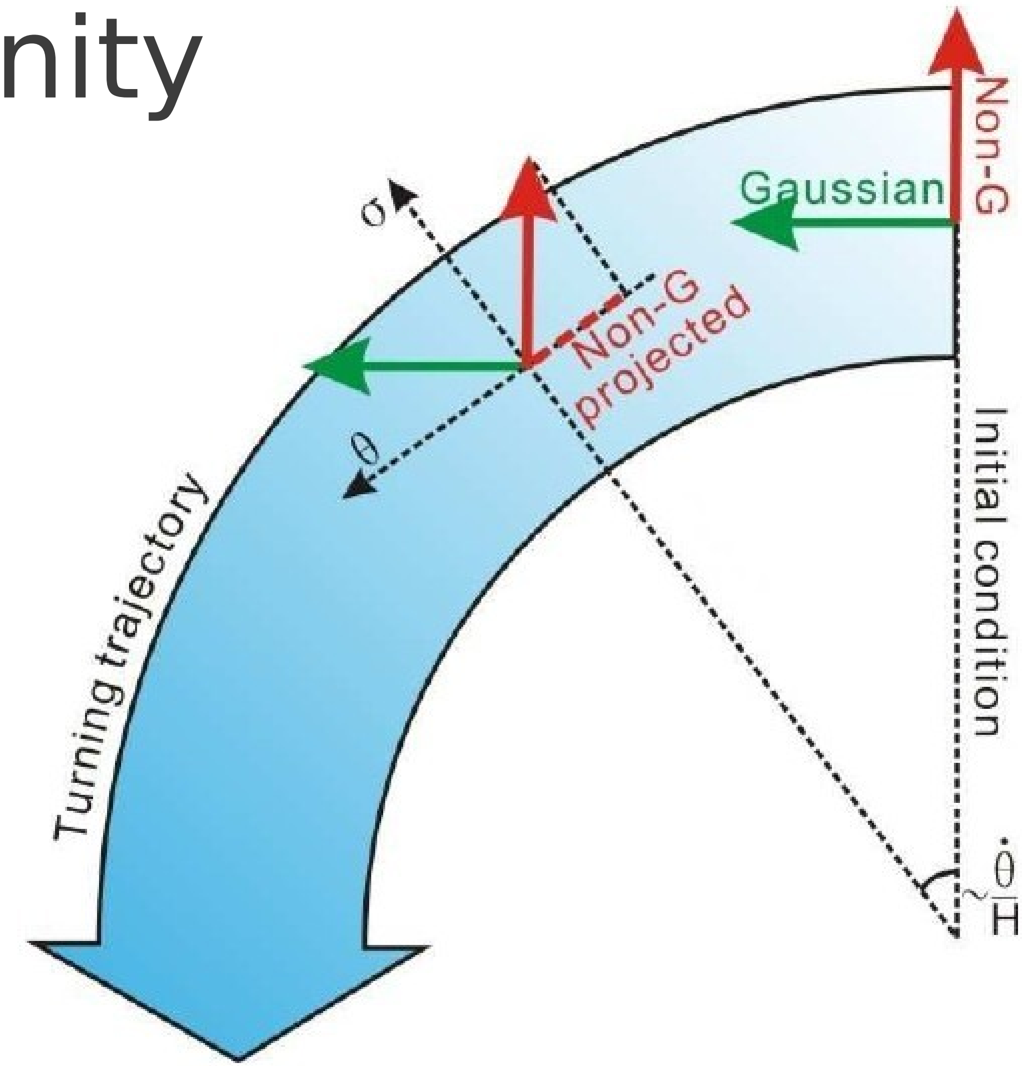


Preview: power spectrum

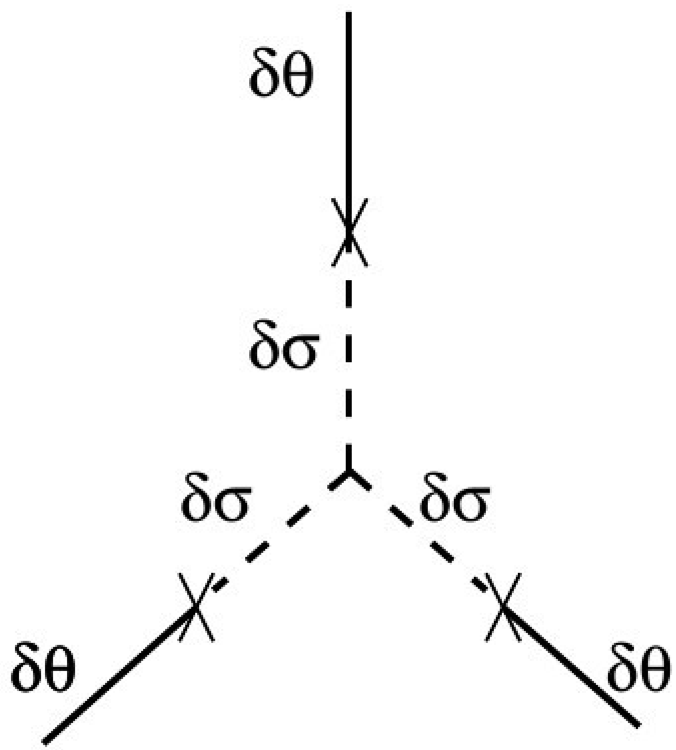


$$\delta P_\zeta \sim \left(\dot{\theta}/H\right)^2 P_\zeta$$

Preview: non-Gaussianity

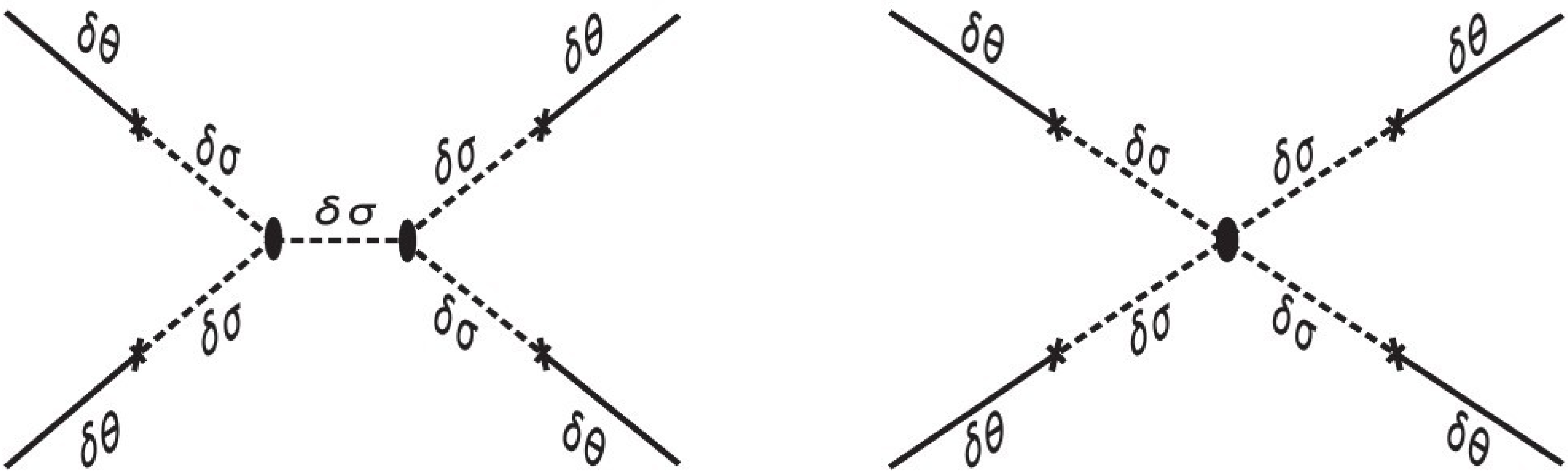


Preview: bispectrum



$$f_{NL} \sim P_\zeta^{-1/2} \left(\dot{\theta}/H\right)^3 (V'''/H)$$

Preview: trispectrum



$$t_{NL} \sim \max\left\{ P_\zeta^{-1}\left(\dot\theta/H\right)^4 (V'''/H)^2 \ , \ P_\zeta^{-1}\left(\dot\theta/H\right)^4 V'''' \right\}$$

$$t_{NL} \gg f_{NL}^2 \text{ for } \dot\theta/H \ll 1$$

Preview:
higher point correlations

$$h_{NL} \sim P_\zeta^{-3/2} \left(\dot{\theta}/H\right)^5 (V'''/H)^3 \sim (\dot{\theta}/H)^{-4} f_{NL}^3 \ ,$$

$$i_{NL} \sim P_\zeta^{-2} \left(\dot{\theta}/H\right)^6 (V'''/H)^4 \sim (\dot{\theta}/H)^{-6} f_{NL}^4 \ .$$

Might be difficult to probe. But ... who knows?

Chunshan Lin and YW, 2010

Model: bkgd dynamics

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$

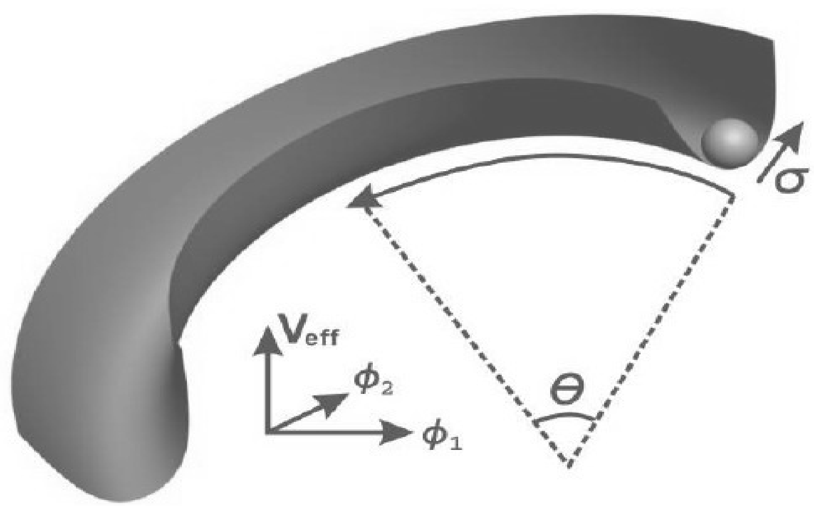
$$3M_p^2H^2 = \frac{1}{2}R^2\dot{\theta}_0^2 + V + V_{\rm sr}$$

$$-2M_p^2\dot{H} = R^2\dot{\theta}_0^2\ .$$

$$\sigma_0 = \text{const.}\ , \quad V'(\sigma_0) = R\dot{\theta}_0^2$$

$$R^2\ddot{\theta}_0 + 3R^2H\dot{\theta}_0 + V_{\rm sr}' = 0\ .$$

$$R \equiv \tilde{R} + \sigma_0.$$



Perturbations:

in-in formalism (factorized)

$$\langle Q(t)\rangle \equiv \langle 0| \left[\bar{T}\exp\left(i\int_{t_0}^{t} dt' H_I(t')\right)\right] Q_I(t) \left[T\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right] |0\rangle$$

Perturbations (action):

$$\mathcal{H}_0 = a^3 \left[\frac{1}{2} R^2 \delta\dot{\theta}_I{}^2 + \frac{R^2}{2a^2} (\partial_i \delta\theta_I)^2 + \frac{1}{2} \delta\dot{\sigma}_I{}^2 + \frac{1}{2a^2} (\partial_i \delta\sigma_I)^2 + \frac{1}{2} m^2 \delta\sigma_I^2 \right]$$

$$\begin{aligned} \mathcal{H}_2^I &= -c_2 a^3 \delta\sigma_I \delta\dot{\theta}_I \\ \mathcal{H}_3^I &= c_3 a^3 \delta\sigma_I^3 \ , \end{aligned}$$

$$c_2 = 2R\dot{\theta}_0 \ , \quad c_3 = \frac{1}{6} V''' \ , \quad m^2 = V'' + 7\dot{\theta}_0^2$$

Perturbations (quantization):

$$
\begin{aligned}
\delta\theta^I_{\mathbf{k}} &= u_{\mathbf{k}} a_{\mathbf{k}} + u^*_{-\mathbf{k}} a^\dagger_{-\mathbf{k}} \\
\delta\sigma^I_{\mathbf{k}} &= v_{\mathbf{k}} b_{\mathbf{k}} + v^*_{-\mathbf{k}} b^\dagger_{-\mathbf{k}}
\end{aligned}
$$

$$
[a_{\mathbf{k}}, a^\dagger_{-\mathbf{k}'}] = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \ , \quad [b_{\mathbf{k}}, b^\dagger_{-\mathbf{k}'}] = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}')
$$

Perturbations (eom):

$$
\begin{aligned}
u''_{\mathbf{k}} - \frac{2}{\tau}u'_{\mathbf{k}} + k^2 u_{\mathbf{k}} &= 0 \\
v''_{\mathbf{k}} - \frac{2}{\tau}v'_{\mathbf{k}} + k^2 v_{\mathbf{k}} + \frac{m^2}{H^2\tau^2}v_{\mathbf{k}} &= 0
\end{aligned}
$$

$$u_{\mathbf{k}} = \frac{H}{R\sqrt{2k^3}}(1 + ik\tau)e^{-ik\tau}$$

$$v_{\mathbf{k}} = -ie^{i(\nu+\frac{1}{2})\frac{\pi}{2}}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{\nu}^{(1)}(-k\tau)$$

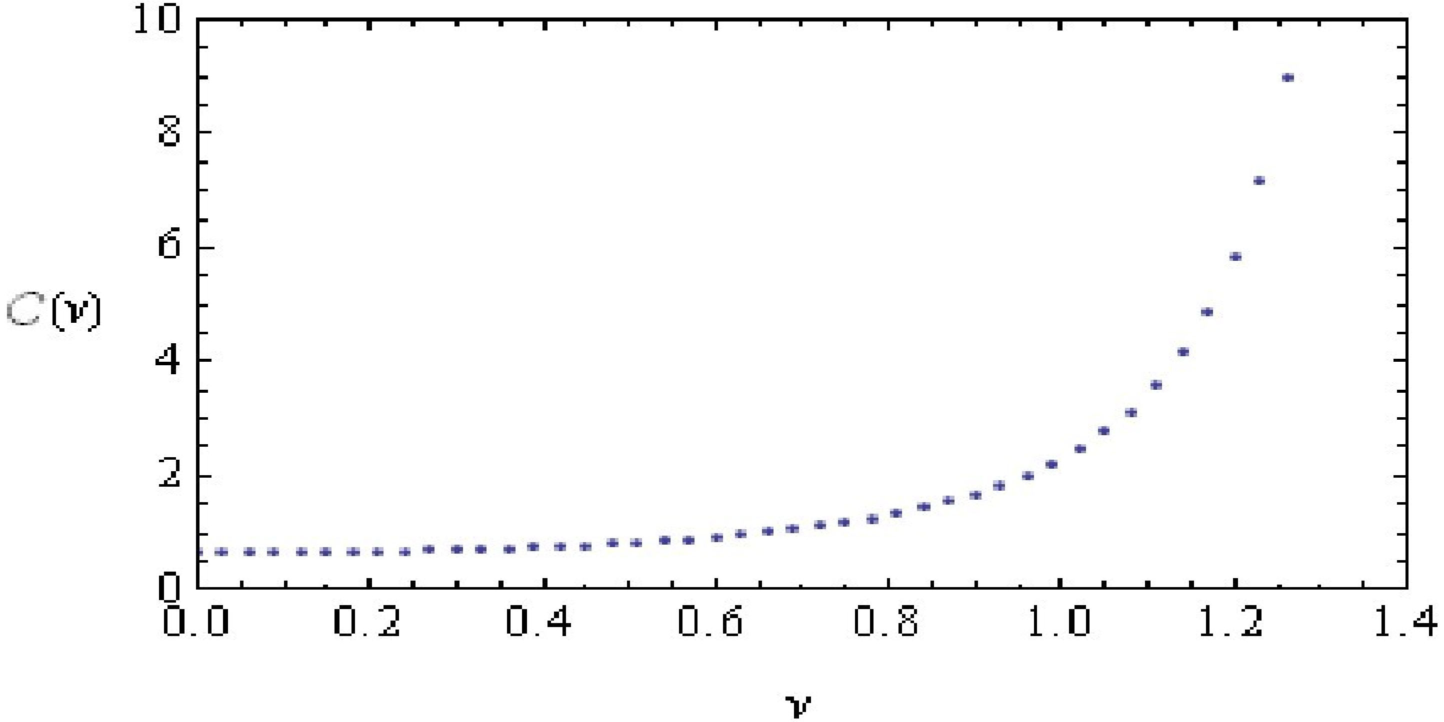
$$\nu = \sqrt{9/4 - m^2/H^2}$$

Power spectrum:

$$
\begin{aligned}
\langle \delta\theta^2 \rangle &\equiv \langle 0| \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t')\right)\right] \delta\theta_I^2(t) \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t')\right)\right] |0\rangle \\
&= \langle 0|\delta\theta_I^2|0\rangle \\
&+ \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{t} dt_1 \langle 0|H_I(\tilde{t}_1)\ \delta\theta_I^2\ H_I(t_1)|0\rangle \\
&- 2\ \mathrm{Re}\left[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \langle 0|\delta\theta_I^2\ H_I(t_1) H_I(t_2)|0\rangle\right] \\
&+ \cdots .
\end{aligned}
$$

IR divergence cancels in the two terms

Power spectrum: $P_\zeta = \frac{H^4}{4\pi^2 R^2 \dot\theta_0^2}\left[1 + 8\mathcal{C}\left(\frac{\dot\theta_0}{H}\right)^2\right]$



$$\mathcal{C}(\nu) \equiv \frac{\pi}{4}\mathrm{Re}\left[\int_0^\infty dx_1 \int_{x_1}^\infty dx_2 \left(x_1^{-1/2} H_\nu^{(1)}(x_1) e^{ix_1} x_2^{-1/2} H_\nu^{(2)}(x_2) e^{-ix_2} - x_1^{-1/2} H_\nu^{(1)}(x_1) e^{-ix_1} x_2^{-1/2} H_\nu^{(2)}(x_2) e^{-ix_2}\right)\right]$$

Bispectrum:

We met UV / IR divergences in
both known forms of in-in formalism

Let us review and generalize in-in

Factorized form:

$$\langle \delta\theta^3 \rangle \equiv \langle 0| \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t') \right) \right] \delta\theta_I^3(t) \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t') \right) \right] |0\rangle$$

$$\begin{aligned} \langle \delta\theta^3 \rangle &= \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, \langle H_I(\tilde{t}_2) H_I(\tilde{t}_1) \; \delta\theta_I^3 \; H_I(t_1) H_I(t_2) \rangle \\ &- 2\,\mathrm{Re}\left[\int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 \, \langle H_I(\tilde{t}_1) \; \delta\theta_I^3 \; H_I(t_1) H_I(t_2) H_I(t_3) \rangle \right] \\ &+ 2\,\mathrm{Re}\left[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 \int_{t_0}^{t_3} dt_4 \, \langle \delta\theta_I^3 \; H_I(t_1) H_I(t_2) H_I(t_3) H_I(t_4) \rangle \right] \end{aligned}$$

$$
\begin{aligned}
- \quad & 12 c_2^3 c_3 u^*_{p_1}(0) u_{p_2}(0) u_{p_3}(0) \\
\times \quad & \mathrm{Re}\left[\int_{-\infty}^{0} d\tilde{\tau}_1 \; a^3(\tilde{\tau}_1) v^*_{p_1}(\tilde{\tau}_1) u'_{p_1}(\tilde{\tau}_1) \int_{-\infty}^{\tilde{\tau}_1} d\tilde{\tau}_2 \; a^4(\tilde{\tau}_2) v_{p_1}(\tilde{\tau}_2) v_{p_2}(\tilde{\tau}_2) v_{p_3}(\tilde{\tau}_2)\right. \\
\times \quad & \left.\int_{-\infty}^{0} d\tau_1 \; a^3(\tau_1) v^*_{p_2}(\tau_1) u'^*_{p_2}(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 \; a^3(\tau_2) v^*_{p_3}(\tau_2) u'^*_{p_3}(\tau_2)\right] \\
\times \quad & (2\pi)^3 \delta^3\Big(\sum_i \mathbf{p}_i\Big) + 9 \text{ other similar terms} \\
+ \quad & 5 \text{ permutations of } \mathbf{p}_i \; .
\end{aligned}
$$

In the IR:

$$
\begin{aligned}
u'_{p_i}(\tau) & \propto (-\tau)(1 - i p_i \tau + \cdots) \,, \\
v_{p_i}(\tau) & \propto (-\tau)^{\frac{3}{2}-\nu} \left(1 + \alpha_1(-\tau)^2 + \alpha_2(-\tau)^{2\nu} + \cdots\right)
\end{aligned}
$$

IR Divergence
when ν≥1/2

$$\nu = \sqrt{9/4 - m^2/H^2}$$

Commutator form:

$$\langle \delta\theta^3 \rangle = \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 \int_{t_0}^{t_3} dt_4 \langle [H_I(t_4), [H_I(t_3), [H_I(t_2), [H_I(t_1), \delta\theta_I(t)^3]]]] \rangle$$

$$\begin{aligned}
\langle \delta\theta^3 \rangle &= 12 c_2^3 c_3 u_{p_1}(0) u_{p_2}(0) u_{p_3}(0) \\
&\times \mathrm{Re}\left[\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \int_{-\infty}^{\tau_3} d\tau_4 \prod_{i=1}^{4} a^3(\tau_i) \right. \\
&\times \left. (a(\tau_2) A + a(\tau_3) B + a(\tau_4) C) \right] (2\pi)^3 \delta^3 (\sum \mathbf{p}_i) \\
&+ 5 \text{ perm.} \; .
\end{aligned}$$

$$\begin{aligned}
A &= \left(u'_{p_1}(\tau_1) - c.c.\right) \left(v_{p_1}(\tau_1) v^*_{p_1}(\tau_2) - c.c.\right) \left(v_{p_3}(\tau_2) v^*_{p_3}(\tau_4) u'^*_{p_3}(\tau_4) - c.c.\right) \\
&\quad v_{p_2}(\tau_2) v^*_{p_2}(\tau_3) u'^*_{p_2}(\tau_3) \; , \\
B &= \left(u'_{p_1}(\tau_1) - c.c.\right) \left(u'_{p_2}(\tau_2) - c.c.\right) \left(v^*_{p_1}(\tau_1) v^*_{p_2}(\tau_2) v_{p_1}(\tau_3) v_{p_2}(\tau_3) - c.c.\right) \\
&\quad v_{p_3}(\tau_3) v^*_{p_3}(\tau_4) u'^*_{p_3}(\tau_4) \; , \\
C &= -\left(u'_{p_1}(\tau_1) - c.c.\right) \left(u'_{p_2}(\tau_2) - c.c.\right) \left(u'_{p_3}(\tau_3) - c.c.\right) \\
&\quad v^*_{p_1}(\tau_1) v^*_{p_2}(\tau_2) v^*_{p_3}(\tau_3) v_{p_1}(\tau_4) v_{p_2}(\tau_4) v_{p_3}(\tau_4) \; .
\end{aligned}$$

$$
\begin{aligned}
A &= \left(u'_{p_1}(\tau_1) - c.c.\right)\left(v_{p_1}(\tau_1)v^*_{p_1}(\tau_2) - c.c.\right)\left(v_{p_3}(\tau_2)v^*_{p_3}(\tau_4)u'^*_{p_3}(\tau_4) - c.c.\right) \\
&\quad v_{p_2}(\tau_2)v^*_{p_2}(\tau_3)u'^*_{p_2}(\tau_3) \ , \\
B &= \left(u'_{p_1}(\tau_1) - c.c.\right)\left(u'_{p_2}(\tau_2) - c.c.\right)\left(v^*_{p_1}(\tau_1)v^*_{p_2}(\tau_2)v_{p_1}(\tau_3)v_{p_2}(\tau_3) - c.c.\right) \\
&\quad v_{p_3}(\tau_3)v^*_{p_3}(\tau_4)u'^*_{p_3}(\tau_4) \ , \\
C &= -\left(u'_{p_1}(\tau_1) - c.c.\right)\left(u'_{p_2}(\tau_2) - c.c.\right)\left(u'_{p_3}(\tau_3) - c.c.\right) \\
&\quad v^*_{p_1}(\tau_1)v^*_{p_2}(\tau_2)v^*_{p_3}(\tau_3)v_{p_1}(\tau_4)v_{p_2}(\tau_4)v_{p_3}(\tau_4) \ .
\end{aligned}
$$

In the UV:

Interaction vacuum: $\exp(-ik\tau+\varepsilon\tau)$ when $\tau\to-\infty$

But now: $\sin(k\tau+i\varepsilon\tau)\to\infty$, when $\tau\to-\infty$

UV Divergence!

Mixed form:

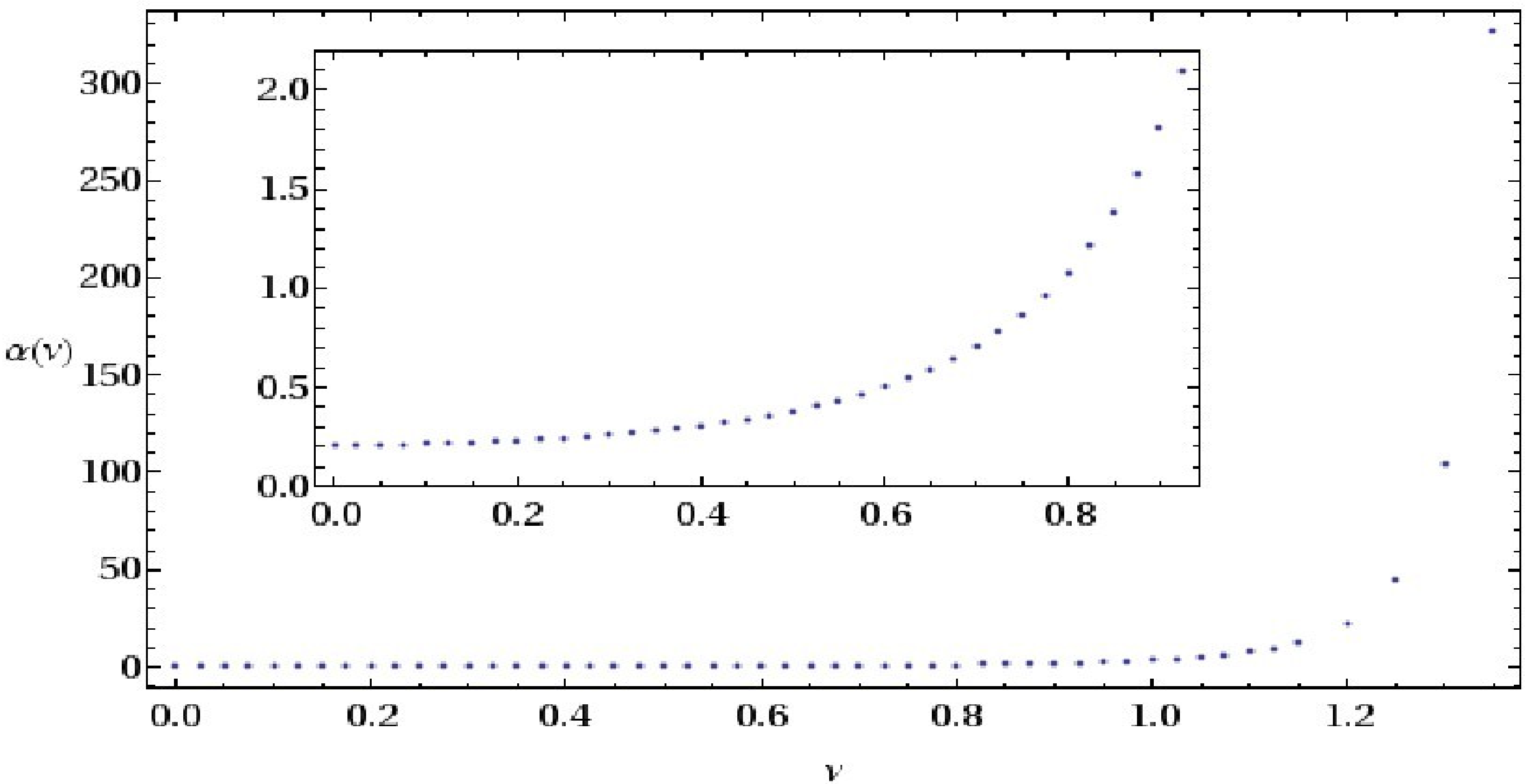
Impose an (independent) cutoff τ_c

$$\sum_i \int_{\tau_c}^{0} d\tau_1 \cdots \int_{\tau_c}^{\tau_{i-1}} d\tau_i \,\{\text{commutator form}\} \int_{-\infty}^{\tau_c} d\tau_{i+1} \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n \,\{\text{factorized form}\}$$

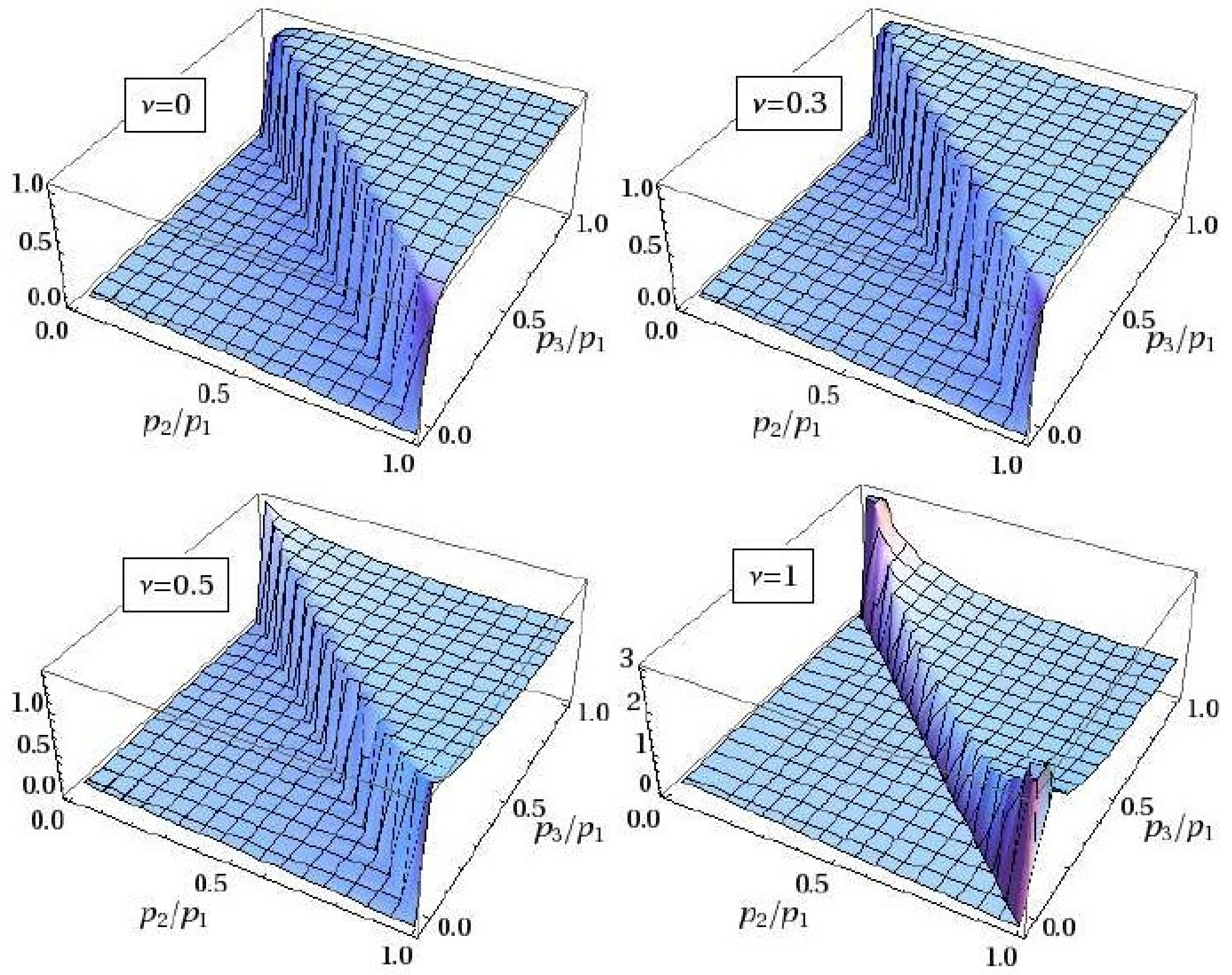
UV / IR well behaved

Bispectrum: amplitude

$$f_{NL}^{\text{int}} = \alpha(\nu) P_\zeta^{-1/2} \left(\dot{\theta}_0/H\right)^3 (-V'''/H)$$



Bispectrum: shape



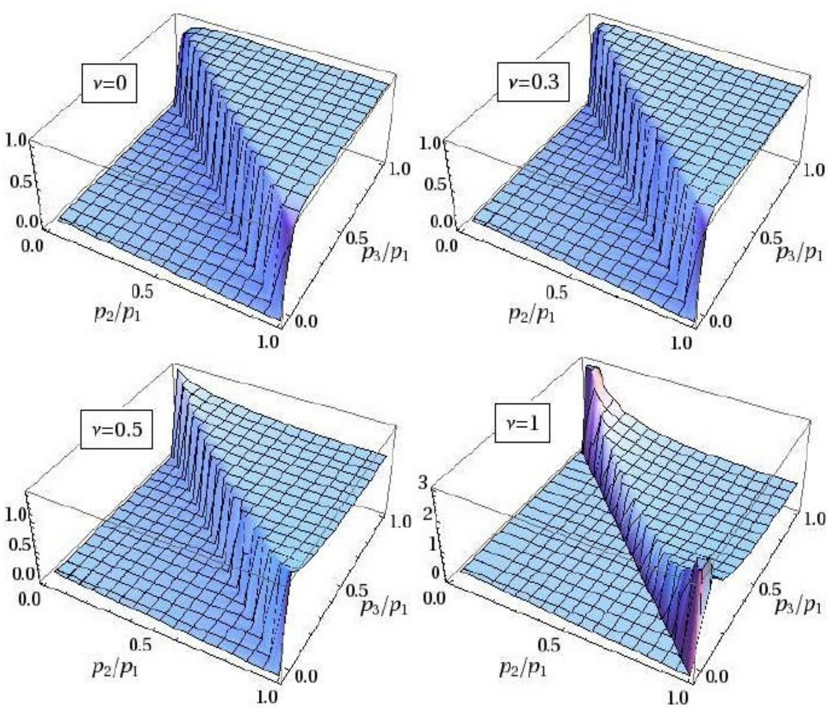
Bispectrum: squeezed limit

$$\frac{3\pi^2\Gamma(\nu)}{2^{5-\nu}}\frac{c_2^3 c_3}{HR^6}\frac{1}{p_1^{\frac{7}{2}-\nu}\,p_2\,p_3^{\frac{3}{2}+\nu}}$$

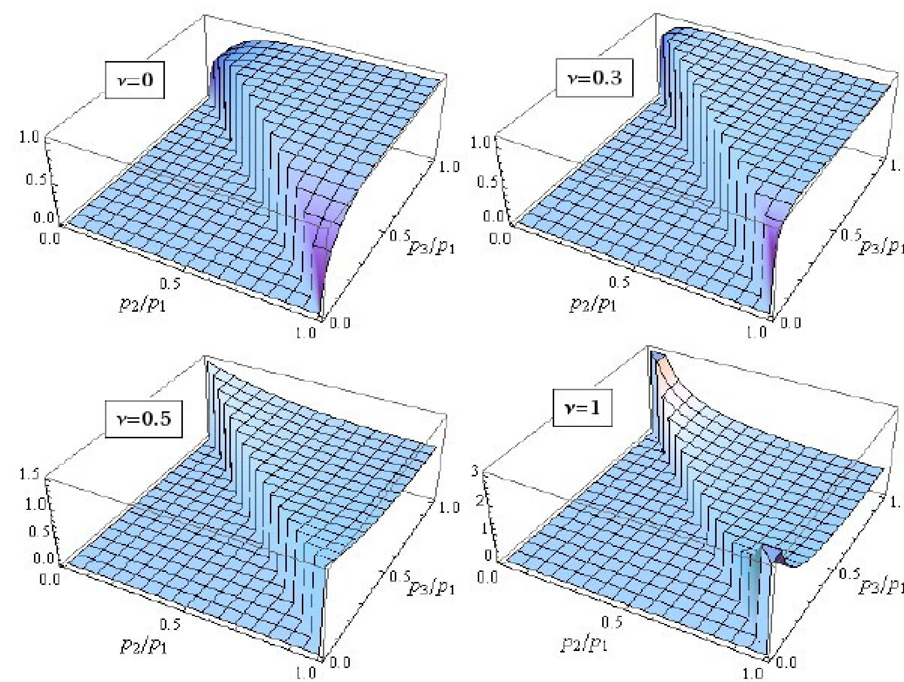
Shape changed during projection

Bispectrum: shape ansatz

$$F = \frac{3^{\frac{9}{2}-3\nu}}{10} \frac{f_{NL}^{\text{int}}(p_1^2 + p_2^2 + p_3^2)}{(p_1 p_2 p_3)^{\frac{3}{2}+\nu}(p_1 + p_2 + p_3)^{\frac{7}{2}-3\nu}}$$



True numeric



Shape ansatz

Higher points:

$$t_{NL} \sim \max\left\{ P_\zeta^{-1}\left(\dot{\theta}/H\right)^4 (V'''/H)^2 \ ,\ P_\zeta^{-1}\left(\dot{\theta}/H\right)^4 V'''' \right\}$$

$$t_{NL} \gg f_{NL}^2 \text{ for } \dot{\theta}/H \ll 1$$

$$h_{NL} \sim P_\zeta^{-3/2}\left(\dot{\theta}/H\right)^5 (V'''/H)^3 \sim (\dot{\theta}/H)^{-4} f_{NL}^3 \ ,$$

$$i_{NL} \sim P_\zeta^{-2}\left(\dot{\theta}/H\right)^6 (V'''/H)^4 \sim (\dot{\theta}/H)^{-6} f_{NL}^4 \ .$$

Summary

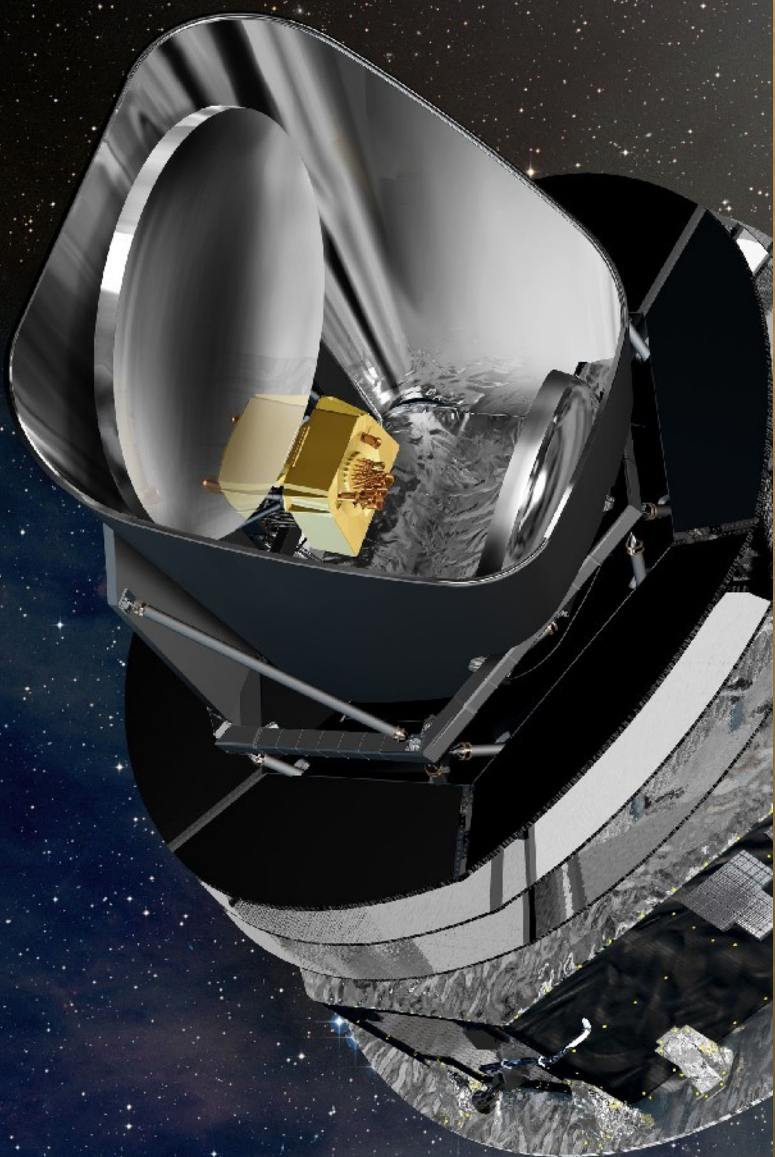
- ➔ Motivation
- ➔ Preview
- ➔ Bkgd dynamics
- ➔ Power spectrum
- ➔ Bispectrum
- ➔ Higher order

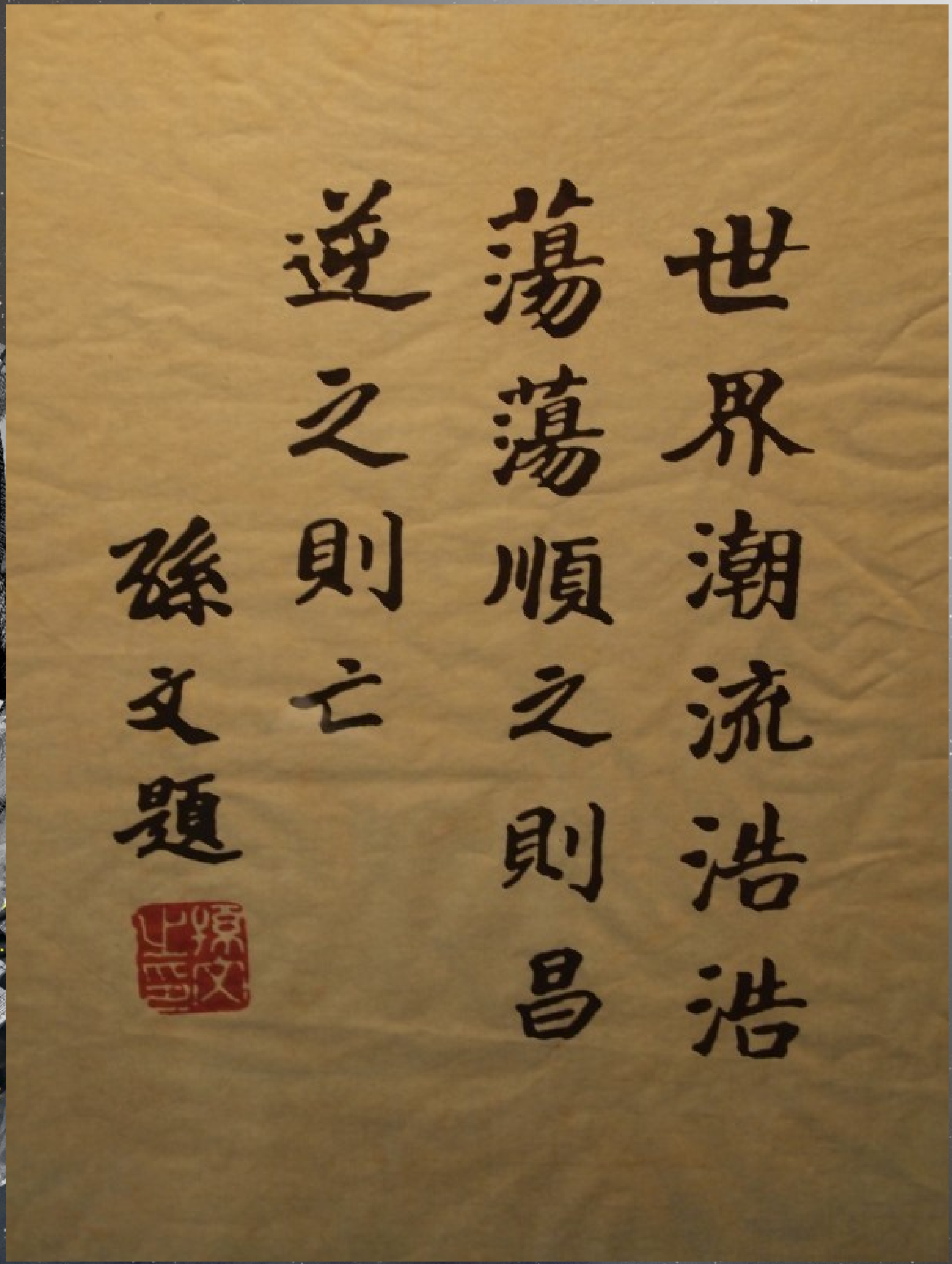
Who knows …

inflation is like

Or …

inflation is like





We look forward to the future

THANK YOU

新年快乐！