Energy Correlators in QCD

Jet Physics Meet Conformal Field Theory

朱华星 （Hua Xing Zhu）
浙江大学

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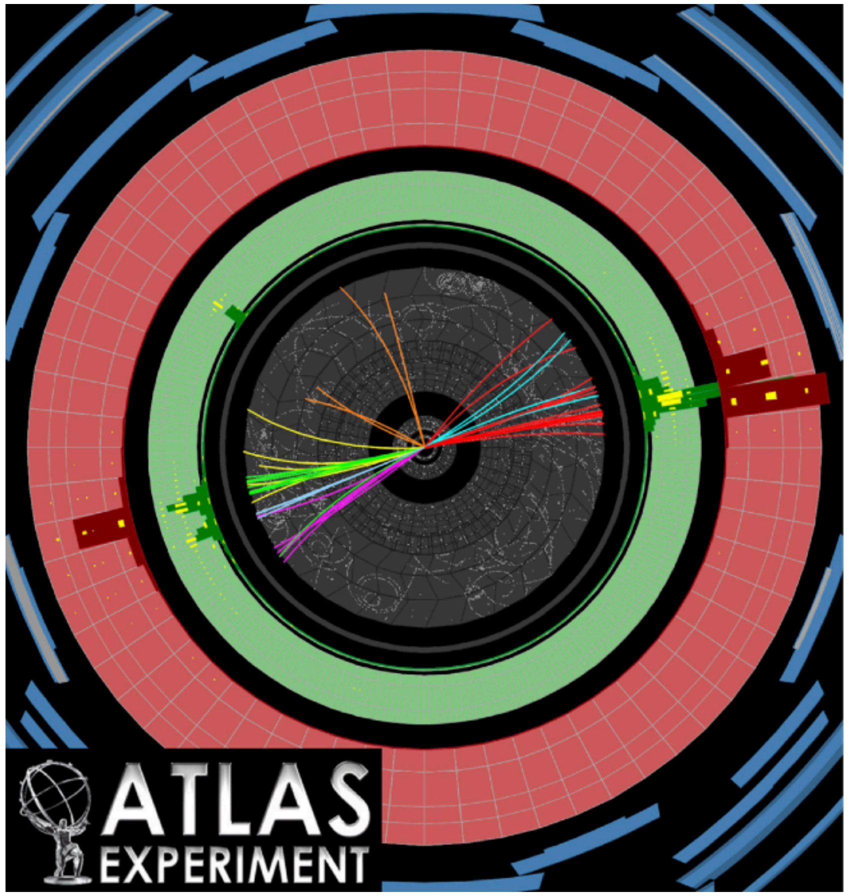
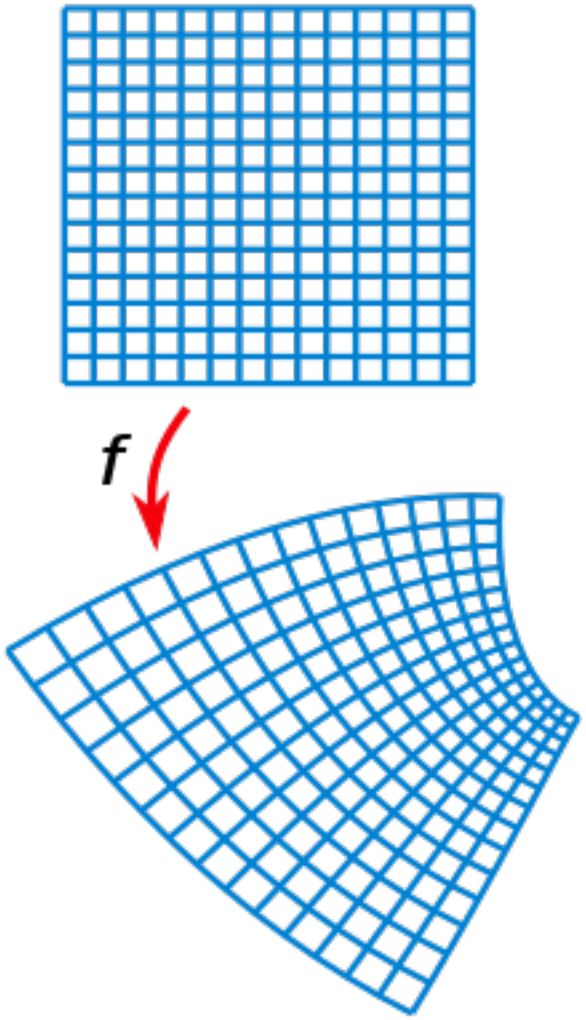
Jet Physics v.s. Conformal Field Theory



v.s.

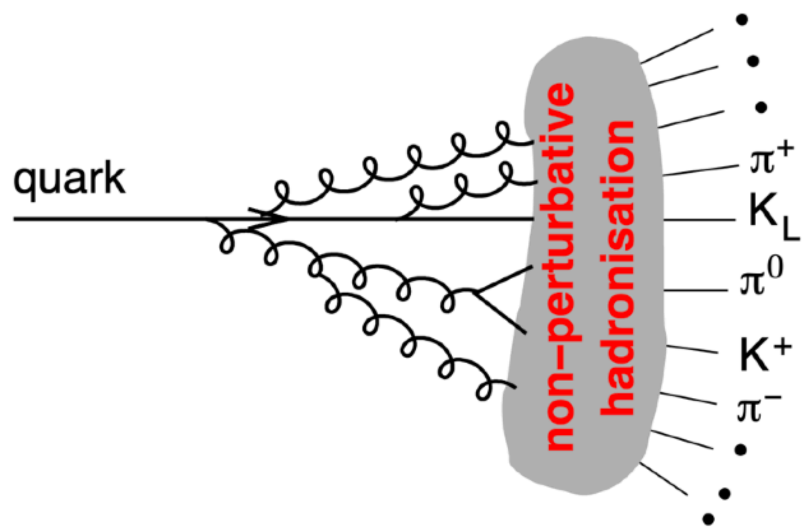


Outline

- Brief introduction to jets
- Energy correlators in QCD: what they are and some recent progress
- Energy correlators in conformal field theory
- Interplay between QCD and N=4 sYM

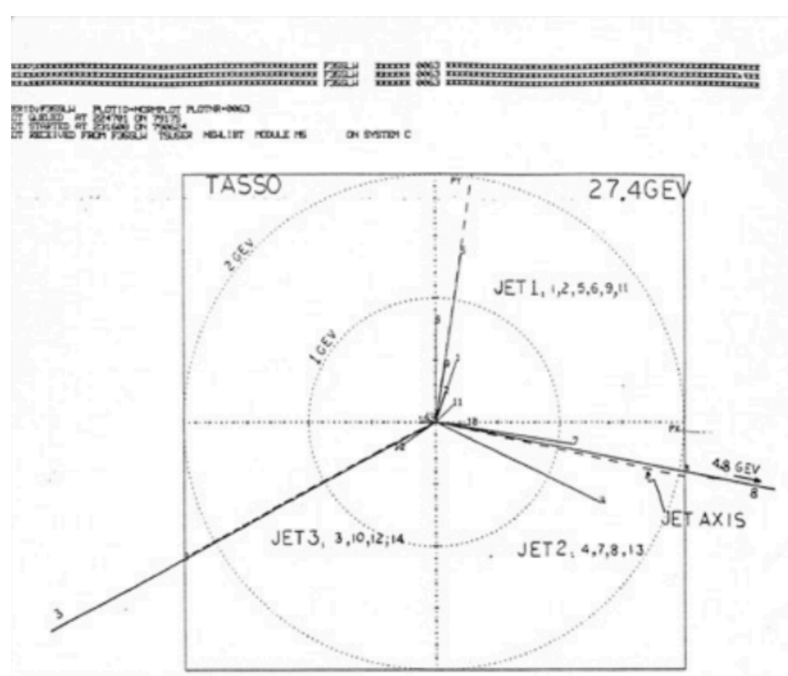
What are jets

- Jets are collimated spray of energetic particles



from G. Salam

- Historically, gluon was discovered in three jet events in e+e-

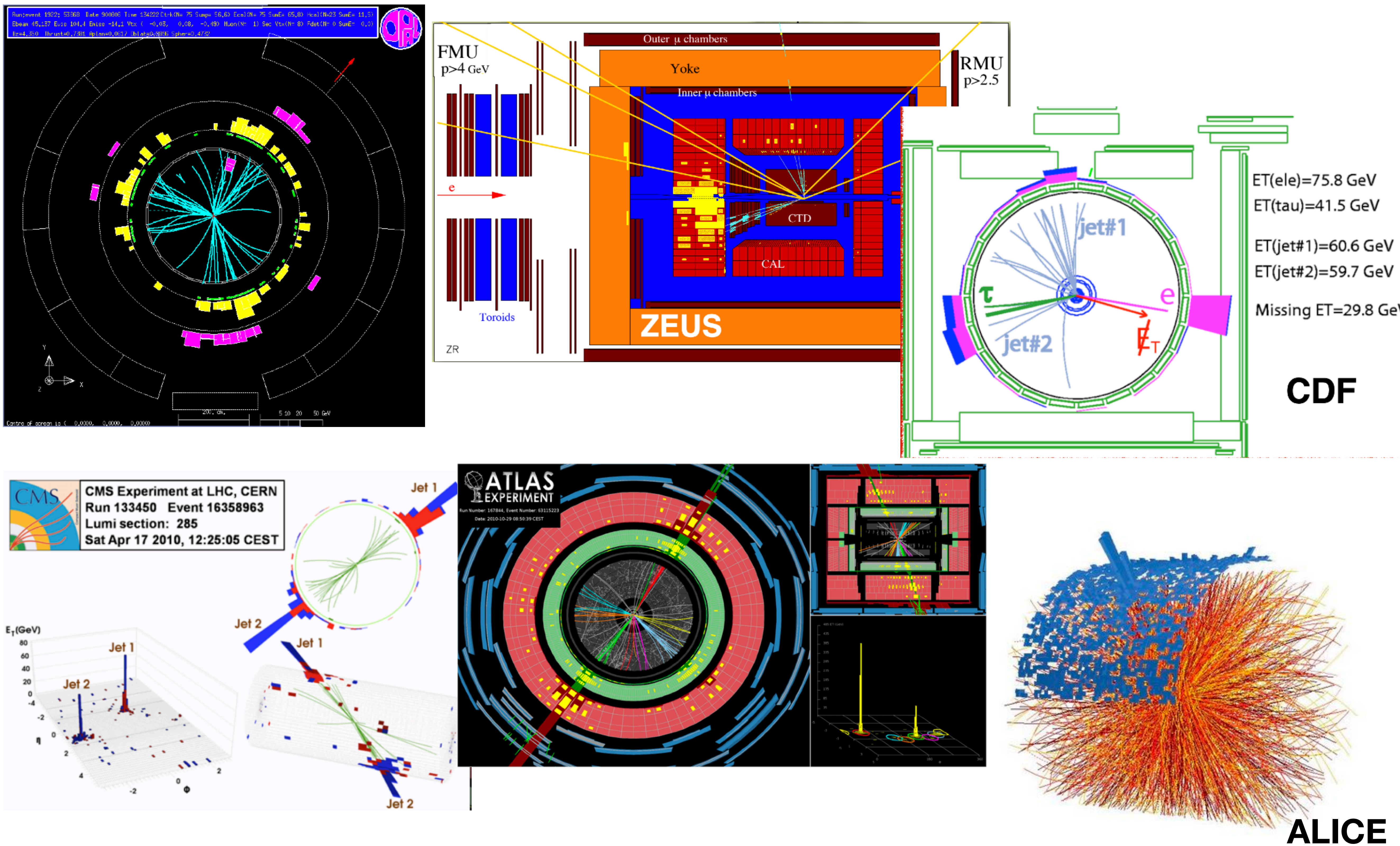


Gluon discovery

$\sqrt{s}$ = 27.4 GeV, in 1979

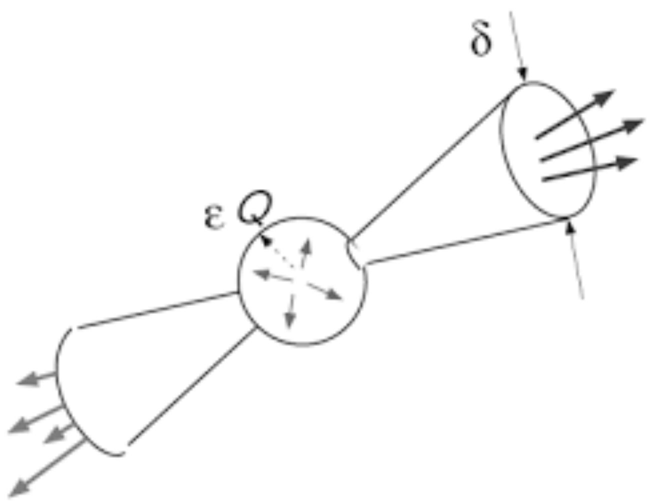
Nowadays jets are everywhere

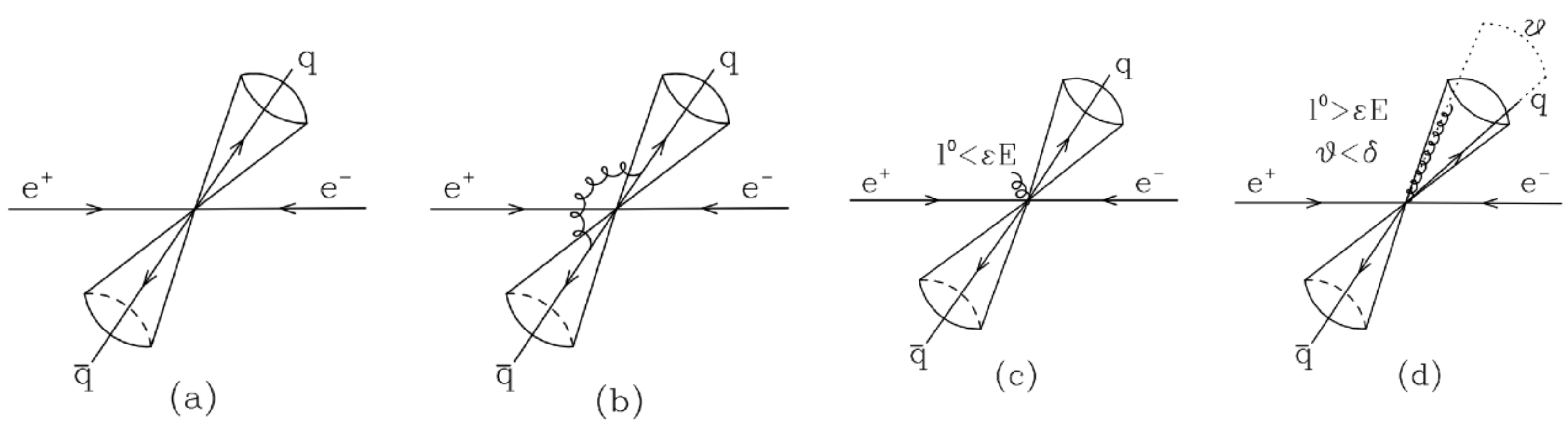
from G. Salam



First IR safe definition for jet

- Sterman-Weinberg jet, 1979



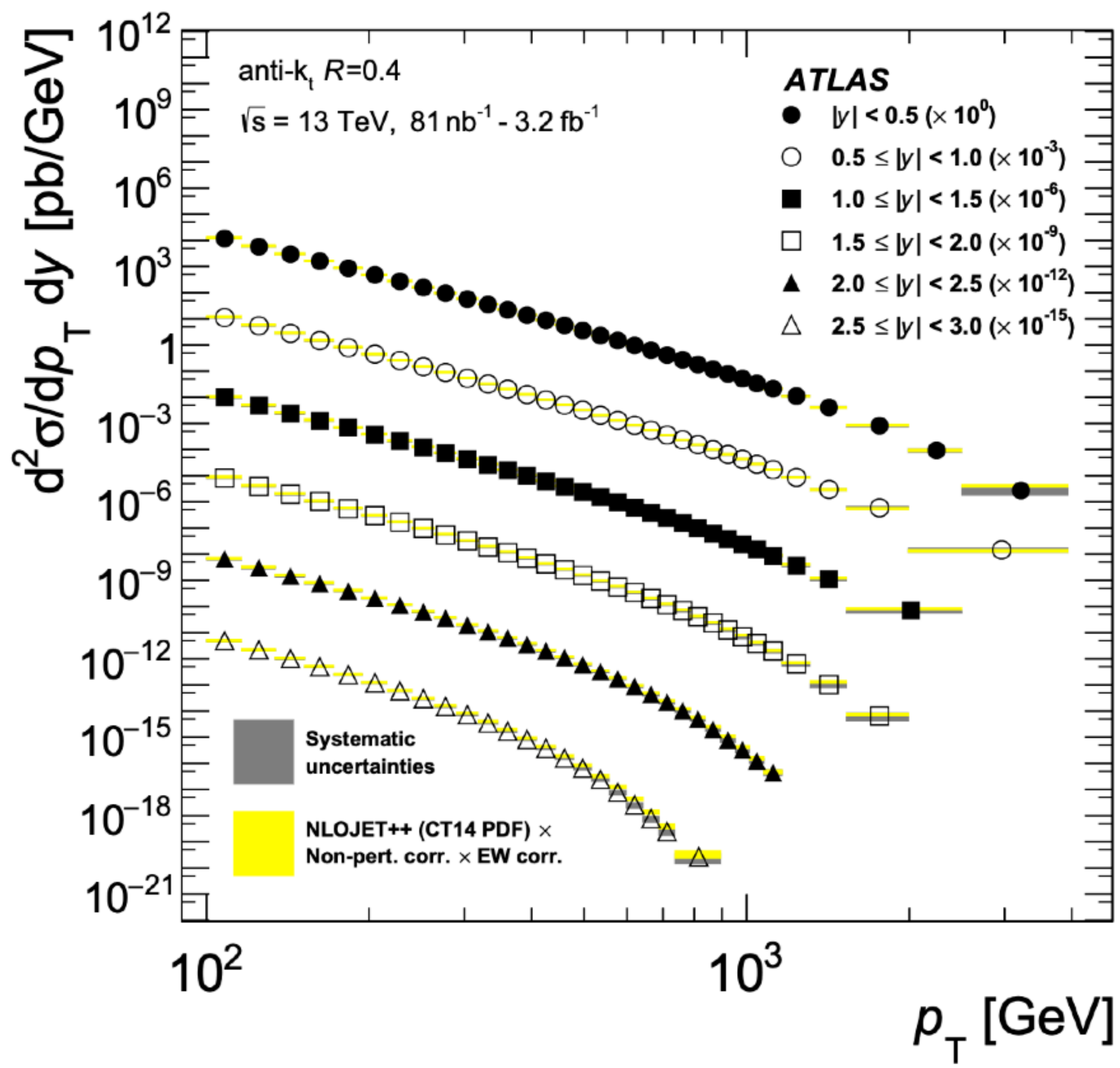


LO **NLO-virtual** **NLO-real out-of-jet** **NLO-real in jet**

- For the first time an operational definition for jet beyond LO in perturbation theory

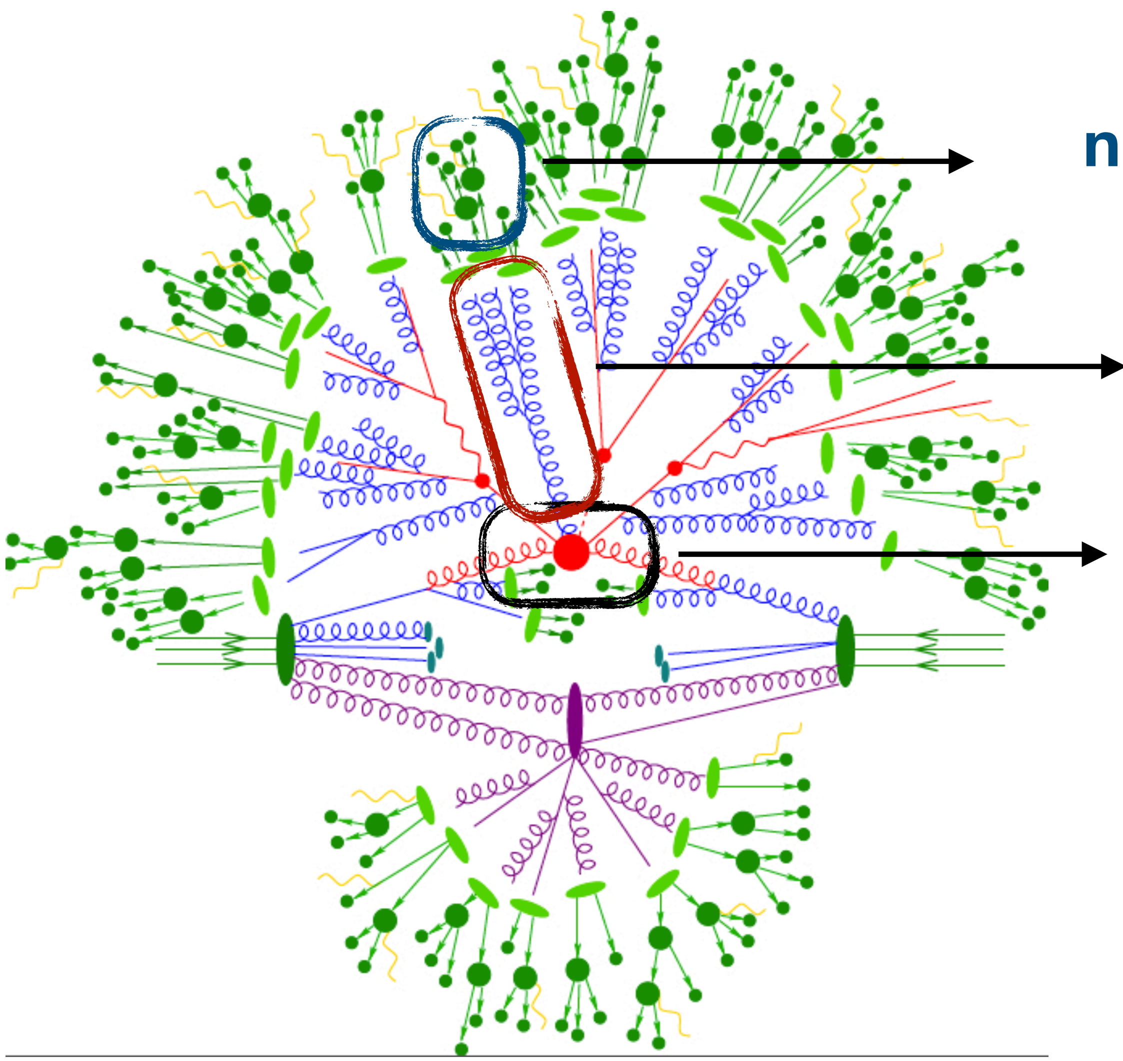
Jet measurements at the LHC

- Since then, a significant theoretical efforts in making precision predictions for jets production
- Resulting in the remarkable success in comparing theory and measurements at the LHC



What we want to learn from jets?

- What do you expect to learn? Isn't it just the good old QCD?



nonperturbative hadronization: soft physics

jet evolution: semi-hard physics

Scattering amplitudes : hard physics

- QCD in several order of magnitude in energy
- Jet evolution is a link between hard and soft physics

JET CALCULUS: A SIMPLE ALGORITHM FOR RESOLVING QCD JETS

K. KONISHI
Rutherford Laboratory, Chilton, Didcot, England

A. UKAWA * and G. VENEZIANO
CERN, Geneva, Switzerland

Summarizing, besides the "classical" ($e^+e^- \to$ hadrons, $ep \to e + X$) and the "standard" (i.e., hard processes involving *one* large scale) applications of IPT, we can imagine "new" applications of IPT to processes involving several large ($\gg 1$ GeV) mass scales. These processes cover most of phase space and involve relatively large cross sections, hence they are phenomenologically very important. Their importance is also theoretical though: understanding semi-hard hadron physics could provide the long sought bridge between hard hadron physics (where parton degrees of freedom are relevant) and soft hadron physics (where resonances, reggeons and pomerons become the right degrees of freedom). It could eventually lead to important hints as to how confinement itself might originate.

- I hope I have convinced you that there is something interesting to learn from jets
- There are many different observables “designed” to probe the structure of jets
- In this talk, I will focus on the simplest one: the energy correlators

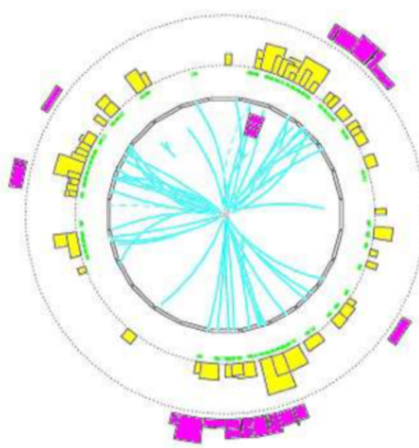
Correlation functions

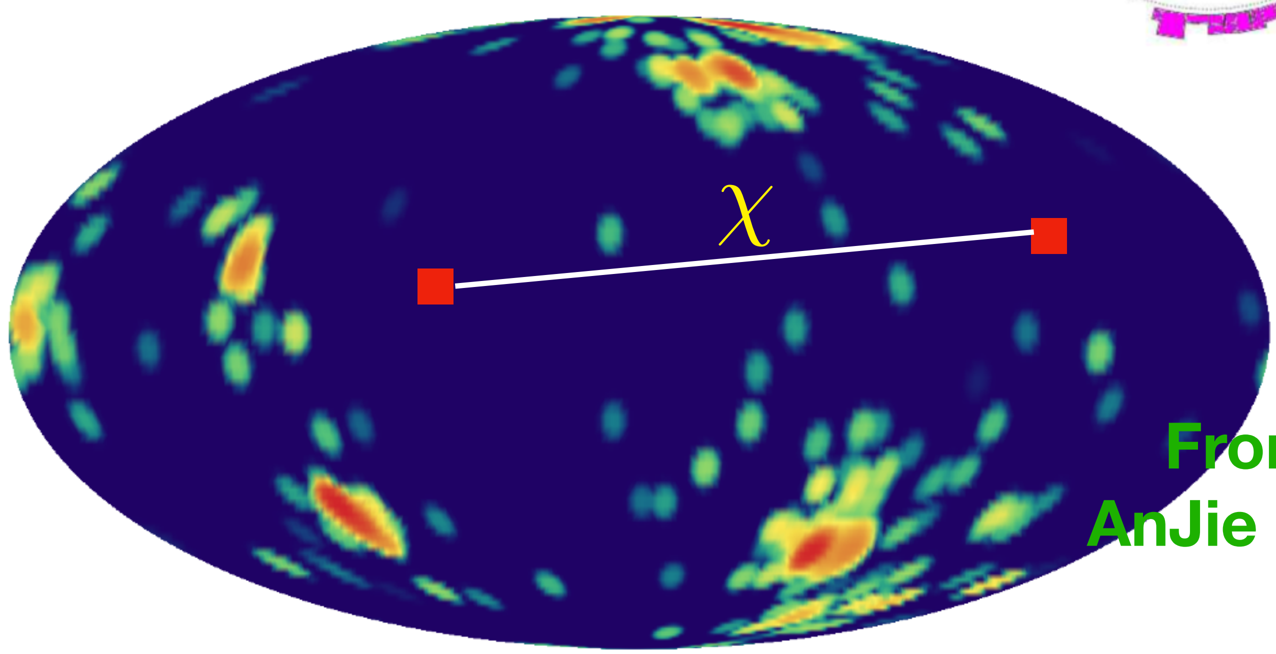
- Correlations are the most basic observables in physics
- Perhaps the most well-known example is the spin-spin correlation in Ising model
- Scattering amplitudes in QFT are related to correlation functions through LSZ reduction
- But amplitudes in gauge theory are not directly observable beyond tree-level, due to the appearance of IR divergences
- Energy Correlators are a class of observables, which are both physical, and infrared finite

Energy-Energy Correlator

- The simplest energy correlator is the two-point energy correlator (EEC)

- Consider e+e- -> anything

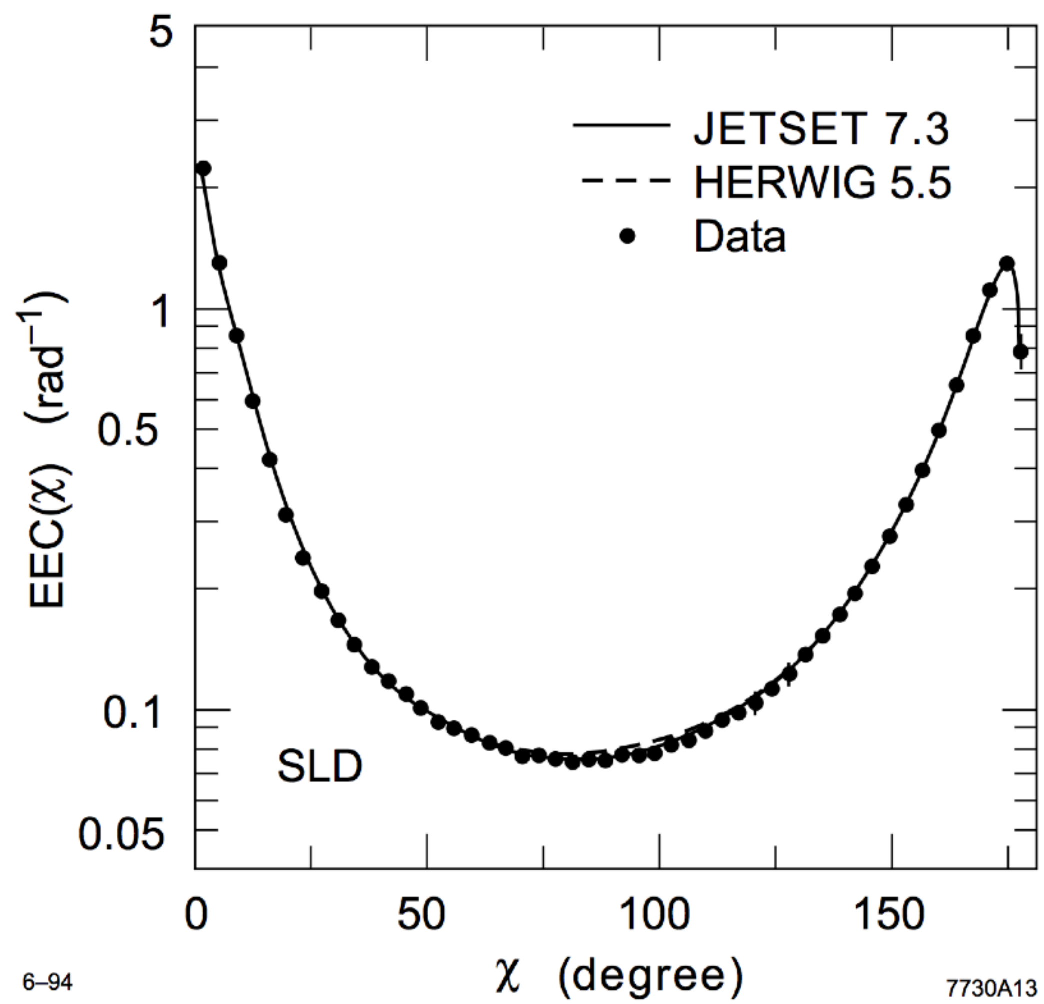




From
AnJie Gao

$$\frac{1}{N_{\text{events}}} \sum_{\text{events}} \int d\Omega_a \int d\Omega_b E_a E_b \delta(\cos\chi - \cos\theta_{ab})$$

- EEC was proposed 40 years ago **Basham, Brown, Ellis, Love, 1978**
- Since then dedicated theoretical and experimental study
- Very accurate experimental measurement!



Analytical calculation beyond Leading Order

- Suppose we want to compute some physical observable O, which is a function of final state momenta:

$$O = \hat{O}(\{k_i\})$$

- Standard textbook teach us that O can be computed from integrating the phase space of k_i

$$\frac{d\sigma}{dO} = \frac{1}{\mathcal{I}} \sum_{n=2}^{\infty} \int \frac{d^3k_1}{(2\pi)^3 2E_1} \cdots \int \frac{d^3k_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum_{i=1}^{n} k_i) |M_{p_1 p_2 \to k_1 \ldots k_n}|^2 \delta(O - \hat{O}(\{k_i\}))$$

- Complications:
  - For individual n, the phase space integrals are usually not IR finite. But the sum is (KLN theorem). Regularization for IR is needed
  - The observable define a very complicated codimension 1 sub-manifold in the 3n-4 dimension phase space

Previous analytical jet observable

- Historically, only very few jet observable are known even just at leading order
- Progress is very slow in a long period

Observable	Full analytic result at LO	Full analytic result at NLO
C-parameter	No[1]	No
Thrust	Yes [De Rujula et al., 1978]	No
Heavy jet mass	Yes[2]	No
EEC	Yes [Basham et al., 1978]	**Not until now**

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(3)}}{\mathrm{d}C}=\frac{\alpha_s}{2\pi}C_F\int_{x_2^-(C)}^{x_2^+(C)}\mathrm{d}x$$

Integral representation for C parameter@LO

$$\times\frac{6x\left[C\left(x^3+(x-2)^2\right)-6(1-x)(1+x^2)\right]}{C(C+6)^2(x-6/(C+6))\sqrt{(6/(C+6)-x)(x_2^+-x)(x-x_2^-)x}}$$

Ellis, Ross, Terrano, 1981

The methods

- Reverse unitarity (phase space becomes "loops")

Anastasiou, Melnikov, 2002

$$\int \frac{d^3 k_1}{(2\pi)^3 2E_1} = \int \frac{d^4 k_1}{(2\pi)^4}(2\pi)\delta_+(k_1^2) = \int \frac{d^4 k_1}{(2\pi)^4}\frac{1}{i}\left(\frac{1}{k_1^2 - i0} - \frac{1}{k_1^2 + i0}\right)$$

- Nonlinear (in Mandelstam variable) propagator

$$\delta(1 - \cos\chi_{ij} - \frac{p_i \cdot p_j}{p_i \cdot Q p_i \cdot Q})$$
$$= \frac{p_i \cdot Q p_i \cdot Q}{2\pi i}\left(\frac{1}{(1-\cos\chi_{ij})p_i \cdot Q p_i \cdot Q - p_i \cdot p_i - i0} - \frac{1}{(1-\cos\chi_{ij})p_i \cdot Q p_i \cdot Q - p_i \cdot p_j + i0}\right)$$

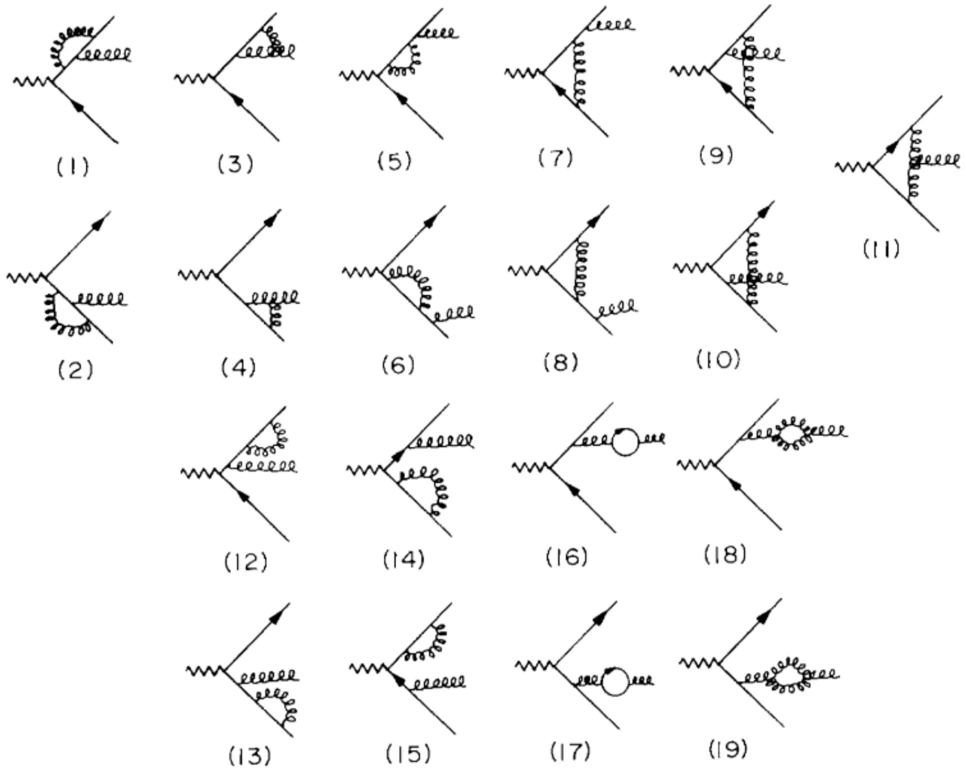
- Integration-By-Parts relation **Tkachov, 1981**

$$\int d^d k \frac{d}{dk^\mu}\mathcal{I}(k) = 0$$

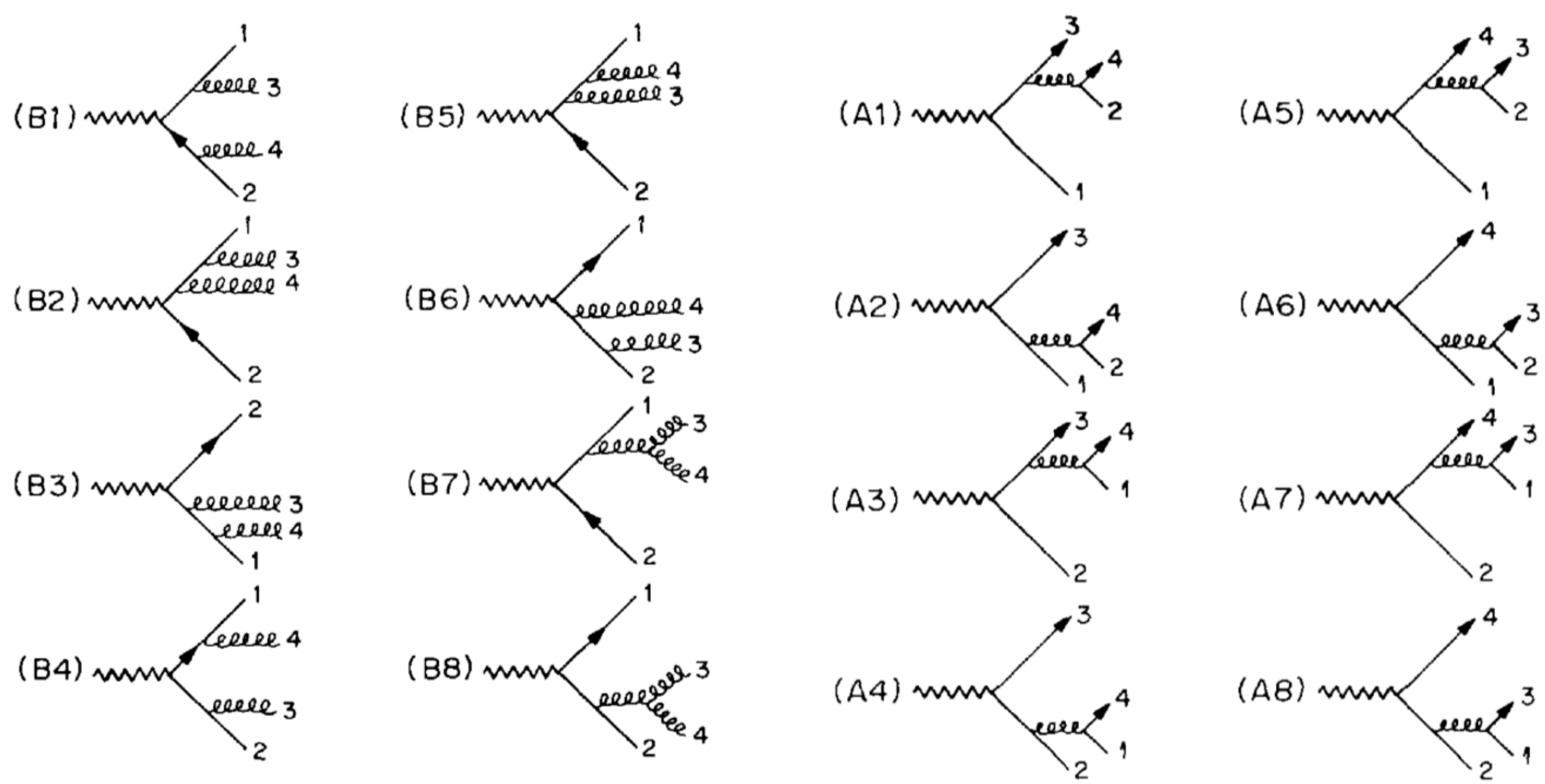
- Differential equation for Feynman integral **Gehrmann, Remiddi, 1997**

$$\frac{d}{dm^2}\int d^d k \frac{1}{k^2 - m^2 + i0} = \int d^d k \frac{d}{dm^2}\left(\frac{1}{k^2 - m^2 + i0}\right)$$

Virtual corrections



real corrections



A little bit detail

- Thanks to IBP reductions, O(10000) integrals reduced to O(40) (initially takes 3 days on 18 core server with 128G RAM. Now takes only 1 hours)

- System of differential equations can be casted into canonical form **Henn, 2013**

$$d\vec{f}(z,\epsilon) = \epsilon\left[d\sum_k A_k \ln \alpha_k(z)\right]\vec{f}(z,\epsilon) \qquad z = \frac{1-\cos\chi}{2}$$

- Alphabet:

$$\alpha_k(z) \in \{z, 1-z, x, 1-x, y, 1-y, 1+y\}$$

$$x = \sqrt{z}, \quad y = i\frac{\sqrt{z}}{\sqrt{1-z}}$$

The space of function

- At the NLO, EEC is fully described by the following set the uniform weight special function

$$g_1^{(1)} = \log(1-z)\,, \qquad g_2^{(1)} = \log(z)\,, \qquad g_1^{(2)} = 2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\,,$$

$$g_2^{(2)} = \mathrm{Li}_2(1-z) - \mathrm{Li}_2(z)\,,$$

$$g_3^{(2)} = -2\,\mathrm{Li}_2\left(-\sqrt{z}\right) + 2\,\mathrm{Li}_2\left(\sqrt{z}\right) + \log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z)\,, \qquad g_4^{(2)} = \zeta_2\,,$$

$$g_1^{(3)} = -6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right) - \zeta_3\right] - \log\left(\frac{z}{1-z}\right)\left(2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\right)\,,$$

$$g_2^{(3)} = -12\left[\mathrm{Li}_3(z) + \mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right] + 6\,\mathrm{Li}_2(z)\log(1-z) + \log^3(1-z)\,,$$

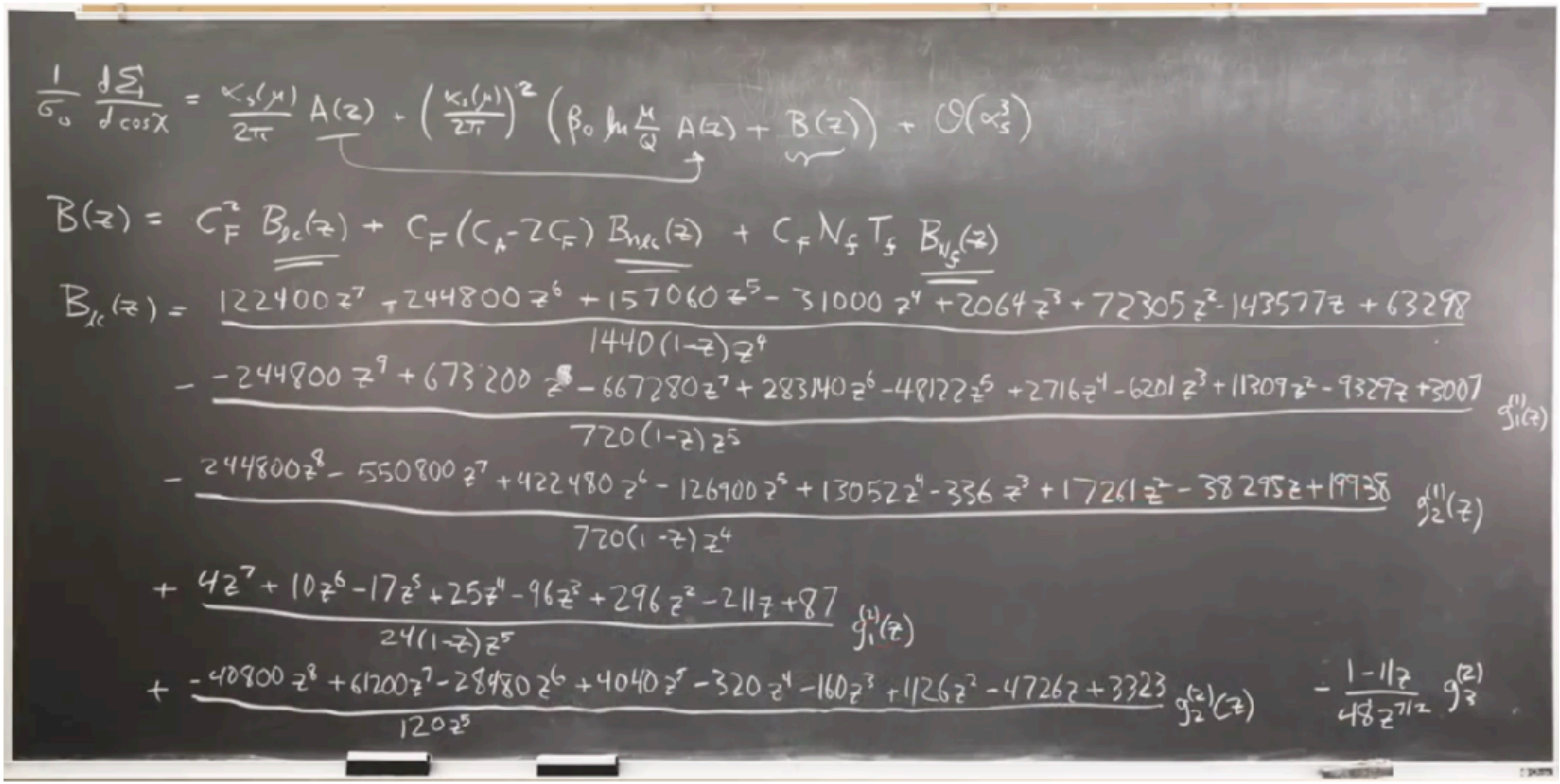
$$g_3^{(3)} = 6\log(1-z)\left(\mathrm{Li}_2(z) - \zeta_2\right) - 12\,\mathrm{Li}_3(z) + \log^3(1-z)\,,$$

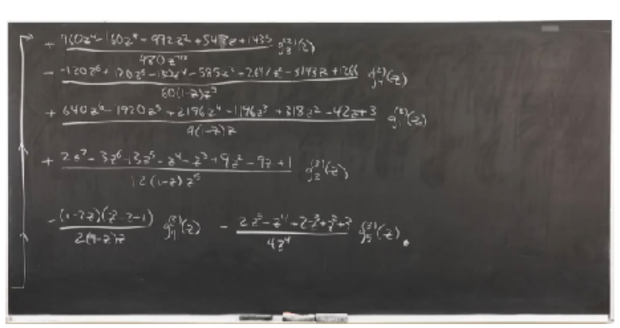
$$g_4^{(3)} = \mathrm{Li}_3\left(-\frac{z}{1-z}\right) - 3\,\zeta_2\log(z) + 8\,\zeta_3\,,$$

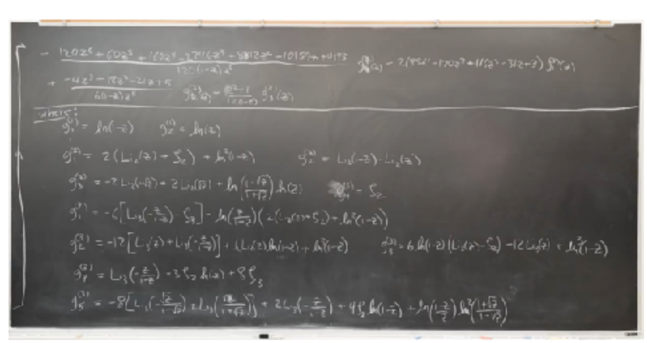
$$g_5^{(3)} = -8\left[\mathrm{Li}_3\left(-\frac{\sqrt{z}}{1-\sqrt{z}}\right) + \mathrm{Li}_3\left(\frac{\sqrt{z}}{1+\sqrt{z}}\right)\right] + 2\mathrm{Li}_3\left(-\frac{z}{1-z}\right)$$
$$+ 4\zeta_2\log(1-z) + \log\left(\frac{1-z}{z}\right)\log^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\,.$$

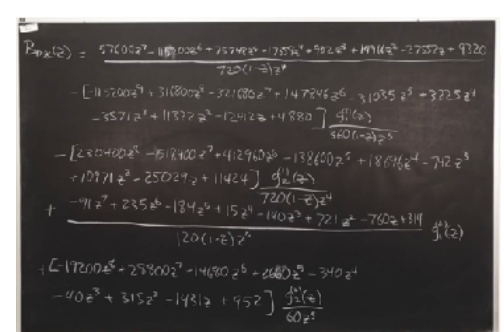
The analytical NLO results

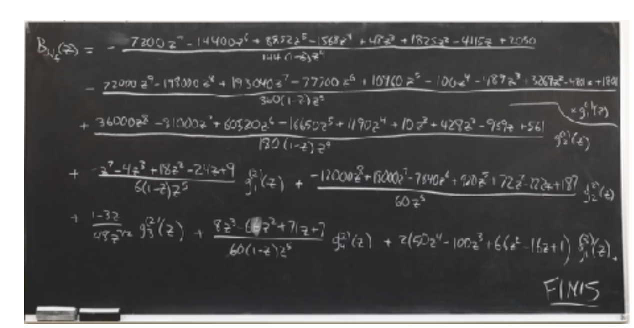
Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018





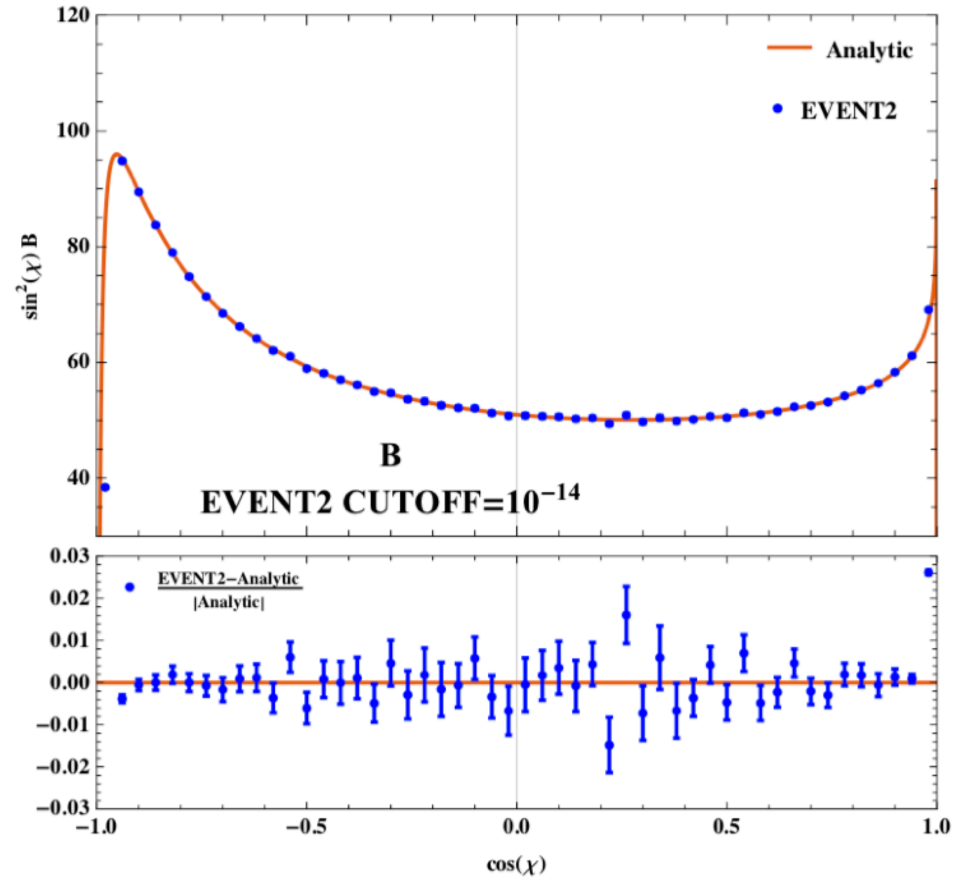


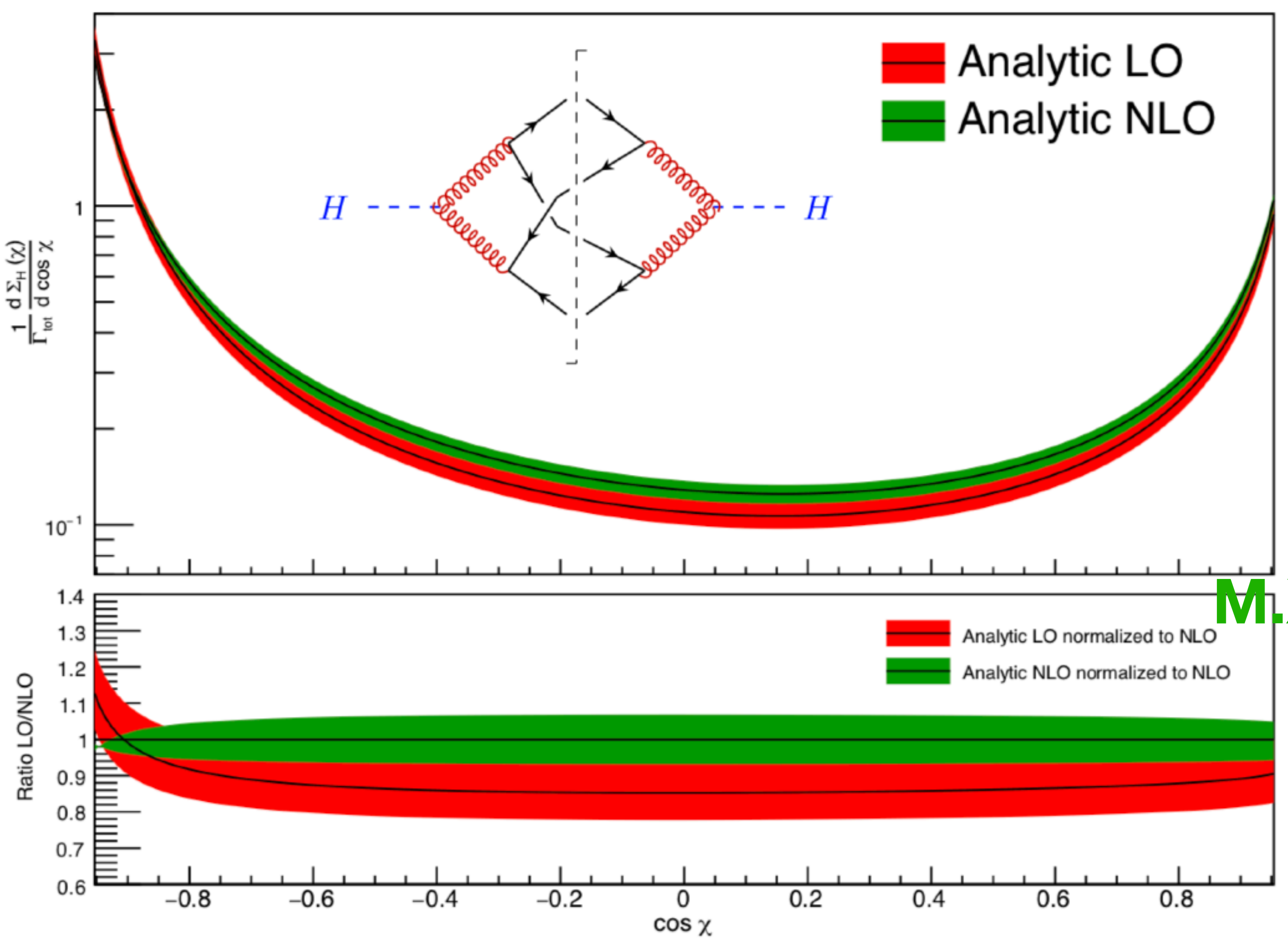




Did we get it right?

- Excellent agreement with numerical calculation (done some 13 years ago!)

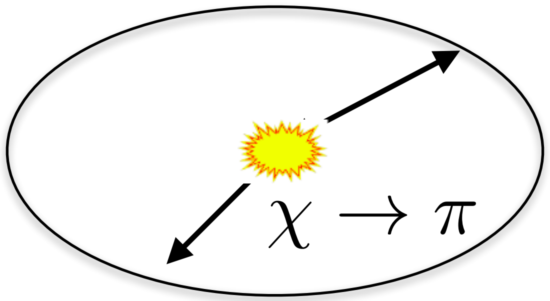




- Using the same method, we also calculate the EEC distribution in hadronic Higgs decay

M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2019

- No numerical results to compared with this time!

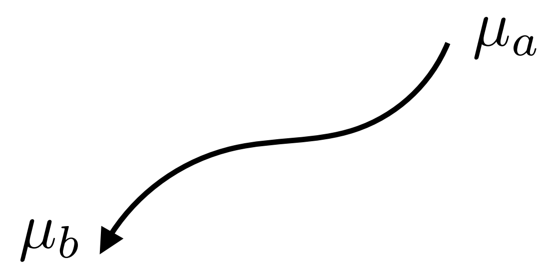


Back-to-back limit

Appearance of large logarithms

- One benefit of having analytical results is making investigation of the structure easy

- In QFT, one typically encounters large logarithms due to RG running between two different scales



$$c_1 \alpha \ln \frac{\mu_a}{\mu_b} + c_2 \alpha^2 \ln^2 \frac{\mu_a}{\mu_b} + \ldots + c_k \alpha^k \ln^k \frac{\mu_a}{\mu_b} + \ldots$$

- They comes from UV divergence in the theory, and are single logarithmic. Can be resummed using RG equation

$$\left(\frac{\mu_a}{\mu_b} \right)^{\gamma(\alpha)}$$

Sudakov double logarithms

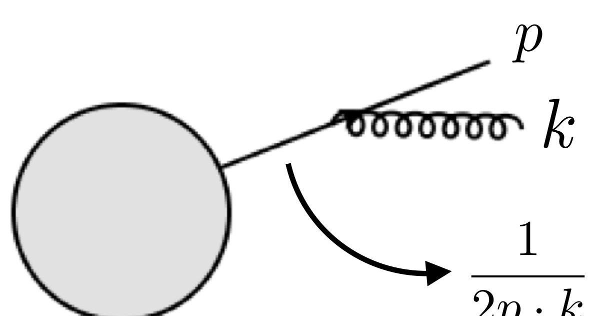
- Instead of single logarithms, we encounter double logarithms in the back-to-back limit

$$\frac{1}{\sigma_0}\frac{d\sigma}{dz} = \frac{\alpha_s}{4\pi}C_F\left(-\frac{\ln(1-z)}{1-z} + \ldots\right) + (\frac{\alpha_s}{4\pi})^2 C_F^2 \left(4\frac{\ln(1-z)}{1-z} + \ldots\right)$$

$$z = \frac{1-\cos\chi}{2}$$

$$\int_0^z dz' \frac{\ln(1-z')}{1-z'} \sim \ln^2(1-z)$$

- They are Sudakov double logarithms. They arise from soft-collinear radiation of massless gauge theory



$$\frac{1}{2p\cdot k} = \frac{1}{2E_q E_g(1-\cos\theta)}$$

$$\int_{E_0} \frac{dE_g}{E_g}\int_{\theta_0}\frac{d\theta}{\theta} \sim \ln E_0 \ln\theta_0$$

- The proper effective theory for resumming soft/collienar logarithms is Soft-Collinear Effective Theory (SCET)

Brief introduction to SCET

- SCET is an EFT for separating different modes with hierarchy in virtuality (offshellness)

$$\lambda \ll 1$$

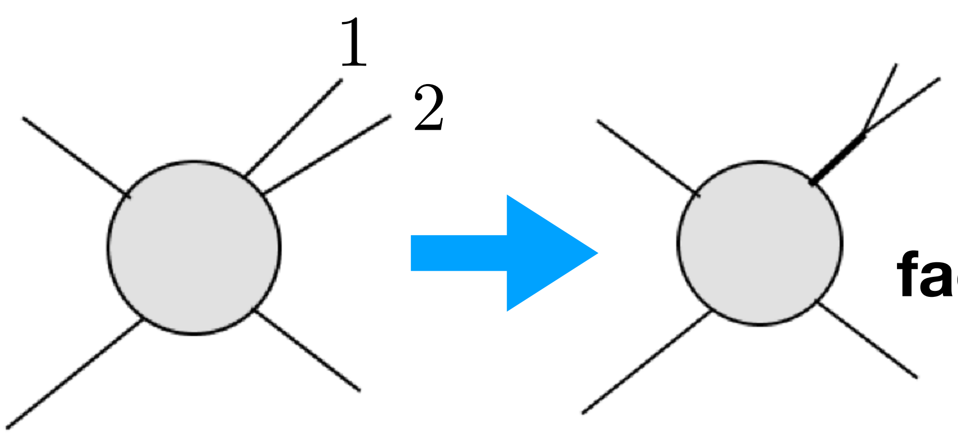
$$n^\mu = (1, \vec{n}), \quad \bar{n}^\mu = (1, -\vec{n})$$

$$p^\mu = \frac{1}{2}\bar{n}\cdot p n^\mu + \frac{1}{2} n \cdot p \bar{n}^\mu = (p^+, p^-, p_\perp)$$

hard mode: $p_h = Q(1,1,1)$

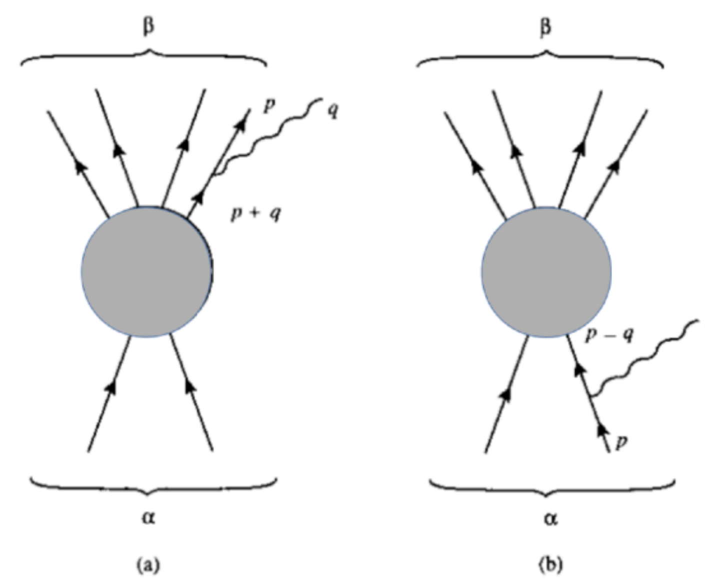
collinear mode: $p_c = Q(1, \lambda^2, \lambda)$

soft mode: $p_s = Q(\lambda, \lambda, \lambda)$



collinear factorization

$$A_n(\ldots, 1^{\lambda_1}, 2^{\lambda_2}, \ldots) \overset{1||2}{\Rightarrow} \sum_{\lambda\pm} \mathrm{Split}_{-\lambda}^{P\to 12}(x, 1^{\lambda_1}, 2^{\lambda_2}) A_{n-1}(\ldots, P^\lambda, \ldots)$$



Weinberg's soft theorem

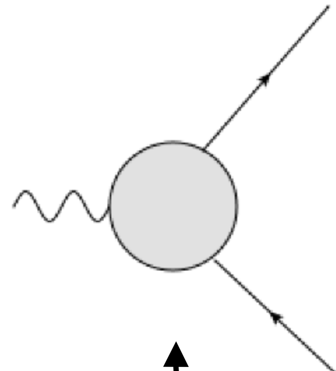
$$\varepsilon_\mu(q) M^\mu_{n+1}(q) \overset{E_q\to 0}{=} M_n \varepsilon_\mu(q) \sum_n \frac{e_n p_n^\mu}{p_n \cdot q + i\epsilon}$$

$$\mathcal{L} = \bar{\hat{\xi}}_n \left(i n\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \hat{\xi}_n$$

$$S_{n+}(z) = P \exp\left[ig \int_{-\infty}^{0} ds\; n\cdot A_{us}(z + sn) \right]$$

Back-to-back factorization for EEC

Moult, HXZ, 2018

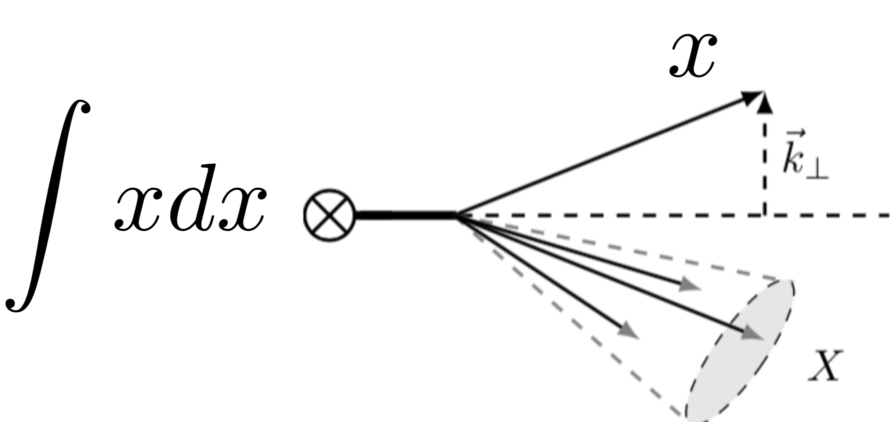


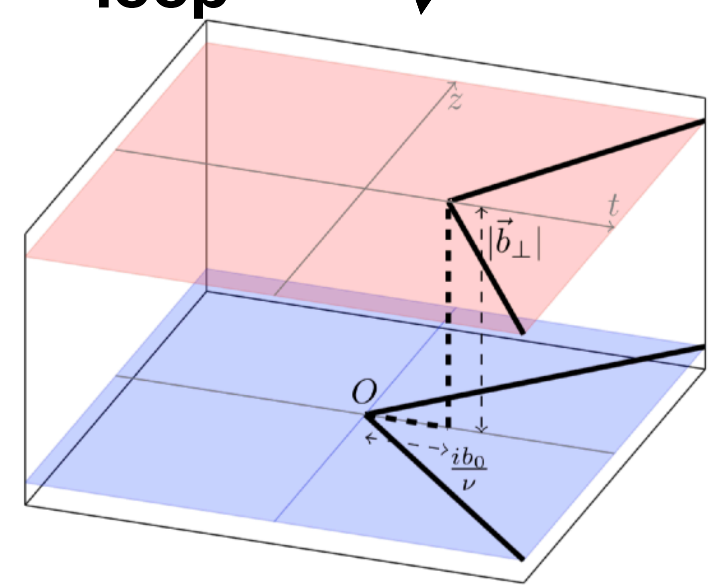
hard function:
form factor for vector current

$$\frac{d\sigma}{dz} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J^q_{\rm EEC}(\vec{b}_\perp,\mu,\nu) J^{\bar{q}}_{\rm EEC}(\vec{b}_\perp,\mu,\nu) S_{\rm EEC}(\vec{b}_\perp,\mu,\nu)\,\delta\left(1-z-\frac{\vec{k}_\perp^2}{Q^2}\right)$$

jet function:
N=2 moment of TMD fragmentation function

Soft function:
semi-infinite light-like Wilson loop



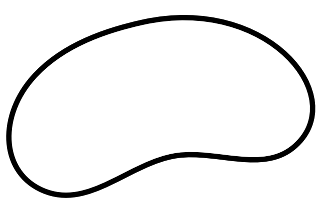


Where does the Sudakov logs come from?

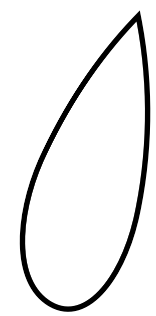
- Double logs appears in hard, jet, and soft functions

- Take soft function as an example. Renormalization of Wilson loop

Polyakov, 1980



$$\approx \gamma_s(\alpha)\ln(L\Lambda_{\mathrm{UV}})$$

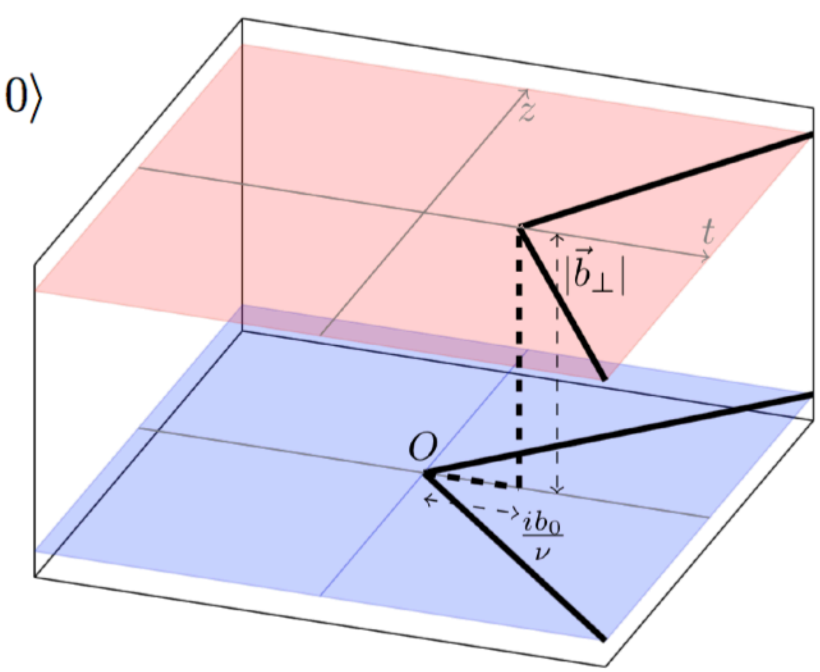


$$\approx \Gamma_{\mathrm{cusp}}(\alpha_s,\phi)\ln^2(L\Lambda_{\mathrm{UV}}) + \gamma_s(\alpha)\ln(L\Lambda_{\mathrm{UV}})$$

$$S_{\mathrm{EEC}}(\vec{b}_\perp,\mu,\nu) = \lim_{\nu\to+\infty}\frac{1}{N_c}\mathrm{tr}\langle 0|T\left[S^\dagger_{\bar{n}+}(0)S_{n-}(0)\right]\bar{T}\left[S^\dagger_{n+}\left(y_\nu(\vec{b}_\perp)\right)S_{\bar{n}-}\left(y_\nu(\vec{b}_\perp)\right)\right]|0\rangle$$

semi-infinite light-like Wilson line

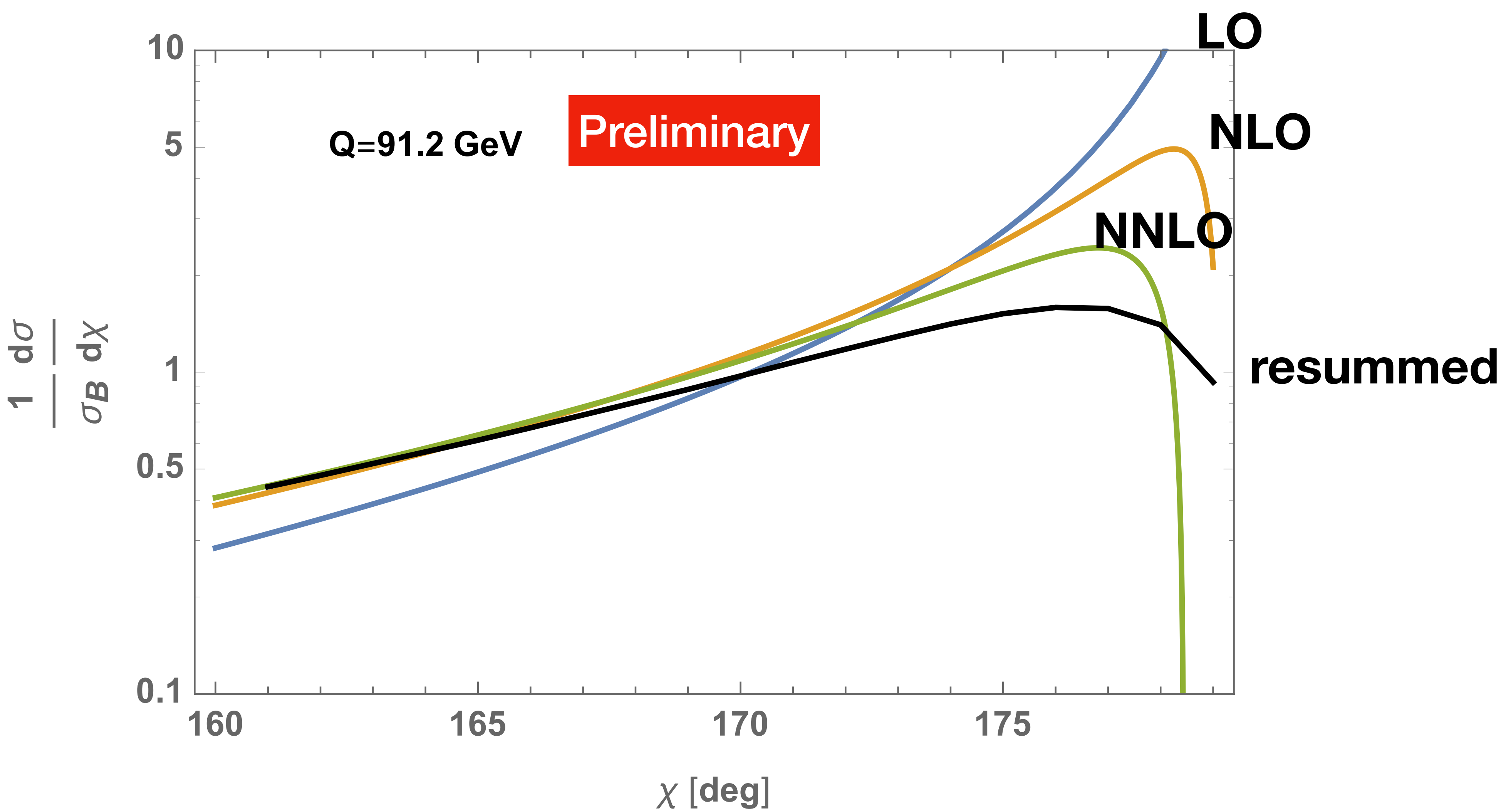
$$\mu\frac{dS_{\mathrm{EEC}}(\vec{b}_\perp,\mu,\nu)}{d\mu} = \left[2\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{\mu^2}{\nu^2} - 2\gamma^s_{\mathrm{EEC}}(\alpha_s)\right]S_{\mathrm{EEC}}(\vec{b}_\perp,\mu,\nu)$$

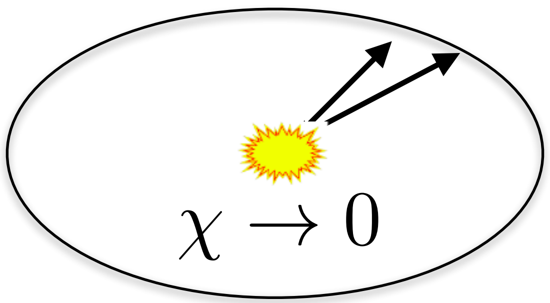


From factorization to resummation

- Solving the RG equation for the hard, jet, and soft function resum the Sudakov logarithms to all order

- Such a resummation is important to compare theory with experiment





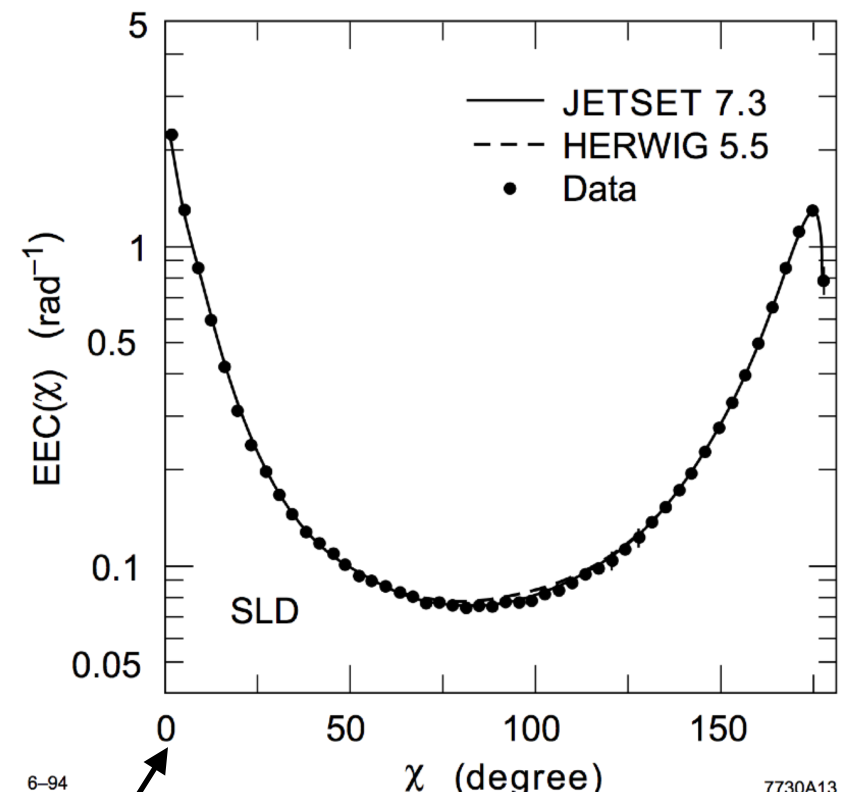
Collinear limit from factorization approach

Single logarithmic collinear divergence

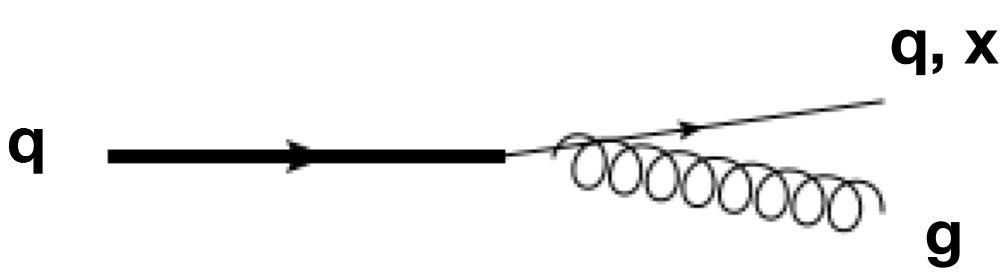
$$\chi^2 \sim z$$

- In the small angle limit, cross section diverges as 1/z

$$\frac{d\sigma}{dz} = \alpha_s \frac{1}{z} + \alpha_s^2 \frac{\ln z}{z} + \ldots + \alpha_s^3 \frac{\ln^2 z}{z} + \ldots$$



collinear limit



$$P_{T,qq}(x) \sim \frac{1}{\chi_{qg}^2}\left(\frac{1+x^2}{1-x}\right)$$

time-like splitting kernel

Leading Log resummation

- LL series resummed long time ago by jet calculus

Konishi, Ukawa, Veneziano, 1979

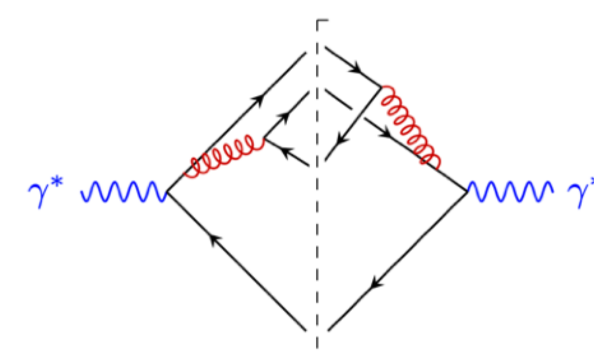
$$\sum_{a_1 a_2} D_{a_1 a_2, i}(1, 1; p_T^2, Q^2) = \frac{\alpha_s(4p_T^2)}{p_T^2\, 2\pi} \sum_{a,j} (-A_2)_{aj} \left[\frac{\alpha_s(4p_T^2)}{\alpha_s(Q^2)}\right]_{ji}^{A_2/2\pi b}, \quad (5.17)$$

and the same with $4p_T^2 \to \delta^2 Q^2$.

- Attempt to go beyond LL with limited success

Kalinowski, Konishi, Scharbach, Taylor, 1980

$$E(x_1,x_2) = 2\left[-2 + \frac{1+x_1}{1-x_2} + \frac{1+x_2}{1-x_1} - \frac{2x_1}{(1-x_2)^2} - \frac{2x_2}{(1-x_1)^2} + \left(2 - \frac{1+x_1}{1-x_2} - \frac{1+x_2}{1-x_1} + \frac{2}{(1-x_1)(1-x_2)}\right)\log\left(\frac{(1-x_1)(1-x_2)}{1-x_1-x_2}\right)\right]$$



Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018

$$\int_0^1 dx_1 \int_0^{1-x_2} dx_2\, E(x_1,x_2)\,[x_1 x_2 + (x_1+x_2)(1-x_1-x_2)] = -4\zeta_3 + \frac{43}{3}\zeta_2 - \frac{8011}{432}$$

1/z Coefficient of 4 quark interference

NLL resummation not available in general!

All-order factorization for z→0

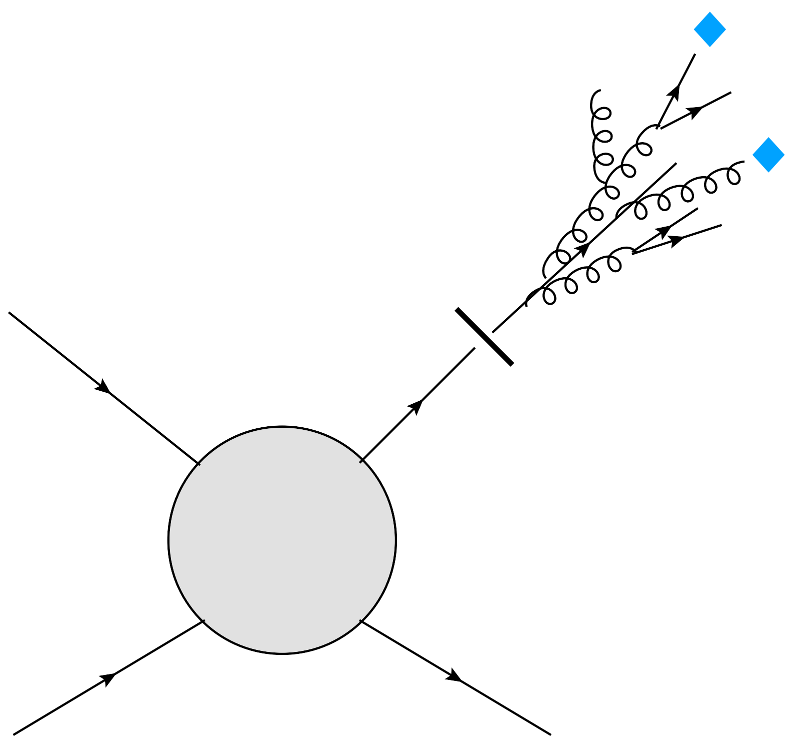
Dixon, Moult, HXZ, to appear

- Cumulant $$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^z dz' \, \Sigma(z', \ln\frac{Q^2}{\mu^2}, \mu)$$

$$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}^T(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)$$

$$z = \frac{q_T^2}{x^2Q^2}$$

Fixed by kinematics and dimension analysis

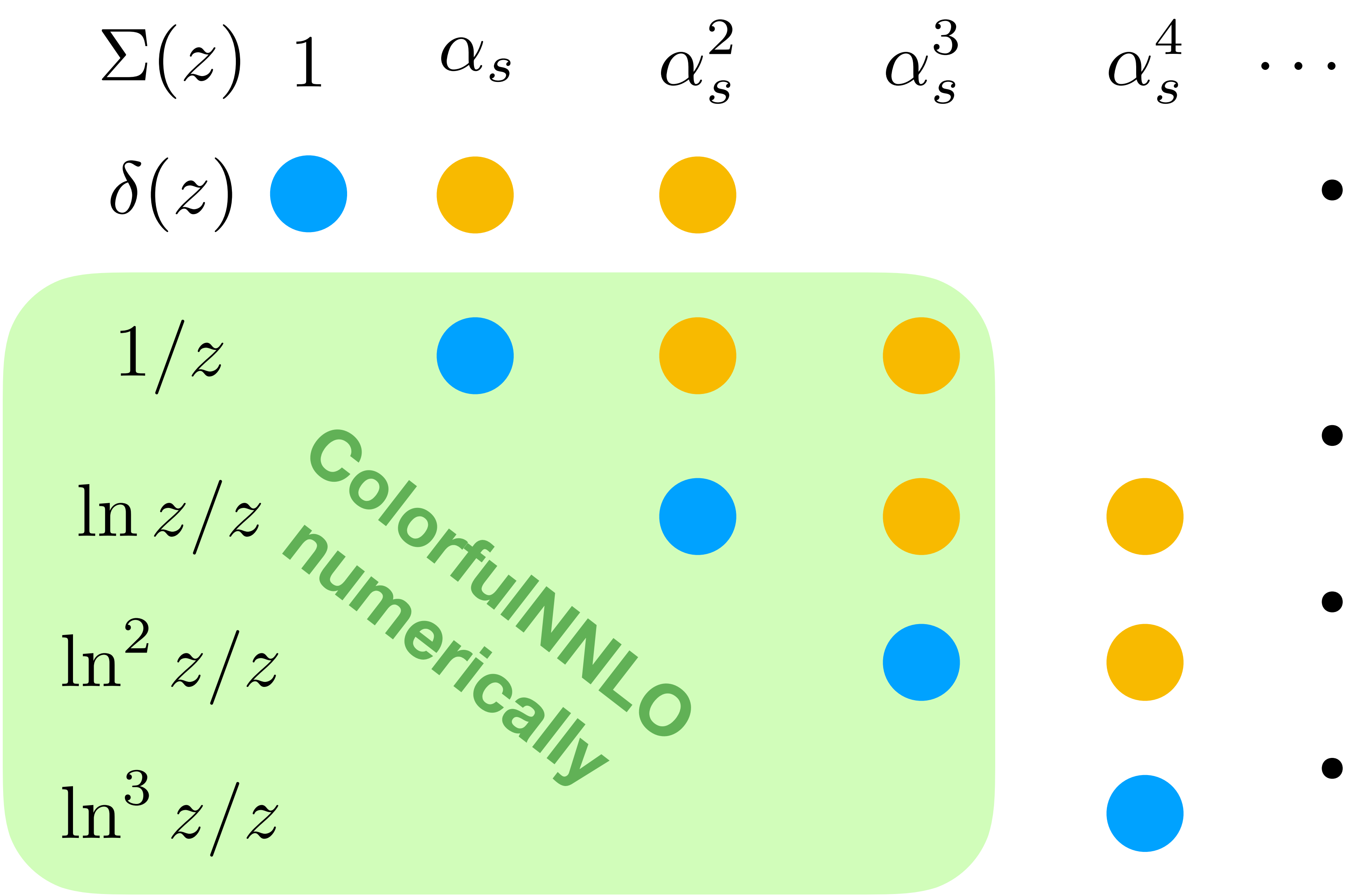


Full interference effects retained in H and J, separately

- Both jet and hard function are vector in flavor space
- H_q (H_g) : probability of finding a quark (gluon) with momentum fraction x
- J_q (J_g) : probability of finding two parton with momentum fraction y_1, y_2 and relative transverse momentum q_T in quark (gluon) initiated jet, weighted by y_1*y_2

Counting the order

- LL **Konishi, Ukawa, Veneziano, 1979**
- NLL + NNLL **Dixon, Moult, HXZ, 2019, to appear**



- To get to NNLL require:

- NNLO splitting kernel
  Moch, Vermasseren, Vogt
- NNLO hard function
  Mitov, Moch, 2006;
  Almasy, Moch, Vogt, 2011
- NNLO jet function
  Very challenging!

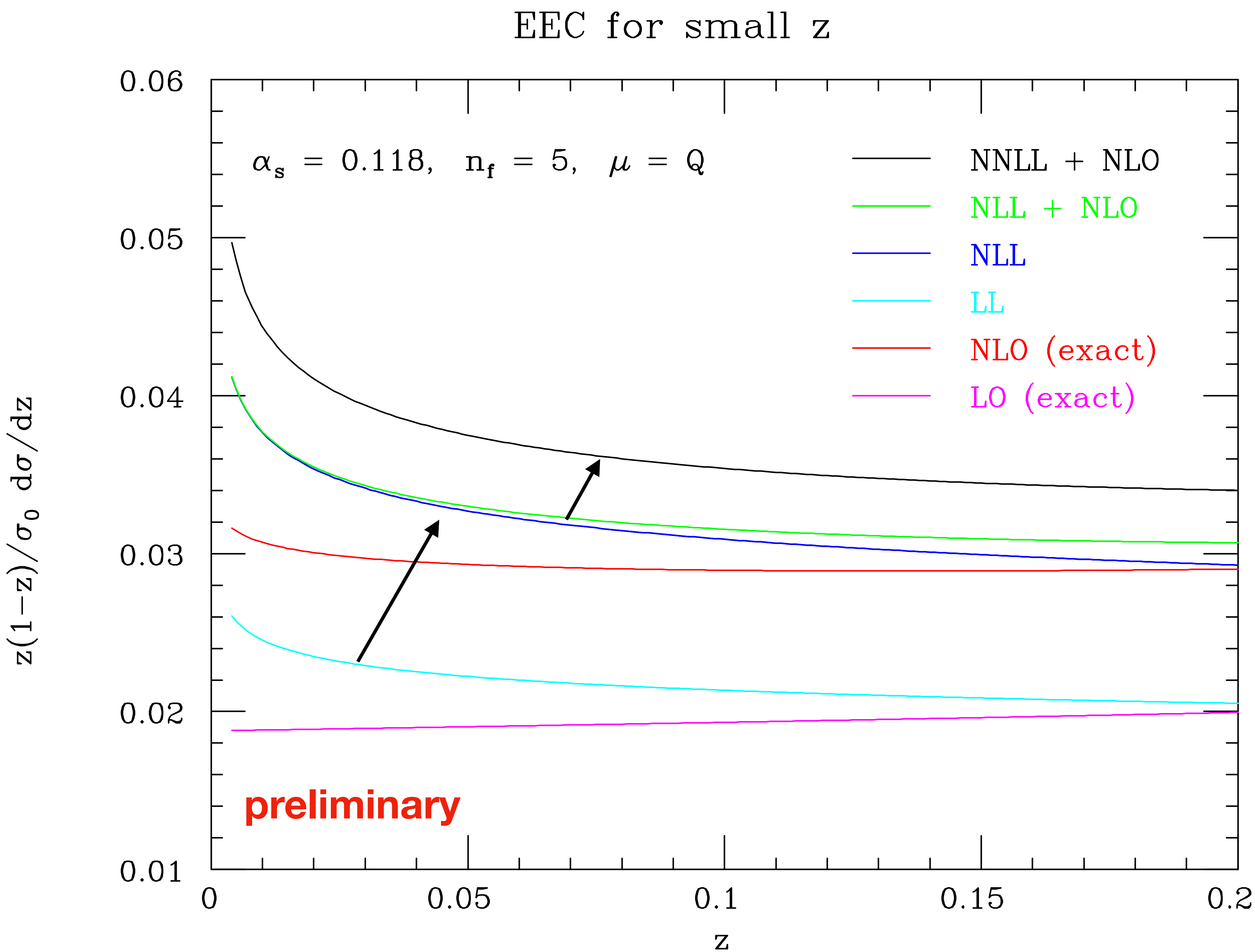
$$
\begin{aligned}
z\Sigma(z) = \ & \frac{\alpha_s}{4\pi}\frac{3C_F}{2} \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg[C_A C_F \left(-\frac{50\zeta_2}{3} + 4\zeta_3 - \frac{107\log(z)}{15} + \frac{35366}{675}\right) + C_F n_f \left(\frac{53\log(z)}{60} - \frac{4913}{900}\right) \\
& + C_F^2 \left(\frac{86\zeta_2}{3} - 8\zeta_3 + \frac{25\log(z)}{4} - \frac{8263}{216}\right) \Bigg] \\
& + \left(\frac{\alpha_s}{4\pi}\right)^3 \Bigg[C_A C_F n_f \left(\frac{379579\zeta_2}{5400} + \frac{3679\zeta_3}{30} - \frac{118\zeta_4}{3} + \left(-\frac{108\zeta_2}{5} + \frac{16\zeta_3}{3} + \frac{6644267}{54000}\right)\log(z) \right. \\
& \left. - \frac{16259\log^2(z)}{1800} - \frac{1025118113}{2160000}\right) \\
& + C_A C_F^2 \left(-\frac{400\zeta_2^2}{3} + \frac{137305\zeta_2}{216} - 72\zeta_2\zeta_3 + \frac{10604\zeta_3}{15} + \frac{4541\zeta_4}{6} - 216\zeta_5 \right. \\
& \left. + \left(-\frac{1100\zeta_2}{3} + \frac{262\zeta_3}{3} + \frac{105425}{144}\right)\log(z) - \frac{340}{9}\log^2(z) - \frac{105395741}{51840}\right) \\
& + C_A^2 C_F \left(-\frac{906257\zeta_2}{2700} + 24\zeta_2\zeta_3 - \frac{47483\zeta_3}{90} - \frac{481\zeta_4}{6} + 56\zeta_5 + \left(\frac{503\zeta_2}{5} - \frac{74\zeta_3}{3} - \frac{2916859}{6750}\right)\log(z) \right. \\
& \left. + \frac{8059\log^2(z)}{300} + \frac{964892417}{540000}\right) \\
& + C_F^2 n_f \left(-\frac{15161\zeta_2}{120} - \frac{7994\zeta_3}{45} + \frac{236\zeta_4}{3} + \left(\frac{416\zeta_2}{9} - \frac{32\zeta_3}{3} - \frac{6760183}{64800}\right)\log(z) + \frac{4619\log^2(z)}{720} \right. \\
& \left. + \frac{164829499}{486000}\right) + C_F n_f^2 \left(\frac{6\zeta_2}{5} + \frac{23\log^2(z)}{45} - \frac{8867\log(z)}{1350} + \frac{88031}{4500}\right) \\
& + C_F^3 \left(\frac{688\zeta_2^2}{3} - \frac{18805\zeta_2}{216} + 48\zeta_2\zeta_3 + 52\zeta_3 - 1130\zeta_4 + 208\zeta_5 + \left(\frac{1849\zeta_2}{9} - \frac{172\zeta_3}{3} - \frac{723533}{2592}\right)\log(z) \right. \\
& \left. + \frac{625\log^2(z)}{48} + \frac{742433}{1944}\right) \Bigg] \\
& + \mathcal{O}(\alpha_s^4)
\end{aligned}
$$

- The results obey leading transcendental principle: setting CF=CA, the leading transcendental series agree with N=4 SYM.

- Furthermore, this leading transcendental series is simply AD of twist-two spin 1 anomalous dimension in N=4 SYM $\gamma_{\text{uni}}(1)$. Maybe the first all order prediction in QCD cross section for the leading transcendental part!

Numerical impact of NNLL

Dixon, Moult, HXZ, to appear



EEC in N=4 sYM

EEC as four point correlator

- The nice thing about EEC is that it can be defined directly in terms of field theory correlator **Hofman, Maldacena, 2008**

$$\langle \mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)\rangle_Q = \sigma^{-1}\int \mathrm{d}^4x\, e^{ix\cdot Q}\langle 0|O^\dagger(x)\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)O(0)|0\rangle$$

$$\mathcal{E}(\boldsymbol{n}) = \int_0^\infty \mathrm{d}t \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\boldsymbol{n})$$

energy flow operator

- The operator O produce localized excitation from vacuum

- In N=4 sYM, one can choose the operator O to be in the same supermultiplet of the energy-momentum tensor

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_E = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2}\Phi(u,v;a) \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{23}^2 x_{41}^2}{x_{13}^2 x_{24}^2}$$

↑ **known to very high order!**

EEC in N=4 SYM

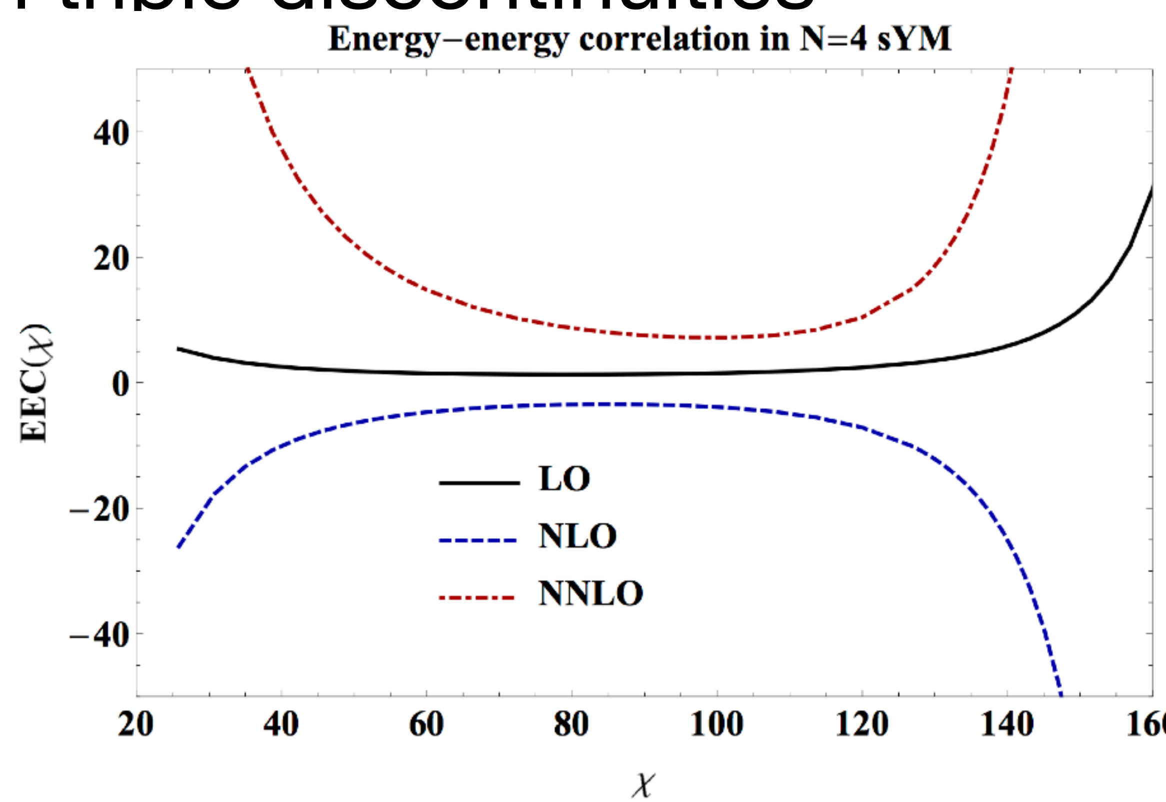
- The only difficulty is that EEC is defined as Wightman correlation with Minkowski signature, while known results are for Euclidean correlator. Require very non-trivial analytical continuation

- This has been done at NLO by Mellin space techniques

- Very recently, exploring the Lorentzian inversion formula, EEC can be computed through triple discontinuities

Henn, Sokatchev, Y. Kai, Zhibodove, 2019

$$\mathrm{EEC}(\zeta) = \frac{1}{4\pi^3\zeta^2} \lim_{\epsilon\to 0} \int_0^1 d\bar{z} \int_0^{\bar{z}} dt \, \frac{1}{t(\zeta-\bar{z})+(1-\zeta)\bar{z}} \times \mathrm{dDisc}_{\bar{z}=1}\mathrm{Disc}_{z=0}\left[(z-\bar{z})\mathcal{G}_\epsilon(z,\bar{z})\right] \,, \quad (7)$$



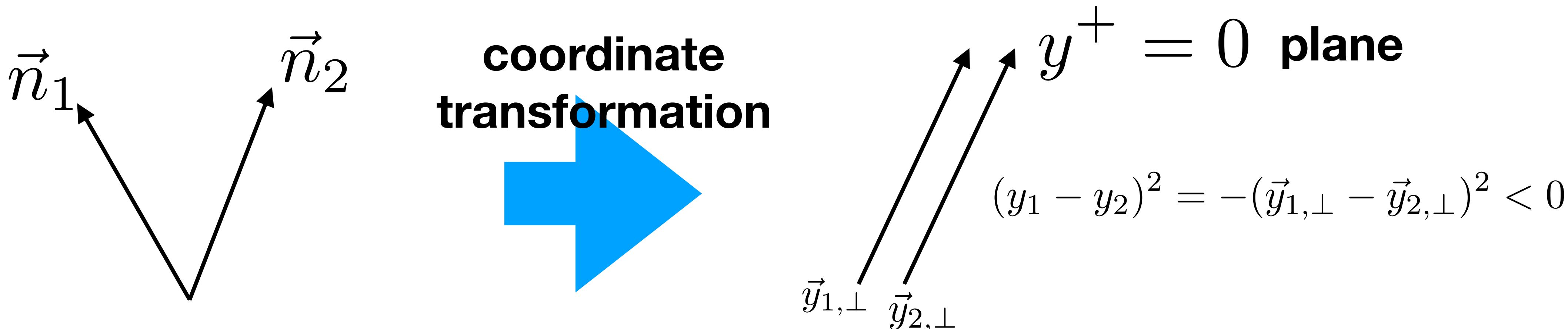
Collinear limit from field theory

- Collinear limit in CFT has a particular interesting interpretation as operator product expansion

- For a jet in z direction, useful to change coordinate $x^+=x^0+x^3$, $x^-=x^0-x^3$ **Hofman, Maldacena, 2008**

$$y^+ = -\frac{1}{x^+}, \quad y^- = x^- - \frac{x_1^2 + x_2^2}{x^+}, \quad y^1 = \frac{x^1}{x^+}, \quad y^2 = \frac{x^2}{x^+}$$

$$\mathcal{E}(y_1, y_2) \equiv 2\int_{-\infty}^{\infty} \mathrm{d}y^- T_{--}(y^-, y^+ = 0, y^1, y^2)$$

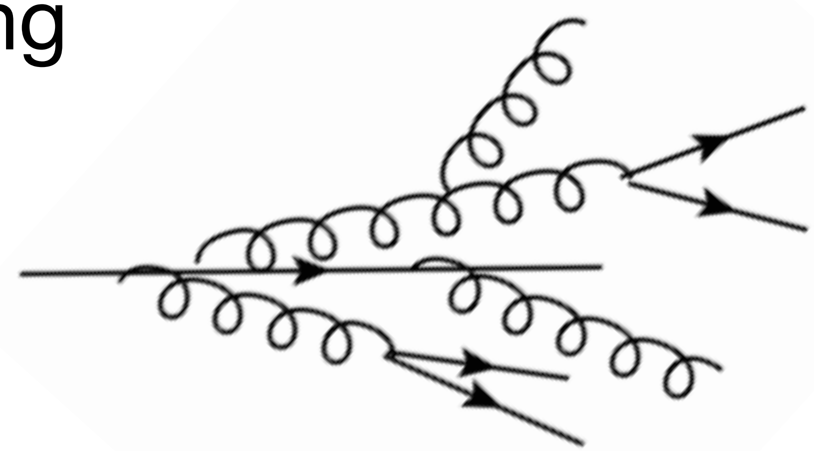


leading term: $\sim 1/z$

$$\mathcal{E}(\vec{y}_\perp)\mathcal{E}(\vec{0}_\perp) \sim \int dy^- T_{--}(y^-, y^+ = 0, \vec{y}_\perp) \int dy'^- T_{--}(y'^-, y^+ = 0, \vec{0}_\perp) \sim \sum_n |\vec{y}_\perp|^{\tau_n - 4} \mathcal{U}_{j-1,n}\Big|_{j=3}$$

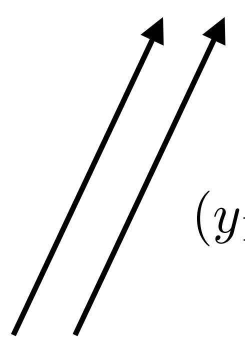
Two very different picture

- In the factorization approach, small z limit determined by time-like splitting



$t \rightarrow$

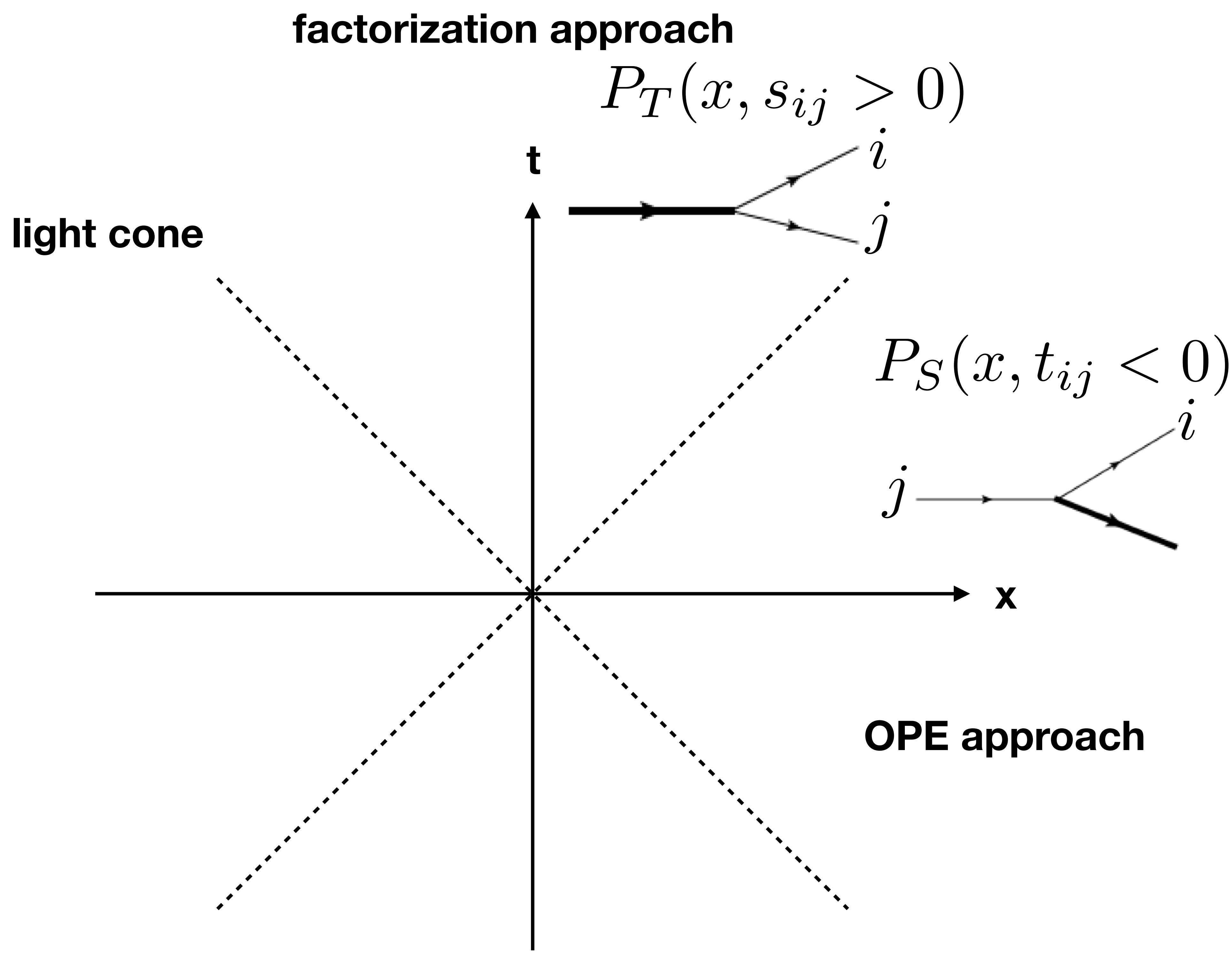
- In the field theory correlator approach, small z limit corresponds to space-like OPE



$y^+ = 0$ **plane**

$$(y_1 - y_2)^2 = -(\vec{y}_{1,\perp} - \vec{y}_{2,\perp})^2 < 0$$

$$\vec{y}_{2,\perp} \rightarrow \vec{y}_{1,\perp}$$



- How comes that the time-like picture and space-like picture describe the same physics?

Time-like space-like connection from factorization equation

Dixon, Moult, HXZ, to appear

- We can demonstrate the
- Power-law ansatz for the jet function

$$J(zQ^2,\mu) = C_J(\alpha_s)\left(\frac{zQ^2}{\mu^2}\right)^{\gamma^{\mathcal{N}=4}(\alpha_s)}$$

time-like kernel

$$\frac{dJ_{\mathcal{N}=4}(\ln\frac{zQ^2}{\mu^2})}{d\ln\mu^2} = \int_0^1 dy\, y^2 J_{\mathcal{N}=4}(\ln\frac{zy^2Q^2}{\mu^2}) P_{uni}(y)$$

$$2\gamma^{\mathcal{N}=4}(\alpha_s) = -2\int_0^1 dy\, y^{2+2\gamma^{\mathcal{N}=4}(\alpha_s)} P_{T,uni.}(x,\alpha_s)$$
$$= 2\gamma_T^{\mathcal{N}=4}(3+2\gamma^{\mathcal{N}=4},\alpha_s)$$

- Reciprocity $2\gamma_S^{\mathcal{N}=4}(N,\alpha_s) = 2\gamma_T^{\mathcal{N}=4}(N+2\gamma_S^{\mathcal{N}=4}(N,\alpha_s),\alpha_s)$

Basso, Korchemsky, 2006

$$\Rightarrow \quad \gamma^{\mathcal{N}=4}(\alpha_s) = \gamma_S^{\mathcal{N}=4}(3,\alpha_s)$$

- We can also derive a reciprocity relation in pure Yang-Mills

$$2\gamma_S^{\text{pure YM}}(N,\alpha_s(\mu)) = 2\gamma_T^{\text{pure YM}}\left(N + \frac{2\gamma_S^{\text{pure YM}}(N,\alpha_s(\mu))}{1+\frac{\alpha_s(Q)}{4\pi}\beta_0\ln(z)},\alpha_s(\mu)\right)$$

Summary

- Jets provides excellent probe for connecting perturbative hard physics and non-perturbative soft physics
- Energy correlators are perhaps the simplest observable to understand structure of jets
- Recent progress in understanding energy correlators: fixed order calculation, all order resummation
- Energy correlators in N=4 sYM: interesting connection between time-like and space-like physics