Real-time Correlators and Hidden Conformal Symmetries in Kerr/CFT Correspondence

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with Jiang Long, 1004.5039 , Chong-sun Chu, 1001.3208
see also the work with Zhi-bo Xu, 0901.3588, 0908.0057,
with Zhi-bo Xu and Bo Ning, 0911.0167

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Outline

- Brief review of Kerr/CFT correspondence
- (Near-)$NHEK/CFT_2$ correspondence
- Green's functions in 2D CFT
- Retarded Green's functions in NHEK/CFT correspondence
- Hidden conformal symmetry and real-time correlators
- Conclusion and discussions

Kerr black holes

A Kerr black hole is characterized by the mass M and angular momentum $J = aM$. It could be described by the metric of the following form

$$ds^2 = -\frac{\Delta}{\hat{\rho}^2}(d\hat{t} - a\sin^2\theta d\hat{\phi})^2 + \frac{\sin^2\theta}{\hat{\rho}^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t}\right)^2 + \frac{\hat{\rho}^2}{\Delta}d\hat{r}^2 + \hat{\rho}^2 d\theta^2, \tag{1.1}$$

with

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2, \qquad \hat{\rho}^2 = \hat{r}^2 + a^2\cos^2\theta, \tag{1.2}$$

where we have used the unit $G = \hbar = c = 1$.

- Two horizons: $r_\pm = M \pm \sqrt{M^2 - a^2}$;
- The Hawking temperature, the angular velocity of the horizon and the entropy of the Kerr black hole are

$$T_H = \frac{r_+ - r_-}{8\pi M r_+}, \qquad \Omega_H = \frac{a}{2Mr_+}, \qquad S_{BH} = 2\pi M r_+.$$

NHEK/CFT correspondence M.Guica et.al. 0809.4266

- Conjecture: the quantum gravity in the near-horizon extreme Kerr (NHEK) geometry with certain boundary conditions is dual to a (1+1) dimensional chiral conformal field theory;
- The correspondence was inspired by the properties of the asymptotic symmetry group of near horizon geometry of the extremal Kerr (NHEK)
  - $SL(2,R)_R \times U(1)_L$ isometry group;
  - Under a certain set of boundary condition on the asymptotic behaviour of the metric, the $U(1)_L$ get enhanced into a Virasoro algebra with central charge $c_L = 12J$;
  - Further studies showed that there's right central charge $c_R = 12J$;
- Perfect match of the macroscopic Berenstein-Hawking entropy of the black hole with the conformal field theory entropy computed by the Cardy formula.
- This has been generalized to many other cases, including Kerr BH in higher dimensions, ...

Superradiant scattering

- For a scalar incident scattering an extreme Kerr BH, if $0 \leq \omega \leq m\Omega_H$ where $\Omega_H = 1/2M$, then the scattering is superradiant. Zeldovich, Misner 1970s
- Superradiance: the classical reflected wave is more energetic than the incident one;
- Quantum mechanically, superradiant modes with $\omega \leq m\Omega_H$ are spontaneously emitted by the BH;
- Hawking radiation:

$$\Gamma = \frac{1}{e^{(\omega - m\Omega_H)/T_H} - 1}\sigma_{abs}, \tag{1.3}$$

- σ_{abs} is called greybody factor, or absorptive cross section, which modified the spectrum observed at ∞ from that of a blackbody;

Near-NHEK/CFT

- The scattering brings the Kerr a little away from extremality;
- Namely we have to consider near-extremal Kerr;
- In this case, the right sector is excited;
- Same kind of scaling argument gives near-NHEK geometry:
  1. Locally, this geometry is isometric to NHEK;
  2. However, the coordinate transformation is singular;
  3. The physics is different: there exists right temperature;
  4. This is quite similar to BTZ BH to AdS_3, and warped AdS_3 BH to global warped AdS_3;
- In the superradiant scattering of near-extreme Kerr(-Newman) black hole, the absorption cross section has been shown to be in perfect match with CFT prediction;(more details later)Bredberg et.al. 0907.3477, Hartman et.al. 0908.3909, Cvetic et.al. 0908.1136, BC and Chong-sun Chu 1001.3208

Hidden conformal invariance Castro et.al. 1004.0996

- Kerr/CFT should be true for generic non-extremal Kerr black holes;
- Conjectures: a generic non-extremal Kerr black hole is dual to a 2D CFT with central charges $c_L = c_R = 12J$ and temperatures $T_L = M^2/2\pi J$ and $T_R = \sqrt{M^4 - J^2}/2\pi J$;
- Consider the low frequency scalar scattering off the Kerr BH, one can find that there exists a local conformal invariance in the solution space of the wave equation;
  1. It is not globally defined;
  2. It is sufficient to associate a CFT to Kerr BH;

Motivation

- Real-time correlators in warped AdS/CFT correspondence;BC, Bo Ning and Zhi-bo Xu, 0911.0167
- NHEK is in fact a warped AdS_3 spacetime with a warping factor being a function of the angular variable;
- If the separation constant is independent of the frequency, then we can apply the Minkowski prescription to compute the real-time correlators;
- We will show that this happens at least in two cases:
  1. The frequency is near the superradiant bound;BC and Chong-sun Chu 1001.3208
  2. The very low frequency region;BC and Jiang Long, 1004.5039
- In the low frequency limit, even though there is no near horizon geometry, the existence of local conformal invariance is essential to apply the prescription;

Near-NHEK geometry

The near-extremal near-horizon metric

$$ds^2 = 2J\Gamma\left(-r(r+2\alpha)dt^2 + \frac{dr^2}{r(r+2\alpha)} + d\theta^2 + \Lambda^2(d\phi + (r+\alpha)dt)^2\right),$$

where $\alpha = 2\pi T_R$, $\Gamma(\theta) = \frac{1+\cos^2\theta}{2}$, $\Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$, and $\phi \sim \phi + 2\pi, 0 \leq \theta \leq \pi$.

- If $\alpha = 0$, it reduces to NHEK;
- In global coordinates, the NHEK metric is

$$ds^2 = 2J\Gamma\left(-(1+\rho^2)d\tau^2 + \frac{d\rho^2}{1+\rho^2} + d\theta^2 + \Lambda^2(d\varphi + \rho d\tau)^2\right).$$

- The NHEK geometry has an isometry $SL(2,R)_R \times U(1)_L$;
- For fixed polar angle θ, it is a global warped AdS_3 spacetime, while the NHEK geometry is the quotient of warped AdS_3.
- The global warped AdS spacetime can be taken as the vacuum with the NHEK geometry (resp. near-NHEK) being taken as an extreme warped AdS_3 black hole (resp. as an non-extremal warped AdS_3 black hole)

(Near-)NHEK/CFT correspondence

- $U(1)_L$ is enhanced to a Virasoro algebra with $c_L = 12J$;
- Another set of Virasoro algebra with $c_R = 12J$;
- The entropy could be recovered from the Cardy's formula

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R). \tag{2.1}$$

- Originally, it was suggested that $c_R = 0$ with a finite T_R;
- Now it seems reasonable to let T_R to be very small to keep the entropy not far from the extreme case;
- Conjecture: a generic non-extremal Kerr black hole is dual to a 2D CFT with central charges $c_L = c_R = 12J$ and temperatures $T_L = M^2/2\pi J$ and $T_R = \sqrt{M^4 - J^2}/2\pi J$;

Finite temperature AdS/CFT

- BH in AdS $\sim$ finite temperature CFT;
- Taking BH as a thermodynamical system, the thermal equil. in BH system could be compared to thermal equil. of finite T CFT;
- Quasi-normal modes in BH correspond to the poles of retarded Green's function;
- Real-time correlators from gravity is subtle
  - Q: Boundary conditions at black hole horizon?
  - A: Purely ingoing one corresponds to retarded Green's function;
  - Analytic continuation? Not clear.

Prescriptions for retarded Green's functions

- A prescription first suggested by A. Son et.al.(2005);
- Its modern version:Gubser et.al.(2008), H.Liu et.al.(2009)

$$G_R(\omega,\vec{k}) = \lim_{r\to\infty} \frac{\Pi(r,\omega,\vec{k})|_{\phi_R}}{\phi_R(r,\omega,\vec{k})}, \tag{2.2}$$

where Π is the canonical momentum conjugate to ϕ, taking r as the "time" direction. Now ϕ_R is the classical solution, which should be purely in-falling at the black hole horizon.

- Subtlety: plug in appropriate terms proportional to the power of r to cancel the divergence;
- In practice, one can get the retarded Green's function from the asymptotic behavior of the solution;
- For example, for a scalar field with the asymptotic behaviour

$$\phi \sim A(\omega,\vec{k})r^{-n_A} + B(\omega,\vec{k})r^{-n_B}, \tag{2.3}$$

with $n_A > n_B$, the real-time correlator of the scalar field is given by $A(\omega,\vec{k})/B(\omega,\vec{k})$, up to a constant factor

2D CFT

- Two independent sectors: left-moving one and right-moving one, possibly with different central charges and temperatures;
- Retarded Green's function and Matsubara propagator:

$$G_R(i\omega_L, i\omega_R) = G_E(\omega_{L,E}, \omega_{R,E}), \tag{3.1}$$

at

$$\omega_{L,E} = 2\pi n_L T_L, \qquad \omega_{R,E} = 2\pi n_R T_R \tag{3.2}$$

with n_L, n_R being integers.

- Two-point function:

$$G(t^+, t^-) = \langle \mathcal{O}_\phi^\dagger(t^+, t^-)\mathcal{O}_\phi(0)\rangle, \tag{3.3}$$

where t^+, t^- are the left and right moving coordinates of 2d worldsheet, and $\mathcal{O}_\phi$ is the operator corresponding to the field perturbing the black hole.

Green's functions in 2D CFT: continued

- Consider an operator of conformal dimensions (h_L, h_R), right charge q_R, at temperature (T_L, T_R) and chemical potential Ω_R.
- Correlators in 2D CFT are very much decided by conformal invariance: J.Cardy (1984)

$$G(t^+, t^-) \sim \left(\frac{\pi T_L}{\sinh(\pi T_L t^+)}\right)^{2h_L} \left(\frac{\pi T_R}{\sinh(\pi T_R t^-)}\right)^{2h_R} e^{iq_R \Omega_R t^-}. \tag{3.4}$$

- Left-mover:

$$G_E(\omega_{L,E}) \sim \frac{T_L^{2h_L-2} e^{i\omega_{L,E}/2T_L} \Gamma(1-2h_L)}{\Gamma(1-h_L+\frac{\omega_{L,E}}{2\pi T_L})\Gamma(1-h_L-\frac{\omega_{L,E}}{2\pi T_L})}. \tag{3.5}$$

Green's functions in 2D CFT: continued

- Right-mover:

$$G_E(\omega_{R,E}) \sim \frac{T_R^{2h_R-2} e^{(i\omega_{R,E}+q_R\Omega_R)/2T_R}\Gamma(1-2h_R)}{\Gamma(1-h_R+\frac{\omega_{R,E}-iq_R\Omega_R}{2\pi T_R})\Gamma(1-h_R-\frac{\omega_{R,E}-iq_R\Omega_R}{2\pi T_R})}. \tag{3.6}$$

- The total contribution is the product of the left-mover's (3.5) and the right-mover's (3.6):

$$G_E(\omega_{L,E},\omega_{R,E}) = G_E(\omega_{L,E})G_E(\omega_{R,E}). \tag{3.7}$$

Cross section

- Cross section: following Fermi's golden rule:

$$\sigma_{abs} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+}[G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)] \quad (3.8)$$

- In momentum space:

$$\sigma \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh(\frac{\omega_L}{2T_L} + \frac{\omega_R - q_R\Omega_R}{2T_R}) |\Gamma(h_L + i\frac{\omega_L}{2\pi T_L})|^2 |\Gamma(h_R + i\frac{\omega_R - q_R\Omega_R}{2\pi T_R})|^2. \quad (3.9)$$

Superradiant scattering of scalar in near-extreme Kerr

- Consider the scalar field Φ of mass μ in the near-NHEK background.
- Ansatz: $\Phi = e^{-i\omega t + im\phi}\mathcal{R}(r)\mathcal{S}(\theta)$, where ω and m are the quantum numbers;
- The angular part $\mathcal{S}(\theta)$ satisfies the spheroidal harmonic equation:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\mathcal{S}\right)+\left(\Lambda_{lm}-(\frac{m^2}{4}-J\mu^2)\sin^2\theta-\frac{m^2}{\sin^2\theta}\right)\mathcal{S}=0, \tag{3.10}$$

where Λ_{lm} is the eigenvalue, which can be computed numerically.

- The radial part $\mathcal{R}$ satisfies the equation

$$\frac{d}{dr}\left(r(r+2\alpha)\frac{d}{dr}\right)\mathcal{R}(r)=\left(\Lambda_{lm}-m^2+2J\mu^2-\frac{(\omega+m(r+\alpha))^2}{r(r+2\alpha)}\right)\mathcal{R}(r \tag{3.11}$$

- Key point: Λ_{lm} is independent of the frequency;

Scalar retarded Green's function

Taking into account of the ingoing boundary condition at the horizon, the radial wave function could be written in terms of hypergeometric function. At asymptotic infinity, the radial eigenfunction has the behaviour

$$\mathcal{R}(r) \sim A r^{-\frac{1}{2}-\beta} + B r^{-\frac{1}{2}+\beta} \tag{3.12}$$

where

$$\begin{aligned} \beta^2 &= \frac{1}{4} + \Lambda_{lm} - 2m^2 + 2J\mu^2. \\ A &= N \frac{\Gamma(-2\beta)\Gamma(1 - i(m + \frac{\omega}{\alpha}))}{\Gamma(\frac{1}{2} - \beta - im)\Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})} (2\alpha)^{\frac{1}{2}+\beta-\frac{i}{2}(m+\frac{\omega}{\alpha})}, \quad (3.13) \\ B &= A(\beta \to -\beta) \quad (3.14) \end{aligned}$$

and N is an arbitrary constant.

Scalar retarded Green's function

1. Consider a real $\beta > 0$, then

$$G_R \sim \frac{A}{B} = (2\alpha)^{2\beta}\frac{\Gamma(-2\beta)}{\Gamma(2\beta)}\frac{\Gamma(\frac{1}{2}+\beta-im)\Gamma(\frac{1}{2}+\beta-i\frac{\omega}{\alpha})}{\Gamma(\frac{1}{2}-\beta-im)\Gamma(\frac{1}{2}-\beta-i\frac{\omega}{\alpha})}. \quad (3.15)$$

2. With the identification:

$$h_L = h_R = \frac{1}{2}+\beta, \quad \omega_L = m, \quad \bar{\omega}_R = \omega, \quad T_L = \frac{1}{2\pi}, \quad T_R = T_R, \quad (3.16)$$

at the Matsubara frequencies, the expression on scalar retarded Green's function agrees precisely with CFT predition up to an irrelevant normalization factor which depends only on β, q_L and q_R and can be absorbed into the normalization of the operator.

Absorption cross section

- The cross section can also be read out from the retarded correlator directly

$$\sigma = \mathrm{Im}(G_R) = \frac{(2\alpha)^{2\beta}}{2\beta\pi(\Gamma(2\beta))^2}\sinh(\pi(m+\frac{\omega}{\alpha}))|\Gamma(\frac{1}{2}+\beta-im)\Gamma(\frac{1}{2}+\beta-i\frac{\omega}{\alpha})|^2. \tag{3.17}$$

  This agree, up to an irrelevant normalization factor, with CFT prediction as it should be.

- The quasi-normal modes frequencies could be read from the poles of the retarded Green's function

$$\begin{aligned}\bar{\omega}_L &= -i2\pi T_L(n_L + h_L)\\ \bar{\omega}_R &= -i2\pi T_R(n_R + h_R)\end{aligned} \tag{3.18}$$

  with n_L, n_R being non-negative integers. The left part is not actually the quasi-normal modes since it is related to the quantum number of rotation. The right part gives the contribution.

Newman-Penrose null tetrad of near-NHEK

- For other kinds of perturbations with nonzero spin, we have to use Newman-Penrose formalism;
- The NP null tetrad of near-NHEK is $e_a^\mu = (l^\mu, n^\mu, m^\mu, m^{*\mu})$, where in coordinate basis

$$
\begin{aligned}
l^\mu &= \frac{1}{r(r+2\alpha)}(1, r(r+2\alpha), 0, -(r+\alpha)),\\
n^\mu &= \frac{1}{4J\Gamma(\theta)}(1, -r(r+2\alpha), 0, -(r+\alpha)),\\
m^\mu &= \frac{1}{2\sqrt{J\Gamma(\theta)}}(0, 0, 1, i\Lambda^{-1}(\theta)),
\end{aligned}
\tag{3.19}
$$

satisfy the normalization and orthogonal condition with nonvanishing inner products

$$l \cdot n = -m \cdot m^* = -1. \tag{3.20}$$

Ingoing solution

The wave function could be decomposed into the form

$$\Psi^s = e^{-i\omega t + im\phi}(r(r+2\alpha))^{-s}\mathcal{R}^s(r)\mathcal{S}^s(\theta). \tag{3.21}$$

The asymptotic behaviour of the ingoing solution is

$$\mathcal{R}^s(r) \sim A^s r^{-\frac{1}{2}-\beta} + B^s r^{-\frac{1}{2}+\beta}, \tag{3.22}$$

where

$$\begin{aligned}
\beta^2 &= \frac{1}{4} + \Lambda^s_{lm} - 2m^2. \\
A^s &= N\frac{\Gamma(-2\beta)\Gamma(1-i(q+\frac{\omega}{\alpha}))}{\Gamma(\frac{1}{2}-\beta-iq)\Gamma(\frac{1}{2}-\beta-i\frac{\omega}{\alpha})}(2\alpha)^{\frac{1}{2}+\beta-\frac{i}{2}(q+\frac{\omega}{\alpha})} \qquad (3.23) \\
B^s &= A^s(\beta \to -\beta) \qquad (3.24)
\end{aligned}$$

Prescriptions for retarded Green's functions

- For $|s| = 1, 2$, Ψ^s are related to the gauge field strength and the Weyl tensor of the tensor field;
- It is not appropriate to identify Ψ^s as the perturbations themselves.
- We can inversely obtain the vector and gravitational perturbations from the wave functions in terms of the Newman-Penrose complex spin coefficients;
- Note that in determining the retarded Green's function from gravity, it is the asymptotic behaviours of the source and the response that matter;
- In other words, once the source term of the field is decided, its field strength has the same Gamma function dependence, up to a factor;
- The working prescription for computing the retarded Green's function for general perturbations with spin s:

$$G_R^s \sim (-1)^s \frac{A^{-s}}{B^s}. \tag{3.25}$$

Retarded Green's functions

- With the above prescription,

$$G_R^s \sim (-1)^s \frac{A^{-s}}{B^s}$$
$$\sim (-1)^s (2\alpha)^{2\beta} \frac{\Gamma(-2\beta)\Gamma(\frac{1}{2}+\beta-s-im)\Gamma(\frac{1}{2}+\beta-i\frac{\omega}{\alpha})}{\Gamma(2\beta)\Gamma(\frac{1}{2}-\beta+s-im)\Gamma(\frac{1}{2}-\beta-i\frac{\omega}{\alpha})}.$$

- With the conformal weights of the field identified as

$$h_R^s = \frac{1}{2}+\beta, \qquad h_L^s = h_R^s - s, \tag{3.26}$$

the retarded Green's function can be rewritten as

$$G_R^s \sim (-1)^s T_R^{2h_R^s-1} \frac{\Gamma(1-2h_R^s)\Gamma(h_L^s-im)\Gamma(h_R^s-i\frac{\omega}{\alpha})}{\Gamma(2h_R^s-1)\Gamma(1-h_L^s-im)\Gamma(1-h_R^s-i\frac{\omega}{\alpha})}.$$

- The above retarded Green's function agrees precisely, up to a frequencies independent normalization factor, with the CFT result if the frequencies and the temperatures are identified as before:

$$\omega_L = m, \quad \omega_R = \omega, \quad T_L = \frac{1}{2\pi}, \quad T_R = T_R. \tag{3.27}$$

Cross sections

1. The cross section can be read directly from the Green's function by the relation $\sigma \sim \mathrm{Im}(G_R)$.
2. for the fermion, the cross section is

$$\sigma \sim \frac{(2\alpha)^{2h_R-1}}{\Gamma(2h_R-1)^2}\cosh((m+\frac{\omega}{\alpha})\pi)|\Gamma(h_L-im)\Gamma(h_R-i\frac{\omega}{\alpha})|^2. \tag{3.28}$$

3. for the gauge field and the graviton,

$$\sigma \sim \frac{(2\alpha)^{2h_R-1}}{\Gamma(2h_R-1)^2}\sinh((m+\frac{\omega}{\alpha})\pi)|\Gamma(h_L-im)\Gamma(h_R-i\frac{\omega}{\alpha})|^2. \tag{3.29}$$

4. They agree with the CFT result.
5. As for the quasi-normal modes, their frequencies are simply

$$\begin{aligned} \omega_L^s &= -i2\pi T_L(h_L^s+n_L) \\ \omega_R^s &= m\Omega_H - i2\pi T_R(h_R^s+n_R) \end{aligned} \tag{3.30}$$

with n_L, n_R being non-negative integers.

Remarks

1. NHEK itself is dual to the 2D chiral CFT with a temperature in the left-moving sector;
2. It is thus more natural to take NHEK as a limiting case of near-NHEK.
3. The scalar scattering supports the picture that NHEK is an extreme BH;
4. The same analysis has been applied to the scattering of charged scalar by (near-)extreme Kerr-Newman BH;

Kerr-Newman black hole

For the Kerr-Newman black hole with mass M, angular momentum $J = aM$ and electric charge Q, its metric takes the following form

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{1}{\rho^2}\sin^2\theta\left(adt - (r^2+a^2)d\phi\right)^2, \tag{4.1}$$

where

$$\begin{aligned} \Delta &= (r^2+a^2) - 2Mr + Q^2, \\ \rho^2 &= r^2 + a^2\cos^2\theta. \end{aligned} \tag{4.2}$$

The gauge field is $A = -\frac{Qr}{\rho^2}(dt - a\sin^2\theta d\phi)$.

- There are two horizons $r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}$.

Kerr-Newman black hole

The Hawking temperature, entropy, angular velocity of the horizon and the electric potential are respectively

$$
\begin{aligned}
T_H &= \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)}, \\
S &= \pi(r_+^2 + a^2), \\
\Omega_H &= \frac{a}{r_+^2 + a^2}, \\
\Phi &= \frac{Q r_+}{r_+^2 + a^2}.
\end{aligned}
\tag{4.3}
$$

Charged scalar scattering

- Let us consider the complex scalar field with mass μ and charge e scattering with the Kerr-Newman black hole;
- The Klein-Gordon equation is

$$(\nabla_\mu + ieA_\mu)(\nabla^\mu + ieA^\mu)\Phi - \mu^2\Phi = 0. \tag{4.4}$$

- The ansatz $\Phi = e^{-i\omega t + im\phi}\mathcal{R}(r)\mathcal{S}(\theta)$;
- The angular part is of the form

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\mathcal{S}\right) + \left(\Lambda_{lm} - a^2(\omega^2 - \mu^2)\sin^2\theta - \frac{m^2}{\sin^2\theta}\right)\mathcal{S} = 0.$$

- The radial part of the wave function is of the form

$$\partial_r(\Delta\partial_r\mathcal{R}) + V_R\mathcal{R} = 0 \tag{4.5}$$

with

$$V_R = -\Lambda_{lm} + 2am\omega + \frac{H^2}{\Delta} - \mu^2(r^2 + a^2), \tag{4.6}$$

$$H = \omega(r^2 + a^2) - eQr - am. \tag{4.7}$$

Low-frequency limit

- In the low frequency limit,

$$\omega M << 1, \tag{4.8}$$

  the ω^2 term in the angular equation could be neglected.
- Note that the low frequency limit (4.8) is very different from the near-extreme case, where only the frequencies near the superradiant bound were studied;
- To simplify our discussion, we focus on the massless scalar.
- The angular equation is just the Laplacian on the 2-sphere with the separation constants taking values $\Lambda_{lm} = l(l+1)$.

Radial wavefunction

In the near region, $r\omega << 1$, the radial equation could be simplified even more

$$\partial_r \Delta \partial_r \mathcal{R}(r) + \frac{(ma - \omega(2Mr_+ - Q^2) + eQr_+)^2}{(r - r_+)(r_+ - r_-)}\mathcal{R}(r)$$

$$- \frac{(ma - \omega(2Mr_- - Q^2) + eQr_-)^2}{(r_+ - r_-)(r - r_-)}\mathcal{R}(r) = \left(l(l+1) - e^2Q^2\right)\mathcal{R}(r)$$

With the ingoing boundary condition at the BH horizon, we have the radial wave function

$$\mathcal{R}(z) = z^\alpha (1-z)^\beta F(a, b, c; z) \tag{4.9}$$

with

$$\alpha = -i\sqrt{A_1}, \qquad \beta = \frac{1}{2}(1 - \sqrt{1 - 4A_3}), \tag{4.10}$$

and

$$c = 1 + 2\alpha, \tag{4.11}$$

$$a = \alpha + \beta + i\sqrt{-A_2}, \tag{4.12}$$

$$b = \alpha + \beta - i\sqrt{-A_2}. \tag{4.13}$$

Conformal coordinates

- Let's introduce the conformal coordinates

$$\omega^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R\phi+2n_Rt},$$
$$\omega^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L\phi+2n_Lt},$$
$$y = \sqrt{\frac{r_+-r_-}{r-r_-}}e^{\pi(T_L+T_R)\phi+(n_L+n_R)t},$$

- Define locally the vector fields

$$H_1 = i\partial_+$$
$$H_0 = i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right)$$
$$H_{-1} = i(\omega^{+2}\partial_+ + \omega^+y\partial_y - y^2\partial_-) \tag{4.15}$$

which obey the $SL(2,R)$ Lie algebra: $[H_0, H_{\pm1}] = \mp iH_{\pm1}$;
- Similarly we can define another set of vector fields $(\tilde{H}_0, \tilde{H}_{\pm1})$ with $+\leftrightarrow-$;

Casimir

- The quadratic Casimir is

$$\mathcal{H}^2 = \tilde{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) = \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-. \quad (4.16)$$

- The key point: the neutral scalar Laplacian is just the $SL(2,R)$ Casimir

$$\tilde{\mathcal{H}}^2\mathcal{R}(r) = \mathcal{H}^2\mathcal{R}(r) = l(l+1)\mathcal{R}(r), \quad (4.17)$$

with the following identification:

$$n_R = 0, \qquad T_R = \frac{r_+ - r_-}{4\pi a}$$
$$n_L = -\frac{1}{4M}, \qquad T_L = \frac{(r_+ + r_-) - Q^2/M}{4\pi a}, \quad (4.18)$$

Remarks

- The vector fields are only defined locally;
- The $SL(2,R) \times SL(2,R)$ symmetry is spontaneously broken down to $U(1)_L \times U(1)_R$ subgroup by the periodic identification

$$\phi \sim \phi + 2\pi. \tag{4.19}$$

- The identification (4.18) reflects the nature of the underlying geometry. It is universal to all kinds of perturbations;
- In the $Q \to 0$ limit, it reduces to the one in the Kerr case.
- The relation between conformal coordinates with original coordinates is remniscent of Rindler coordinates;

Conjecture on Kerr-Newmann BH

- Conjecture: the Kerr-Newman black hole is dual to a CFT with central charges

$$c_L = c_R = 12J \tag{4.20}$$

at finite temperature (T_L, T_R) given in (4.18).

- Entropy: from Cardy's formula

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R), \tag{4.21}$$

we get the microscopic entropy

$$S = \pi(r_+^2 + a^2) \tag{4.22}$$

which is in perfect agreement with the macroscopic Bekenstein-Hawking area law for the entropy of the Kerr-Newman black hole.

Identification of charged scalar

- First law: $T_H \delta S = \delta M - \Omega \delta J - \Phi \delta Q$.
- We are looking for the conjugate charges δE_L and δE_R such that

$$\delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \tag{4.23}$$

- The solution:

$$\begin{aligned} \delta E_L &= \frac{(2M^2 - Q^2)M}{J}\delta M - \frac{QM^2 \delta Q}{J} + \frac{Q^3 \delta Q}{2J}, \\ \delta E_R &= \frac{(2M^2 - Q^2)M}{J}\delta M - \frac{QM^2 \delta Q}{J} - \delta J, \end{aligned} \tag{4.24}$$

- If we identify

$$\begin{aligned} &\delta M = \omega, \quad \delta J = m, \quad \delta Q = e, \\ &\omega_L = \frac{(2M^2 - Q^2)M}{J}\omega, \quad \omega_R = \frac{(2M^2 - Q^2)M}{J}\omega - m, \\ &q_L = q_R = \delta Q = e, \end{aligned} \tag{4.25}$$

$$\mu_L = \frac{QM^2}{J} - \frac{Q^3}{2J}, \quad \mu_R = \frac{QM^2}{J} \tag{4.26}$$

Scalar retarded correlator

- The radial wavefunction behaves asymptotically as

$$\mathcal{R}(r) \simeq Ar^{h-1} + Br^{-h} \tag{4.28}$$

where h is the conformal weight of the scalar field

$$h = 1 - \beta = \frac{1}{2}(1 + \sqrt{(2l+1)^2 - 4e^2Q^2}), \tag{4.29}$$

and

$$A = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \qquad B = \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \tag{4.30}$$

- From the Minkowski prescription, the two-point retarded correlator is just

$$G_R \sim \frac{B}{A} \tag{4.31}$$

Scalar retarded correlator

- For the charged scalar scattering off the Kerr-Newman black hole,

$$G_R \sim \frac{\Gamma(1-2h)}{\Gamma(2h-1)} \frac{\Gamma\left(h+i\frac{\omega_L-q_L\mu_L}{2\pi T_L}\right)\Gamma\left(h+i\frac{\omega_R-q_R\mu_R}{2\pi T_R}\right)}{\Gamma\left(1-h+i\frac{\omega_L-q_L\mu_L}{2\pi T_L}\right)\Gamma\left(1-h+i\frac{\omega_R-q_R\mu_R}{2\pi T_R}\right)}$$

- The absorption cross section:

$$P_{abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L-q_L\mu_L}{2T_L}+\frac{\omega_R-q_R\mu_R}{2T_R}\right)\times \left|\Gamma\left(h_L+i\frac{\omega_L-q_L\mu_L}{2\pi T_L}\right)\right|^2 \cdot \left|\Gamma\left(h_R+i\frac{\omega_R-q_R\mu_R}{2\pi T_R}\right)\right|^2$$

Photons and gravitons in Kerr black hole

- Now we focus on the low-frequency limit, instead of the frequencies near the super-radiant bound;
- Similar to the near-extreme case, the wave function could be solved exactly in terms of hypergeometric functions;
- Due to the existence of hidden conformal invariance, we apply the Minkowski prescription directly and compute the retarded Green's functions;
- Results: the retarded Green's function and the absorption cross sections are both in good agreement with the CFT prediction;
- This gives strong support to Kerr/CFT correspondence for generic non-extremal Kerr BH;

Conclusion and discussions

- Scattering off the Kerr(-Newman) BH is an important way to check Kerr/CFT correspondence;
- The retarded Green's function could be computed via Minkowski prescription;
- For the scattering near the super-radiant bound, this is guaranteed by the fact that the (near)-NHEK looks like warped AdS_3 spacetime;
- For the low frequency limit, the effectiveness of the prescription is due to the hidden conformal invariance;
- The study supports the picture that Kerr/CFT should be true for any value of J;

Thank you!