Accelerating in 5 dimensions

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0910.3358; 1005.2444

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Introduction

C-metric is a kind of exact Einstein metric in which all observers held at fixed spacial coordinate experience uniform accelerations.

- It was found during the very early stage in the studies of Einstein gravity. Some source points to 1917 $\sim$ 1919, but the earliest source I can trace to is this: pp.73 in L.Witten: *Gravitation: an introduction to current research*, John Wesley & Sons (1962)

TABLE 2–3.1. Degenerate Static Vacuum Fields

1	2	3	4	5	6	7	8	9	10
Class	First Fundamental Form G	Coordinate Ranges	α	ν_S	ν_T	K_S	K_T	r	s
A1	$r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + \frac{dr^2}{1 - b/r} - \left(1 - \frac{b}{r}\right) dt^2$	$0 \leqslant \vartheta \leqslant \pi$, φ mod 2π $0 < b < r < \infty$ or $b < 0 < r < \infty$	$-\frac{b}{r^3}$		$\left(\frac{\alpha}{b}\right)^{2/3}$	$\frac{1}{r^2}$			
A2	$z^2(dr^2 + \sinh^2 r\, d\varphi^2) + \frac{dz^2}{b/z - 1} - \left(\frac{b}{z} - 1\right) dt^2$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < b$	$\frac{b}{z^3}$	0	$-\left(\frac{\alpha}{b}\right)^{2/3}$	$\frac{-1}{z^2}$	$-\alpha$		1S
A3	$z^2(dr^2 + r^2\, d\varphi^2) + z\, dz^2 - \frac{dt^2}{z}$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < \infty$	$\frac{1}{z^3}$		0	0			
B1	$\frac{dt^2}{1 - b/r} + \left(1 - \frac{b}{r}\right) d\varphi^2 + r^2(d\vartheta^2 - \sin^2\vartheta\, dt^2)$	$0 < b < r < \infty$ or $b < 0 < r < \infty$ $0 < \vartheta < \pi$	$-\frac{b}{r^3}$	$-\left(\frac{\alpha}{b}\right)^{2/3}$			$\frac{1}{r^2}$	4	
B2	$\frac{dz^2}{b/z - 1} + \left(\frac{b}{z} - 1\right) d\varphi^2 + z^2(dr^2 - \sinh^2 r\, dt^2)$	$0 < z < b$ $0 < r < \infty$	$\frac{b}{z^3}$	$-\left(\frac{\alpha}{b}\right)^{2/3}$	0	$-\alpha$	$\frac{-1}{z^2}$		1T
B3	$z\, dz^2 + \frac{d\varphi^2}{z} + z^2(dr^2 - r^2\, dt^2)$	$0 < z < \infty$ $0 < r < \infty$	$\frac{1}{z^3}$	0			0		
C	$\frac{1}{(x + y)^2}\left(\frac{dx^2}{f(x)} + f(x)\, d\varphi^2 + \frac{dy^2}{\lvert f(-y)\rvert}\lvert f(-y)\rvert\, dt^2\right)$ $f(u) \equiv \pm(u^3 + au + b)$	$0 < x + y$ $f(-y) < 0$ $0 < f(x)$	$\pm(x + y)^3$	$f(x) - \alpha$	$-f(-y) - \alpha$	ν_T	ν_s	2	0

Static Vacuum Fields

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Introduction

- C-metric is usually written in an unconventional coordinate system. In the absence of rotational parameters, the metric can be written as

$$ds^2 = [A(x+y)]^{-2}(-\mathcal{F}dt^2 + \mathcal{F}^{-1}dy^2 + \mathcal{G}^{-1}dx^2 + \mathcal{G}dz^2),$$
$$\mathcal{F} = -\left(\pm\frac{1}{\ell^2 A^2} + 1\right) + y^2 - 2mAy^3 + q^2A^2y^4,$$
$$\mathcal{G} = 1 - x^2 - 2mAx^3 - q^2A^2x^4.$$

The parameters ℓ, m, q correspond to (A)dS radius, mass and charge respectively. Black hole horizons appear at roots of $\mathcal{F}$. It was known that there are 2 black holes in this metric and they accelerate apart with acceleration related to the value of A.

Introduction

- Physical properties of C-metric was extensively studied by W. Kinnersley and M. Walker, in "Uniformly accelerating charged mass in general relativity," Phys. Rev. D 2 (1970) 1359.

PHYSICAL REVIEW D

PARTICLES AND FIELDS

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Uniformly Accelerating Charged Mass in General Relativity*

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(Received 25 May 1970)

Introduction

- Essentially, the C-metric describes two black holes accelerating apart;
- In the presence of charge, K. Hong and E. Teo wrote two papers gr-qc/0305089 and gr-qc/0410002, mainly using a factorized coordinate choice;
- J. B. Griffiths, P. Krtous, and J. Podolsky, gr-qc/0609056 gave a thorough analysis on the metric in the case $\ell \to \infty$ and $q = 0$, i.e. in the absence of charge and cosmological constant;
- The cosmological constant in C-metric was analyzed in detail by O. J. C. Dias and J. P. S. Lemos in hep-th/0210065 (AdS) and hep-th/0301046 (dS);

Why is C-metric so interesting?

- It provides a laboratory to understand how accelerated motion in Einstein spacetimes affects the observed structure of the spacetime itself. It turns out that different accelerating observers can observe different causal structures and perceives different global structure of the spacetime;
- The physical explanation of the acceleration between the two black holes can be either cosmic strings pulling the holes apart or strut extending in between;

Introduction

- C-metric to black ring is just as bricks to a house (R. Emparan, H. Real, hep-th/0110258, hep-th/0110260, ...

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Generalized Weyl solutions

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- Even the most trivial limit of C-metric (with $\ell \to \infty$, $m, q \to 0$, i.e. empty C-metric) has found important role in black ring construction, see e.g. Emparan *et al*: hep-th/0407065;

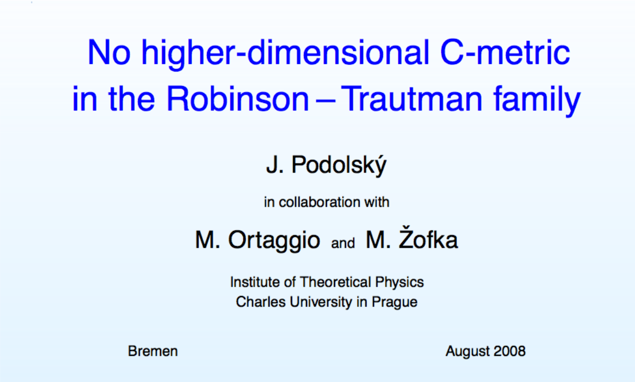
Introduction

Higher dimensional generalizations of C-metric are needed if we

- wish to understand the structure of higher dimensional Einstein spacetimes perceived by observers at uniform accelerations;
- want to construct black rings in dimensions higher than 5;
- try to understand some deeper respects of the C-metric which may be too degenerated in its 4D form;

Introduction

- Unfortunately, the search of higher dimensional C-metric have so far lead to no success, some even claim that the higher dimensional analogue just doesn't exist!



Introduction

- The major reason for the difficulties in finding higher dimensional C-metric may be stemmed from the complicated coordinate structures for the C-metric;
- Alternative approximate methods (matched asymptotic expansion etc) have lead to the new concept of black folds (arXiv: 0708.2081, 0902.0427, 0910.1601), but this way of research can only touch the asymptotic and approximate properties of the objects;
- In this talk, I'll report two possible 5-dimensional analogue of the massless uncharged C-metric (i.e. *empty* C-metrics in 5-dimensions). Though 5D C-metrics with black hole horizons are yet to be found, the result presented here at least allows for playing with the 5D analogues of the C-metric coordinates and thus can be regarded as warming up exercises before arriving at the final goal.

The metrics

We have two 5D empty C-metrics at hand:

- Metric A:

$$\begin{aligned} ds^2 &= \frac{1}{\alpha^2(x+y)^2}\left[-G(y)dt^2 + \frac{dy^2}{G(y)} + \frac{dx^2}{F(x)}\right. \\ &\qquad \left. +F(x)\left(\frac{dz^2}{H(z)} + H(z)d\phi^2\right)\right], \end{aligned}$$

  where

$$F(x) = 1 - x^2, \quad G(y) = -1 - \frac{\Lambda}{6\alpha^2} + y^2, \quad H(z) = 1 - z^2.$$

The metrics

- Metric B:

$$ds^2 = \frac{1}{\alpha^2(x+y)^2}\left[-G(y)H(z)dt^2 + G(y)\frac{dz^2}{H(z)} + \frac{dy^2}{G(y)} + \frac{dx^2}{F(x)} + F(x)d\phi^2\right],$$

where

$$F(x) = 1 - x^2, \quad G(y) = -1 - \frac{\Lambda}{6\alpha^2} + y^2, \quad H(z) = 1 - \left(1 + \frac{\Lambda}{6\alpha^2}\right) z^2.$$

The metrics

- Both are exact solutions to the 5D Einstein equation

$$R_{MN} - \frac{1}{2} g_{MN} R + \Lambda g_{MN} = 0.$$

- I'll be mainly talking about metric A and making very brief comments on the metric B if time permits.

Metric A: coordinate ranges

- t is the coordinate time, which can take values in $(-\infty, \infty)$;
- y is the "radial" coordinate, its range is not restricted by the zeros of the metric function $G(y)$, but it is restricted by the conformal factor $\frac{1}{\alpha^2(x+y)^2}$. Since $x+y=0$ is the conformal infinity, y can only take values on one side of the surface $x+y=0$, so we take $y \in [-x, \infty)$;
- x, z are considered as "angular" coordinates, whose values are restricted by the zeros of the metric functions $F(x)$ and $H(z)$;
- ϕ is also restricted by the condition of the absence of conical singularity.

Finally, we have

$$
\begin{aligned}
&t \in (-\infty, \infty), \quad y \in [-x, \infty) \\
&x \in [-1, 1], \quad z \in [-1, 1], \\
&\phi \in [0, 2\pi].
\end{aligned}
$$

Metric A: coordinate ranges

The metric function $G(y) = -1 - \frac{\Lambda}{6\alpha^2} + y^2$ has rich zero structures depending on the value of the cosmological constant Λ:

1. for $\Lambda > 0$ (dS case), $G(y)$ has two zeros

$$y = \pm y_0, \quad y_0 = \sqrt{1 + \frac{\Lambda}{6\alpha^2}} > 1;$$

2. for $\Lambda = 0$ (Minkowski case), $G(y)$ has two zeros $y = \pm y_0 = \pm 1$;
3. for $\Lambda < 0$ (AdS case), the situation is a little more complicated:
   - if $-6\alpha^2 < \Lambda < 0$, $G(y)$ has two zeros

$$y = \pm y_0, \quad y_0 = \sqrt{1 + \frac{\Lambda}{6\alpha^2}} < 1;$$

   - if $\Lambda = -6\alpha^2$, $G(y)$ has a double zero

$$y = 0;$$

   - if $\Lambda < -6\alpha^2$, $G(y)$ has no zeros.

Metric A: coordinate ranges

The zeros of $G(y)$ reflects the possible horizon structures in the metric. Whether a zero of $G(y)$ is a horizon depends if it is located inside the physical region of y:

- if $x + y_0 > 0$ (or $x - y_0 > 0$ respectively), then $y = y_0$ ($y = -y_0$) is a horizon;
- if $x + y_0 = 0$ (or $x - y_0 = 0$ respectively), then $y = y_0$ ($y = -y_0$) coincides with the causal infinity and is not a horizon;
- if both $x + y_0$ and $x - y_0$ are negative, then $y = \pm y_0$ are both not horizons.

Causal structure

To construct the causal structure of the metric, several coordinate transformations are to be made:

- $x \to \theta_1 = \arccos(-x), z \to \theta_2 = \arccos(z)$;
- tortoise coordinates: $y \to y^* = \int G^{-1} \mathrm{d}y$;
- $(t, y^*) \to u = t - y^*, v = t + y^*$;
- Kruskal like coordinates: $\tilde{u} = \pm \exp(-y_0 u), \tilde{v} = \pm \exp(y_0 v)$ [This step may be omitted if y_0 doesn't exist];
- Carter-Penrose coordinates:

$$
\begin{aligned}
U &= \arctan \tilde{u}, \quad V = \arctan \tilde{v}, \\
T &\equiv U + V, \quad R \equiv U - V,
\end{aligned}
$$

Causal structure

After these steps, the metric becomes

$$\begin{aligned} ds^2 &= \frac{1}{\alpha^2(y_0\cos T - \cos\theta_1\cos R)^2}\left[-dT^2 + dR^2 + \cos^2(R)\ d\Omega_3^2\right], \\ d\Omega_3^2 &= d\theta_1^2 + \sin^2\theta_1(d\theta_2^2 + \sin^2\theta_2 d\phi^2). \end{aligned}$$

Note that at constant T, the metric is conformal to a 4-sphere.

The Carter-Penrose diagrams:

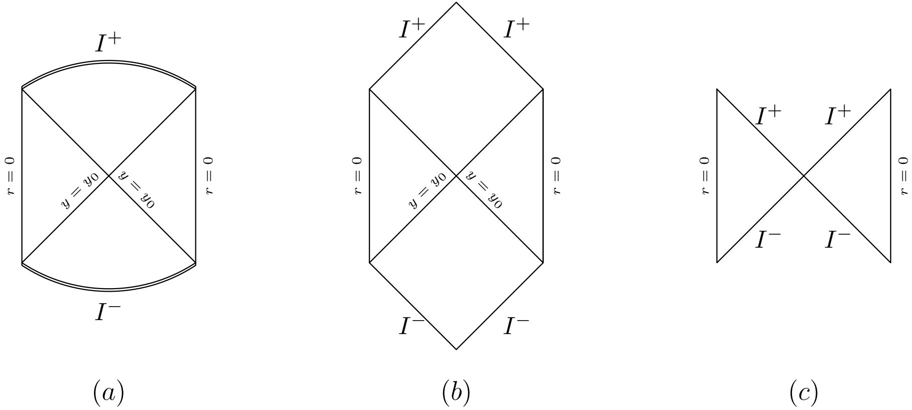


Figure: Carter-Penrose diagrams for $\Lambda \geq 0$: (a) corresponds to both $\Lambda > 0$ and the $-1 < x < 1$ case of $\Lambda = 0$; (b) and (c) respectively corresponds to $\Lambda = 0$ with $x = 1$ and $x = -1$.

Causal structure

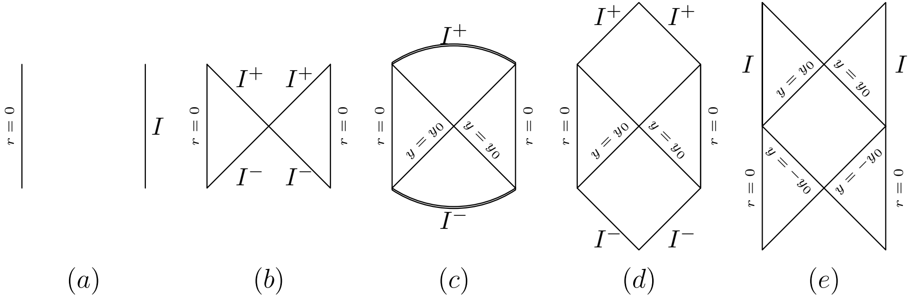


Figure: Carter-Penrose diagrams for $-6\alpha^2 < \Lambda < 0$: (a) $-1 \leq x < -y_0$; (b) $x = -y_0$; (c) $-y_0 < x < y_0$; (d) $x = y_0$; (e) $y_0 < x < 1$.

Causal structure

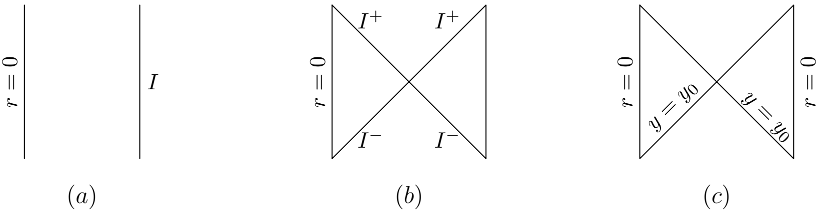


Figure: Carter-Penrose diagrams for $\Lambda \leq -6\alpha^2$: (a) $\Lambda < -6\alpha^2$ and $-1 < x < 0$ for $\Lambda = -6\alpha^2$; (b) $x = 0$ for $\Lambda = -6\alpha^2$; (c) $0 < x < 1$ for $\Lambda = -6\alpha^2$

Accelerating nature of the horizons

All horizons described above are acceleration horizons, and the parameter α is just the acceleration of the observer at the origin.

- $\Lambda > 0$. Coordinate transform:

$$\begin{aligned} \tau &= \frac{\sqrt{1+\alpha^2\ell^2}}{\alpha} t, \quad \rho = \frac{\sqrt{1+\alpha^2\ell^2}}{\alpha}\frac{1}{y}, \\ \theta_1 &= \arccos(-x), \quad \theta_2 = \arccos z, \quad \ell^2 = 6/\Lambda. \end{aligned}$$

  Metric:

$$\begin{aligned} ds^2 &= \frac{1}{\gamma^2}\left[-(1-\rho^2/\ell^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2/\ell^2} + \rho^2 d\Omega_3^2\right], \\ \gamma &= \sqrt{1+\alpha^2\ell^2} - \alpha\rho\cos\theta_1. \end{aligned}$$

Accelerating nature of the horizons

Timelike observer (λ is the proper time):

$$x^\mu(\lambda) = (\gamma\ell\lambda/\sqrt{\ell^2 - \rho^2}, \rho, \theta_1, \theta_2, \phi),$$

magnitude of the proper acceleration $a^\mu = (\nabla_\nu u^\mu)u^\nu$:

$$|a| = \sqrt{a_\mu a^\mu} = \frac{\rho\sqrt{1+\alpha^2\ell^2} + \alpha\ell^2}{\ell\sqrt{\ell^2 - \rho^2}}.$$

We see that the acceleration is α at $\rho = 0$ and ∞ at $\rho = \ell$.

Accelerating nature of the horizons

- $\Lambda = 0$. Coordinate transform:

$$\begin{aligned} \tau &= t, \quad \rho = \frac{1}{y}, \\ \theta_1 &= \arccos(-x), \quad \theta_2 = \arccos z. \end{aligned}$$

Metric:

$$\begin{aligned} \mathrm{d}s^2 &= \frac{1}{\gamma^2}\left[-(1-\rho^2)\mathrm{d}\tau^2 + \frac{\mathrm{d}\rho^2}{1-\rho^2} + \rho^2 \mathrm{d}\Omega_3^2\right], \\ \gamma &= \alpha(1-\rho\cos\theta_1). \end{aligned}$$

Accelerating nature of the horizons

Observer:

$$x^{\mu}(\lambda) = (\gamma\lambda/\sqrt{1-\rho^2}, \rho, \theta_1, \theta_2, \phi)$$

Acceleration:

$$|a| = \frac{\alpha(1+\rho)}{\sqrt{1-\rho^2}}.$$

Accelerating nature of the horizons

- $\Lambda < 0$.
  - $-6\alpha^2 < \Lambda < 0$:

$$
\begin{aligned}
ds^2 &= \frac{1}{\gamma^2}\left[-(1-\rho^2/\ell^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2/\ell^2} + \rho^2 d\Omega_3^2\right], \\
|a| &= \frac{\rho\sqrt{\alpha^2\ell^2 - 1} + \alpha\ell^2}{\ell\sqrt{\ell^2-\rho^2}};
\end{aligned}
$$

  - $\Lambda = -6\alpha^2$;

$$
ds^2 = \frac{1}{\gamma^2}\left[-d\tau^2 + d\rho^2 + \rho^2 d\Omega_3^2\right], \quad |a| = \alpha(1+\rho);
$$

  - $\Lambda < -6\alpha^2$:

$$
\begin{aligned}
ds^2 &= \frac{1}{\gamma^2}\left[-(1+\rho^2/\ell^2)d\tau^2 + \frac{d\rho^2}{1+\rho^2/\ell^2} + \rho^2 d\Omega_3^2\right], \\
|a| &= \frac{-\rho\sqrt{1-\alpha^2\ell^2} + \alpha\ell^2}{\ell\sqrt{\ell^2+\rho^2}}.
\end{aligned}
$$

Relations to standard dS, Minkowski and AdS spacetimes

- $\Lambda > 0$: The metric follows from the following parametrization

$$\begin{aligned} X_0 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\sinh(\tau/\ell), \quad X_2 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\sin\phi, \\ X_1 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\cosh(\tau/\ell), \quad X_3 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\cos\phi, \\ X_5 &= \gamma^{-1}\left[-\rho\sqrt{1+\alpha^2\ell^2}\cos\theta_1+\alpha\ell^2\right], \quad X_4 = \gamma^{-1}\rho\sin\theta_1\cos\theta_2 \end{aligned}$$

of the embedding equation

$$-(X_0)^2+(X_1)^2+(X_2)^2+(X_3)^2+(X_4)^2+(X_5)^2 = \ell^2$$

of de Sitter space in 6D Minkowski spacetime

$$ds^2 = -(dX_0)^2+(dX_1)^2+(dX_2)^2+(dX_3)^2+(dX_4)^2+(dX_5)^2.$$

- $\Lambda = 0$: The metric is a rewriting of the 5D Minkowski metric

$$ds^2 = dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2 - dX_5^2$$

by use of the coordinate transform

$$\begin{aligned} X_1 &= \frac{\alpha}{B} \sin\theta \sin\chi \cos\phi, \quad X_2 = \frac{\alpha}{B} \sin\theta \sin\chi \sin\phi, \\ X_3 &= \frac{\alpha}{B} \sin\theta \cos\chi, \\ X_4 &= \frac{\alpha}{B} \sinh\eta \cosh\psi, \quad X_5 = \frac{\alpha}{B} \sinh\eta \sinh\psi, \end{aligned}$$

where

$$B \equiv \cosh\eta - \cos\theta.$$

Relations to standard dS, Minkowski and AdS spacetimes

- $-6\alpha^2 < \Lambda < 0$: The metric follows from the following parametrization

$$
\begin{aligned}
X_0 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\sinh(\tau/\ell), \quad X_2 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\sin\phi,\\
X_1 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\cosh(\tau/\ell), \quad X_3 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\cos\phi,\\
X_5 &= \gamma^{-1}\left[-\rho\sqrt{\alpha^2\ell^2-1}\cos\theta_1+\alpha\ell^2\right], \quad X_4 = \gamma^{-1}\rho\sin\theta_1\cos\theta_2,
\end{aligned}
$$

of the 5D hyperboloid

$$-(X_0)^2+(X_1)^2+(X_2)^2+(X_3)^2+(X_4)^2-(X_5)^2=-\ell^2$$

embedded in the 6D hyperbolic spacetime

$$ds^2=-(dX_0)^2+(dX_1)^2+(dX_2)^2+(dX_3)^2+(dX_4)^2-(dX_5)^2.$$

Relations to standard dS, Minkowski and AdS spacetimes

- $\Lambda = -6\alpha^2$: same as previous case, with the parametrization of the 5D hyperboloid changed into

$$
\begin{aligned}
X_0 &= \eta^{-1}\tau, \\
X_1 &= \frac{1}{2}\eta^{-1}\left[1 - (\rho^2 \sin^2\theta_1 + \ell^2\eta^2 - \tau^2)\right], \\
X_2 &= \eta^{-1}\rho \sin\theta_1 \sin\theta_2 \sin\phi, \\
X_3 &= \eta^{-1}\rho \sin\theta_1 \sin\theta_2 \cos\phi, \\
X_4 &= \eta^{-1}\rho \sin\theta_1 \cos\theta_2, \\
X_5 &= \frac{1}{2}\eta^{-1}\left[1 + (\rho^2 \sin^2\theta_1 + \ell^2\eta^2 - \tau^2)\right].
\end{aligned}
$$

Relations to standard dS, Minkowski and AdS spacetimes

- $\Lambda < -6\alpha^2$: same as the previous case, with the parametrization changed into

$$
\begin{aligned}
X_0 &= \gamma^{-1}\sqrt{\ell^2+\rho^2}\sinh(\tau/\ell), \quad X_2 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\sin\phi,\\
X_5 &= \gamma^{-1}\sqrt{\ell^2+\rho^2}\cosh(\tau/\ell), \quad X_3 = \gamma^{-1}\rho\sin\theta_1\sin\theta_2\cos\phi,\\
X_1 &= \gamma^{-1}\left[-\rho\sqrt{1-\alpha^2\ell^2}\cos\theta_1 - \alpha\ell^2\right], \quad X_4 = \gamma^{-1}\rho\sin\theta_1\cos\theta_2.
\end{aligned}
$$

4D interpretation

With a boost and KK reduction, the original Metric A can be reduced into a 4D metric coupled with Maxwell and Liouville fields. The reduced action reads

$$S_4 = \int \mathrm{d}^4x \sqrt{-g_{(4)}} \left(R_{(4)} - \frac{1}{2} (\partial\varphi)^2 - \Lambda e^{\varphi/\sqrt{3}} - \frac{1}{4} e^{\varphi/\sqrt{3}} F_{\mu\nu} F^{\mu\nu} \right),$$

where

$$F = F_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \equiv \mathrm{d}\mathcal{A}.$$

4D interpretation

The fields $g_{(4)}$, $\mathcal{A}$ and φ are given exactly as follows:

$$
\begin{aligned}
\mathrm{d}\tilde{s}_4^2 &= \frac{1}{\alpha^3(x+y)^3}\left(\frac{F(x)H(z)-k^2G(y)}{1-k^2}\right)^{1/2} \\
&\times \left[-\frac{F(x)G(y)H(z)(1-k^2)}{F(x)H(z)-k^2G(y)}\mathrm{d}T^2+\frac{\mathrm{d}y^2}{G(y)}+\frac{\mathrm{d}x^2}{F(x)}+F(x)\mathrm{d}\theta_2^2\right], \\
\mathcal{A} &= \frac{k[F(x)H(z)-G(y)]}{F(x)H(z)-k^2G(y)}\mathrm{d}T, \\
e^{-2\varphi/\sqrt{3}} &= \frac{1}{\alpha^2(x+y)^2}\frac{F(x)H(z)-k^2G(y)}{1-k^2}.
\end{aligned}
$$

In these expressions, k is the boost velocity.

Discussions

◇ What we have learnt so far:

- Accelerating observers observe very different structures from static observers;
- Acceleration horizons can exist even in AdS spacetimes, showing that a simple observation of accelerating expansion does not necessarily imply an asymptotic dS behavior;
- dS/AdS metrics are anisotropic in accelerating coordinates (anisotropy appears in the conformal factor), so probably the cosmological observation of large scale homogeneity means our own coordinate system is not accelerating. In that case, it is necessary to ask why we are in such a special coordinate system;

Discussions

◇ Potential generalizations:

- can we add black holes into the 5D metric? Possibly yes, but still difficult;
- is it possible to embed such 5D metrics in a proper way into 6D to get 6D black rings? No clue yet;
- is it possible to consider analogues of (A)dS/CFT in the accelerating framework? [Remark: in the $(\tau, \rho, \theta_1, \theta_2, \phi)$ coordinates, $\Lambda = 0$ and $-6\alpha^2 < \Lambda < 0$ cases of the metric look exactly like the $\Lambda > 0$ metric, the only difference lies in the conformal factor]

Discussions

- The solution (to Einstein equation $R_{MN} - \frac{1}{2}g_{MN}R + \Lambda g_{MN} = 0$)

$$ds^2 = \frac{1}{\alpha^2(x+y)^2}\left[-G(y)H(z)dt^2 + G(y)\frac{dz^2}{H(z)} + \frac{dy^2}{G(y)} + \frac{dx^2}{F(x)} + F(x)d\phi^2\right],$$

$$F(x) = 1 - x^2, \quad G(y) = -1 - \frac{\Lambda}{6\alpha^2} + y^2, \quad H(z) = 1 - \left(1 + \frac{\Lambda}{6\alpha^2}\right)z^2.$$

The C-metric-ish of the metric is apparent from its coordinate description, but this one has no black holes in it (no m, q parameters present).

Discussions

- two Killing coordinates t, ϕ, each can be taken as time (to make the spacetime possessing a static patch). We take t;
- signature change in $H(z)$ would result in changing the roles of t and z — horizons occur at zeros $\pm z_0$ of $H(z)$ — z plays the role of a "radial" coordinate;
- $G(y)$ appears 3 times in the metric, changing its signature corresponds to triple Wick rotations in t, z, y, resulting in a metric with two timelike variables — give up this choice and let the signature of $G(y)$ remain fixed (unless $G(y) = 0$);
- for the same reason the signature of $F(x)$ must be kept fixed unless it takes the value 0;
- the hypersurface $x + y = 0$ lies at conformal infinity, thus the spacetime must sit in one side of this hypersurface. We take $x + y \geq 0$.

Discussions

- Physical ranges of the coordinates:

$$\begin{aligned} t &\in (-\infty, \infty), \\ z &\in (-\infty, \infty), \\ y &\in [y_0, \infty), \\ x &\in [-1, 1], \\ \phi &\in [0, 2\pi), \end{aligned}$$

where

$$y_0 = \sqrt{1 + \frac{\Lambda}{6\alpha^2}}, \quad z_0 = \frac{1}{\sqrt{1 + \frac{\Lambda}{6\alpha^2}}} = \frac{1}{y_0}.$$

Horizons appear at $z = \pm z_0$ — they cease to exist for $\Lambda < -6\alpha^2$!
(Will only consider $\Lambda \geq 0$ later)

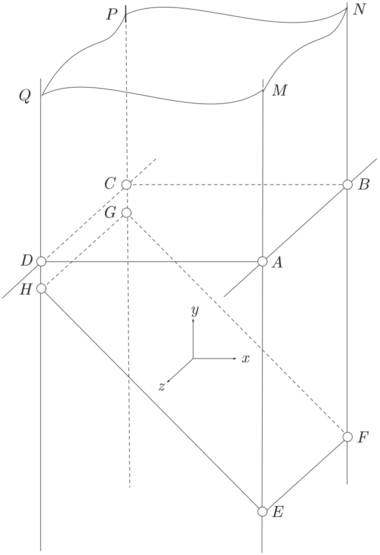
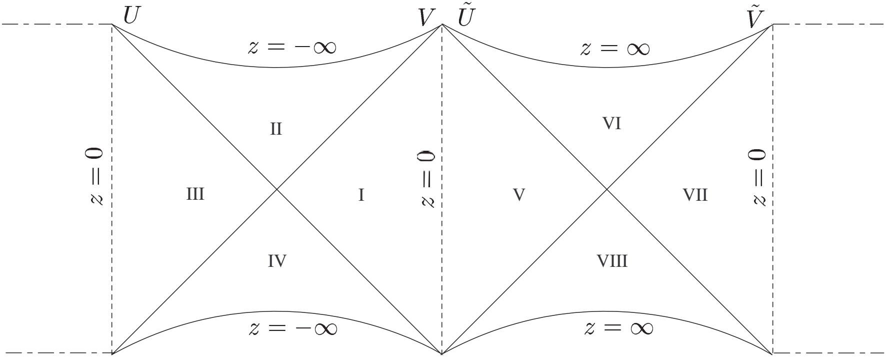


Figure: The (x, y, z) slice of the physical region of the spacetime: the static patch of the spacetime is the region bounded by the planes (A, B, C, D), (A, B, N, M), (C, D, Q, P), (A, D, Q, M) and (B, C, P, N).



Penrose diagram for $\Lambda \geq 0$: Horizontal slashed lines represent repeated occurrences of the eight zones depicted in the middle.

Discussions

- For all values of Λ, the origin of the coordinate system are all accelerating with magnitude α;
- There are always 2 horizons for $\Lambda > -6\alpha^2$;
- For $\Lambda < -6\alpha^2$, no horizons;
- Also possible to add black holes.