On Warped AdS/CFT correspondence

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Outline

- Warped AdS_3/CFT_2 correspondence
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- Retarded Green's functions in warped spacetime
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AdS/CFT correspondence

Quantum gravity (string theory/M-theory) in $(D+1)$-dim. anti-de-Sitter(AdS) spacetime is equivalent (dual) to D-dim. conformal field theory at AdS boundary

- Encodes rich physics: holographic principle, duality, ...;
- Relies heavily on string technology;
- Strong evidences from various points of view;
- Profound physical implications in both quantum gravity and QFT;

Implications of AdS/CFT

- A novel definition of quantum gravity: is every gauge theory a kind of string theory?
- Resolve the black hole information paradox;
- Emergent spacetime;
- Provide a tool to study the problems in quantum gravity;

1. A completely new way to study the strong coupling problems in field theory;
2. AdS/QCD;
3. RHIC physics: QGP;
4. Condensed matter physics: superfluid, superconductor, ultra-cold atoms;
5. ...

Beyond AdS spacetime?

- Holographic principle is essential;
- How to generalize the correspondence?
  - dS/CFT?;
  - Minkowski/CFT?
  - Other possibility?
    - Dual of field theory with Schrodinger or Galilean symmetry;
    - Dual of field theory with anisotropic scaling;
- Can we get rid of string technology?

AdS_3/CFT_2 correspondence

- 3D pure gravity with a negative C.C;
- AdS_3 as vacuum solution;
- Asymptotical symmetry group is $SL(2,R) \times SL(2,R)$;
- Quantum gravity asymp. to AdS_3 is holographically dual to a two-dim. CFT;
- A first example of AdS/CFT correspondence;
- The discovery of the correspondence actually has nothing to do with string theory. It is based on the analysis of the symmetry of the perturbations at asym. boundary;
- The BTZ black hole entropy could be read from Cardy formula in dual CFT;

3D Massive Gravity S.Deser et.al. (1981,1982)

- Action:

$$I_{TMG}=\frac{1}{16\pi G}[\int d^3x\sqrt{-g}(R+\frac{2}{l^2})+\frac{1}{\mu}I_{CS}]. \qquad (1.1)$$

$(\mu>0,G>0)$

- I_{CS}: gravitational Chern-Simons term

$$I_{CS}=\frac{1}{2}\int d^3x\sqrt{-g}\varepsilon^{\lambda\mu\nu}\Gamma^\rho_{\lambda\sigma}\left(\partial_\mu\Gamma^\sigma_{\rho\nu}+\frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu\rho}\right) \qquad (1.2)$$

$==>$ a new massive, propagating d.o.f;

- AdS_3 vacuum: $SL(2)_L\times SL(2)_R$ invariant. It is only well-defined at $\mu l=1$. This defines so-called chiral gravity. Furthermore, it was conjectured that quantum topological massive gravity in AdS_3 vacuum is dual to a (chiral) CFT. Li, Song and Strominger (2008), ...

Other vacua at $\mu l \neq 1$?

YES, warped AdS_3 with $SL(2) \times U(1)$ isometry. A. Strominger et.al.(2008)

- Spacelike one: $v = \frac{\mu l}{3}$

$$ds^2 = \frac{l^2}{v^2+3}[-(1+r^2)dr^2 + \frac{dr^2}{1+r^2} + \frac{4v^2}{v^2+3}(dx + rd\tau)^2], \tag{1.3}$$

  where $v^2 > 1$ stretched, $v^2 < 1$ squashed. Only stretched one is stable. Anninos, et.al. (2009)

- Timelike one;
- Null one:

$$ds^2 = l^2[\frac{du^2}{u^2} + \frac{dx^+dx^-}{u^2} \pm (\frac{dx^-}{u^2})^2], \tag{1.4}$$

  only a solution of TMG at $v^2 = 1$.

Warped AdS/CFT correspondence: A. Strominger et.al.(2008)

$v > 1$ quantum topological massive gravity with asymptotical spacelike stretched AdS_3 geometry is holographically dual to a two-dimensional conformal field theory with central charges

$$c_L = \frac{l}{G}\frac{4v}{v^2+3}, \qquad c_R = \frac{l}{G}\frac{5v^2+3}{v(v^2+3)}. \tag{1.5}$$

- Inspired by the study of BTZ black hole, the warped BH could be obtained from discrete identifications of warped AdS_3;
- For the spacelike stretched and null warped AdS_3 blackholes, there exist dual CFT descriptions;
- Thermodynamics: entropy, ...;
- Asym. symmetry analysis and Stability; Compere et.al., D. Anninos et.al., Blagojevic et.al.
- No string theory technology;
- Applied to Kerr/CFT correspondence; M. Guica et.al.(2008)
- It is intriguing: warped AdS_3 has very different conformal boundary;

Motivations

- More evidence?
- Conformal weights of various pert.;
- Quasi-normal modes;
- Real-time correlators;

Finite temperature AdS/CFT

- BH in AdS $\sim$ finite temperature CFT;
- Taking BH as a thermodynamical system, the thermal equil. in BH system could be compared to thermal equil. of finite T CFT;
- Quasi-normal modes in BH correspond to the poles of retarded Green's function in the momentum space;
- Real-time correlators from gravity is subtle
    - Q: Boundary conditions at black hole horizon?
    - A: Purely ingoing one correspond to retarded Green's function;
    - Analytic continuation? Not clear.

Prescriptions

- A prescription first suggested by A. Son et.al.(2005);
- Its modern version:Gubser et.al.(2008), H.Liu et.al.(2009)

$$G_R(\omega,\vec{k}) = \lim_{r\to\infty} \frac{\Pi(r,\omega,\vec{k})|_{\phi_R}}{\phi_R(r,\omega,\vec{k})}, \tag{2.1}$$

where Π is the canonical momentum conjugate to ϕ, taking r as the "time" direction. Now ϕ_R is the classical solution, which should be purely in-falling at the black hole horizon and turns to $\phi_0(\vec{k})$ asymptotically.

- Subtlety: plug in appropriate terms proportional to the power of r to cancel the divergence;
- Holographic renormalization? not clear;

2D CFT

- Two independent sectors: left-moving one and right-moving one, possibly with different central charges and temperatures;
- Retarded Green's function and Matsubara propagator:

$$G_R(i\omega_L, i\omega_R) = G_E(\omega_{L,E}, \omega_{R,E}), \tag{3.1}$$

at

$$\omega_{L,E} = 2\pi n_L T_L, \qquad \omega_{R,E} = 2\pi n_R T_R \tag{3.2}$$

with n_L, n_R being integers.

- Two-point function:

$$G(t^+, t^-) = \langle \mathcal{O}_\phi^\dagger(t^+, t^-)\mathcal{O}_\phi(0)\rangle, \tag{3.3}$$

where t^+, t^- are the left and right moving coordinates of 2d worldsheet, and $\mathcal{O}_\phi$ is the operator corresponding to the field perturbing the black hole.

Green's functions in 2D CFT: continued

- Consider an operator of conformal dimensions (h_L, h_R), right charge q_R, at temperature (T_L, T_R) and chemical potential Ω_R.
- Correlators in 2D CFT are very much decided by conformal invariance:J.Cardy (1984)

$$G(t^+, t^-) \sim \left(\frac{\pi T_L}{\sinh(\pi T_L t^+)}\right)^{2h_L}\left(\frac{\pi T_R}{\sinh(\pi T_R t^-)}\right)^{2h_R} e^{iq_R\Omega_R t^-}. \tag{3.4}$$

- Left-mover in the momentum space:

$$G_E(\omega_{L,E}) \sim \frac{T_L^{2h_L-2} e^{i\omega_{L,E}/2T_L}\Gamma(1-2h_L)}{\Gamma(1-h_L+\frac{\omega_{L,E}}{2\pi T_L})\Gamma(1-h_L-\frac{\omega_{L,E}}{2\pi T_L})}. \tag{3.5}$$

Green's functions in 2D CFT: continued

- Right-mover:

$$G_E(\omega_{R,E}) \sim \frac{T_R^{2h_R-2} e^{(i\omega_{R,E}+q_R\Omega_R)/2T_R}\Gamma(1-2h_R)}{\Gamma(1-h_R+\frac{\omega_{R,E}-iq_R\Omega_R}{2\pi T_R})\Gamma(1-h_R-\frac{\omega_{R,E}-iq_R\Omega_R}{2\pi T_R})}. \tag{3.6}$$

- The total contribution is the product of the left-mover's (3.5) and the right-mover's (3.6):

$$G_E(\omega_{L,E},\omega_{R,E}) = G_E(\omega_{L,E})G_E(\omega_{R,E}). \tag{3.7}$$

Cross section

- Cross section: following Fermi's golden rule:

$$\sigma_{abs} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+}[G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)] \quad (3.8)$$

- In momentum space:

$$\sigma \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R - q_R\Omega_R}{2T_R}\right)\left|\Gamma\left(h_L + i\frac{\omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R - q_R\Omega_R}{2\pi T_R}\right)\right|^2. \quad (3.9)$$

Spacelike Stretched Black holes

- The metric in terms of Schwarzschild coordinates:

$$ds^2 = -N(r)^2 dt^2 + R(r)^2[d\theta + N^\theta(r)dt]^2 + \frac{l^2 dr^2}{4R(r)^2 N(r)^2},$$

where

$$R(r)^2 = \frac{r}{4}\Big(3(v^2-1)r + (v^2+3)(r_+ + r_-) - 4v\sqrt{r_+ r_-(v^2+3)}$$

$$N^2(r) = \frac{(v^2+3)(r-r_+)(r-r_-)}{4R(r)^2},$$

$$N^\theta(r) = \frac{2vr - \sqrt{r_+ r_-(v^2+3)}}{2R(r)^2},$$

where $-l^{-2}$ is the negative cosmological constant and the parameter $v = \mu l/3$ with μ being the mass of the graviton.

- Two horizons $r = r_\pm$;
- Physical one: $v \geq 1$;
- $v = 1$ case: BTZ BH in rotational coordinates.

Scalar pert. in spacelike stretched BH

1. Linearized equation:

$$(\nabla_\mu \nabla^\mu - m^2)\Phi = 0$$

2. Two translational Killing symmetry along t and θ in the background so

$$\Phi = e^{-i\omega t + ik\theta}\phi,$$

3. Another trick: define $z = \frac{r-r_+}{r-r_-}$;

The equation is of the form

$$z(1-z)\frac{d^2\phi}{dz^2} + (1-z)\frac{d\phi}{dz} + \frac{1}{(v^2+3)^2}\left(\frac{A}{z} + B + \frac{C}{1-z}\right)\phi = 0,$$

Near the horizon $z = 0$ or $r = r_+$, there are two independent solutions. The ingoing B.C. takes the solution

$$\phi_1 = z^\alpha (1-z)^\beta F(a,b,c,z)$$

Conformal weights BC and Xu (2009)

- The spacelike stretched AdS$_3$ has $U(1)_L \times SL(2)_R$ isometry, with the generators L_0 and $\bar{L}_0, \bar{L}_1, \bar{L}_{-1}$;
- The scalar eq.:

$$\left\{-[\frac{1}{2}(\bar{L}_1\bar{L}_{-1}+\bar{L}_{-1}\bar{L}_1)-\bar{L}_0^2]+\frac{3(v^2-1)}{4v^2}L_0^2-\frac{m^2l^2}{v^2+3}\right\}\Phi=0.$$

- Highest weight mode:

$$\bar{L}_1\Phi=0, \qquad \bar{L}_0\Phi=h_R\Phi, \qquad L_0\Phi=\tilde{k}\Phi.$$

- The conformal weight of the scalar of mass m:

$$h_R^{\pm}==\frac{1}{2}\pm\sqrt{\frac{1}{4}+s_s}, \tag{4.1}$$

with

$$s_s=\frac{3(1-v^2)}{4v^2}\tilde{k}^2+\frac{l^2}{v^2+3}m^2.$$

Quantum number $\tilde{k}$

- The presence of the quantum number $\tilde{k}$ in the conformal weight;
- Even though the mass-square of the scalar field satisfies the Breitenlohner-Freedman bound for three-dimensional AdS spacetime, the perturbation could still be unstable;
- It also induce a right-charge w.r.t. chemical potential;

Asym. behaviors

1. The asym. behaviors of the scalar field is like:

$$\phi \sim A r^{h_R - 1} + B r^{-h_R}.$$

2. If $s_s > 0$, $h_R > 1$, then the source should be the first term;
3. If $s_s < 0$, $0 < h_R^{\pm} < 1$, then both terms could be the source. One needs special care to treat the problem.

$h_R > 1$ case

- The conjugate momentum:

$$\Pi_s = -\frac{1}{2}(r_+ - r_-)(v^2+3)z\partial_z\phi$$

- Asymptotically the dominant contribution in ϕ is proportional to r^{h_R-1}, so the terms in Π_s proportional to r^{1-h_R} should be picked out.
- The retarded correlator is just

$$G_R \sim \frac{\Gamma(a+b-c+1)}{\Gamma(c-a-b)}\frac{\Gamma(c-a)\Gamma(c-b)}{\Gamma(a)\Gamma(b)} \qquad (4.2)$$

Identification of quantum numbers

- The asym. geometry of spacelike AdS$_3$ BH is only same to the ones of spacelike AdS$_3$, after local coordinate transf.:

$$\tau \leftrightarrow -\frac{v^2+3}{2}\theta, \qquad x \leftrightarrow -\frac{v^2+3}{2v}t.$$

- It induces the identification of quantum numbers:

$$\tilde{\omega} = \frac{2}{v^2+3}k, \qquad \tilde{k} = \frac{2v}{v^2+3}\omega, \tag{4.3}$$

where $\tilde{\omega}, \tilde{k}$ are the quantum numbers in the global spacelike AdS_3.

- Key point: It is essential to use the quantum numbers in global warped AdS$_3$ spacetime rather than the ones in the black holes to set up the dictionary.

Retarded Green's function

In terms of $\tilde{\omega}, \tilde{k}$, the correlator could be rewritten in the following form:

$$G_R \sim \mathcal{N}' \frac{\Gamma(1-2h_R)\Gamma(1-2h_L)}{\Gamma(1-h_L+\frac{i\omega_L}{2\pi T_L})\Gamma(1-h_L-\frac{i\omega_L}{2\pi T_L})\Gamma(1-h_R+\frac{i(\omega_R-q_R\Omega_R)}{2\pi T_R})\Gamma(}$$

with $\mathcal{N}'$ being a normalization factor. Here $h_L = h_R$ and

$$\omega_L = 2\pi T_L \tilde{k}, \quad \omega_R = \frac{v^2+3}{2l}\tilde{\omega}, \quad q_R = -\tilde{k}, \quad \Omega_R = 2\pi T_L. \tag{4.5}$$

Obviously the retarded correlator (4.4) is proportional to (3.7), up to a normalization factor.

Quasi-normal modes

1. Quasi-normal modes as the “Sound” of a BH: purely ingoing at horizon and “damped” oscillations due to complex frequency;
2. QNM $\sim$ tiny deviations $\mathcal{O}_i$ of thermal equilibrium in dual field theory;
3. The frequencies of QNM $\sim$ the poles in the retarded green function of the perturbations $\mathcal{O}_i$ in momentum space;
4. For the scalar pert. about spacelike AdS$_3$ BH:

$$\begin{aligned} \omega_L &= -i2\pi T_L(n_L + h_L) \\ \omega_R &= q_R\Omega_R - i2\pi T_R(n_R + h_R) \end{aligned}$$

where n_L and n_R are the non-negative integers. This is in precise match with the result from direct computation.

Cross section

- The imaginary part of real-time retarded correlators could be identified as the cross section;
- For the scalar scattering BH:

$$\sigma \sim \sinh\left(\left(\frac{\omega_L}{2\pi T_L}+\frac{\omega_R-q_R\Omega_R}{2\pi T_R}\right)\pi\right)|\Gamma(h_L^+ + i\frac{\omega_L}{2\pi T_L})|^2 |\Gamma(h_R^+ + i\frac{\omega_R-q_R\Omega_R}{2\pi T_R})|^2, \tag{4.6}$$

- It is well consistent with the CFT prediction.
- $m=k=0, \omega\to 0$ limit, $\sigma\propto\omega$, consistent with universal statement;

Remarks:

- For the conformal weight $0 < h_R < 1$, the analysis is tricky, but the picture is the same;
- For vector, the similar agreement with CFT prediction;
- For spin-$1/2$ fermion, the QNMs could be calculated analytically, but the real-time correlator is subtler;
- For graviton pert., the calculation is too involved;
- For the extremal BH, it behaves like “chiral”, much like null BH;

Null warped AdS/CFT correspondence

- Conjecture: $v = 1$ quantum topological massive gravity with asymptotical null warped AdS_3 geometry is holographically dual to a 2D boundary CFT with $c_L = \frac{l}{G}$ and $c_R = \frac{2l}{G}$.
- The null warped BH is extremal;
- The thermodynamics of null warped BH: $T_R = \frac{\alpha}{\pi l}$.
- The study of QNMs and real-time correlators support this conjecture;

Self-dual solutions

The equation of motion in 3D TMG:

$$G_{\mu\nu} - \frac{1}{\ell^2} g_{\mu\nu} + \frac{\ell}{3\nu} C_{\mu\nu} = 0 \,, \tag{5.1}$$

where $G_{\mu\nu}$ is the Einstein tensor and $C_{\mu\nu}$ is the Cotton tensor:

$$C_{\mu\nu} = \varepsilon_{\mu}{}^{\alpha\beta} \nabla_\alpha \left(R_{\beta\nu} - \frac{1}{4} g_{\beta\nu} R \right) \,. \tag{5.2}$$

The self-dual solution is of the form

$$\begin{aligned} ds^2 \;=\; & \frac{\ell^2}{\nu^2+3} \Bigg(-(x-x_+)(x-x_-)\, d\tau^2 + \frac{1}{(x-x_+)(x-x_-)}\, dx^2 \\ & + \frac{4\nu^2}{\nu^2+3} (\alpha d\phi + (x - \frac{x_+ + x_-}{2})\, d\tau)^2 \Bigg) \,, \end{aligned} \tag{5.3}$$

where the coordinates range as $\tau \in [-\infty, \infty]$, $x \in [-\infty, \infty]$ and $\phi \sim \phi + 2\pi$.

Remarks on the self-dual solutions

- There seems to be two horizon at x_+ and x_-;
- Extremal one with $x_+ = x_-$, which appears as part of the near-horizon geometry of extremal Kerr black hole;
- Inspired by the study in the Kerr/CFT: near-NHEK counterpart in TMG?
- It turns out to be the case;
- Virtue: asymptotical to the global warped AdS_3 without local coordinate transformation;
- Locally it is diffeomorphic to the global spacetime;
- However, it could not be obtained by the discrete identification, except the extremal one, unlike the other black holes;

Thermodynamics of the self-dual solutions

- Conserved charges:

$$\mathcal{M}^{ADT} = 0\,, \qquad \mathcal{J}^{ADT} = \frac{(\alpha^2-1)\nu\ell}{6G(\nu^2+3)}\,. \tag{5.4}$$

- Entropy: with the Chern-Simons correction

$$S = S_E + S_{CS} = \frac{2\pi\alpha\nu\ell}{3G(\nu^2+3)}\,. \tag{5.5}$$

- the Hawking temperature T_H and the angular velocity of the horizon Ω_h ,

$$T_H = \frac{x_+ - x_-}{4\pi\ell}\,, \qquad \Omega_h = -\frac{x_+ - x_-}{2\alpha\ell}\,. \tag{5.6}$$

- First law:

$$d\mathcal{M}^{ADT} = T_H\, dS + \Omega_h\, d\mathcal{J}^{ADT}\,, \tag{5.7}$$

is satisfied for a variation of the black hole parameter α.

Temperatures in dual CFT

- The vacuum is a diagonal density matrix in the energy-angular momentum eigenbasis with a Boltzmann weighting factor

$$\exp\left(-\hbar\frac{\omega - k\Omega_H}{T_H}\right). \tag{5.8}$$

- The Boltzman factor should be identified with CFT

$$\exp\left(-\hbar\frac{\omega - k\Omega_H}{T_H}\right) = \exp\left(-\frac{n_L}{T_L} - \frac{n_R}{T_R}\right) \tag{5.9}$$

- The left and right charges n_L , n_R associated to ∂_ϕ and ∂_τ are

$$n_L \equiv k\,, \qquad n_R \equiv \omega\,. \tag{5.10}$$

- This defines the left and right temperatures:

$$T_L = \frac{\alpha}{2\pi\ell}\,, \qquad T_R = \frac{x_+ - x_-}{4\pi\ell}\,. \tag{5.11}$$

Asymptotic behavior

We impose the following boundary conditions

$$\begin{pmatrix} h_{\tau\tau} = O(1) & h_{\tau x} = O(1/x^3) & h_{\tau\phi} = O(x) \\ h_{x\tau} = h_{\tau x} & h_{xx} = O(1/x^3) & h_{x\phi} = O(1/x) \\ h_{\phi\tau} = h_{\tau\phi} & h_{\phi x} = h_{x\phi} & h_{\phi\phi} = O(1) \end{pmatrix} \quad (5.12)$$

where $h_{\mu\nu}$ is the deviation of the full metric from the vacuum.

- The allowed deviations $h_{\tau\phi}$ and $h_{\phi\phi}$ are of the same order as the leading terms;
- The boundary conditions differ from both the ones in the spacelike warped AdS_3 and the Kerr black holes;

Asymptotically conserved charges

- Asymp. symmetry group: preserving the above boundary condition;
- In our case, we have only one copy of the conformal group of the circle generated by

$$\xi_\epsilon \;=\; \epsilon\,(\phi)\,\partial_\phi\,. \tag{5.13}$$

- Since $\phi \sim \phi + 2\pi$, it is convenient to define $\epsilon_n(\phi) = e^{in\phi}$ and $\xi_n = \xi(\epsilon_n)$. These generators admit the following commutators

$$i\,[\xi_m,\,\xi_n] \;=\; (m-n)\,\xi_{m+n}\,, \tag{5.14}$$

and ξ_0 generates the $U(1)$ rotational isometry.

- This Virasora algebra become centrally extended via Poisson bracket, with

$$c_L \;=\; \frac{4\nu\ell}{G(\nu^2+3)} \tag{5.15}$$

- There is no right-moving central charge;

Holographic picture

- Conjecture: our self-dual black hole solution is dual to a 2D CFT with the temperatures and central charges;

$$T_L = \frac{\alpha}{2\pi\ell}\,, \qquad T_R = \frac{x_+ - x_-}{4\pi\ell} \qquad c_L = \frac{4\nu\ell}{G(\nu^2+3)}\,. \tag{5.16}$$

- The entropy could be recovered via the Cardy formula:

$$S = \frac{2\pi\alpha\nu\ell}{3G(\nu^2+3)} = \frac{\pi^2\ell}{3}\frac{4\nu\ell}{G(\nu^2+3)}\frac{\alpha}{2\pi\ell} \equiv \frac{\pi^2\ell}{3}c_L T_L\,, \tag{5.17}$$

- Another support comes from the scalar perturbation in the background: the retarded Green's function is in perfect match with the CFT prediction;

Conclusion and discussions

- Scalar, vector and spinor wave-functions for the spacelike stretched BH and the null warped BH can be calculated analytically;
- Scalar and vector real-time correlators are well consistent with the CFT prediction, so are QNMs and the cross sections;
- The superradiance may happen, just like Kerr BH;
- The discrepancy resides in the normalization factor, which ask for extra nontrivial temperature dependence;
- Null warped correspondence?

The relation to Kerr/CFT correspondence

- Kerr/CFT correspondence is actually NHEK/CFT;
- The NHEK region is a warped AdS_3 spacetime;
- In Kerr case, asym. flat so well-defined ingoing and outgoing waves. This fact allows the determination of cross section;
- In warped AdS_3 case, the ingoing and outgoing wave are not well-defined. However the prescription in AdS/CFT correspondence allows us to determine the real-time correlators, and then the cross sections;
- The similarities in two correspondences are not unexpected;

Another implication

- The prescription is still effective for the spacetime with nontrivial conformal boundary;
- Null backgrounds are actually related to the gravity dual to the field theory with Lifshitz point, AdS/CMT;
- Our study provide evidence to apply the prescription to these area;

On self-dual warped black holes

- The self-dual warped BH are the TMG analogs of the near-NHEK;
- There exist consistent boundary conditions, for which the asymptotic symmetry generators form one sector of Virasoro algebra with central charge $c_L = \frac{4\nu\ell}{G(\nu^2+3)}$;
- The black hole quantum states can be identified with those of a chiral(left) half of a two-dimensional conformal field theory;
- Our investigation suggests that the quantum topological massive gravity asymptotic to the same spacelike warped AdS_3 in different consistent ways may dual to different 2D CFTs.

Thank you!