# Brane Cosmology in String/M-Theory

Baylor Science Building


Anzhong Wang

GCAP-CASPER, Phys. Dept.

Baylor Univ., Waco, Texas 76798


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# Members of Gravity, Cosmology and Astroparticle Physics Group - GCAP

Dr. Anzhong Wang (Head, DOE)

Dr. K. Kirsten (Math, NSF)

Dr. Qin (Tim) Sheng (Math, Air Forces)

Michael Devin

Janie Hoormann (Undergrad)

Yongqing (Steve) Huang

Satheeshkumar Veerahanumak

Raziyeh Yousefi

Ahmad Borzou

Dr. Rong-Gen Cai, ITP, Chinese Academy

Dr. Yun-Gui Gong, Chongqing Univ. of Posts & Telecommunications

Dr. Jianxin Lu, Interdisciplinary Center for Theoretical Study, USTC

Dr. N. O. Santos, Queen Mary College, London University

# Collaborators

- Dr. Greg Benesh, Phys. Dept., Baylor University, USA
- Dr. Gerald Cleaver, Phys. Dept., Baylor University, USA
- Jared Greenwald, Phys. Dept., Baylor University, USA

- Dr. R. Chan, Brazilian National Observatory, Rio de Janeiro, Brazil
- Dr. M.F. da Silva, the State University of Rio de Janeiro, Brazil
- Dr. Eric Hirschmann, BYU, Utah, USA
- Dr. Mustapha Ishak, UT Dallas, USA
- Dr. Roy Maartens, ICG, Portsmouth University, England
- Dr. Antonios Papazoglou, ICG, Portsmouth University, England
- Dr. David Wands, ICG, Portsmouth University, England
- Dr. Qiang Wu, ZUT, Hangzhou, China

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- Radion Stability & Mass
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- The Hierarchy Problem
- The Cosmological Constant
- Current Acceleration of the Universe
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③ **Brane Cosmology in M-Theory**

④ **Conclusions & Future Work**

# 0. A Brief Intro to Baylor University

- Chartered in 1845 by the Republic of Texas **- the oldest university in Texas.** Right now we have about 15,000 undergraduate & graduate students.

http://www.baylor.edu/

# 0. A Brief Intro to Baylor University (Cont.)

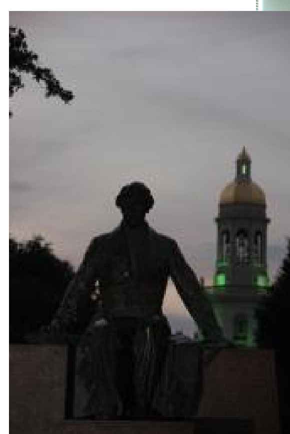
- Physics Department has 20 faculty members, and about $\sim$ 35 graduate and $\sim$ 35 undergraduate students.

Phys. Dept. Located at 3th Floor

http://www.baylor.edu/physics/

# 0. A Brief Intro to Baylor University (Cont.)

- Undergrad Majors: BA & BSc. in (a) Physics; (b) Astrophysics; (c) Astronomy.
- Graduate Degrees: MA, MSc. & Ph. D. Physics.

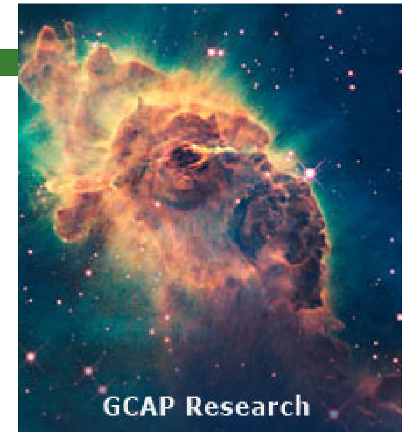


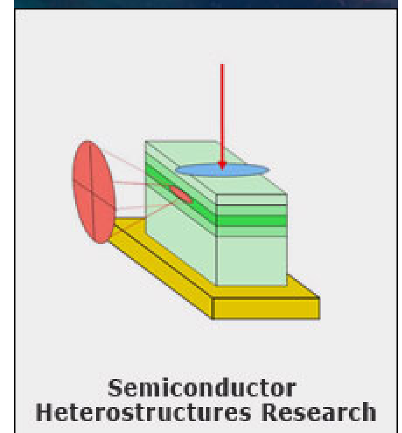
Some Physics Faculty & Students

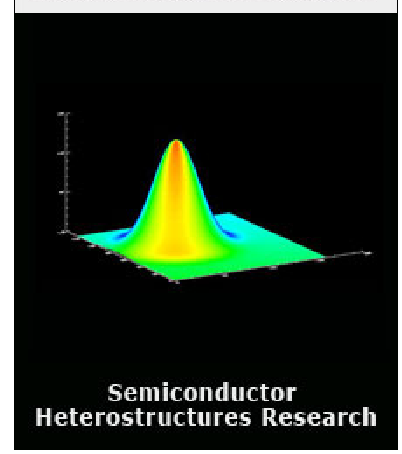
http://www.baylor.edu/physics/

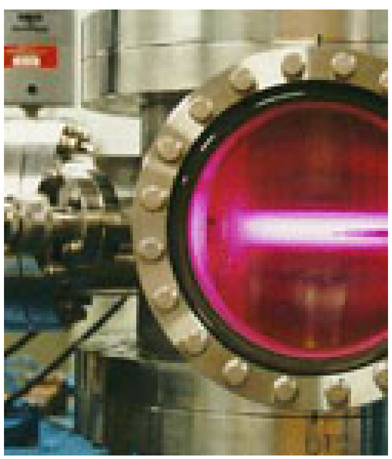
# 0. A Brief Intro to Baylor University (Cont.)

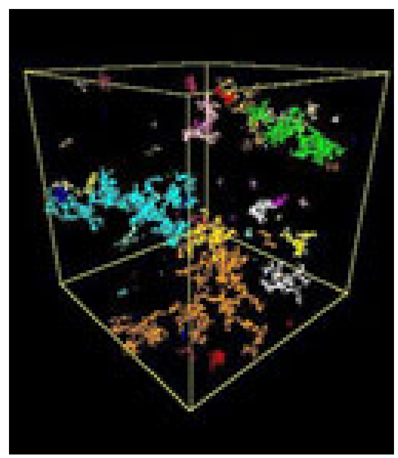
- Astrophysics and Space Sciences
- Atomic and Molecular Physics
- **Condensed Matter Physics**
- Elementary Particle Physics
- Gravitation and Cosmology
- Non-Linear Dynamics
- **Plasma Physics**
- Surface Chemical Physics



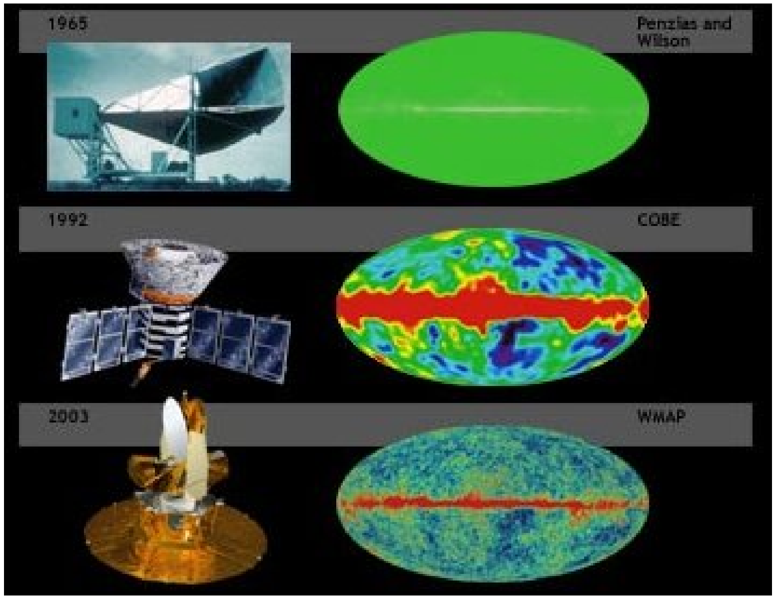





# 0. A Brief Intro to Baylor University (Cont.)

GCAP - Gravity, Cosmology and Astroparticle Physics Group:

- Horava-Lifshtiz theory of quantum gravity & applications
- Formation & thermodynamics of black holes
- Cosmology in String/M Theory
- **Brane Cosmology**




# I. Motivations

## A. The Hierarchy Problem:

- Classical gravity is described by

$$S_g = \frac{M_{pl}^2}{16\pi}\int d^4x\sqrt{-g}R[g],$$

$M_{pl}$: the Planck scale,

$$M_{pl} \equiv G^{-1/2} \sim 1.22 \times 10^{16}\ \text{TeV},$$

which determines the gravitational scale at which the quantum gravity is required.

# I. Introduction (Cont.)

- When $E \ll M_{pl}$, gravity is Newtonian,

$$V(r) = -\frac{G_4 m}{r}.$$

  Latest Table Top Experiments show Newtonian gravity is valid down to $r \sim 44\mu m$ [D. Kapner, *et al*, PRL98 (2007) 021101].

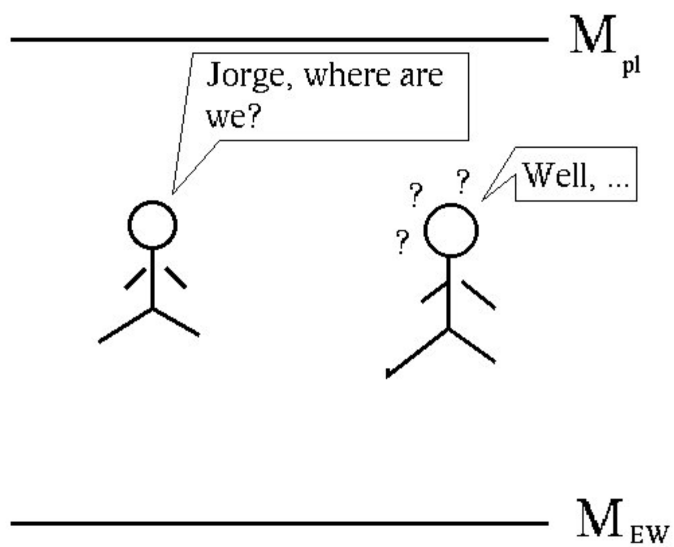
- Electroweak Scale: $M_{EW} \sim 1\ TeV$.

# I. Introduction (Cont.)

- A “Desert” (**Hierarchy Problem**):

$$\frac{M_{pl}}{M_{EW}} \sim 10^{16},$$

but no new physics between them!!!



# I. Introduction (Cont.)

## B. Current Acceleration of the Universe:

**Recent observations show:**

- The total matter is

$$\Omega_{total} = 1.02 \pm 0.02,$$

$\Rightarrow$ our universe is **FLAT.**

- It is accelerating,

$$\ddot{a} > 0!!! \tag{1}$$

# I. Introduction (Cont.)

- So far, all observations are consistent with the cosmological constant,

$$\rho_{\Lambda,ob} \simeq 10^{-47}\ GeV^4.$$

[E. Komatsu, et al., arXiv:1001.4538].

- The Cosmological Constant Problem: Zel'dovich (1968) first realized

$$\langle T_{\mu\nu} \rangle = \Lambda g_{\mu\nu},$$

but divergent,

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} = \langle T_{00} \rangle \sim \int_0^\infty \sqrt{k^2+m^2}k^2 dk \sim k^4\Big|_0^\infty.$$

## I. Introduction (Cont.)

- The ultraviolet cutoff at the Planck scale yields,

$$\rho_{\Lambda,pl} \sim 10^{123} \rho_{\Lambda,ob}$$

- Even the cutoff is at the QCD scale, we still have

$$\rho_{\Lambda,QCD} \sim 10^{40} \rho_{\Lambda,ob}$$

**— the cosmological problem.**

# II. Orbifold Branes in String Theory

[AW & Santos, PLB669,127 (2008) [arXiv:0712.3938]; IJMPA25, 1661 (2010) [arXiv:0808.2055]]

Consider the NS-NS sector in (D+d) dimensions,

$$M_{D+d} = M_D \times \mathcal{T}_d,$$

$\mathcal{T}_d$: a d-dimensional torus. The action

$$\hat{S}_{D+d} = -\frac{1}{2\kappa^2_{D+d}} \int d^{D+d}x \sqrt{|\hat{g}_{D+d}|} e^{-\hat{\Phi}} \left\{ \hat{R}_{D+d}[\hat{g}] + \hat{g}^{AB} \left(\hat{\nabla}_A \hat{\Phi}\right) \left(\hat{\nabla}_B \hat{\Phi}\right) - \frac{1}{12} \hat{H}^2 \right\}, \qquad (2)$$

$\hat{\Phi}$: the dilaton field; $\hat{H}_{ABC}$: the 3-form.

# II. Orbifold Branes in String (Cont.)

Consider the spacetime,

$$
\begin{aligned}
d\hat{s}^2_{D+d} &= \hat{g}_{AB} dx^A dx^B \\
&= e^{-\sqrt{\frac{2}{D-2}}\phi} g_{ab}(x)\, dx^a dx^b \\
&\quad + e^{\sqrt{\frac{2}{d}}\psi(x)} \delta_{ij} dz^i dz^j ,
\end{aligned}
$$

with $\hat{\Phi} = \hat{\Phi}(x^a)$, $\hat{B}_{CD} = \hat{B}_{CD}(x^a)$,

$$
\phi = \hat{\Phi}(x^a) - \sqrt{\frac{d}{2}}\, \psi(x^a),
$$

# II. Orbifold Branes in String (Cont.)

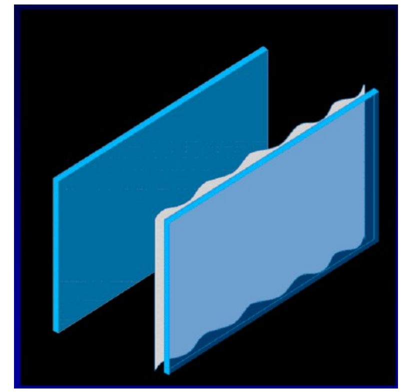
$$\left(\hat{B}_{MN}\right) = \begin{pmatrix} \tilde{B}_{ab}(x) & 0 \\ 0 & B_{ij}(x) \end{pmatrix}. \tag{3}$$

After dimensional reduction & conformal transformation, the action reduces,

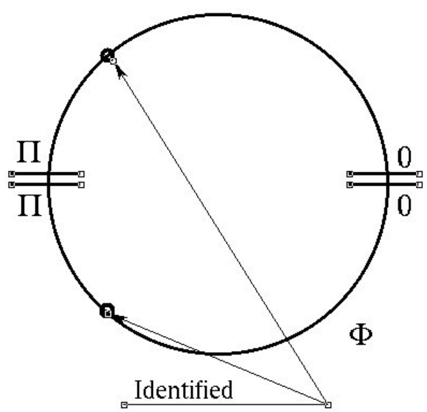
$$\begin{aligned} S_D^{(E)} &= -\frac{1}{2\kappa_D^2}\int d^D x\sqrt{|g_D|}\,\Big\{R_D[g] \\ &\quad -\frac{1}{2}\left[(\nabla\phi)^2 + (\nabla\psi)^2 - 2V_D(\phi,\psi)\right] \\ &\quad -\frac{1}{4}e^{-\sqrt{\frac{8}{d}}\,\psi}\left(\nabla_a B_{ij}\right)\left(\nabla^a B^{ij}\right) \\ &\quad -\frac{1}{12}e^{-\sqrt{\frac{8}{D-2}}\,\phi}H_{abc}H^{abc}\Big\}. \end{aligned}$$

# II. Orbifold Branes in String (Cont.)

Compactify one of the $(D-1)$ dimensions by putting two orbifold branes,



$$
\begin{aligned}
S^{(E,I)}_{D-1,m} = & -\int_{M^{(I)}_{D-1}} \sqrt{\left|g^{(I)}_{D-1}\right|} \\
& \times \Big( \epsilon_I V^{(I)}_{D-1}(\phi, \psi) \\
& -\mathcal{L}^{(I)}_{D-1,m}(\phi, \psi, B, \chi) \\
& + \mathrm{g}^{(\mathrm{I})}_{\kappa} \Big) \, d^{D-1}\xi_{(I)}
\end{aligned} \tag{4}
$$




$\epsilon_1 = -\epsilon_2 = 1$. $\chi$: matter fields; $g^{(I)}_{\kappa}$: brane tensions.

## II. Orbifold Branes in String (Cont.)

The total action is given by,

$$S^{(E)}_{total} = S^{(E)}_{D} + S^{(E)}_{D,m} + \sum_{I=1}^{2} S^{(E,I)}_{D-1,m}, \qquad (5)$$

$\Rightarrow$

$$G^{(D)}_{ab} = \kappa^2_D T^{(D)}_{ab} + \kappa^2_D \sum_{I=1}^{2} \mathcal{T}^{(I)}_{\mu\nu} e^{(I,\,\mu)}_{a} e^{(I,\,\nu)}_{b} \times \sqrt{\left|\frac{g^{(I)}_{D-1}}{g_D}\right|}\, \delta\left(\Phi_I\right), \qquad (6)$$

where

# II. Orbifold Branes in String (Cont.)

$$
\begin{aligned}
\kappa_D^2 T_{ab}^{(D)} &\equiv \frac{1}{2}\left[(\nabla_a\phi)(\nabla_b\phi) + (\nabla_a\psi)(\nabla_b\psi)\right. \\
&+ \frac{1}{2}e^{-\sqrt{\frac{8}{d}}\,\psi}\left(\nabla_a B^{ij}\right)\left(\nabla_b B_{ij} + \frac{1}{2}e^{\sqrt{\frac{8}{D-2}}\,\phi} H_{acd}H_b{}^{cd}\right] \\
&- \frac{1}{4}g_{ab}\left[(\nabla\phi)^2 + (\nabla\psi)^2 - 2V_D \frac{1}{2}e^{-\sqrt{\frac{8}{d}}\,\psi}\left(\nabla_c B^{ij}\right)\left(\nabla^c B_{ij}\right)\right. \\
&\left. + \frac{1}{6}e^{\sqrt{\frac{8}{D-2}}\,\phi} H_{cde}H^{cde}\right], \\
\mathcal{T}_{\mu\nu}^{(I)} &\equiv \tau_{\mu\nu}^{(I)} + \left(g_\kappa^{(I)} + \tau_{(\phi,\psi)}^{(I)}\right) g_{\mu\nu}^{(I)}, \\
\tau_{\mu\nu}^{(I)} &\equiv 2\frac{\delta \mathcal{L}_{D-1,m}^{(I)}}{\delta g^{(I)\,\mu\nu}} - g_{\mu\nu}^{(I)}\mathcal{L}_{D-1,m}^{(I)}, \\
\tau_{(\phi,\psi)}^{(I)} &\equiv \epsilon_I V_{D-1}^{(I)}(\phi,\psi), \quad e_a^{(I,\mu)} \equiv g^{(I)\,\mu\nu} e_{(\nu)}^{(I)\,b} g_{ab}\Big|_{M_{D-1}^{(I)}}, \\
& g^{(I)\,\mu\nu} g_{\lambda\nu}^{(I)} = \delta_\lambda^\mu. 
\end{aligned} \tag{7}
$$

# II. Orbifold Branes in String (Cont.)

$$\begin{aligned}\Box\phi &= -\frac{\partial V_D}{\partial\phi} - \frac{1}{12}\sqrt{\frac{8}{D-2}}e^{-\sqrt{\frac{8}{D-2}}\,\phi}H_{abc}H^{abc} \\ &\quad -2\kappa_D^2\sum_{i=1}^{2}\left(\epsilon_I\frac{\partial V_{D-1}^{(I)}}{\partial\phi}+\sigma_\phi^{(I)}\right)\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right), \qquad (8)\\ \Box\psi &= -\frac{\partial V_D}{\partial\psi} - \sqrt{\frac{1}{2d}}e^{-\sqrt{\frac{8}{d}}\,\psi}\left(\nabla_a B^{ij}\right)\left(\nabla^a B_{ij}\right) \\ &\quad -2\kappa_D^2\sum_{i=1}^{2}\left(\epsilon_I\frac{\partial V_{D-1}^{(I)}}{\partial\psi}+\sigma_\psi^{(I)}\right)\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right), \qquad (9)\\ \Box B_{ij} &= \sqrt{\frac{8}{d}}\left(\nabla_a\psi\right)\left(\nabla^a B_{ij}\right) - \sum_{i=1}^{2}\sigma_{ij}^{(I)}\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right), \qquad (10)\\ \nabla^c H_{cab} &= \sqrt{\frac{8}{D-2}}\,H_{cab}\nabla^c\phi - \sum_{i=1}^{2}\sigma_{ab}^{(I)}\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right), \qquad (11)\end{aligned}$$

$\Box \equiv g^{ab}\nabla_a\nabla_b$ and

## II. Orbifold Branes in String (Cont.)

$$\sigma_\phi^{(I)} \equiv -\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta\phi}, \quad \sigma_\psi^{(I)} \equiv -\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta\psi},$$

$$\sigma_{ij}^{(I)} \equiv -4\kappa_D^2 e^{\sqrt{\frac{8}{d}}\,\psi}\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta B^{ij}},$$

$$\sigma_{ab}^{(I)} \equiv -4\kappa_D^2 e^{\sqrt{\frac{8}{D-2}}\,\phi}\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta B^{ab}}. \qquad (12)$$

# II. Orbifold Branes in String (Cont.)

## A. Radion Stability & Mass:

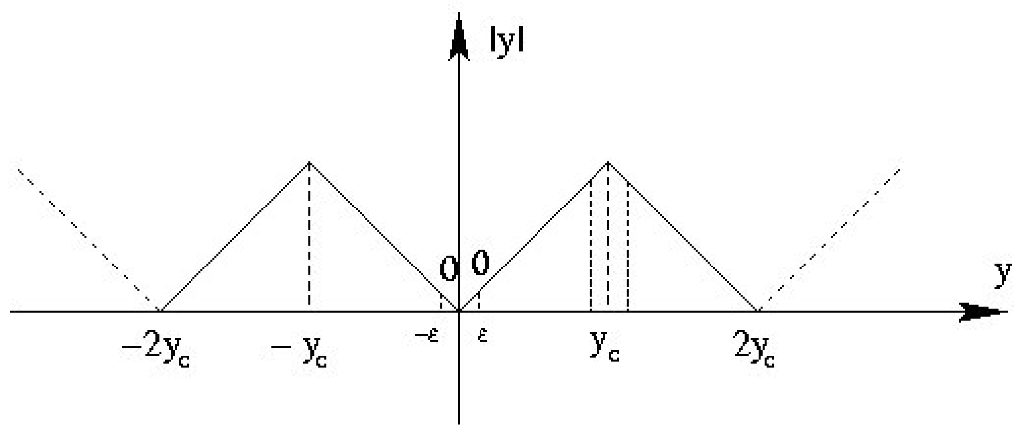
Static Solutions with Poincaré symmetry:

$$\begin{aligned}
ds_5^2 &= e^{2\sigma(y)}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\right),\\
\sigma(y) &= \frac{1}{9}\ln\left(\frac{|y|+y_0}{L}\right),\\
\phi(y) &= -\sqrt{\frac{25}{54}}\ln\left(\frac{|y|+y_0}{L}\right)+\phi_0,\\
\psi(y) &= -\sqrt{\frac{5}{18}}\ln\left(\frac{|y|+y_0}{L}\right)+\psi_0,\\
B_{ij} &= 0 = B_{ab},
\end{aligned}$$

$L,\ y_0,\ \phi_0,\ \psi_0$: integration constants, and

# II. Orbifold Branes in String (Cont.)

$|y|$ is defined as:




$$(0 \leq y \leq y_c)$$

[AW, R.-G. Cai & N.O. Santos, NPB797 (2008) 395 [ arXiv:astro-ph/0607371]]

# II. Orbifold Branes in String (Cont.)

Consider a massive scalar field $\Phi$ [Goldberger & Wise, PRL83 (1999) 4922]:

$$
\begin{aligned}
S_{bulk} &= \int d^4x \int_0^{Y_c} dY \sqrt{-g_5}\left((\nabla\Phi)^2 - M^2\Phi^2\right),\\
S^{(I)}_{brane} &= -\alpha_I \int_{M_4^{(I)}} d^4x \sqrt{-g_4^{(I)}}\left(\Phi^2 - v_I^2\right)^2,
\end{aligned}
$$

$\alpha_I,\ v_I$: real constants. The Klein-Gordon Equation:

$$
\Phi'' - 4A'\Phi' - M^2\Phi = \sum_{I=1}^{2} 2\alpha_I \Phi\left(\Phi^2 - v_I^2\right)\delta(Y - Y_I),
$$

# II. Orbifold Branes in String (Cont.)

with the boundary conditions,

$$\left.\frac{d\Phi(Y)}{dY}\right|_{Y_I-\epsilon}^{Y_I+\epsilon} = 2\alpha_I\Phi_I\left(\Phi_I^2 - v_I^2\right).$$

The solutions are

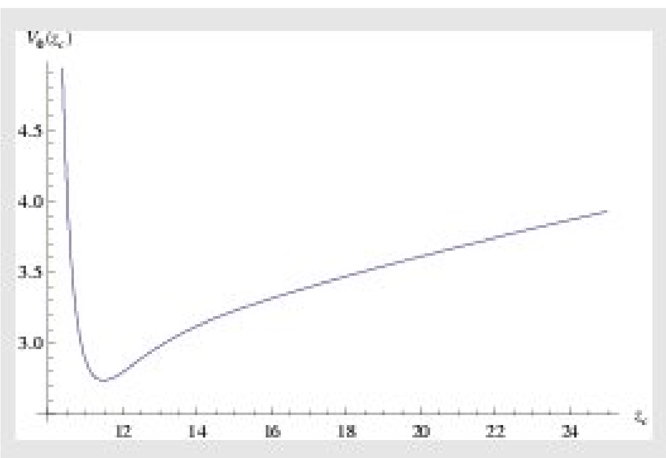
$$\Phi = z^{\nu}\left(aI_{\nu}(z) + bK_{\nu}(z)\right),$$

$\nu \equiv 3/10;\ a,\ b$: constants; $I_{\nu}(z),\ K_{\nu}(z)$: the modified Bessel functions, and $z \equiv M(Y + Y_0)$.

# II. Orbifold Branes in String (Cont.)

Then, the radion potential is given by

$$
\begin{aligned}
V_\Phi\left(Y_c\right) &\equiv -\int_{0+\epsilon}^{Y_c-\epsilon} dY \sqrt{|g_5|}\left((\nabla\Phi)^2 - m^2\Phi^2\right) \\
&\quad + \sum_{I=1}^{2} \alpha_I \int_{Y_I-\epsilon}^{Y_I+\epsilon} dY \sqrt{\left|g_4^{(I)}\right|}\left(\Phi^2 - v_I^2\right)^2 \delta\left(Y - Y_I\right) \\
&= e^{-4A(Y)}\Phi(Y)\Phi'(Y)\Big|_0^{Y_c} + \sum_{I=1}^{2} \alpha_I \left(\Phi_I^2 - v_I^2\right)^2 e^{-4A(Y_I)}.
\end{aligned}
$$




$\Rightarrow$ the radion is Stable!!!

# II. Orbifold Branes in String (Cont.)

Radion Mass: The radion field $\varphi$ is related to $Y_c$ by [Q. Wu, Y. Gong & AW, JCAP, **06**, 015 (2009) [arXiv:0810.5377]]

$$\varphi\left(Y_c\right) = \sqrt{12 f\left(Y_c\right)}, \tag{13}$$

where

$$f \equiv \frac{1}{\kappa_5^2}\int_0^{Y_c} e^{-2A(Y)}dY = \frac{5L}{6\kappa_5^2}\left(\frac{10}{9}\right)^{1/5} \times \left\{\left(\frac{Y_c+Y_0}{L}\right)^{6/5} - \left(\frac{Y_0}{L}\right)^{6/5}\right\}. \tag{14}$$

# II. Orbifold Branes in String (Cont.)

Then, we find that

$$
\begin{aligned}
m_\varphi^2 &= \frac{1}{2}\frac{\partial^2 V_\Phi(Y_c)}{\partial\varphi^2}\bigg|_{Y_c=Y_c^{min}} \\
&= \left(\frac{10Y_0}{9L}\right)^{1/5}\frac{2M^5}{3M_5^3} \\
&\quad \times \tilde{v}_1^2\tilde{v}_2^2\left|\frac{\ln(\tilde{v}_1/\tilde{v}_2)}{\tilde{v}_1^2-\tilde{v}_2^2}\right|,
\end{aligned}
\qquad (15)
$$

$v_i = M^{3/2}\tilde{v}_i$. Since $v_i$ has the dimension $[m]^{3/2}$, we can see that $\tilde{v}_i$ is dimensionless.

# II. Orbifold Branes in String (Cont.)

In addition, $M$ and $v_i$ are all 5-dimensional quantities, we expect that

$$M \sim M_5, \quad \tilde{v}_i \sim \mathcal{O}(1). \tag{16}$$

Without introducing new hierarchy, we also expect that

$$\left(\frac{Y_0}{L}\right)^{1/5} \sim \mathcal{O}(1), \quad \frac{Y_c}{Y_0} \sim \mathcal{O}(1).$$

## II. Orbifold Branes in String (Cont.)

Then, we find

$$m_\varphi \simeq \left(\frac{M_{10}}{M_{pl}}\right)^{8/3} \left(\frac{R}{l_{pl}}\right)^{5/3} M_{pl}. \tag{17}$$

For $M_{10} \sim \ TeV$, we find that

$$m_\varphi \simeq 10^{5(17-n)/3}\, TeV, \tag{18}$$

where $R = 10^{-5n/3}\, m$. Note that the experimental limit is $m_\varphi > 10^{-3}\, eV$.

# II. Orbifold Branes in String (Cont.)

B. 4D Newtonian Potential & Yukawa Corrections: [AW, PRD66 (2002) 024024 [ arXiv:hep-th/0201051]]

Consider the following tensor perturbations,

$$ds_5^2 = e^{2\sigma(y)}\left[\left(\eta_{\mu\nu} + h_{\mu\nu}(x,y)\right)dx^\mu dx^\nu - dy^2\right],$$

with the gauge

$$h_\lambda^\lambda = 0 = \partial^\lambda h_{\mu\lambda}.$$

Setting

$$h_{\mu\nu}(x,y) = e^{-3\sigma/2}\hat{h}_{\mu\nu}(x)\psi(y),$$

$$\Box_5 = \Box_4 - \nabla_y^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu - \partial_y^2,$$

$$\Box_4\hat{h}_{\mu\nu}(x) = -m^2\hat{h}_{\mu\nu}(x),$$

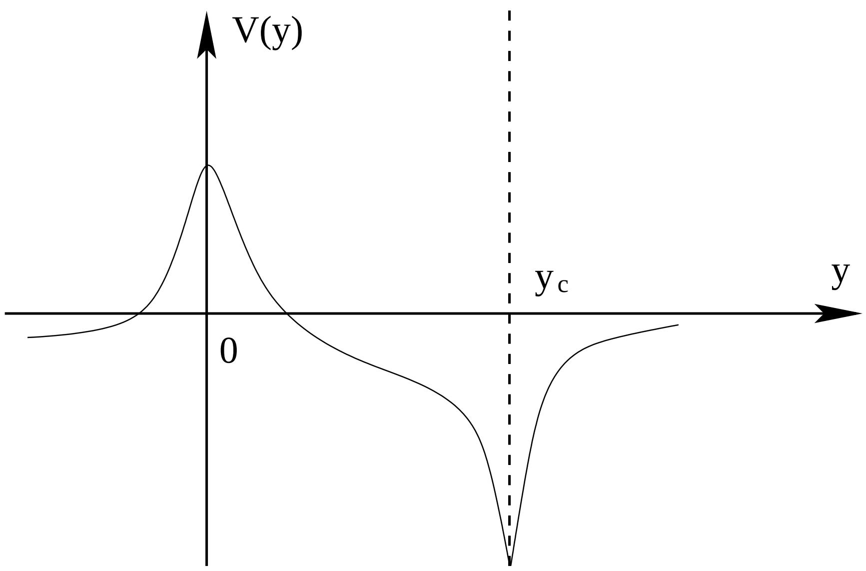
# II. Orbifold Branes in String (Cont.)

$\psi$ satisfies the Schrödinger equation:

$$\left(-\nabla_y^2 + V\right)\psi = m^2\psi,$$

$$V \equiv \frac{3}{2}\left(\sigma'' + \frac{3}{2}\sigma'^2\right) = -\frac{5}{36\left(|y| + y_0\right)^2} + \frac{\delta(y)}{3y_0}$$

$$-\frac{\delta(y - y_c)}{3(y_c + y_0)}.$$



## II. Orbifold Branes in String (Cont.)

Introducing the operators,

$$Q \equiv \nabla_y - \frac{3}{2}\sigma', \quad Q^\dagger \equiv -\nabla_y - \frac{3}{2}\sigma',$$

the Schrödinger equation takes the form,

$$Q^\dagger \cdot Q\,\psi = m^2\psi.$$

This equation itself does not ensure that the operator $Q^\dagger \cdot Q$ is Hermitian, because now it is defined only on a finite interval, $y \in [0, y_c]$.

## II. Orbifold Branes in String (Cont.)

To ensure its Hermiticity, in addition to writing it in the supersymmetric form, one also needs to require the Hermitian boundary conditions,

$$\begin{aligned} {\psi'}_n(0)\psi_m(0) \;-\; \psi_n(0){\psi'}_m(0) &= {\psi'}_n\left(y_c\right)\psi_m\left(y_c\right) \\ &- \psi_n\left(y_c\right){\psi'}_m\left(y_c\right), \end{aligned}$$

for any two solutions.

Boundary Conditions:

$$\begin{aligned} \lim_{y\to y_c^-} \psi'(y) &= \frac{1}{6\left(y_c + y_0\right)} \lim_{y\to y_c^-} \psi(y), \\ \lim_{y\to 0^+} \psi'(y) &= \frac{1}{6y_0} \lim_{y\to 0^+} \psi(y). \end{aligned}$$

# II. Orbifold Branes in String (Cont.)

*So, all the wave functions with* $m^2 < 0$ *are not normalizable, and must be discarded.*

- Zero-mode is normalized:

$$\begin{aligned}
\psi_0(y) &= N_0 \left(\frac{|y| + y_0}{L}\right)^{1/6}, \\
N_0 &\equiv 2\left\{3L\left[\left(\frac{y_c + y_0}{L}\right)^{4/3} - \left(\frac{y_0}{L}\right)^{4/3}\right]\right\}^{-1/2}.
\end{aligned}$$

$\Rightarrow$ **Gravity is localized at** $y = y_c$.

# II. Orbifold Branes in String (Cont.)

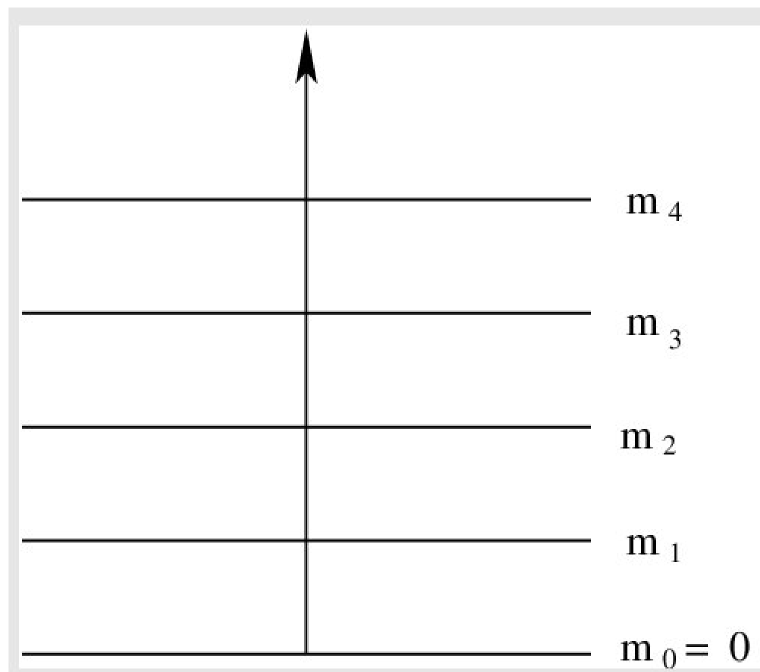
- The spectrum of $m_n$ is discrete:

$$m_n \simeq \frac{(2n-1)\,\pi}{2y_c}, \quad (n = 1, 2, 3, \ldots)$$

For $y_c \simeq 10^{-19}\ m$, we have

$$m_1 \simeq \frac{\pi}{2y_c}$$

$$\sim TeV.$$



# II. Orbifold Branes in String (Cont.)

It should be noted that the mass $m_n$ calculated above is measured by the observer with the metric $\eta_{\mu\nu}$. However, since the warped factor $e^{\sigma(y)}$ is not one at $y = y_c$, the physical mass on the visible brane should be given by

$$m_n^{obs} = e^{-\sigma(y_c)} m_n = \left(\frac{y_c + y_0}{L}\right)^{-1/9} m_n. \qquad (19)$$

## II. Orbifold Branes in String (Cont.)

Without introducing any new hierarchy, we expect that $[(y_c + y_0)/L]^{-1/9} \simeq \mathcal{O}(1)$. As a result, we have

$$m_n^{obs} = \left(\frac{y_c + y_0}{L}\right)^{-1/9} m_n \simeq m_n. \qquad (20)$$

## II. Orbifold Branes in String (Cont.)

- Yukawa corrections are suppressed exponentially:

$$U(r) = G_4 \frac{M_1 M_2}{r} \left( 1 + \frac{M_{pl}^2}{M_5^3} \times \sum_{n=1}^{\infty} e^{-m_n r} \left| \psi_n(z_c) \right|^2 \right).$$

# II. Orbifold Branes in String (Cont.)

## C.The Hierarchy Problem:

One of the main motivations of the brane worlds is to resolve the long standing *hierarchy problem*. But, it should be noted that when ADD [Arkani-Hamed, Dimopoulos, and Dvali, PLB429, 263 (1998)] derived the relation,

$$M = \left(\frac{M_{pl}^2}{V_n}\right)^{1/(2+n)}, \tag{21}$$

## II. Orbifold Branes in String (Cont.)

and RS [L. Randall and Sundrum, PRL83, 3370 (1999)] derived the one

$$M_{pl}^2 = M^3 k^{-1}\left(1 - e^{-2ky_c}\right) \simeq M_5^2, \tag{22}$$

they all implicitly assumed that the 4-dimensional effective Einstein-Hilbert action couples with matter directly in the form,

$$S_g^{eff.} + S_m = \int \sqrt{-g} d^4x \left(-\frac{1}{2\kappa_4^2} R + \mathcal{L}_m\right), \tag{23}$$

from which one obtains the Einstein field equations, $G_{\mu\nu} = \kappa_4^2 \tau_{\mu\nu}$.

## II. Orbifold Branes in String (Cont.)

In the weak field limit, one arrives at

- $g_{00} = 1 + \frac{2}{c^2}\phi$;
- $G_{ab} = \kappa_4^2\, \rho u_a u_b \quad \Rightarrow \nabla^2\phi = \frac{1}{2}c^4\kappa_4^2\rho$.
- Newtonian Law: $\nabla^2\phi = 4\pi G\rho; \Rightarrow$

$$\kappa_4^2 = \frac{8\pi G}{c^4}.$$

## II. Orbifold Branes in String (Cont.)

However, in the brane-world scenarios, the coupling between the effective Einstein-Hilbert action and matter is much more complicated than that given by Eq.(23). In particular, we have

$$G_{ab} \propto \tau_{\mu\lambda}\tau_{\nu}^{\lambda} + ...$$

$$\Rightarrow \qquad H^2 = \frac{1}{36}\kappa_5^4(\rho + g_k)^2 + ... \simeq \frac{1}{18}\kappa_5^2 g_k \rho + ... \tag{24}$$

## II. Orbifold Branes in String (Cont.)

In the weak-field regime, the quadratic terms are negligible, and the linear term dominates. Then, under the weak-field limit, one can show that the Newtonian constant is now is related to the tension of the brane by that

$$\frac{8\pi G}{c^4} = \frac{1}{6} g_\kappa \kappa_5^4. \tag{25}$$

Note that the above result is quite general, and applicable to a large class of brane-world scenarios.

# II. Orbifold Branes in String (Cont.)

In our present setup, we have $\kappa_5^2 = M_{10}^{-8} R^{-5}$, where $R$ is the typical size of the extra dimensions. Then, we find that

$$g_\kappa = 3 \left( \frac{R}{l_{pl}} \right)^{10} \left( \frac{M_{10}}{M_{pl}} \right)^{16} {M_{pl}}^4 \simeq 10^{-42}\, GeV^4, \tag{26}$$

for $R \simeq 10^{-20}\, m$.

## II. Orbifold Branes in String (Cont.)

**D. The Cosmological Constant:** It is surprised that the cosmological constant can be also expressed in terms of $M$ and $R$ [AW, arXiv:1003.4991],

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} = \left(\frac{M}{M_{pl}}\right)^{16} \left(\frac{R}{l_{pl}}\right)^{10} M_{pl}^4 \simeq 10^{-47}\ GeV^4,$$

for $M \sim TeV,\ R \sim 10^{-22} m$.

# II. Orbifold Branes in String (Cont.)

## E. Current Acceleration of the Universe:

[Wu, Santos, Vo & AW, JCAP, 09, 004 (2008) [arXiv:0804.0620]]

Consider a class of solutions in the bulk given by,

$$
\begin{aligned}
\sigma(t) &= \frac{1}{9}\ \ln(t) + \frac{1}{2}\ln\left(\frac{7}{6}\right), \\
\omega(t) &= \frac{10}{9}\ \ln(t), \\
\phi(t) &= -\frac{5}{18}\sqrt{6}\ \ln(t) + \phi_0, \\
\psi(t) &= -\frac{\sqrt{10}}{6}\ \ln(t) + \psi_0, \\
B_{ij} &= 0 = B_{\mu\nu}.
\end{aligned} \tag{27}
$$

## II. Orbifold Branes in String (Cont.)

Assuming that the spacetime on each of the two branes is given by the FRW universe,

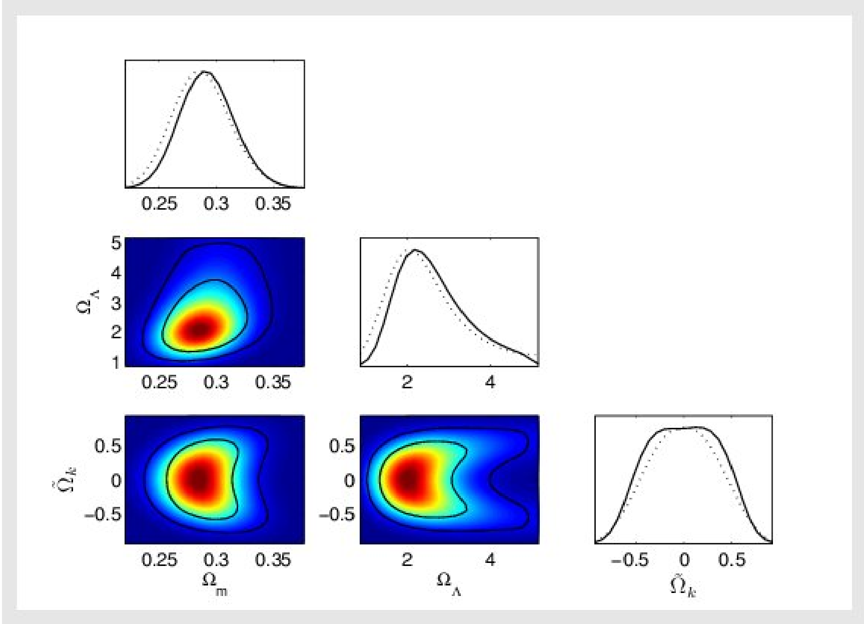
$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-k^2} + r^2 d^2\Omega\right\},$$

we obtained two generalized Friedmann equations on the branes with four free parameters,

$$\left\{\Omega_\Lambda, \Omega_m^{(0)}, \Omega_k^{(0)}, v_1\right\}.$$

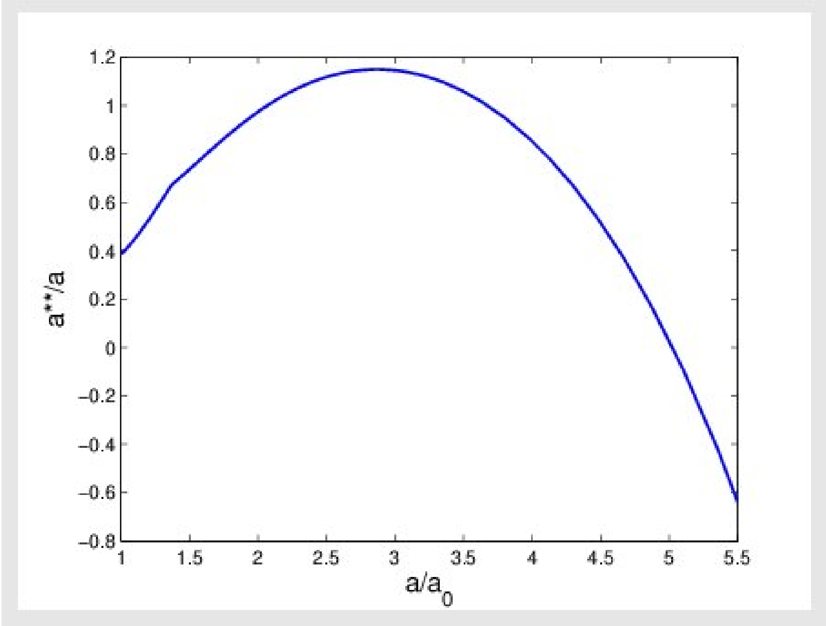
# II. Orbifold Branes in String (Cont.)

Fitting the model to the 182 gold supernova Ia data and the BAO parameter from SDSS, using our Monte-Carlo Markov Chain (MCMC) code, we find $\left\{\Omega_\Lambda, \Omega_m^{(0)}, \Omega_k^{(0)}, v_I\right\}$ $= \{1.93, 0.28, 0.008, 3.0\}$.



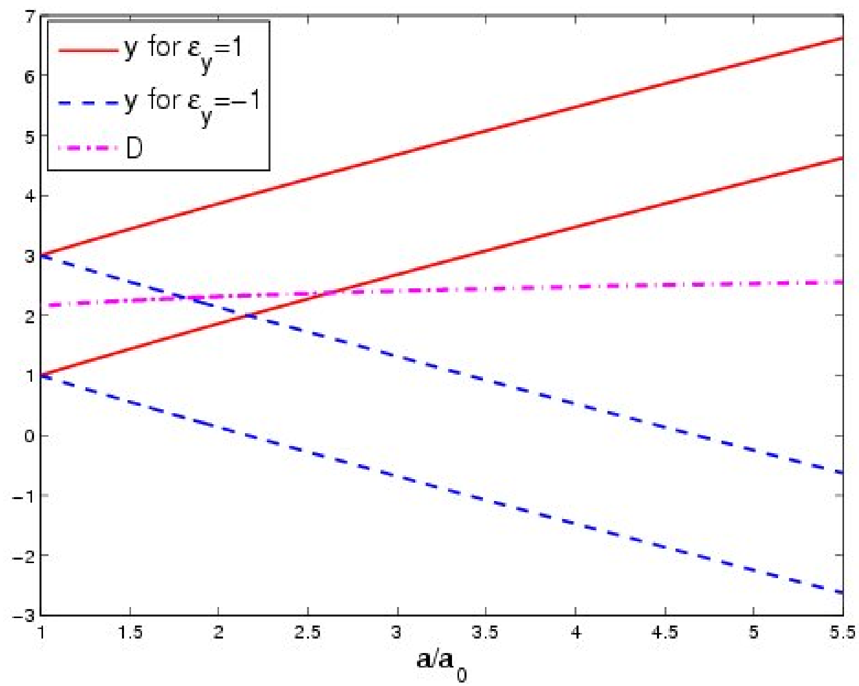
# II. Orbifold Branes in String (Cont.)

With the above best fitting data as the initial conditions, we integrate the generalized Friedmann equations numerically, and found that *the acceleration of the universe will become negative again in the future, whereby all the problems with a far future de Sitter space are solved.*



# II. Orbifold Branes in String (Cont.)

Note that the distance, D, between the two branes remains almost constant, which indicates that the two branes are indeed stable.



# II. Orbifold Branes in String (Cont.)

F. Bouncing Universe: When we studied radion stability and localization of gravity, we worked with static branes. If we allow the branes to move,

$$y_I = y_I(\tau_I), \tag{28}$$

while the bulk is still described by the same 5D metric, we find that

## II. Orbifold Branes in String (Cont.)

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left(\rho + \tau_\phi + \rho_\Lambda\right) \\ &+ \frac{2\pi G}{3\rho_\Lambda}\left(\rho + \tau_\phi\right)^2 - \frac{1}{25L^2a^{12}}, \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(13\rho + 3p + 10\tau_\phi + 10\rho_\Lambda\right) \\ &- \frac{2\pi G}{3\rho_\Lambda}\left[8\rho + \left(13\rho + 5\tau_\phi\right)\tau_\phi + 3p\left(\rho + \tau_\phi\right)\right] \\ &+ \frac{1}{5L^2a^{12}}. \end{aligned} \tag{30}$$

When $a \to 0$, the term $-a^{-12}$ will soon dominates, where by a bouncing universe is resulted.

# II. Orbifold Branes in String (Cont.)

G. Further generalization[M. Devin et al, JHEP10, 095 (2009)

[arXiv:0907.1756]: $M_N = M_D \times M_{d_+} \times M_{d_-}$,

$$\begin{aligned} d\hat{s}_N^2 &= \hat{g}_{AB} dx^A dx^B \\ &= \tilde{g}_{ab}(x)\, dx^a dx^b \\ &\quad + e^{\sqrt{\frac{2}{d_+}}\psi_+(x)} h^+_{ij}(z_+)\, dz_+^i dz_+^j \\ &\quad + e^{\sqrt{\frac{2}{d_-}}\psi_-(x)} h^-_{pq}(z_-)\, dz_-^p dz_-^q, \end{aligned} \tag{31}$$

with $a, b, c = 0, 1, ..., D-1,\ i, j = D, D+1, ..., D+d_+ - 1$, and

$p, q = D + d_+, D + d_+ + 1, ..., N - 1.$

# II. Orbifold Branes in String (Cont.)

We assume,

$$\left(\hat{B}_{CD}\right) = \begin{pmatrix} B_{ab}(x) & 0 & 0 \\ 0 & e^{\xi_+(x)} B_{ij}\left(z_+\right) & 0 \\ 0 & 0 & e^{\xi_-(x)} B_{pq}\left(z_-\right) \end{pmatrix}. \tag{32}$$

After conformation transformation and dimensional reduction, the D-dimensional effective action reads,

$$S_D^{(E)} = -\frac{1}{2\kappa_D^2}\int \sqrt{|g_D|}d^D x\left(R_D[g] - \mathcal{L}_D^{(E)}\left(\phi_n, \xi_\pm\right)\right), \tag{33}$$

$\phi_n = \{\phi, \psi_\pm\}$, and

# II. Orbifold Branes in String (Cont.)

$$\begin{aligned}
\kappa_D^2 &\equiv \frac{\kappa_N^2}{V_{d_+}V_{d_-}}, \quad V_{d_\pm} \equiv \int \sqrt{\left|h^{\pm}\right|}\, d^{d_\pm} z_\pm, \\
\mathcal{L}_D^{(E)} &= \frac{1}{2}\sum_n (\nabla\phi_n)^2 + \frac{1}{12} e^{-\sqrt{\frac{8}{D-2}}\phi} H^2 \\
&+\alpha_+ e^{2\xi_+ - \sqrt{\frac{8}{d_+}}\psi_+} (\nabla\xi_+)^2 + \alpha_- e^{2\xi_- - \sqrt{\frac{8}{d_-}}\psi_-} (\nabla\xi_-)^2 \\
&-e^{\sqrt{\frac{2}{D-2}}\phi}\left(\beta_+ e^{-\sqrt{\frac{2}{d_+}}\psi_+} + \beta_- e^{-\sqrt{\frac{2}{d_-}}\psi_-} - \gamma_+ e^{2\xi_+ - \sqrt{\frac{18}{d_+}}\psi_+} - \gamma_- e^{2\xi_- - \sqrt{\frac{18}{d_-}}\psi_-}\right), \\
\phi &\equiv \sqrt{\frac{2}{D-2}}\left(\hat{\Phi} - Q\right), \quad Q \equiv \sqrt{\frac{d_+}{2}}\,\psi_+ + \sqrt{\frac{d_-}{2}}\,\psi_-, \qquad (34) \\
\alpha_\pm &\equiv \frac{1}{4V_{d_\pm}}\int d^{d_\pm} z_\pm \sqrt{\left|h^{\pm}\right|} B_\pm^2 (z_\pm), \\
\beta_\pm &\equiv \frac{1}{V_{d_\pm}}\int d^{d_\pm} z_\pm \sqrt{\left|h^{\pm}\right|} R_{d_\pm} (z_\pm), \\
\gamma_\pm &\equiv \frac{1}{12V_{d_\pm}}\int d^{d_\pm} z_\pm \sqrt{\left|h^{\pm}\right|} H_\pm^2 (z_\pm). \qquad (35)
\end{aligned}$$

## II. Orbifold Branes in String (Cont.)

Compactify one of the $(D-1)$ spatial dimensions by placing two orbifold branes as its boundaries,

$$\begin{aligned} S^{(E,I)}_{D-1,m} &= -\epsilon_I \int_{M^{(I)}_{D-1}} \sqrt{\left|g^{(I)}_{D-1}\right|} V^{(I)}_{D-1}\left(\phi_n, \xi_\pm\right) d^{D-1}\xi_{(I)} \\ &+ \int_{M^{(I)}_{D-1}} d^{D-1}\xi_{(I)} \sqrt{\left|g^{(I)}_{D-1}\right|} \mathcal{L}^{(I)}_{D-1,m}\left(\phi_n, \xi_\pm, \chi\right), \end{aligned} \tag{36}$$

$I = 1, 2,$ $V^{(I)}_{D-1}\left(\phi_n, \xi_\pm\right)$: the potential of the scalar fields $\phi_n$. $\xi^{\mu}_{(I)}$: the intrinsic coordinates of the branes with $\mu, \nu = 0, 1, 2, ..., D-2,$ $\epsilon_1 = -\epsilon_2 = 1.$ $\chi$: collectively the matter fields. The two branes are localized on the surfaces,

$$\Phi_I\left(x^a\right) = 0. \tag{37}$$

# II. Orbifold Branes in String (Cont.)

Restricting to the cases $D = d = 5$, we find similar conclusions:

- The radion is stable and its mass is about $1\ GeV$.
- the hierarchy problem can be properly addressed.
- The zero-mode is normalizable, and the gravity is localized on the $y = y_c$ brane.

# III. Brane Cosmology in M-Theory

[Y.-G. Gong, AW & Q. Wu, PLB663 (2008) 147 [arXiv:0711.1597]; JCAP06, 015 (2009) [arXiv:0810.5377]]

*The Model*: The 11D spacetime of the Horava-Witten M Theory:

$$ds^2_{11} = V^{-2/3}\gamma_{\alpha\beta}dx^\alpha dx^\beta - V^{1/3}\Omega_{ab}dz^a dz^b,$$

$ds^2_{CY,6} \equiv \Omega_{ab}dz^a dz^b$: the Calabi-Yau 3-fold; $V$: the Calabi-Yau volume modulus.

# III. Brane Cosmology in M-Theory

Then, the 5D effective action reduces to:

$$S_5 = \frac{1}{2\kappa_5^2}\int_{M_5}\sqrt{\gamma}\left(R[\gamma] - \frac{1}{2}(\nabla\phi)^2 + 6\alpha^2 e^{-2\phi}\right)$$
$$+ \sum_{i=1}^{2}\epsilon_i \frac{6\alpha}{\kappa_5^2}\int_{M_4^{(i)}}\sqrt{-g^{(i)}}\,e^{-\phi}, \qquad (38)$$

$\epsilon_1 = -\epsilon_2 = -1,\ \phi \equiv \ln(V),\ \kappa_5^2 \equiv \kappa_{11}^2/v_{CY,6}$, with $v_{CY,6}$: the volume of the Calabi-Yau space, defined as

$$v_{CY,6} \equiv \int_X \sqrt{\Omega}.$$

$\alpha$: related to the internal four-form that has to be included in the dimensional reduction.

# III. Brany Cosmology in M-Theory (Cont.)

To study cosmology, we add matter fields on each of the two branes,

$$S^{m}_{(I)} = -\int_{M_4^{(I)}} \sqrt{-g^{(I)}}\mathcal{L}_m^{(I)}(\phi, \chi). \tag{39}$$

This in general makes the two branes no longer supersymmetric, although the bulk still is.

# III. Brany Cosmology in M-Theory (Cont.)

Similarly to the string case, we find:

- The radion is stable and its mass is about $GeV$.
- By properly choosing the free parameters involved in the model, the hierarchy problem can be resolved.
- The zero-mode is normalizable, and the gravity is localized on the $y = y_c$ brane.

# III. Brany Cosmology in M-Theory (Cont.)

- The spectrum of the KK towers is discrete, with $m_1 \simeq \ TeV$.
- The Yukawa corrections to the Newtonian potential are exponentially suppressed.
- A bouncing universe can also be constructed.

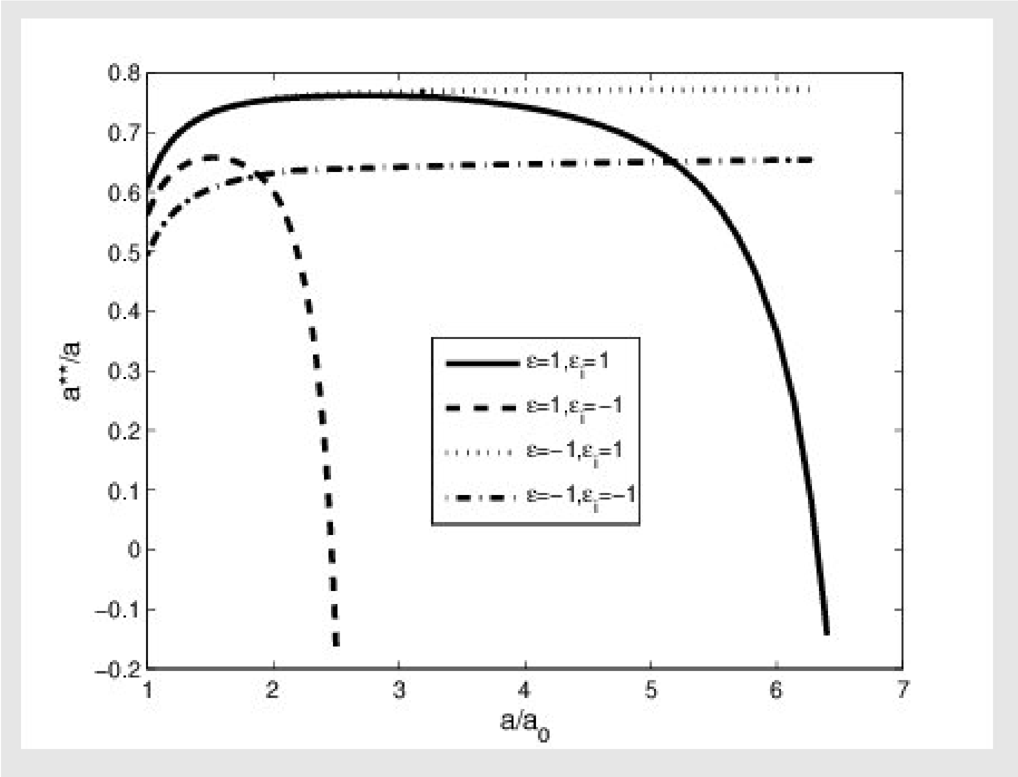
- The Cosmological Constant is given by

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} = \left(\frac{M}{M_{pl}}\right)^{18} \left(\frac{R}{l_{pl}}\right)^{12} M_{pl}^4 \simeq 10^{-47}\, GeV^4,$$

for $M \sim TeV,\ R \sim 10^{-21} m$.

# III. Brany Cosmology in M-Theory (Cont.)

- The current acceleration of the universe is temporary:




[Gong, AW & Wu, PLB663 (2008) 147 [arXiv:0711.1597]]

# IV. Conclusions & Future Work

Conclusions:

- Brane worlds in string/M-Theory provide a possible mechanism for solving the long-standing hierarchy problem.

- The domination of $\Lambda$ over the evolution of the universe is only temporarily. Due to the interaction of the bulk and the branes, the universe will be in its decelerating expansion phase again in the future, whereby all problems connected with a far future de Sitter universe are resolved.

# IV. Conclusions & Future Work

## Doing/do:

- couplings among the KK modes of gravity, radion and SM
- possible observational signals in LHC
- linear perturbations and the formation of the large scale structure
- the early universe, including inflationary models, nucleosynthesis and non-Gaussianity.

# IV. Conclusions & Future Work

## Doing/do (Cont.):

- Supersymmetric embedding or the breaking of supersymmetry. Our approach so far is partly string theoretic and partly phenomenological. We have not investigated what sort of gauged supergravity theories could realize our setup.
- Dark Matter.
- ...

THANK YOU !!!