Open string pair production & its consequences

Jian-Xin Lu

The Interdisciplinary Center for Theoretical Study (ICTS)
University of Science & Technology of China

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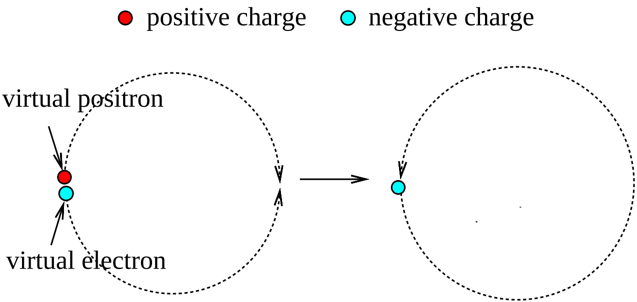
Outline

- Motivation
- A lightening review on D-branes
- The open string pair production rate calculations
- The production rate enhancement by a magnetic flux
- Discussion and Summary

Vacuum polarization

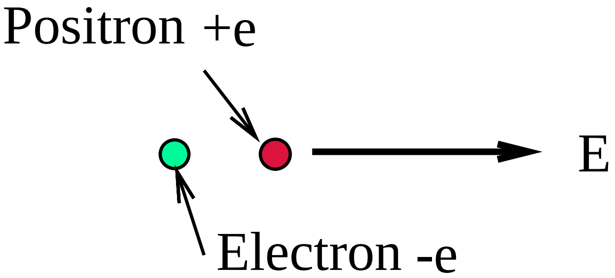
VACUUM FLUCTUATION!

An anti-charge moving forward in time equivalent to a charge moving backward in time



Vacuum polarization

Applying a constant electric field to QED vacuum, there is certain probability to create real electron and positron pairs from the vacuum fluctuation, called Schwinger pair production.



Stringy process

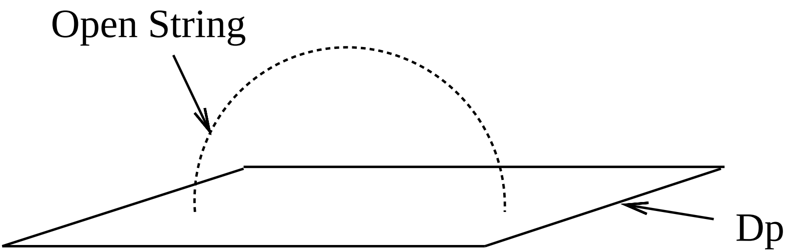
- Does there exist an analogous process in string theory?
- If so, can it have observational consequences?
- Then what are the possible implications?

A lightening review on D-branes

Among the p-branes, there is one particular useful type called D_p branes.

A D_p brane

- can have two equivalent descriptions, one by a closed string and the other by an open string,

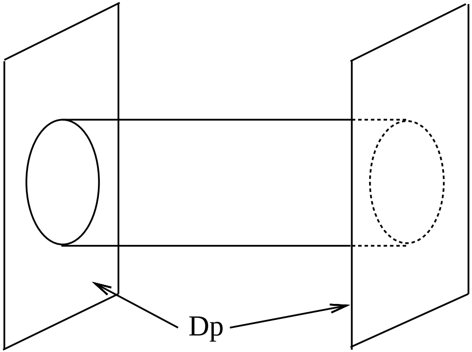


A lightening review on D-branes

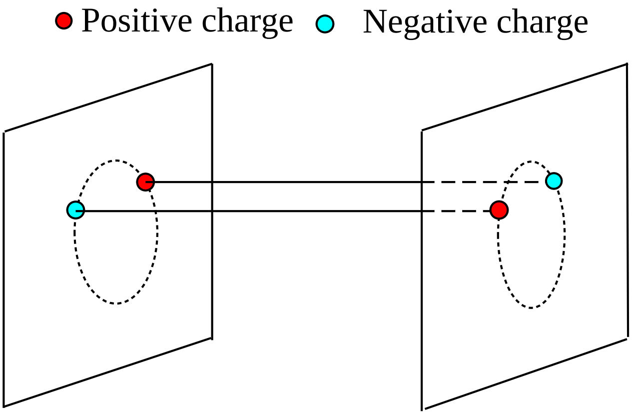
- is a non-perturbative object of string theory $(T_p \sim 1/g_s)$, but a fundamental building block of string/M-theory,
- is the so-called 1/2-BPS object, therefore stable,
- has spatial dimensionality $p = 0, 1, 2, \cdots, 9$
- each carries a mass (due to its tension) and a so-called R-R charge

A lightening review on D-branes

As such, the net static force between two Dp branes is zero!

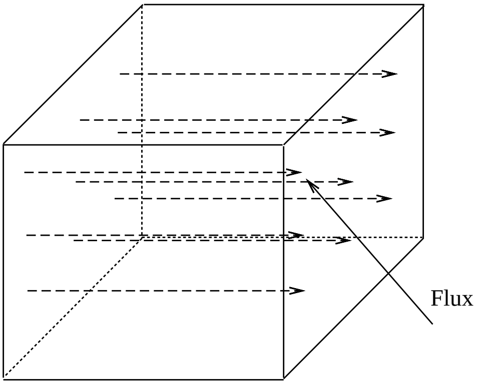


A lightening review on D-branes



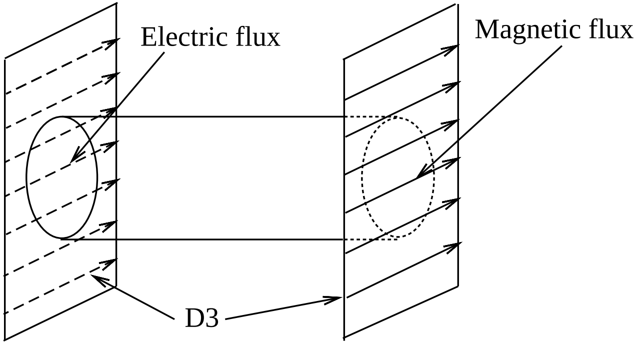
The brane interaction

Consider a particular D3 brane, relevant to our own world, carrying a flux, electric or magnetic,



The brane interaction

The static force between two D3 brane, with one carrying an electric flux and the other a magnetic flux, is now non-zero,



The brane interaction

We choose the electric flux $\hat{F}'$ in one D3 brane and the magnetic flux on the other, respectively, as

$$\hat{F}' = \begin{pmatrix} 0 & -f' & 0 & 0 \\ f' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \hat{F} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -f \\ 0 & 0 & f & 0 \end{pmatrix}. \quad (3.1)$$

The boundary state

In the closed string operator formalism, 1/2 BPS D-branes of Type II theories can be described by means of boundary states. The GSO projected boundary state in either NS-NS or R-R sector is given

$$|B\rangle_{\mathrm{NS-NS}} = \frac{1}{2}\left[|B,+\rangle_{\mathrm{NS-NS}} - |B,-\rangle_{\mathrm{NS-NS}}\right], \qquad (3.2)$$

$$|B\rangle_{\mathrm{R-R}} = \frac{1}{2}\left[|B,+\rangle_{\mathrm{R-R}} + |B,-\rangle_{\mathrm{R-R}}\right], \qquad (3.3)$$

where

$$|B,\eta\rangle = \frac{c_p}{2}|B_{\mathrm{mat}},\eta\rangle|B_{\mathrm{g}},\eta\rangle, \qquad (3.4)$$

with $\eta = \pm$, $c_p = \sqrt{\pi}\left(2\pi\sqrt{\alpha'}\right)^{3-p}$ and

$$|B_{\mathrm{mat}},\eta\rangle = |B_X\rangle|B_\psi,\eta\rangle, \qquad |B_{\mathrm{g}},\eta\rangle = |B_{gh}\rangle|B_{sgh},\eta\rangle. \qquad (3.5)$$

The boundary state

$$|B_X\rangle = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\cdot S\cdot\tilde{\alpha}_{-n}\right]|B_X\rangle^{(0)}, \qquad (3.6)$$

$$|B_\psi,\eta\rangle_{\mathrm{NS-NS}} = -\mathrm{i}\,\exp\left[i\eta\sum_{m=1/2}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|0\rangle, \qquad (3.7)$$

$$|B_\psi,\eta\rangle_{\mathrm{R-R}} = -\exp\left[i\eta\sum_{m=1}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|B,\eta\rangle^{(0)}_{\mathrm{R-R}} \qquad (3.8)$$

The boundary state

$$S = \left(\left[(\eta - \hat{F})(\eta + \hat{F})^{-1} \right]_{\alpha\beta}, -\delta_{ij} \right), \quad (3.9)$$

$$|B_X\rangle^{(0)} = \sqrt{-\det\left(\eta + \hat{F}\right)}\, \delta^{9-p}(q^i - y^i) \prod_{\mu=0}^{9} |k^\mu = 0\rangle, \quad (3.10)$$

$$|B_\psi, \eta\rangle^{(0)}_{\mathrm{R-R}} = \left(C\Gamma^0\Gamma^1 \cdots \Gamma^p \frac{1 + \mathrm{i}\eta\Gamma_{11}}{1 + \mathrm{i}\eta} U \right)_{AB} |A\rangle|\tilde{B}\rangle, \quad (3.11)$$

with $\hat{F} = 2\pi\alpha' F$ and

$$U = \frac{1}{\sqrt{-\det(\eta + \hat{F})}}; \exp\left(-\frac{1}{2} \hat{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta \right); \quad (3.12)$$

(for example, see Di Vecchia & Licciardo hep-th/9912161)

The boundary state

Note that

$$S^T{}_\mu{}^\rho S_\rho{}^\nu = \delta_\mu{}^\nu, \tag{3.13}$$

and $\mathcal{S} = S_2 S_1^T$ satisfies also the same relation but its determinant is always unity, therefore it can be diagonalized via a proper unitary transformation.

This makes the evaluation of the following respective matrix elements no more difficult than the corresponding case when there is no flux present.

The stringy interaction

The interaction can be calculated via the vacuum amplitude

$$\Gamma = \Gamma_{\rm NS-NS} + \Gamma_{\rm R-R} \tag{3.14}$$

where

$$\Gamma_{\rm NS-NS/R-R} = {}_{NS-NS/R-R}\langle B^1 | D | B^2 \rangle_{NS-NS/R-R}, \tag{3.15}$$

with the propagator

$$D = \frac{\alpha'}{4\pi} \int_{|z|\leq 1} \frac{d^2 z}{|z|^2} z^{L_0} \bar{z}^{\tilde{L}_0}, \tag{3.16}$$

for example, $L_0 = L_0^X + L_0^\psi + L_0^{gh} + L_0^{sgh}$.

The stringy interaction

To calculate $\Gamma_{\mathrm{NS-NS/R-R}}$, need to calculate the following first

$$\begin{aligned}\Gamma(\eta',\eta) &= \langle B^1,\eta'|D|B^2,\eta\rangle\\ &= \frac{n_1 n_2 c_p^2}{4}\frac{\alpha'}{4\pi}\int_{|z|\le 1}\frac{d^2 z}{|z|^2}A^X\,A^{bc}\,A^\psi(\eta',\eta)\,A^{\beta\gamma}(\eta',\eta),\end{aligned} \tag{3.17}$$

where we have replaced c_p by nc_p to count the multiplicity of D$_p$ branes in the bound state and $\eta\,\eta' = \pm$. The above respective matrix elements, considering $\tilde{L}_0|B\rangle = L_0|B\rangle$, are

$$\begin{aligned} A^X &= \langle B^1_X||z|^{2L_0^X}|B^2_X\rangle, & A^\psi(\eta',\eta) &= \langle B^1_\psi,\eta'||z|^{2L_0^\psi}|B^2_\psi,\eta\rangle,\\ A^{bc} &= \langle B^1_{gh}||z|^{2L_0^{gh}}|B^2_{gh}\rangle, & A^{\beta\gamma}(\eta',\eta) &= \langle B^1_{sgh},\eta'||z|^{2L_0^{sgh}}|B^2_{sgh},\eta\rangle.\end{aligned} \tag{3.18}$$

The brane interaction

The closed string tree-level cylinder or the open string one-loop annulus amplitude per unit 3-brane worldvolume can be calculated to be

$$\begin{aligned}\Gamma &= \frac{4\,n'n\,\tanh\pi\nu'\,\tan\pi\nu}{(8\pi^2\alpha')^2}\int_0^\infty \frac{dt}{t}\,e^{-\frac{Y^2 t}{2\pi\alpha'}}\\ &\times\frac{(\cos\pi\nu' t-\cosh\pi\nu t)^2}{\sin(\pi\nu' t)\sinh(\pi\nu t)}\\ &\times\prod_{n=1}^{\infty}\frac{1}{(1-|z|^{2n})^4(1-e^{2\pi\nu t}|z|^{2n})(1-e^{-2\pi\nu t}|z|^{2n})}\\ &\times\frac{\prod_{j=1}^{2}(1-e^{\pi(i\nu'+(-)^j\nu)t}|z|^{2n})^2(1-e^{-\pi(i\nu'+(-)^j\nu)t}|z|^{2n})^2}{1-2|z|^{2n}\cos 2\pi\nu' t+|z|^{4n}}\end{aligned} \tag{3.19}$$

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The brane interaction

In the above, we have $|z| = e^{-\pi t} < 1$, $2\pi\alpha' = 1/T$ with T the string tension and

$$\tanh \pi\nu' = |f'|, \quad \tan \pi\nu = |f| \tag{3.20}$$

where $0 < \nu' < \infty$ for an electric flux $0 < |f'| < 1$, and for a magnetic flux $0 < \nu < 1/2$ since $0 < |f| < \infty$.

The tachyonic instability

$$\lim_{t\to\infty} e^{-\frac{Y^2 t}{2\pi\alpha'}} \times \frac{(\cos\pi\nu' t - \cosh\pi\nu t)^2}{\sin(\pi\nu' t)\sinh(\pi\nu t)} \sim \lim_{t\to\infty} e^{-\frac{t}{2\pi\alpha'}(Y^2 - 2\pi^2\nu\alpha')} \to \infty, \quad (3.21)$$

when $Y < \pi\sqrt{2\nu\alpha'}$.

Note also $\alpha' M^2 \sim \frac{1}{2\pi\alpha'}(Y^2 - 2\pi^2\nu\alpha')$.

The open string pair production rate

- This amplitude has an infinite number of simple poles occurring on the positive real t-axis at $t_k = k/\nu'$ with $k = 1, 2, \cdots$.
- Therefore this amplitude has an imaginary part which is given as sum of the residues of these simple poles. It gives the non-perturbative rate of pair production of open strings per unit worldvolume as

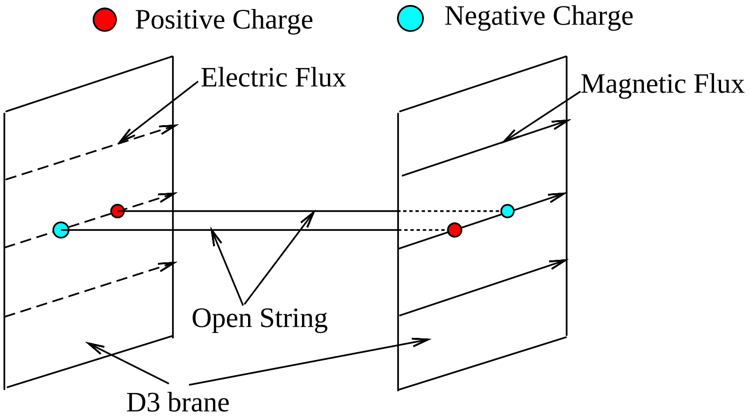
The open string pair production rate

$$
\begin{aligned}
\mathcal{W} &\equiv -\frac{2\mathrm{Im}\Gamma}{V_3} \\
&= \frac{8n'n\tanh\pi\nu'\tan\pi\nu}{\nu'}\sum_{k=1}^{\infty}(-)^{k+1}\left(\frac{\nu'}{8k\pi^2\alpha'}\right)^2 \\
&\times e^{-\frac{kY^2}{2\pi\nu'\alpha'}}\frac{\left[\cosh\frac{k\pi\nu}{\nu'}-(-)^k\right]^2}{\frac{\nu'}{k}\sinh\frac{k\pi\nu}{\nu'}} \\
&\times\prod_{n=1}^{\infty}\frac{\left[1-2(-)^k e^{-\frac{2nk\pi}{\nu'}}\cosh\frac{k\pi\nu}{\nu'}+e^{-\frac{4nk\pi}{\nu'}}\right]^4}{\left[1-e^{-\frac{2nk\pi}{\nu'}}\right]^6\left[1-e^{-\frac{2k\pi}{\nu'}(n-\nu)}\right]\left[1-e^{-\frac{2k\pi}{\nu'}(n+\nu)}\right]},
\end{aligned}
\tag{3.22}
$$

The open string pair production rate

- The rate is highly suppressed by the separation and the integer k and for each given k the corresponding term appears likely enhanced by both ν' and ν.
- The latter is particularly evident for large magnetic flux for which $\nu \to 1/2$ and the front factor $\tan \pi\nu \to \infty$.
- The odd k gives positive contribution while the even k gives negative contribution to the above rate. $k = 1$ term gives the leading positive contribution to the rate.
- The presence of magnetic flux appears to enhance the rate

The open string pair production rate



The rate enhancement

Consider now small fluxes, realistic case!

Let us consider two separate cases for showing the enhancement by the presence of a magnetic flux:

- The rate for small ν' and $\nu = 0$ can be given approximately by the $k = 1$ term as

$$(2\pi\alpha')^2 \mathcal{W}_{\nu=0} \approx 32\pi n' n \left(\frac{\nu'}{4\pi}\right)^2 e^{-\frac{Y^2}{2\pi\nu'\alpha'}} \tag{4.1}$$

  Vanishingly small in general!

- Similarly, the rate for small ν' and fixed ν is

$$(2\pi\alpha')^2 \mathcal{W}_{\nu\neq 0} \approx 4\pi n' n \left(\frac{\nu'}{4\pi}\right)^2 e^{-\frac{Y^2}{2\pi\nu'\alpha'}} \frac{e^{\frac{\pi\nu}{\nu'}}}{\nu'} \tan \pi\nu, \tag{4.2}$$

The rate enhancement

The following ratio

$$\frac{\mathcal{W}_{\nu\neq 0}}{\mathcal{W}_{\nu=0}} = \frac{e^{\frac{\pi\nu}{\nu'}}}{8\nu'}\tan\pi\nu \qquad (4.3)$$

gives the rate enhancement of the magnetic flux, which can be very significant!

The rate enhancement

To have a better sense of the enhancement, let us make the following numerical estimations for illustration.

Take $\nu' = 0.02$ and $\nu = 0.4$, the enhancement given above is then

$$\frac{e^{\frac{\pi\nu}{\nu'}}}{8\,\nu'}\tan\pi\nu = e^{20\pi}\frac{25\tan 0.4\pi}{4} \sim 3.6\times 10^{28}!!! \qquad (4.4)$$

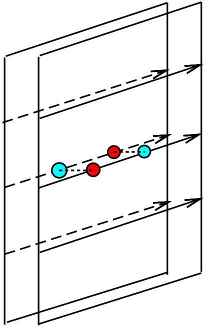
A huge number!

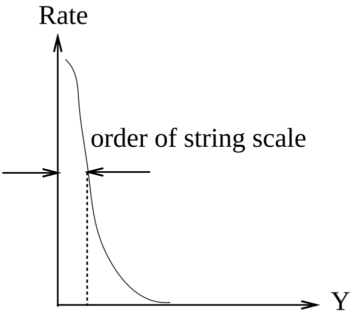
The rate enhancement

We have then

- The above significant enhancement in the presence of a magnetic flux indicates the possibility that the dimensionless rate itself can become significant
- Note that the rate is highly suppressed by the brane separation
- So only at small brane separation $Y = \pi\sqrt{2\nu\alpha'} + \triangle\sqrt{\alpha'}$ with $\triangle \ll \nu'/\sqrt{2\nu}$, this rate can become significant indeed!
- Any brane separation significant away from the above will make the rate vanishingly small

The rate enhancement





The rate enhancement

Given $\nu' = 0.02$ and $\nu = 0.40$, let us estimate the dimensionless rate

$$\begin{aligned}(2\pi\alpha')^2 \mathcal{W}_{\nu=0.4} &\approx n' n \frac{\nu'}{4\pi} \tan 0.4\pi \\ &\approx 0.49\,!!!, \end{aligned} \qquad (4.5)$$

where we have taken the brane separation given above and also $n' = n = 10$.

Note that we need to have $p \geq 3$ for this enhancement to occur and the $p = 3$ case gives the largest rate and the rate for $p = 4$ is one order of magnitude smaller and so on.

Summary

- When one flux is electric and the other magnetic, the pair production rate is greatly enhanced by the presence of the magnetic flux and can become significant even for a small electric flux
- This can occur only for $p \geq 3$
- When the brane separation is on the order of string scale, this rate can be very significant, is the largest for $p = 3$ and decreases rapidly with the value of $p > 3$
- For example, the rate of $p = 3$ is larger than that of $p = 4$ by at least one order of magnitude and the rate becomes insignificant for $p > 4$ for reasonable large brane numbers of n', n

Discussion

- If string theory is indeed relevant to our real world, the above pair production should have potentially observational consequences, most likely from the sky
- For example, in quantum gravity stage or even after the reheating epoch or later, the two sets of D$_p$ branes ($p \geq 3$) with one set carrying an electric flux and the other carrying a magnetic flux can experience dynamics by approaching each other to produce highly energetic open string pairs and then annihilate to give rise to other particles such as the high energy photons or other particles. Now one expects the brane numbers of n' and n not to be very large

Discussion

- Also, in the late stage of our Universe, one can take either of the above mentioned sets of D$_p$ branes as a macroscopic object in our Universe for which the brane number should be fairly large. Then when the two objects approach to each other in a small separation, one expects a large number of open string pairs to be produced

- This should be most significant for $p = 3$ and the produced open string pairs are mostly confined along the $p = 3$ brane directions since the rate only becomes significant when the brane separation is very small

Speculation

- If such effects can indeed be observed, it would imply the existence of extra dimensions since we need to have $p \geq 3$ for this process to occur
- It would also imply the selection of three extra large dimensions since the $p = 3$ gives the largest effect and the pair production is mostly confined along the $p = 3$ directions (Does this imply further why our world is (1 + 3)-dimensions?)

Speculation

- As mentioned above, when the two sets of D_p branes are taken as macroscopic objects in our Universe, the open string pair production can be very significant in particular for $p = 3$ since now n' and n can be very large. In analogy to the annihilation of electron-positron into photons, we expect the annihilation of large number of open string pairs to produce huge high energy photons in a short time of period. Can this be used to explain the recently observed γ-ray burst?

THANK YOU!