Phenomenological Study of the wave functions of B and D Mesons

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Based on the work:

Mao-Zhi Yang, Eur. Phys. J. C72 (2012) 1880

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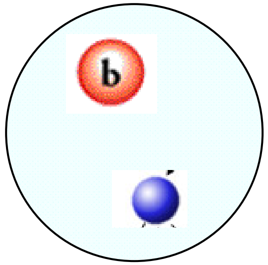
I Introduction

- The wave function of the meson is determined by the strong interaction between quark and antiquarks



- The wave function is important for investigating the property of strong interaction

- It is also important for studying the heavy-meson decays

For B meson decays, the factorization of the decay amplitude can be written as

$$T = \int dx_1 dk_{1\perp} dx_2 dk_{2\perp} dx_3 dk_{3\perp}$$
$$\times \phi_B(x_1, k_{1\perp}) H(x_i, k_{i\perp}) \phi_2(x_2, k_{2\perp}) \phi_3(x_3, k_{3\perp})$$

The light-cone wave function through the matrix

$$\int_0^1 \frac{\mathrm{d}^4 z}{(2\pi)^4} \mathrm{e}^{\mathrm{i}\boldsymbol{k_1}\cdot z} \langle 0|\bar{b}_\alpha(0) d_\beta(z)|B(p_B)\rangle$$
$$= -\frac{\mathrm{i}}{\sqrt{2N_c}} \left\{ (\not{p}_B + m_B)\gamma_5 \left[\phi_B(\boldsymbol{k}_1) - \frac{\not{n} - \not{v}}{\sqrt{2}} \bar{\phi}_B(\boldsymbol{k}_1) \right] \right\}_{\beta\alpha}$$

The light-cone wave function of B meson has been extensively studied in the recent several years:

1) The constraint of equation of motion

H. Kawamura, J. Kodaira, C.F. Qiao, K. Tanaka, PLB523 (2001)111

2) Renormalization-group equation

The evolution and asymptotic behavior of the B meson wave function is obtained

B. Lange and M. Neubert, PRL91(2003)102001

3) QCD sum rule

Only moments of the wave function can be calculated

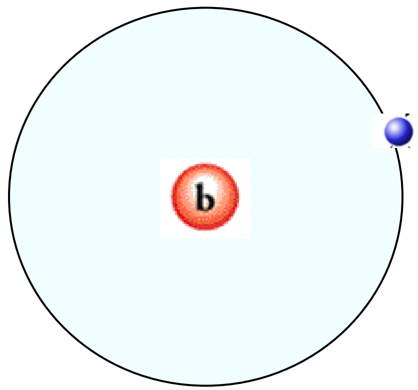
V. Braun, D. Ivanov, G.P. Korchemsky , PRD69(2004) 034014;

- Methods to obtain the light-cone wave function exactly from QCD is still under investigation

- Alternatively, obtaining the wave function of the heavy-meson by solving the bound state wave equation is also an effective way

II The Picture of the heavy meson in the Heavy Quark Limit

1) The heavy quark can be viewed as a static color source



2) The light antiquark is bound around the heavy quark by an effective color potential

3) The heavy quark spin decouples from the interactions as $m_Q \to \infty$

$$m_B \approx m_{B^*}$$
$$m_D \approx m_{D^*}$$

4) The interaction relevant to quark spin can be treated as perturbative corrections

III. Bound State Equation in the Relativistic Potential Model

$$\left[\sqrt{-\hbar^2\nabla_1^2 + m_1^2} + \sqrt{-\hbar^2\nabla_2^2 + m_2^2} + V(r)\right]\psi(\vec{r})$$
$$= E\psi(\vec{r}),$$

$$\vec{r} = \vec{x}_1 - \vec{x}_2$$

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + b\,r + c.$$

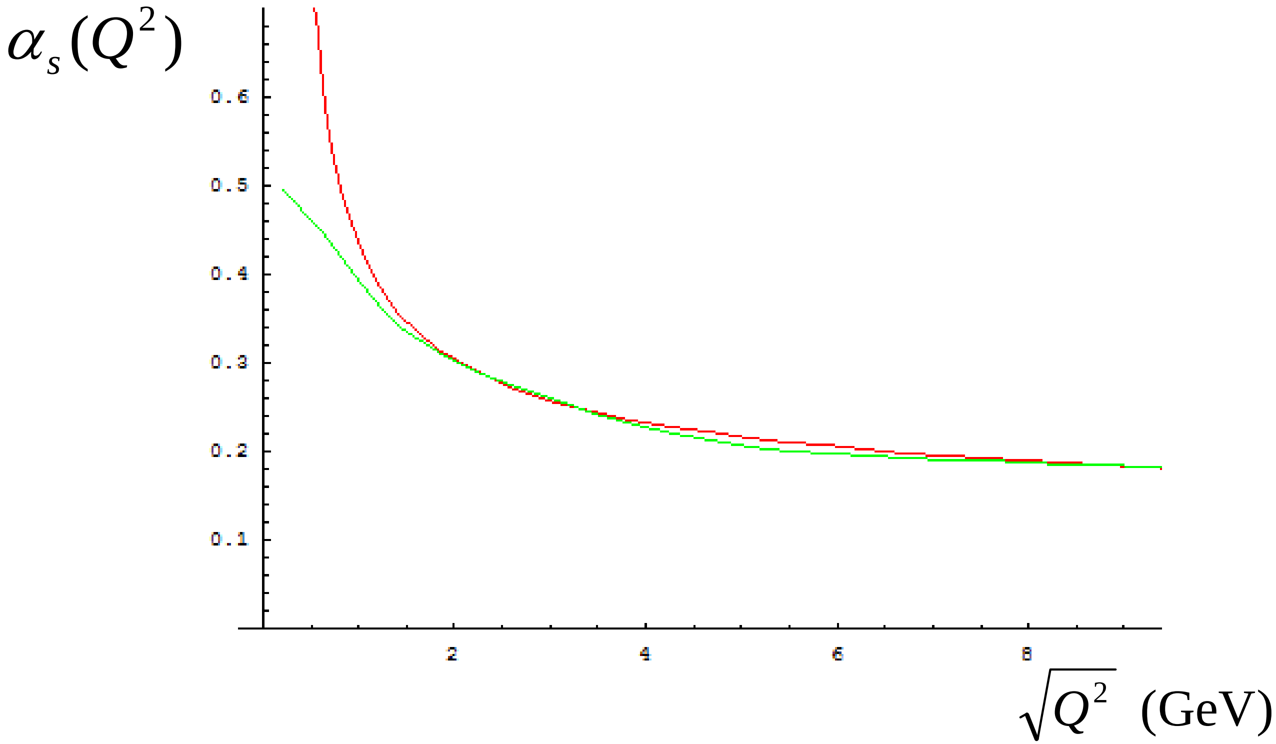
The running conpling constant in momentum space in lowest order in QCD

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)\ln(Q^2/\Lambda^2)}$$

It should be transformed into coordinate space, to get $\alpha_s(r)$

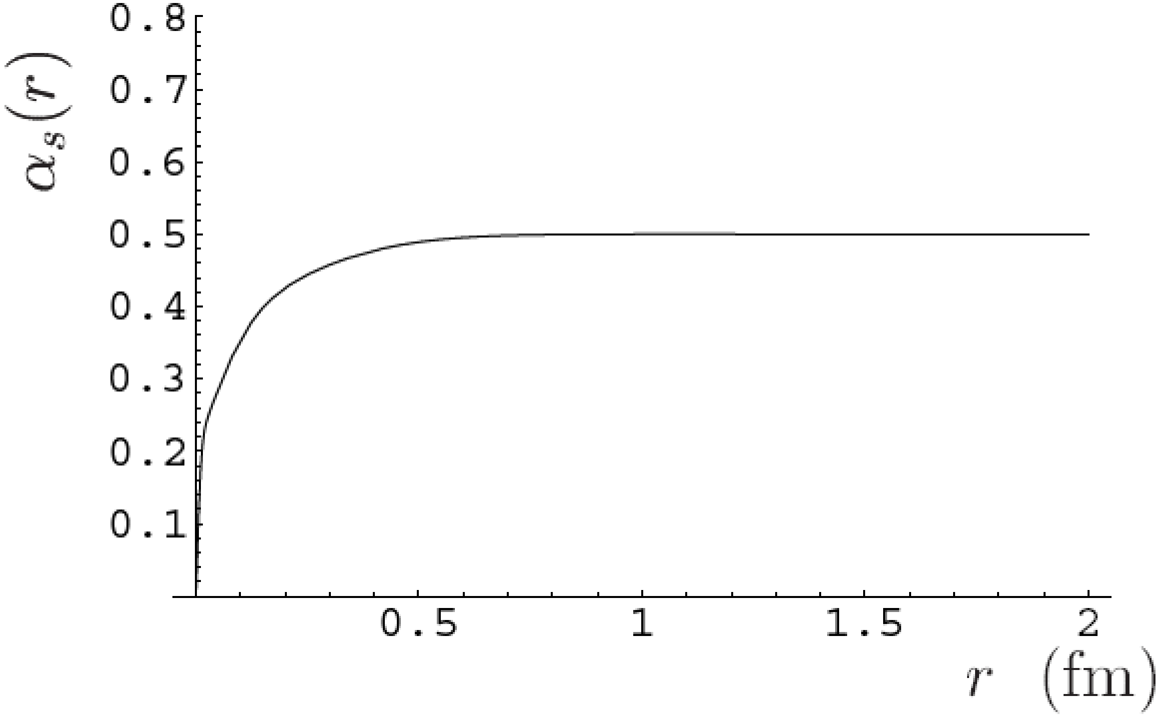
A new form for α_s which can be conveni-ently transformed into r-space is

$$\alpha_s(r) = \sum_i \alpha_i e^{-Q^2/4\gamma_i^2}$$



$$\alpha_1 = 0.15 \quad \alpha_2 = 0.15 \quad \alpha_3 = 0.20$$

$$\gamma_1 = 1/2 \quad \gamma_2 = \sqrt{10}/2 \quad \gamma_3 = \sqrt{1000}/2$$



$$\alpha_s(r) = \sum_i \alpha_i \frac{2}{\sqrt{\pi}} \int_0^{\gamma_i r} e^{-x^2} dx$$

IV The Solution of the Wave Equation

The wave function can be expressed in terms of the spectrum integration

$$
\begin{aligned}
\psi(\vec{r}) &= \int d^3r' \delta^3(\vec{r} - \vec{r}\,')\psi(\vec{r}\,') \\
&= \int d^3r' \int \frac{d^3k}{(2\pi\hbar)^3} e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar}\psi(\vec{r}\,')
\end{aligned}
$$

Then the wave equation becomes

$$\int \frac{d^3k}{(2\pi\hbar)^3} d^3r' \left(\sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\,\right)$$
$$\times e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar}\psi(\vec{r}\,') = (E-V(r))\psi(\vec{r}).$$

$$e^{i\vec{k}\cdot\vec{r}/\hbar} = 4\pi\sum_{ln} i^l j_l\left(\frac{kr}{\hbar}\right)Y^*_{ln}(\hat{k})Y_{ln}(\hat{r})$$

Factorize the wave function into two parts: radial and angular

$$\psi(\vec{r}) = \Phi_l(r) Y_{ln}(\hat{r})$$

The wave equation

$$V(r)\Phi_l(r) + \frac{2}{\pi\hbar}\int dk \frac{k^2}{\hbar^2}\int dr' r'^2 (\sqrt{k^2 + m_1^2}$$
$$+\sqrt{k^2 + m_2^2}\,) j_l(\frac{kr}{\hbar}) j_l(\frac{kr'}{\hbar}) \Phi_l(r') = E\Phi_l(r)$$

Define the reduced radial wave function by

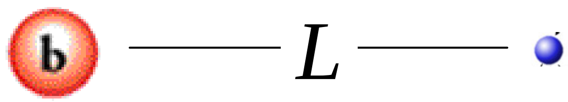
$$\Phi_l(r) = \frac{u_l(r)}{r}$$

Then for l =0, the wave equation

$$V(r)u_0(r) + \frac{2}{\pi\hbar c}\int_0^\infty dk \int_0^\infty dr'(\sqrt{k^2+m_1^2}$$

$$+\sqrt{k^2+m_2^2}\,)\sin(\frac{kr}{\hbar c})\sin(\frac{kr'}{\hbar c})u_0(r') = Eu_0(r)$$

For a bound state, if the distance between quark and antiquark is large enough, the wave function will effectively vanish



Then the bound state can be treated as if the quark and antiquark are restricted in a limited space

$$0 < r < L$$

In the limited space, the Fourior transform of the reduced wave function is

$$u_0(r) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{L} r\right),$$

In the limited space, the momentum is discretized

$$\frac{k}{\hbar c} \to \frac{n\pi}{L}, \qquad \int \frac{dk}{\hbar c} \to \frac{\pi}{L}.$$

With the above replacement, and the integration over the distance r' being limited within $0 < r' < L$, Eq.(11) becomes

$$
\begin{aligned}
& V(r)u_0(r) + \sum_n \frac{2}{L} \int_0^L dr' \, (\sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_1^2} \\
& + \sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_2^2} \Bigg) \sin(\frac{n\pi}{L} r) \sin(\frac{n\pi}{L} r') u_0(r') \\
& = E u_0(r). \qquad (15)
\end{aligned}
$$

Truncate the series of the Fourier expansion of the wave function $u_0(r)$ as

$$u_0(r) = \sum_{n=1}^{N} c_n \sin\left(\frac{n\pi}{L} r\right), \tag{16}$$

Where N is a large integer

The wave equation finally becomes

$$\left(\sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_1^2} + \sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_2^2}\right) c_n$$
$$+ \sum_{m=1}^{N} \frac{2}{L} \int_0^L drV(r) \sin\left(\frac{n\pi}{L} r\right) \sin\left(\frac{m\pi}{L} r\right) c_m$$
$$= Ec_n. \qquad (17)$$

V. The Bound State and the Decay Constant

In the rest frame of the meson:

$$|P(\vec{p}=0)\rangle = \frac{1}{\sqrt{3}}\sum_i \int d^3k\,\Psi_0(k)\frac{1}{\sqrt{2}}[b_Q^{i+}(\vec{k},\uparrow)d_q^{i+}(-\vec{k},\downarrow)$$
$$-b_Q^{i+}(\vec{k},\downarrow)d_q^{i+}(-\vec{k},\uparrow)]|0\rangle, \qquad (23)$$

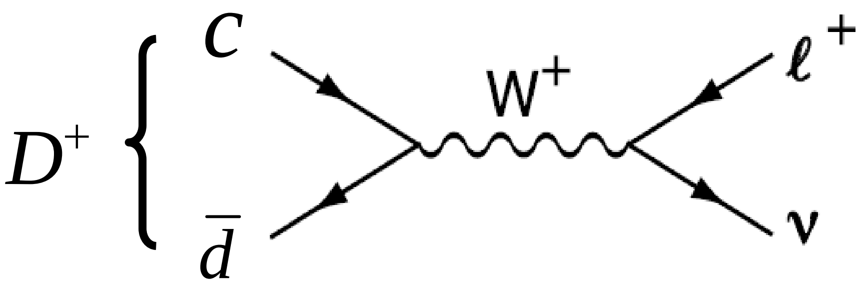
The decay constant is defined by

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P(p)\rangle = if_P\, p_\mu$$

The decay constant can be obtained to be

$$\begin{aligned} f_P &= \sqrt{\frac{3}{2}}\frac{1}{2\pi^2}\frac{1}{m_P}\int_0^\infty dk|\vec{k}|^2\Psi_0(k) \\ &\times\frac{(E_q+m_q)(E_Q+m_Q)-|\vec{k}|^2}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}}, \end{aligned}$$

The decay constant describes the possibility amplitude for the quark and antiquark to come together



$$\Gamma(P \to l\nu) = \frac{G_F^2}{8\pi} f_P^2 m_l^2 M_P (1 - \frac{m_l^2}{M_P^2})^2 |V_{ij}|^2$$

Experimental measurement of the decay constant

CLEO-c 818 pb^{-1} 2008:

$$f_D = 205.8 \pm 8.5 \pm 2.5 \,\text{MeV}$$

CLEO-c 2009:

$$f_{D_s} = 252.5 \pm 11.1 \pm 5.2 \,\text{MeV}$$ PRD79,052002 (2009)

$$f_{D_s} = 259.7 \pm 7.8 \pm 3.4 \,\text{MeV}$$ PRD80,112004 (2009)

***BABAR* 2010:**

$$f_{D_s} = 258.6 \pm 6.4 \pm 7.5 \,\text{MeV}$$ PRD82,091103(2010)

$$f_{D_s} = 233 \pm 13 \pm 10 \pm 7 \,\text{MeV}$$ 1003.3063

VI Numerical Result and Discussion

The parameters in the model can be selected by comparing the predicted meson masses with the exp. data

The decay constants give a further constraint

Input parameters:

$$b = 0.10\ \text{GeV}^2, \quad c = -0.19\ \text{GeV}^2,$$
$$m_b = 4.98\ \text{GeV}, \quad m_c = 1.54\ \text{GeV},$$
$$m_s = 0.30\ \text{GeV}, \quad m_u = m_d = 0.08\ \text{GeV}$$
$$\alpha_s^{\text{critical}} = 0.5,$$

For *N* and *L*, we take

$$L = 10\ \text{fm}$$

$$N = 100$$

The masses obtained

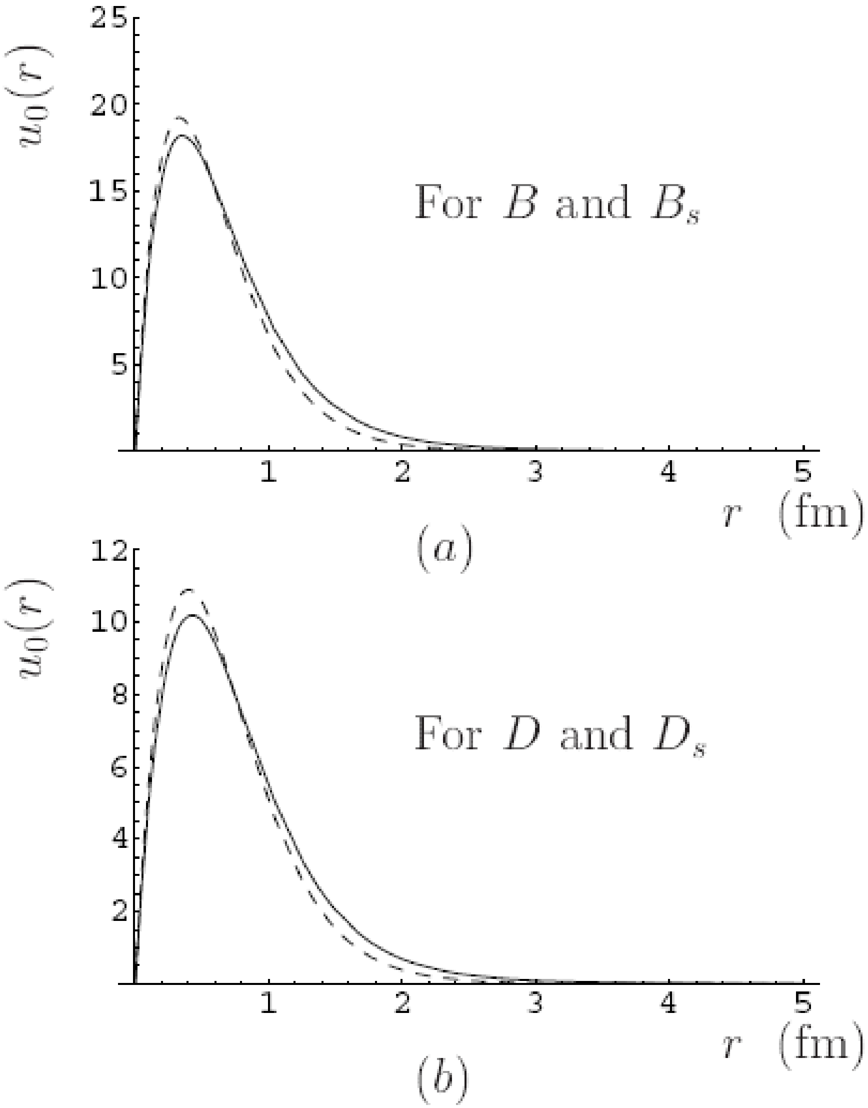
	m_B	m_{B_s}	m_D	m_{D_s}
this work (GeV)	5.25 ± 0.37	5.34 ± 0.37	1.86 ± 0.19	1.96 ± 0.20
Exp. (MeV)	5279.17 $\pm$0.29	5366.3 $\pm$0.6	1869.6 $\pm$0.16	1968.47 $\pm$0.33

The decay constants

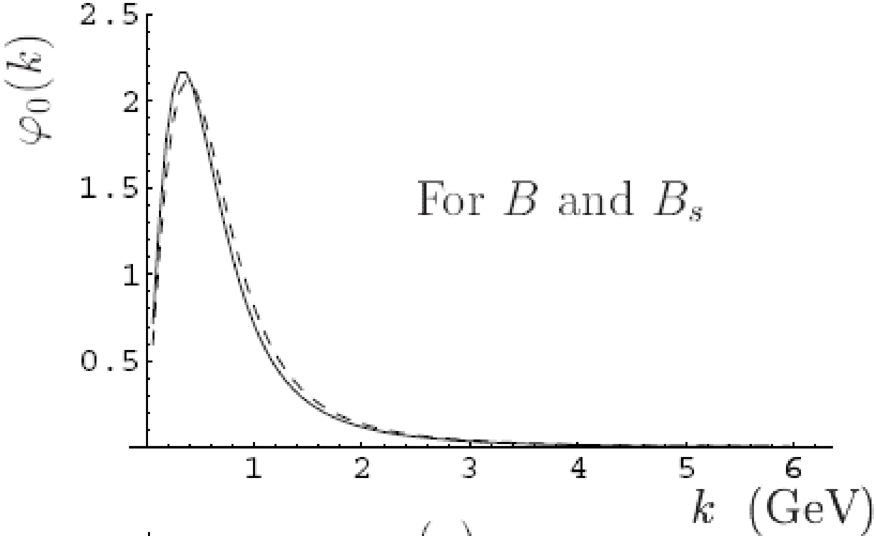
$$f_B = 198 \pm 14 \text{ MeV}, \quad f_{B_s} = 237 \pm 17 \text{ MeV},$$
$$f_D = 208 \pm 21 \text{ MeV}, \quad f_{D_s} = 256 \pm 26 \text{ MeV}.$$

Comparison of the decay constants with exp.

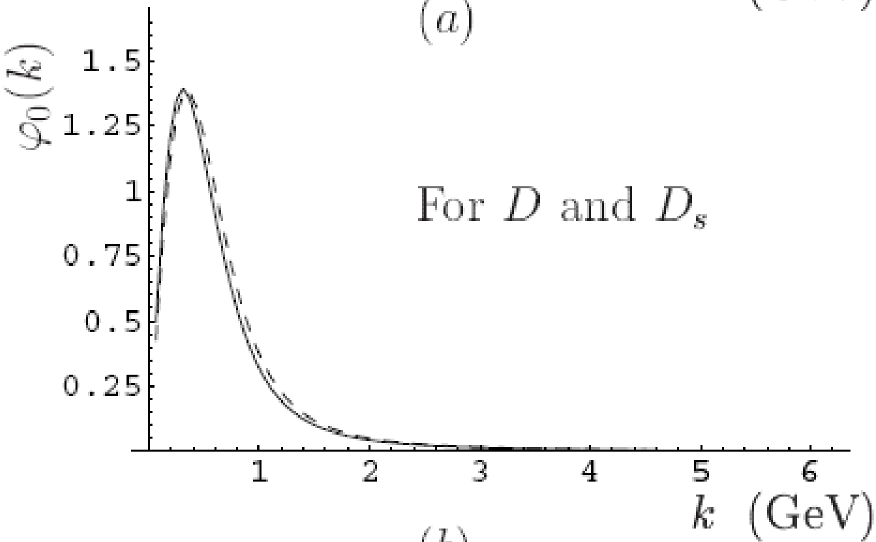
	f_B	f_{B_s}	f_D	f_{D_s}
this work	198 ± 14	237 ± 17	208 ± 21	256 ± 26
Exp. [19, 26]	$-$	$-$	$205.8 \pm 8.5 \pm 2.5$	254.6 ± 5.9



Wave function in coordinate space



(a)



(b)

Wave function in momentum space

Semileptonic decays

$$Br(B^+ \to e^+\nu_e) = (1.11 \pm 0.26) \times 10^{-11},$$

$$Br(B^+ \to \mu^+\nu_\mu) = (4.7 \pm 1.1) \times 10^{-7},$$

$$Br(B^+ \to \tau^+\nu_\tau) = (1.1 \pm 0.2) \times 10^{-4},$$

SuperB

$$\mathcal{B}(B^- \to \tau^-\overline{\nu})$$

PRL 97, 251802 (2006)

$$= (1.79^{+0.56}_{-0.49}\,{}^{+0.46}_{-0.51}) \times 10^{-4} \quad \text{Belle;}$$

$$= (1.2 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4} \quad \text{BaBar;}$$

PRD76,052002 (2007)

$$Br(D^+ \to e^+\nu) = 9.5\times10^{-9};$$

$$Br(D^+ \to \mu^+\nu) = 4.0\times10^{-4};$$

$$Br(D^+ \to \tau^+\nu) = 1.1\times10^{-3};$$

CLEO-c 2008:

$$Br(D^+ \to \mu^+\nu)$$

$$= (3.8 \pm 0.32 \pm 0.09)\times10^{-4};$$

$$Br(D_S^+ \to e^+\nu) = 1.4\times10^{-7};$$

$$Br(D_S^+ \to \mu^+\nu) = 5.7\times10^{-3};$$

$$Br(D_S^+ \to \tau^+\nu) = 5.6\times10^{-2};$$

CLEO-c 2009 :

$$Br(D_S^+ \to \tau^+\nu)$$

$$= (5.3 \pm 0.47 \pm 0.22)\times10^{-2};$$

VII Summary

- D constants and wave functions of B and D mesons are studied in relativistic potential model

- The masses and decay constants can be well consistent with experiment

- The wave functions obtained here will be useful for studying B and D decays