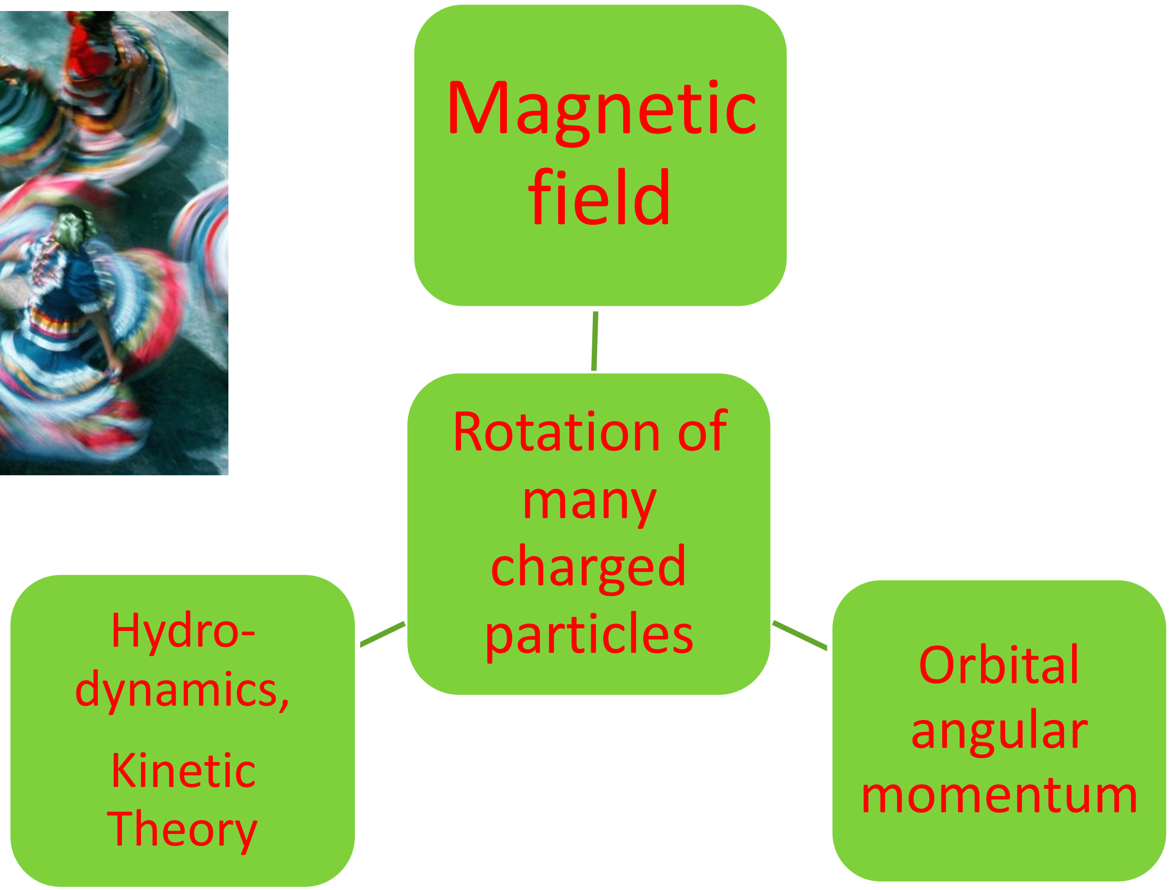
An Introduction to Chiral Magnetic Effect

Qun Wang

Department of Modern Physics
University of Science and Technology of China

ICTS-seminar, March 7, 2013

A game of collective rotation



Outline

- A little (pre-)history
- Chiral Magnetic Effect: an introduction
- Approaches to CME/CVE: Field theory, Hydrodynamics, Quantum kinetic theory
- Other related and new phenomena

A little (pre-)history

- Existence topologically non-trivial configurations of gauge fields [Belavin, Polyakov, Shvarts, Tyupkin, Phys. Lett. B 59 (1975) 85]
- Tunneling between topologically different configurations ['t Hooft, Phys. Rev. Lett. 37 (1976) 8; 't Hooft, Phys. Rev. D 14 (1976) 3432; 't Hooft, Phys. Rev. D 18 (1978) 2199, Erratum]
- Transition between different configuration state $\rightarrow$ changes in the topological charge $\rightarrow$ anomalous processes (processes forbidden by the classical process are allowed in quantum theory) [Adler, Phys. Rev. 177 (1969) 2246; Bell, Jackiw, Nuovo Cimento A 60 (1969) 47]

A little (pre-)history

- Quantum tunneling between topologcally different configurations as non-perturbative phenomena is suppressed by $e^{-2\pi/\alpha}$ where α is the interaction strength of the gauge theory. High temperature $\longrightarrow$ disappearance of exponential suppression (there is sufficient energy to pass classically over the barrier)
[Manton, Phys. Rev. D 28 (1983) 2019; Klinkhamer, Manton, Phys. Rev. D 30 (1984) 2212]

A little (pre-)history

- The rate is sufficiently large: important for electroweak baryogenesis [Kuzmin, Rubakov, Shaposhnikov, Phys. Lett. B155(1985)36; Shaposhnikov, Nucl. Phys. B287(1987)757; Arnold, McLerran, Phys. Rev. D36(1987)581; Arnold, McLerran, Phys. Rev. D37(1988)1020]
- Electroweak baryon number violation $\leftrightarrow$ Helicity non-conservation in QCD [McLerran, E. Mottola, M.E. Shaposhnikov, Phys. Rev. D 43 (1991) 2027]

A little (pre-)history

- Topological charge changing process involves P-odd and CP-odd field configurations → Existence of axion [Peccei, Quinn, Phys. Rev. D16 (1977) 1791; Weinberg, Phys. Rev. Lett. 40 (1978) 223; Wilczek, Phys. Rev. Lett. 40 (1978) 279; Kim, Phys. Rev. Lett. 43 (1979) 103; Shifman, Vainshtein, Zakharov, Nucl. Phys. B 166 (1980) 493]
- It is thus possible that an excited vacuum domain which may be produced in heavy ion collisions can break P and CP spontaneously [T.D. Lee, Phys. Rev. D 8 (1973) 1226]

A little (pre-)history

- In deconfinement phase transition, QCD vacuum can possess metastable domains or P-odd bubbles (space–time domains with non-trivial winding number). [Kharzeev, Pisarski, M.H.G. Tytgat, Phys. Rev. Lett. 81 (1998) 512]
- P-odd bubbles does not contradict the Vafa–Witten theorem (P and CP cannot be broken in the true ground state of QCD for $\theta = 0$), which does not apply to QCD dense and hot matter where Lorentz-non-invariant P-odd operators have non-zero expectation values. [Vafa, Witten, Phys. Rev. Lett. 53 (1984) 535; Vafa, Witten, Nucl. Phys. B 234 (1984) 173; Son, Stephanov, Phys. Rev. Lett. 86 (2001) 592]

A little (pre-)history

- Examples of P-odd or C-odd ground states in hot or dense matter: P-odd pion-condensation phase, P-odd color superconducting phase, P-even but C-odd metastable states in hot gauge theory. [Migdal, Rev. Mod. Phys. 50 (1978) 107; Pisarski, Rischke, Phys. Rev. Lett. 83 (1999) 37; Bronoff, Korthals Altes, Phys. Lett. B 448 (1999) 85]
- Non-zero angular momentum (or equivalently of magnetic field) in heavy ion collisions → P-odd and CP-odd domains induce charge separation in the produced particles, because they carry net chirality and break symmetry between right-hand and left-hand fermions. [Voloshin, Phys. Rev. C 70 (2004) 057901; Kharzeev, Phys. Lett. B 633 (2006) 260]

Chirality and Helicity

- Chiraltiy $\psi_L = \frac{1}{2}(1-\gamma^5)\psi, \quad \psi_R = \frac{1}{2}(1+\gamma^5)\psi$
- Helicity $h = \boldsymbol{\sigma} \cdot \frac{\mathbf{p}}{|\mathbf{p}|}$
- In the chiral limit (massless quark) with $m_f = 0$

Helicity	RH chirality	LH chirality
Particle	+1	-1
Anti-particle	-1	+1

Axial Anomaly and Winding number

- All gauge field configurations are classified by the topological winding numbers ($\tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma} F^a_{\rho\sigma}$)

$$Q_w = \frac{g^2}{32\pi^2}\int d^4x F_{\alpha\beta}\tilde{F}^{\alpha\beta}$$
$$= N_{CS}(\mathrm{t}=\infty) - N_{CS}(t=-\infty)$$

- Axial anomaly

$$j_\mu^5 = \sum_f \langle \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f \rangle_A$$

Average over gluon field configuration

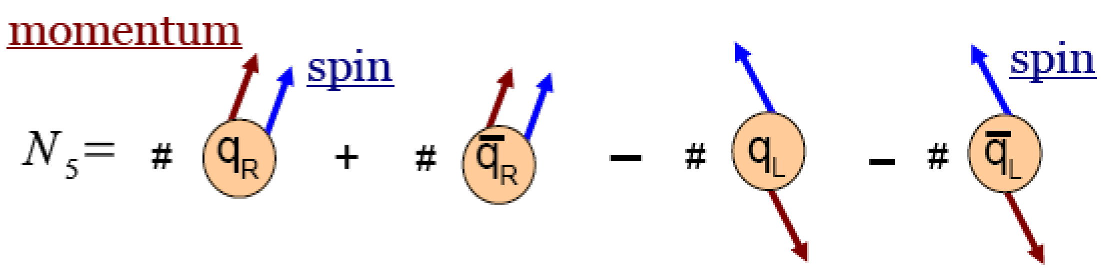
$$\partial^\mu j_\mu^5 = 2\sum_f m_f \langle \bar{\psi}_f i \gamma_5 \psi_f \rangle_A - \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}_a^{\mu\nu}$$

Axial Anomaly

- Chiral charge number at chiral limit:

$$N_5 = N_R - N_L = (n_R - \bar{n}_R) - (n_L - \bar{n}_L)$$
$$= (n_R + \bar{n}_L) - (n_L + \bar{n}_R)$$
$$= n(h = +1) - n(h = -1)$$

R,L denote chirality



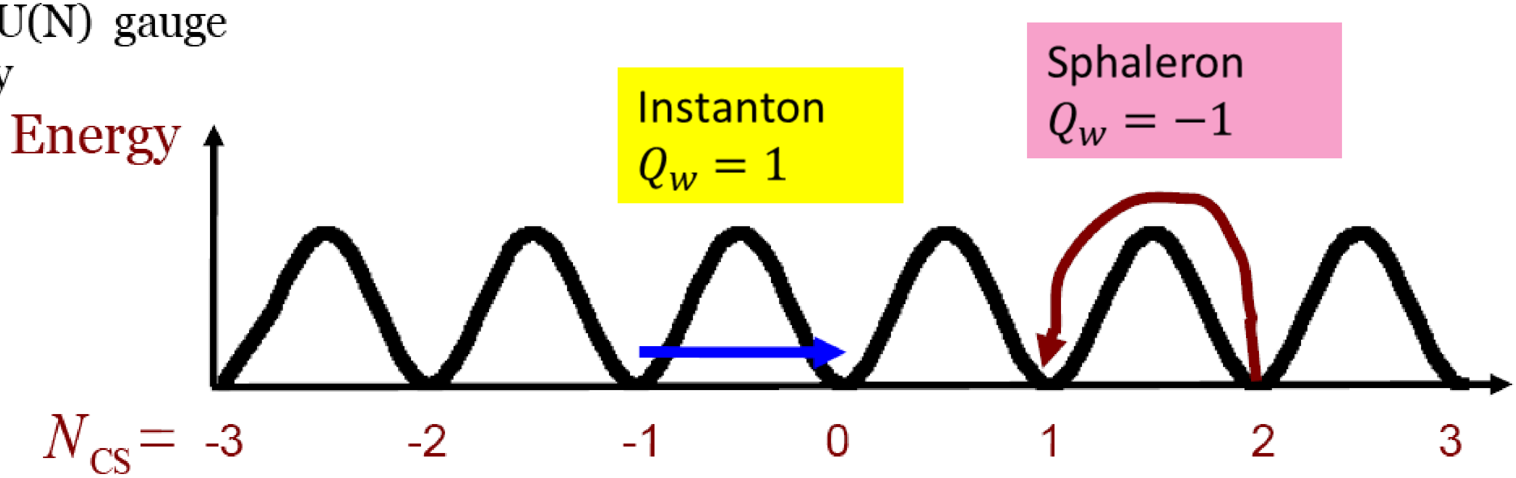
R,L denote helicity

- Assuming $N_R(t = 0) = N_L(t = 0)$, then we have

$$N_5(t = \infty) = -2N_f Q_w = -2N_f \Delta N_{CS}$$

Chern-Simons number and instanton/sphaleron transition

The nontrivial vacuum structure of a SU(N) gauge theory



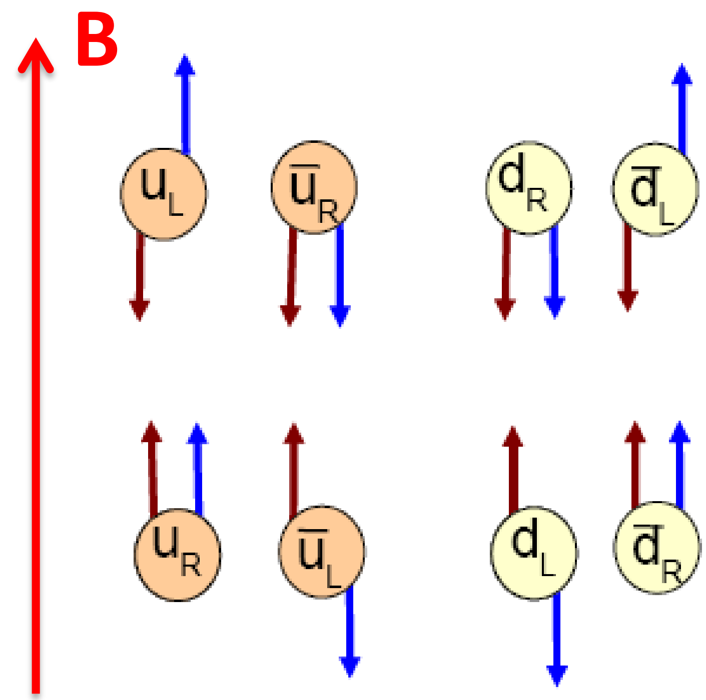
$\langle Q_w{}^2 \rangle \sim m^2(\eta') + m^2(\eta) - 2m^2(K) \neq 0$ ['t Hooft, Witten, Veneziano]

$\langle Q_w \rangle = 0$, neutron dipole moment, [Baker et al, PRL97,131801(2006)]

$|\theta| < 10^{-10}$ $\rightarrow$ no P and CP violation, but Q_w can induce P- and CP-odd effect

Chiral Magnetic Effect

- Magnetic field aligns spin depending on electric charge; The momenta of quarks and antiquarks align along the magnetic field.
- Quarks with RH-helicity move opposite to those with LH-helicity



- Momentum-down:

$d_R + \bar{u}_R \ (Q_e = -)$

$u_L + \bar{d}_L \ (Q_e = +)$

- Momentum-up:

$u_R + \bar{d}_R \ (Q_e = +)$

$d_L + \bar{u}_L \ (Q_e = -)$

Chiral Magnetic Effect

- Meganetic field locks

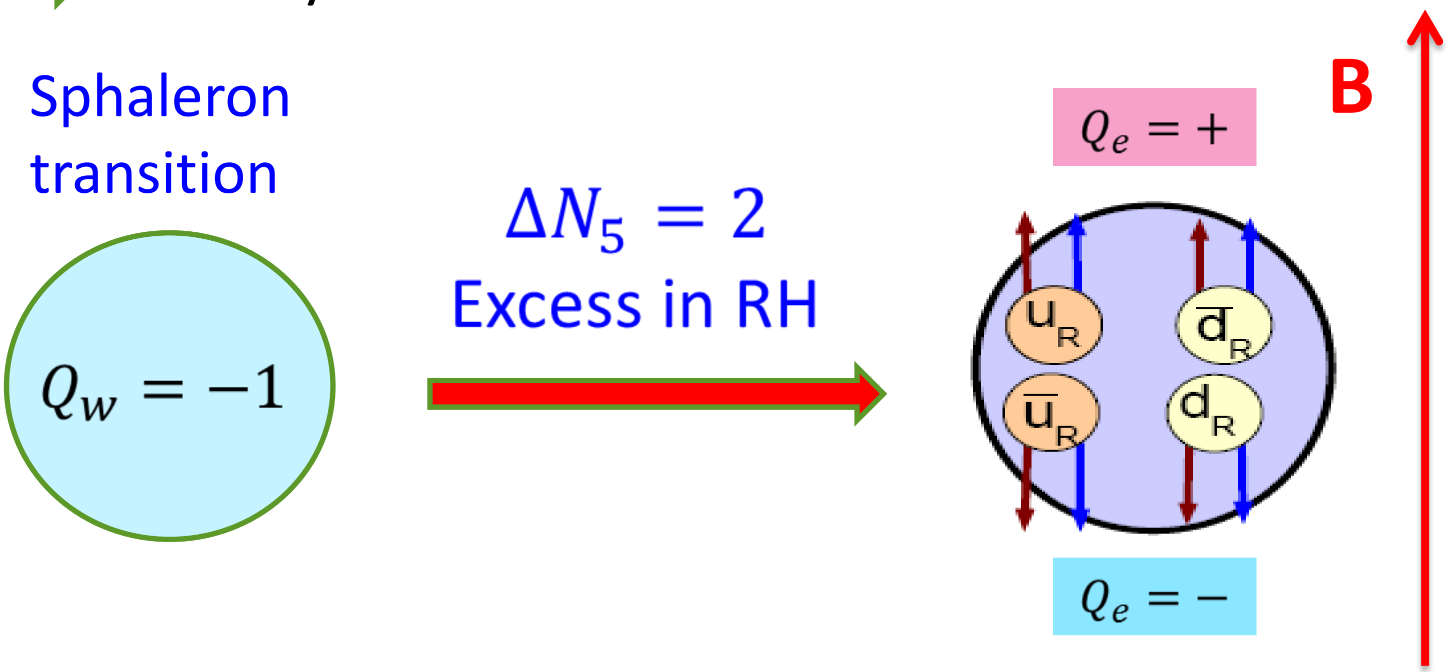
  charge $\leftrightarrow$ helicity $\leftrightarrow$ momentum

- The asymmetry between RH- and LH-helicity from anomaly will lead to charge asymmetry (charge separation) along magnetic field
- This is called Chiral Magnetic Effect

Charge Separation

- Topological charge + Magnetic field

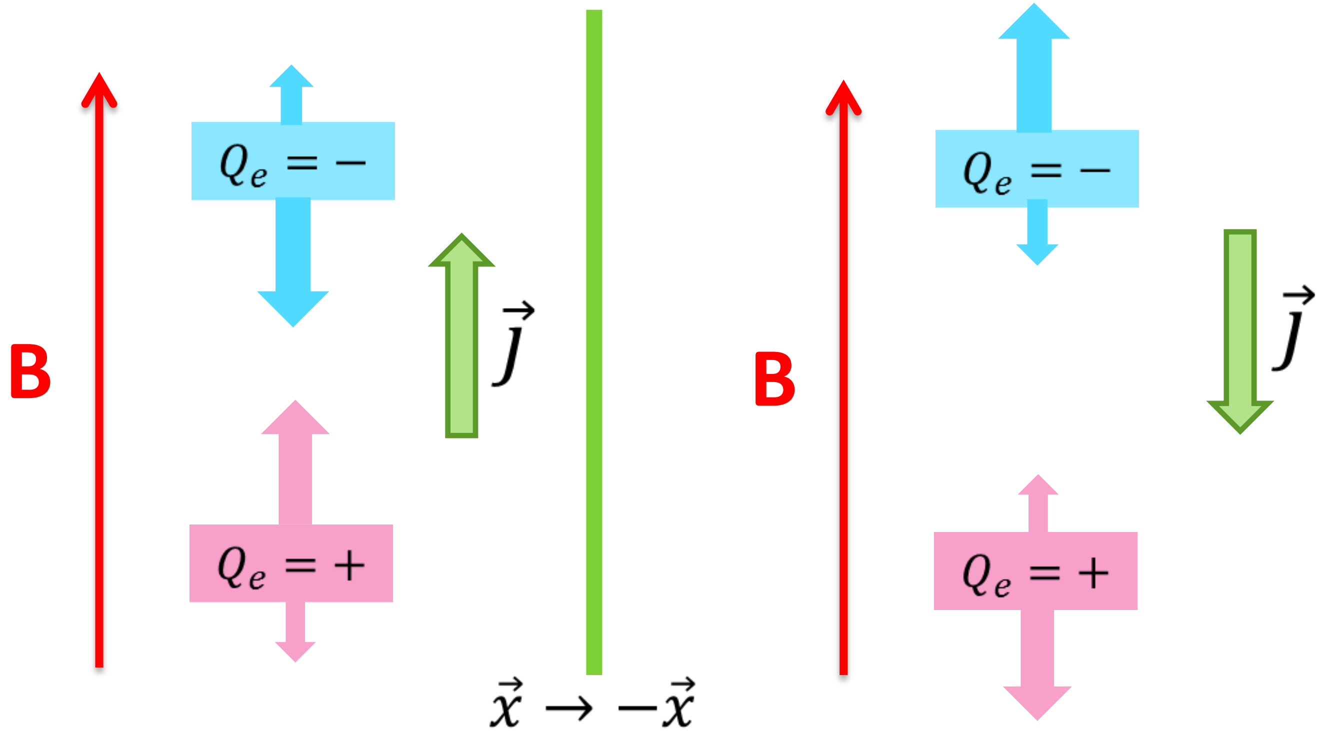
  ➡ Chirality + Polarization



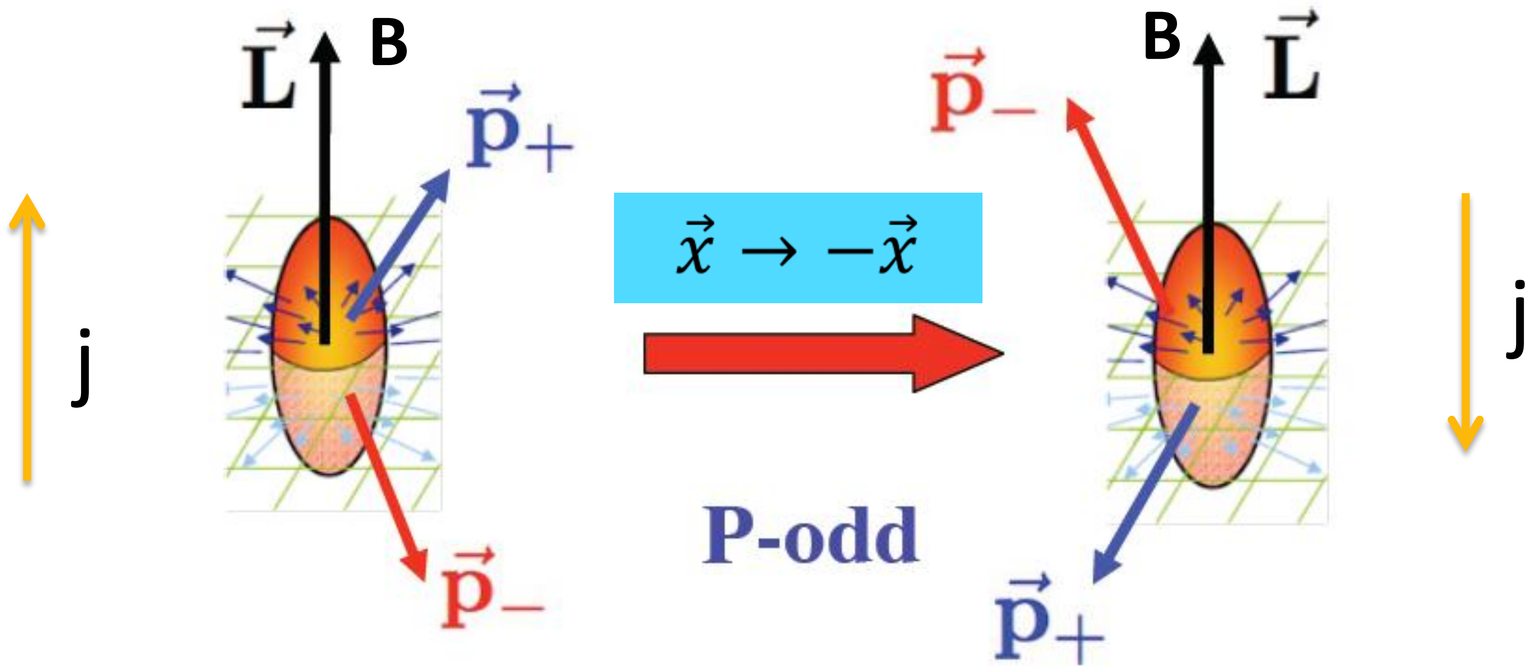
Electric charge current induced by magnetic field

Charge Separation means P-violation

- Separation of charge along L_z or B_z means P-violation ($\vec{L}$ and $\vec{B}$ are axial vector)



Charge Separation means P-violation



$$
\begin{array}{lcl}
\mathbf{L} = \mathbf{r} \times \mathbf{p} & \to & \mathbf{L} \\
\mathbf{B} & \to & \mathbf{B} \\
\mathbf{E} & \to & -\mathbf{E} \\
j^\mu & \to & -j^\mu
\end{array}
$$

$$j \to -j, \psi_L \leftrightarrow \psi_R \qquad j_5^\mu = j_R^\mu - j_L^\mu \to j_5^\mu$$

$$\partial_\mu j_5^\mu = CE \cdot B \quad \to \quad \partial_\mu j_5^\mu = -CE \cdot B$$

$$N_R - N_L = Q_W \quad \to \quad N_R - N_L = -Q_W$$

Induced current by magnetic field

- Chiral chemical potential $\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$
- Induced current

$$j = \frac{N_c \sum q_f{}^2}{2\pi^2} \mu_5 B$$

- Derivation:

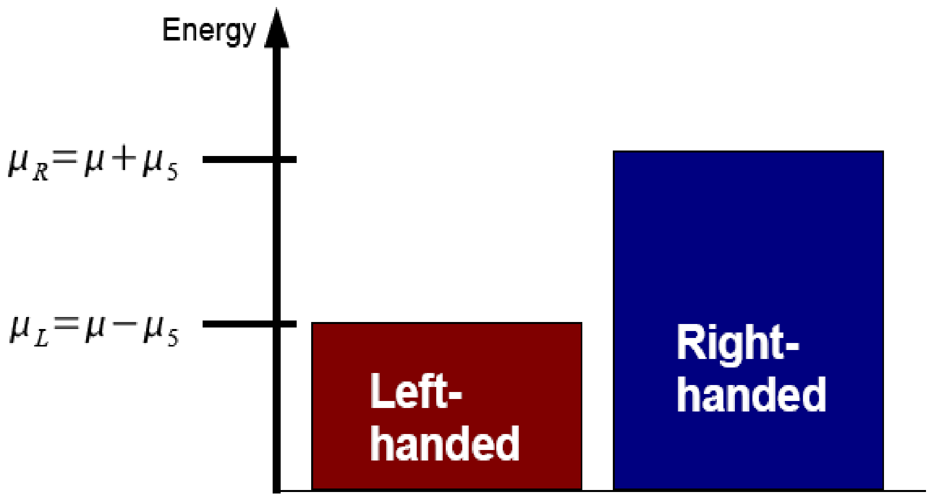
Thermodynamic potential

Linear response theory

Propagator in magnetic field

Kubo-Formula

Chern-Simons term

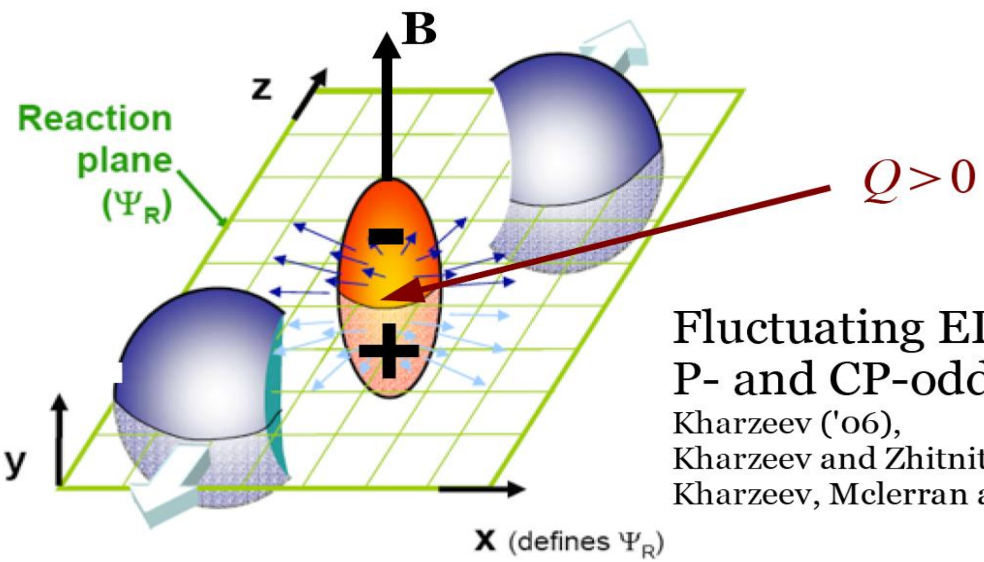


Chiral Magnetic Effect and Charge Separation

- Average over charge and charge squared
  $\langle Q_e \rangle = 0, \langle {Q_e}^2 \rangle \neq 0$

$\langle Q \rangle = 0$

$\langle Q^2 \rangle \neq 0$



Fluctuating EDM of QGP
P- and CP-odd effect
Kharzeev ('06),
Kharzeev and Zhitnitsky ('07)
Kharzeev, Mclerran and Warringa ('08)

About unit for magnetic field: from cgs to natural unit

$$
\begin{aligned}
1\,c &= 3\times 10^{10}\,\mathrm{cm}\cdot\mathrm{s}^{-1}\\
1\,\hbar &= 1.05\times 10^{-27}\,\mathrm{g}\cdot\mathrm{cm}^2\cdot\mathrm{s}^{-1}\\
1\,\mathrm{eV} &= 1.6\times 10^{-12}\,\mathrm{g}\cdot\mathrm{cm}^2\cdot\mathrm{s}^{-2}
\end{aligned}
$$

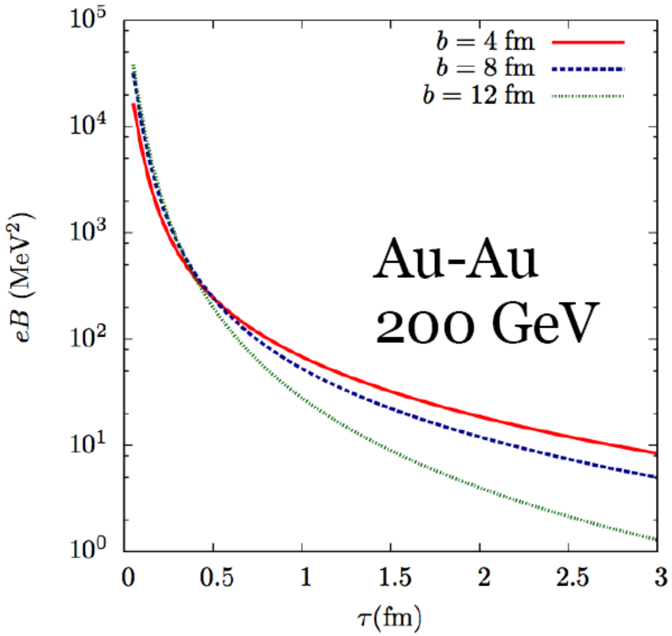
$$
\begin{aligned}
1\,\mathrm{s} &= 1.52\times 10^{15}\,\hbar\cdot\mathrm{eV}^{-1}\\
1\,\mathrm{cm} &= 5.06\times 10^{4}\,\hbar\cdot\mathrm{eV}^{-1}\cdot c\\
1\,\mathrm{g} &= 5.6\times 10^{32}\,\mathrm{eV}\cdot c^{-2}
\end{aligned}
$$

$$E = \frac{1}{8\pi}\int d^3x B^2$$

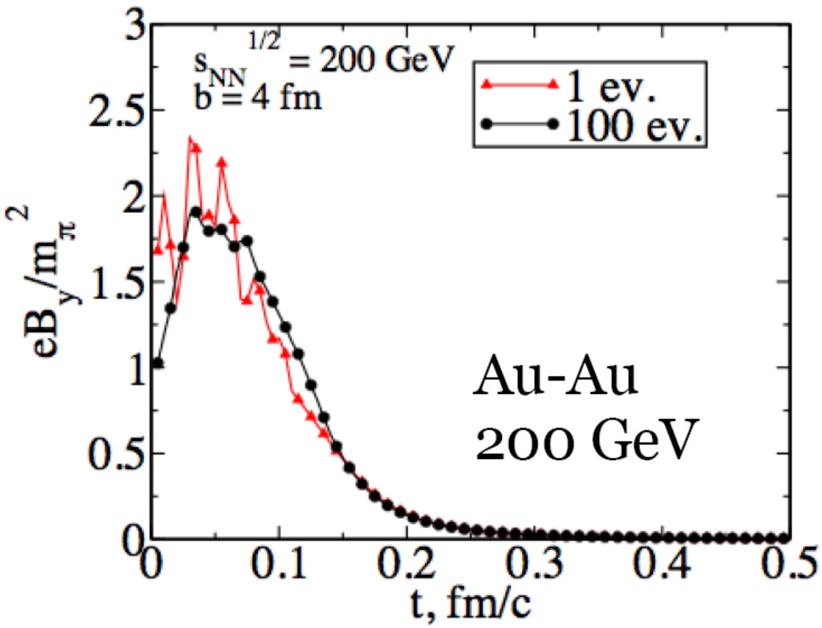
$$
\begin{aligned}
1\,\mathrm{Gauss} &= 10^{-4}\,\mathrm{T}\\
&= 1\,\mathrm{g}^{1/2}\mathrm{cm}^{-1/2}\mathrm{s}^{-1}\\
&= 6.92\times 10^{-2}\,(\hbar c)^{-3/2}\cdot\mathrm{eV}^2
\end{aligned}
$$

$$
\begin{aligned}
1\,\mathrm{MeV}^2 &= 1.44\times 10^{13}\,\mathrm{Gauss}\\
m_\pi^2 &\sim 2.8\times 10^{17}\,\mathrm{Gauss}
\end{aligned}
$$

Ultra-high Magnetic field



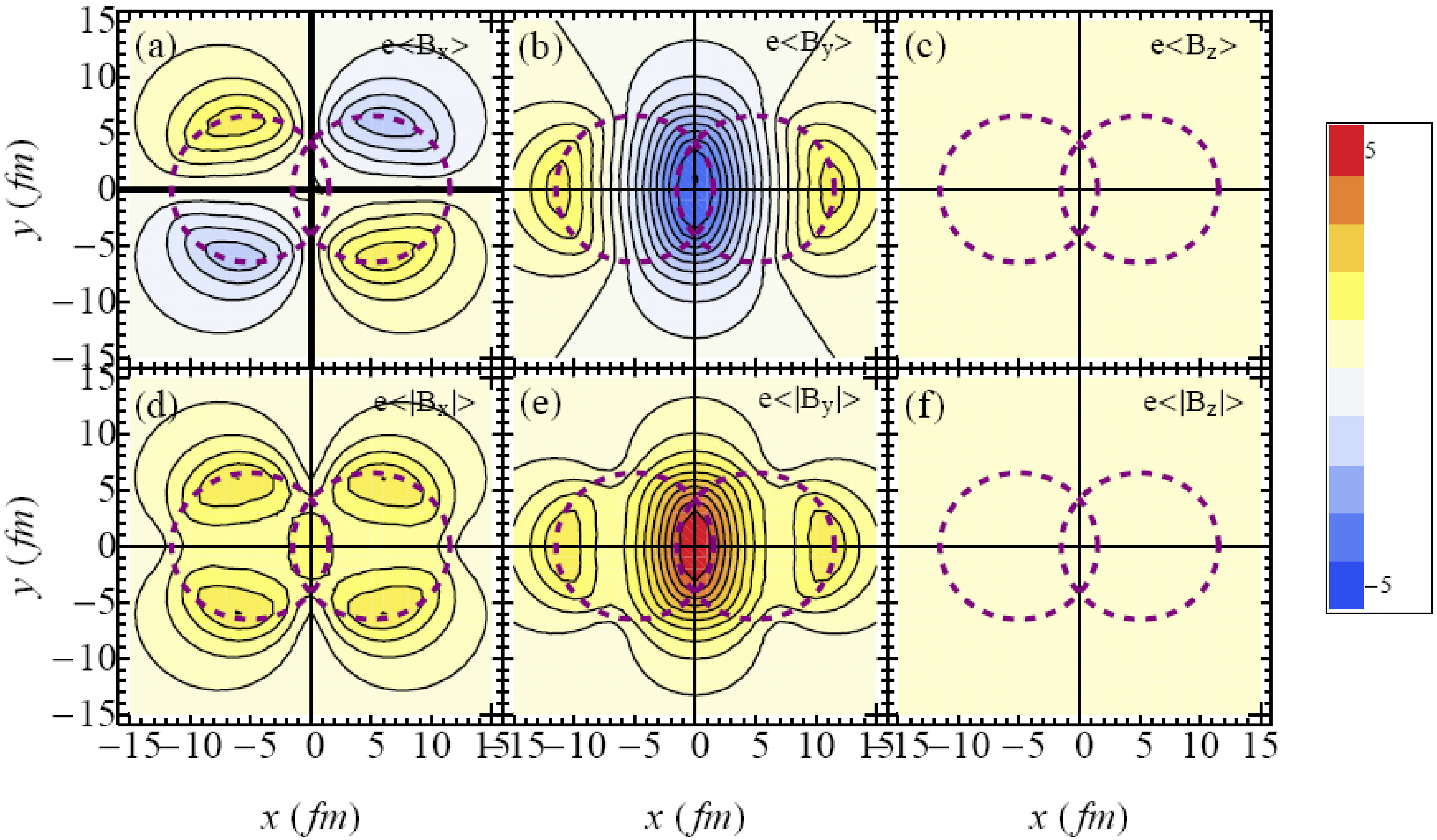
Pancake approximation
Kharzeev, McLerran & HJW ('08)
See also Minakata and Müller ('96)



URQMD calculation
Skokov, Illarionov, Toneev ('09)

$$eB(\tau = 0.2\ \mathrm{fm/c}) \approx 10^3 \sim 10^4\,\mathrm{MeV}^2 \approx 10^{18}\,\mathrm{G}$$

Ultra-high Magnetic field

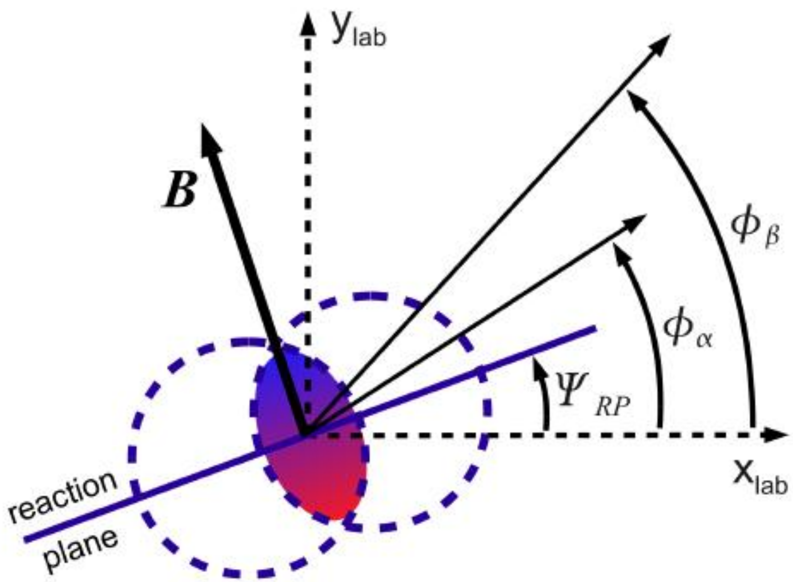


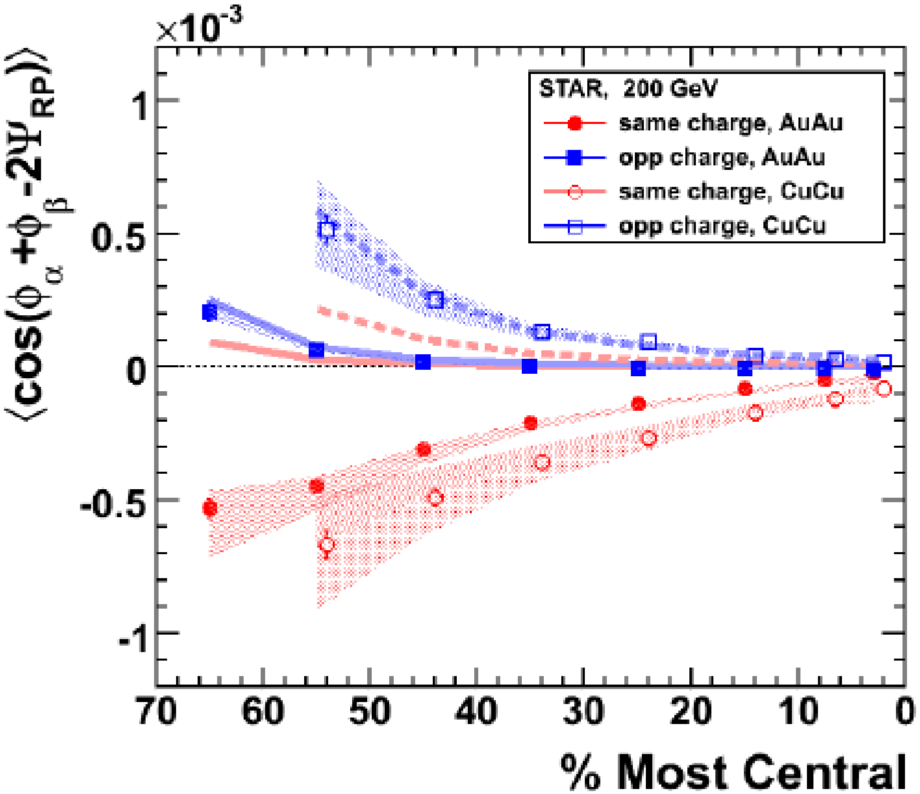
The spatial distribution of magnetic fields for b=10 fm at 200 GeV.
Unit: pion mass squared. ---- W. Deng, X. Huang, PRC85, 044907(2012)

Charge correlation at RHIC

STAR Collab., PRL 103, 251601(2009); PRC 81, 054908 (2010)

Au+Au, Cu+Cu at 200 GeV/A
Minimum bias, $|\eta| < 1$,
$p_t \in [0.15,2]$ GeV





Sign of local parity violation?

Approaches to CME/CVE

- Gauge/Gravity correspondence

  Erdmenger, et. al., JHEP 0901, 055(2009); Banerjee, et. al., JHEP 1101, 094 (2011); Torabian and Yee, JHEP 0908, 020(2009); Rebhan, Schmitt and Stricker, JHEP 1001, 026 (2010); Kalaydzhyan and Kirsch, PRL 106, 211601 (2011); ……

- Hydrodynamics with Entropy/T-invariance Principle

  Son and Surowka, PRL 103, 191601(2009); Kharzeev and Yee, PRD 84, 045025(2011); Pu, Gao and Q.W., PRD 83, 094017(2011); ……

- Field Theory

  Metlitski and Zhitnitsky, PRD 72, 045011(2005); Newman and Son, PRD 73, 045006(2006); Lublinsky and Zahed, PLB 684, 119(2010); Asakawa, Majumder and Muller, PRC 81, 064912(2010); Landsteiner, Megias and Pena-Benitez, PRL 107, 021601(2011); Hou, Liu and Ren, JHEP 1105, 046(2011); ……

- Quantum Kinetic Approach

  Gao, Liang, Pu, Q. Wang and X.-N. Wang, PRL 109, 232301(2012); Son, Yamamoto, arXiv:1210.8158;

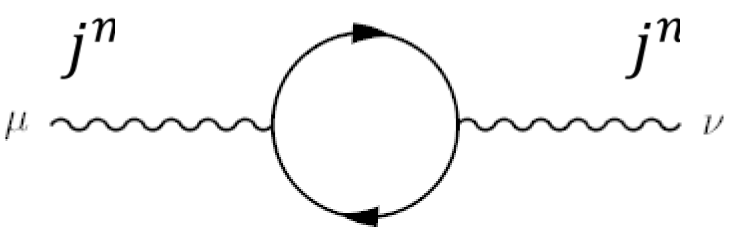
Approaches to CME/CVE

- Field theory
- Hydrodynamics with entropy principle
- Quantum kinetic theory with Wigner function

Kubo formula

$$j^{\mu} = \sigma_{\chi} B^{\mu}$$

$$\sigma_{\chi} = \lim_{\omega\to 0}\lim_{k\to 0}\frac{1}{2ik^{l}}\epsilon^{lmn}\underbrace{<[j^{m}(\omega,k),j^{n}(0,k)]>}_{G_R^{mn}}$$



$$\sigma_{\chi} = \frac{e^2}{2\pi^2}\mu_5$$

Although the chiral magnetic conductivity is given from one loop diagram, the CME is related to the chiral anomaly, which is guaranteed no more high order contributions by the Ward identity. Therefore, at least now, there is no more correlation found from the high orders. [For higher order correction of CVE, see Golkar, Son 2012, Hou, Ren, 2012]

Lattice calculation

- The chiral chemical potential has no sign problem, so the lattice simulations are available [Fukushima, Kharzeev, Warringa, 2008]

$$\det \mathcal{M}(\mu_5) \equiv \det(\not{D} + \mu_5 \gamma_E^0 \gamma^5 + m)$$

  - a direction study of chiral magnetic current as a function of chiral chemical potential [Yamamoto, 2011]
  - Buividoivch, Luschevskaya, Polikarpov et.al, 2008, 2009, 2010
  - Abramczy, Blum, Petropoulos, Zhou, 2009

Anomaly in Hydrodynamics with Entropy Principle

- Hydrodynamic equation with anomaly

$$E^\mu = F^{\mu\nu} u_\nu ,$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \, , \qquad \partial_\mu j^\mu = C E \cdot B$$

- Entropy production satisfies

$$s^\mu = s u^\mu - \frac{\mu}{T} \nu^\mu$$

$$\begin{aligned} \partial_\mu s^\mu &= -\frac{1}{T} \partial_\mu u_\nu \tau^{\mu\nu} - \nu^\mu \left(\partial_\mu \frac{\mu}{T} - \frac{E_\mu}{T} \right) - C \frac{\mu}{T} E \cdot B \\ \tau^{\mu\nu} &= -\eta \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha \right) - \left(\zeta - \frac{2}{3} \right) \Delta^{\mu\nu} \partial \cdot u \\ \nu^\mu &= -\sigma T \left(\Delta^{\mu\nu} \partial_\nu \frac{\mu}{T} - \frac{E^\mu}{T} \right) , \end{aligned}$$

- The new (E.B) term violates the positive entropy production

[Son and Surowka, PRL 103, 191601(2009)]

Anomaly in Hydrodynamics with Entropy Principle

- Requiring positive entropy production, new terms are needed

$\omega^\mu, \quad B^\mu$

co-moving frame

$$\nu^\mu = -\sigma T \Delta^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi\omega^\mu + \xi_B B^\mu \qquad (0, \nabla\times\mathbf{u})$$

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu$$

$$\begin{aligned} E^\mu &= F^{\mu\nu}u_\nu \\ B^\mu &= \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu F_{\lambda\rho} \\ \omega^\mu &= \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho \\ &= (0, \nabla\times\mathbf{u}) \end{aligned}$$

- The new coefficients can be fixed

$$D = \frac{1}{3}C\frac{\mu^3}{T}, \qquad D_B = \frac{1}{2}C\frac{\mu^2}{T},$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right), \quad \xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right)$$

[Son and Surowka, PRL 103, 191601(2009)]

With two currents

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$
$$\partial_\mu j^\mu = CE \cdot B$$

$$j^\sigma = nu^\sigma + \nu^\sigma, \; j_5^\sigma = n_5 u^\sigma + \nu_5^\sigma$$
$$\nu^\sigma = -\sigma T(\Delta^{\sigma\rho}\partial_\rho \frac{\mu}{T} - \frac{E^\sigma}{T}) + \xi\omega^\sigma + \xi_B B^\sigma$$
$$\nu_5^\sigma = -\sigma_5 T \Delta^{\sigma\rho}\partial_\rho \frac{\mu_5}{T} + \xi_5\omega^\sigma + \xi_{5B} B^\sigma$$
$$s^\sigma = su^\sigma - \nu^\sigma \frac{\mu}{T} - \nu_5^\sigma \frac{\mu_5}{T} + D\omega^\sigma + D_B B^\sigma$$

$$\partial_\mu s^\mu \geq 0$$

$$\xi = 2\mu\mu_5 C, \; \xi_5 = \mu^2 C$$
$$\xi_B = \mu_5 C, \quad \xi_{5B} = \mu C$$
$$D = -\frac{\mu^2\mu_5}{T}, \; D_B = -\frac{\mu\mu_5}{T}$$

Pu, Gao, Q.Wang, PRD83, 094017(2011)

Quantum Kinetic Theory: an approach to CME/CVE

- How to describe CME/CVE as quantum effects in microscopic theory?
- Quantum kinetic theory is quite a natural choice. It can bridge between the microscopic and macroscopic approach.
- We will show that CME/CVE are natural consequences of the quantum kinetic equations. Chiral anomaly appears naturally in a remarkable way and all the other conservation laws are also automatically satisfied.

Wigner function

Gauge invariant Wigner operator/function

$$W(x,p) = <: \hat{W}(x,p) :>$$

$$\hat{W}_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta \left(x + \frac{1}{2}y\right) \mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \psi_\alpha \left(x - \frac{1}{2}y\right)$$

Gauge link $\mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \equiv \mathcal{P}\mathrm{Exp}\left(-iey^\mu \int_0^1 ds A_\mu \left(x - \frac{1}{2}y + sy\right)\right)$

Dirac equation

$$[i\gamma^\mu D_\mu(x) - m]\,\psi(x) = 0, \qquad \bar{\psi}(x)\left[i\gamma^\mu D_\mu^\dagger(x) + m\right] = 0$$

Quantum Kinetic Equation for Wigner function for massless fermion in homogeneous magnetic field

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i \nabla^\mu\right) W(x,p) = 0 \qquad \nabla^\mu \equiv \partial_x^\mu - QF^{\mu\nu}\partial_\nu^p$$

Wigner function

Decomposition of Wigner function:

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

Coupled equations for vector and axial-vector

$$\begin{aligned} p^\mu\mathscr{V}_\mu &= 0, \quad p^\mu\mathscr{A}_\mu = 0, \\ \nabla^\mu\mathscr{V}_\mu &= 0, \quad \nabla^\mu\mathscr{A}_\mu = 0, \\ \epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathscr{A}^\sigma &= -2\left(p_\mu\mathscr{V}_\nu - p_\nu\mathscr{V}_\mu\right), \\ \epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathscr{V}^\sigma &= -2\left(p_\mu\mathscr{A}_\nu - p_\nu\mathscr{A}_\mu\right). \end{aligned}$$

Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986).

16 equations for 8 components of $\mathscr{V}_\mu$ and $\mathscr{A}_\mu$ $\Rightarrow$ these equations must be highly consistent with each other. At this point, $\mathscr{V}_\mu$ and $\mathscr{A}_\mu$ can be any functions of x and p that satisfy the above equations.

Perturbation method for solving quantum kinetic equation

Expand vector and axial vector in powers of ∂_x^μ and $F^{\mu\nu}$

$$\mathscr{V}_\mu = \mathscr{V}_\mu^0 + \mathscr{V}_\mu^1 + \ldots\ , \qquad \mathscr{A}_\mu = \mathscr{A}_\mu^0 + \mathscr{A}_\mu^1 + \ldots$$

0-th order $\quad \mathscr{V}_\mu^0,\ \ \mathscr{A}_\mu^0\ :\ (\partial_x^\mu)^0,\ \ (F^{\mu\nu})^0$

1-st order $\quad \mathscr{V}_\mu^1,\ \ \mathscr{A}_\mu^1\ :\ (\partial_x^\mu)^1,\ \ (F^{\mu\nu})^1$

Gao, Liang, Pu, Q.Wang, X.N.Wang, PRL 109, 232301(2012)

Iterative equations:

$$\begin{aligned}
\epsilon_{\mu\nu}{}^{\rho\sigma}\nabla_\rho \mathscr{A}_\sigma^n &= -2\left(p_\mu \mathscr{V}_\nu^{n+1} - p_\nu \mathscr{V}_\mu^{n+1}\right), \\
\epsilon_{\mu\nu}{}^{\rho\sigma}\nabla_\rho \mathscr{V}_\sigma^n &= -2\left(p_\mu \mathscr{A}_\nu^{n+1} - p_\nu \mathscr{A}_\mu^{n+1}\right).
\end{aligned}$$

CME/CVE as Result of Solution to Coupled Quantum Kinetic Equations

$$
\begin{aligned}
j^{\mu} &= \int d^4p \mathscr{V}^{\mu} = nu^{\mu} + \xi\omega^{\mu} + \xi_B B^{\mu}, \\
j_5^{\mu} &= \int d^4p \mathscr{A}^{\mu} = n_5 u^{\mu} + \xi_5\omega^{\mu} + \xi_{B5} B^{\mu},
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu} &= \frac{1}{2}\int d^4p (p^{\mu}\mathscr{V}^{\nu} + p^{\nu}\mathscr{V}^{\mu}) \\
&= (\epsilon + P)u^{\mu}u^{\nu} - Pg^{\mu\nu} + n_5(u^{\mu}\omega^{\nu} + u^{\nu}\omega^{\mu}) \\
&\quad + \frac{1}{2}Q\xi(u^{\mu}B^{\nu} + u^{\nu}B^{\mu})
\end{aligned}
$$

CME/CVE as Result of Solution to Coupled Quantum Kinetic Equations

Transport coefficients

Absent in hydro-derivation

$$
\begin{aligned}
\xi &= \frac{1}{\pi^2}\mu\mu_5, \\
\xi_B &= \frac{Q}{2\pi^2}\mu_5, \\
\xi_5 &= \boxed{\frac{1}{6}T^2} + \frac{1}{2\pi^2}\left(\mu^2 + \boxed{\mu_5^2}\right), \\
\xi_{B5} &= \frac{Q}{2\pi^2}\mu.
\end{aligned}
$$

$$
\begin{aligned}
n &= \frac{1}{3\pi^2}\mu\left(\pi^2T^2 + \mu^2 + 3\mu_5^2\right) \\
n_5 &= \frac{1}{3\pi^2}\mu_5\left(\pi^2T^2 + \mu_5^2 + 3\mu^2\right) \\
\epsilon &= \frac{1}{2\pi^2}\left[\frac{7\pi^4}{30}T^4 + \pi^2T^2(\mu^2 + \mu_5^2) \right. \\
&\quad \left. + \frac{1}{2}(\mu^4 + \mu_5^4) + 3\mu^2\mu_5^2\right]
\end{aligned}
$$

comes out naturally

All the conservation laws and anomaly can be derived naturally

$$
\partial_\sigma j_5^\sigma = -\boxed{\frac{Q^2}{2\pi^2}}E\cdot B \qquad \partial_\sigma j^\sigma = 0 \qquad \partial_\sigma T^{\sigma\nu} = QF^{\nu\rho}j_\rho
$$

An Independent derivation of chiral anomaly from quantum kinetic theory !

With more flavors

Consider 3-flavor quark matter (u,d,s), the vector current can be electromagnetic or baryonic

$$\xi^{\text{baryon}} = \frac{N_c}{\pi^2}\mu\mu_5, \quad \boxed{\xi_B^{\text{baryon}} = \frac{N_c}{6\pi^2}\mu_5 \sum_f Q_f,}$$

Baryonic current is blind to B

$$\boxed{\xi^{\text{EM}} = \frac{N_c}{\pi^2}\mu\mu_5 \sum_f Q_f,} \quad \xi_B^{\text{EM}} = \frac{N_c}{2\pi^2}\mu_5 \sum_f Q_f^2.$$

EM-current is blind to ω

Since $\sum_f Q_f = 0$ for the three-flavor quark matter, we have

$$\xi_B^{\text{baryon}} = \xi^{\text{EM}} = 0$$

[Kharzeev and Son, PRL 106, 062301(2011); Gao, Liang, Pu, Q.Wang, X.N.Wang, PRL 109, 232301(2012)]

Local Polarization Effect

Consider 3-flavor quark matter (u,d,s), the axial baryonic current

$$j_5^\sigma = n_5 u^\sigma + +\xi_5 \omega^\sigma + \xi_{5B} B^\sigma$$

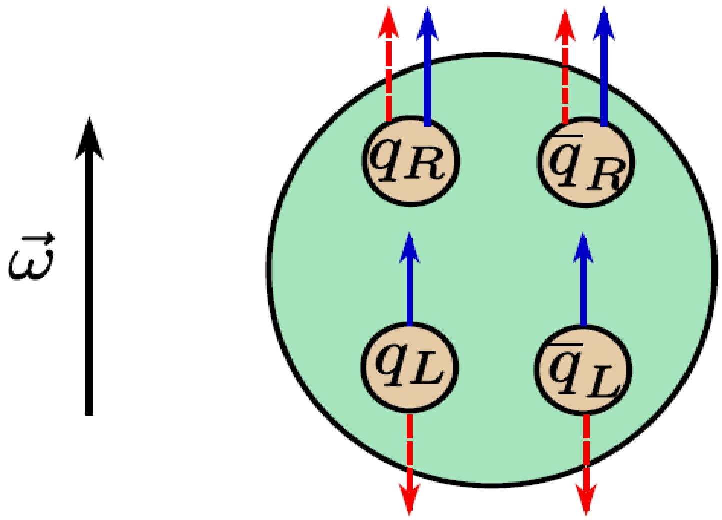
$$\xi_5 = N_c \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right],$$

$$\xi_{B5} = \frac{N_c}{6\pi^2} \mu \sum_f Q_f = 0.$$

Quadratic in temperature, chemical potential, chiral chemical potential
→
No cancellation!

Leading to Local Polarization Effects!
(either for high or low energy HIC)

The LPE can be measured in heavy ion collisions by the hadron (e.g. hyperon) polarization along the vorticity direction once it is fixed in the event.



Related and new phenomena

- Recently, there are a lot of phenomena related to CME, e.g. chiral magnetic spirals [Basar, Dunne, Kharzeev, 2010, chiral shear wave, Sahoo, Yee, 2009, etc]

- Chiral magnetic wave (CMW) [Burnier, Kharzeev, Liao, and Yee, 2011,2012].

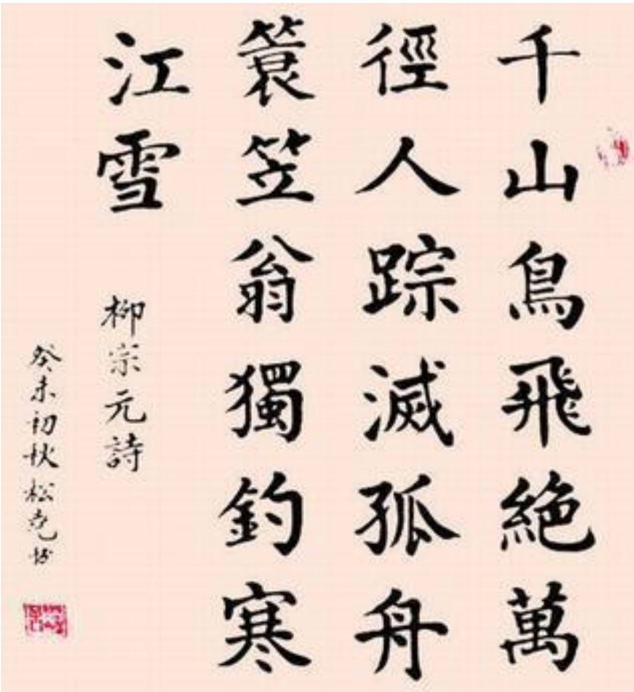
- Berry phase and chiral kinetic equation can be given within quantum kinetic approach [Chen, Pu, Q.Wang, X.N.Wang, 2012; Yamamoto, Son, 2012; Stenphanov, Yin, 2012].

- Other related phenomena are disscussed in the physics of neutrions [Vilenkin, 1979, 1980], primordial electroweak plasma [Giovannini , Shaposhnikov 1998], quantum wires [Alekseev, Cheianov, Frolich, 1998].

Long March to Vacuum

- If spontaneous breaking of Parity in QCD-vacumm under magnetic field is confirmed, it would be a big event in fundamental physics!

A poem for beauty and loneliness



Loneliness seems to be luxury in turbulent China now ……