Noval Half-BPS Wilson Loops in 3d N=4 Chern-Simons-matter theories

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Outline

- Background – classification of BPS Wilson loops in CSM theories
- M2-branes in AdS_4*S^7/ Γ
- Constructions of half-BPS Wilson loops in 3d N=4 CSM theories
- Discussions

Backgrounds

- The **classification** of BPS Wilson loops (WLs) in 3d super Chern-Simons theories is more complicated than similar problems in 4d SYM.
- There are two types: Gaiotto-Yin type and Drukker-Trancanelli type.
- The construction of DT type WLs involves fermions.

Backgrounds

- GY type WLs exist in CSM theories with at least $\mathcal{N}$=2 SUSY.
- DT type WLs exist for N=5, 6 CSM theories. *[Drukker etal 09][Lee, Lee, 10]*
- We have strong evidence that BPS WLs can be at most 1/3 BPS. ***Either DT type WL does not exist here or it can be at most 1/3 BPS.***

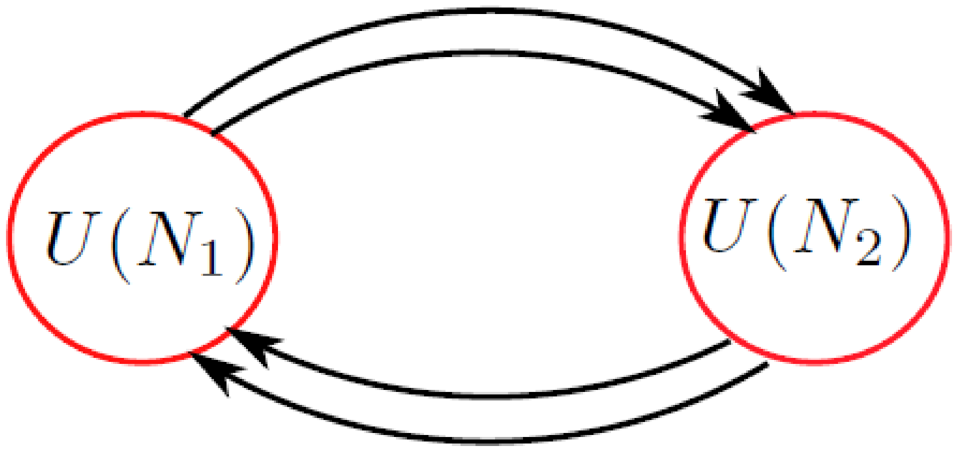
  [Chen, JW, Zhu, 14]
- How about the case of N=4 theories?

Reviews

- ABJM theory is 3d *N*=6 **Chern-Simons-matter** theory with gauge group U(N)*U(N) and Chern-Simons level (k, -k).

- The scalar fields are four complex scalars in the bifundemental representation $(N, \bar{N})$

(N1=N2=N in the fig.)



Supersymmetry transformation

$$\delta A_\mu = \frac{4\pi}{k}\left(\phi_I \bar{\psi}_J \gamma_\mu \theta^{IJ} + \bar{\theta}_{IJ}\gamma_\mu \psi^J \bar{\phi}^I\right),$$

$$\delta \hat{A}_\mu = \frac{4\pi}{k}\left(\bar{\psi}_J \gamma_\mu \phi_I \theta^{IJ} + \bar{\theta}_{IJ}\bar{\phi}^I \gamma_\mu \psi^J\right),$$

$$\delta \phi_I = 2i\bar{\theta}_{IJ}\psi^J, \quad \delta\bar{\phi}^I = 2i\bar{\psi}_J \theta^{IJ},$$

$$\delta \psi^I = 2\gamma^\mu \theta^{IJ} D_\mu \phi_J - \frac{4\pi}{k}\theta^{IJ}\left(\phi_J \bar{\phi}^K \phi_K - \phi_K \bar{\phi}^K \phi_J\right) - \frac{8\pi}{k}\theta^{KL}\phi_K \bar{\phi}^I \phi_L,$$

$$\delta \bar{\psi}_I = -2\bar{\theta}_{IJ}\gamma^\mu D_\mu \bar{\phi}^J + \frac{4\pi}{k}\bar{\theta}_{IJ}\left(\bar{\phi}^J \phi_K \bar{\phi}^K - \bar{\phi}^K \phi_K \bar{\phi}^J\right) + \frac{8\pi}{k}\bar{\theta}_{KL}\bar{\phi}^K \phi_I \bar{\phi}^L.$$

GY type Wilson loops in ABJM theory

- Following GY, we consider the WLs (before take the trace)

$$W = \mathcal{P}e^{-i\int \mathcal{A}},$$
$$\mathcal{A} = A_0 + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J .$$

- δA=0 gives only 1/6-BPS WLs with M=diag(1, 1, -1, -1).
- $SU(4)_R$ is broken to SU(2)*SU(2)*U(1).
- Non-constant M leads to less supersymmetry (general rule).
- *[Drukker, Plefka, Young 08] [Chen, JW, 08]*
  [Rey, Suyama, Yamaguchi, 08]

Gravity dual

- The simplest string solution dual to WL is AdS_2 in AdS_4 and which is a point in $\mathbf{CP}^3$.
- This solution is half-BPS and the isometry of $\mathbf{CP}^3$ is broken to SU(3)*U(1).

- So the immediate problem is to construct the half-BPS WLs in ABJM theory.

Half-BPS Wilson loops

- In 2009, Drukker and Trancanelli constructed the following WL (we consider the **timelike** Wilson line going from infinite to infinite)

$$W = \mathcal{P} e^{-i\int L},$$

$$L = \begin{pmatrix} \mathcal{A} & f_1 \\ \bar{f}_2 & \hat{\mathcal{A}} \end{pmatrix}.$$

$$\mathcal{A} = A_0 + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J,$$

$$\hat{\mathcal{A}} = \hat{A}_0 + \frac{2\pi}{k} N_I{}^J \bar{\phi}^I \phi_J,$$

$$f_1 = \sqrt{\frac{2\pi}{k}} \bar{\eta}_I \psi^I,$$

$$\bar{f}_2 = \sqrt{\frac{2\pi}{k}} \bar{\psi}_I \eta^I.$$

Half-BPS Wilson loops

- We only need $\delta L = \mathcal{D}_0 G \equiv \partial_0 G + i[L, G],$ for the WL to be BPS.

- Recall that though the gauge potential is not gauge covariant, the WL is gauge invariant.

Parameters for half-BPS WL

- M=N=diag(-1, 1, 1, 1)

$$\bar{\eta}_I = \bar{\eta}\delta_I^1, \quad \eta^I = \eta\delta_1^I,$$

$$\gamma_0\eta = i\eta, \quad \bar{\eta}\gamma_0 = i\bar{\eta}.$$

$$\eta\bar{\eta} = -i - \gamma_0,$$

M2-branes in $AdS_4 * S^7/\Gamma$

Set-up

- We put multi- membranes at C^4/Γ, where the discrete group Γ is generated by

$$(z_1, z_2, z_3, z_4) \to (\omega_m z_1, \omega_m z_2, z_3, z_4),$$

$$(z_1, z_2, z_3, z_4) \to (\omega_{mk} z_1, \omega_{mk} z_2, \omega_{mk} z_3, \omega_{mk} z_4),$$

$$\omega_n \equiv \exp(2i\pi/n)$$

- The near horizon limit gives M-theory on $AdS_4 * S^7/\Gamma$. This background is dual to 3d *N*=4 SCFT.

M2-brane solution

- The simplest M2 brane dual to WL has topology $AdS_2 * S^1$.
- AdS_2 part is inside AdS_4. S^1 part is inside S^7/Γ and is along the **M-theory circle**.

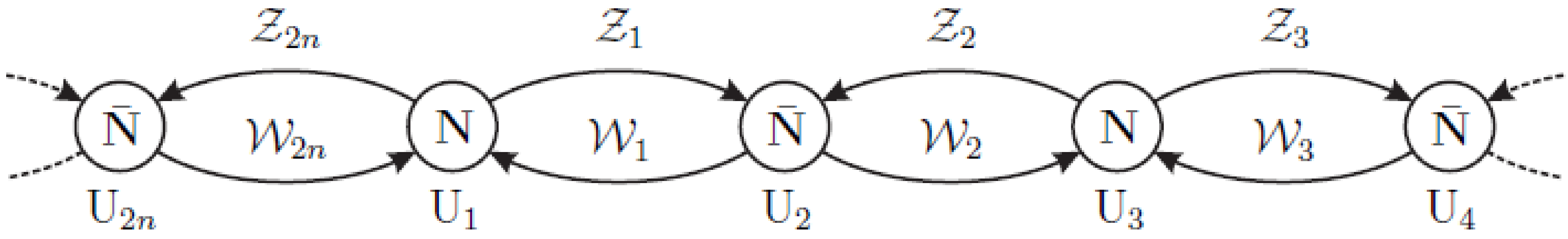
- After some computations, we found that this probe M2-brane is **half-BPS** w. r. t. the background.

- So there should be half-BPS WL in the dual field theory.

Half BPS Wilson Loops in 3d $\mathcal{N}$=4 Chern-Simons-matter theories

Dual field theory

- The conformal field theory dual to M-theory on $AdS_4 * S^7/\Gamma$ can be obtained from non-chiral orbifold of ABJM theory. The obtained theory has *N*=4 supersymmetries. *[Benna, Klebanov, Klose, Smedback, 08].*



Supersymmetry transformation

$$\delta A_\mu^{(2\ell+1)} = \frac{4\pi}{k}\left[\left(\phi_i^{(2\ell+1)}\bar{\psi}_{\hat{\imath}}^{(2\ell+1)} - \phi_{\hat{\imath}}^{(2\ell)}\bar{\psi}_i^{(2\ell)}\right)\gamma_\mu\theta^{i\hat{\imath}} + \bar{\theta}_{i\hat{\imath}}\gamma_\mu\left(\psi_{(2\ell+1)}^{\hat{\imath}}\bar{\phi}_{(2\ell+1)}^{i} - \psi_{(2\ell)}^{i}\bar{\phi}_{(2\ell)}^{\hat{\imath}}\right)\right],$$

$$\delta \hat{A}_\mu^{(2\ell)} = \frac{4\pi}{k}\left[\left(\bar{\psi}_{\hat{\imath}}^{(2\ell-1)}\phi_i^{(2\ell-1)} - \bar{\psi}_i^{(2\ell)}\phi_{\hat{\imath}}^{(2\ell)}\right)\gamma_\mu\theta^{i\hat{\imath}} + \bar{\theta}_{i\hat{\imath}}\gamma_\mu\left(\bar{\phi}_{(2\ell-1)}^{i}\psi_{(2\ell-1)}^{\hat{\imath}} - \bar{\phi}_{(2\ell)}^{\hat{\imath}}\psi_{(2\ell)}^{i}\right)\right],$$

$$\delta\phi_i^{(2\ell+1)} = 2i\bar{\theta}_{i\hat{\imath}}\psi_{(2\ell+1)}^{\hat{\imath}}, \quad \delta\phi_{\hat{\imath}}^{(2\ell)} = -2i\bar{\theta}_{i\hat{\imath}}\psi_{(2\ell)}^{i},$$

$$\delta\bar{\phi}_{(2\ell+1)}^{i} = 2i\bar{\psi}_{\hat{\imath}}^{(2\ell+1)}\theta^{i\hat{\imath}}, \quad \delta\bar{\phi}_{(2\ell)}^{\hat{\imath}} = -2i\bar{\psi}_i^{(2\ell)}\theta^{i\hat{\imath}},$$

$$(\theta^{i\hat{\imath}})^* = \bar{\theta}_{i\hat{\imath}}, \quad \bar{\theta}_{i\hat{\imath}} = \epsilon_{ij}\epsilon_{\hat{\imath}\hat{\jmath}}\theta^{j\hat{\jmath}}.$$

Supersymmetry transformation

$$
\begin{aligned}
\delta\psi^{i}_{(2\ell)} = 2\gamma^{\mu}\theta^{i\hat{i}}D_{\mu}\phi^{(2\ell)}_{\hat{i}} - \frac{4\pi}{k}\theta^{i\hat{i}}\Big(&\phi^{(2\ell)}_{\hat{i}}\bar{\phi}^{j}_{(2\ell-1)}\phi^{(2\ell-1)}_{j} + \phi^{(2\ell)}_{\hat{i}}\bar{\phi}^{\hat{j}}_{(2\ell)}\phi^{(2\ell)}_{\hat{j}} \\
&-\phi^{(2\ell+1)}_{j}\bar{\phi}^{j}_{(2\ell+1)}\phi^{(2\ell)}_{\hat{i}} - \phi^{(2\ell)}_{\hat{j}}\bar{\phi}^{\hat{j}}_{(2\ell)}\phi^{(2\ell)}_{\hat{i}}\Big) \\
- \frac{8\pi}{k}\theta^{j\hat{j}}\Big(\phi^{(2\ell+1)}_{j}\bar{\phi}^{i}_{(2\ell+1)}\phi^{(2\ell)}_{\hat{j}} - \phi^{(2\ell)}_{\hat{j}}\bar{\phi}^{i}_{(2\ell-1)}\phi^{(2\ell-1)}_{j}\Big)&,
\end{aligned}
$$

$$
\begin{aligned}
\delta\psi^{\hat{i}}_{(2\ell+1)} = -2\gamma^{\mu}\theta^{i\hat{i}}D_{\mu}\phi^{(2\ell+1)}_{i} + \frac{4\pi}{k}\theta^{i\hat{i}}\Big(&\phi^{(2\ell+1)}_{i}\bar{\phi}^{j}_{(2\ell+1)}\phi^{(2\ell+1)}_{j} + \phi^{(2\ell+1)}_{i}\bar{\phi}^{\hat{j}}_{(2\ell+2)}\phi^{(2\ell+2)}_{\hat{j}} \\
&-\phi^{(2\ell+1)}_{j}\bar{\phi}^{j}_{(2\ell+1)}\phi^{(2\ell+1)}_{i} - \phi^{(2\ell)}_{\hat{j}}\bar{\phi}^{\hat{j}}_{(2\ell)}\phi^{(2\ell+1)}_{i}\Big) \\
- \frac{8\pi}{k}\theta^{j\hat{j}}\Big(\phi^{(2\ell+1)}_{j}\bar{\phi}^{\hat{i}}_{(2\ell+2)}\phi^{(2\ell+2)}_{\hat{j}} - \phi^{(2\ell)}_{\hat{j}}\bar{\phi}^{\hat{i}}_{(2\ell)}\phi^{(2\ell+1)}_{j}\Big)&,
\end{aligned}
$$

Supersymmetry transformation

$$\delta\bar{\psi}_i^{(2\ell)} = -2\bar{\theta}_{i\hat{\imath}}\gamma^\mu D_\mu \bar{\phi}^{\hat{\imath}}_{(2\ell)} + \frac{4\pi}{k}\bar{\theta}_{i\hat{\imath}}\left(\bar{\phi}^{\hat{\imath}}_{(2\ell)}\phi_j^{(2\ell+1)}\bar{\phi}^j_{(2\ell+1)} + \bar{\phi}^{\hat{\imath}}_{(2\ell)}\phi_{\hat{\jmath}}^{(2\ell)}\bar{\phi}^{\hat{\jmath}}_{(2\ell)} \right.$$
$$\left. - \bar{\phi}^j_{(2\ell-1)}\phi_j^{(2\ell-1)}\bar{\phi}^{\hat{\imath}}_{(2\ell)} - \bar{\phi}^{\hat{\jmath}}_{(2\ell)}\phi_{\hat{\jmath}}^{(2\ell)}\bar{\phi}^{\hat{\imath}}_{(2\ell)}\right)$$
$$+ \frac{8\pi}{k}\bar{\theta}_{j\hat{\jmath}}\left(\bar{\phi}^j_{(2\ell-1)}\phi_i^{(2\ell-1)}\bar{\phi}^{\hat{\jmath}}_{(2\ell)} - \bar{\phi}^{\hat{\jmath}}_{(2\ell)}\phi_i^{(2\ell+1)}\bar{\phi}^j_{(2\ell+1)}\right),$$

$$\delta\bar{\psi}_{\hat{\imath}}^{(2\ell+1)} = 2\bar{\theta}_{i\hat{\imath}}\gamma^\mu D_\mu \bar{\phi}^i_{(2\ell+1)} - \frac{4\pi}{k}\bar{\theta}_{i\hat{\imath}}\left(\bar{\phi}^i_{(2\ell+1)}\phi_j^{(2\ell+1)}\bar{\phi}^j_{(2\ell+1)} + \bar{\phi}^i_{(2\ell+1)}\phi_{\hat{\jmath}}^{(2\ell)}\bar{\phi}^{\hat{\jmath}}_{(2\ell)} \right.$$
$$\left. - \bar{\phi}^j_{(2\ell+1)}\phi_j^{(2\ell+1)}\bar{\phi}^i_{(2\ell+1)} - \bar{\phi}^{\hat{\jmath}}_{(2\ell+2)}\phi_{\hat{\jmath}}^{(2\ell+2)}\bar{\phi}^i_{(2\ell+1)}\right)$$
$$+ \frac{8\pi}{k}\bar{\theta}_{j\hat{\jmath}}\left(\bar{\phi}^j_{(2\ell+1)}\phi_{\hat{\imath}}^{(2\ell)}\bar{\phi}^{\hat{\jmath}}_{(2\ell)} - \bar{\phi}^{\hat{\jmath}}_{(2\ell+2)}\phi_{\hat{\imath}}^{(2\ell+2)}\bar{\phi}^j_{(2\ell+1)}\right).$$

¼-BPS GY type Wilson loops

$$W^{(2\ell+1)} = \mathcal{P}\exp\left(-i\int \mathcal{A}^{(2\ell+1)}\right),$$

$$\mathcal{A}^{(2\ell+1)} = A_0^{(2\ell+1)} + \frac{2\pi}{k}\left(M^i{}_j\phi_i^{(2\ell+1)}\bar{\phi}^j_{(2\ell+1)} + M^{\hat{i}}{}_{\hat{j}}\phi_{\hat{i}}^{(2\ell)}\bar{\phi}^{\hat{j}}_{(2\ell)}\right).$$

$$\hat{W}^{(2\ell)} = \mathcal{P}\exp\left(-i\int \hat{\mathcal{A}}^{(2\ell)}\right),$$

$$\hat{\mathcal{A}}^{(2\ell)} = \hat{A}_0^{(2\ell)} + \frac{2\pi}{k}\left(N_i{}^j\bar{\phi}^i_{(2\ell-1)}\phi_j^{(2\ell-1)} + N_{\hat{i}}{}^{\hat{j}}\bar{\phi}^{\hat{i}}_{(2\ell)}\phi_{\hat{j}}^{(2\ell)}\right),$$

- All the matrices M and N are diag(-1, 1).

Half-BPS DT type Wilson loops

$$W = \mathcal{P} e^{-i \int L}, \qquad L = \begin{pmatrix} \mathcal{A} & F_1 \\ \bar{F}_2 & \hat{\mathcal{A}} \end{pmatrix}.$$

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}^{(1)} & & & \\ & \mathcal{A}^{(3)} & & \\ & & \ddots & \\ & & & \mathcal{A}^{(2n-1)} \end{pmatrix},$$

$$\mathcal{A}^{(2\ell+1)} = A_0^{(2\ell+1)} + \frac{2\pi}{k} \left(M^i{}_j \phi_i^{(2\ell+1)} \bar{\phi}^j_{(2\ell+1)} + M^{\hat{i}}{}_{\hat{j}} \phi_{\hat{i}}^{(2\ell)} \bar{\phi}^{\hat{j}}_{(2\ell)} \right),$$

Half-BPS WL

$$\hat{\mathcal{A}} = \begin{pmatrix} \hat{\mathcal{A}}^{(0)} & & & \\ & \hat{\mathcal{A}}^{(2)} & & \\ & & \ddots & \\ & & & \hat{\mathcal{A}}^{(2n-2)} \end{pmatrix},$$

$$\hat{\mathcal{A}}^{(2\ell)} = \hat{A}_0^{(2\ell)} + \frac{2\pi}{k}\left(N_i{}^j \bar{\phi}^i_{(2\ell-1)} \phi_j^{(2\ell-1)} + N_{\hat{\imath}}{}^{\hat{\jmath}} \bar{\phi}^{\hat{\imath}}_{(2\ell)} \phi_{\hat{\jmath}}^{(2\ell)} \right),$$

Half-BPS WL

$$F_1 = \begin{pmatrix} f_1^{(0)} & f_1^{(1)} & & & & \\ & f_1^{(2)} & f_1^{(3)} & & & \\ & & \ddots & \ddots & & \\ & & & f_1^{(2n-4)} & f_1^{(2n-3)} \\ f_1^{(2n-1)} & & & & f_1^{(2n-2)} \end{pmatrix},$$

$$f_1^{(2\ell+1)} = \sqrt{\frac{2\pi}{k}}\bar{\eta}_{\hat{\imath}}^{(2\ell+1)}\psi^{\hat{\imath}}_{(2\ell+1)}, \quad f_1^{(2\ell)} = \sqrt{\frac{2\pi}{k}}\bar{\eta}_i^{(2\ell)}\psi^i_{(2\ell)},$$

Half-BPS WL

$$\bar{F}_2 = \begin{pmatrix} \bar{f}_2^{(0)} & & & & \bar{f}_2^{(2n-1)} \\ \bar{f}_2^{(1)} & \bar{f}_2^{(2)} & & & \\ & \bar{f}_2^{(3)} & \ddots & & \\ & & \ddots & \bar{f}_2^{(2n-4)} & \\ & & & \bar{f}_2^{(2n-3)} & \bar{f}_2^{(2n-2)} \end{pmatrix},$$

$$\bar{f}_2^{(2\ell+1)} = \sqrt{\frac{2\pi}{k}}\bar{\psi}_{\hat{i}}^{(2\ell+1)}\eta^{\hat{i}}_{(2\ell+1)}, \quad \bar{f}_2^{(2\ell)} = \sqrt{\frac{2\pi}{k}}\bar{\psi}_{i}^{(2\ell)}\eta^{i}_{(2\ell)},$$

Half-BPS WL

$$\bar{\eta}_i = \bar{\eta}\delta_i^1, \quad \eta^i = \eta\delta_1^i, \quad \bar{\eta}_{\hat{\imath}} = \eta^{\hat{\imath}} = 0$$

$$M^i{}_j = \begin{pmatrix} m_1 & \\ & m_2 \end{pmatrix}, \quad M^{\hat{\imath}}{}_{\hat{\jmath}} = \begin{pmatrix} m_{\hat{1}} & \\ & m_{\hat{1}} \end{pmatrix},$$

$$N_i{}^j = \begin{pmatrix} n_1 & \\ & n_2 \end{pmatrix}, \quad N_{\hat{\imath}}{}^{\hat{\jmath}} = \begin{pmatrix} n_{\hat{1}} & \\ & n_{\hat{1}} \end{pmatrix}.$$

$$m_1 = n_1 = -1 \qquad m_2 = m_{\hat{\imath}} = n_2 = n_{\hat{\imath}} = 1,$$

$$\gamma_0 \eta_{(2\ell)} = i\eta_{(2\ell)}, \quad \bar{\eta}^{(2\ell)}\gamma_0 = i\bar{\eta}^{(2\ell)}. \qquad \eta_{(2l)}\bar{\eta}^{(2l)} = -iI_{2\times 2} - \gamma_0$$

Side Remark on a subtle point

- All of these BPS Wilson loops in Minkowski spacetime are **time-like**.
- There are no space-like BPS Wilson loops due to some real conditions.
- Anybody wrote down this claim clearly?
- (Same results found in von Dennis Muller's M. Sc. Thesis, but he did not give the clear statement.)
- In Euclidean case, we can relax the real condition and can define the BPS WL.

- Put the theory on S^3 and perform the supersymmetry localization.
- Check if the half-BPS DT-type WL is in the same Q-cohomology as special ¼-BPS GY-type WL. (Q is the supercharge used for localization).

- If so, compute the vev of half-BPS WL using localization.

What will we do? – continued

- There are many other 3d $\mathcal{N}$=4 Chern-Simons-matter theories.

[Gaiotto, Witten, 08] [Hosomichi, Lee, Lee, Lee, Park, 08]

[Fa-Min Chen, Yong-Shi Wu, 12]

- Are there DT type WL in these theories?
- How about the complete story in $\mathcal{N}$=3 case.

Thanks for your time!