

Intersecting Branes & Closed String Tachyon Condensation

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Outline

- Motivation
- The Intersecting Branes
- The Closed String Tachyon Condensation
- Some Related Issues
- Summary

The final state of collapsing matter in 5-d KK Theory?

- In 80's, **Gross & Perry** (NPB 226(1983)29) observed:

$$\begin{aligned}
 ds^2 = & - \left(\frac{1 - m/r}{1 + m/r} \right)^{2/\alpha} dt^2 + \left(\frac{1 - m/r}{1 + m/r} \right)^{2\beta/\alpha} dx_5^2 \\
 & + \left(1 + \frac{m}{r} \right)^4 \left(\frac{1 - m/r}{1 + m/r} \right)^{2(\alpha - \beta - 1)/\alpha} (dr^2 + r^2 d\Omega_2^2),
 \end{aligned} \tag{1.1}$$

with $\alpha = \sqrt{\beta^2 + \beta + 1}$

- This solution (defined for $r \geq m$ in general) interpolates between the singular Schwarzschild metric ($\beta = 0, \alpha = 1; r \geq 0$) and a regular soliton ($\alpha = \beta = \infty; r \geq m$), a variant of Witten's "bubble of nothing". In between, the configuration has a naked singularity at $r = m$, interpreted as describing the exterior geometry of a massive object in the KK theory.

The final state of collapsing matter in 5-d KK Theory?

- The non-uniqueness of the exterior geometry of a massive object in 5-d KK theory raises an interesting question as to the final state of collapsing matter: a black hole described by the Schwarzschild metric or a soliton described by a totally non-singular metric?

Implications of similar interpolation in type II string theories?

- A similar family of solutions exist also in low energy string theories, namely, the charged non-susy p -brane ($1 \leq p \leq 6$) intersecting with chargeless non-susy 1-brane and 0-brane of type II string theories. They interpolate between black p -branes and the non-singular static KK "bubble of nothing". Again in between, the configuration has a naked singularity.
- What is the implication or/and a physical interpretation of this interpolation?

Two puzzles or questions:

- What is the possible final state of collapsing matter (or branes)?
- What causes the transition from black hole (branes) to the static KK "bubble of nothing"?

Partial Answers

In a separate development, [Horowitz & Silverstein](#) ([PRD73\(2006\)064016](#)) provides answer to the question of non-uniqueness of collapsing matter but doesn't provide answer to the existence of the interpolation or topological transition, based on their previous work ([Adams et al JHEP0510\(2005\)033](#), [Horowitz JHEP 0508 \(2005\)091](#)) of closed string tachyon condensation causing a topological transition.

Partial Answers

A black p -brane with one brane direction (say x_1) compactified as a circle

$$\begin{aligned}
 i s d s^2 &= \bar{H}^{\frac{1}{2}} \left(\frac{d\rho^2}{f} + \rho^2 d\Omega_{8-p}^2 \right) + \bar{H}^{-\frac{1}{2}} \left(-f dt^2 + dx_1^2 + \sum_{i=2}^p (dx^i)^2 \right) \\
 e^{2\phi} &= \bar{H}^{\frac{3-p}{2}}, \quad F_{[8-p]} = b \text{Vol}(\Omega_{8-p})
 \end{aligned} \tag{1.2}$$

where $\bar{H} = 1 + \sinh^2 \theta \rho_0^{7-p} / \rho^{7-p}$ and $f = 1 - \rho_0^{7-p} / \rho^{7-p}$. It has a topology:

$$R^{1,p-1} \times S^1 \times R_+ \times S^{8-p}. \tag{1.3}$$

The radius of the circle is dependent of ρ as

$$L(\rho) = \bar{H}^{-1/4}(\rho) L_\infty. \tag{1.4}$$

Charged Dp intersecting with chargeless D(p - 1) up to D0

$$\begin{aligned}
 ds^2 &= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}} \right)^{\frac{p\delta_1}{8} + \frac{(3-p)\sum_{i=1}^p \delta_{i+1}}{2(7-p)}} (dr^2 + r^2 d\Omega_{8-p}^2) \\
 &+ F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}} \right)^{\frac{p\delta_1}{8} + \sum_{i=1}^p \frac{\delta_{i+1}}{2}} (-dt^2) \\
 &+ F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}} \right)^{\frac{(p-8)\delta_1}{8} + \sum_{j=1}^p \frac{\delta_{j+1}}{2}} \sum_{i=1}^p \left(\frac{H}{\tilde{H}} \right)^{-2\delta_{i+1}} (dx^i)^2 \\
 e^{2\phi} &= F^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}} \right)^{\frac{\delta_1}{2}(4-p) - \sum_{i=1}^p 2\delta_{i+1}} \\
 F_{[8-p]} &= b\text{Vol}(\Omega_{8-p})
 \end{aligned} \tag{2.1}$$

Charged D_p intersecting with chargeless $D(p-1)$ up to D_0

$$\begin{aligned}
 F &= \left(\frac{H}{\tilde{H}}\right)^\alpha \cosh^2 \theta - \left(\frac{\tilde{H}}{H}\right)^\beta \sinh^2 \theta \\
 H &= 1 + \frac{\omega^{7-p}}{r^{7-p}}, \quad \tilde{H} = 1 - \frac{\omega^{7-p}}{r^{7-p}}
 \end{aligned} \tag{2.2}$$

where $\alpha, \beta, \theta, \delta_1, \delta_2, \dots, \delta_{p+1}$ and ω are $(p+5)$ integration constants and b is the charge parameter.

$$\begin{aligned}
 \alpha - \beta &= -\frac{3}{2}\delta_1 \\
 \frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha + \frac{3}{2}\delta_1) + \frac{2\sum_{i>j=2}^{p+1} \delta_i \delta_j}{7-p} &= \left(1 - \sum_{i=2}^{p+1} \delta_i^2\right) \frac{8-p}{7-p} \\
 b &= (7-p)\omega^{7-p}(\alpha + \beta) \sinh 2\theta
 \end{aligned} \tag{2.3}$$

$(p+3)$ independent parameters!

Charged D_p intersecting with chargeless $D(p - 1)$ up to D_0

The above configuration represents **charged non-susy p -branes intersecting chargeless non-susy $(p - 1)$ -brane, $(p - 2)$ -brane, \dots , upto 0 -brane**. For the purpose of this talk, we need to consider only **charged non-susy p -brane intersecting with chargeless non-susy 1 -brane and 0 -brane**. This can be achieved by putting $\delta_3 = \delta_4 = \dots = \delta_{p+1} = \delta_0$ in the above and now the only anisotropic directions are x^1 and t .

Charged p-brane intersecting chargeless 1-brane and 0-brane

The configuration is now

$$\begin{aligned}
 ds^2 &= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}} \right)^{\frac{p\delta_1}{8} + \frac{2(3-p)\bar{\delta}}{(7-p)}} (dr^2 + r^2 d\Omega_{8-p}^2) \\
 &+ F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}} \right)^{\frac{p\delta_1}{8} + 2\bar{\delta}} (-dt^2) + F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}} \right)^{\frac{(p-8)\delta_1}{8} + 2\bar{\delta} - 2\delta_2} (dx^1)^2 \\
 &+ F^{-\frac{7-p}{8}} \left(\frac{H}{\tilde{H}} \right)^{\frac{(p-8)\delta_1}{8} + 2\bar{\delta} - 2\delta_0} \sum_{i=2}^p (dx^i)^2 \\
 e^{2\phi} &= F^{\frac{3-p}{2}} \left(\frac{H}{\tilde{H}} \right)^{\frac{\delta_1}{2}(4-p) - 8\bar{\delta}} \\
 F_{[8-p]} &= b \text{Vol}(\Omega_{8-p})
 \end{aligned} \tag{2.4}$$

Charged p-brane intersecting chargeless 1-brane and 0-brane

In the above, we define

$$\bar{\delta} = \frac{1}{4} \sum_{i=1}^p \delta_{i+1} = \frac{1}{4} \delta_2 + \frac{(p-1)}{4} \delta_0 \quad (2.5)$$

and the previous parameter relations are simplified to

$$\begin{aligned} \alpha - \beta &= -\frac{3}{2} \delta_1 \\ \frac{1}{2} \delta_1^2 + \frac{1}{2} \alpha (\alpha + \frac{3}{2} \delta_1) + \frac{(p-1)[2\delta_2 + (p-2)\delta_0] \delta_0}{7-p} \\ &= (1 - \delta_2^2 - (p-1)\delta_0^2) \frac{8-p}{7-p} \\ b &= (7-p) \omega^{7-p} (\alpha + \beta) \sinh 2\theta \end{aligned} \quad (2.6)$$

Charged p -brane intersecting chargeless 1-brane and 0-brane

Note for $p = 1$, the configuration has 4 parameters ($\delta_1, \delta_2, \omega$ and θ) while for $1 < p \leq 6$, the configuration has 5 parameters ($\delta_0, \delta_1, \delta_2, \omega$ and θ).

Charged p-brane intersecting chargeless 1-brane and 0-brane

Passing r to ρ as $r = \rho \left(\frac{1+\sqrt{f}}{2} \right)^{\frac{2}{7-p}}$ with

$$f = 1 - \frac{4\omega^{7-p}}{\rho^{7-p}} \equiv 1 - \frac{\rho_0^{7-p}}{\rho^{7-p}},$$

$$\begin{aligned}
 ds^2 = & G^{\frac{p+1}{8}} f^{-\frac{\alpha(p+1)}{16} - \frac{p\delta_1}{16} - \frac{(3-p)\bar{\delta}}{(7-p)} + \frac{1}{7-p}} \left(\frac{d\rho^2}{f} + \rho^2 d\Omega_{8-p}^2 \right) \\
 & + G^{-\frac{7-p}{8}} f^{\frac{\alpha(7-p)}{16} - \frac{p\delta_1}{16} - \bar{\delta}} (-dt^2) + G^{-\frac{7-p}{8}} f^{\frac{\alpha(7-p)}{16} - \frac{(p-8)\delta_1}{16} - \bar{\delta} + \delta_2} (dx^1)^2 \\
 & + G^{-\frac{7-p}{8}} f^{\frac{\alpha(7-p)}{16} - \frac{(p-8)\delta_1}{16} - \bar{\delta} + \delta_0} \sum_{i=2}^p (dx^i)^2 \\
 e^{2\phi} = & G^{\frac{3-p}{2}} f^{-\frac{\alpha(3-p)}{4} - \frac{\delta_1}{4}(4-p) + 4\bar{\delta}} \\
 F_{[8-p]} = & b \text{Vol}(\Omega_{8-p})
 \end{aligned} \tag{2.7}$$

$$G(\rho) = \cosh^2 \theta - f^{\frac{\alpha+\beta}{2}} \sinh^2 \theta.$$

Interpolation between black p-brane and KK bubble

When

$$\delta_1 = -\frac{12}{7}, \quad \delta_0 = \delta_2 = -\frac{1}{7} \quad (2.8)$$

implying also

$$\alpha = \frac{16}{7}, \quad \beta = -\frac{2}{7}, \quad \text{and} \quad \bar{\delta} = -\frac{p}{28} \quad (2.9)$$

the above becomes in string frame as

$$\begin{aligned} ds^2 &= \bar{H}^{\frac{1}{2}} \left(\frac{d\rho^2}{f} + \rho^2 d\Omega_{8-p}^2 \right) + \bar{H}^{-\frac{1}{2}} \left(-f dt^2 + \sum_{i=1}^p (dx^i)^2 \right) \\ e^{2\phi} &= \bar{H}^{\frac{3-p}{2}}, \quad F_{[8-p]} = b \text{Vol}(\Omega_{8-p}) \end{aligned} \quad (2.10)$$

where $\bar{H} = 1 + \sinh^2 \theta \rho_0^{7-p} / \rho^{7-p}$ and $f = 1 - \rho_0^{7-p} / \rho^{7-p}$.**The standard black Dp-brane solutions!**

Interpolation between black p-brane and KK bubble

While

$$\delta_0 = -\frac{2}{7}, \quad \delta_1 = \frac{4}{7} \quad \text{and} \quad \delta_2 = \frac{5}{7} \quad (2.11)$$

implying

$$\alpha = \frac{4}{7}, \quad \beta = \frac{10}{7} \quad \text{and} \quad \bar{\delta} = -\frac{2p-7}{28} \quad (2.12)$$

the above now reduces to,

$$ds^2 = \bar{H}^{\frac{1}{2}} \left(\frac{d\rho^2}{f} + \rho^2 d\Omega_{8-p}^2 \right) + \bar{H}^{-\frac{1}{2}} \left(-dt^2 + f(dx^1)^2 + \sum_{i=2}^p (dx^i)^2 \right)$$

$$e^{2\phi} = \bar{H}^{\frac{3-p}{2}}, \quad F_{[8-p]} = b \text{Vol}(\Omega_{8-p}). \quad (2.13)$$

The KK bubble solutions!

Interpolation between black p-brane and KK bubble

- The above clearly indicates that the intersecting brane configuration continuously interpolates between the black p-branes and the static KK bubbles through a 2-parameter (for $p = 1$) or 3-parameter (for $p > 1$) family of solutions.
- Both the black p-branes and the KK bubble can be well described by the corresponding classical geometry but in between, the configuration has a naked singularity at $\rho = \rho_0$ as in Gross & Perry case. So it can be viewed to either represent long-distance behavior of violent quantum gravity region in the closed tachyon condensation phase or to represent the exterior geometry of a spherical shell of branes.

Interpolation between black p-brane and KK bubble

- The existence of this interpolation indicates a possibility that a black p-brane can decay into a static KK bubble, a topology change transition.

Remark: the KK bubble can also be obtained from the black p-brane through the so-called double Wick rotations but here we have it through an interpolation.

Closed string tachyon condensation

- The above possibility can become a reality when certain conditions are satisfied.
- Closed string tachyons are known to develop when the fermions in the theory satisfy anti-periodic boundary condition around a circle and when the size of this circle reaches string scale.
- **Adams et al** ([JHEP0510\(2005\)033](#)) argued that these localized closed string tachyons can trigger a topology changing transition, causing certain region of spacetime to pinch off, as a consequence of closed string tachyon condensation.
- **Horowitz** ([JHEP 0508 \(2005\)091](#)) made use of this and argued that black p-branes in type II string theories have a dramatic endpoints to Hawking radiation in the form of KK bubbles through closed string tachyon condensation.

Closed string tachyon condensation

- For black p -branes, the size of the string wound along the compact circle (x^1) varies monotonically from the circle period L to zero as we move along the radial direction from infinity to the singularity point, due to the harmonic function appearing in the metric. So at some point in between, the size of circle becomes of the order of string scale and the closed string tachyon develops. This can occur either outside or inside the horizon.
- In order to make this transition to occur, the curvature when the circle reaches the string scale must remain small, otherwise **Horowitz-Polchinski correspondence point** is reached and the black branes make transition to quantum string states, not to our intended classical KK bubbles. So a semi-classical consideration is adequate here.

Closed string tachyon condensation

Two cases:

(1) Closed string tachyon condensation occurs outside (on) the horizon.

(2) Closed string tachyon condensation occurs inside the horizon.

Tachyon condensation on the horizon

Free of a conical singularity at $\rho = \rho_0$ for the bubble (2.13),

$$L_b = (4\pi\rho_0 \cosh \theta)/(7 - p). \quad (3.1)$$

while for a black p-brane (2.10), the tachyon condensation occurring on the horizon gives $L\bar{H}^{-1/4} = L \cosh^{-1/2} \theta = l_s$, therefore the size of the compact circle x^1 as

$$L = l_s \cosh^{1/2} \theta. \quad (3.2)$$

Note $L \gg l_s$ implies large θ .

Tachyon condensation on the horizon

The black Dp-brane has RR charge

$$Q \sim \rho_0^{7-p} \sinh 2\theta \quad (3.3)$$

and its horizon size

$$Z = \rho_0 \sinh^{\frac{1}{2}} \theta \quad (3.4)$$

while for the static bubble, its charge

$$Q_b \sim \rho_0^{7-p} \sinh 2\theta. \quad (3.5)$$

and size

$$Z_b = \rho_0 \sinh^{\frac{1}{2}} \theta \quad (3.6)$$

Tachyon condensation on the horizon

The transition from the black D p -brane to the bubble requires $Q = Q_b$ and $Z = Z_b$ in addition to $L = L_b$. This implies both θ and ρ_0 must be the same for the black D p and the bubble except for $p = 3$ case for which they can be different. For large θ , we have $\sinh \theta \approx \cosh \theta \approx e^\theta$ and $L = L_b$ gives $\rho_0 e^{\theta/2} = l_s$, implying that the curvature at the horizon becomes of the order of string scale, therefore reaching the Horowitz-Polchinski correspondence point where supergravity description breaks down and the black D p branes will transform to excited string states, not to the static bubble. In other words, if such a transition occurs at all, the final bubble cannot be our static bubble as argued by Horowitz but a dynamical one which expands out.

Tachyon condensation on the horizon

For $p = 3$ case, the ρ_0 and θ need not to be same in both cases and we now denote them for the bubble with a 'tilde'. The conservation of Z and Q gives $\rho_0 e^{\theta/2} = \tilde{\rho}_0 e^{\tilde{\theta}/2}$ and combing this with the following

$$l_s e^{\theta/2} \sim \tilde{\rho}_0 e^{\tilde{\theta}} \quad (3.7)$$

obtained from equating L and L_b , we have

$$\rho_0 e^{\theta/2} = l_s \left(\frac{e^\theta}{e^{\tilde{\theta}}} \right)^{1/2} \quad (3.8)$$

which implies

$$l_s^2 R \ll 1 \quad (3.9)$$

if $\theta \gg \tilde{\theta}$.

Tachyon condensation on the horizon

Therefore, for $p = 3$ case, we can still trust supergravity description and the black D3 brane can decay into the static bubble through the closed string tachyon condensation outside (on) the horizon.

If the above is the whole story, one would be very surprised about the existence of interpolation between black D_p and the static KK bubble. For this, we turn to the case (2).

Tachyon condensation inside the horizon

Assume now the tachyon condensation occurs at some $\rho_i < \rho_0$, i.e.,

$$\bar{H}^{-1/4}(\rho_i)L = l_s. \quad (3.10)$$

But need also $l_s^2 R \ll 1$ at this point. $L \gg l_s$ says
 $\bar{H}(\rho_i) = 1 + \rho_0^{7-p} \sinh^2 \theta / \rho_i^{7-p} \equiv 1 + l^{7-p} / \rho_i^{7-p} \gg 1 \Rightarrow$

$$\frac{l}{\rho_i} = \left(\frac{L}{l_s} \right)^{4/(7-p)} \quad (3.11)$$

$L \sim \rho_0 \cosh \theta$ and $l = \rho_0 \sinh^{2/(7-p)} \theta \Rightarrow l < L$, a constraint except for $p = 6$ case which will be excluded from the other constraints. Small Ricci curvature at $\rho = \rho_i$ requires

$$l_s^2 R \sim \frac{l_s^2}{\rho_i^2 \bar{H}^{1/2}(\rho_i)} \ll 1, \quad (3.12)$$

when combined with (3.11) gives

Tachyon condensation inside the horizon

$$\left(\frac{L}{l_s}\right)^{(p-3)/(7-p)} \ll \frac{l}{l_s}. \quad (3.13)$$

Considering $l_s \ll l < L$, the above can hold good only for $p \leq 4$. Further the Weyl curvature could be large inside the horizon and we need also

$$l_s^2 W \sim \frac{l_s^2}{l^2 \left(\frac{l}{\rho_i}\right)^{(3-p)/2}} \left(\frac{\rho_0}{\rho_i}\right)^{7-p} \ll 1, \quad (3.14)$$

when combined with (3.11) gives

$$\frac{\rho_0}{l_s} \ll a^{\frac{2}{7-p} + \frac{(3-p)(9-p)}{(7-p)^2}} \left(\frac{l}{L}\right)^{2(11-p)/(7-p)^2} \quad (3.15)$$

where $l = al_s$ for some large number $a \gg 1$.

Tachyon condensation inside the horizon

It can also hold good for $p \leq 4$ even if we keep $l/L < 1$ but finite and for large enough l . Note that we can still have $\rho_0 \gg l_s$ while keeping $l/\rho_0 \gg 1$, which implies large θ .

A consistent check: need to ensure $\rho_i < \rho_0$. Using (3.11),

$$\begin{aligned} \frac{\rho_i}{l_s} &= a \left(\frac{l_s}{L} \right)^{4/(7-p)} = a^{\frac{3-p}{7-p}} \left(\frac{l}{L} \right)^{4/(7-p)} \\ &< \frac{\rho_0}{l_s} \ll a^{\frac{2}{7-p} + \frac{(3-p)(9-p)}{(7-p)^2}} \left(\frac{l}{L} \right)^{2(11-p)/(7-p)^2} \Rightarrow \end{aligned} \quad (3.16)$$

$$1 \ll a^{\frac{4(5-p)}{(7-p)^2}} \left(\frac{l}{L} \right)^{2(p-3)/(7-p)^2} \quad (3.17)$$

which again holds good for $p \leq 4$ and for $a \gg 1$.

Tachyon condensation inside the horizon

- We have shown the closed string tachyon condensation can occur inside the horizon of black D_p branes where the curvatures can remain smaller than the string scale, causing the black branes to decay into static bubbles for $p \leq 4$.
- When the tachyon condenses, one first produces a non-static bubble which expands out and eventually settles down to the static bubble plus radiation as discussed by [Horowitz and Silverstein](#) (PRD73(2006)064016).

F-string responsible for the tachyon mode

Where is the F-string in our setup which when winds around the periodic x^1 gives rise to tachyon mode when the size reaches the string scale outside or inside the horizon?

There are many ways to see how such F-strings can appear. For example, the original non-isotropic solutions contain chargeless $(p - 1)$ -brane, $(p - 2)$ -brane, upto 0-brane, which are either the brane-antibrane or non-BPS brane in the theory. The process of making the non-isotropic directions x^p, x^{p-1}, \dots, x^2 isotropic is actually the process of elimination of these chargeless branes from the solutions. This elimination can also be viewed as the end of annihilation process for these branes and will give rise to F-strings non-perturbatively based on previous work ([Yi NPB550\(1999\)214](#), [Bergman et al NPB580\(2000\)289](#), [Lu&Roy PLB637\(2006\)013](#)). In addition, the chargeless part of non-susy Dp as well as part of the chargeless D-strings in this configuration can also give rise to F-strings when they annihilate.

F-string responsible for the tachyon mode

The chargeless non-susy F-strings intersecting with chargeless non-susy Dp-branes are described by

$$\begin{aligned}
 ds^2 &= (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\bar{\alpha}}{4} - \frac{2(p-1)\gamma_0}{(7-p)}} (dr^2 + r^2 d\Omega_{8-p}^2) \\
 &\quad + \left(\frac{H}{\tilde{H}}\right)^{-\frac{3\bar{\alpha}}{4}} (-dt^2 + (dx^1)^2) + \left(\frac{H}{\tilde{H}}\right)^{\frac{\bar{\alpha}}{4} + 2\gamma_0} \sum_{i=2}^p (dx^i)^2 \\
 e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-\bar{\alpha} + 2\gamma_1}, \tag{4.1}
 \end{aligned}$$

with the parameter relation

$$\frac{1}{2}\gamma_1^2 + \frac{1}{2}\bar{\alpha}(\bar{\alpha} - \gamma_1) + \frac{(p-1)(p-2)\gamma_0^2}{(7-p)} = (1 - (p-1)\gamma_0^2) \frac{8-p}{7-p} \tag{4.2}$$

F-string responsible for the tachyon mode

To show the presence of F-strings in our system, we need to connect our system to the above which amounts to set vanish the Dp charge and to remove the presence of D0 branes. This can be achieved by setting $\theta = 0$ and $\delta_2 = -\delta_1/2$ in (2.4)-(2.6). Then we end up with

$$\begin{aligned}
 ds^2 &= (H\tilde{H})^{\frac{2}{7-p}} \left(\frac{H}{\tilde{H}}\right)^{\alpha\frac{p+1}{8} + \frac{p\delta_1}{8} + \frac{2(3-p)\bar{\delta}}{(7-p)}} (dr^2 + r^2 d\Omega_{8-p}^2) \\
 &+ \left(\frac{H}{\tilde{H}}\right)^{-\alpha\frac{7-p}{8} + \frac{p\delta_1}{8} + 2\bar{\delta}} (-dt^2 + (dx^1)^2) \\
 &+ \left(\frac{H}{\tilde{H}}\right)^{-\alpha\frac{7-p}{8} \frac{(p-8)\delta_1}{8} + 2\bar{\delta} - 2\delta_0} \sum_{i=2}^p (dx^i)^2 \\
 e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{\alpha\frac{3-p}{2} + \frac{\delta_1}{2}(4-p) - 8\bar{\delta}}
 \end{aligned} \tag{4.3}$$

F-string responsible for the tachyon mode

with the parameter relation

$$\begin{aligned}
 & \frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha + \frac{3}{2}\delta_1) + \frac{(p-1)[- \delta_1 + (p-2)\delta_0]\delta_0}{7-p} \\
 &= (1 - \delta_1^2/4 - (p-1)\delta_0^2) \frac{8-p}{7-p} \tag{4.4}
 \end{aligned}$$

F-string responsible for the tachyon mode

One can show indeed that the above two can be identified with the following parameter relations:

$$\begin{aligned}
 \bar{\alpha} &= -\frac{2(p-1)\delta_0}{3} + \frac{(2-p)\delta_1}{6} + \frac{7-p}{6}\alpha \\
 \gamma_0 &= \frac{p-4}{3}\delta_0 + \frac{p-8}{12}\delta_1 - \frac{7-p}{12}\alpha \\
 \gamma_1 &= -\frac{4(p-1)}{3}\delta_0 + \frac{5-p}{3}\delta_1 + \frac{4-p}{3}\alpha.
 \end{aligned} \tag{4.5}$$

Therefore showing the existence of F-strings along x^1 , responsible for the presence of tachyon mode when the circle reaches string scale.

Summary

- We have shown that a 4-parameter (for $p = 1$) or 5-parameter (for $p > 1$) family of configurations, representing the charged non-susy charged D p branes intersecting with chargeless non-susy 1-branes and 0-branes, interpolates between black p -branes and the corresponding static KK "bubble of nothing" by continuously varying two ($p = 1$) or three ($p > 1$) parameters characterizing the configuration.
- Such an interpolation can be understood as a topological transition from the black p -brane to the bubble through the closed string tachyon condensation for $p \leq 4$.
- The wound F-string around the compact circle, responsible for the tachyon condensation leading to the transition to occur, is shown to exist in the configuration.

Thank you!