Role of the three-body/left-hand cut on the pole extraction of the $T_{cc}(3875)$

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Based on PRD **105**, 014024 (2022) and PRL **131**, 131903 (2023)

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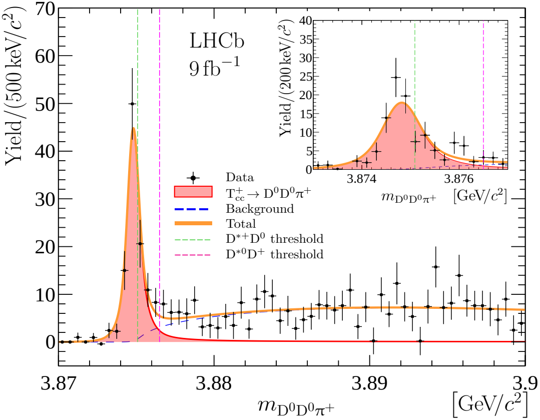
Three-body cut: the physical world!

Doubly charmed tetraquark T_{cc}^{+} $(cc\bar{u}\bar{d})$

Breit-Wigner fit

LHCb, Nature Phys. 18, (2022) 751



Parameter	Value
N	117 ± 16
δm_{BW}	-273 ± 61 keV
Γ_{BW}	410 ± 165 keV

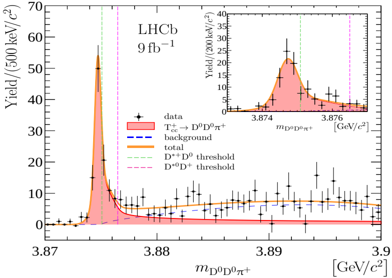
$\delta m = m_{T_{cc}^{+}} - m_{D^{*+}} - m_{D^0}$

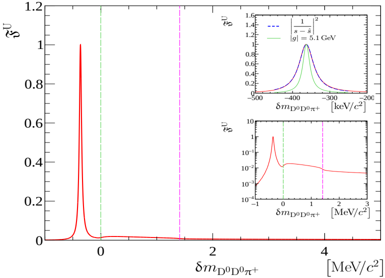
- significance $> 10\sigma$
- $\delta m_{\mathrm{BW}} < 0$ (4.3σ)

☞ $\Re \sim 400$ keV.

Doubly charmed tetraquark T_{cc}^{+}

Unitarized and analytical model fit to the spectrum





LHCb, Nature Commun. 13 (2022), 3351

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

☞ $I = 0$: isoscalar

$\hookrightarrow D^+ D^0 \pi^+,\ D^+ D^+$ ✗

☞ Weinberg compositeness:

$$1 - Z = \sqrt{\frac{1}{1 + 2|r/\Re a|}}$$

$$a = \left[-(7.16 \pm 0.51) + i(1.85 \pm 0.28)\right] \text{ fm},$$

$$-r < 11.9(16.9) \text{ fm at } 90\ (95)\% \text{ CL},$$

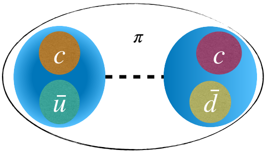
$$Z < 0.52(0.58) \text{ at } 90\ (95)\% \text{ CL}.$$

$Z = 0$: composite,

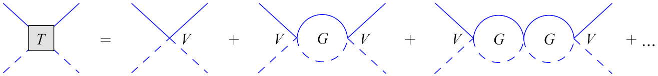
$Z = 1$: elementary.

☞ Consistent with molecule

T_{cc}^{+} as a hadronic molecule



☞ T_{cc}^{+} resides near D^*D thresholds LHCb, Nature Commun. 13 (2022)
$\hookrightarrow$ approximate 90% of $D^0D^0\pi^+$ events contain a D^{*+}.

☞ 

☞ D^*D isoscalar ($I = 0$) and isovector ($I = 1$)

$$|D^*D, I = 0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$

$$|D^*D, I = 1\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+),$$

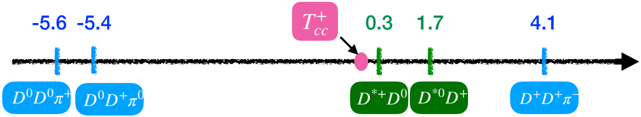
$$V_{\mathrm{CT}}^{I=0}(D^*D \to D^*D; J^P = 1^+) = v_0,$$
$$V_{\mathrm{CT}}^{I=1}(D^*D \to D^*D; J^P = 1^+) = v_1.$$

☞ In the particle basis $\{D^{*+}D^0, D^{*0}D^+\}$

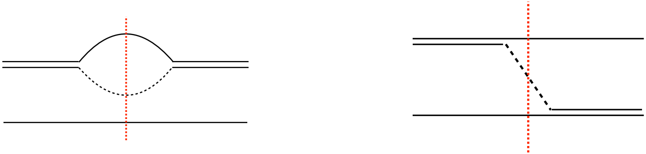
$$V_{\mathrm{CT}}^{I=0}[D^*D, 1^+] = \frac{1}{2}\begin{pmatrix} v_0 & -v_0 \\ -v_0 & v_0 \end{pmatrix}$$

Including three-body cuts

☞ A coupled-channel analysis using an EFT approach



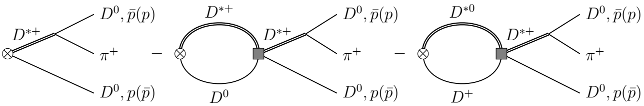
☞ Three-body cuts



☞ LO Chiral Lagrangian (g determined from $D^* \to D\pi$)

$$\mathcal{L} = \frac{1}{4} g \,\mathrm{Tr}\left(\vec{\sigma}\cdot\vec{u}_{ab} H_b H_a^\dagger\right)$$

$D^0 D^0 \pi^+$ mass distribution



☞ $$U_\alpha(M,p) = P_\alpha - \sum_\beta \int \frac{d^3\vec{q}}{(2\pi)^3} V_{\alpha\beta}(M,p,q) G_\beta(M,q) U_\beta(M,q)$$

$$\hookrightarrow \quad G_\alpha(M,p) = \frac{1}{m_\alpha^* + m_\alpha + \frac{p^2}{2\mu_\alpha} - M - \frac{i}{2}\Gamma_\alpha(M,p)}$$

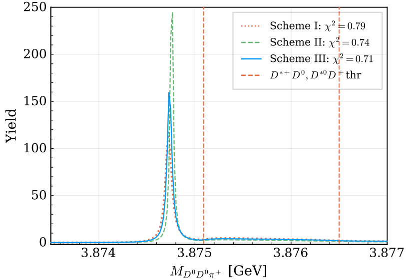
☞ Fit Schemes:

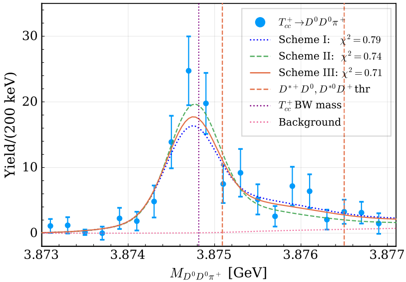
- Scheme I (No 3-body cut):
  $\hookrightarrow$ no OPE, $\Gamma_c(M,p) = 82.5$ keV, $\Gamma_0(M,p) = 53.7$ keV
- Scheme II (partial 3-body cut):
  $\hookrightarrow$ no OPE, dynamical widths of D^* (self-energy)
- Scheme III (complete 3-body cut):
  $\hookrightarrow$ OPE + dynamical widths of D^*

☞ Only two free parameters: $\mathcal{N}$, v_0

checked for $\Lambda = [0.5 - 1.2]$ GeV

Fit to the $D^0 D^0 \pi^+$ mass distribution $\Lambda = 0.5$ GeV





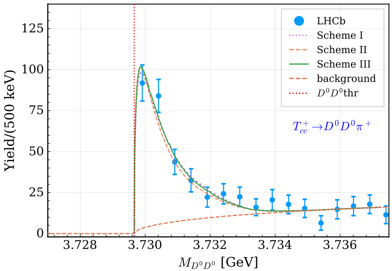
Scheme	III	II	I
Pole [keV]	$-356^{+39}_{-38} - i(28 \pm 1)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-368^{+43}_{-42} - i(37 \pm 0)$

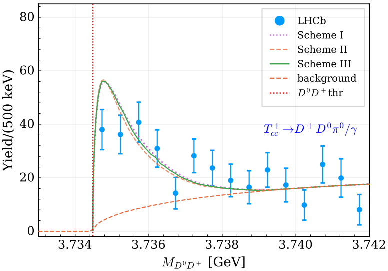
☞ The width of T_{cc}^+

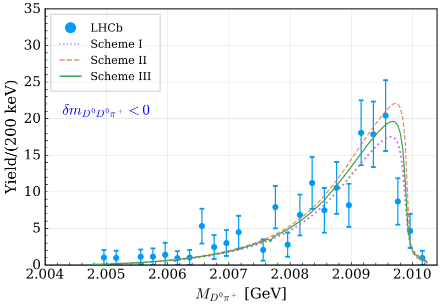
$$56 \text{ keV} \xrightarrow[\text{OPE}]{\text{remove}} 36 \text{ keV} \xrightarrow[\text{M-dep. of } \Gamma^*]{\text{remove}} 74 \text{ keV}$$

$$\text{Scheme III} \longrightarrow \text{Scheme II} \longrightarrow \text{Scheme I}$$

Post-predictions for the D^0D^0 and D^0D^+ line shapes





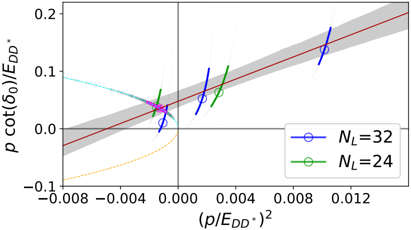


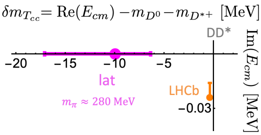
Left-hand cut: the Lattice world!

Doubly Charm Tetraquark on the Lattice

Padmanath *et al*, PRL129,032002(2022)

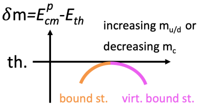
	m_D (MeV)	m_{D^*} (MeV)	M_{av} (MeV)	$a_{l=0}^{(J=1)}$ (fm)	$r_{l=0}^{(J=1)}$ (fm)	$\delta m_{T_{cc}}$ (MeV)	T_{cc}
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	Virtual bound st.
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	Virtual bound st.
Experiment [2,41]	1864.85(5)	2010.26(5)	3068.6(1)	−7.15(51)	[−11.9(16.9), 0]	−0.36(4)	Bound st.



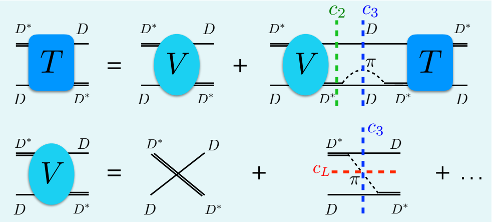


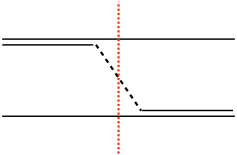
$$t = \frac{E_{\rm cm}}{2}\frac{1}{p\cot\delta - ip},$$

$$p\cot\delta = \frac{1}{a_0} + \frac{1}{2}r_0 p^2,$$



three-body cut vs. left-hand cut





$$G_\pi(E,\boldsymbol{k},\boldsymbol{k}') = \frac{1}{E - E_D(k^2) - E_D(k'^2) - \omega_\pi(q^2)}$$
$$\approx \frac{1}{E - 2M_D - \frac{k^2+k'^2}{2M_D} - \omega_\pi(k^2+k'^2-2kk'\cos\theta)}$$

☞ three-body cut

$$E > M_D + M_D + M_\pi$$

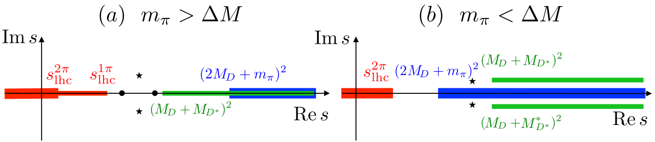
☞ left-hand cut

$$\int_{-1}^{1} d\cos\theta G_\pi(E,p,p)$$

$$G_\pi^{-1}(E,\mathrm{k},\mathrm{k}') \xrightarrow[\text{on shell: } k=k'=p]{\cos\theta=\pm 1}$$
$$E_{D*}(p^2) - E_D(p^2) - \omega_\pi(4p^2/0) = 0$$

$$\left(p_{\rm lhc}^{1\pi}\right)^2 \approx \frac{\left(\Delta M^2 - m_\pi^2\right)}{4},$$
$$\left(\tilde{p}_{\rm lhc}^{1\pi}\right)^2 \approx \frac{\left(\Delta M^2 - m_\pi^2\right)}{4}\frac{4M_D^2}{m_\pi^2}$$

three-body cut vs. left-hand cut



$m_\pi = 280$ MeV

☞ two-body branch point:

$E = M_D + M_{D^*}$

$\Longrightarrow p^2_{\rm rhc_2} = 0$

☞ three-body branch point:

$E = M_D + M_D + m_\pi$

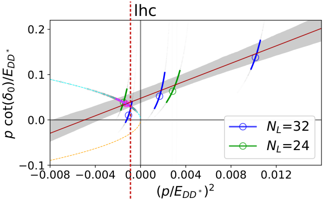
$\Longrightarrow \left(\frac{p_{\rm rhc_3}}{E_{DD^*}}\right)^2 = +0.019$

☞ left-hand cut branch point:

$\Longrightarrow \left(\frac{p^{1\pi}_{\rm lhc}}{E_{DD^*}}\right)^2 = -0.001$

$\left(\frac{\tilde{p}^{1\pi}_{\rm lhc}}{E_{DD^*}}\right)^2 = -0.190$

$m_\pi = 280$ MeV



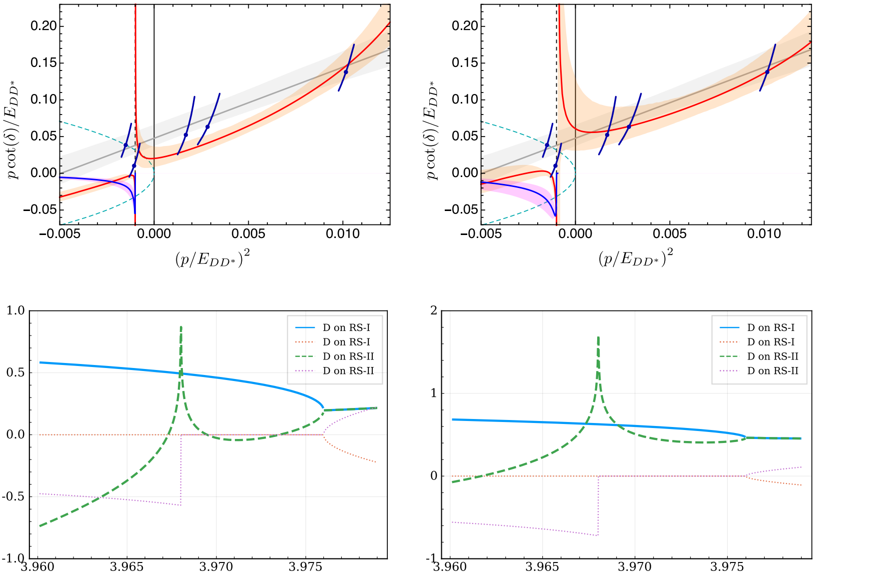
$m_\pi \longrightarrow \Delta M = M_{D^*} - M_D$

$p^2_{\rm rhc_2} \longrightarrow p^2_{\rm rhc_3}$

$(p^{1\pi}_{\rm lhc})^2 \longrightarrow (\tilde{p}^{1\pi}_{\rm lhc})^2$

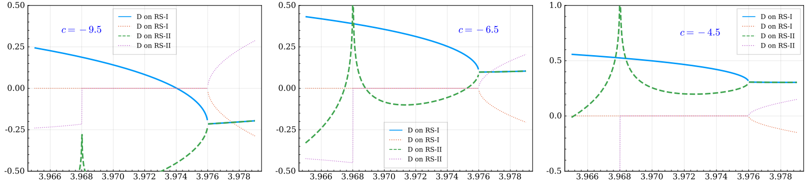
Phase shift with the left-hand cut $v_0 = 2c + 2c_2(k^2 + k'^2)$

$M_D = 1927$ MeV, $M_{D^*} = 2049$ MeV, $m_\pi = 280$ MeV

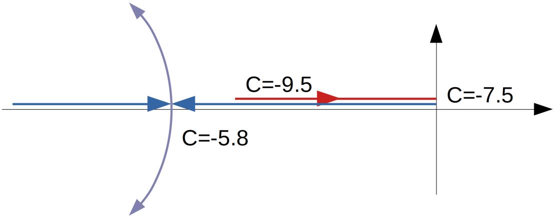


Pole trajectory $(v_0 = 2c)$

$M_D = 1927$ MeV, $M_{D^*} = 2049$ MeV, $m_\pi = 280$ MeV



bound state $\longrightarrow$ virtual state $\longrightarrow$ resonances below threshold



Summary

- T_{cc}^{+} is the first doubly charmed (heavy quark) meson $(cc\bar{u}\bar{d})$
- $m_{T_{cc}^{+}} > m_{DD\pi} \rightarrow$ three-body cuts
  $\hookrightarrow$ one-pion exchange + self-energy of D^*
- The width of T_{cc}^{+} is sensitive to the details

$$56\ \text{keV} \xrightarrow[\text{OPE}]{\text{remove}} 36\ \text{keV} \xrightarrow[\text{M-dep. of }\Gamma^*]{\text{remove}} 74\ \text{keV}$$

★ Unphysical pion mass on the Lattice
  $M_D = 1927$ MeV, $M_{D^*} = 2049$ MeV, $m_\pi = 280$ MeV
  $\hookrightarrow$ the three-body cut above the two-body cut
  $\hookrightarrow$ the left-hand cut: $\sqrt{s_{\text{lhc}}} = 3968$ MeV

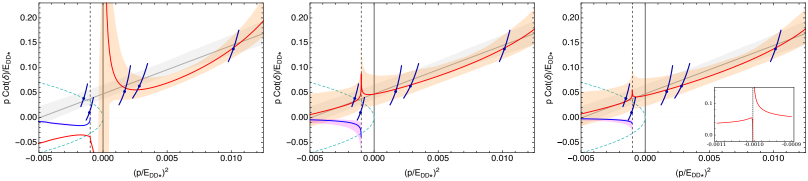
★ ERE valid only in a very limited range
  $\hookrightarrow$ An accurate extraction of the pole requires the OPE implemented

★ The similar lhc appear: BB^*, $B\bar{B}^*$, $D\bar{D}^*$, etc.

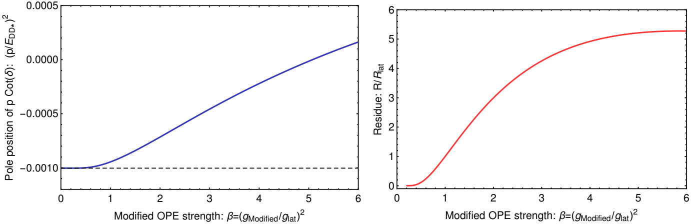
★ A direct comparison of the energy levels predicted in a finite volume with the lattice results.

Thank you very much for your attention!

The impact of the OPE: Modified coupling g



$(g_{\mathrm{Modified}}/g_{\mathrm{lat}})^2 = 5$ $(g_{\mathrm{Modified}}/g_{\mathrm{lat}})^2 = 1/5$ $(g_{\mathrm{Modified}}/g_{\mathrm{lat}})^2 = 1/10$



Low energy expansion of the scattering amplitude

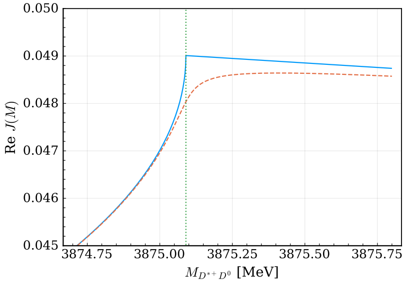
$$T_{D^{*+}D^0\to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0}+\frac{1}{2}r_0k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$

☞ A single-channel with a constant potential:

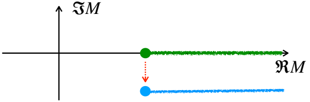
$$T^{-1}(M) = V_{\rm CT}^{-1} + J(M), \quad J(M) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} G(M,p)$$

☞ $r_0 \propto -\Re \frac{\mathrm{d}J(M)}{\mathrm{d}M}\bigg|_{M=M_{\rm thr}+0^+}$

$\propto \frac{1}{\sqrt{\Gamma_{D^*}}}$



☞ $M = m_c^* - i\Gamma_c/2 + m_0 + \frac{k^2}{2\mu_{c0}}$



Isospin violation: infinite narrow D^* limit

☞ Only contact potential (Scheme I):

$$T^{-1}_{D^{*+}D^0\to D^{*+}D^0}(M) = \frac{2}{v_0} + \big(J_1(M)+J_2(M)\big)$$

☞ $r_0 = -\frac{2\pi}{\mu^2}\frac{\mathrm{d}\big(J_1(M)+J_2(M)\big)}{\mathrm{d}M}\Big|_{M=M_{\mathrm{thr},1}+0^+}$ $\quad E = M - M_{\mathrm{thr},1}$

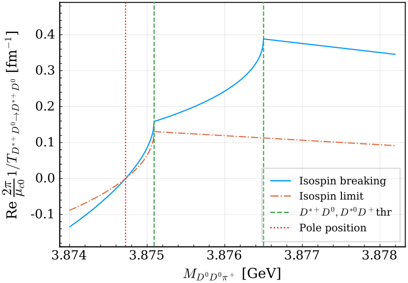
$$\begin{aligned} J_1(E) &= \frac{\Lambda\mu}{\pi^2} - \frac{2\mu^2 E}{\pi^2\Lambda} + i\frac{\sqrt{2\mu E}\mu}{2\pi} + \mathcal{O}(E^2), \\ J_2(E) &= \frac{\Lambda\mu}{\pi^2} - \frac{2\mu^2 E}{\pi^2\Lambda} + \frac{2\Delta\mu^2}{\pi^2\Lambda} - \frac{\mu\sqrt{2\mu\Delta}}{2\pi} + \frac{\mu E\sqrt{2\mu\Delta}}{4\pi\Delta} + \mathcal{O}(E^2) \end{aligned}$$

☞ $r_0 = \frac{8}{\pi\Lambda} - \frac{1}{\sqrt{2\mu\Delta}}$

$1 - Z > 0.48$ LHCb: 2109.01056

$\downarrow$

$1 - Z \gtrsim 0.6$ Baru et al: 2110.13765



Composite or Compact? $(\Delta r_{\mathrm{IV}} = -\sqrt{\frac{1}{2\mu\Delta}} = -3.78\ \mathrm{fm})$

- Compositeness $\bar{X}_A = 1 - Z$

$$\bar{X}_A = \left(1 + 2\left|\frac{r_0'}{\Re a_0}\right|\right)^{-1/2}, \quad r_0' = r_0 - \Delta r_{\mathrm{IV}}$$

- Note here $T_{\mathrm{thr}} \neq -\frac{2\pi}{\mu} a_0$

	a_0 [fm]	r_0 [fm]	$\bar{X}_A$	X_1	X_2
I	$\left(-6.31^{+0.36}_{-0.45}{}_{\pm 0.27}\right) + i\left(0.05^{+0.01}_{-0.01}{}_{\pm 0.00}\right)$	$-2.78 \pm 0.01 \pm 0.66$	$0.87 \pm 0.01 \pm 0.07$	$0.71 \pm 0.01 \pm 0.02$	$0.29 \pm 0.01 \pm 0.02$
II	$\left(-6.64^{+0.36}_{-0.50}{}_{\pm 0.27}\right) - i\left(0.10^{+0.01}_{-0.02}{}_{\pm 0.01}\right)$	$-2.80 \pm 0.01 \pm 0.59$	$0.88 \pm 0.01 \pm 0.06$	$0.71 \pm 0.01 \pm 0.02$	$0.29 \pm 0.01 \pm 0.02$
III	$\left(-6.72^{+0.36}_{-0.45}{}_{\pm 0.27}\right) - i\left(0.10^{+0.03}_{-0.03}{}_{\pm 0.03}\right)$	$-2.40 \pm 0.01 \pm 0.85$	$0.84 \pm 0.01 \pm 0.06$	$0.73 \pm 0.01 \pm 0.11$	$0.27 \pm 0.01 \pm 0.02$

- compositeness to each coupled-channel

Hyodo et al, PRC85(2012)015201
Aceti et al, PRD86(2012)014012

$$X_i = g_i \frac{\mathrm{d}G_i(M_{\mathrm{pole}})}{\mathrm{d}M_{\mathrm{pole}}^2}$$

Prediction: HQSS partner

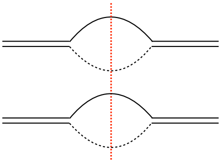
- In the heavy quark limit

$$V^{I=0}(D^*D^* \to D^*D^*, J^P = 1^+) = V^{I=0}(D^*D \to D^*D, J^P = 1^+)$$

- ? $V^{I=0}(D^*D \to D^*D^*, J^P = 1^+)$
- renormalization of OPE
  $\hookrightarrow$ regulator dependent $\leftrightarrow$ higher order counter terms

Baru et al, PRD99(2019)074023; Du et al,JHEP08(2021)157

- Four-body cuts



- Neglecting D^*D-D^*D^* and widths of D^* ($\Lambda = 0.5$ GeV)

Scheme I:	$\delta_{cc}^{*+} = -1444(61)$ keV,
Scheme II:	$\delta_{cc}^{*+} = -1138(50)$ keV,
Scheme III:	$\delta_{cc}^{*+} = -503(40)$ keV.

two-body approx., two Λ's, M. Albaladejo, 2110.02944
width & strangeness Dai, 2110.15270

Left-hand cuts from one-pion exchange

Propagator in TOPT,

$$D^{\pi}(q) = \frac{1}{q_\mu q^\mu - m_\pi^2} = \frac{1}{q_0^2 - \omega_\pi^2(q^2)} = \frac{1}{2\omega_\pi(q^2)}\left(\frac{1}{q_0 - \omega_\pi(q^2)} - \frac{1}{q_0 + \omega_\pi(q^2)}\right)$$
$$= \frac{1}{2\omega_\pi(q^2)}\left[\frac{1}{E - E_D(k^2) - E_D(k'^2) - \omega_\pi(q^2)} + \frac{1}{E - E_{D^*}(k^2) - E_{D^*}(k'^2) - \omega_\pi(q^2)}\right].$$

Partial wave decomposition in the Feynman propagator,

$$\frac{1}{2}\int_{-1}^{+1} d\cos\theta \, \frac{1}{u - m_\pi^2} = \frac{s}{\sqrt{\lambda(s, m_1^2, m_2^2)}\sqrt{\lambda(s, m_3^2, m_4^2)}} \log\left(\frac{z-1}{z+1}\right),$$
$$z = \frac{2s(m_1^2 + m_3^2 - m_\pi^2) - (s + m_1^2 - m_2^2)(s + m_3^2 - m_4^2)}{\lambda^{1/2}(s, m_1^2, m_2^2)\lambda^{1/2}(s, m_3^2, m_4^2)}. \quad (1)$$

$z = \pm$ gives the branch points of the left-hand cut,

$$s^{(\pm)} = \frac{1}{2m_\pi^2}\Big[(m_3^2 - m_1^2)(m_2^2 - m_4^2) - m_\pi^4 + m_\pi^2(m_1^2 + m_2^2 + m_3^2 + m_4^2)$$
$$\pm\lambda^{1/2}(m_\pi^2, m_3^2, m_1^2)\lambda^{1/2}(m_\pi^2, m_2^2, m_4^2)\Big]. \quad (2)$$

T-matrix

Potential

$$V_C^{I=0}(k,k') = c_0 + c_2(k^2 + k'^2),$$
$$V_{\text{OPE}}^{I=0}(E,k,k') = \frac{g^2}{8f_\pi^2}\int_{-1}^{1} dz D^\pi(E,k,k',z)(k^2 + k'^2 - 2kk'z), \quad (3)$$

The Lippmann-Schwinger equation

$$T(E,k,k') = V(E,k,k') - \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} V(E,k,q)G(E,q)T(E,q,k'),$$

The DD^* propagator is expressed as

$$G(E,q) = \left[M_{D^*} + M_D + \frac{q^2}{2\mu} - E - \frac{i}{2}\Gamma(E,q)\right]^{-1}, \quad (4)$$

$$\Gamma(E,q) = \frac{g^2 M_D}{8\pi f_\pi^2 M_{D^*}}\left[\Sigma(s) - \Sigma_0(s)\theta(M_D + m_\pi - M_{D^*})\right],$$

with $s = [E - M_D - q^2/(2\mu)]^2$,

$$\Sigma(s) = \left[\frac{\sqrt{\lambda(s, M_D^2, m_\pi^2)}}{2\sqrt{s}}\right]^3, \quad (5)$$

and $\Sigma_0(s) = \Sigma(M_{D^*}^2) + 2M_{D^*}(E - M_{D^*} - M_D - \frac{q^2}{2\mu})\Sigma'(M_{D^*}^2)$.

Chiral extrapolation of f_π and g

Upto the one-loop chiral perturbation theory,

$$f_\pi(\xi) = f_\pi^{\rm ph}\left[1 + \left(1 - \frac{f_0}{f_\pi^{\rm ph}}\right)(\xi^2 - 1) - \frac{(m_\pi^{\rm ph})^2}{8\pi^2 f_0^2}\xi^2 \log\xi\right],$$

where $\xi = m_\pi/m_\pi^{\rm ph}$, $f_0 \equiv f_\pi(m_\pi = 0) = 85$ MeV and $f_\pi^{\rm ph} = 92.1$ MeV.

$$g(\xi) = g^{\rm ph}\left[1 + C_1(\xi^2 - 1) + C_2\xi^2 \log\xi\right], \qquad g^{\rm ph} = 0.57. \tag{6}$$

$$C_1 = 1 - \left[1 - \frac{1 + 2g_0^2}{8\pi^2 f_0^2}(m_\pi^{\rm ph})^2 \log\frac{m_\pi^{\rm ph}}{\mu_{\rm lat}} + \alpha_{\rm lat}(m_\pi^{\rm ph})^2\right]^{-1},$$

$$C_2 = -\frac{1 + 2g_0^2}{8\pi^2 f_0^2}(m_\pi^{\rm ph})^2(1 - C_1),$$

where $g_0 = 0.46$, $\quad \alpha_{\rm lat} = -0.16\ {\rm GeV}^{-2}$, $\quad \mu_{\rm lat} = 1$ GeV.

Specifically, for $m_\pi = 280$ MeV this gives $g(m_\pi = 280\,{\rm MeV}) = 0.65$.

LHCb model

Lippmann-Schwinger equation: $T = V + VGT$, $V = V_0 + V_{\rm OPE}$.

$$T = T_{\rm OPE} + (1 + T_{\rm OPE}G)\hat{T}(1 + GT_{\rm OPE}),$$

where

$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}GT_{\rm OPE}, \quad \hat{T} = V_0 + G_3\hat{T},$$

with $G_3 = G + GT_{\rm OPE}G$.

$$2i\Im G_3 = (1 + G^\dagger T_{\rm OPE}^\dagger)\left[G - G^\dagger + G^\dagger\left(V_{\rm OPE} - V_{\rm OPE}^\dagger\right)G\right](1 + GT_{\rm OPE}).$$

$$G_3^{(\rm LHCb)} = \frac{1}{2\pi i}\int_{\rm th}^{\infty} ds' \, \frac{G - G^\dagger + G^\dagger(V_{\rm OPE} - V_{\rm OPE}^\dagger)G}{s' - s - i\epsilon},$$

Then

$$\hat{T}^{(\rm LHCb)} = V_0 + V_0 G_3^{(\rm LHCb)}\hat{T}^{(\rm LHCb)},$$

with $V_{0,ij} = g_i g_j/(m^2 - s)$ and $g_1 = -g_2 = g$. The solution is

$$\hat{T}^{(\rm LHCb)} = \frac{1}{m^2 - s - g^2\Sigma}(m^2 - s)V_0, \quad \Sigma = (1 \;\; -1)G_3^{(\rm LHCb)}\begin{pmatrix}1\\-1\end{pmatrix}.$$