量子场中的相与相变在新物理中的运用：从渐近安全到引力波

王志伟

电子科技大学

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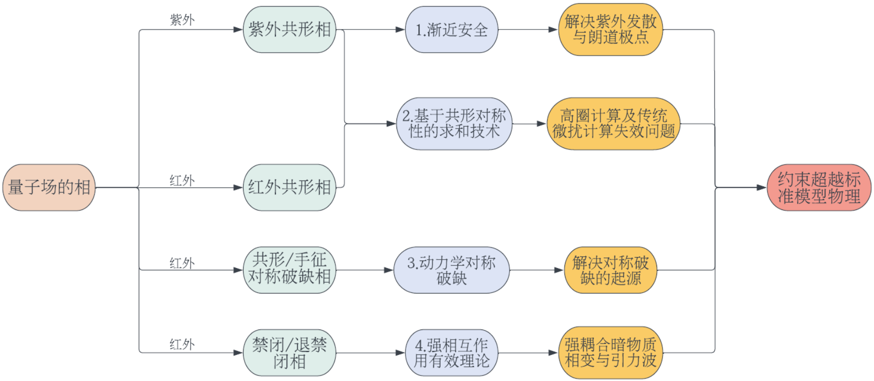
彭桓武高能基础理论研究中心，中国科学技术大学

A Landscape of Phases in QFT and its Relation to BSM Physics



An Example of Phase Diagram

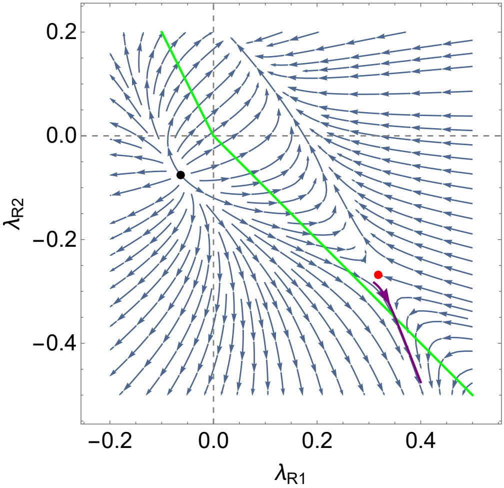


图: Stream plot presents RG flows in Pati-Salam Model. λ_{R1}, λ_{R2} represent couplings for the $(4,1,2)$ scalar potential which triggers symmetry breaking:
$SU(4)\otimes SU(2)_L\otimes SU(2)_R \xrightarrow{v_R} SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$

Part 1: Towards Asymptotically Safe Standard Model

- The Standard Model is not a fundamental theory since it runs into Landau Pole at UV due to the abelian $U(1)$ gauge group.
- Question 1: how to make the Standard Model UV complete without gravity?
- Many GUTs are even worse.
- Due to the presence of large representations, the RGE of the unified coupling will hit Landau pole right above the unification scale.
- Question 2: can we make the Standard Model UV complete via a GUT embedding?
- Question 3: can UV completion provides an alternative guiding principle to BSM like naturalness/fine-tuning?

Fundamental Theory $\Longleftrightarrow$ UV Fixed Point (UV Conformal Phase)

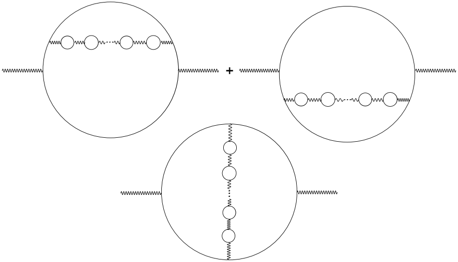
- A fundamental theory has an UV fixed point (UV conformal phase) (K. G. Wilson, Phys. Rev. B **4** (1971) 3174.)
- Couplings stop running with the energy scale at the fixed point
- Thus, the Standard Model is not a fundamental theory.
- Asymptotically Free: non-interacting (Gaussian) fixed point (D. J. Gross and F. Wilczek, Phys. Rev. D **8** (1973) 3633; D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.)
  - non-interacting in the UV
  - coupling runs with logarithmic scale dependence
  - Perturbation theory in UV
- Asymptotically Safe: Interacting fixed point (S. Weinberg(1979). "Ultraviolet divergences in quantum theories of gravitation".)
  - interacting in the UV
  - coupling runs with power law scale dependence closed to the safe fixed point (much faster runnning compared with the free case)
  - Perturbative/Non perturbative theory in UV
  - Smaller critical surface dimension $\Rightarrow$ more IR predictiveness

Large N_f Expansion

- $1/N_f$ expansion in Abelian/non-Abelian gauge theory was firstly developed respectively by Pascual and Gracey and later on summarized by Bob Holdom with initial analysis of the pole structure

  A. Palanques-Mestre and P. Pascual, Commun. Math. Phys. **95** (1984) 277; J. A. Gracey, Phys. Lett. B **373** (1996) 178; B. Holdom, Phys. Lett. B **694**, 74 (2011).

- Pascual noticed that it is possible to sum up a subset of the diagrams and the resulting power series is so well behaved to provide a closed-form expression at $1/N_f$ order



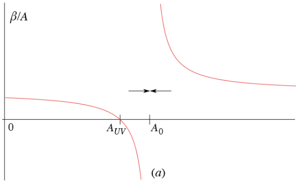
The Summation Function and its Pole Structure

- The resummed U(1) beta function reads (with summation function $F_1(A)$):

$$\beta_A = \frac{2A^2}{3}\left[1 + \frac{1}{N_f}F_1(A)\right], \qquad A \equiv 4N_f\alpha = 4N_f\frac{g_1^2}{(4\pi)^2}$$

$$F_1(A) = \frac{3}{4}\int_0^A \mathrm{d}x\, \tilde{F}\left(0, \tfrac{2}{3}x\right), \quad \tilde{F}(0,x) = \frac{(1-x)(1-\frac{x}{3})(1+\frac{x}{2})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}$$

- $1/N_f$ expansion encodes all order loop contributions.
- $F_1(A)$ has a pole structure at $A = 15/2$ (Non-abelian at $A = 3$).
- $\beta - A$ diagram(B. Holdom, Phys. Lett. B **694**, 74 (2011)).



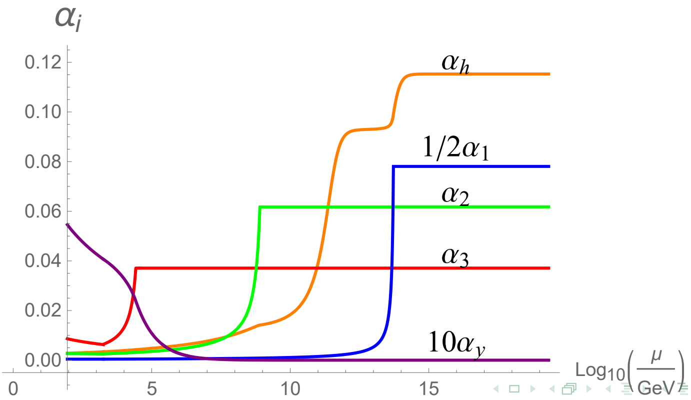
- The pole structure guarantees the **UV fixed point** of the gauge coupling.

Mann, Meffe, Sannino, Steele, Z. W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802

Safe Standard Model: $SU(3) \times SU(2) \times U(1)$

Mann, Meffe, Sannino, Steele, Z.W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802

- We introduce vector-like fermions under the SM group $SU(3) \times SU_L(2) \times U(1)$: $N_{F3}\ (3,1,0) \oplus N_{F2}\ (1,3,0) \oplus N_{F1}\ (1,1,1)$
- The gauge couplings $\alpha_1,\ \alpha_2,\ \alpha_3$ and Higgs quartic coupling α_h are safe while the top Yukawa coupling α_y is free
- The transition scale of the interacting fixed point is dependent on N_f



Conformal Window 2.0 O. Antipin and F. Sannino, Phys. Rev. D **97** (2018) 116007 .

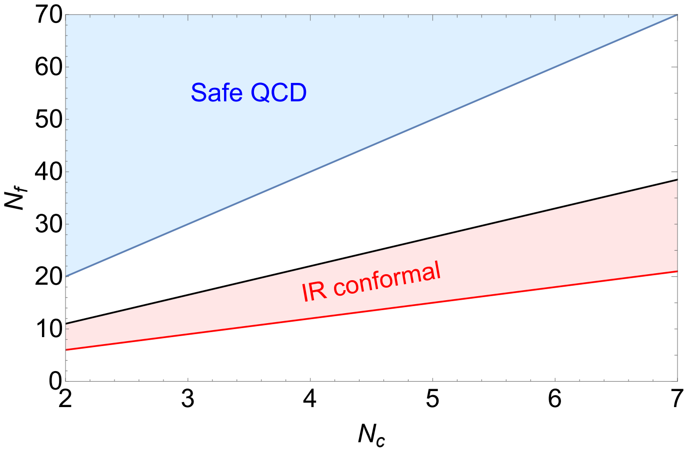


图: Phase diagram of $SU(N_c)$ gauge theories with fermionic matter in the fundamental representation. The shaded areas depict the corresponding conformal windows where the theories develop an IRFP (light red region) or an UVFP (light blue region).

Further Testing: Lattice and Beyond $1/N_f$ Calculations

- The existence of UV fixed point is based on the truncation of diagram resummation at $1/N_f$ order.
- It is important to check whether higher order results say $1/N_f^2$ order will destablize the fixed point or not. Dondi, Dunne, Reichert and Sannino, "Towards the QED beta function and renormalons at $1/N_f^2$ and $1/N_f^3$," PRD **102** (2020) 035005.
- Helsinki Lattice group has studied $SU(2)$ case with 24 and 48 Dirac fermions by using the gradient flow method. Leino, Rindlisbacher, Rummukainen, Sannino and Tuominen, "Safety versus triviality on the lattice," PRD **101** (2020) 074508.
- However, they found the current lattice actions is unable to explore the deep ultraviolet region where safety might emerge.
- Their work constitutes an **essential** step towards determining the UV conformal phase of non asymptotically free gauge theories (lattice group lead by Oliver Witzel at Siegen University is also studying this).

Question 2：Can we make the Standard Model UV safe through GUT embedding?

- Safe Pati-Salam: $G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R$

  (Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.)

- Safe Trinification: $G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$

  (Z.W. Wang, Balushi, Mann and Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.)

Generalize the Large N_f beta Functions

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- We generalize the Large N_f gauge beta functions to more general semi-simple gauge groups.
- We insert the bubble chain to quartic and Yukawa interactions and have obtained large N_f beta functions for the first time for quartic and Yukawa couplings.

Large N_f beta Functions: Semi-Simple Gauge

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Vector-like fermions charged
  under only simple gauge group (PRL **119** (2017) 261802)
  $\Rightarrow$ semi-simple gauge group (Note: two different gauge lines below)

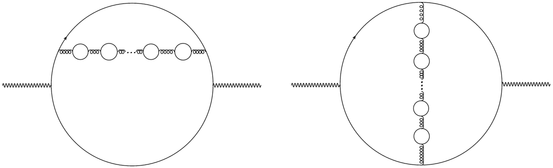


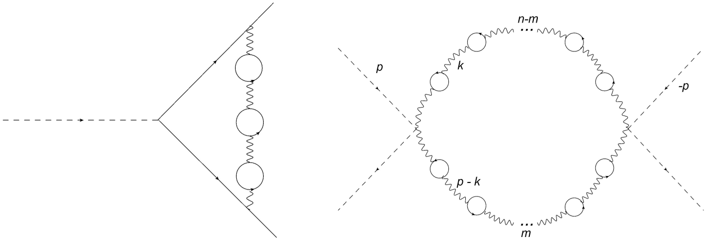
图: Two extra Feynman diagrams for the 2-point functions giving mixed terms to the beta functions.

- $$\beta_i^{\mathrm{ho}} = \frac{2A_i\alpha_i}{3}\left(\frac{d(G_i)H_{1_i}(A_i)}{N_f\prod_k d\left(R_\psi^k\right)} + \frac{\sum_j d(G_j)\,F_{1_j}(A_j)}{N_f\prod_k d\left(R_\psi^k\right)}\right)$$

Large N_f beta Functions: Yukawa and Quartic

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Bubble chain insertion only for gauge couplings (PRL **119** (2017) 261802)
  $\Rightarrow$ all gauge, Yukawa and Quartic couplings (PRD 98 (2018) 016003)



Recipe of Bubble Improved RG Function: Yukawa

(Antipin, Dondi, Sannino, Thomsen and Z.W. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

See also (Kowalska, Sessolo, JHEP 1804 (2018) 027; Phys.Rev. D97 (2018) 095013)

- Recipe for Yukawa

$$\beta_y = c_1 y^3 + y \sum_\alpha c_\alpha A_\alpha I_y\left(A_\alpha\right), \text{ where}$$

$$I_y\left(A_\alpha\right) = H_\phi\left(0, \tfrac{2}{3} A_\alpha\right)\left(1 + A_\alpha \frac{C_2\left(R_\phi^\alpha\right)}{6\left(C_2\left(R_\chi^\alpha\right) + C_2\left(R_\xi^\alpha\right)\right)}\right),$$

$$H_\phi(0,x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}.$$

- The summation function H_ϕ has a pole at $x = 5$ corresponding to $A = 15/2$.
- For models where c_1 and c_α are known, we can immediately obtain the bubble diagram contributions following this recipe.

Recipe of Bubble Improved RG Function: Quartic

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.
G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Recipe for Quartic Coupling

$$\beta_\lambda = c_1\lambda^2 + \lambda\sum_\alpha c_\alpha A_\alpha I_{\lambda g^2}\left(A_\alpha\right) + \sum_\alpha c'_\alpha A_\alpha^2 I_{g^4}\left(A_\alpha\right) + \sum_{\alpha<\beta} c_{\alpha\beta} A_\alpha A_\beta I_{g_1^2 g_2^2}\left(A_\alpha,\, A_\beta\right) \,,$$

$$I_{\lambda g^2}\left(A_\alpha\right) = H_\phi\left(0, \tfrac{2}{3}A_\alpha\right)$$

$$I_{g^4}\left(A_\alpha\right) = H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) + A_\alpha \frac{dH_\lambda\left(1, \tfrac{2}{3}A_\alpha\right)}{dA_\alpha}$$

$$I_{g_1^2 g_2^2}\left(A_\alpha,\, A_\beta\right) = \frac{1}{A_\alpha - A_\beta}\left[A_\alpha H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) - A_\beta H_\lambda\left(1, \tfrac{2}{3}A_\beta\right)\right]$$

$$H_\lambda(1,x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{6\Gamma^3(2-\frac{x}{2})\Gamma(1+\frac{x}{2})} \,.$$

- The summation function H_ϕ also has a pole at $x = 5$ corresponding to $A = 15/2$.

Safety of Grand Unified Theory: Pati-Salam Model

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Pati-Salam model is under gauge symmetry group G_{PS}

  J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974) Erratum: [Phys. Rev. D **11**, 703 (1975)].

$$G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \,.$$

- The SM quark and lepton fields are unified into the $G_{\rm PS}$ irreducible representations

$$\psi_{Li} = \begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \end{pmatrix}_i \sim (4,2,1)_i \,,$$
$$\psi_{Ri} = \begin{pmatrix} u_R & u_R & u_R & \nu_R \\ d_R & d_R & d_R & e_R \end{pmatrix}_i \sim (4,1,2)_i \,,$$

- Symmetry breaking pattern: $G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

Pati-Salam Model: Gauge Field Content

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The gauge fields of G_{PS} can be written as follows:

$$\hat{W}_{L\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{L\mu} & \sqrt{2}W^+_{L\mu} \\ \sqrt{2}W^-_{L\mu} & -W^0_{L\mu} \end{pmatrix},$$

$$\hat{W}_{R\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{R\mu} & \sqrt{2}W^+_{R\mu} \\ \sqrt{2}W^-_{R\mu} & -W^0_{R\mu} \end{pmatrix},$$

$$\hat{G}_{\mu} \equiv \frac{1}{2}\begin{pmatrix} G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{12\mu} & \sqrt{2}G^+_{13\mu} & \sqrt{2}X^+_{1\mu} \\ \sqrt{2}G^-_{12\mu} & -G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{23\mu} & \sqrt{2}X^+_{2\mu} \\ \sqrt{2}G^-_{13\mu} & \sqrt{2}G^-_{23\mu} & -\frac{2G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}X^+_{3\mu} \\ \sqrt{2}X^-_{1\mu} & \sqrt{2}X^-_{2\mu} & \sqrt{2}X^-_{3\mu} & -\frac{3B_\mu}{\sqrt{6}} \end{pmatrix}$$

- $W^0_{L\mu}$ and $W^\pm_{L\mu}$ correspond to the electroweak (EW) gauge bosons, $G_{3\mu}$, $G_{8\mu}$, $G^\pm_{12\mu}$, $G^\pm_{13\mu}$ and $G^\pm_{23\mu}$ are the $SU(3)_C$ gluons, B_μ is the $B-L$ gauge field, and $X^\pm_{1\mu}$, $X^\pm_{2\mu}$ and $X^\pm_{3\mu}$ are leptoquarks.

Pati-Salam Model: Scalar Fields and Couplings

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Scalar field $\phi_R \sim (4,1,2)$ triggers Pati-Salam symmetry breaking:

$$\phi_R = \begin{pmatrix} \phi_R^{u_1} & \phi_R^{u_2} & \phi_R^{u_3} & \phi_R^{0} \\ \phi_R^{d_1} & \phi_R^{d_2} & \phi_R^{d_3} & \phi_R^{-} \end{pmatrix}, \quad G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- Scalar bi-doublet $\Phi \sim (1,2,2)$ triggers electroweak symmetry breaking:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi_1 & \Phi_2 \end{pmatrix}$$

- All the couplings:

Gauge Couplings	Yukawa Couplings	Scalar Couplings
$SU(4): g_4$	$\psi_{L/R}: y, y_c$	$\phi_R: \lambda_{R1}, \lambda_{R2}$
$SU(2)_L: g_L$	$N_L\,(1,1,1): y_\nu$	portal: $\lambda_{R\Phi_1}, \lambda_{R\Phi_2}, \lambda_{R\Phi_3}$
$SU(2)_R: g_R$	$F\,(10,1,1): y_F$	$\Phi: \lambda_1, \lambda_2, \lambda_3, \lambda_4$

表: Gauge, Yukawa and scalar quartic couplings of the Pati-Salam model.

Vector-like Fermions Charges & Large N_f Gauge Beta

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- We consider three sets of vector-like fermions charged under $G_{\rm PS}$, with the charge assignment:

$$N_{f_4}\,(4,1,1)\oplus N_{f_{2L}}\,(1,3,1)\oplus N_{f_{2R}}\,(1,1,2)$$

  where the $N_{f_{2L}}$ vector-like fermions are chosen in the adjoint representation of $SU(2)_L$ to avoid fractional electrical charges.
- The charge assignments are chosen to avoid the extra contributions in the summation of semi-simple group
- The large N_f gauge beta functions are given by:

$$\beta^{tot}_{\alpha_{2L}}=\beta^{1loop}_{\alpha_{2L}}+\beta^{\rm ho}_{\alpha_{2L}}=-6\alpha^2_{2L}+\frac{2A_{2L}\alpha_{2L}}{3}\frac{H_{1_{2L}}\left(A_{2L}\right)}{N_{f_{2L}}}$$

$$\beta^{tot}_{\alpha_{2R}}=\beta^{1loop}_{\alpha_{2R}}+\beta^{\rm ho}_{\alpha_{2R}}=-\frac{14}{3}\alpha^2_{2R}+\frac{2A_{2R}\alpha_{2R}}{3}\frac{H_{1_{2R}}\left(A_{2R}\right)}{N_{f_{2R}}}$$

$$\beta^{tot}_{\alpha_4}=\beta^{1loop}_{\alpha_4}+\beta^{\rm ho}_{\alpha_4}=-18\alpha^2_4+\frac{2A_4\alpha_4}{3}\frac{H_{1_4}\left(A_4\right)}{N_{f_4}}\,.$$

Alternative Picture of Gauge Coupling Unification

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- A sample case of gauge unification with $N_{f_{2L}} = 35,\ N_{f_{2R}} = N_{f_4} = 140$:

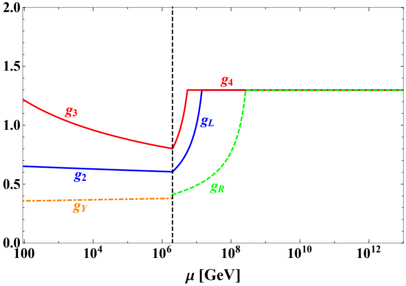


图: The dashed line represents the PS symmetry breaking scale at $2000\,\mathrm{TeV}$ where all the vector-like fermions are introduced. The three couplings g_Y, g_2, g_3 at the left of the dashed line are determined by RGE of the SM gauge couplings.

- This figure **motivates** the idea of asymptotic unification in extra dimension (Cacciapaglia, Cornell, Cot and Deandrea, PRD **104** (2021) 075012)

Classification of UV Fixed Point: Relevant & Irrelevant

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Vacuum stability condition

$$\lambda_{R1} + \lambda_{R2} > 0 \qquad \lambda_1 - \lambda_2 + \lambda_4 > 0, \qquad \lambda_1 > 0$$

λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.12	0.05	0	0.13	0.02	0	0.13	-0.01	0.78	0.78	0.84	0
Irev	Rev	0	Irev	Irev	0	Irev	Rev	Irev	Irev	Irev	0

λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.05	0.02	0	0.01	0.04	0	0.02	0.08	0.24	0.24	0.57	0.74
Irev	Rev	0	Irev	Irev	0	Irev	Irev	Irev	Irev	Irev	Irev

表: These tables summarize the sample UV fixed point solution with two sample values ($N_{f_{2L}} = 40,\ N_{f_{2R}} = 150,\ N_{f_4} = 200;\ N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. The UV fixed point solutions of the couplings are classified with relevant (Rev) and irrelevant (Irev) characteristics. “0” denotes Gaussian Fixed points.

RG Flow: Gauge and Yukawa

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the gauge and Yukawa couplings by using the UV to IR approach.

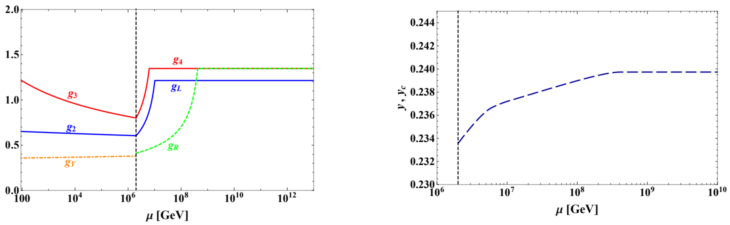


图: We have chosen $N_{f_{2L}} = 40$, $N_{f_{2R}} = 130$, $N_{f_4} = 130$. We have used the matching conditions at IR to set the initial conditions of g_L, g_R, g_4 at IR. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

RG Flow: Quartic Coupling of Bi-doublet Φ

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the Quartic Coupling by using the UV to IR approach.

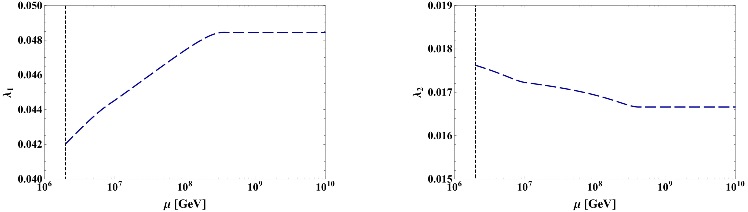


图: We have chosen $N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

RG Flow: Quartic Coupling of ϕ_R

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the Quartic Coupling by using the UV to IR approach.

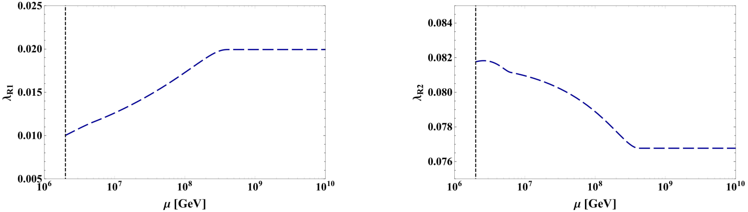


图: We have chosen $N_{f_{2L}} = 40,\ N_{f_{2R}} = 130,\ N_{f_4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

Matching the Standard Model: Top Yukawa Coupling

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The top Yukawa mass term is given by (with CP symmetry $y = y_c$ and $\tan\beta = 1$):

$$m_{\text{top}} = (y\sin\beta + y_c\cos\beta)v \to \sqrt{2}yv = m_{\text{top}}$$

- Thus at electroweak scale, y is smaller than the conventional SM top Yukawa coupling value $\sim \frac{0.93}{\sqrt{2}} \sim 0.66$
- It can be shown that by choosing $N_{f_{2L}} = 32,\ N_{f_{2R}} = 108,\ N_{f_4} = 56$, we obtain $y \sim 0.614$ as required.

Matching the Standard Model: Higgs Mass

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The mass matrix (neutral scalar fields) of the two Higgs doublet model is given by:

$$M^2_{\text{neutral}} = \begin{bmatrix} \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_1 v_1^2 & -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 \\ -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 & \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_2 v_2^2 \end{bmatrix}$$

- This matrix is defined at the electroweak scale. By using the two Higgs doublet beta functions and the matching conditions, we obtain the quartic couplings $\lambda_i \ (i = 1, \cdots 5)$ at the electroweak scale.
- The phenomenological constraint are: both of the eigenvalues of the mass matrix should be positive and the lighter one should close to the $125\,\mathrm{GeV}$ Higgs mass.

Matching the Standard Model

Emiliano, Francesco, Z.W. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- It can be shown that by choosing $N_{F2} = 32,\ N_{F3} = 108,\ N_{F4} = 56$, we obtain:

$$\bar{\lambda}_1 = 0.135,\ \bar{\lambda}_2 = 0.360,\ \bar{\lambda}_3 = 0.25,\ \bar{\lambda}_4 = -0.379,\ \bar{\lambda}_5 = 0.259,\ y = 0.614\,.$$

- Matching of the scalar quartic coupling: two neutral scalar mass with $M_{H1} \sim 125\,\mathrm{GeV}$ (lighter Higgs) and the heavier one $M_{H2} > 238\,\mathrm{GeV}$ with $m_{12} > 150\,\mathrm{GeV}$
- Matching of the top Yukawa coupling: the IR value of y is around 0.66 as required
- Asymptotic Safe Pati-Salam model can roughly match the SM at IR.
- In this minimal model, most of the RG flows lead to much lighter Higgs mass and Pati-Salam symmetry breaking scale above $10000\,\mathrm{TeV}$ is required.

Safety of Grand Unified Theory: Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- Trinification model is under gauge symmetry group G_{TR} (note: without Z_3)
  K. S. Babu, X. G. He and S. Pakvasa, Phys. Rev. D **33** (1986) 763.

$$G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$$

- The coloured fermions are given by $\psi_{Q_L} \sim (3, \bar{3}, 1)$ & $\psi_{Q_R} \sim (3, 1, \bar{3})$:

$$\psi_{Q_L} = \begin{pmatrix} u_L^1 & u_L^2 & u_L^3 \\ \mathscr{D}_L^1 & \mathscr{D}_L^2 & \mathscr{D}_L^3 \\ \mathscr{D}_L'^1 & \mathscr{D}_L'^2 & \mathscr{D}_L'^3 \end{pmatrix}, \; \psi_{Q_R} = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ \mathscr{D}_R^1 & \mathscr{D}_R^2 & \mathscr{D}_R^3 \\ \mathscr{D}_R'^1 & \mathscr{D}_R'^2 & \mathscr{D}_R'^3 \end{pmatrix},$$

  (note: instead of $\psi^c_{Q_L} \sim (\bar{3}, 1, 3)$ we use $\psi_{Q_R} \sim (3, 1, \bar{3})$ since no attempt to unify three gauge group)

- The lepton content in this minimal Trinification model is given by:

$$\psi_E = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ \bar{e}_R & \bar{\nu}_R & \nu' \end{pmatrix} \sim (1, 3, \bar{3}),$$

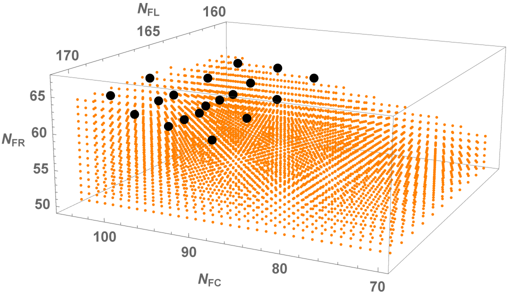
The Matching of the Standard Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- By choosing $N_{FC} = 95,\ N_{FL} = 165,\ N_{FR} = 62$, we obtain:

$$M^{\rm Pre}_{\rm Higgs} = 125\,{\rm GeV} \quad {\rm y}^{\rm Pre}_{\rm top} = 0.806, \quad {\rm y}^{\rm Pre}_{\rm bottom} = 0.019, \quad {\rm y}^{\rm Pre}_{\rm tau} = 0.011$$
$$M^{\rm SM}_{\rm Higgs} = 126\,{\rm GeV} \quad {\rm y}^{\rm SM}_{\rm top} = 0.780, \quad {\rm y}^{\rm SM}_{\rm bottom} = 0.019, \quad {\rm y}^{\rm SM}_{\rm tau} = 0.008\,.$$

- 3D scan of the parameter space



Conclusions So far

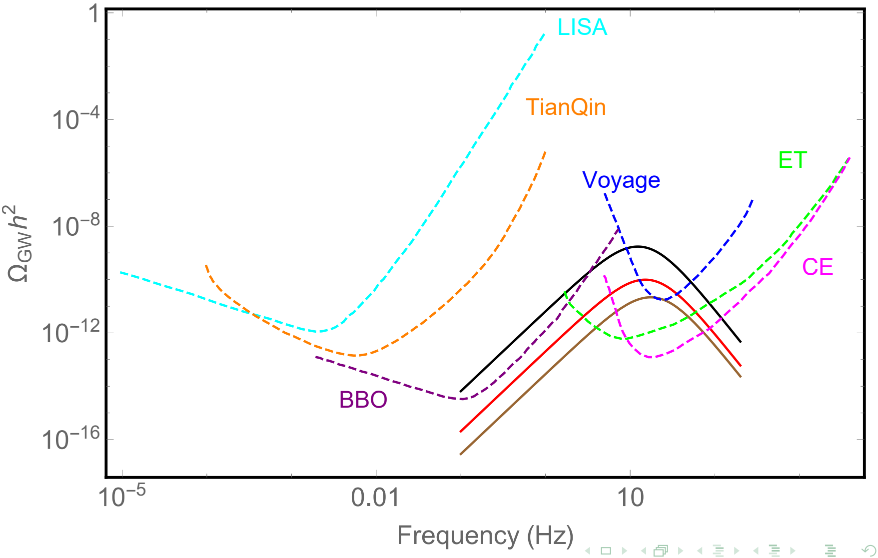
- Large N_f resummation offers a possibility to realize UV completion (asymptotic safety) of the Standard Model via a non-trivial UV fixed point
- Large N_f procedure provides a framework which faciliates the UV completion of various kinds of BSM model building such as GUTs and composite Higgs
- Higher order calculations and in particular, lattice calculations are required to solidfy/confirm the UV fixed point (UV conformal phase)
- Renormalization group flow as a bridge connects UV (boundary condition) and IR physics
- UV completion (asymptotic safety) as a guiding principle provides strong constraints on the RG flows and thus also IR BSM physics

Further Study

- Testing asymptotic safety using gravitational Wave
- From large N_f to large charge Q using CFT
- Alternative way to realize asymptotic safety via extra dimension

Gravitational Wave Signals and Bounds

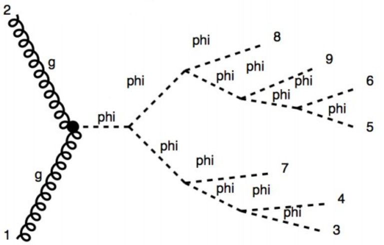
(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)



Multiparticle Production Problem: Higgs Explosion

(Khoze, “Higgs Explosion", indico.cern.ch/event/677640/contributions/2938636)

- It was proposed that multiparitlce production processes are problematic: higgs explosion and instanton-like processes in baryogenesis.
  (V. A. Rubakov, “Nonperturbative aspects of multiparticle production,” hep-ph/9511236.)
- In the process of $A_{gg\to n\times h} = \sum_{\text{polygons}} A^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\cdots+n_k=n} \Pi^k_{i=1} A_{h^*_i\to n_i\times h}$, perturbation theory fails when around 130 Higgses are produced at $O(100\,\text{TeV})$.



Multiparticle Production Problem: Higgs Explosion

(V. A. Rubakov, "Nonperturbative aspects of multiparticle production," hep-ph/9511236.)

- The "exact" result for the tree amplitude at threshold $(E = nm)$ is

$$A^{\text{tree}}_{1\to n} = n!\left(\frac{\lambda}{8m^2}\right)^{\frac{n-1}{2}}$$

- For multiparticle production of $\lambda\phi^4$ theory (mimic Higgs explosion), the amplitude at one loop level:

$$A^{\text{tree}}_{1\to n} + A^{\text{one loop}}_{1\to n} = A^{\text{tree}}_{1\to n}\left(1 + B\lambda n^2\right)\,, \quad A^{\text{tree}}_{1\to n} = n!\left(\frac{\lambda}{8m^2}\right)^{\frac{n-1}{2}}$$

- Two folds problems:
  - The factorial behavior of the tree amplitudes indicates that the cross section also increase with n and at $n \sim 1/\lambda$, the cross section will exceed the unitarity limit at sufficent large n

$$\sigma^{\text{tree}}_{1\to n} \sim \frac{1}{n!}\left|A^{\text{tree}}_{1\to n}\right|^2 \times (\text{phase space}) \sim n!\lambda^n\epsilon^n \qquad \epsilon = \frac{E - nm}{n}$$

  - Loop corrections fails and conventional perturbation theory fails!

Towards the Holy Grail Summation Function $F(\lambda n, \epsilon)$

(V. A. Rubakov, “Nonperturbative aspects of multiparticle production,” hep-ph/9511236.)

- Rubakov's insight: $\sigma_{1\to n}(E) \propto \exp\left[nF(\lambda n, \epsilon)\right]$
- However, only a few terms in the expansion of $F(\lambda n, \epsilon)$ at small λn and ϵ are known:

$$F(\lambda n, \epsilon) = \ln\frac{\lambda n}{16} + \frac{1}{2} + \frac{3}{2}\ln\frac{\epsilon}{3\pi} - \frac{17}{12}\epsilon + B\lambda n + \cdots$$

- Using Large charge method, we can instead calculate LO and NLO scaling dimensions of fixed charge operator $[\phi^n]$ in $U(1)$ symmetric $\lambda\left(\bar{\phi}\phi\right)^2$ theory. (G. Badel, G. Cuomo, A. Monin and R. Rattazzi, JHEP **1911**, 110 (2019), arXiv:1909.01269.)

$$\begin{aligned} &Z_{\phi^n}^2 \lambda_0^n \langle\left[\bar{\phi}^n\right](x_f)\left[\phi^n\right](x_i)\rangle \\ =&\lambda_0^n n! \exp\left[\frac{1}{\lambda_0}\Gamma_{-1}(\lambda_0 n, x_{fi}) + \Gamma_0(\lambda_0 n, x_{fi}) + \Gamma_1(\lambda_0 n, x_{fi}) + \cdots\right] \end{aligned}$$

Comparing Different Large Parameter Expansion

- Large N_c: Planar limit: t'Hooft coupling $A_c \equiv g^2 N_c$ is fixed
- Large N_f: Bubble diagrams: t'Hooft coupling $A_f \equiv g^2 N_f$ is fixed
- Large Q: t'Hooft coupling $A_Q \equiv \lambda Q$ is fixed
- We have:

$$\text{Observable} \sim \sum_{l=\text{loops}} g^l P_l(N) = \sum_k \frac{1}{N_i^k} F_k(\mathcal{A}_i)$$

where $N = \{N_c, N_f, Q\}$ and $\mathcal{A} = \{A_c, A_f, A_Q\}$.

The Results of $O(N)$ Model

(Antipin, Bersini, Sannino, Z.W. Wang and Zhang, “Charging the $O(N)$ model,” PRD 102 (2020) 045011.)

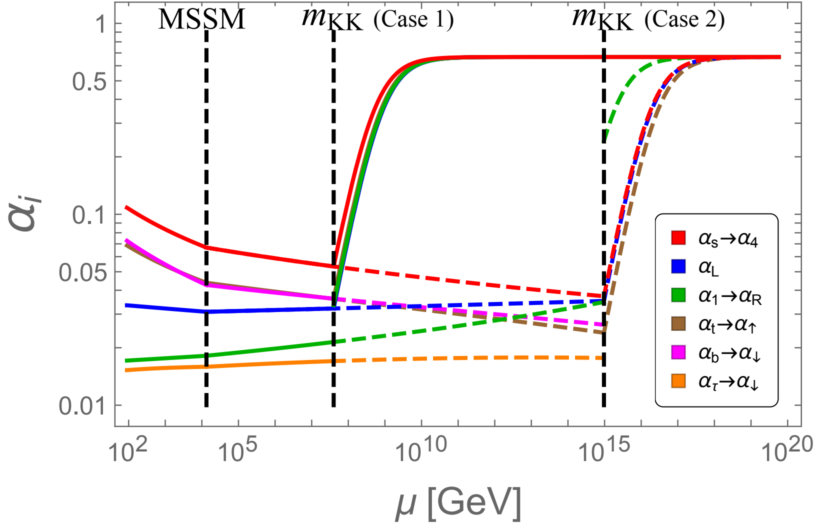
- We calculate the LO and NLO in charge expansion (with small ’t Hooft coupling) but to all order in couplings scaling dimensions of $\bar{Q}$-index traceless symmetric tensor operator $T_{\bar{Q}} \equiv T^{(\bar{Q})}_{i_1 \ldots i_{\bar{Q}}}$ in the $O(N)$ model

$$\Delta_{T_{\bar{Q}}} = \bar{Q} + \left(-\frac{\bar{Q}}{2} + \frac{\bar{Q}(\bar{Q}-1)}{8+N}\right)\epsilon - \left[\frac{184+N(14-3N)}{4(8+N)^3}\bar{Q} + \frac{(N-22)(N+6)}{2(8+N)^3}\bar{Q}^2 + \frac{2}{(8+N)^2}\bar{Q}^3\right]\epsilon^2 + \left[\frac{8}{(8+N)^3}\bar{Q}^4 + \frac{-456-64N+N^2+2(8+N)(14+N)\zeta(3)}{(8+N)^4}\bar{Q}^3 - \frac{-31136-8272N-276N^2+56N^3+N^4+24(N+6)(N+8)(N+26)\zeta(3)}{4(N+8)^5}\bar{Q}^2 + \frac{-65664-8064N+4912N^2+1116N^3+48N^4-N^5+64(N+8)(178+N(37+N))\zeta(3)}{16(N+8)^5}\bar{Q}\right]\epsilon^3 + O\left(\epsilon^4\right) .$$

- At each ϵ order, the semi-classical computation provides term with leading Q and next leading Q shown in red.

Asymptotic Safety via Extra Dimension

(Cacciapaglia, Deandrea, Pasechnik, Z.W.Wang, arXiv:2302.11671,submitted to PRL)



Part II: IR Confinement-Deconfinement Phase Transition

Testing (dark) composite dynamics via gravitational wave

Motivations and what we do

- (Dark) composite dynamics: non perturbative physics, dynamical symmetry breaking, UV completion, naturalness
- (Dark) composite dynamics face challenges to be explored both theoretically and via experiments and thus any extra test is important
- We unify first principle lattice simulations and gravitational wave astronomy to constrain the dark sector

What composes the strongly coupled (hidden) sector?

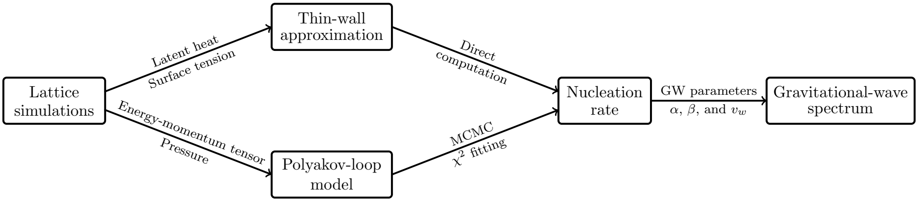
- Dark Yang-Mills theories
- Pure gluons $\Rightarrow$ confinement-deconfinement phase transition
- Gluons + Fermions
  - Fermions in fundamental representation $\Rightarrow$ chiral phase transition
  - Fermions in adjoint rep. $\Rightarrow$ confinement & chiral phase transition
  - Fermions in 2-index symmetric rep. $\Rightarrow$ confinement & chiral phase transition
- Gluons + Fermions + Scalars (not explored yet)

How to describe the strongly coupled sector?

- Pure gluons
  - Polyakov loop model (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005; Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060)
  - Matrix Model (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
  - Holographic QCD model (Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD **105** (2022) 066020; Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRL **128** (2022) 131101)
- Gluons + Fermions
  - Polyakov loop improved Nambu-Jona-Lasinio model (Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003; Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - linear sigma model (Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - Polyakov Quark Meson model (Schaefer, Pawlowski, Wambach, PRD **76** (2007) 074023)

Procedure of pure gluon case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005



Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB 72 (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x}, \tau)\, \mathrm{d}\tau \right]$$

  The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- Convenient to define the trace of the Polyakov loop as an order parameter for the $Z(N)$ symmetry

$$\ell(\vec{x}) = \frac{1}{N}\mathrm{Tr}_c[\mathbf{L}],$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \to e^{i\phi}\ell, \qquad \phi = \frac{2\pi j}{N}, \qquad j = 0, 1, \cdots, (N-1)$$

- The expectation value of ℓ i.e. $< \ell >$ has the important property:

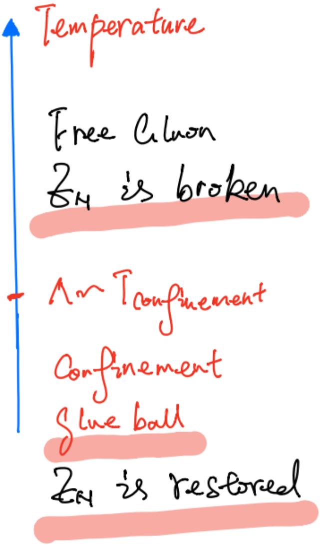
$$\langle \ell \rangle = 0 \quad (T < T_c,\ \mathrm{Confined}); \qquad \langle \ell \rangle > 0 \quad (T > T_c,\ \mathrm{Deconfined})$$

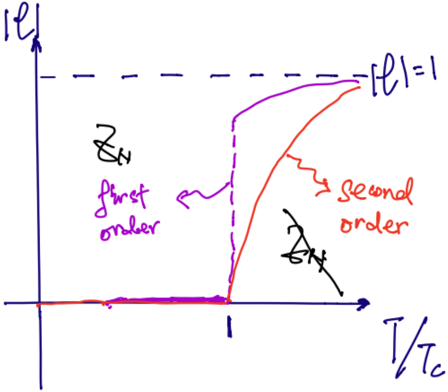
- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle \ell \rangle = \exp\left(i\frac{2\pi j}{N}\right)\ell_0, \qquad j = 0, 1, \cdots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \to 1$ as $T \to \infty$

Summary of Pure Gluon Facts





Second Order for SU(2)

First Order SU(N) (N≥3)

Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\rm PLM}^{(\rm poly)} = T^4\left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \cdots - b_3\left(\ell^N + \ell^{*N}\right)\right)$$

$$\text{where} \quad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 + a_4\left(\frac{T_0}{T}\right)^4$$

"$\cdots$" represent any required lower dimension operator than ℓ^N i.e. $(\ell\ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

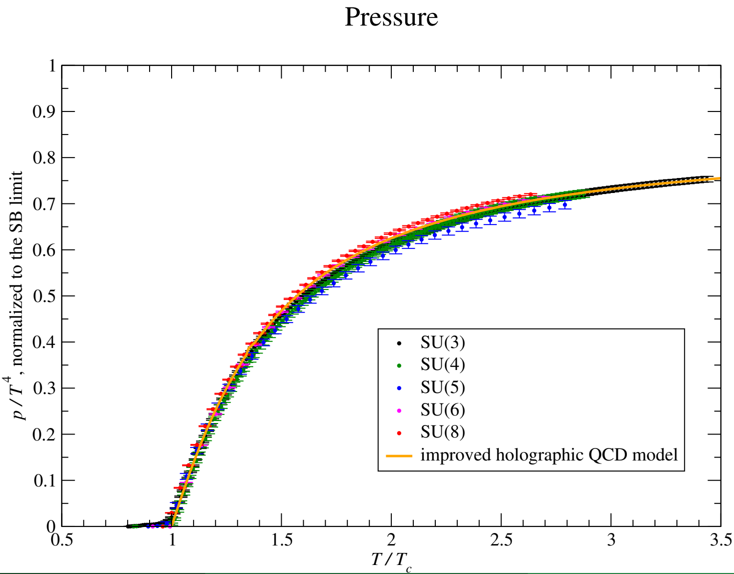
$$V_{\rm PLM}^{(3\log)} = T^4\left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right)\right)$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

- The a_i, b_i coefficients in $V_{\rm PLM}^{(\rm poly)}$ and $V_{\rm PLM}^{(3\log)}$ are determined by fitting the lattice results

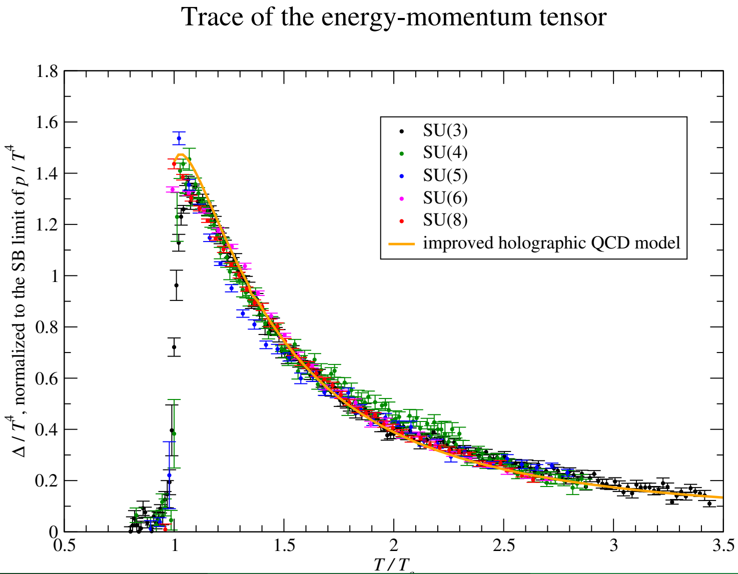
Fitting the Coefficients Using the Lattice Results: I

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

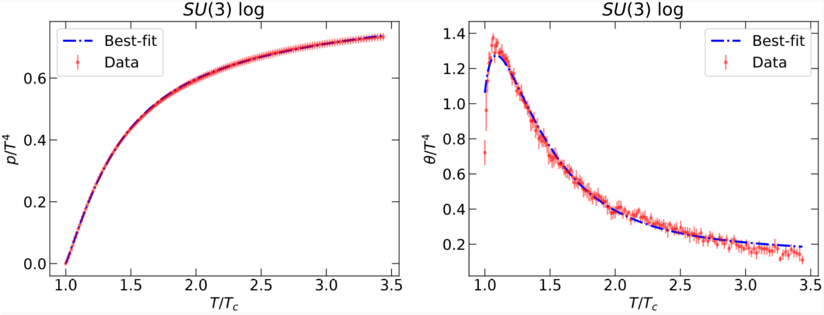
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: III

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Fitted to lattice data of pressure and the trace of energy momentum tensor.



Fitting the Coefficients Using the Lattice Results: IV

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

表: The parameters for the best-fit points.

N	3	3 log	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

Include Fermions

(K. Fukushima, PLB **591** (2004) 277; Ratti, Thaler Weise, PRD **73** (2006) 014019)

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.

- The Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model is used to describe phase-transition dynamics in dark gauge-fermion sectors
- The finite-temperature grand potential of the PNJL models can be generically written as

$$V_{\mathrm{PNJL}} = V_{\mathrm{PLM}}[\ell, \ell^*] + V_{\mathrm{cond}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$$

- $V_{\mathrm{PLM}}[\ell, \ell^*]$ is the Polyakov loop model potential (discussed above)
- $V_{\mathrm{cond}}\left[\langle\bar{\psi}\psi\rangle\right]$ represents the condensate energy
- $V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right]$ denotes the fermion zero-point energy
- The medium potential $V_{\mathrm{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$ encodes the interactions between the chiral and gauge sector which arises from an integration over the quark fields coupled to a background gauge field

PNJL Potential I (Fukushima, Skokov, PPNP **96** (2017) 154)

- The condensate energy is obtained from 4-point and 6-point interaction and using the mean field approximation.
- We obtain the total condensate energy:

$$V_{\rm cond} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3\,, \qquad \sigma \equiv \langle \bar{u}u\rangle = \langle \bar{d}d\rangle = \langle \bar{s}s\rangle = \frac{1}{3}\langle\bar{\psi}\psi\rangle$$

- The total constituent quark mass from $\mathcal{L}_{\rm 4F}$ and $\mathcal{L}_{\rm 6F}$ is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\rm zero}\left[\langle\bar{\psi}\psi\rangle\right] = -\dim({\rm R})\,2N_f\int\frac{{\rm d}^3p}{(2\pi)^3}E_p\,, \qquad E_p = \sqrt{\vec{p}^{\,2}+M^2}$$

  E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$
- The above integration diverges and a regularization is required. We choose a sharp three-momentum cutoff Λ entering the expression for observables and thus also a parameter of the theory.

PNJL Potential II: Medium Potential

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\text{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(\ln\left[1 + e^{-\beta(E-\mu)}\right] + \ln\left[1 + e^{-\beta(E+\mu)}\right] \right)$$

$$\to -2T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \text{Tr}_c \left\{ \left(\ln\left[1 + \mathbf{L}\, e^{-\beta(E-\mu)}\right] + \ln\left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)}\right] \right) \right\}$$

- $\mathbf{L}$ is the Polyakov loop:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x}, \tau)\, d\tau \right]$$

- In this work, we consider chemical potential $\mu = 0$.

Bubble Nucleation and Gravitational Wave

Bubble Nucleation: Generic Discussion

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

- The generic three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\rho}{\mathrm{d}r} \right)^2 + V_{\text{eff}}(\rho, T) \right] ,$$

where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$

Bubble Nucleation: Confinement Phase Transition

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

- Confinement phase transition occurs for pure gluon and adjoint fermions
- $[\ell] = 0$ dimensionless while $[\rho] = 1$, we rewrite ρ as $\rho = \ell\, T$ and convert the radius into a dimensionless quantity $r' = r\, T$:

$$S_3(T) = 4\pi T \int_0^\infty \mathrm{d}r'\, r'^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\ell}{\mathrm{d}r'} \right)^2 + V'_{\text{eff}}(\ell, T) \right] ,$$

which has the same form as the above generic equation.
- The bubble profile (instanton solution) is obtained by solving the E.O.M. of the $S_3(T)$

$$\frac{\mathrm{d}^2\ell(r')}{\mathrm{d}r'^2} + \frac{2}{r'} \frac{\mathrm{d}\ell(r')}{\mathrm{d}r'} - \frac{\partial V'_{\text{eff}}(\ell, T)}{\partial \ell} = 0$$

- The boundary conditions (deconfinement $\to$ confinement) are

$$\frac{\mathrm{d}\ell(r' = 0, T)}{\mathrm{d}r'} = 0\,, \qquad \lim_{r' \to 0} \ell(r', T) = 0$$

- We used the method of overshooting/undershooting (`Python` package)

Bubble Profile of Confinement Phase Transition

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

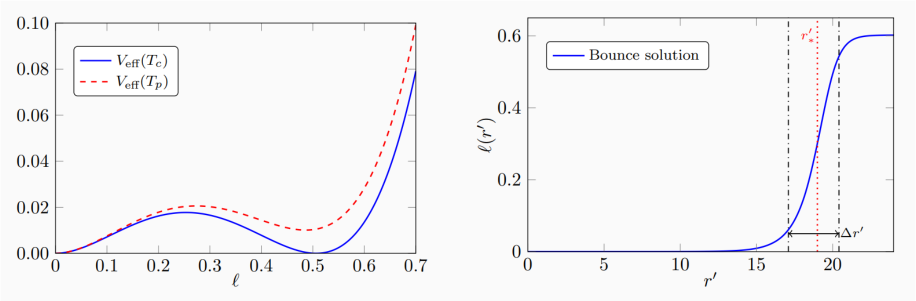


图: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$) lying the confinement phase, the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$) lying the deconfinement phase, the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Bubble Nucleation: Chiral Phase Transition

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552)

- Chiral phase transition occurs when including fermions
- $\bar{\sigma}$ is classically nonpropagating in PNJL and it's kinetic term is induced only via quantum fluctuations
- We thus include its wave-function renormalization Z_σ with

$$Z_\sigma^{-1} = -\frac{\mathrm{d}\Gamma_{\sigma\sigma}(q^0, \mathbf{q}, \bar{\sigma})}{\mathrm{d}\mathbf{q}^2}\bigg|_{q^0=0, \mathbf{q}^2=0}, \qquad \Gamma_{\sigma\sigma} = -i \sum 2\,\text{point 1PI}\,\sigma\sigma\,\text{graph}$$

- The three-dimensional Euclidean action and E.O.M. are modified to:

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{Z_\sigma^{-1}}{2}\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r}\right)^2 + V_{\text{eff}}(\bar{\sigma}, T)\right]$$

$$\frac{\mathrm{d}^2\bar{\sigma}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} - \frac{1}{2}\frac{\partial \log Z_\sigma}{\partial \bar{\sigma}}\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r}\right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{\mathrm{d}\bar{\sigma}(r=0, T)}{\mathrm{d}r} = 0\,, \qquad \lim_{r\to\infty} \bar{\sigma}(r, T) = 0$$

Gravitational Wave Parameters: Inverse Duration Time

- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use percolation temperature T_p: the temperature at which 34% of false vacuum is converted
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*) e^{\beta(t-t_*)}$$

- The inverse duration time then follows as

$$\beta = -\frac{\mathrm{d}}{\mathrm{d}t} \frac{S_3(T)}{T}\bigg|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T}\bigg|_{T=T_*},$$

where we used that $\mathrm{d}T/\mathrm{d}t = -H(T)T$.

Gravitational Wave Parameters: Strength Parameter I

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022)003, arXiv:2109.11552.)

- We define the strength parameter α from the trace of the energy-momentum tensor θ weighted by the enthalpy

$$\alpha = \frac{1}{3}\frac{\Delta\theta}{w_+} = \frac{1}{3}\frac{\Delta e \; - \; 3\Delta p}{w_+}\,, \qquad \Delta X = X^{(+)} - X^{(-)}, \text{ for } X = (\theta, e, p)$$

  $(+)$ denotes the meta-stable phase (outside of the bubble) while $(-)$ denotes the stable phase (inside of the bubble).

- The relations between enthalpy w, pressure p, and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}\,, \qquad e = \frac{\partial p}{\partial \ln T} - p\,,$$

  which are extracted from the effective potential with

$$p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

Gravitational Wave Parameters: Strength Parameter II

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

- α is thus given by

$$\alpha = \frac{1}{3} \frac{4\Delta V_{\text{eff}} - T\frac{\partial \Delta V_{\text{eff}}}{\partial T}}{-T\frac{\partial V_{\text{eff}}^{(+)}}{\partial T}} ,$$

- For confinement phase transition: $\alpha \approx 1/3$ (ΔV_{eff} is negligible since $e_+ \gg p_+$ and $e_- \sim p_- \sim 0$ in PLM potential)
- For chiral phase transition: we find smaller values, $\alpha \sim \mathcal{O}(10^{-2})$, due to the fact that more relativistic d.o.f.s participate in the phase transition
- Relativistic SM d.o.f.s do not contribute to our definition of α since they are fully decoupled from the phase transition but these d.o.f.s will play a role to dilute the GW signals

GW parameters α, β and PNJL observables

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

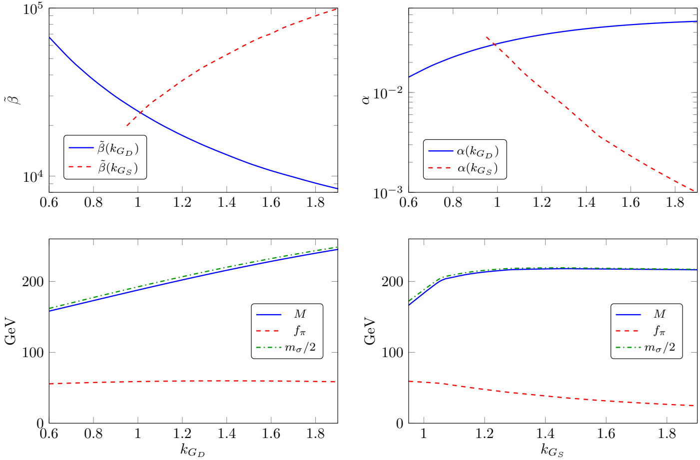


图: The GW parameters $\tilde{\beta}$, α with the observables M, f_π, and m_σ as a function of $G_S = k_{G_S} \cdot 4.6\,\text{GeV}^{-2}$ and $G_D = k_{G_D} \cdot (-743\,\text{GeV}^{-5})$. We use $T_c = 100\,\text{GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S,\text{crit}} = 0.882$, no chiral symmetry breaking occurs.

Gravitational-wave spectrum

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

- Contributions from bubble collision and turbulence are subleading
- The GW spectrum from sound waves is given by

$$h^2\Omega_{\mathrm{GW}}(f) = h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}}\left(\frac{f}{f_{\mathrm{peak}}}\right)^3\left[\frac{4}{7}+\frac{3}{7}\left(\frac{f}{f_{\mathrm{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\mathrm{peak}} \simeq 1.9\cdot 10^{-5}\,\mathrm{Hz}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T}{100\,\mathrm{GeV}}\right)\left(\frac{\tilde{\beta}}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \simeq 2.65\cdot 10^{-6}\left(\frac{v_w}{\tilde{\beta}}\right)\left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Omega_{\mathrm{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.

$$\Omega_{\mathrm{dark}} = \frac{\rho_{\mathrm{rad,dark}}}{\rho_{\mathrm{rad,tot}}} = \frac{g_{*,\mathrm{dark}}}{g_{*,\mathrm{dark}}+g_{*,\mathrm{SM}}}$$

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{sw} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\text{sw}} = \sqrt{\tau_{\text{sw}}}\,\kappa_v$$

- τ_{sw} is dimensionless and measured in units of the Hubble time (H.-K. Guo, Sinha, Vagie and White, JCAP **01** (2021) 001)

$$\tau_{\text{sw}} = 1 - 1/\sqrt{1 + 2\frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f}} \Rightarrow \tau_{\text{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \quad \text{for } \beta >> 1$$

where $\bar{U}_f$ is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} \mathrm{d}\xi\, \xi^2 \frac{v(\xi)^2}{1 - v(\xi)^2} \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w. At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

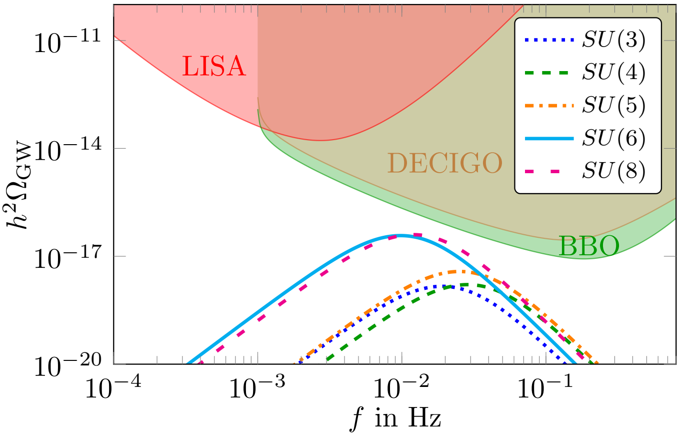


图: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with Tc$= 1\,\mathrm{GeV}$.

A Landscape of GW Signatures with Pure Gluon

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

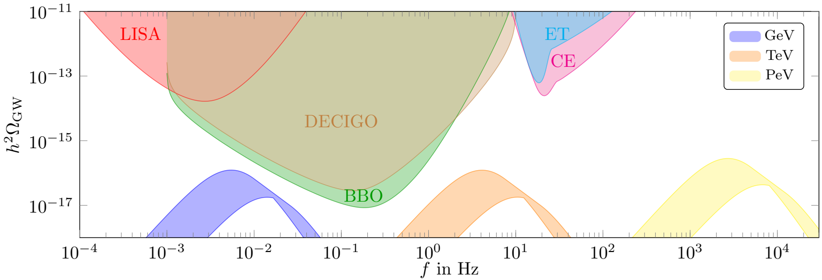


图: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Signal to Noise Ratio

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

$$\mathrm{SNR} = \sqrt{\frac{3\mathrm{year}}{s} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left(\frac{h^2 \Omega_{\mathrm{GW}}}{h^2 \Omega_{\mathrm{det}}} \right)^2 }$$

$h^2\Omega_{\mathrm{GW}}$ is the GW spectrum while Ω_{det} is the sensitivity curve of the detector.

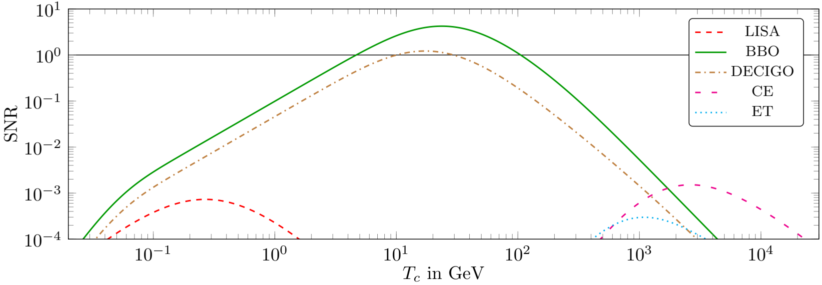


图: We display the SNR for the phase transition in a dark $SU(6)$ sector as a function of the confinement temperature Tc from experiments of LISA, BBO, DECIGO, CE, and ET. We assume an observation time of three years.

Landscape of GW spectrum with three Dirac fermions

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

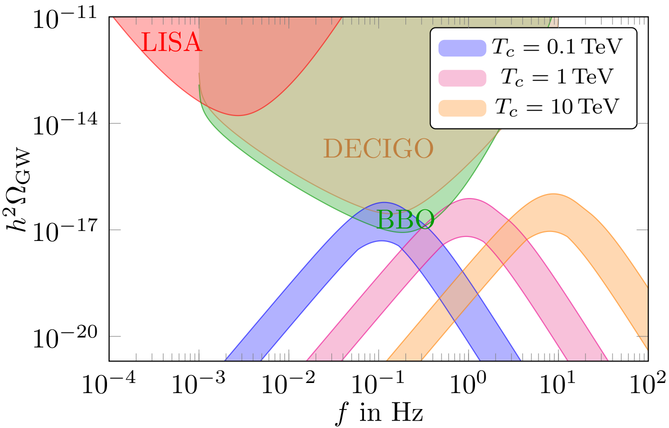


图: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures. The band comes from varying wall velocity $c_s \leq v_w \leq 1$.

Representation Matters

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

Rep.	flavour	chiral PT	conf.-deconf.
Fund.	3	1st	X
adjoint	1	2nd	1st
2-index Sym.	1	2nd	1st

表: Representations versus different phase transitions.

- Need small N_f to remain below the conformal Banks-Zaks window ($N_f \leq 2$ for adjoint and $N_f \leq 3$ for 2-index symmetric under $SU(3)$).

Signal to Noise Ratio for Different Representations

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

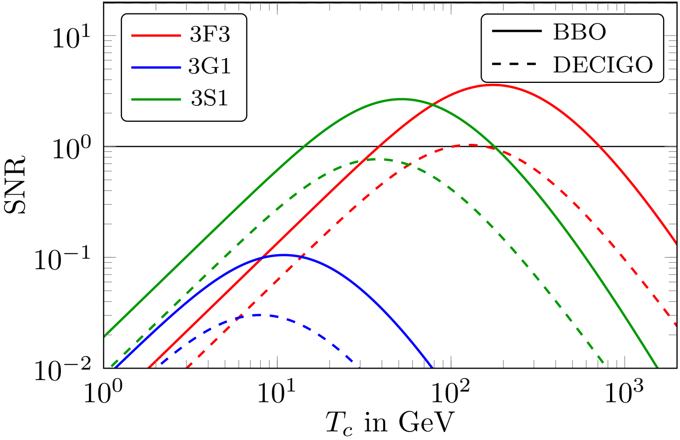


图: Signal-to-noise ratio as a function of the critical temperature for the best-case scenarios of each model at BBO and DECIGO with an observation time of 3 years.

Future Thinking

- The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3} \frac{\sigma(T_c)^3}{L(T_c)^2} \frac{T_c^2}{(T_c - T)^2} \,,$$

- The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T}\bigg|_{T=T_p} \approx \mathcal{O}(10^3) \frac{T_c^{1/2} L}{\sigma^{3/2}} \,.$$

- $\tilde{\beta}$ stems from the competition between the surface tension and latent heat. $L \sim N^2$ while σ can be either $\sigma \sim N$ or $\sigma \sim N^2$ with limited data up to $SU(8)$
- How to construct models with smaller latent heat and larger surface tension to enhance the gravitational wave signals?

Glueball Potential

(Carenza, Pasechnik, Salinas，Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- The dark gluon-glueball dynamics can be effectively described by considering the dimension-4 glueball field $\mathcal{H} \propto \text{tr}(G^{\mu\nu}G_{\mu\nu})$:

$$V\left[\mathcal{H},\ell\right] = \frac{\mathcal{H}}{2}\ln\left[\frac{\mathcal{H}}{\Lambda^4}\right] + T^4\mathcal{V}\left[\ell\right] + \mathcal{H}\mathcal{P}[\ell] + V_T\left[\mathcal{H}\right] \, .$$

- To canonically normalize this field, we introduce ϕ as $\mathcal{H} = 2^{-8}c^{-2}\phi^4$
- We keep the lowest order in $\mathcal{P}[\ell]$ to satisfy the symmetry:

$$\mathcal{P}[\ell] = c_1|\ell|^2 \, ,$$

where c_1 will be determined my the lattice results.

Cosmological evolution of the dark glueball field

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- The glueball field is considered homogeneous and evolves in expanding FLRW universe, with the e.o.m.

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V[\phi, T] = 0\,,$$

- The time variable is found in terms of the photon temperature:

$$t = \frac{1}{2}\sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_\gamma)}}\frac{m_P}{T_\gamma^2}\,, \qquad T_\gamma = \xi_T T$$

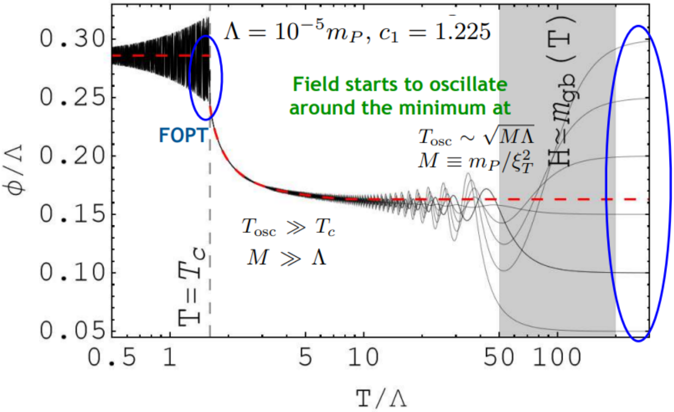
where ξ_T denotes the visible-to-dark sector temperature ratio and $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass and $g_{*,\rho}$ is the number of energy-related degrees of freedom.

- E.o.m. in terms of the dark sector temperature:

$$\frac{4\pi^3 g_{*,\rho}}{45 m_P^2}\xi_T^4 T^6 \frac{d^2\phi}{dT^2} + \frac{2\pi^3}{45 m_P^2}\frac{dg_{*,\rho}}{dT}\xi_T^4 T^6 \frac{d\phi}{dT} + \partial_\phi V[\phi, T] = 0$$

Cosmological Evolution of the Dark Glueball Field

(Carenza, Pasechnik, Salinas，Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)



- In early times in deconfined regime, for different initial conditions the field evolution follows the minimum (red dashed line).
- First order phase transition washes out any dependence on initial conditions.

Glueball Relic Density

(Carenza, Pasechnik, Salinas，Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- Below freeze-out temperature, the predicted glueball relic density is

$$0.12\zeta_T^{-3}\frac{\Lambda}{137.9\,\mathrm{eV}} \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3}\frac{\Lambda}{82.7\,\mathrm{eV}}, \quad 1.035 < c_1 < 1.415$$

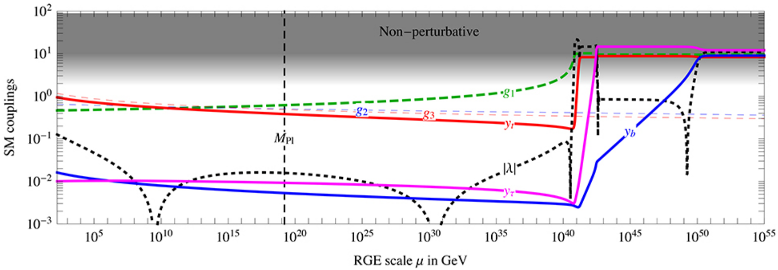
- It is more than a factor of 10 difference compared to the old calculations

$$\Omega h^2 \sim 0.12\,\zeta_T^{-3}\frac{\Lambda}{5.45\,\mathrm{eV}}$$

Thank you for your attention!

- Field: Gauge fields + Fermions + Scalars
- Interactions: Gauge ($SU(3) \times SU(2)_L \times U(1)$) + Yukawa (Fermions Mass) + Scalar self-interaction
- Not UV Complete: the theory is not well defined at very high energy scale
- $U(1)$ gauge coupling runs into Landau Pole

SM RGE at 3 loops in $g_{1,2,3}, y_t, \lambda$ and at 2 loops in $y_{b,\tau}$



G. M.

Pelaggi, F. Sannino, A. Strumia and E. Vigiani, Front. in Phys. **5** (2017) 49

Pole Structure Crisis

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770;

G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Pole in the summation functions:

$$H_\phi\left(0, \tfrac{2}{3}A_\alpha\right) \sim \frac{1}{\frac{15}{2} - A_\alpha}, \; H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) \sim \frac{1}{A_\alpha - \frac{15}{2}},$$

$$\frac{\partial}{\partial A_\alpha} H_\lambda\left(1, \tfrac{2}{3}A_\alpha\right) \sim -\frac{1}{\left(A_\alpha - \frac{15}{2}\right)^2}$$

- Pole structure of Yukawa coupling (multiplicative proportional to y): approaching asymptotically free quickly

$$\beta_y \sim c_1 y^3 + y A_\alpha \left(\frac{1}{A_\alpha - \frac{15}{2}}\right)(c_2 + c_3 A_\alpha)$$

- Pole structure of Quartic Coupling (**Not** multiplicative proportional to λ): blow up to very negative value!

$$\beta_\lambda \sim c_1 \lambda^2 + c_2 \lambda A_\alpha \left(\frac{1}{A_\alpha - \frac{15}{2}}\right) + c_3 A_\alpha^2 \left(\frac{1}{A_\alpha - \frac{15}{2}} - \frac{1}{\left(A_\alpha - \frac{15}{2}\right)^2}\right)$$

$U(1)$ Landau Pole Problem Recap and Alternative Motivation to Study Safe GUT Embedding

- Two ways to address the $U(1)$ problem
  - Embedding in a non-abelian group
  - $U(1)$ safety with large N_f
- $U(1)$ problem is not successfully addressed in the large N_f framework
  - the mass anomalous dimension blows up at the Abelian pole place (Antipin and Sannino, Phys. Rev. D **97** (2018) 116007, arXiv:1709.02354.)
  - the quartic coupling will also blow up at the abelian pole (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Semi-simple gauge does not help (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Yukawa summation does not help (T. Alanne and S. Blasi, Phys. Rev. D **98** (2018) 116004, arXiv:1808.03252.)
- The **incompatibility** between the $U(1)$ and Higgs self coupling motivates the study of a safe GUT theory where U(1) is embedded in a non-abelian group.

Gravitational Waves from Pati-Salam Dynamics

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The detection of stochastic gravitational wave generated through strong 1st order phase transition can help to explore the high energy physics beyond Collider.
- Pati-Salam model is particularly interesting because strong first order phase transition at few $1000\,\mathrm{TeV}$ scale will typically generate gravitational wave with peak frequency at $10-100\,\mathrm{Hz}$ in the detection region of aLIGO.
- We study the stochastic gravitational wave signatures from both safe and non-safe Pati-Salam model.
- Safe scenario has strong predictive power providing a much smaller parameter space which we use as seed values to explore the full parameter space beyond safety.

Relevant Scalar Sector of Pati-Salam Model

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- In order to induce the breaking of $G_{\rm PS}$ to the SM gauge group, we introduce a scalar field ϕ_R which transforms as the fermion multiplet ψ_R, that is, $\phi_R \sim (4,1,2)$:

$$\phi_R = \left(\begin{array}{cccc} \phi_R^{u_1} & \phi_R^{u_2} & \phi_R^{u_3} & \phi_R^{0} \\ \phi_R^{d_1} & \phi_R^{d_2} & \phi_R^{d_3} & \phi_R^{-} \end{array} \right) ,$$

  where the neutral component ϕ_R^0 takes a non-zero vev, $\langle \phi_R^0 \rangle \equiv v_R$, such that $G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.
- The relevant terms in the tree level effective potential can be written as:

$$V_{\rm tree}\left(\phi_R\right) = \lambda_{R1} {\rm Tr}^2 \left(\phi_R^\dagger \phi_R\right) + \lambda_{R2}\, {\rm Tr} \left(\phi_R^\dagger \phi_R \phi_R^\dagger \phi_R\right) \, .$$

- Note that we do not include any explicit mass terms in the tree level potential. The symmetry breaking in this work is induced by Coleman-Weinberg mechanism.

Strong First Oder Phase Transition

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The total finite temperature effective potential of the Pati-Salam model:

$$V_{\rm eff}\left[\phi, T\right] = V_{\rm tree} + V_{\rm 1loop} + V_T^{\rm tot} + V_{\rm ring}^{\rm scalar,tot} + V_{\rm ring}^{\rm gauge,tot} \, .$$

- A positive non-trivial (away from the origin) minimum occurs for $\phi_{Rc} \sim 8400\,{\rm TeV}$ and thus $\phi_{Rc}/T_c \sim 3.13 > 1$. This shows that the associated phase transition is a **strong first order** one.

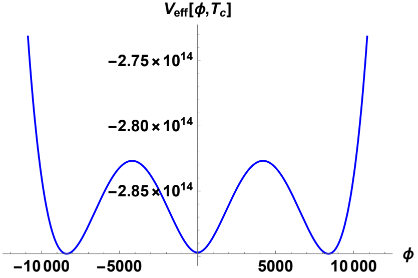


图: We plot the finite temperature effective potential. The renormalization scale μ is set at $5000\,{\rm TeV}$ while the temperature is chosen at $T = T_c = 2680\,{\rm TeV}$ which is the critical temperature.

Using Stream Plot to Locate the Parameter Space

(Huang, Sannino and Z.W.Wang, PRD **102** (2020) 095025, arXiv:2004.02332.)

- The parameter space to have strong first order phase transition is located at the right lower corner!

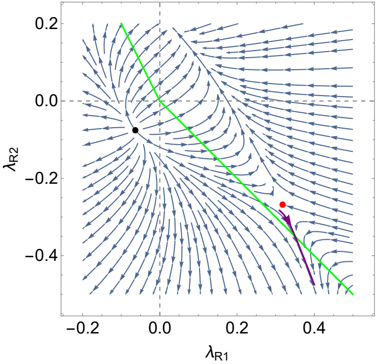


图: The flow direction is defined from UV to IR. The red and black plots are both the fixed point. The two green lines are the symmetry breaking lines which are defined as $\lambda_{R1}+\lambda_{R2}=0$ for $\lambda_{R2}<0$ and $\lambda_{R2}/2+\lambda_{R1}=0$ for $\lambda_{R2}>0$. The purple line is the particular RG flow corresponding to the safe solution.

About Center Symmetry and Confinement

- The standard physical interpretation is that it is related to the free energy of adding an external static color source in the fundamental representation.

$$\ell(\vec{x}) = \exp(-\mathrm{F}\beta)$$

- In the confinement phase, Polyakov loop is zero corresponds to infinity free energy to add a color source and the same time center symmetry is unbroken.

Center Symmetry $Z(N)$ at Nonzero Temperature

- The boundary conditions in imaginary time τ the fields must satisfy are:

$$A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0), \qquad q(\vec{x},\beta) = -q(\vec{x},0) \ ,$$

where gluons as bosons must be periodic in τ while quarks as fermions must be anti-periodic.

- 't Hooft first noticed that one can consider more general gauge transformations which are only periodic up to Ω_c

$$\Omega(\vec{x},\beta) = \Omega_c, \quad \Omega(\vec{x},0) = 1 \qquad \left(\text{here, } \Omega_c = e^{i\phi}I, \ \phi = \frac{2\pi j}{N}\right).$$

- Color adjoint fields are invariant under this transformation, while those in the fundamental representation are not:

$$A^\Omega(\vec{x},\beta) = \Omega_c^\dagger A_\mu(\vec{x},\beta)\,\Omega_c = A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0) \ ,$$
$$q^\Omega(\vec{x},\beta) = \Omega_c^\dagger q(\vec{x},\beta) = e^{-i\phi} q(\vec{x},\beta) \neq -q(\vec{x},0) \ .$$

- Thermal Wilson line transforms like an adjoint field under local $SU(N)$ gauge transformations:

$$L(x) \to \Omega^\dagger(\vec{x},\beta)\, L(\vec{x})\Omega^\dagger(\vec{x},0) \ .$$

About Thin Wall Approximation

The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3}\frac{\sigma(T_c)^3}{L(T_c)^2}\frac{T_c^2}{(T_c - T)^2} ,$$

The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale. From this we infer that

$$T_c - T_p \approx \sqrt{\frac{16\pi\sigma^3 T_c}{3L^2 \cdot \mathcal{O}(150)}} ,$$

and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T\frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T=T_p} \approx \mathcal{O}(10^3)\frac{T_c^{1/2}L}{\sigma^{3/2}} .$$

$\tilde{\beta}$ stems from the competition between the surface tension and latent heat.

The PNJL model Lagrangian (Fukushima, Skokov, PPNP **96** (2017) 154)

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\mathrm{PNJL}} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{\mathrm{4F}} + \mathcal{L}_{\mathrm{6F}} + \mathcal{L}_{k}$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{\mathrm{4F}}$ is the four-quark interaction which reads:

$$\mathcal{L}_{\mathrm{4F}} = G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma^5\lambda^a\psi)^2], \qquad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{\mathrm{6F}}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{\mathrm{6F}} = G_D[\det(\bar{\psi}_{Li}\psi_{Rj}) + \det(\bar{\psi}_{Ri}\psi_{Lj})]$$

The Constituent Quark Mass and Zero Point Energy: I

(Fukushima, Skokov, PPNP **96** (2017) 154)

- In $\mathcal{L}_{6\mathrm{F}}$, there is also $\langle \bar{u}u \rangle^2 \bar{u}u$ term contributes to the constituent quark mass of u
- The total constituent quark mass from $\mathcal{L}_{4\mathrm{F}}$ and $\mathcal{L}_{6\mathrm{F}}$ is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\mathrm{dim}(\mathrm{R})\, 2N_f \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p\,, \qquad E_p = \sqrt{\vec{p}^{\,2} + M^2}$$

  E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$

- The above integration diverges and a regularization is required. We choose a sharp three-momentum cutoff Λ entering the expression for observables and thus also a parameter of the theory.
- Parameters: $G_S,\ G_D,\ \Lambda$; Observables: $M,\ f_\pi,\ m_\sigma$

The Constituent Quark Mass and Zero Point Energy: II

(Fukushima, Skokov, PPNP **96** (2017) 154)

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\frac{\dim(\mathrm{R}) N_f \Lambda^4}{8\pi^2}\left[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right],$$

in which $\xi \equiv \frac{M}{\Lambda}$.