Quark and hadron masses from CLQCD ensembles

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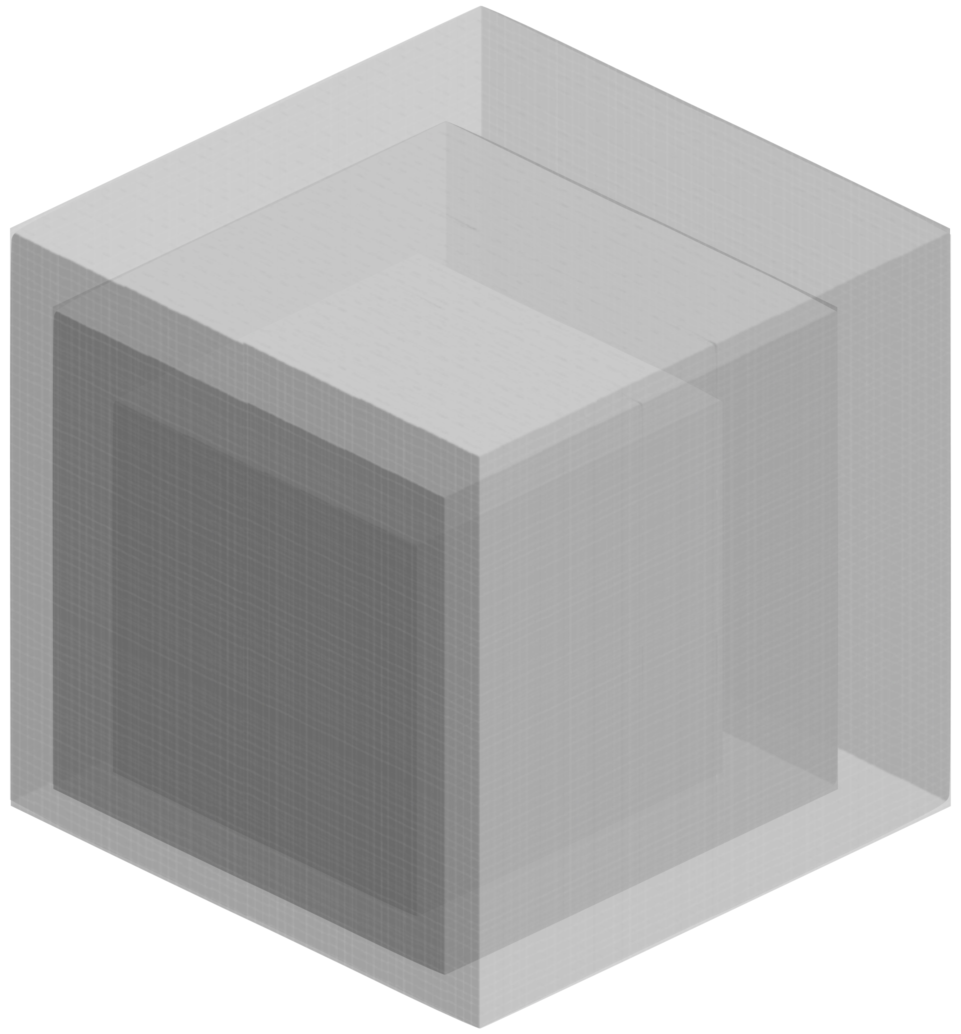
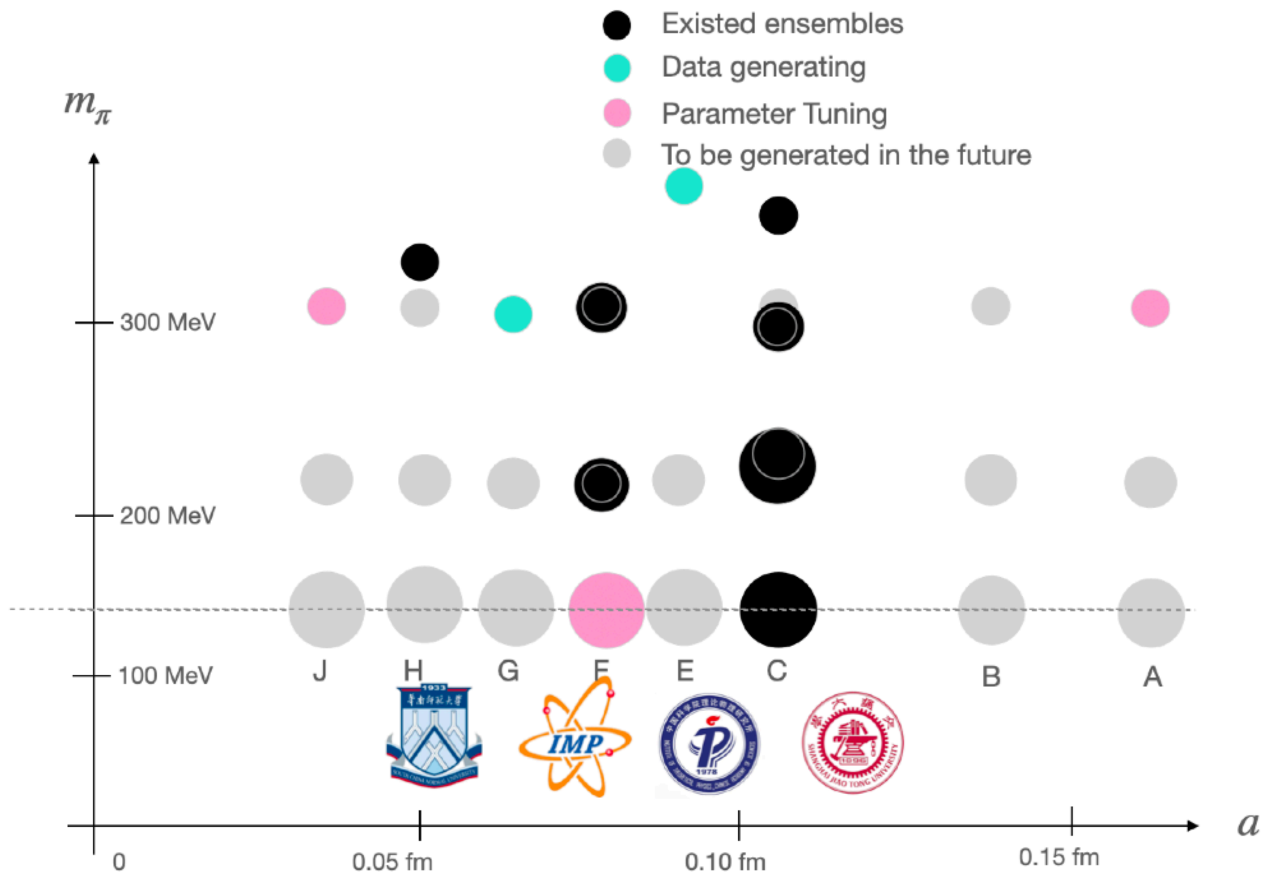
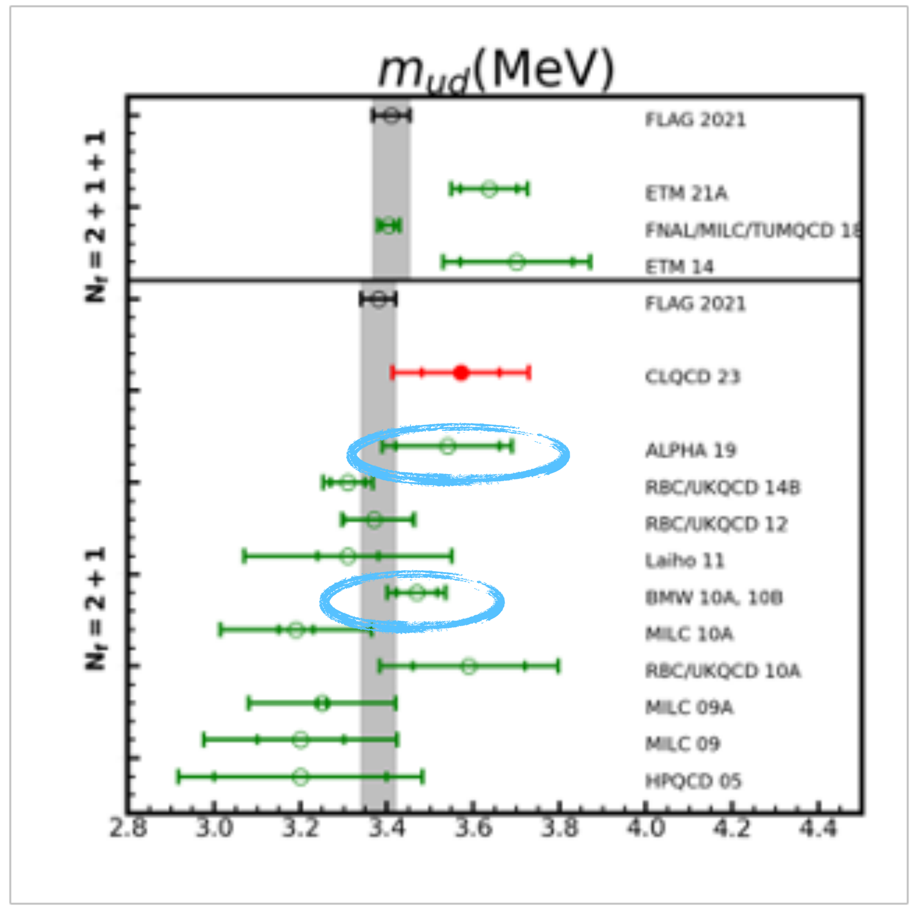
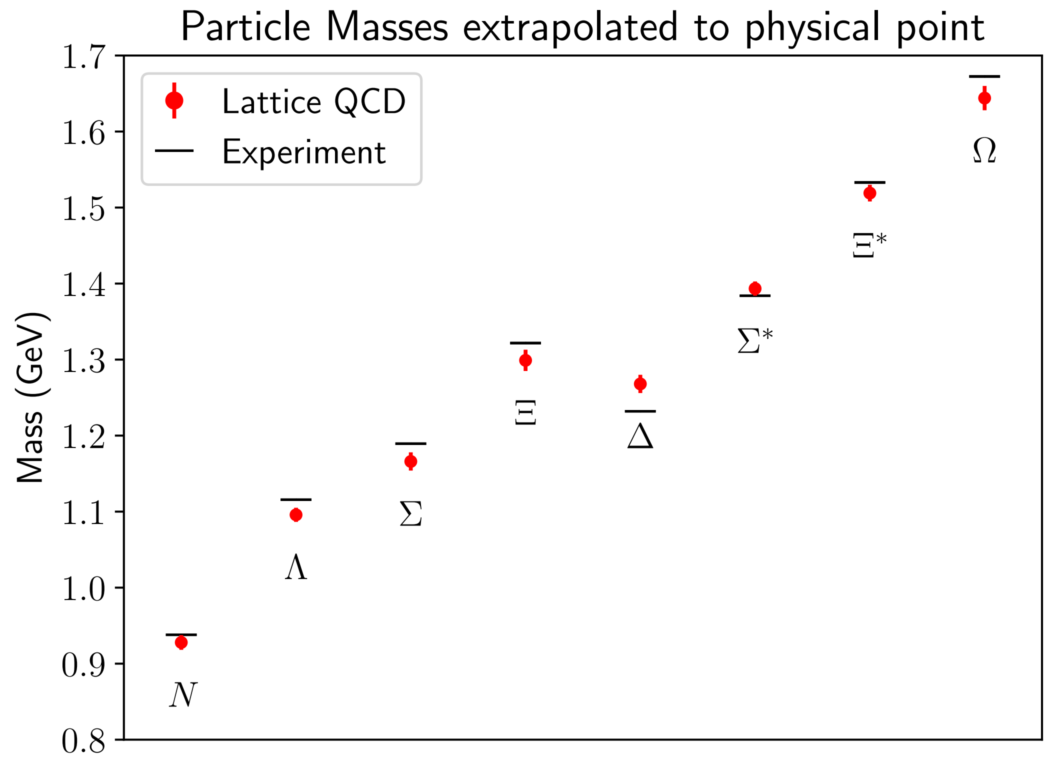
Outline

- **Lattice QCD background**

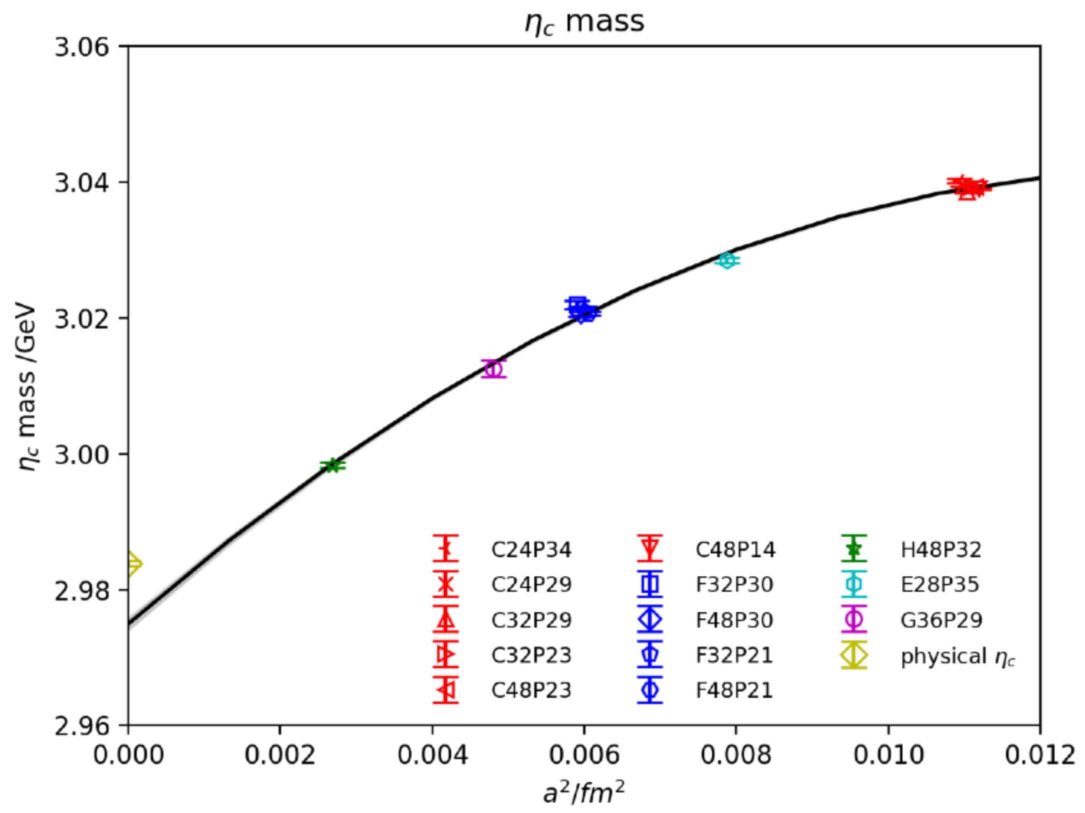


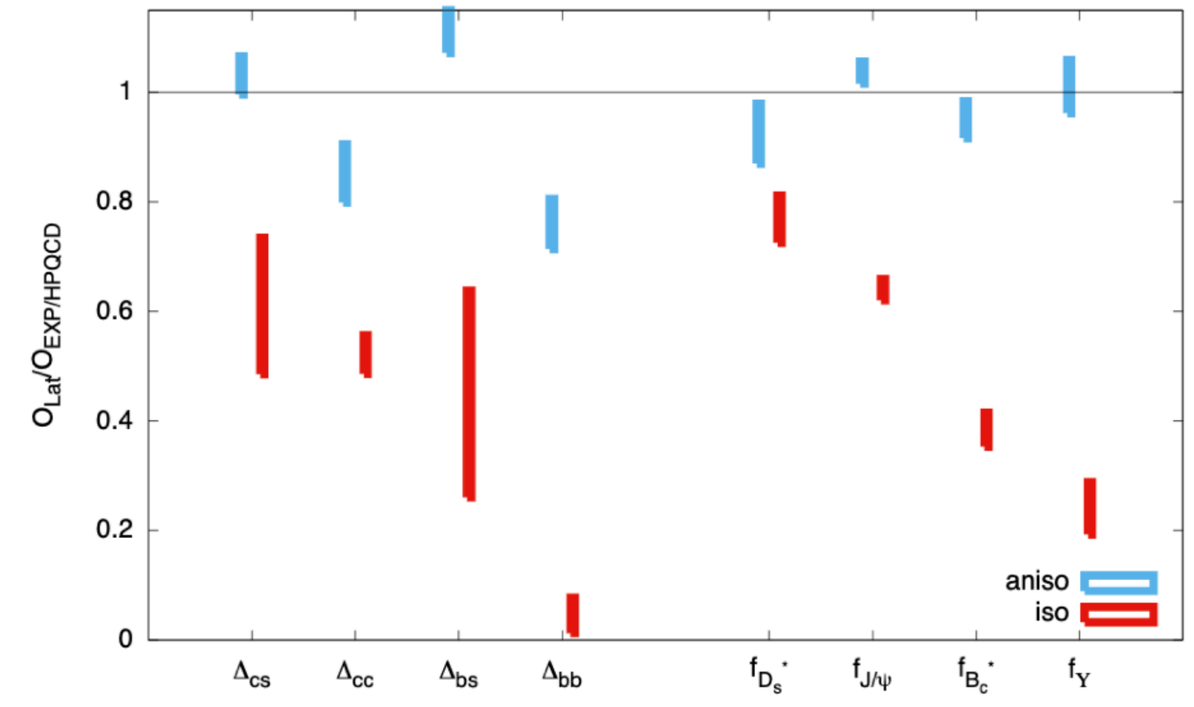






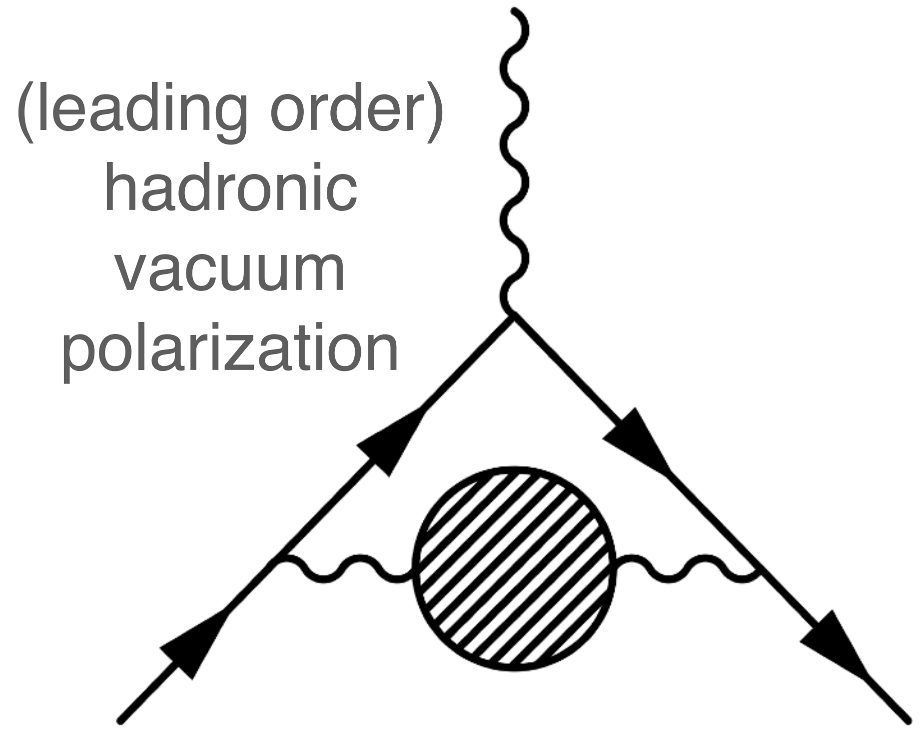
- Light quark and hadron masses



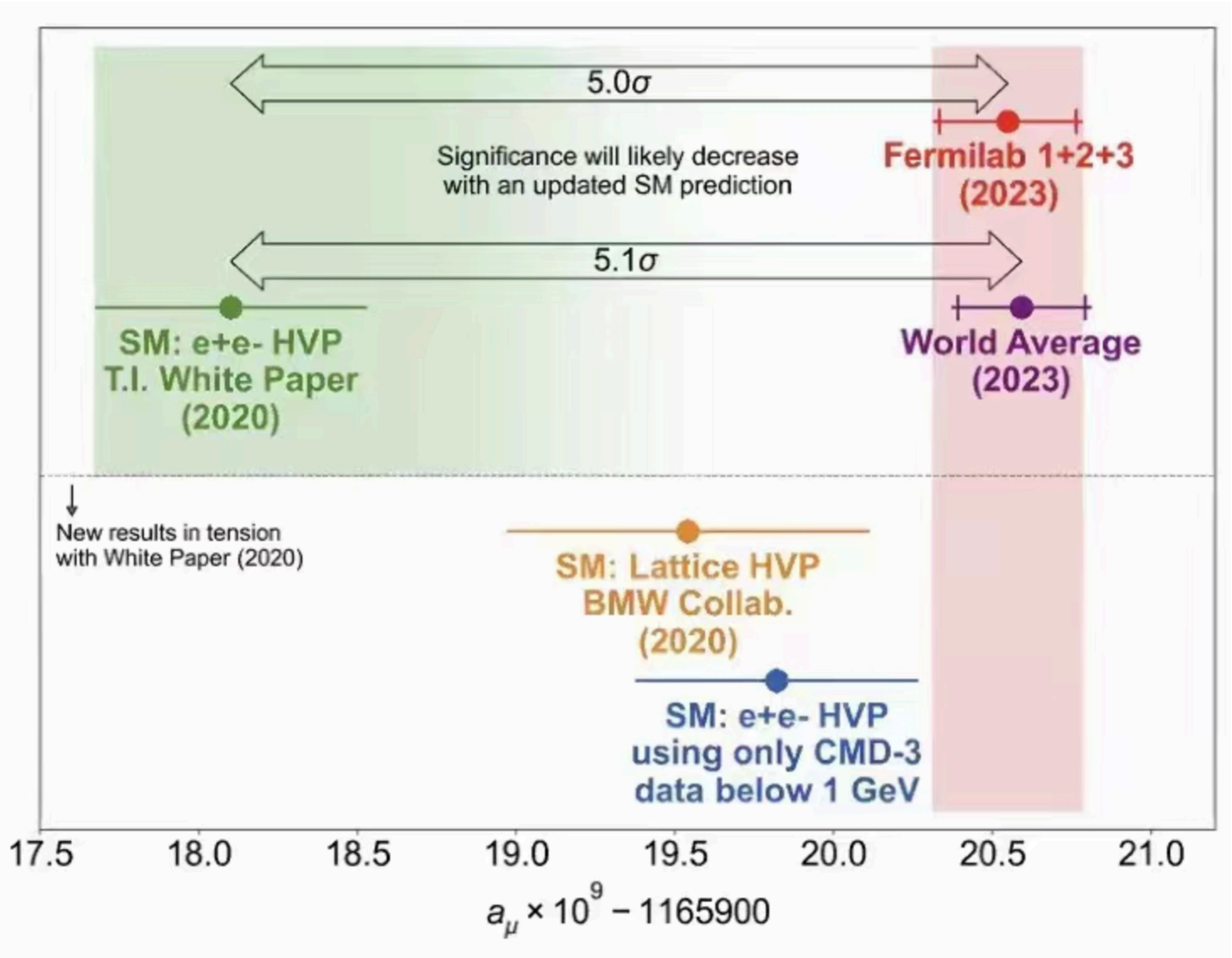


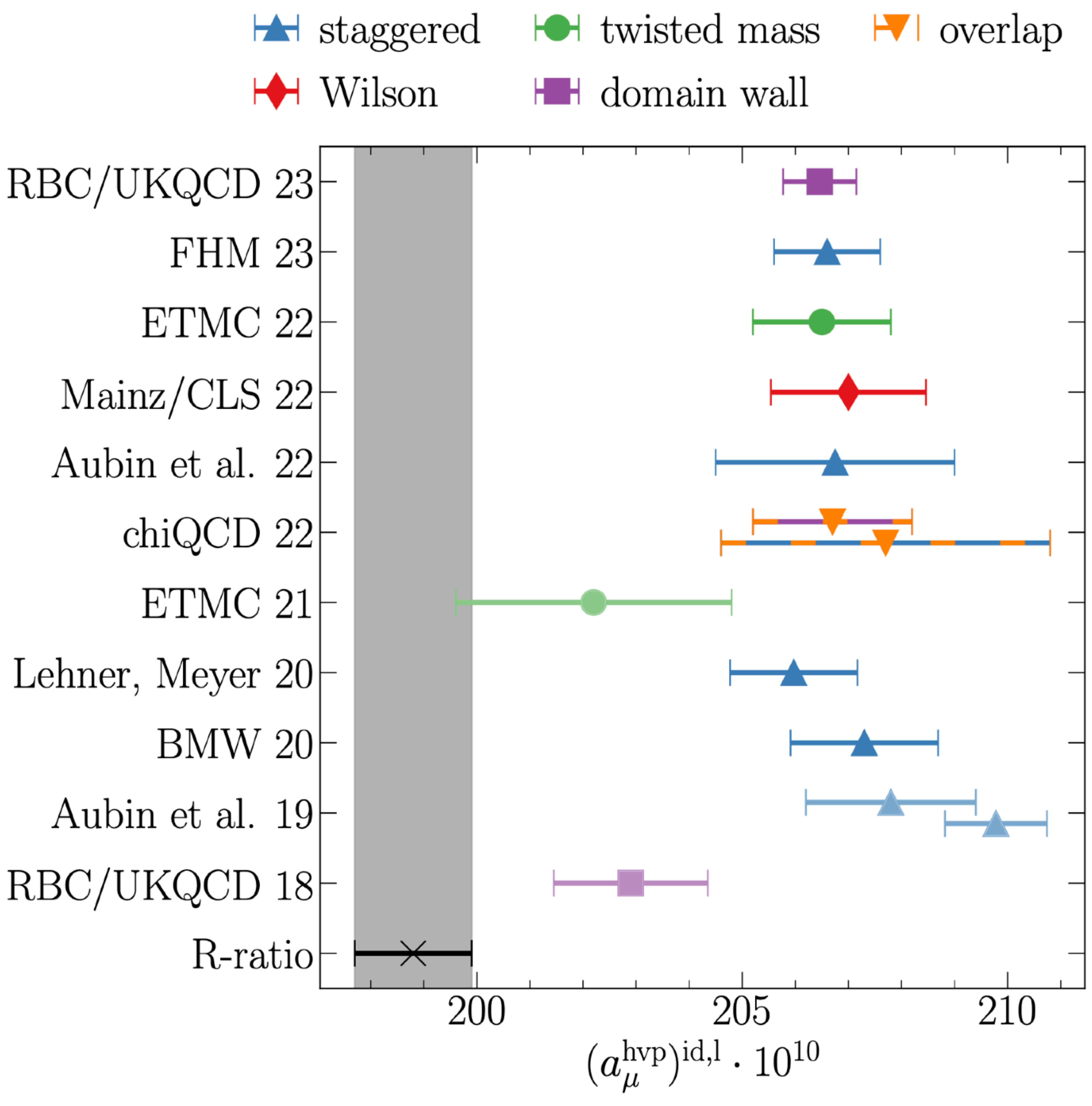
- Investigation on the charm quark

Background



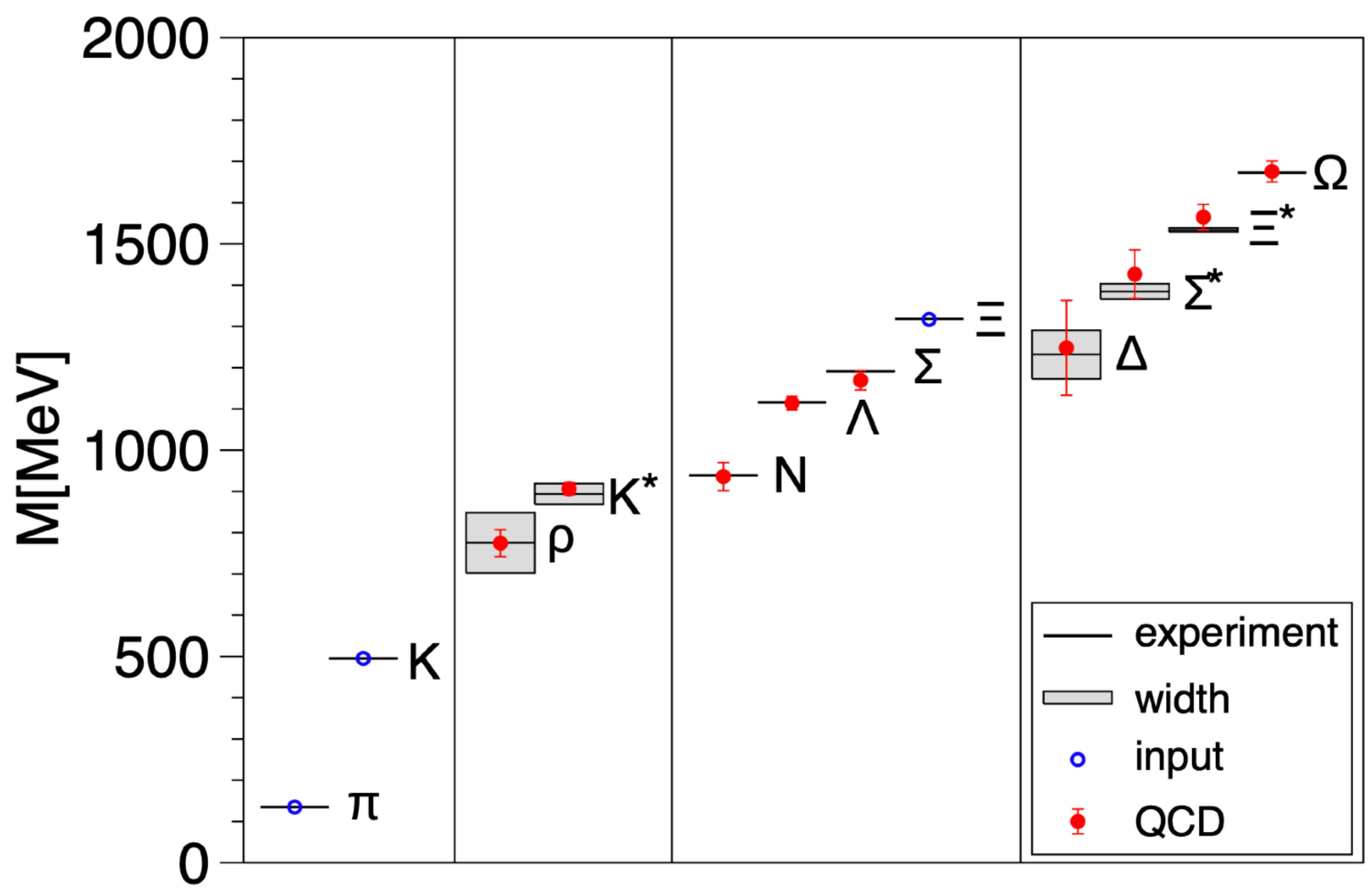
LQCD input for Muon g-2



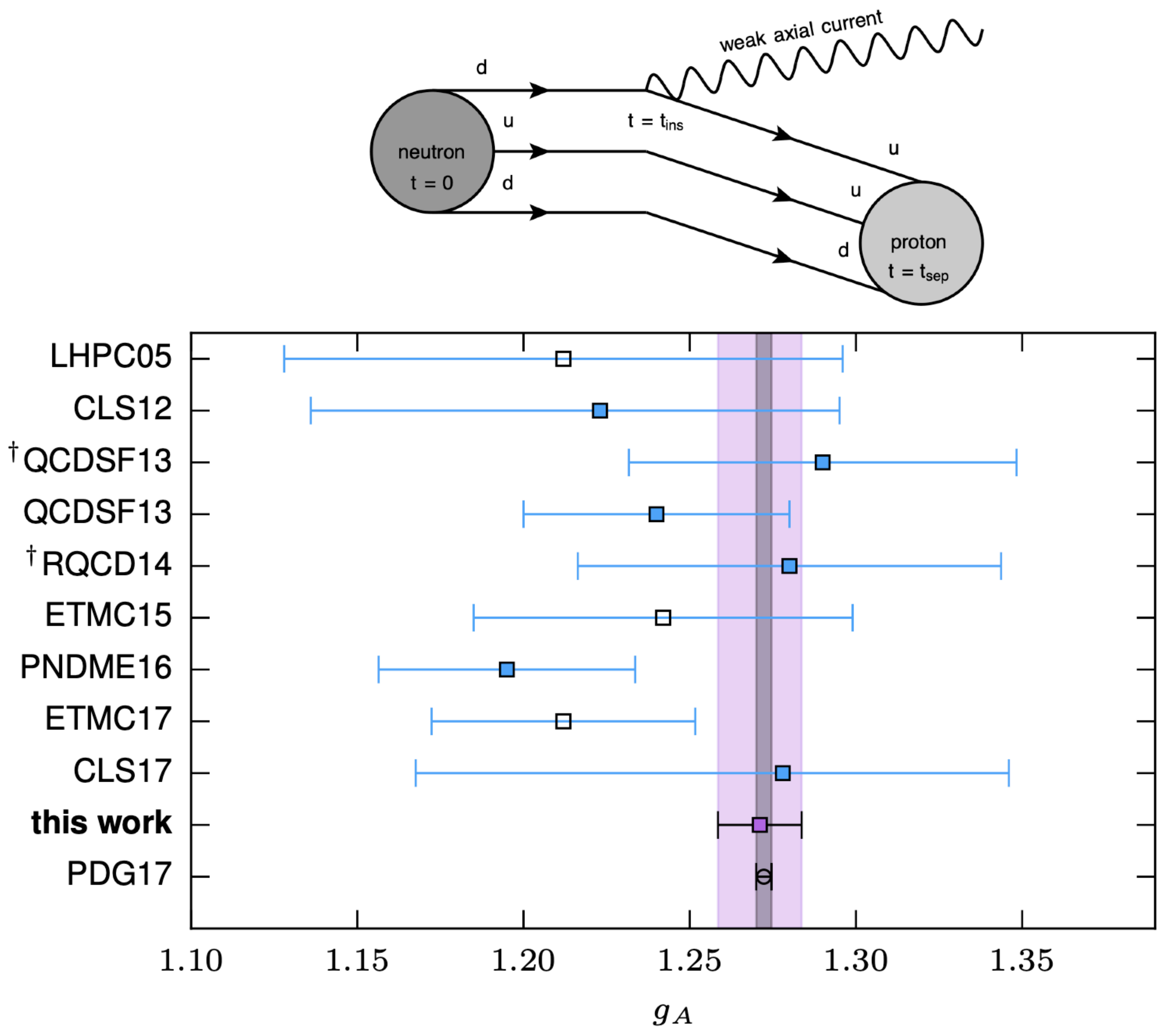


Background

Other high precision LQCD inputs



BMWc, Science 322(2008)1224

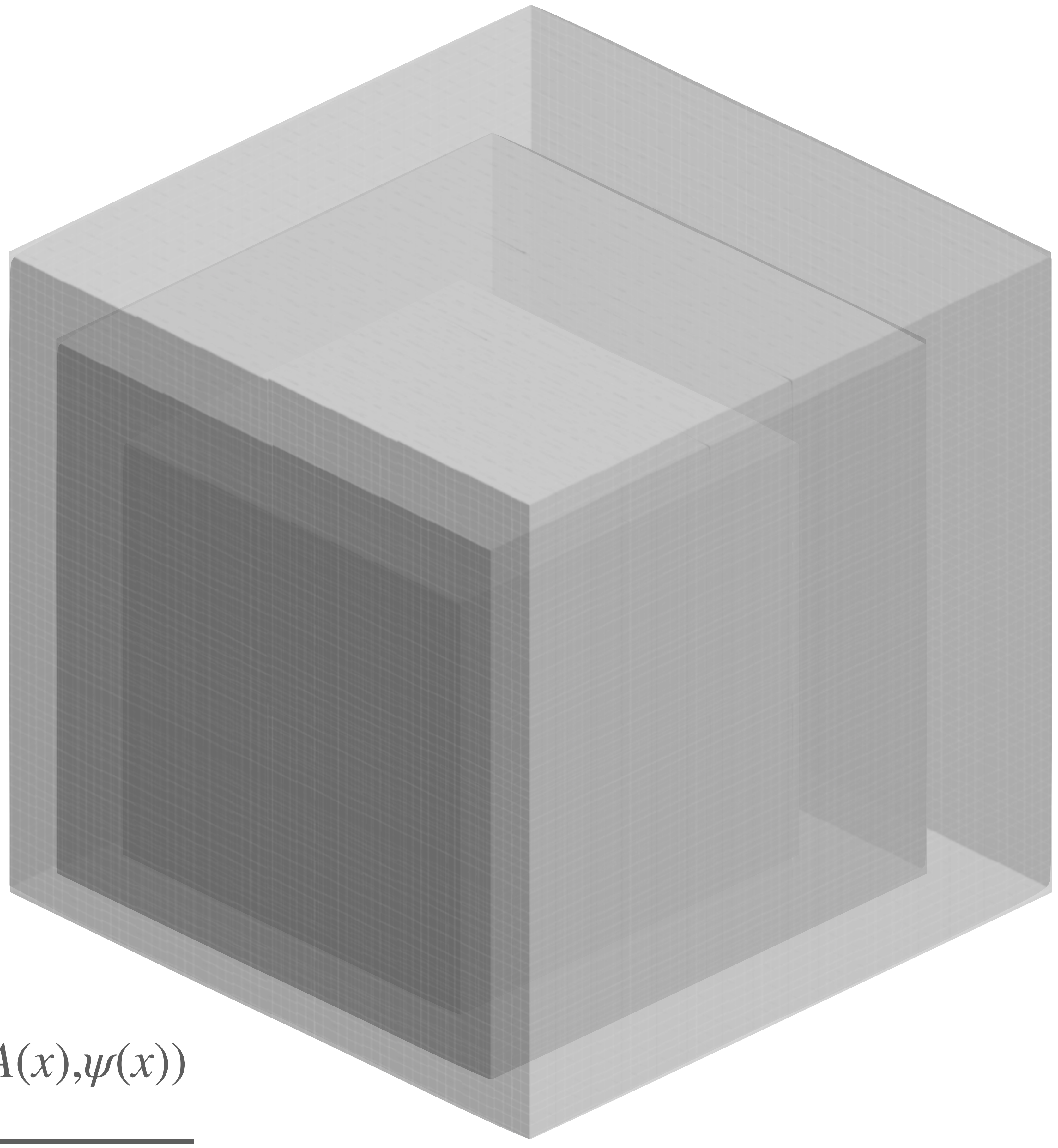


CalLAT, Nature 558(2018)7708,91-94

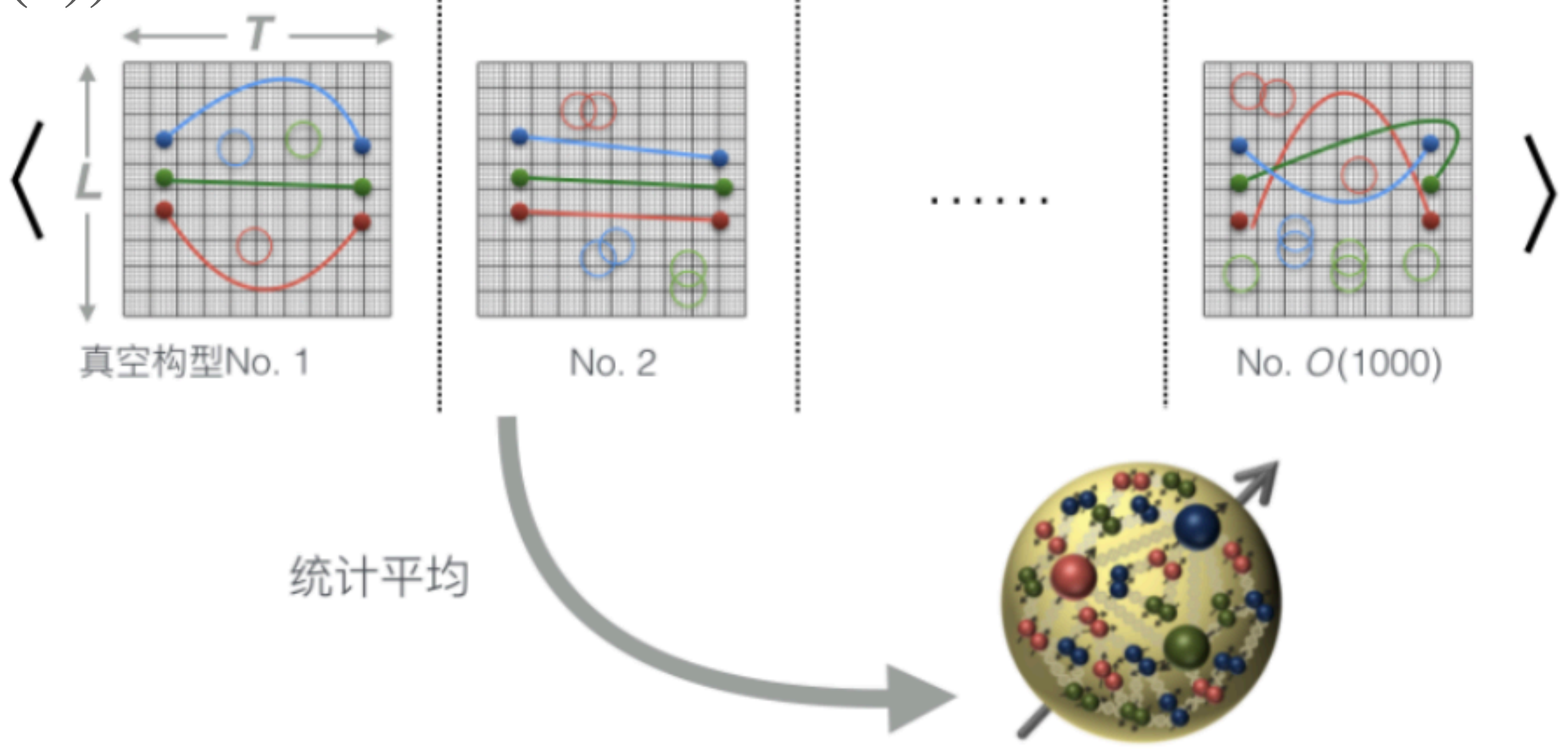
Background

Idea of Lattice QCD

- Discretize the Euclidean space-time into a 4-D lattice with finite size and lattice spacing;
- Sample the QCD path integral with the weights from the QCD action;
- Repeat the calculation at different lattice spacing and volume, and then obtain the result in the continuum/infinite-volume limits.



$$\langle \mathcal{O} \rangle = \frac{\int [\mathrm{d}A\mathrm{d}\psi]\mathcal{O}(A,\psi)e^{-\int \mathrm{d}^4x\mathscr{L}(A(x),\psi(x))}}{\int [\mathrm{d}A\mathrm{d}\psi]e^{-\int \mathrm{d}^4x\mathscr{L}(A(x),\psi(x))}}$$

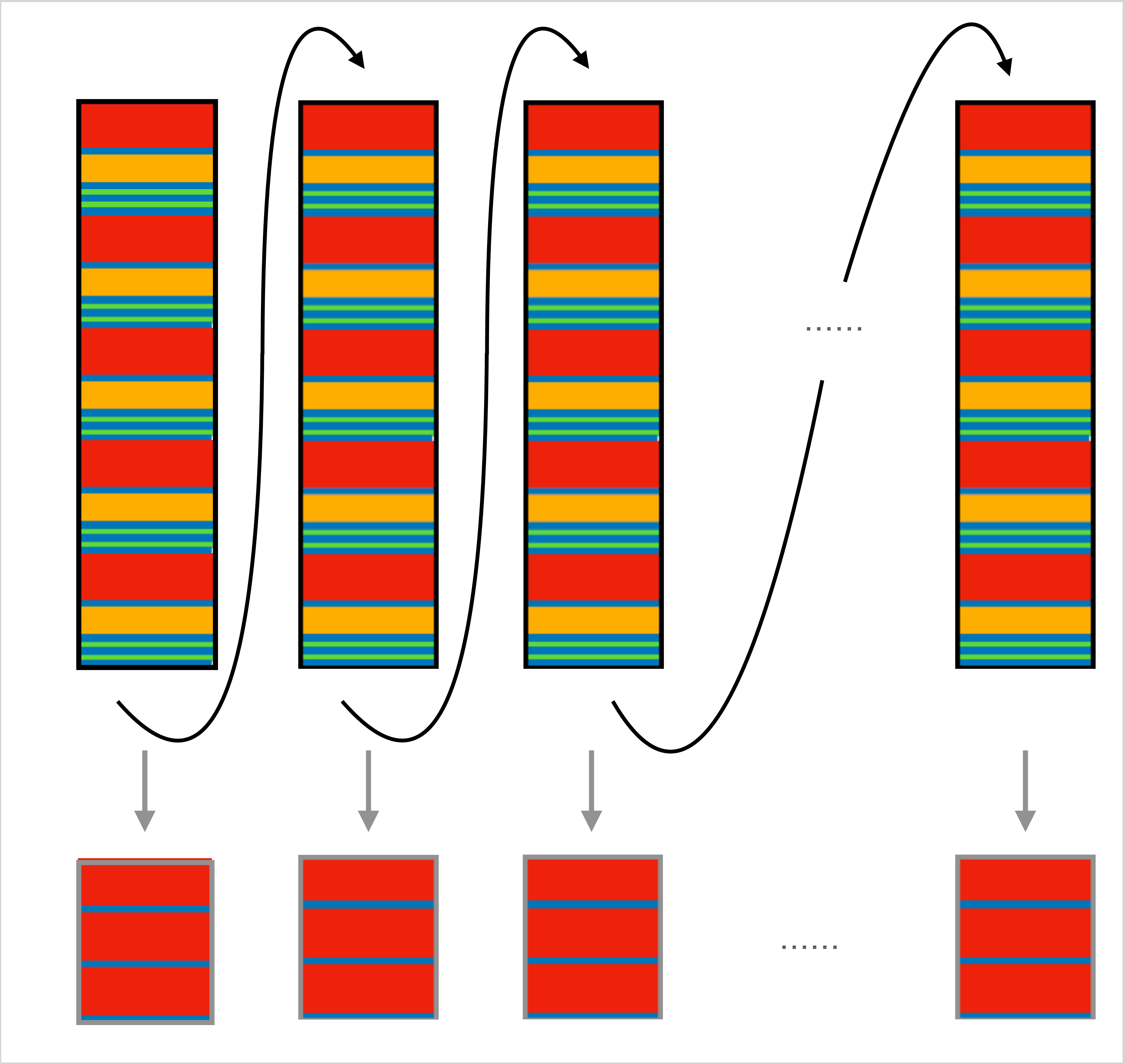


Lattice QCD

hotspot： sparse linear operation,

$$\xi = D\eta$$

Basic flow



Generate **configurations** using important sampling

Analysis the **configurations** to get the physical results

Linear system solver

$$\xi = D^{-1}\eta = \sum_i c_i D^i \eta + \mathcal{O}(10^{-12})$$

Linear algebra operation,

$$\xi = c_1\eta_1 + (\eta_2^\dagger . \eta_3)\eta_4$$

Square root of sparse matrix,

$$\xi = (D+m_0)^{-1/2}\eta = \sum_i \frac{d_i}{D+m_0+e_i}\eta + \mathcal{O}(10^{-12})$$

Derivative of sparse matrix,

$$\xi = \frac{\partial D}{\partial U}\eta = D_1 D_2 D_3 \eta$$

- The major hotspot is linear system solver;
- But after the acceleration of this hotspot, linear algebra operation, square root and derivative of sparse matrix will be the bottlenecks of the performance.
- Configurations are the foundation of all the physical analysis!

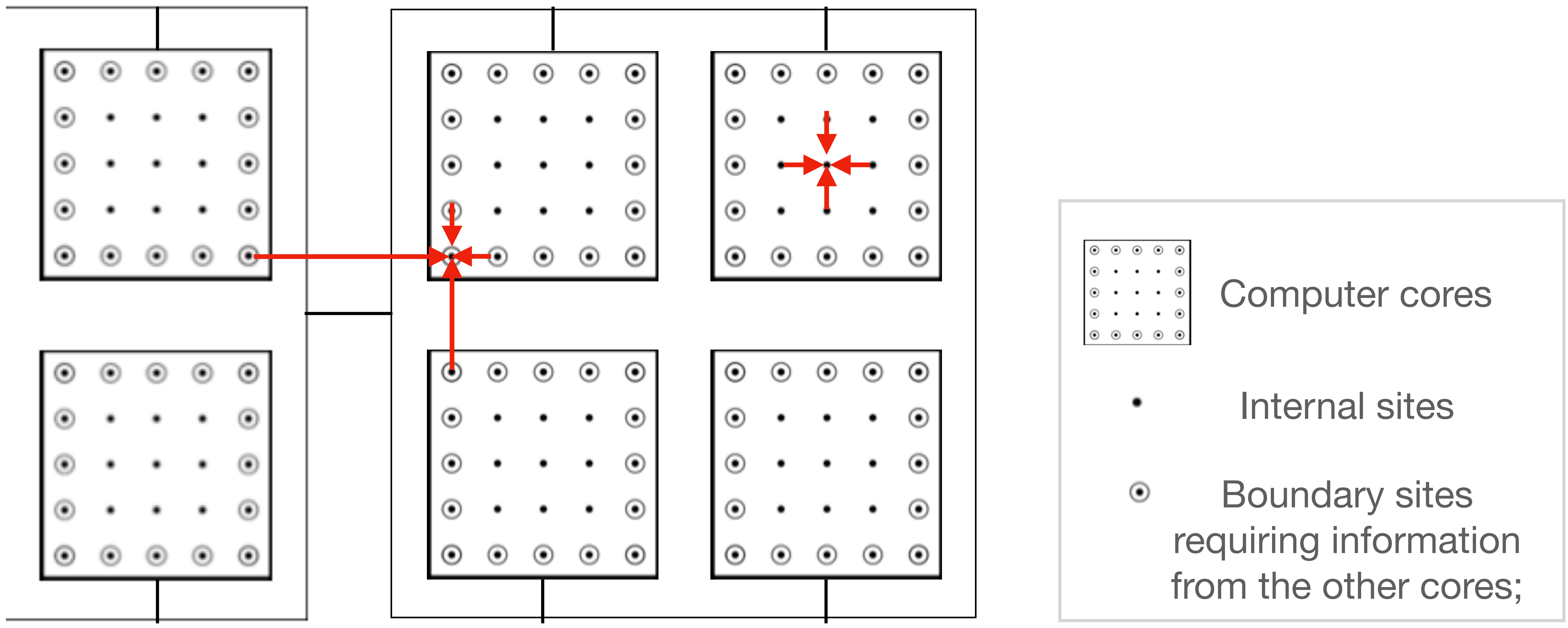
Lattice QCD

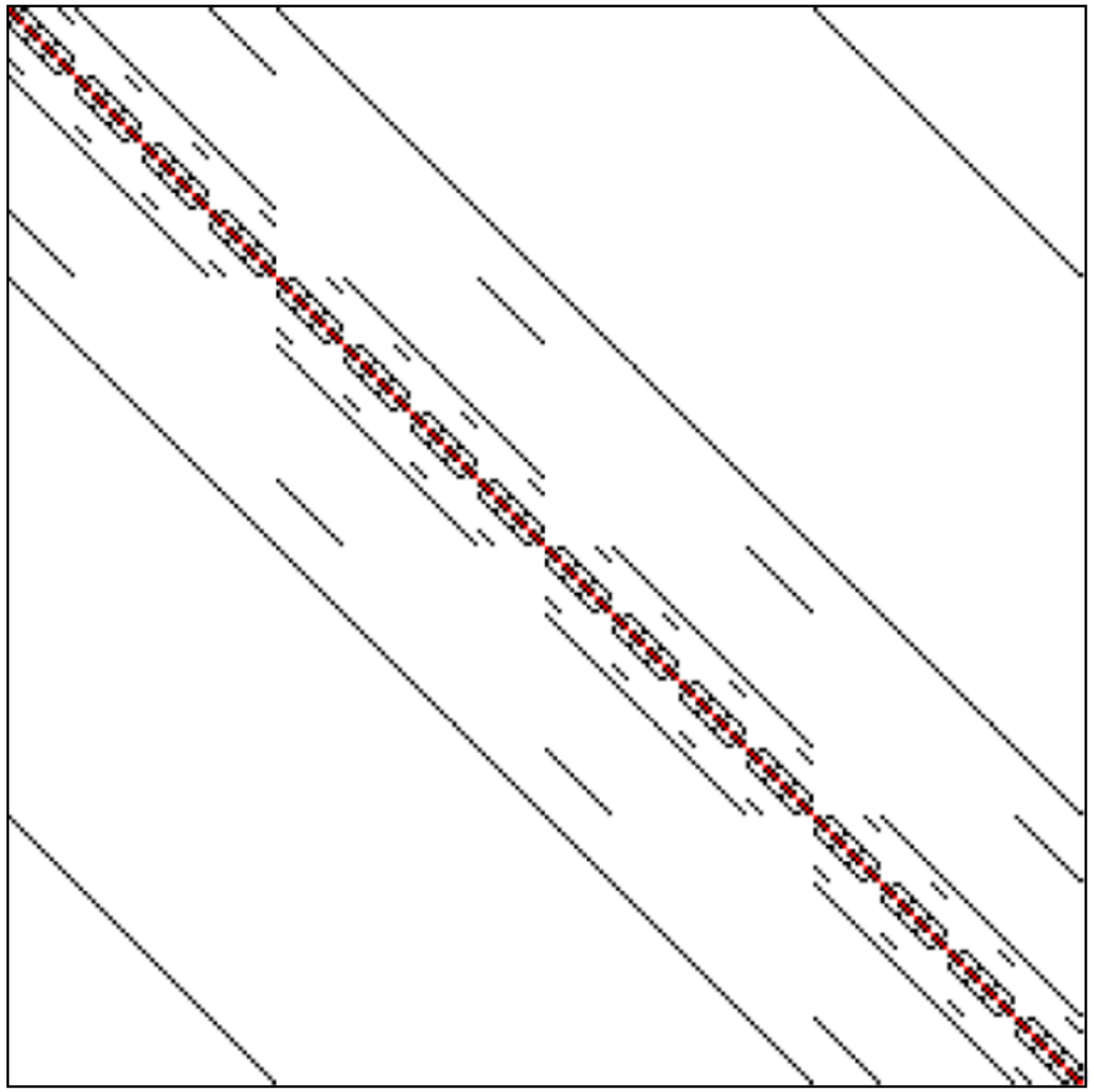
Basic unit

$$\left(\gamma_4(\partial_\tau - igA_4)\psi + \sum (\partial_i - igA_i)\gamma_i - m\right)\psi = 0$$

The discretized Dirac equation with the coupling with the non-abelian SU(3) gauge field:

- $\gamma_{1,2,3,4}$ are 4×4 complex matrices, $A_{1,2,3,4}$ are space-time dependent 3x3 complex matrices;
- Can be converted to a problem of sparse matrix inversion.





$L^3 \times T = 4^3 \times 4$ lattice:

- Red point: 12×12 diagonal matrix
- Black point: 12×12 sparse matrix

Lattice QCD

Naive and staggered fermion

- The naive discretization suffers from the doubling problem：
  - $\mathcal{S}_q^{Naive}(m) = \sum_{x,y} \bar{\psi}(x) D_{Naive}(m;x,y)\psi(y),\ D_{Naive}(m;x,y) = \frac{1}{2a}\sum_{\mu}\gamma_\mu\left(U_\mu(x)\delta_{y,x+a\hat{\mu}} - U_\mu^\dagger(x-a\hat{\mu})\delta_{y,x-a\hat{\mu}}\right) + m\delta_{y,x}$
  - The propagator has $1/m$ IR poles at $pa = (0/\pi, 0/\pi, 0/\pi, 0/\pi)$, which is different from the continuum theory.
- Staggered fermion:
  - $\psi^{\mathrm{st}}(x) = \gamma_4^{x_4}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\psi(x),\ \{\gamma_1^{\mathrm{st}}, \gamma_2^{\mathrm{st}}, \gamma_3^{\mathrm{st}}, \gamma_4^{\mathrm{st}}\} = \{(-1)^{x_4}, (-1)^{x_1+x_4}, (-1)^{x_1+x_2+x_4}, 1\};$
  - 16 IR poles → 4 IR poles, pion mass in the chiral limit can be $\mathcal{O}(a^4)$ and then non-zero at finite a.
  - Mixing between IR poles can be suppressed with kinds of the improvement, likes the so-call highly-improved staggered quark (HISQ).

Staggered/HISQ —Cost x10→ Wilson/Clover —Cost x10→ Domain wall —Cost x10→ Overlap

Lattice QCD

Wilson and clover fermion

- Wilson fermion action：
  - $D + m \rightarrow D + aD^2 + m$
  - It removes the unphysical IR pole at $p_i = \pi/a$, while introduce the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s/a)$.
- Clover fermion action:
  - $D + m \rightarrow D + aD^2 + m + ac_{\rm sw}\sigma_{\mu\nu}F^{\mu\nu}$
  - Suppress the additional chiral symmetry breaking into $\mathcal{O}(\alpha_s^2/a)$.
- The cost of either Wilson or Clover action is $\mathcal{O}(10)$ of the Staggered fermion.

Staggered/HISQ —Cost x10→ Wilson/Clover —Cost x10→ Domain wall —Cost x10→ Overlap

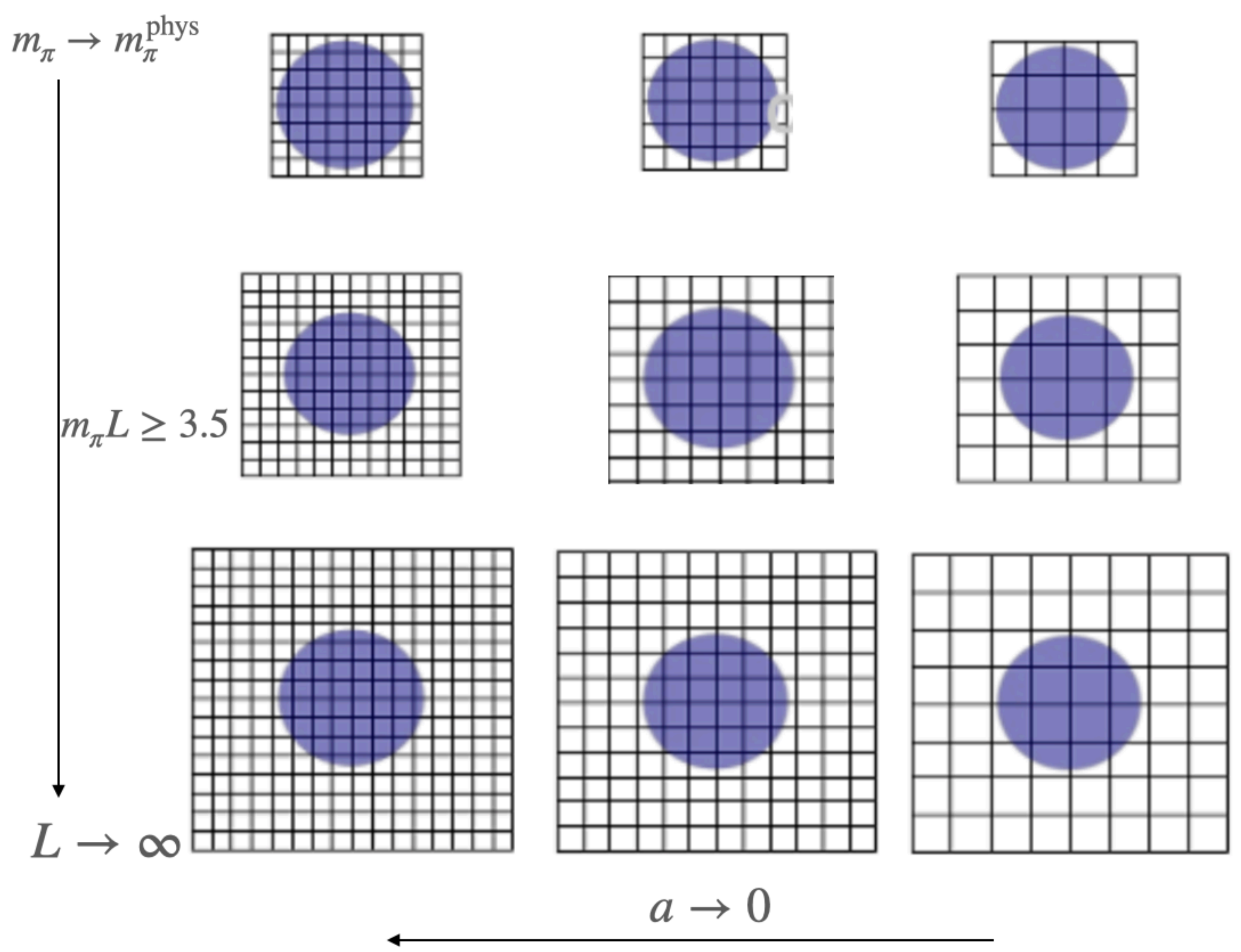
Lattice QCD

Ginsparg-Wilson fermion

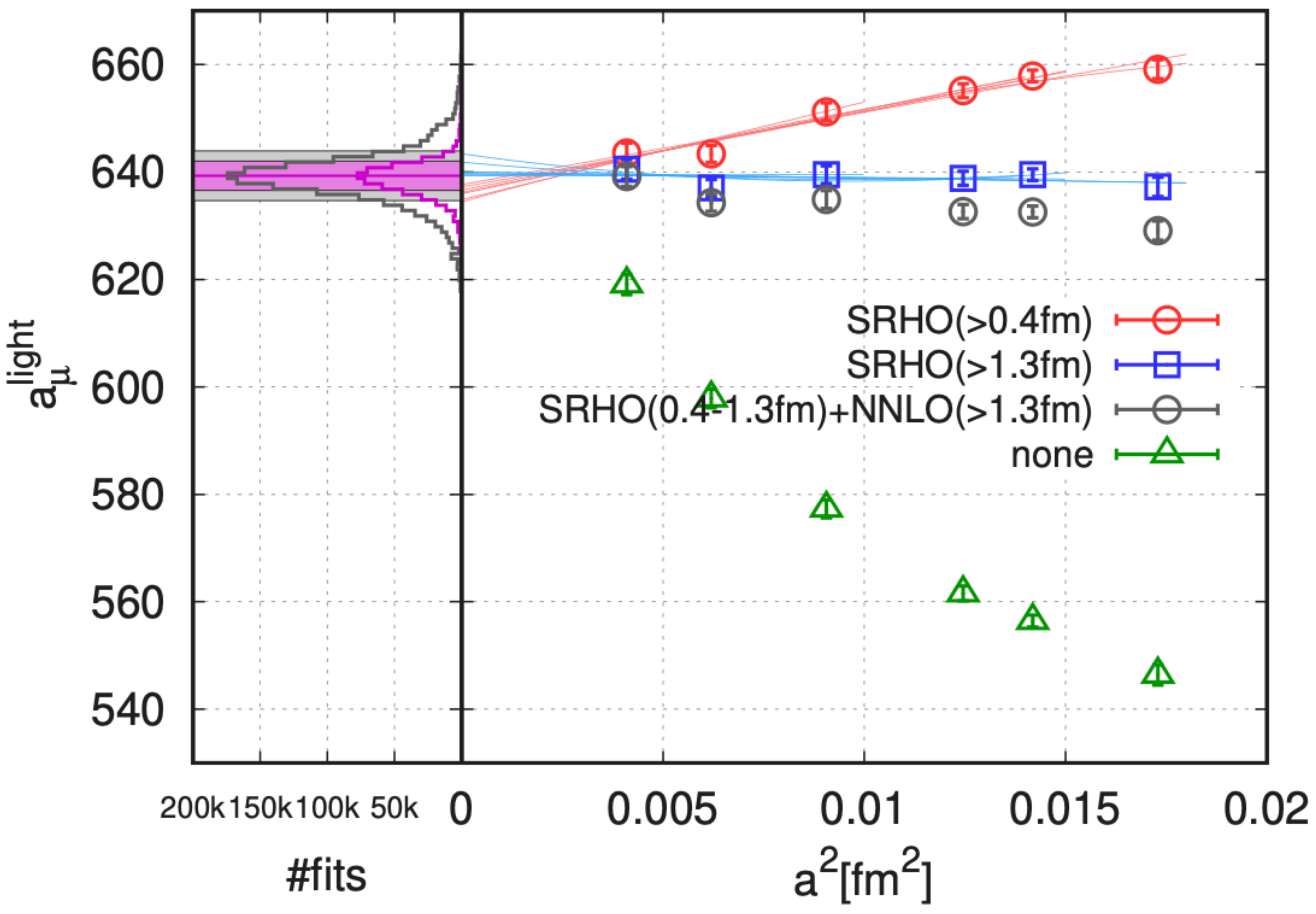
- Ginsparg-Wilson relation: $\gamma_5 D_{\rm GW} + D_{\rm GW}\gamma_5 = \frac{a}{\rho} D_{\rm GW}\gamma_5 D_{\rm GW}$。
- $\gamma_5 D_c^{-1} + D_c^{-1}\gamma_5 = 0$, $D_c^{-1} = D_{GW}^{-1} - \frac{a}{2\rho}$.
- In $p \to 0$ region, $D_{ov} \to a\gamma_\mu p_\mu$;
- In $p \to \pi/a$ region, $D_{ov} \to \mathcal{O}(1)$.
- But approximate the sign function $\frac{\gamma_5 D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}} = \frac{\gamma_5 D_w(-\rho)}{|\gamma_5 D_w(-\rho)|}$ need $\mathcal{O}(100)$ cost of the Wilson/Clover action.
- Domain wall fermion action is an approximation of overlap fermion with $\mathcal{O}(10)$ cost of the Wilson/Clover action.

Staggered/HISQ —Cost x10→ Wilson/Clover —Cost x10→ Domain wall —Cost x10→ Overlap

Lattice QCD



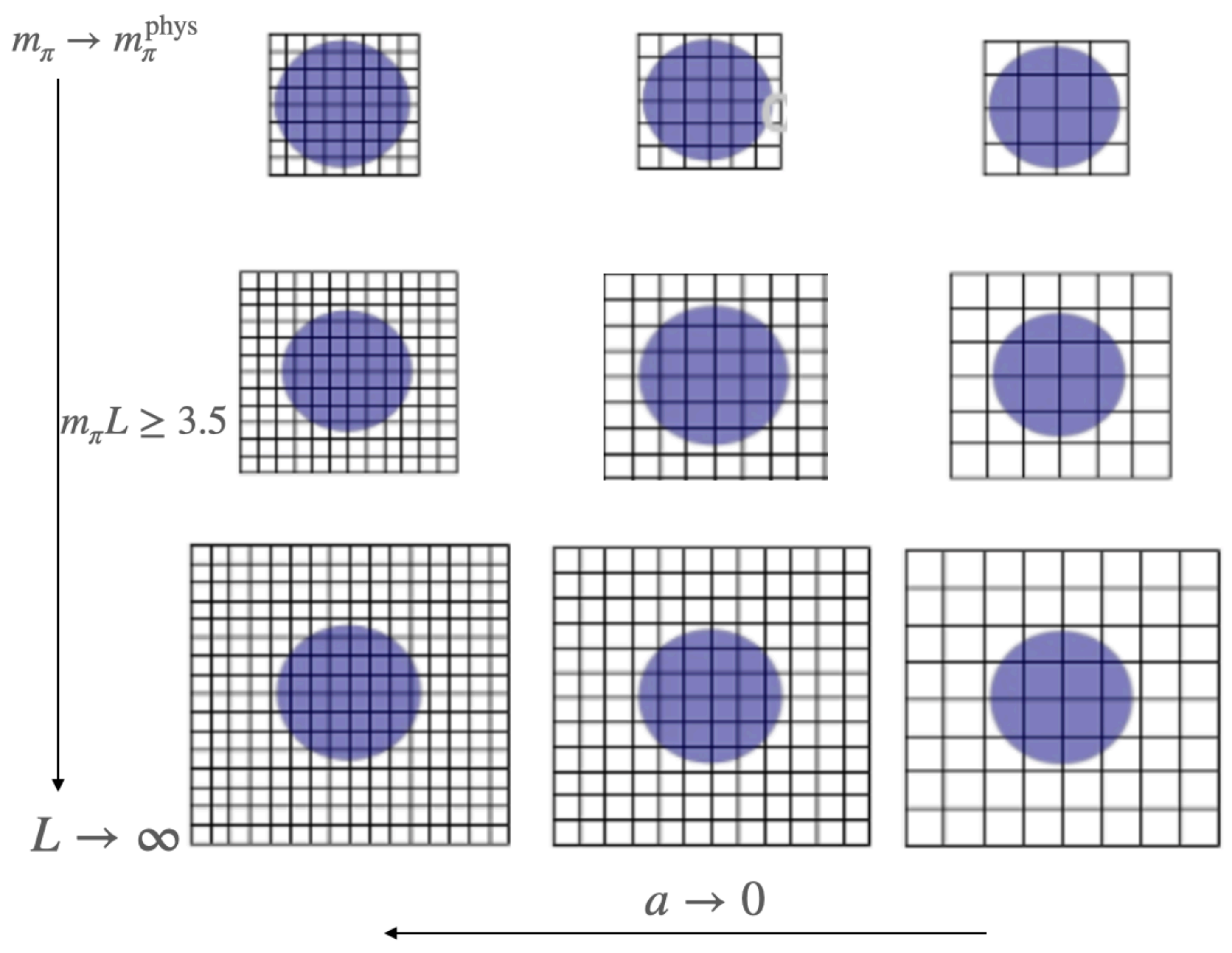
Discretization error



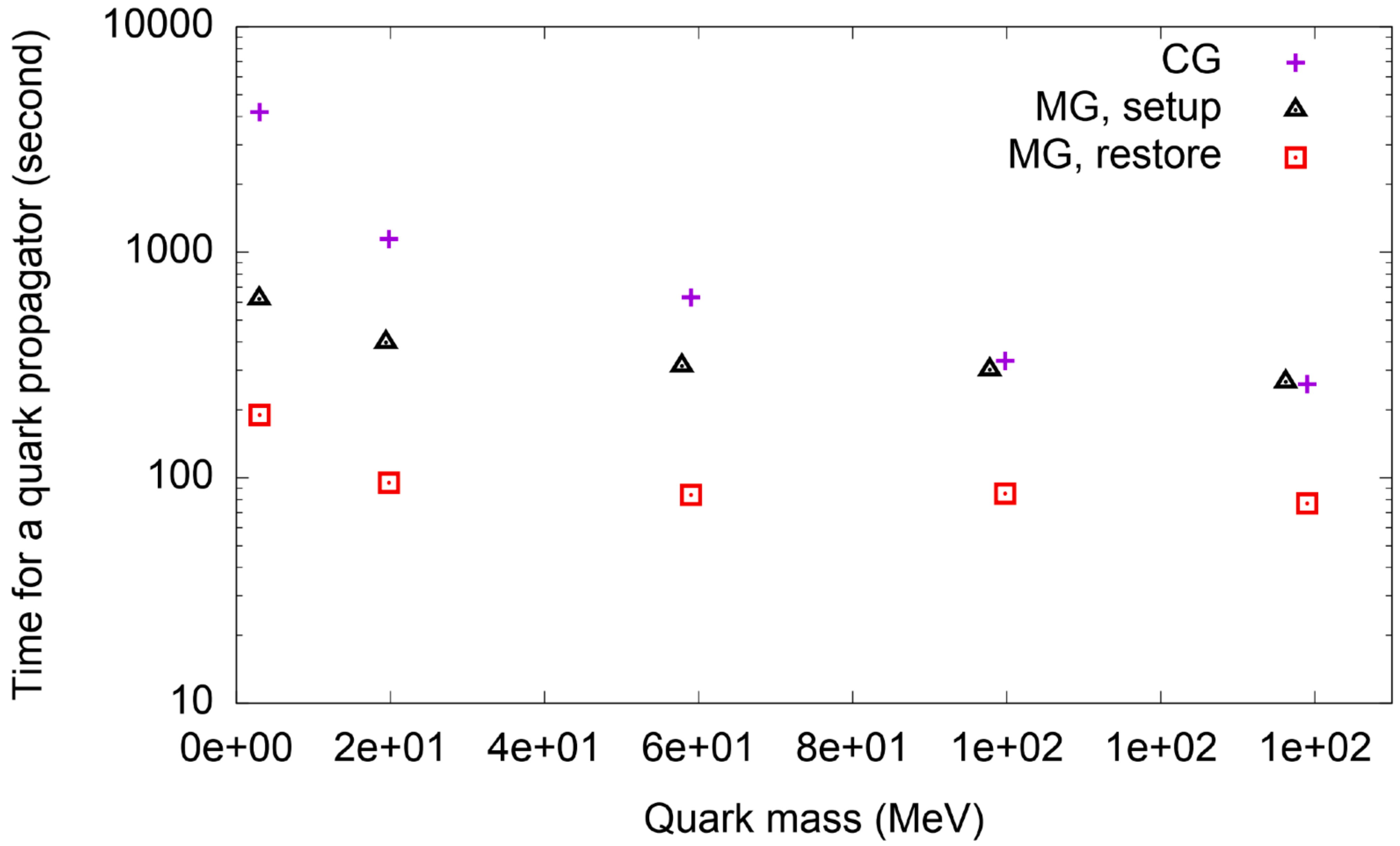
BMWc, Nature 593(2021)51

- Lattice calculation will suffer from the discretization error, which is usually $\mathcal{O}(a^2\Lambda^2_{QCD})$.
- If we reduce the lattice spacing a by a factor of 2, the cost of calculation will increase by a factor of at least 16.
- The current FLAG "green star" requires at least three lattice spacings and at least two points below 0.1 fm and a range of lattice spacings satisfying $a^2_{\max}/a^2_{\min} \geq 2$.

Lattice QCD

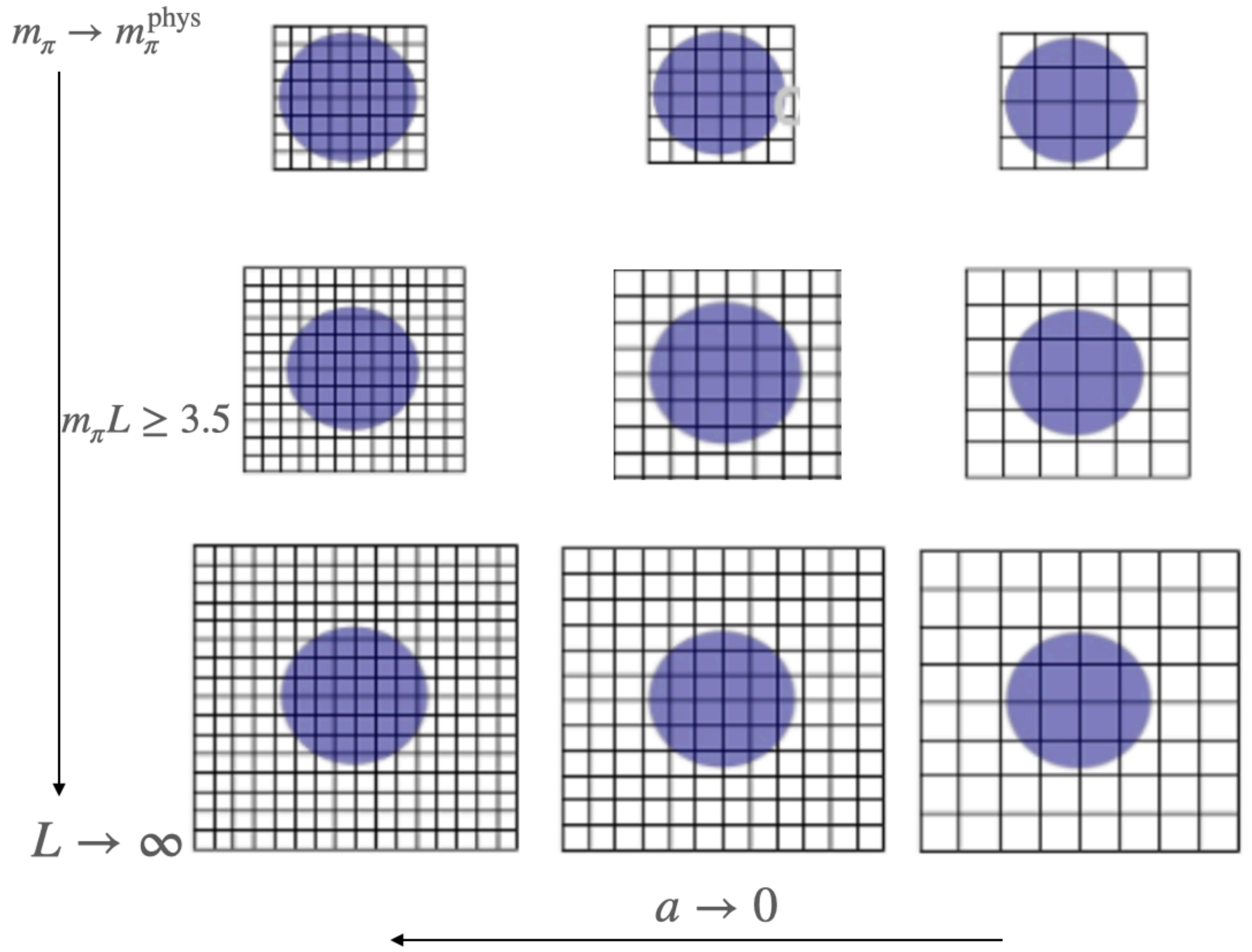


Chiral extrapolation

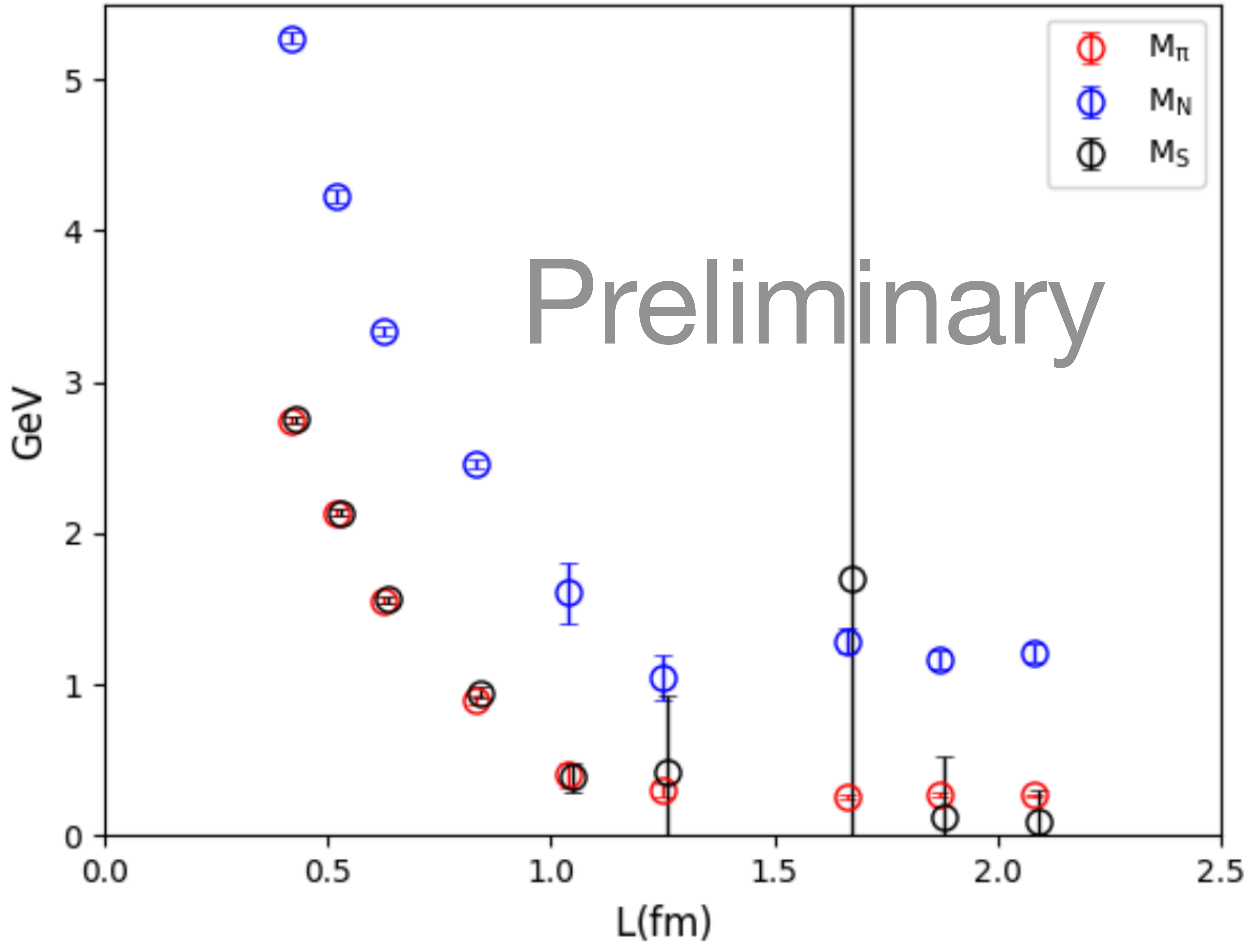


- The cost to simulate light quark can be an order of magnitude larger than that of the strange quark.
- Non-trivial algorithm likes multigrid can speed up the calculation of the light quark for certain fermion actions.
- The current FLAG "green star" requires $m_{\pi,\rm min} < 200$ MeV with at least three m_π in the chiral extrapolation, **or** $m_{\pi,\text{ case1}} = 135 \pm 10$ MeV and $m_{\pi,\text{ case2}} < 200$ MeV.

Lattice QCD

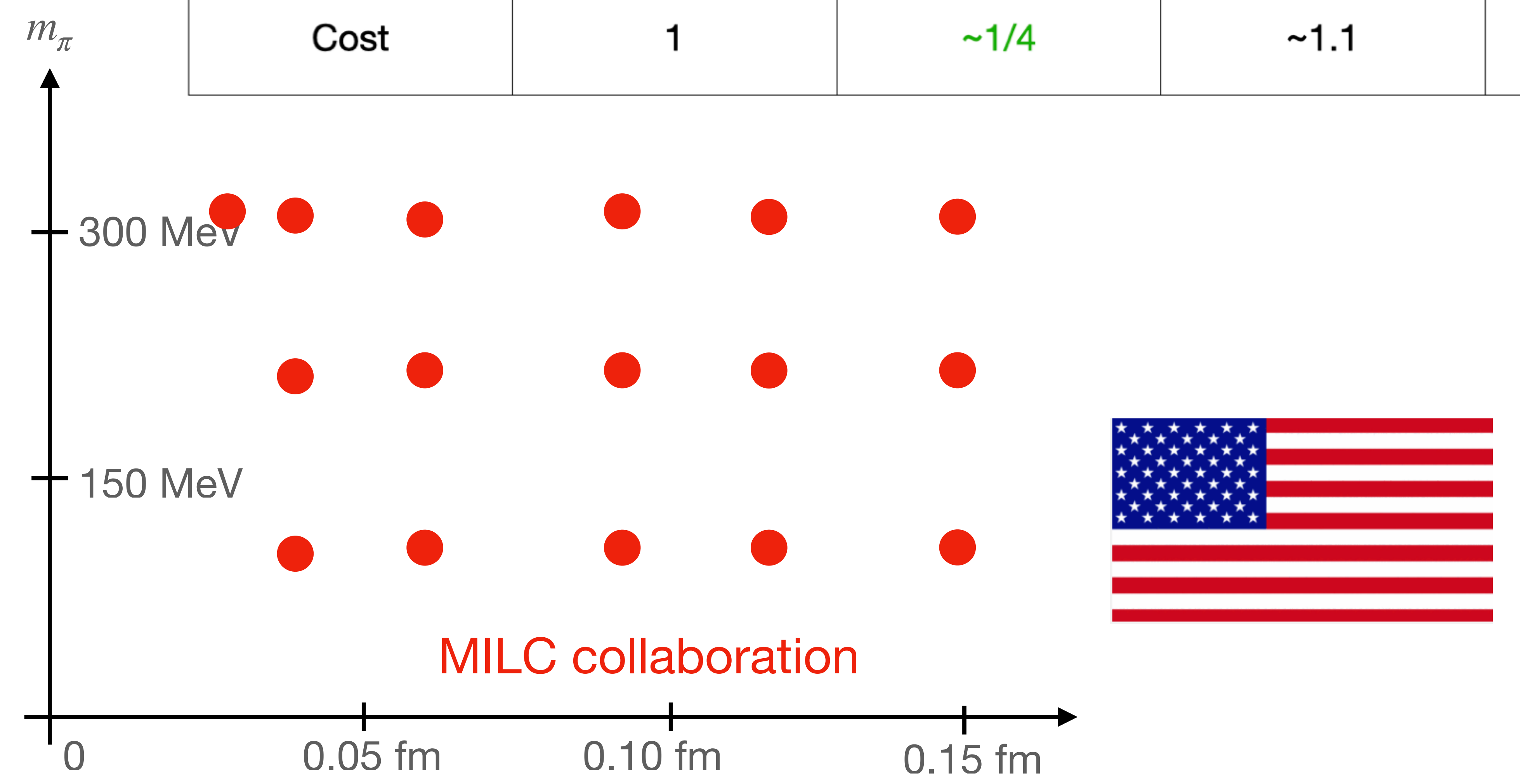


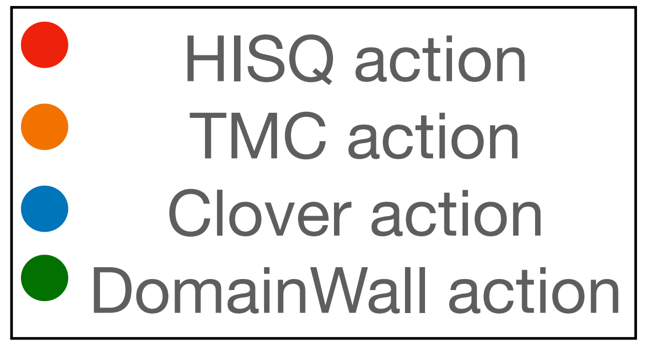
Finite volume effect

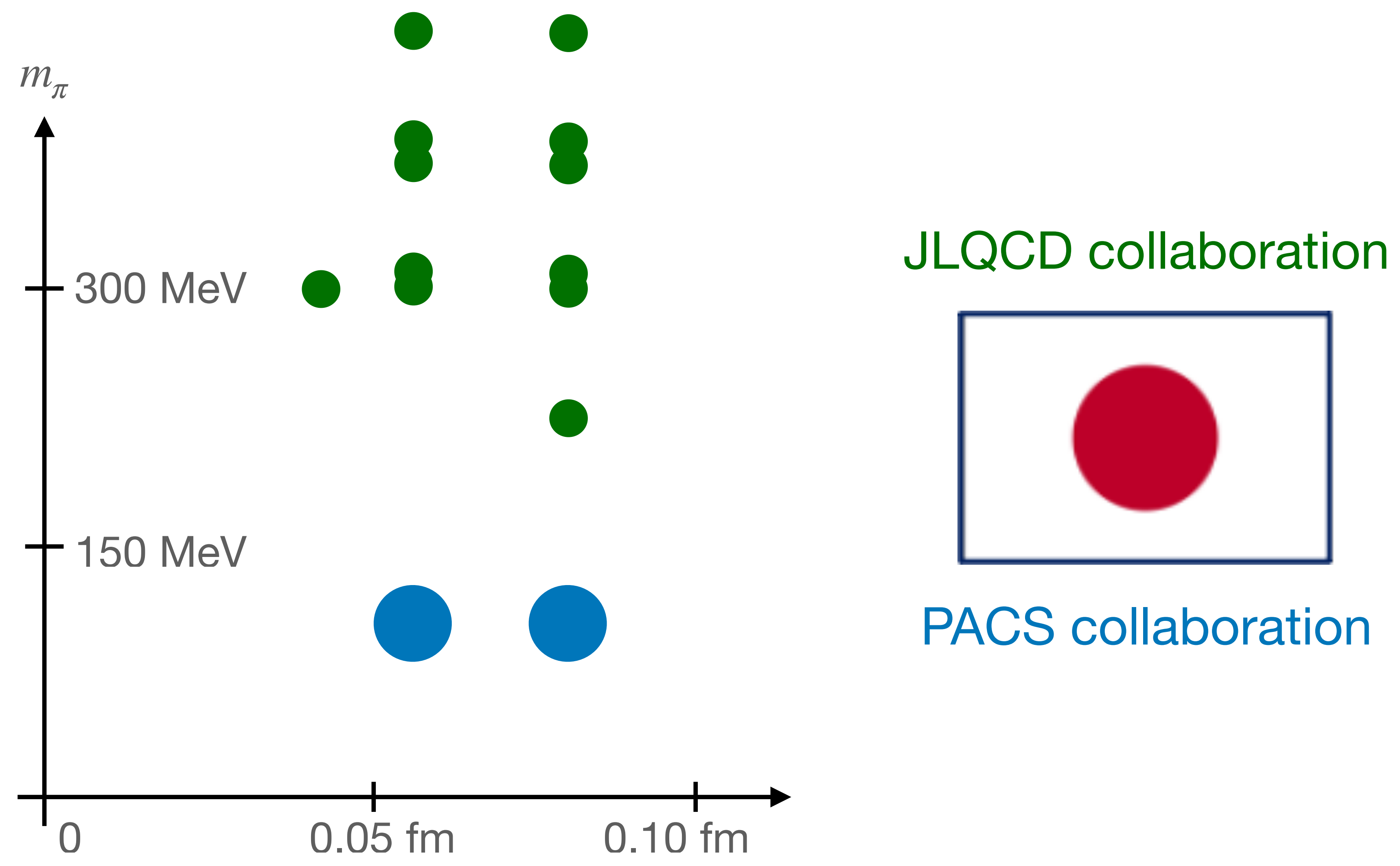


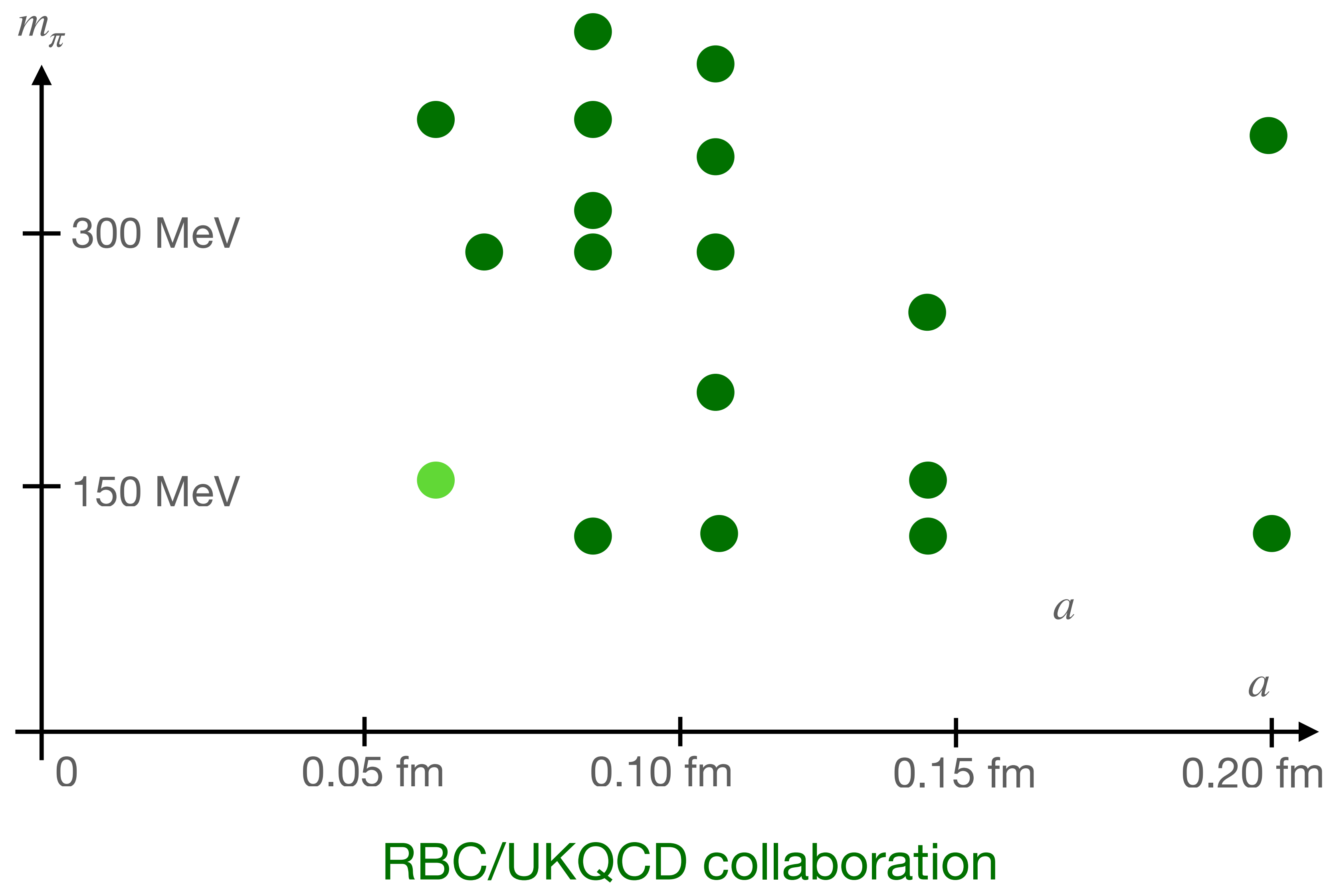
- Hadron mass can have very strong dependence on spatial size L, especially when $L \leq \Lambda_{QCD}^{-1}$;
- The finite volume chiral perturbative theory suggest an $e^{-m_\pi L}$ correction when $m_\pi L \geq 3$, it means that the volume required by $m_\pi \sim$ 135 MeV is ~ 11 times of that required by $m_\pi \sim$ 300 MeV.
- The current FLAG "green star" requires $m_\pi L \sim$ 3.2 for $m_\pi \sim$ 135 MeV, **or** at least three volumes.

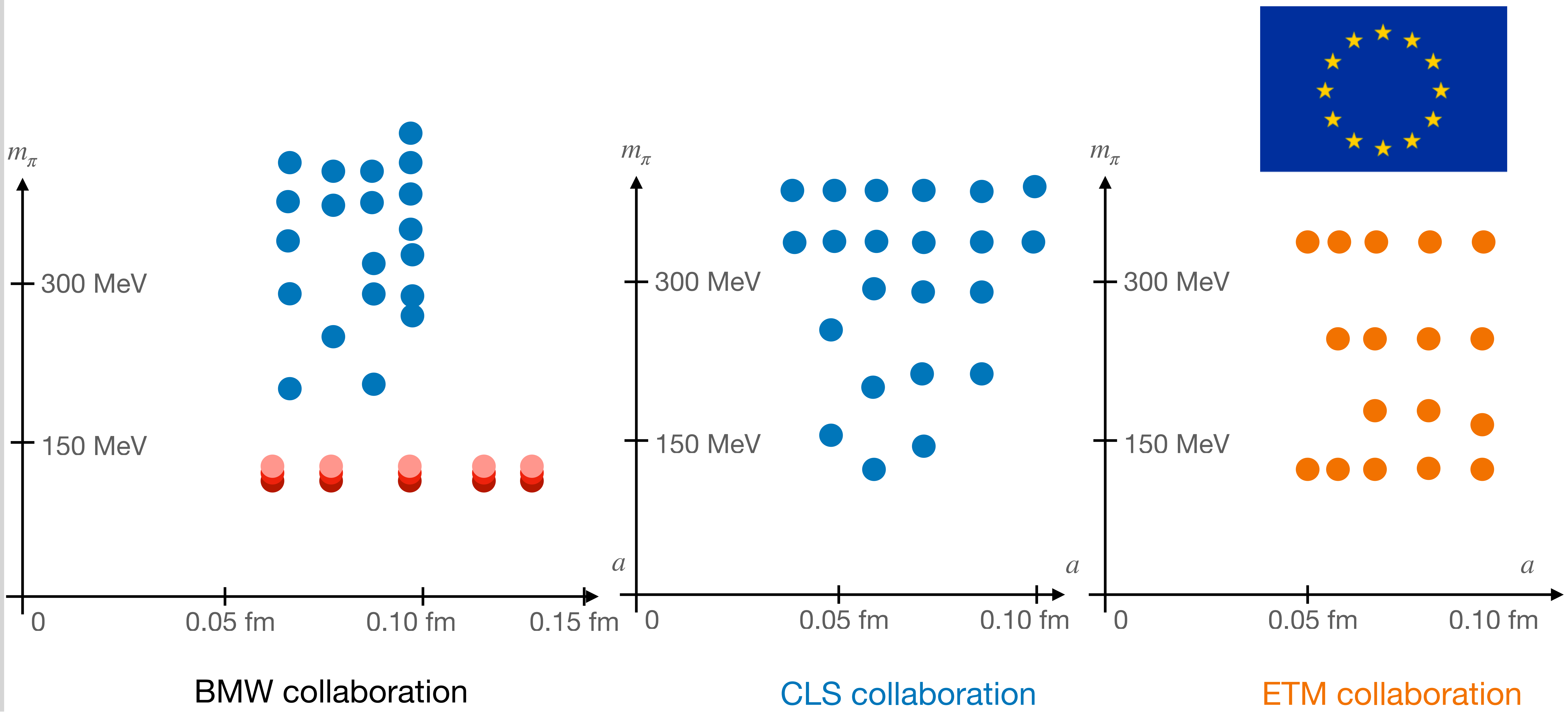
	Naive	Staggered/HISQ	Wilson/Clover	Twisted-mass	Overlap/Domain wall
Form	$D^{\text{naive}} = \gamma_\mu(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{st}} = \gamma_\mu^{\text{st}}(x)(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{clv}} = D + aD^2 + ac_{sw}F_{\mu\nu}\sigma^{\mu\nu}$	$D^{\text{tm}} = D^{\text{clv}} + i\tau_3 m$	$D^{\text{ov}} = [1 + \gamma_5 D(-\rho)/\sqrt{D^\dagger(-\rho)D(-\rho)}]/\rho$
Fermion copies	16	4	1	1	1
Chiral symmetry breaking	N/A	$\mathcal{O}(a^4)$	$\mathcal{O}(\alpha_s/a)$	$\mathcal{O}(\alpha_s)$	N/A
Cost	1	~1/4	~1.1	~1.1	~10-100



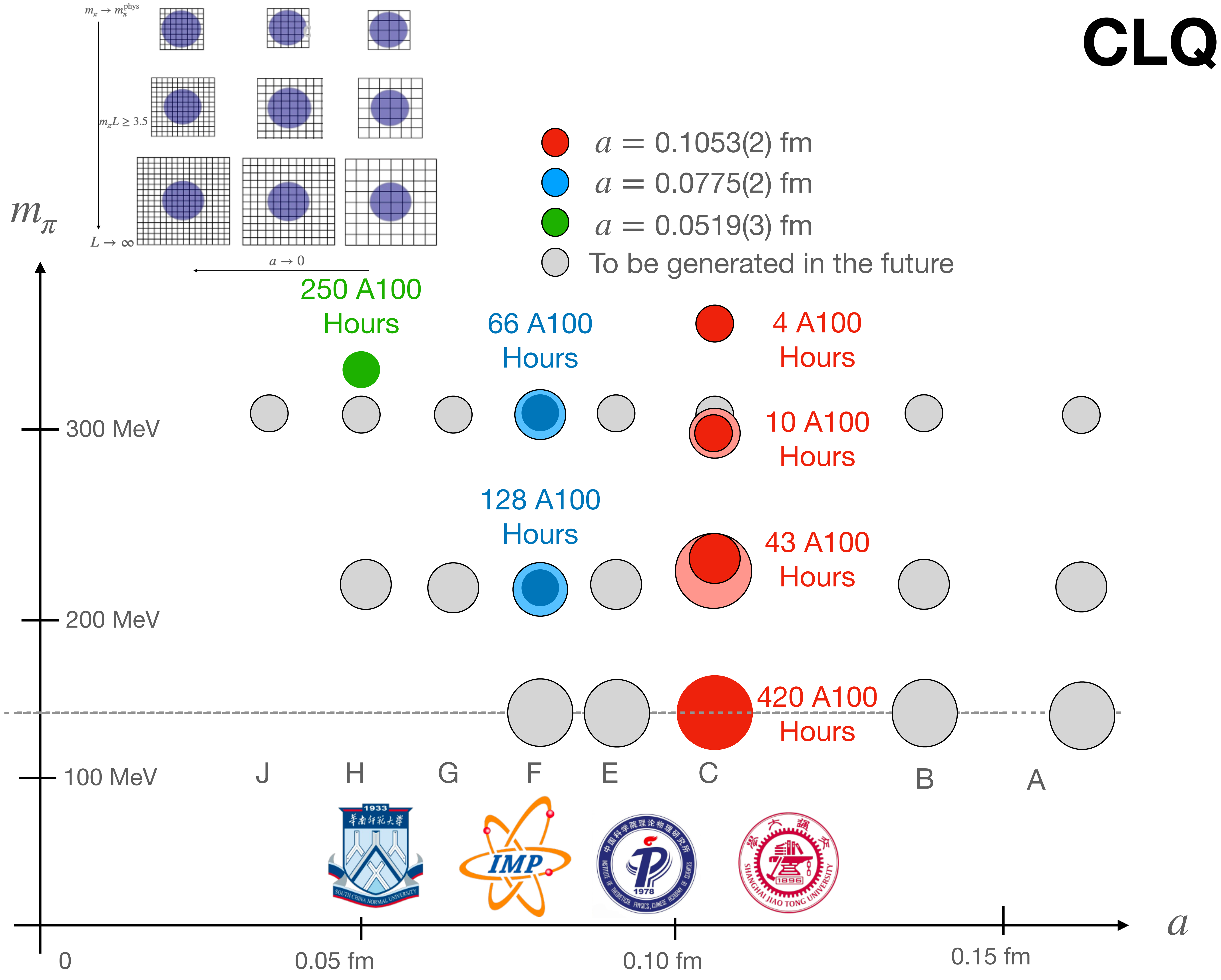








CLQCD ensembles



CLQCD choice and informations

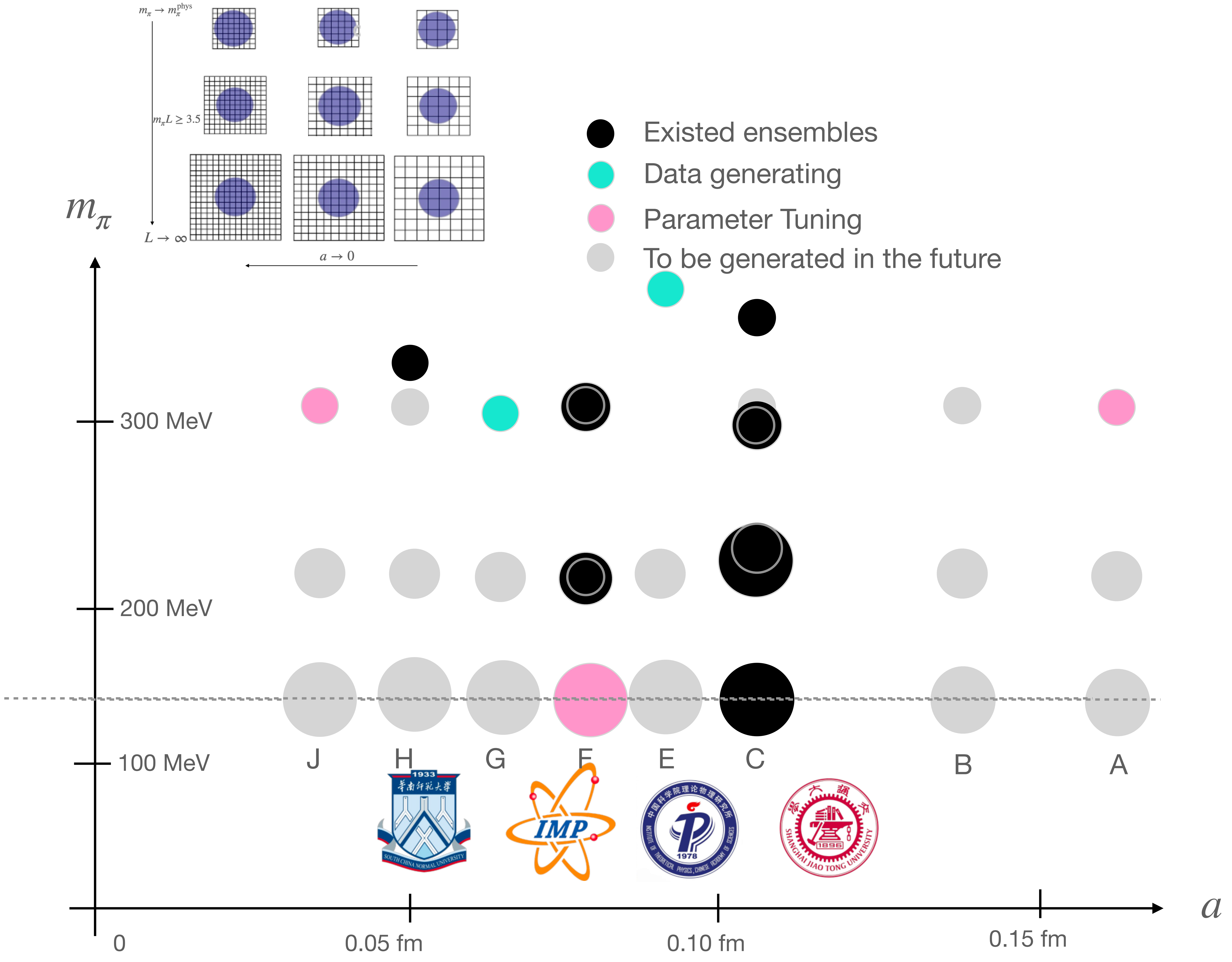
- Features:
- Maximum lattice size $48^3 \times 144$,
- Clover fermion action with stout smearing,
- Similar pion mass and volume at different lattice spacing:
- Cost:
- That of an independent configuration (per 10 traj.'s with $\tau = 1.0$, converted to A100 GPU hours) is shown on the figure;
- Needs ~1,000 configurations per ensemble;
- Working on the Sugon machines to avoid the embargo of A100 GPU.

CLQCD ensembles

Southern Nuclear Science Computing Center

- Start to generate the CLQCD ensembles, when it was still in the container.
- Currently over 50% of the configurations are generated there.

CLQCD ensembles



Current status

★In production：

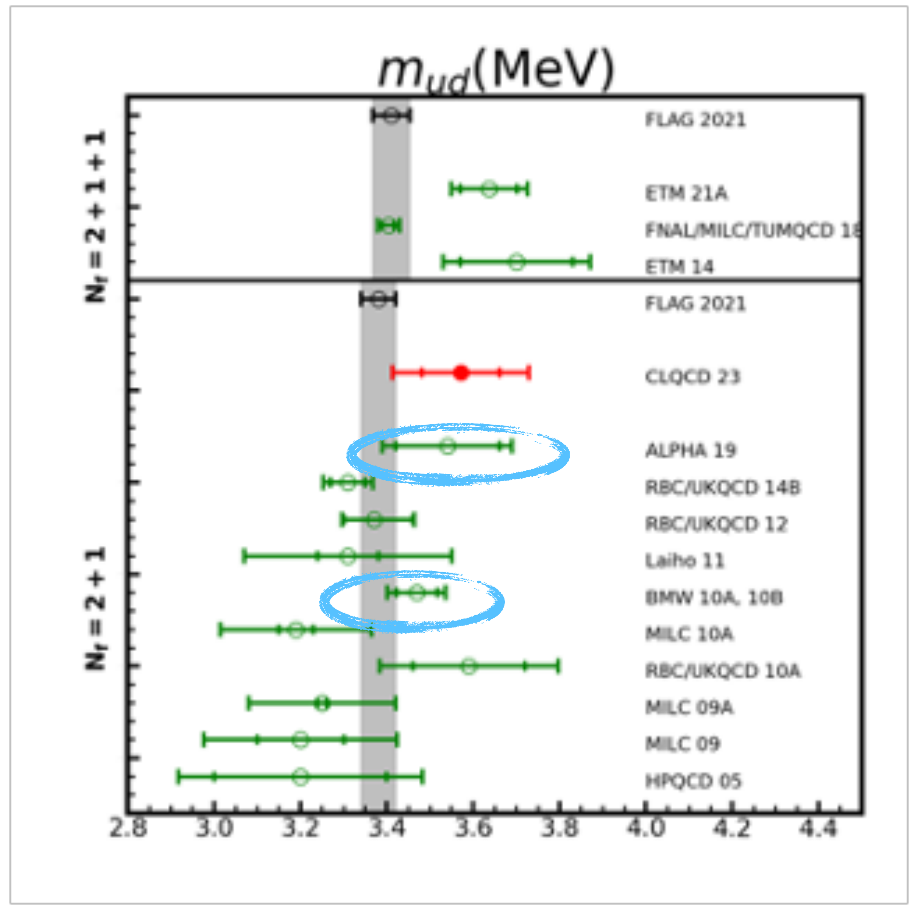
- a=0.0888(3) fm, mpi=349(2) MeV, L=2.49 fm;
- a~0.0683(3) fm, mpi=291(2) MeV, L=2.46 fm;
- Aboot 100 independent configurations each.

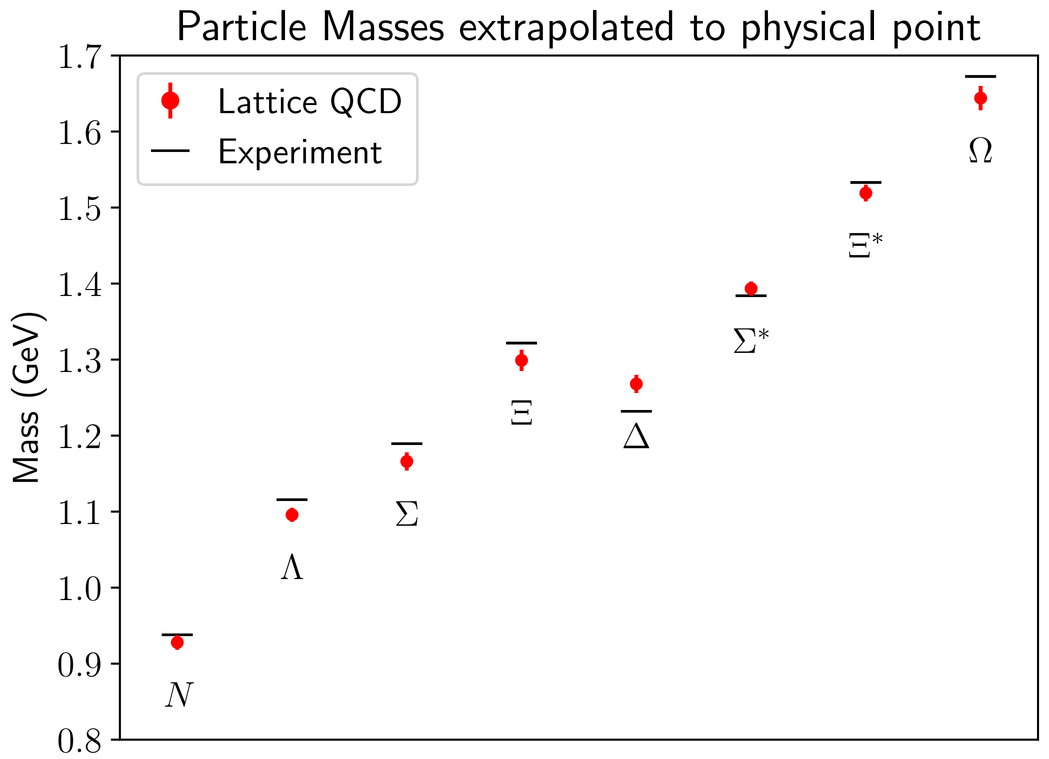
★Parameter tuning：

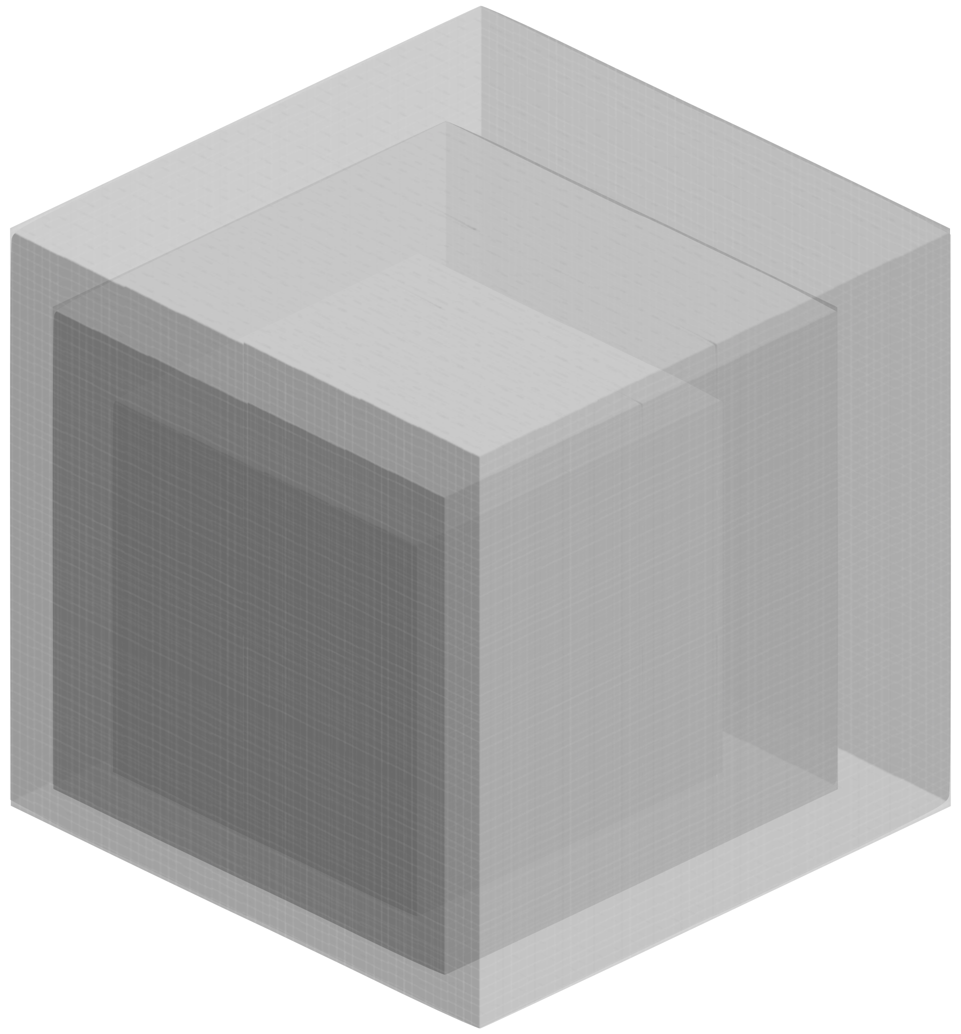
- a~0.04 fm, mpi~300 MeV;
- a~0.20 fm, mpi~300 MeV;
- a~0.08 fm, mpi~135 MeV.

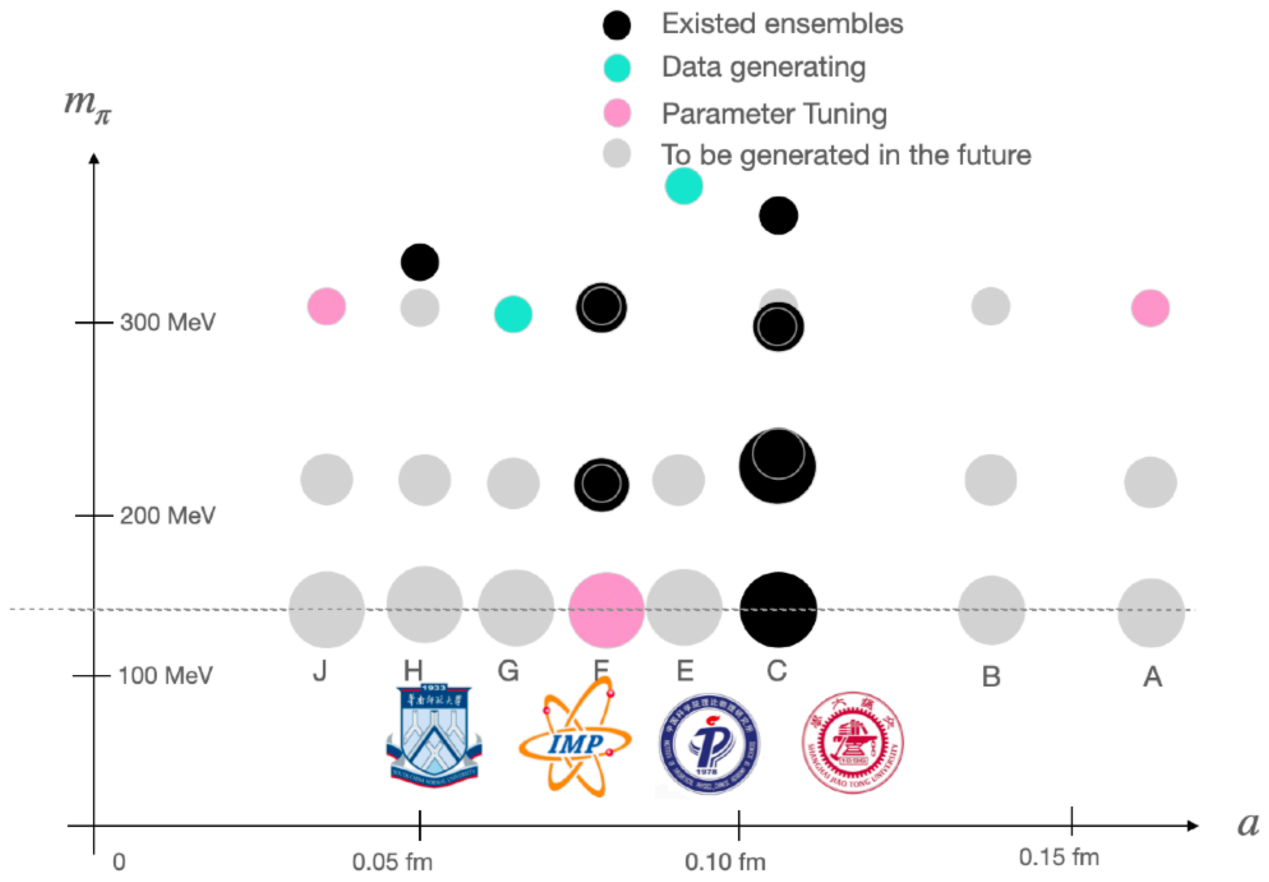
Outline

- Lattice QCD background

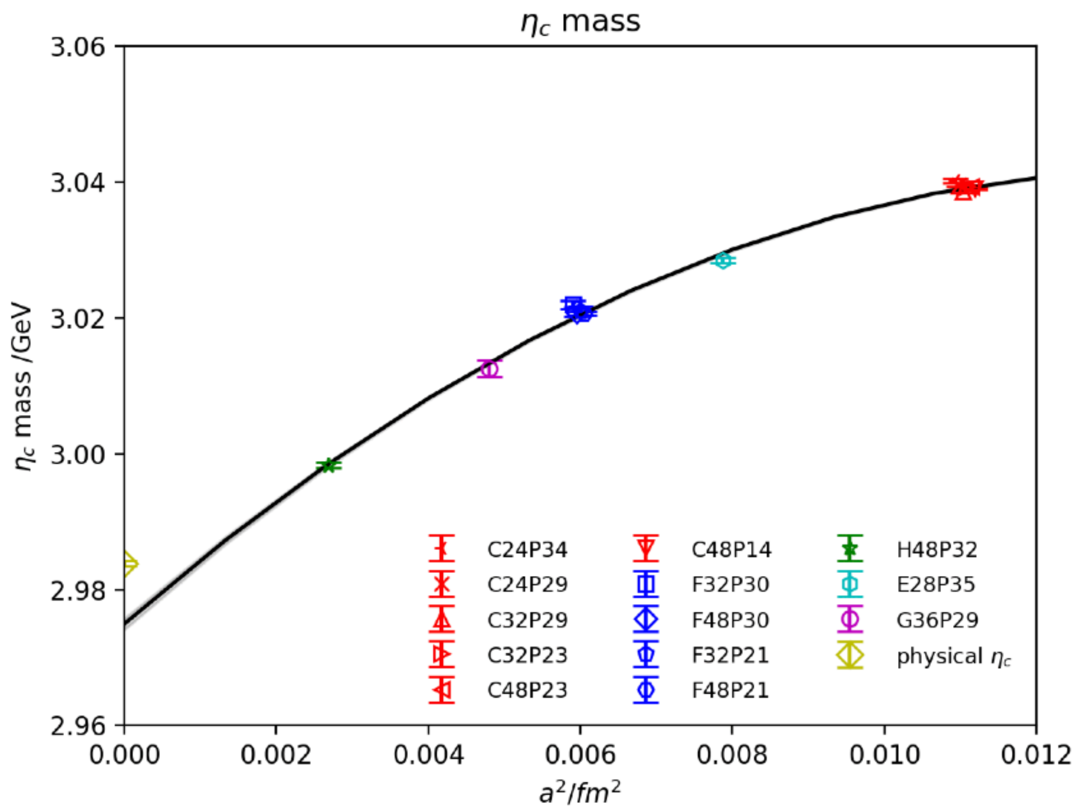


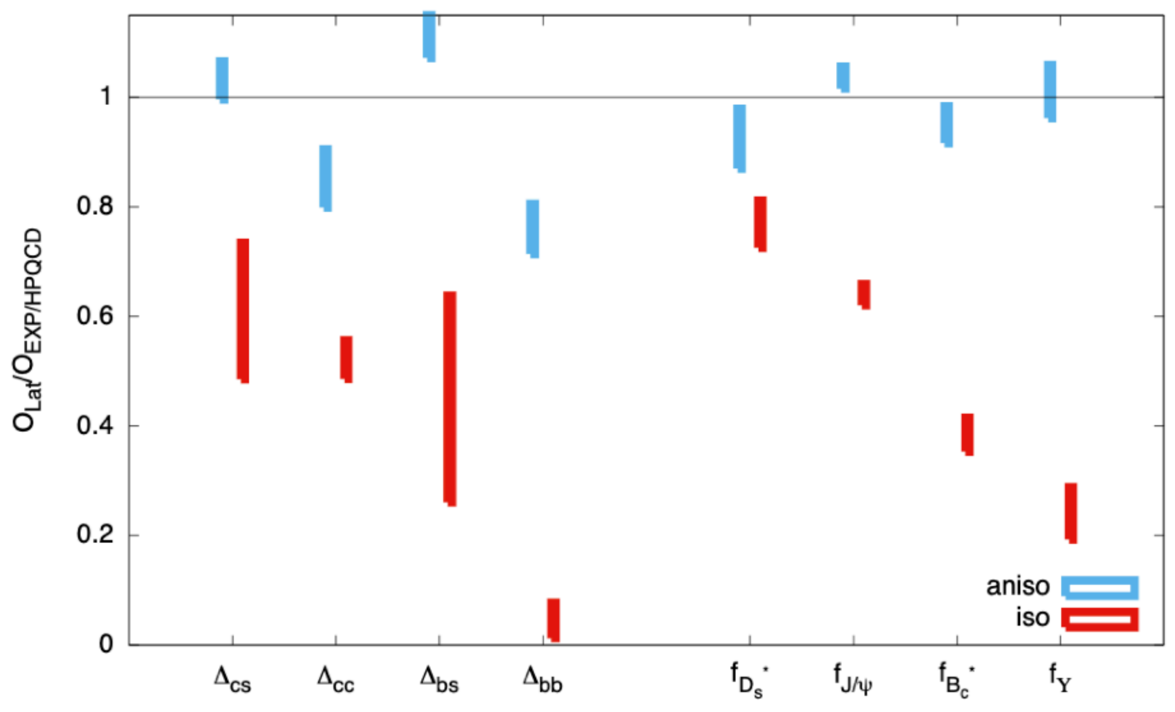






- **Light quark and hadron masses**



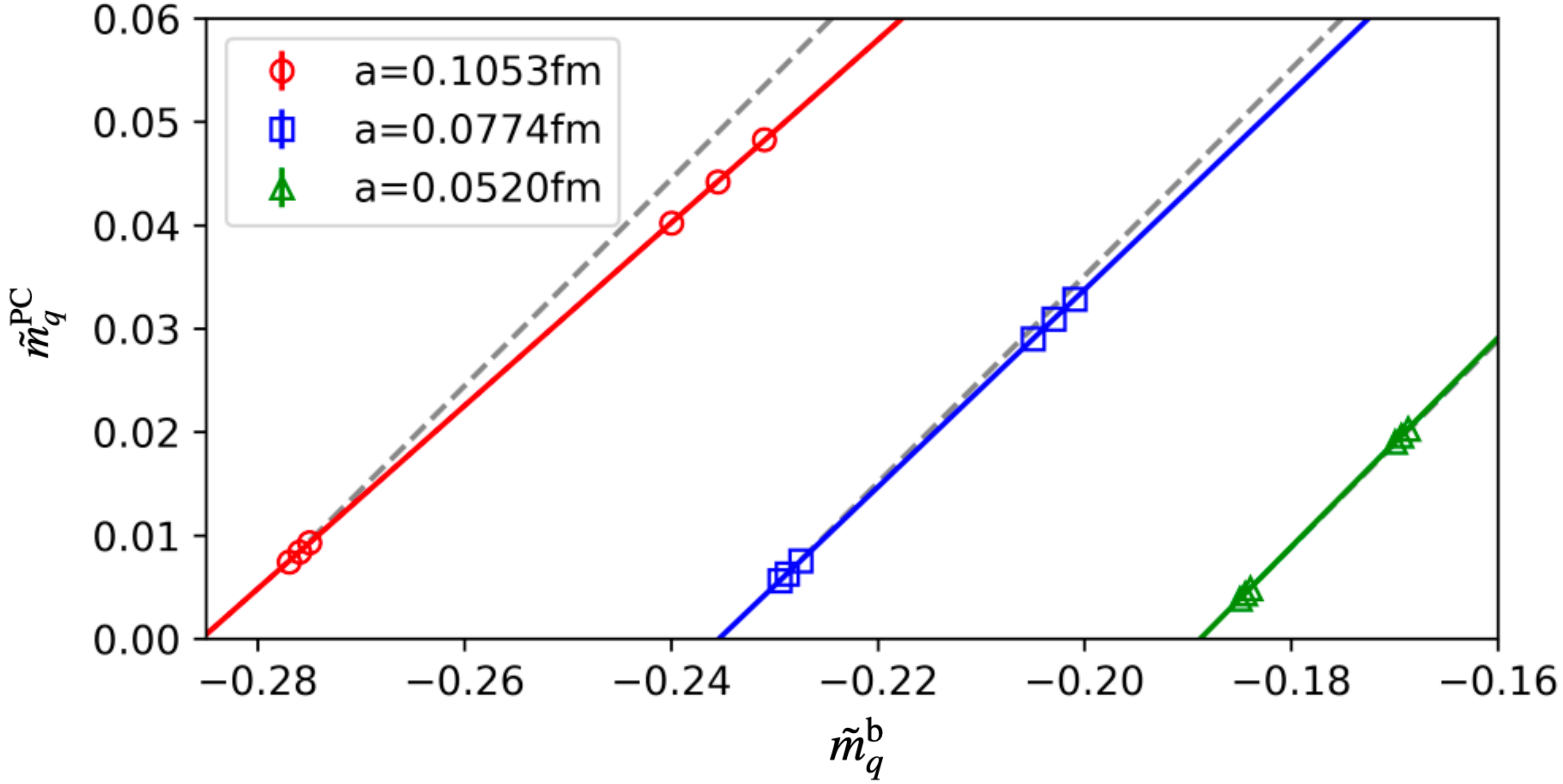


- Investigation on the charm quark

Quark mass

through PCAC

$$m_q^{\rm PC} = \frac{m_\pi^2}{2\Sigma/F^2}(1 + \mathcal{O}(\frac{m_\pi^2}{16\pi^2 F^2})) \sim \frac{m_\pi^2}{5\ \rm GeV}$$



- Due to the additive α_s/a correction, the dimensionless bare quark mass $\tilde{m}_q^{\rm b} = m_q^{\rm b} a$ is negative.
- The renormalized quark mass should be defined as $m_q^R = Z_m(m_q^{\rm b} - m_{\rm crti})$, where $m_{\rm crti}$ is defined as the $m_q^{\rm b}$ which vanishes the pion mass.
- One can avoid this difficulty by defining the quark mass through PCAC relation:

$$\langle 0 | \partial_4 A_4 | {\rm PS} \rangle = (m_q^{\rm PC} + m_{\bar{q}}^{\rm PC}) \langle 0 | P | {\rm PS} \rangle$$

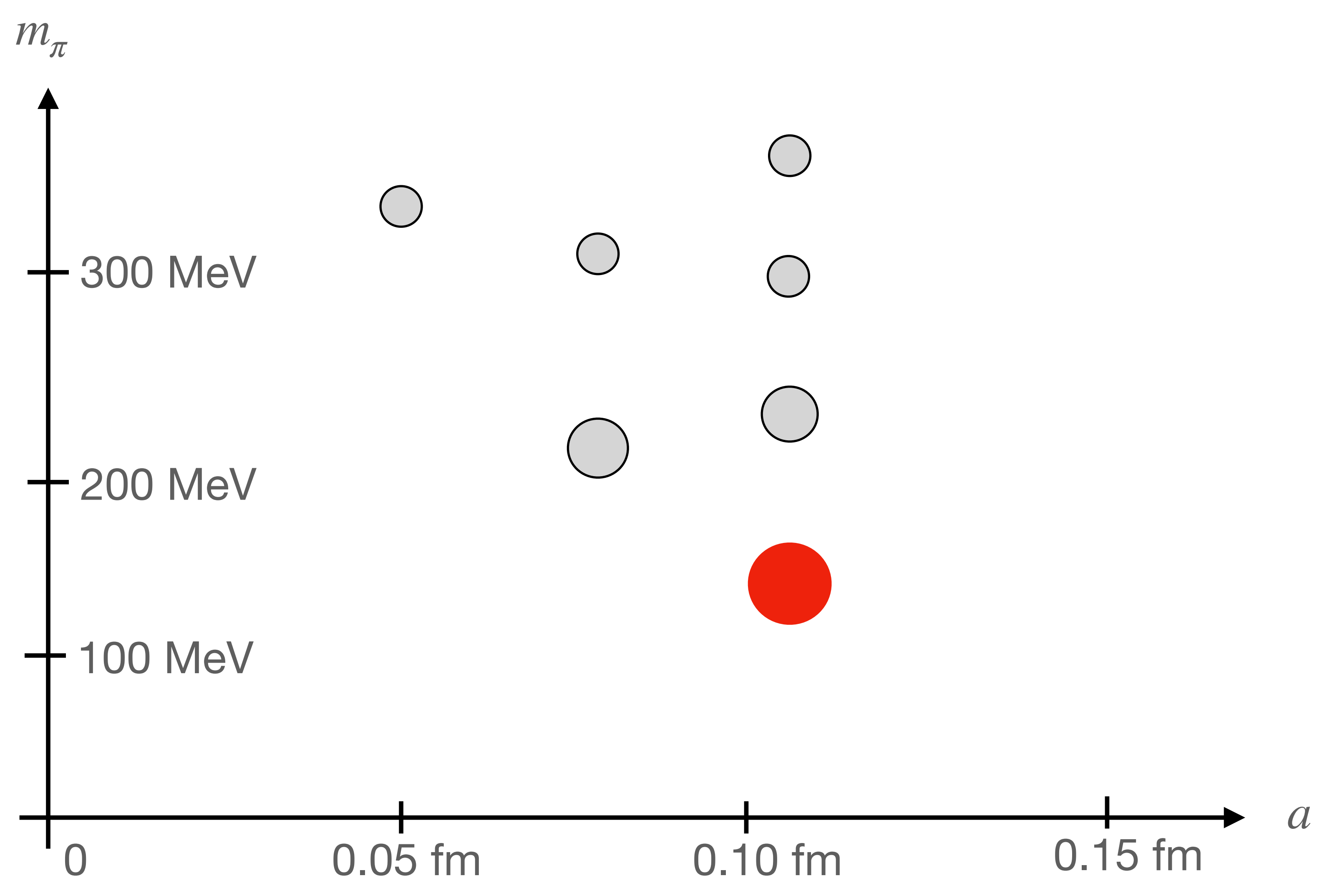
T. Ishikawa, et.al., JLQCD, Phys.Rev.D78 (2008) 011502

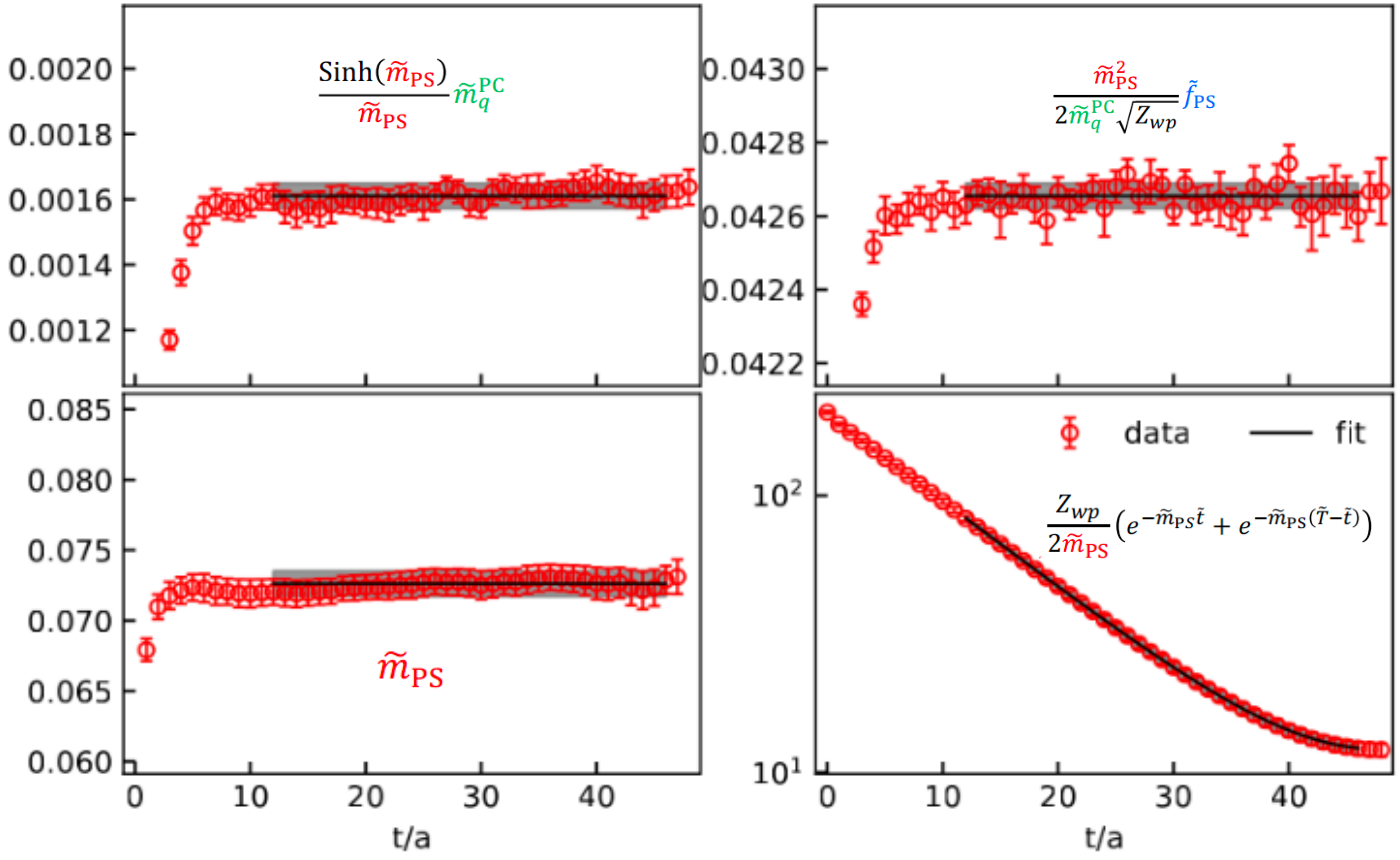
- And then $m_q^{\rm PC}$ is always positive and can be renormalized as $m_q^R = Z_P/Z_A m_q^{\rm PC}$.

Quark mass

Based on joint fit of pion correlators

- Joint fit of $\tilde{m}_q^{\rm PC} = m_q^{\rm PC} a$, $\tilde{f}_{\rm PS} = f_{\rm PS} a$, and $\tilde{m}_{\rm PS} = m_{\rm PS} a$, with several 2pt at $a^{-1} \sim 2$ GeV and physical pion mass;
- Used 48 measurements on each of 203 configurations.





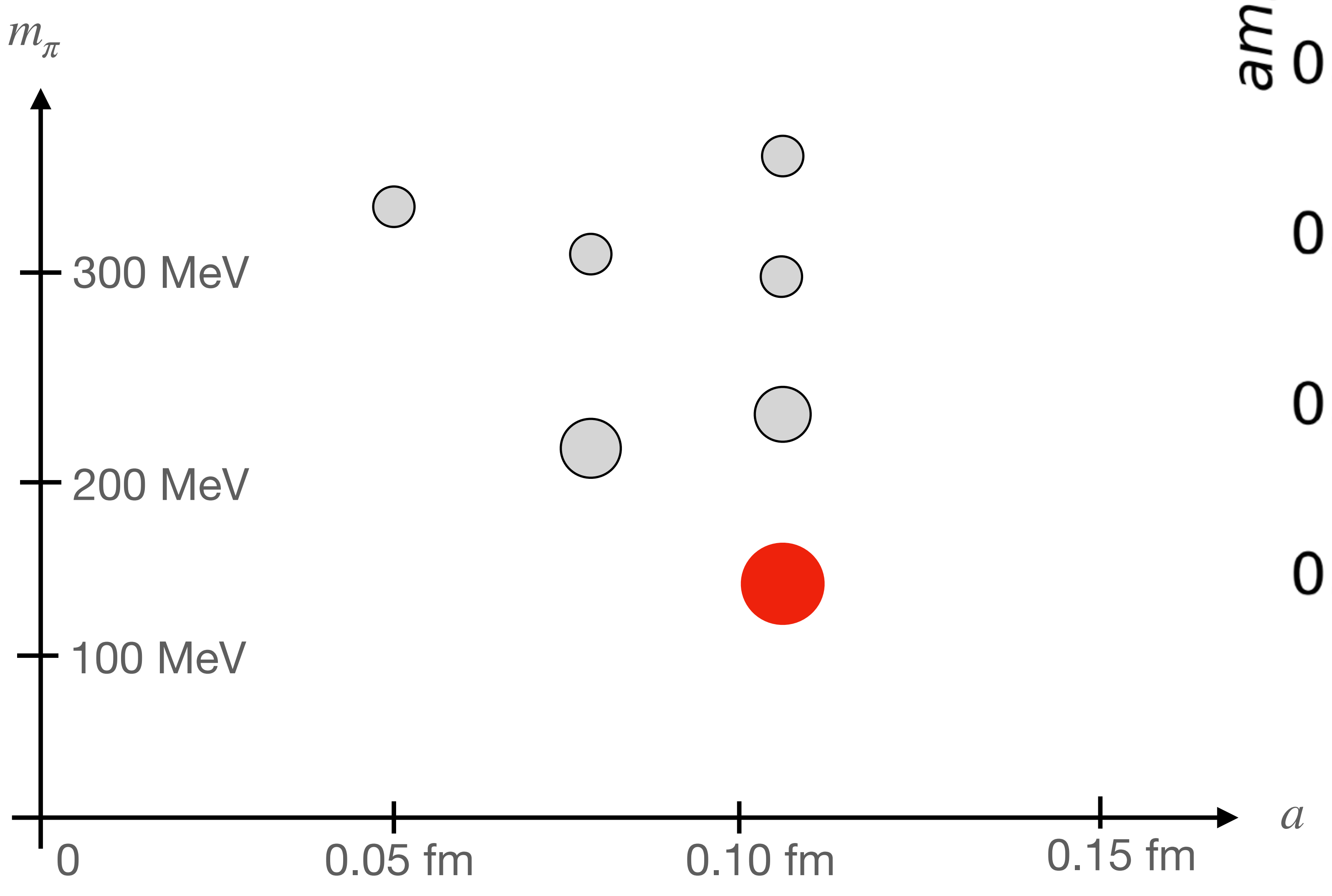
Hadron masses

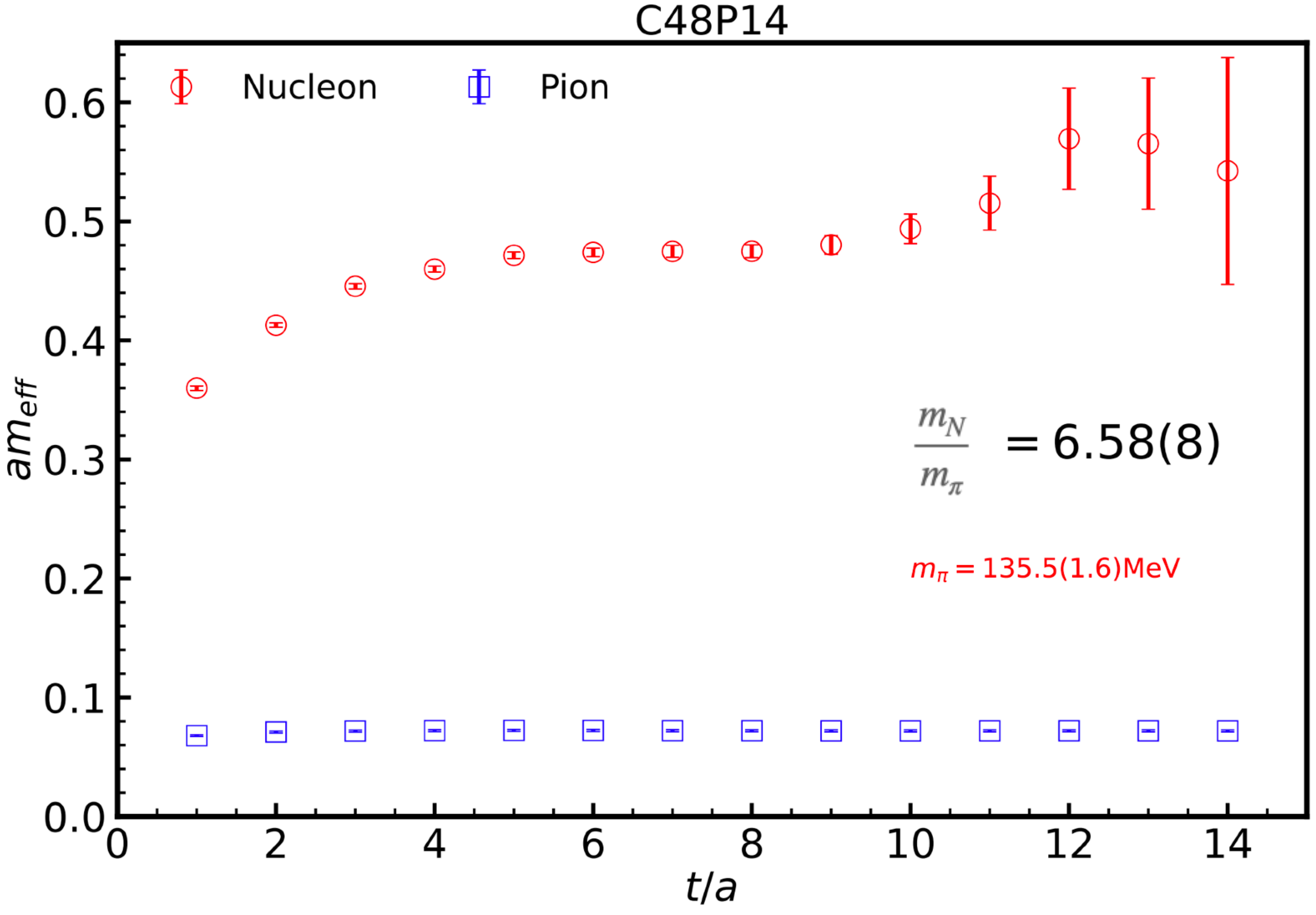
Nucleon mass v.s. pion mass

- With the same quark propagator, the ratio between the nucleon mass and pion mass is

  0.475(6)/0.0723(9)=6.58(8).

- Which is quite close to the physical value 0.939/0.135=6.96.





- Using the lattice spacing determined from the gradient flow, we have

$$m_\pi = 135.5(1.6)\ \text{MeV}, \quad m_N = 890(10)\ \text{MeV}.$$

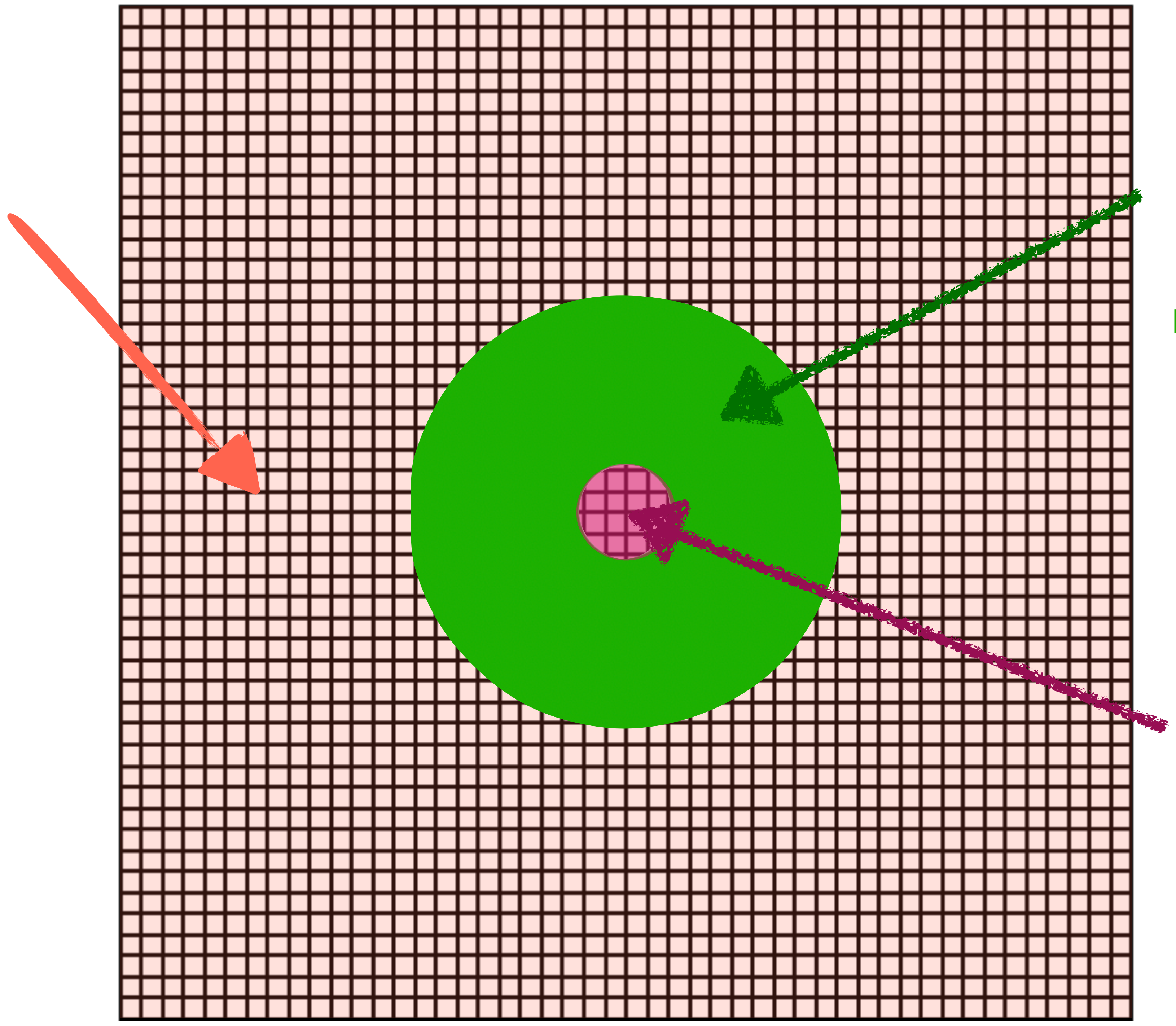
- m_N are ~5% smaller than the physical value, and can be a discretization effect based on the lattice spacing dependence of f_π.

Quark mass

Renormalization through intermediate scheme

$$m_q^{\overline{\rm MS}}(\mu) = \frac{Z_m^{\rm MOM,Lat}(Q,1/a)}{Z_m^{\rm MOM,Dim}(Q,\mu,\epsilon)} Z_m^{\overline{\rm MS},\rm Dim}(\epsilon) m_q^{\rm Lat}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$

- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**



G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

Quark mass

Perturbative renormalization

- The RI/MOM renormalization constant of the quark mass under the lattice regularization is:

$$Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi) = (Z_S^{\mathrm{MOM,Lat}}(Q,1/a,\xi))^{-1} = \langle q|\mathcal{O}|q\rangle^{\mathrm{Lat}} = 1 + \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2);$$

- The RI/MOM and $\overline{\mathrm{MS}}$ renormalization constants under the dimensional regularization are:

$$Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi) = \langle q|\mathcal{O}|q\rangle^{\mathrm{Dim}} = 1 + \frac{\alpha_s C_F}{4\pi}[\frac{3}{\tilde{\epsilon}} - 3\log(\frac{Q^2}{\mu^2}) - \xi + 5] + \mathcal{O}(\alpha_s^2)$$

$$Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\tilde{\epsilon}} + \mathcal{O}(\alpha_s^2)$$

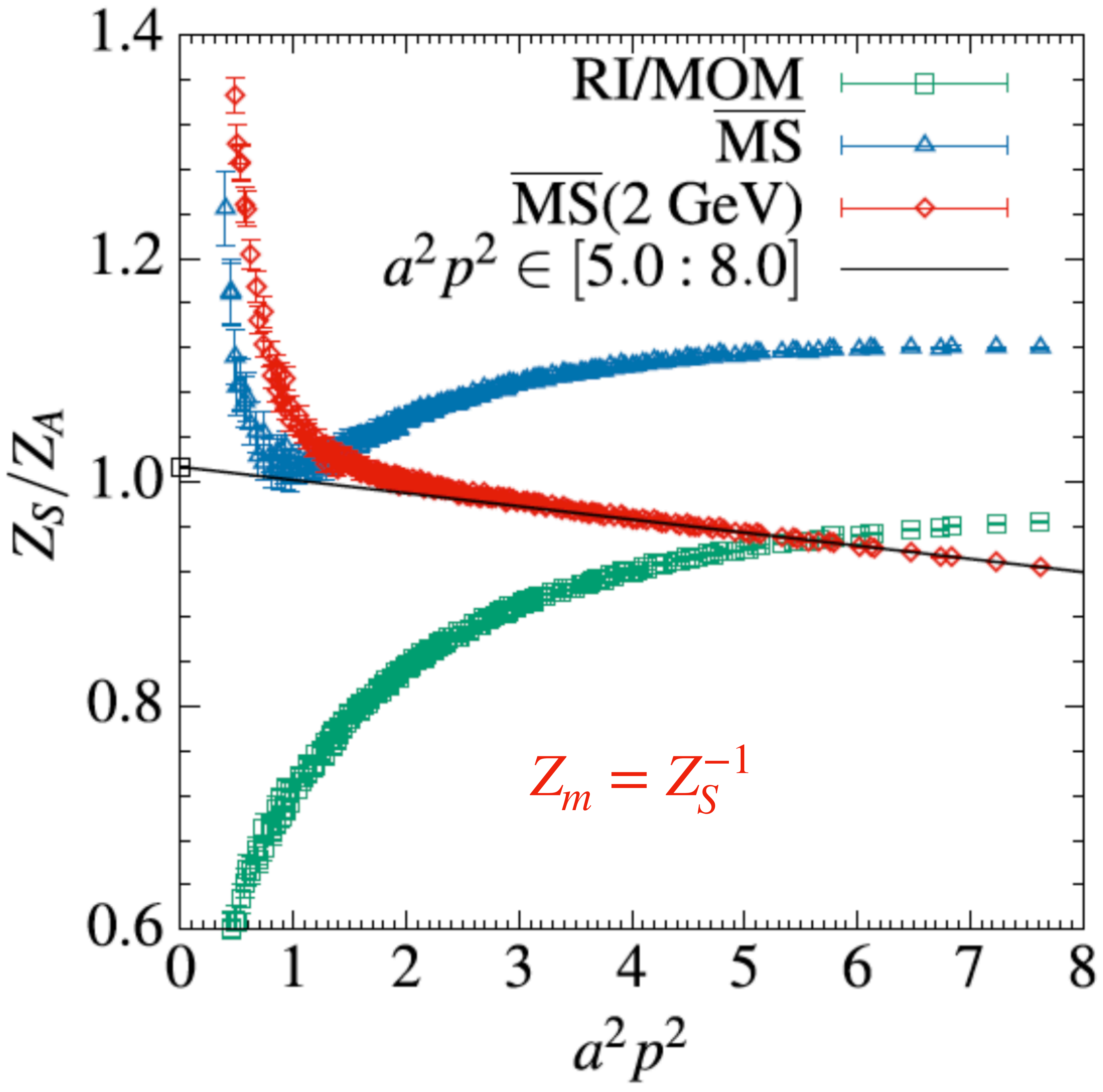
- Thus the renormalized quark mass under the $\overline{\mathrm{MS}}$ scheme can be defined by:

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi)}{Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)$$

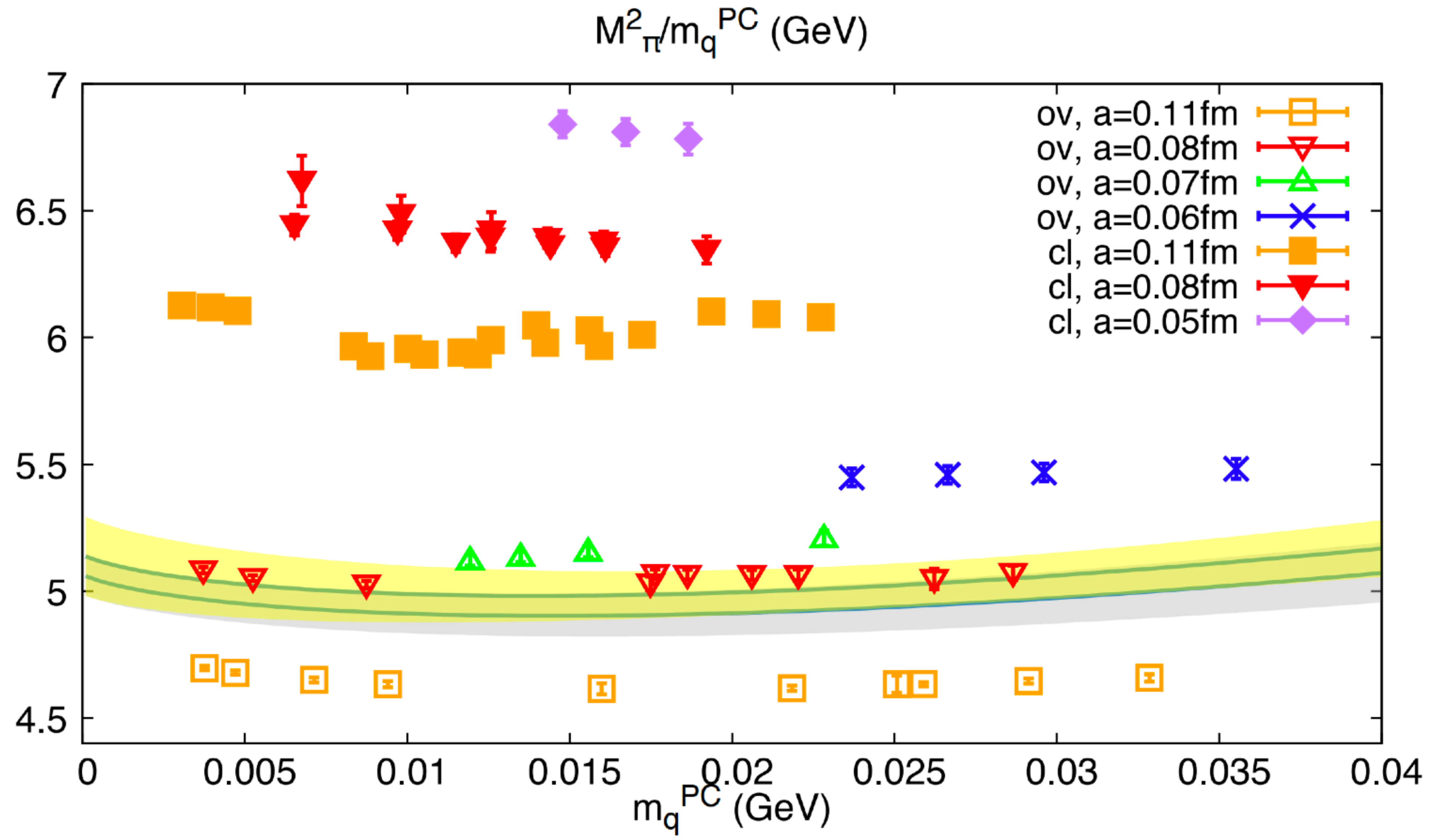
Quark mass

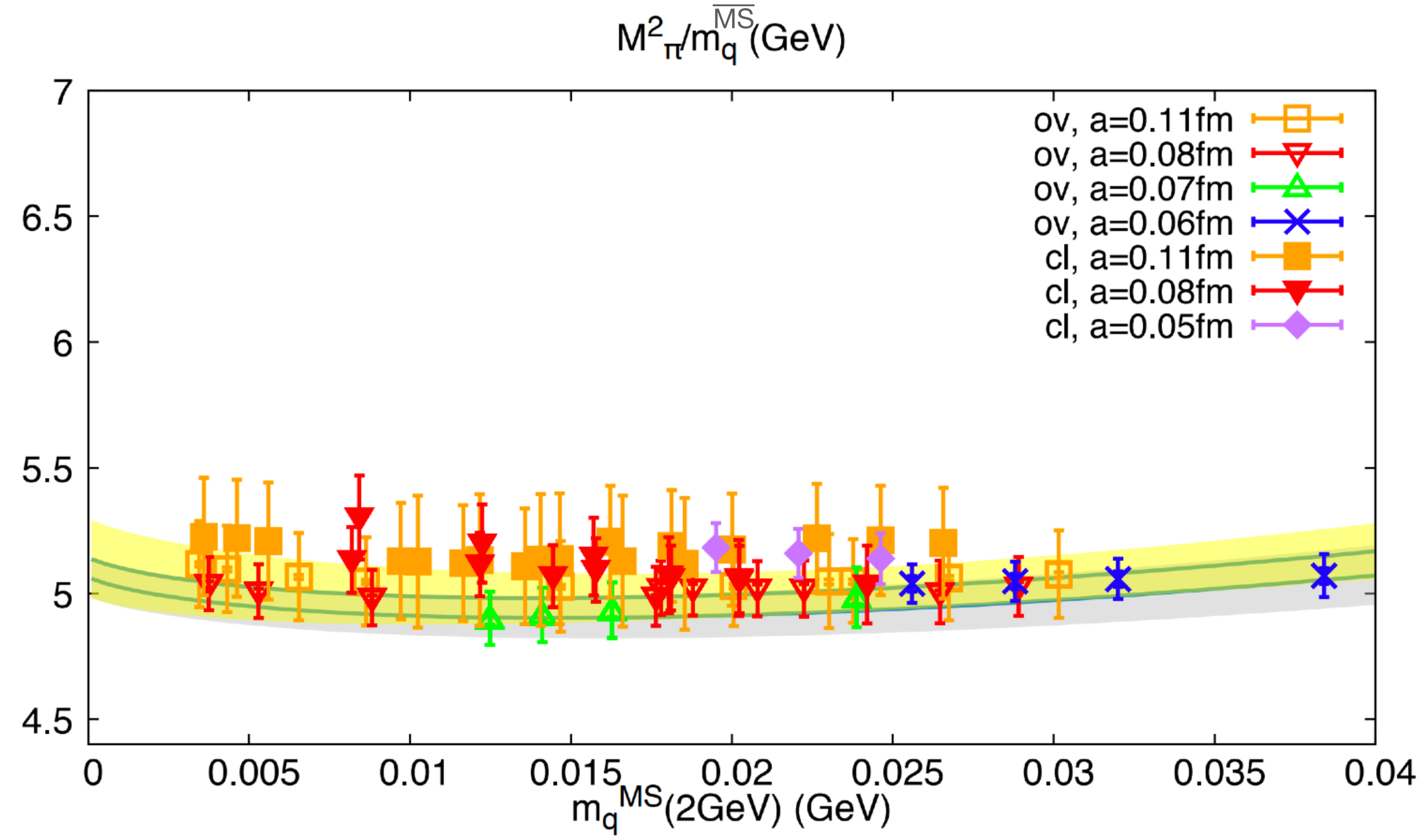
Non-Perturbative renormalization

- Calculate the RI/MOM renormalization constants non-perturbatively;
- Match the RI/MOM results to $\overline{\mathrm{MS}}$ scheme at different scale $\mu^2 = p^2$;
- Evaluate the scale from $\mu^2 = p^2$ to μ = 2 GeV.
- Extraplate to the a^2p^2 limit to eliminate the discretization error.



Quark mass





Renormalize mass using different actions

- Non-perturbative renormalization to $\overline{\text{MS}}$ 2 GeV eliminates the regularization scale $1/a$ dependence of m_π^2/m_q.
- m_π^2/m_q using the clover fermion also turns out to be consistent with that using the overlap fermion.
- The large uncertainty of the renormalized m_π^2/m_q majorly comes from the missing higher order effect of the perturbative matching

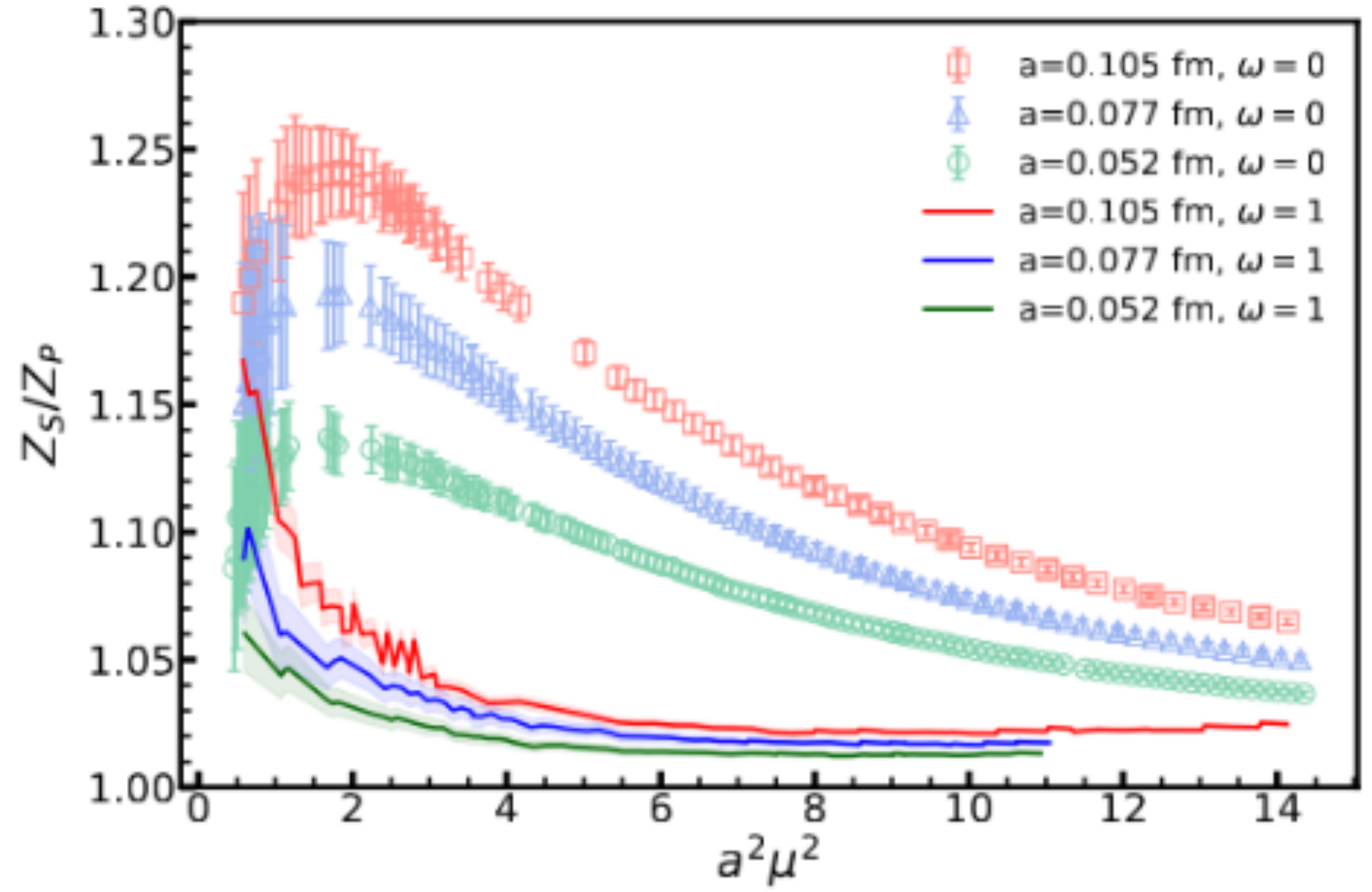
$$\frac{Z_P^{\overline{\text{MS}}}}{Z_P^{\text{MOM}}} = 1 + 0.4244\alpha_s + 1.007\alpha_s^2 + 2.722\alpha_s^3 + 8.263\alpha_s^4 + ?,$$

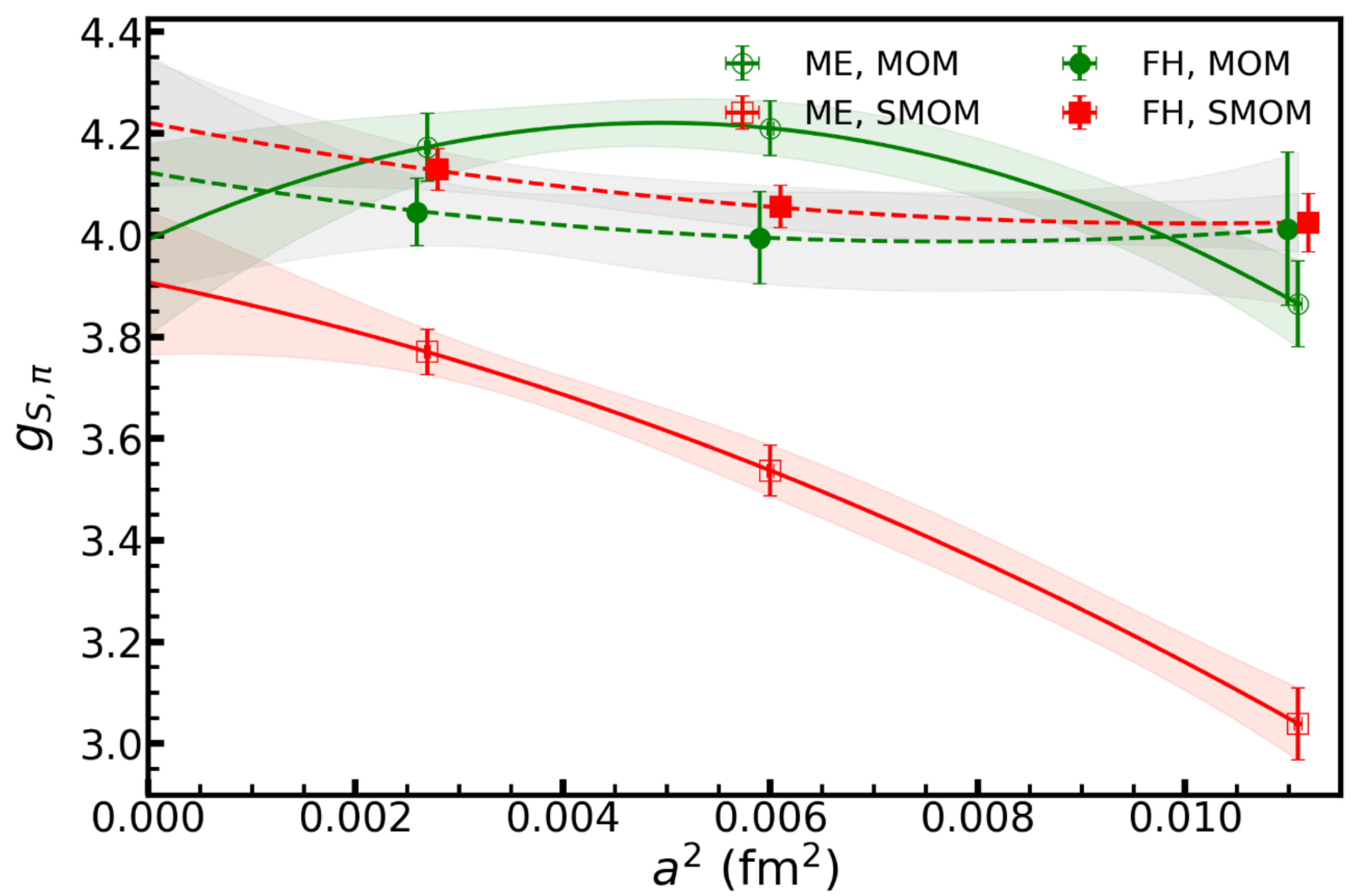
J.A. Gracey, Eur.Phys.J.C83 (2023) 181

- and can be highly suppressed after the continuum extrapolation.

Non-perturbative renormalization

Restore of chiral symmetry in the continuum





- Renormalized quark mass $m_q^R = Z_A/Z_P m_q^{\mathrm{PC}}$ with 317 MeV pion mass at three lattice spacings:
  - The intermediate renormalization scheme dependence is 3.1(1.5)%.
  - RI/MOM scheme has smaller discretization error.
  - Feynman-Hellman theorem can extract $g_{S,\pi}$ as

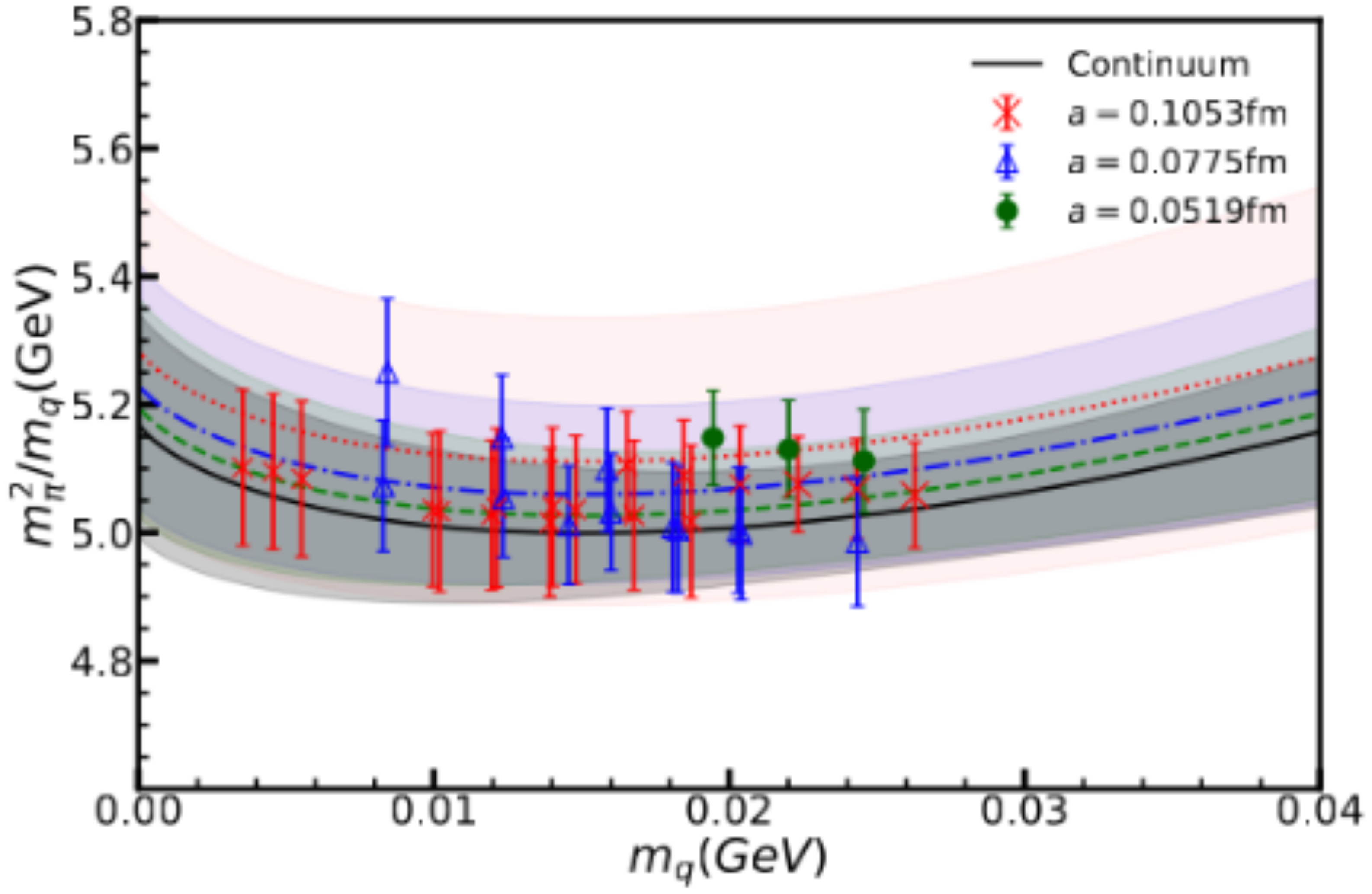
$$g_{S,\pi}^{\mathrm{FH}} = \frac{1}{2}\frac{\partial m_\pi(m_q)}{\partial m_q} \simeq \frac{Z_P}{Z_A}\frac{m_\pi}{4m_q^{\mathrm{PC}}} + \mathcal{O}(m_q, a^2)$$

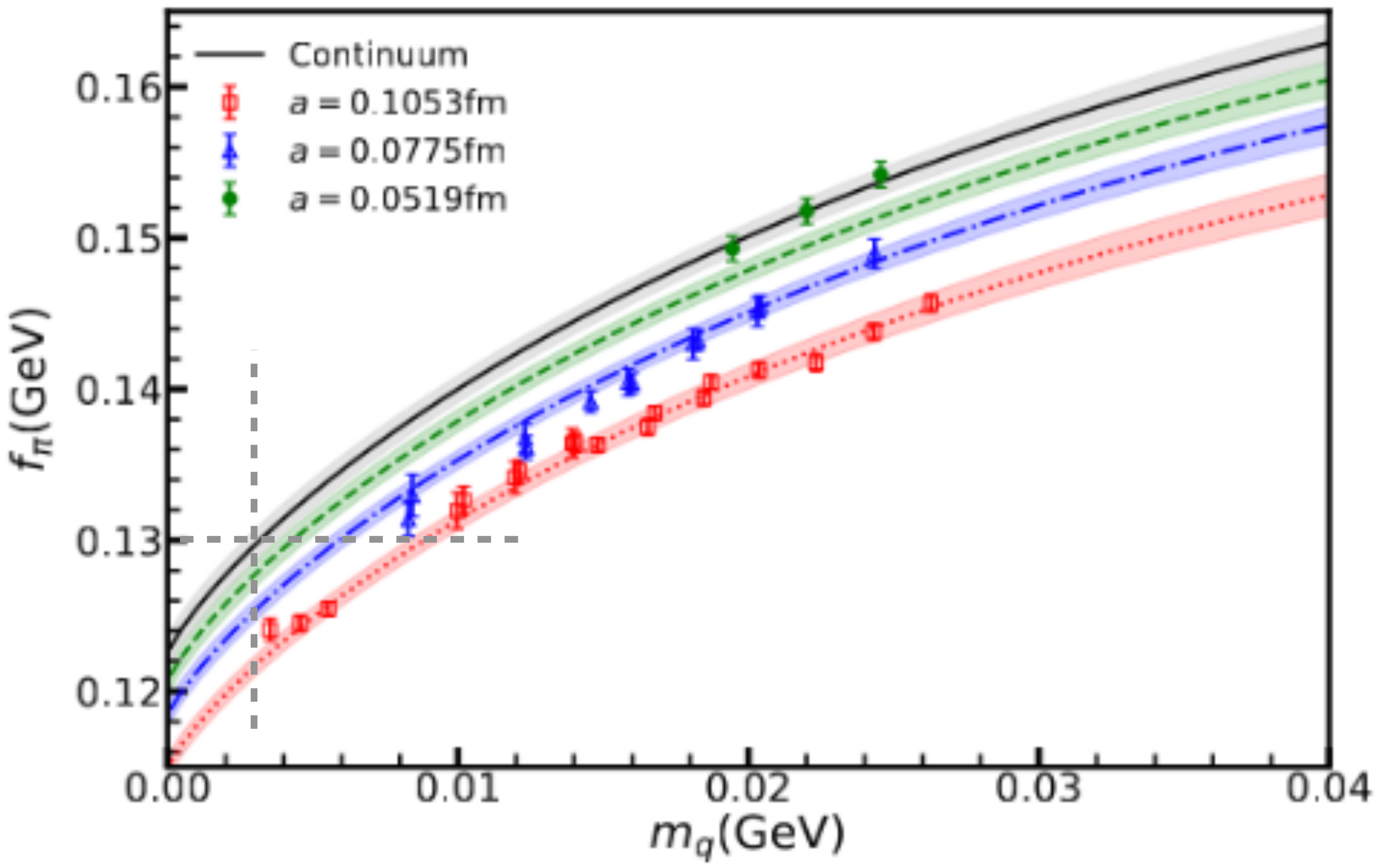
  which is 4.04(6)(12) for $m_\pi = 317\ \mathrm{MeV}$ in the continuum.

- Renormalized $g_{S,\pi}^{\mathrm{R,ME}} = Z_S \frac{\langle \pi | S | \pi \rangle_{\mathrm{conn}}}{\langle \pi | \pi \rangle}$ based on the direct calculation:
  - The intermediate renormalization scheme dependence is 7.6(2.3)% (linear a^2 correction) or 2.0(5.8)% ($a^2 + a^4$ corrections).
  - $g_{S,\pi}^{\mathrm{ME}}$ using RI/MOM scheme has smaller discretization error, and agree with $g_{S,\pi}^{\mathrm{R,FH}}$ within 2σ at all the lattice spacings.

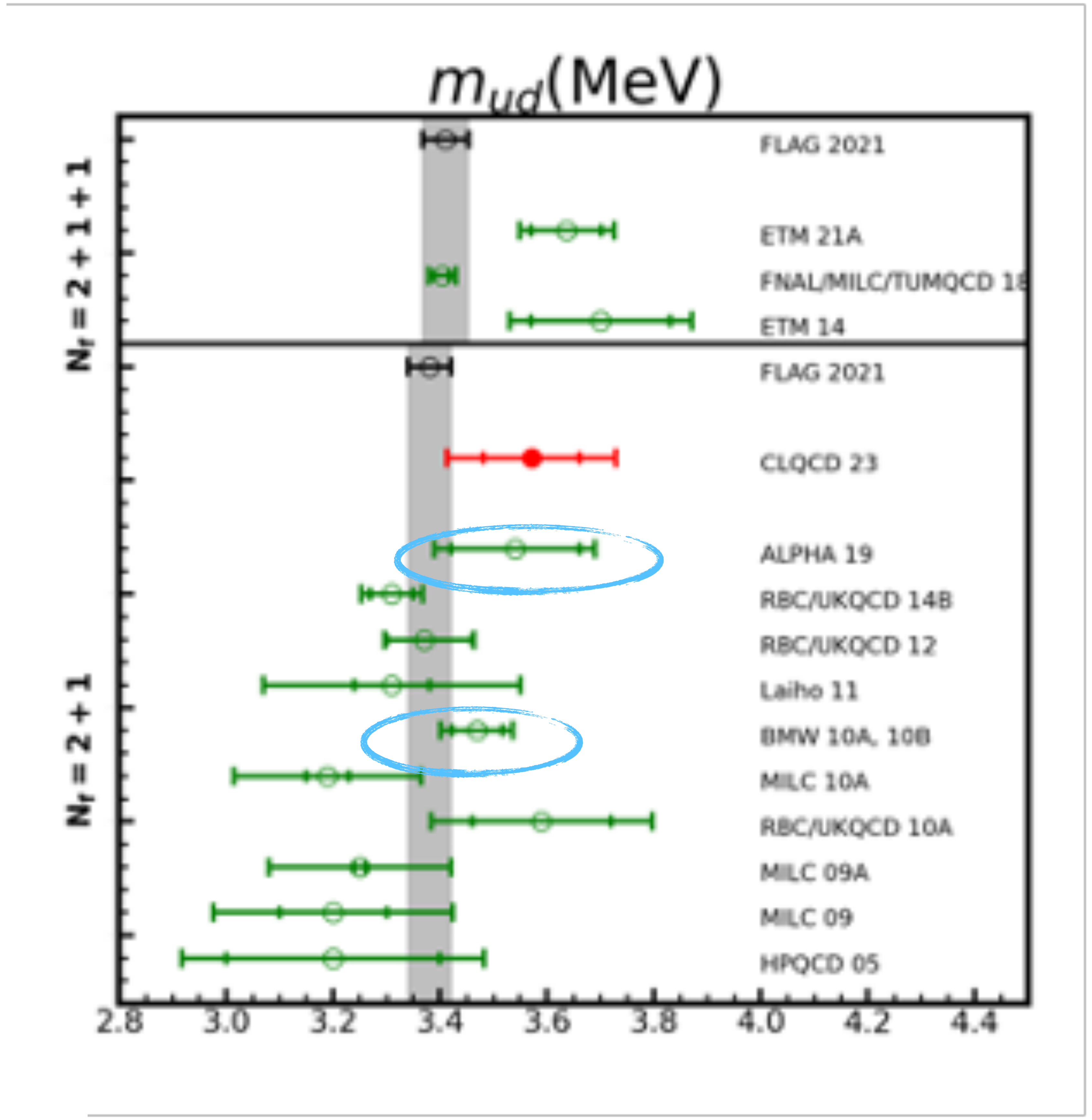
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Quark mass





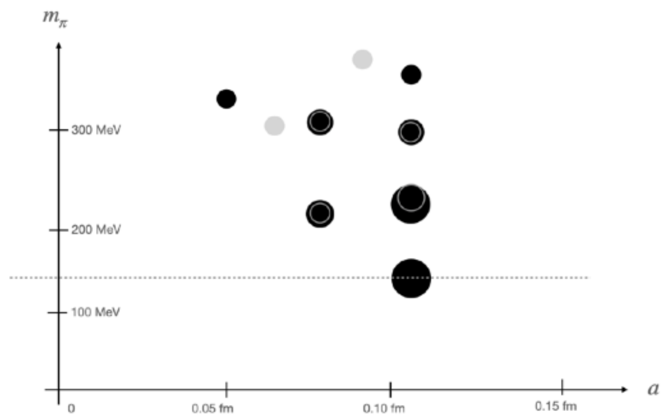
Global fit of the pion mass

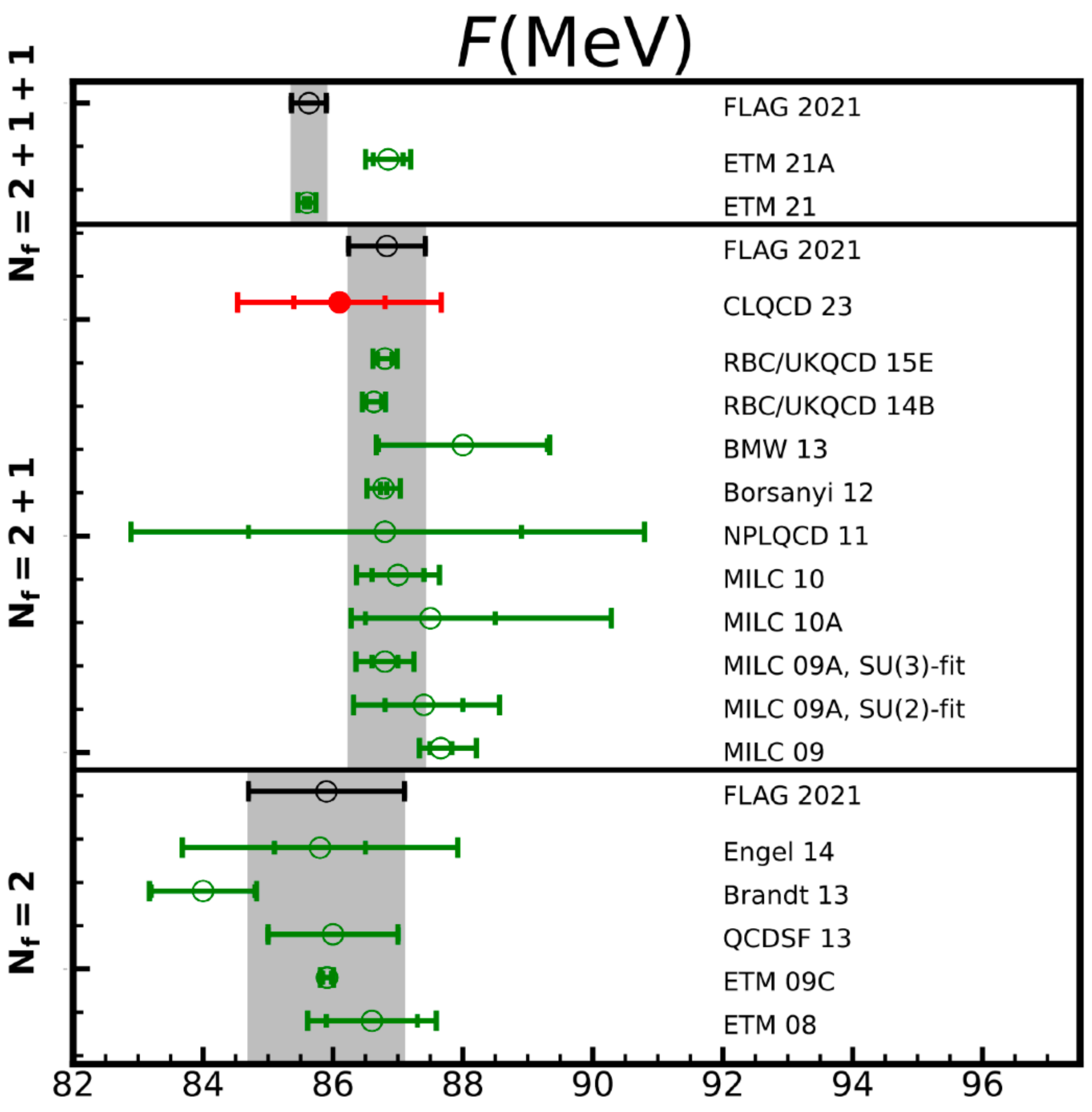


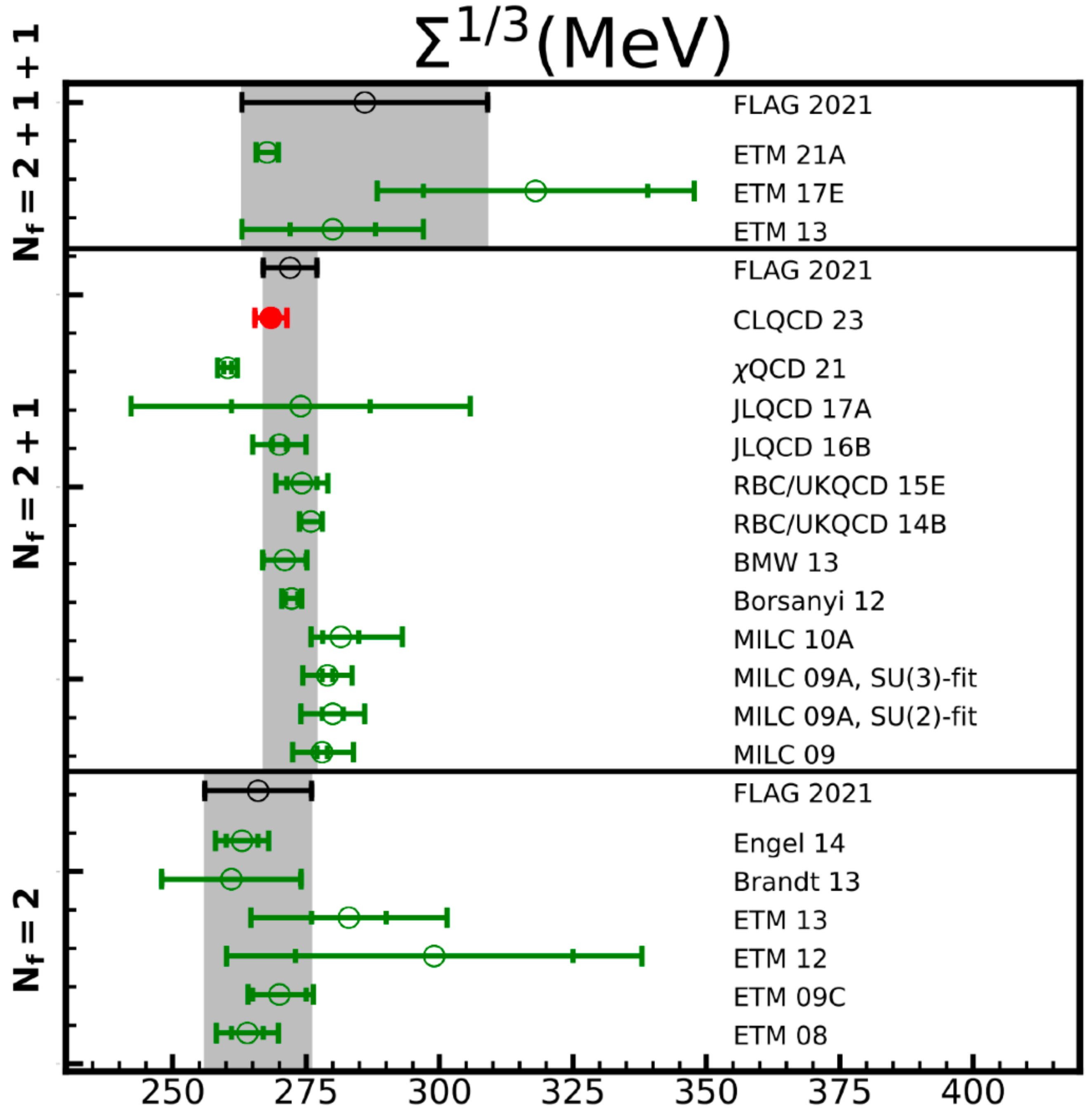
- Present CLQCD prediction of the u-d averaged light quark masses is consistent with the lattice averages within 5% uncertainty.
- Most of the uncertainties come from the non-perturbative renormalization and further improvements are in progress.
- All the finite volume, discretization and sea quark mass effects have been taken into account.

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Low energy constants determination







- The CLQCD prediction on the low energy constants can be more precise.
- The precision of the NLO low energy constants are higher than the present lattice averages.

$$m_\pi^2 = \Lambda_\chi^2 2y\left[1 + y\left(\ln\frac{2y\Lambda_\chi^2}{m_{\pi,\mathrm{phys}}^2} - \ell_3\right) + \mathcal{O}(y^2)\right],$$

$$F_\pi = F\left[1 - 2y\left(\ln\frac{2y\Lambda_\chi^2}{m_{\pi,\mathrm{phys}}^2} - \ell_4\right) + \mathcal{O}(y^2)\right],$$

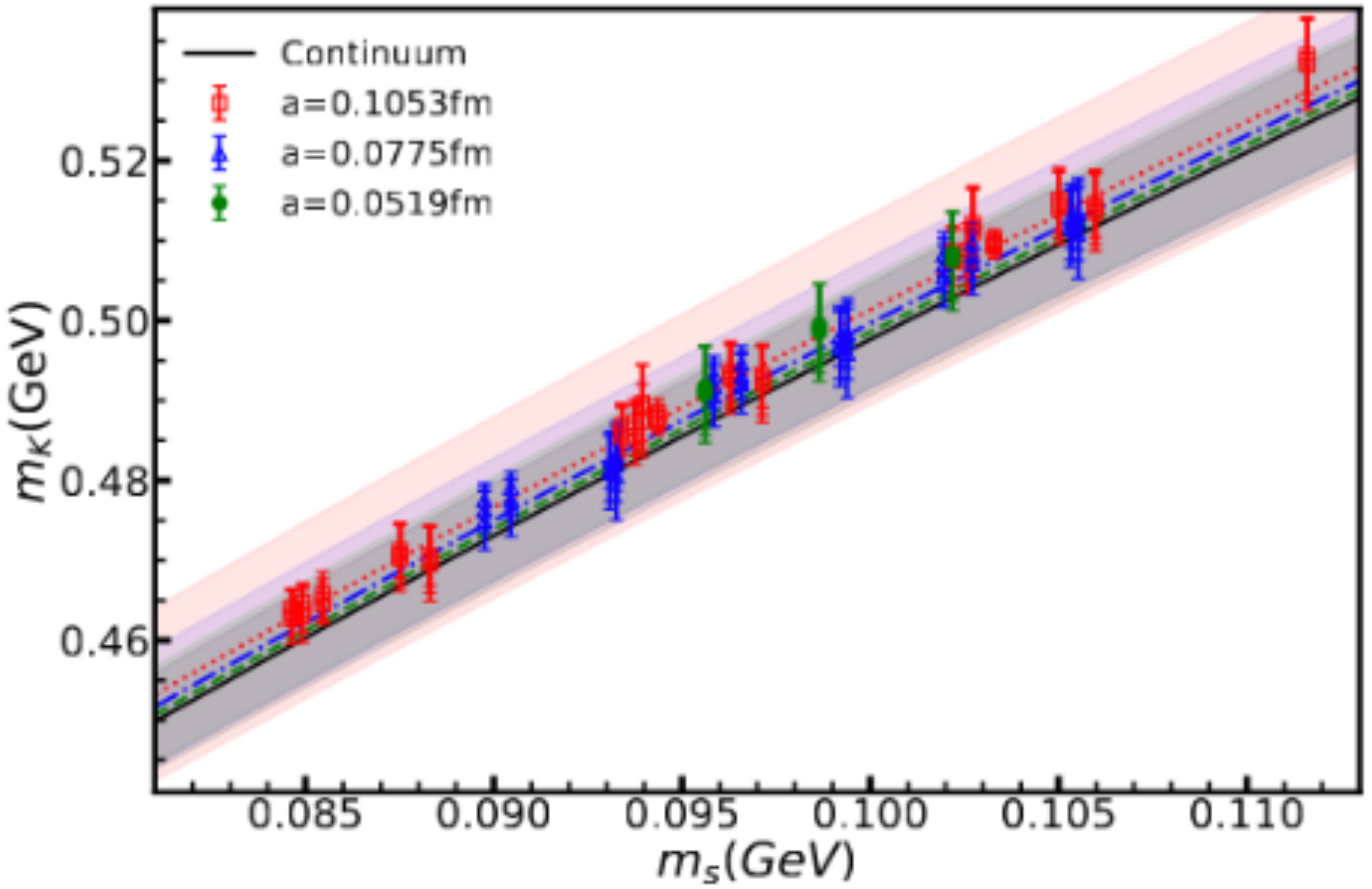
$$y = \frac{\Sigma m_l}{F^2\Lambda_\chi^2} \simeq \frac{m_\pi^2}{32\pi^2F^2}$$

	l_3	l_4
CLQCD	2.43(54)	4.32(12)
FLAG	3.07(64)	4.02(45)

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, Phys.Rev.D109 (2024) 054507

Quark mass

Global fit of the kaon mass



$$m_K^2(m_l^v, m_l^s, m_s^v, m_s^s, a) = (b_s^v m_s^v + b_s^s m_s^s + b_l^v m_l^v + b_l^s m_l^s) \times \left[1 + c_l^K m_l^v + c_m^K a^2 + c_L^K \exp(-m_K L)\right],$$

$$m_u^{\rm phys} + m_d^{\rm phys} = 2m_l^{\rm phys}.$$

$$m_K(m_d^{\rm phys}, m_l^{\rm phys}, m_s^{\rm phys}, m_s^{\rm phys}, 0) = m_{K^0,{\rm QCD}},$$

$$m_K(m_u^{\rm phys}, m_l^{\rm phys}, m_s^{\rm phys}, m_s^{\rm phys}, 0) = m_{K^\pm,{\rm QCD}},$$

val sea val sea

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27$ MeV $= m_{p,\rm QCD} + 1.00(16)$ MeV $+ \ldots$;
- $m_n = 939.57$ MeV;
- $m_\pi^0 = 134.98$ MeV;
- $m_\pi^+ = 139.57$ MeV $= m_\pi^0 + 4.53(6)$ MeV $+ \ldots$;

X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

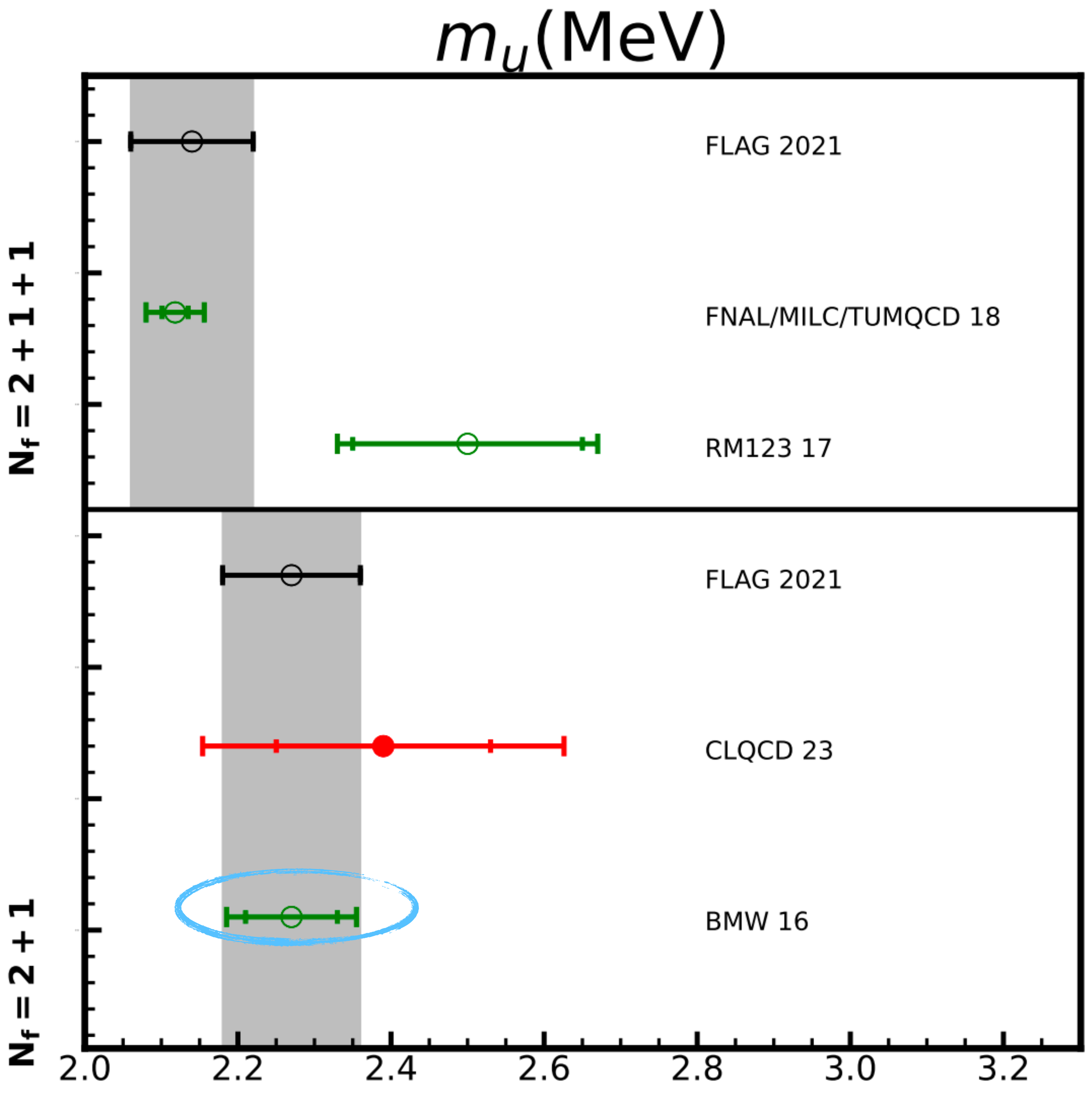
- $m_K^0 = 497.61(1)$ MeV $= m_{K,\rm QCD}^0 + 0.17(02)$ MeV $+ \ldots$;
- $m_K^+ = 493.68(2)$ MeV $= m_{K,\rm QCD}^+ + 2.24(15)$ MeV $+ \ldots$

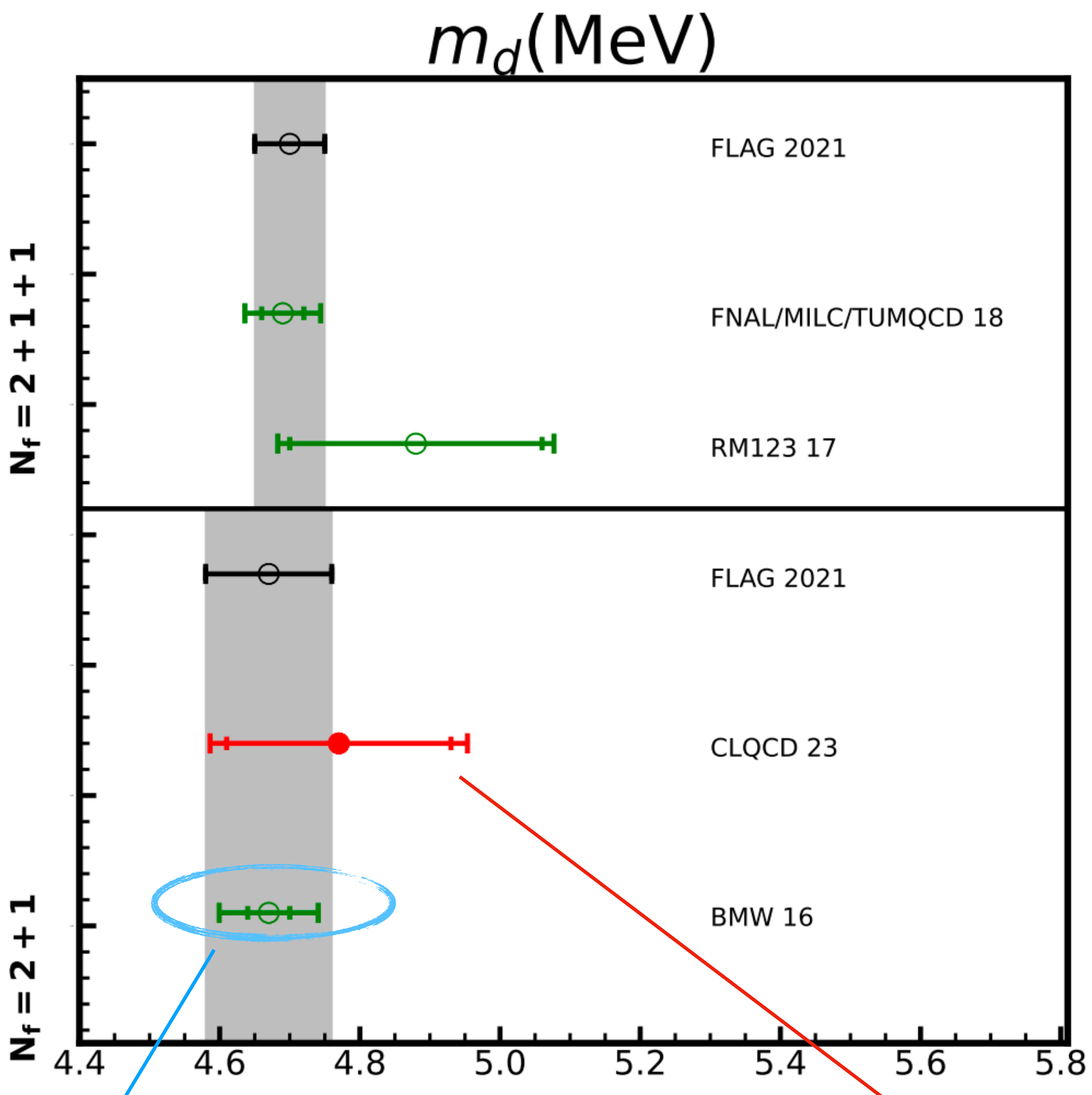
D. Giusti, et,al. PRD95(2017)114504

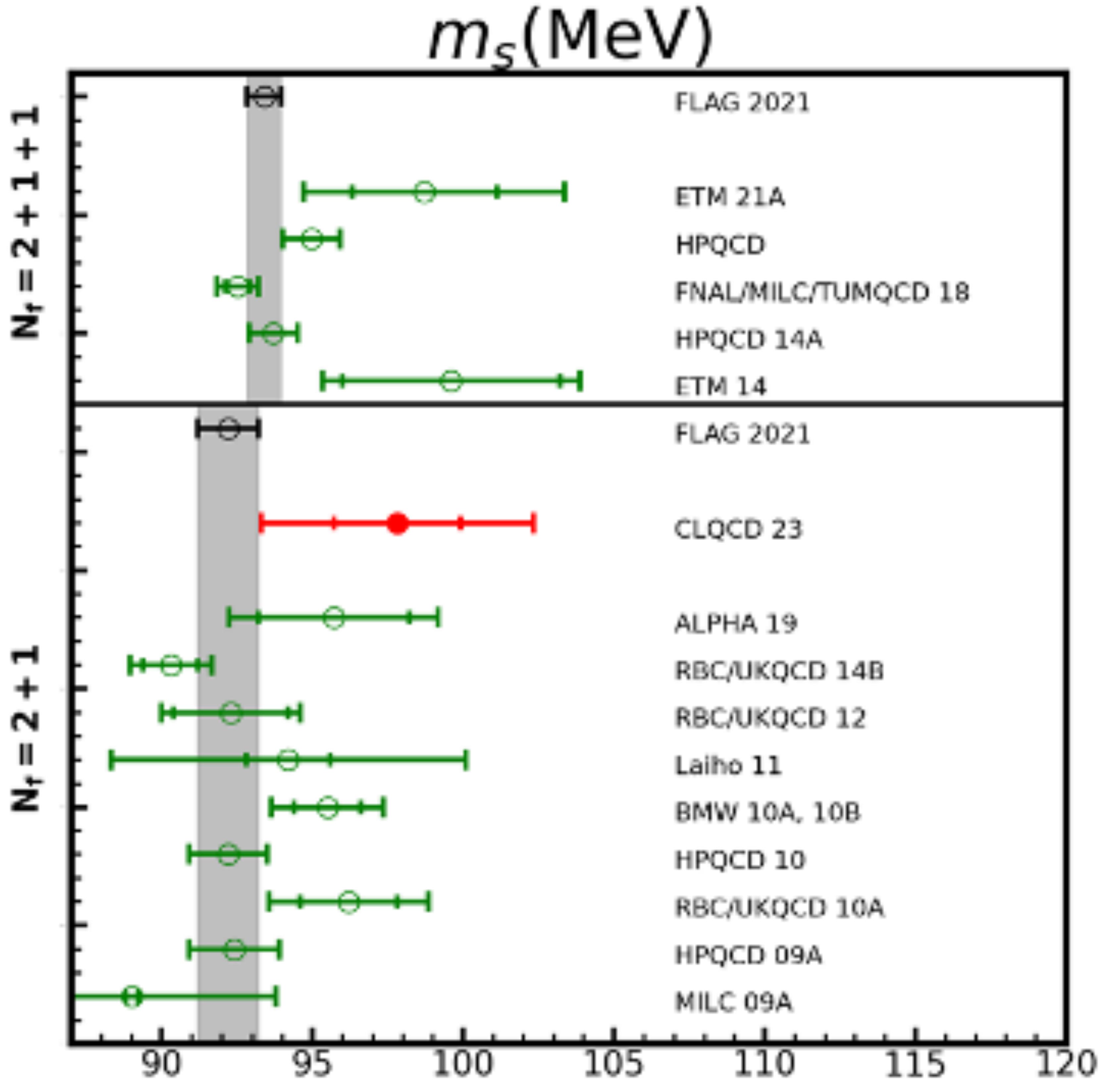
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Quark mass

of three light flavors





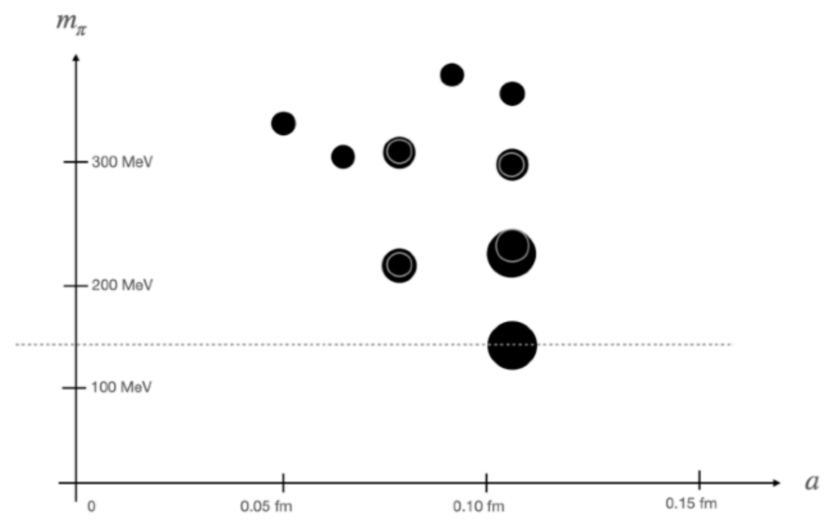


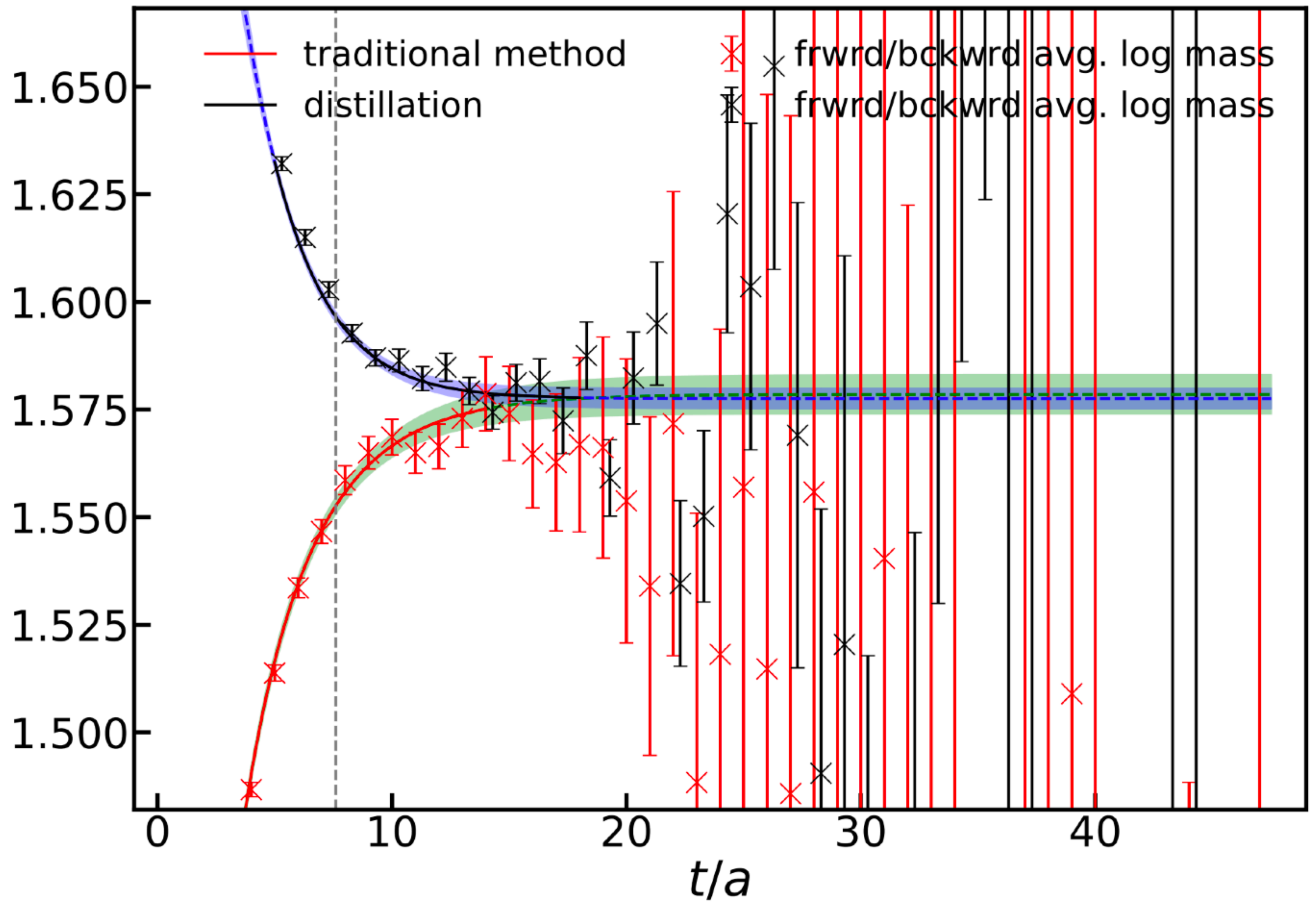
Summary of the lattice methodology.—The lattice setup used for this project is very similar to Ref. [13] and is based on our set of lattice QCD simulations presented in Ref. [6]. It is composed of 47 $N_f = 2 + 1$ QCD ensembles with pion masses down to 120 MeV, 5 lattice spacings down to 0.054 fm, and 16 different volumes up to $(6\ \text{fm})^3$.

BMWc, PRL 117(2016)0820001

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Baryon masses

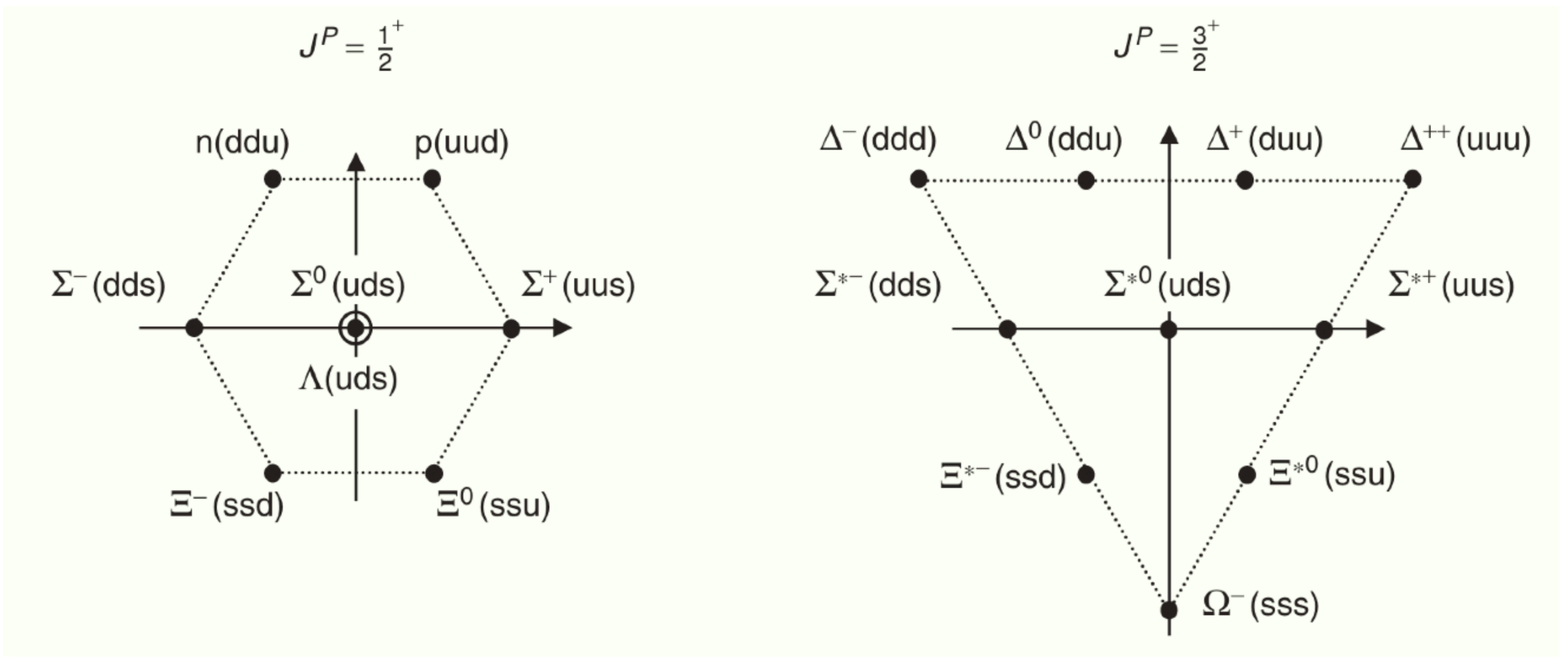


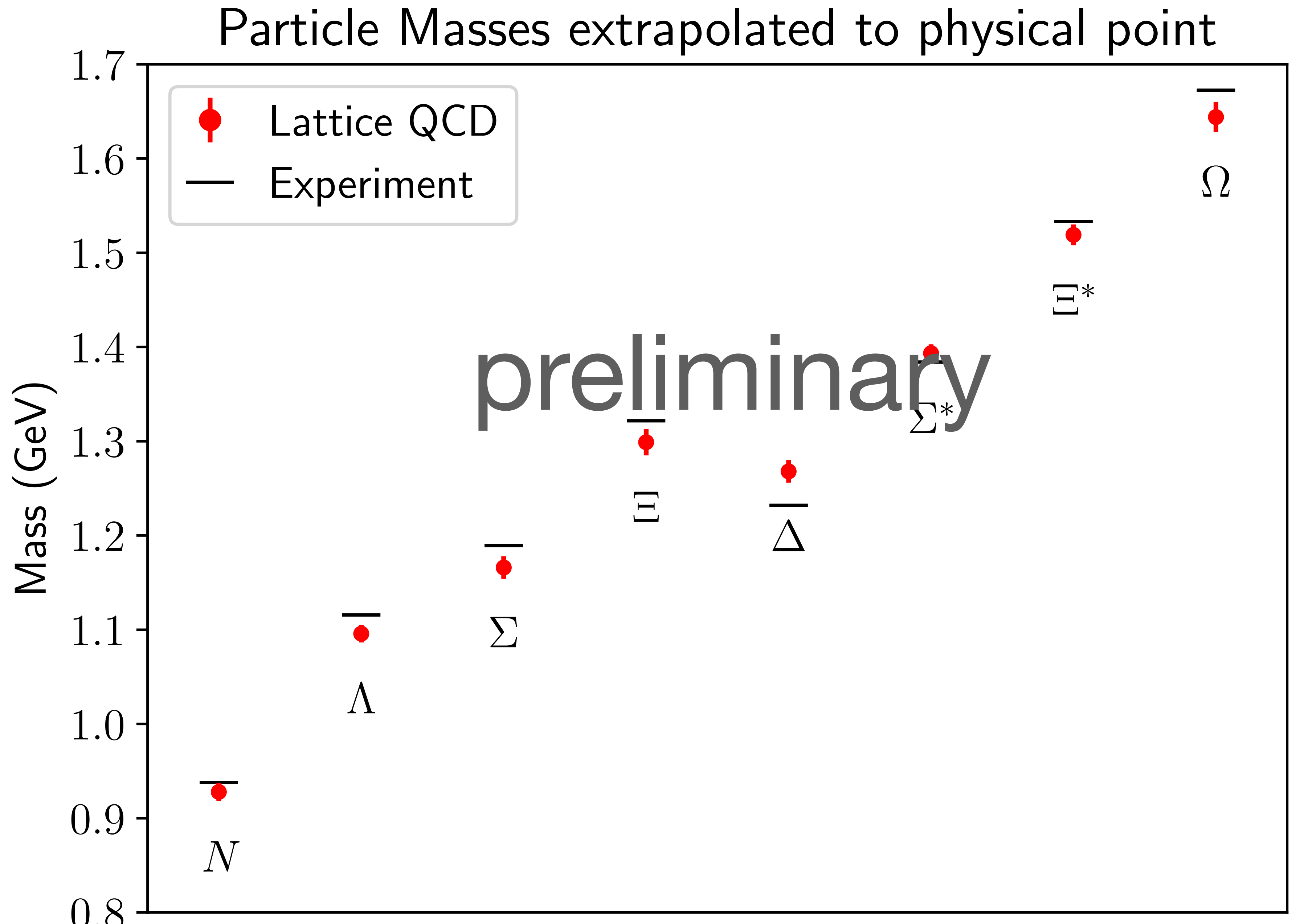


Based on CLQCD ensembles

Extract the ground state mass using multiple interpolation fields:

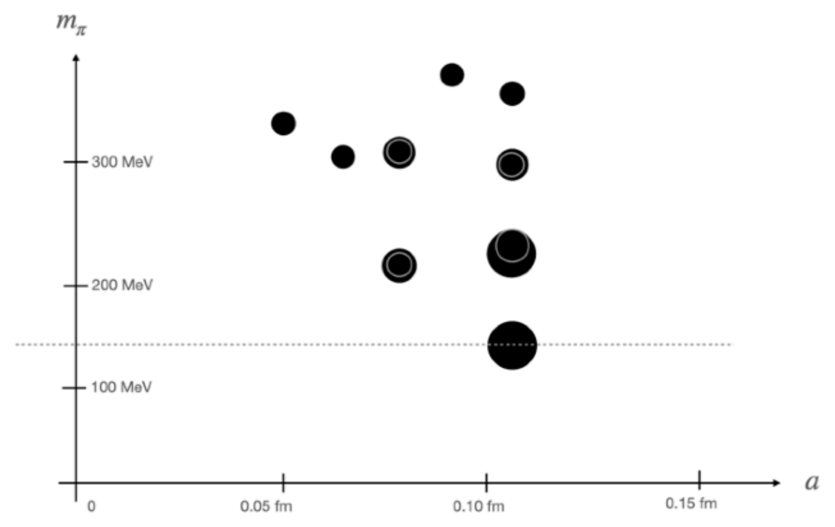
- The extracted mass is independent of the interpolation fields.
- Agree with the experimental value within a few percents;
- The mass difference between octet and decuplet baryon in the $N_f = 3$ chiral limit is 0.31(7) GeV.





B.-L. Hu, et. al., CLQCD, in preparation

Nucleon mass

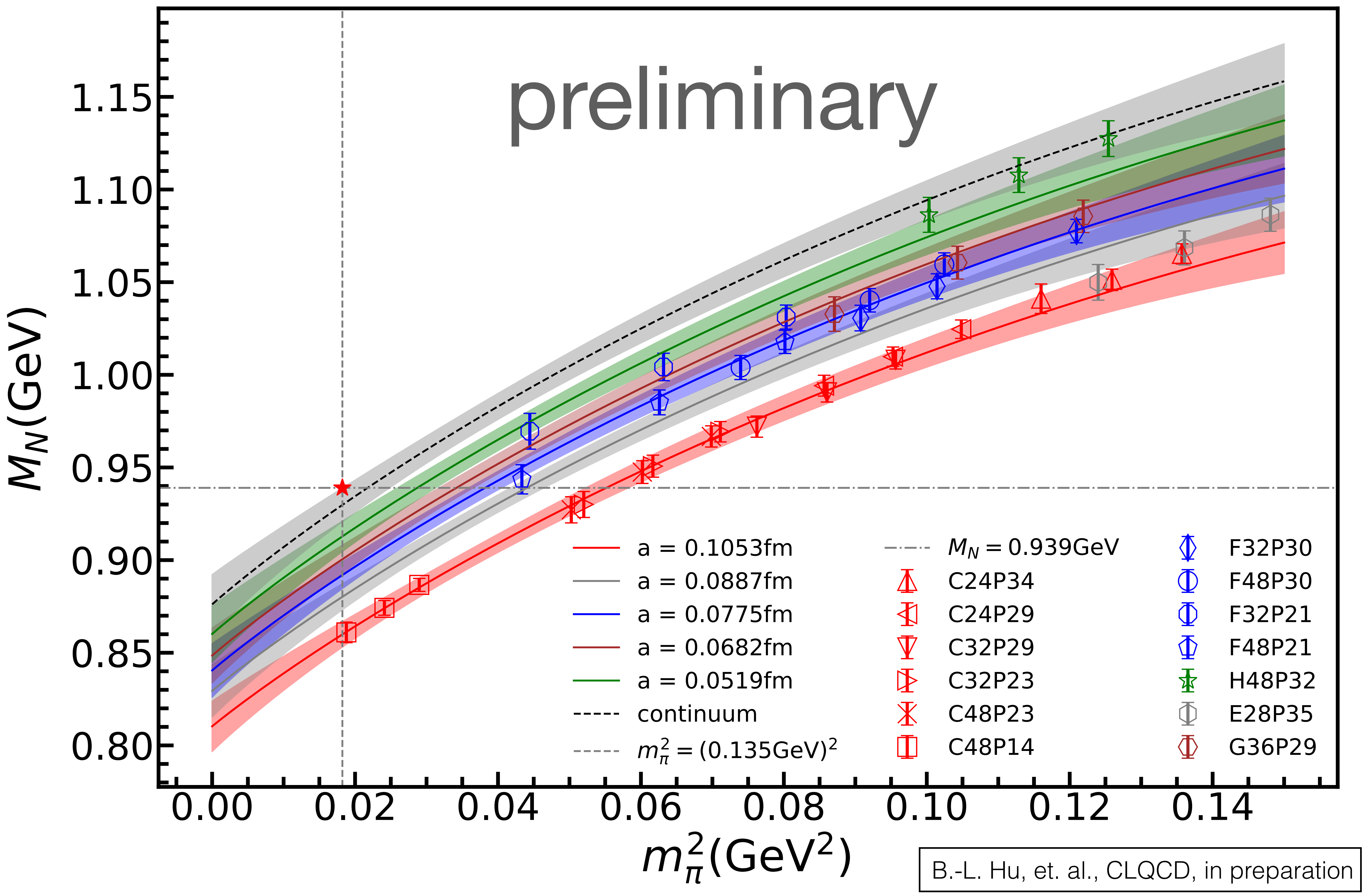


Parameter	Value
M_0	0.876(16)
C_1	2.13(39)
C_2	1.39(59)
C_3	−6.77(57)
C_4	1.85(49)
C_5	0.92(38)
g_A	0.99(27)
g_1	−0.03(51)
M_{phys}	0.9296(91)
χ^2	0.73
Q	0.86

Based on CLQCD ensembles

$$M\left(m_\pi^v, m_\pi^{\text{sea}}, m_s^{\text{sea}}, a, L\right) = \Big[M_0 + C_1\left(m_\pi^v\right)^2 + C_2\left(m_\pi^{\text{sea}}\right)^2 - \frac{\left(g_A^2 - 4g_Ag_1 - 5g_1^2\right)\pi}{3\left(4\pi f_\pi\right)^2}\left(m_\pi^v\right)^3 - \frac{\left(8g_A^2 + 4g_Ag_1 + 5g_1^2\right)\pi}{3\left(4\pi f_\pi\right)^2}\left(m_\pi^{pq}\right)^3 + C_4\frac{\left(m_\pi^v\right)^2}{L}e^{-m_\pi^v L} + C_5(m_s^{\text{sea}} - m_s^{\text{phys}})\Big](1 + C_3 a^2),$$

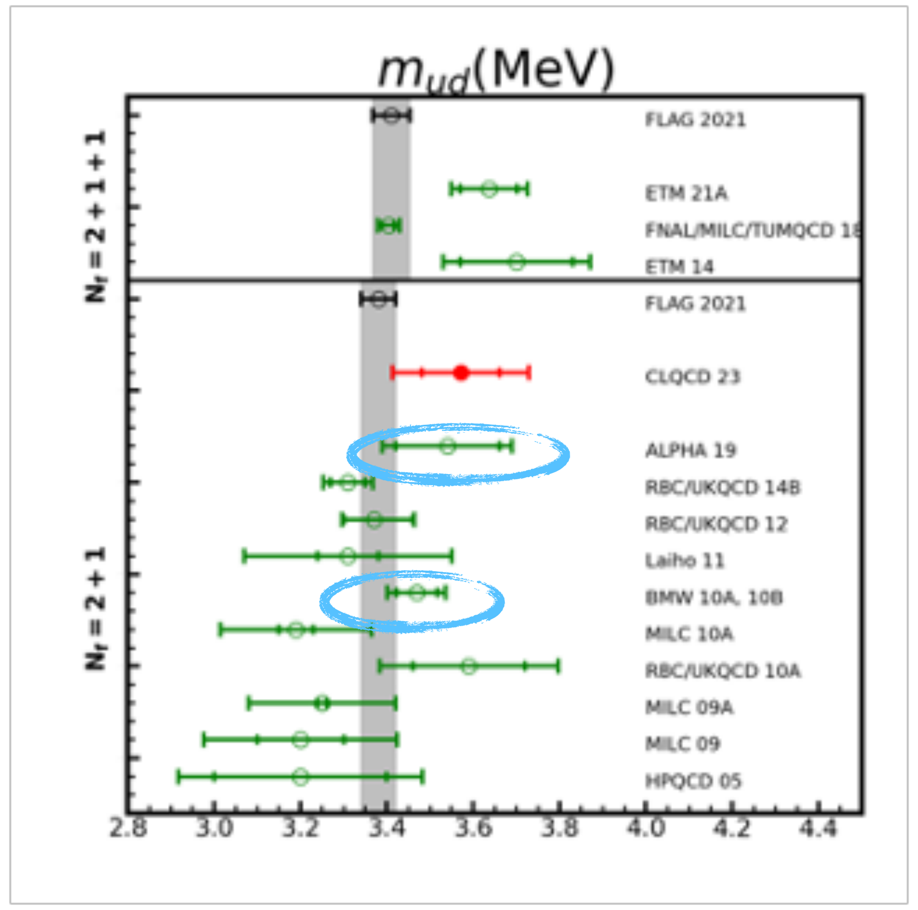
- Sigma term based on FH theorem: $\sigma_{\pi N} \equiv m_l \langle p \,|\, \bar{u}u + \bar{d}d \,|\, p\rangle = m_l \frac{\partial M_N}{\partial m_l}$ =48.8(6.4) MeV;
- Previous Overlap result based on FH theorem: $\sigma_{\pi N}$ = 52(8) MeV;
- Previous Overlap result based on direct ME calculation: $\sigma_{\pi N}$ = 46(7) MeV.

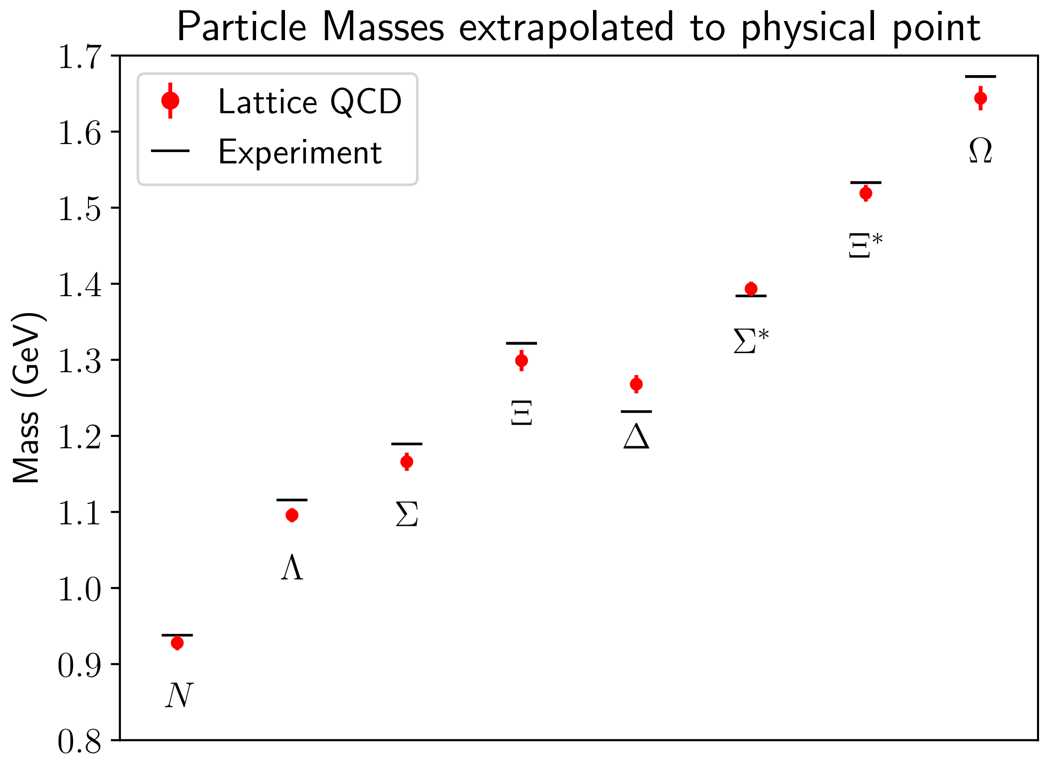


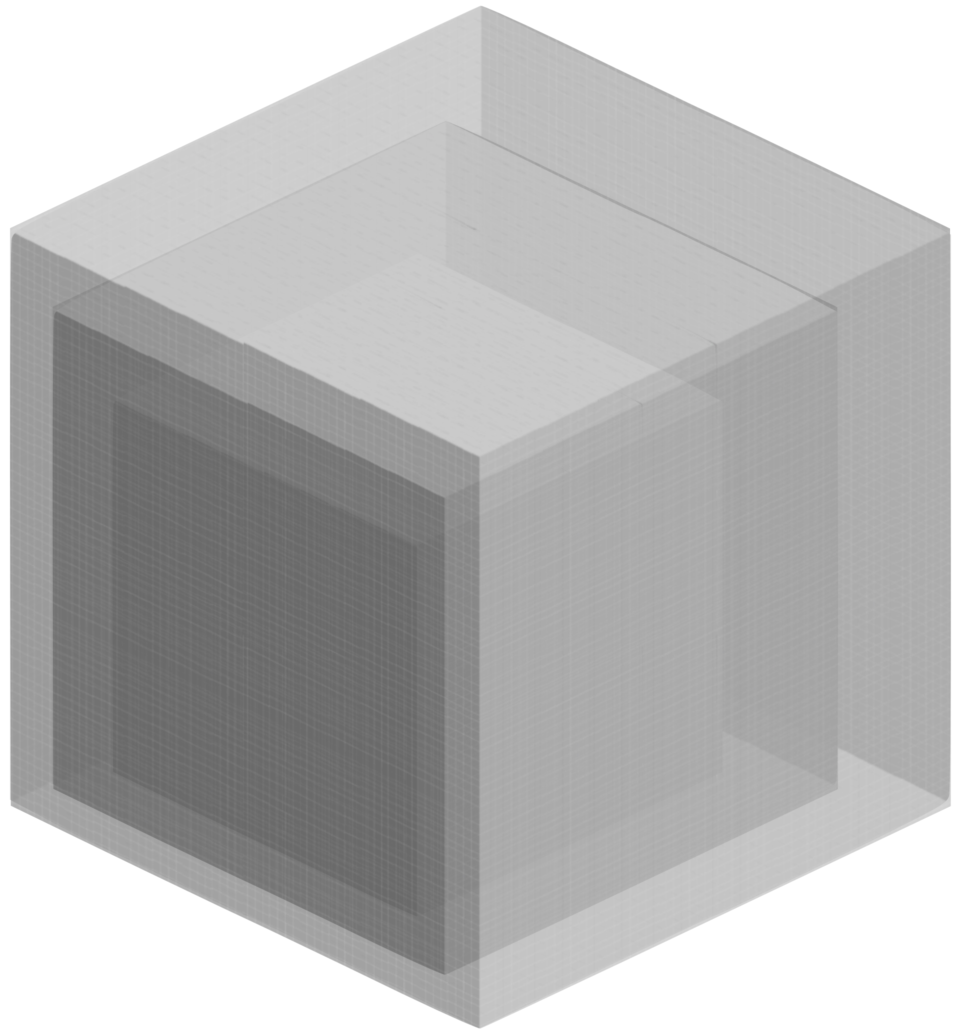
B.-L. Hu, et. al., CLQCD, in preparation

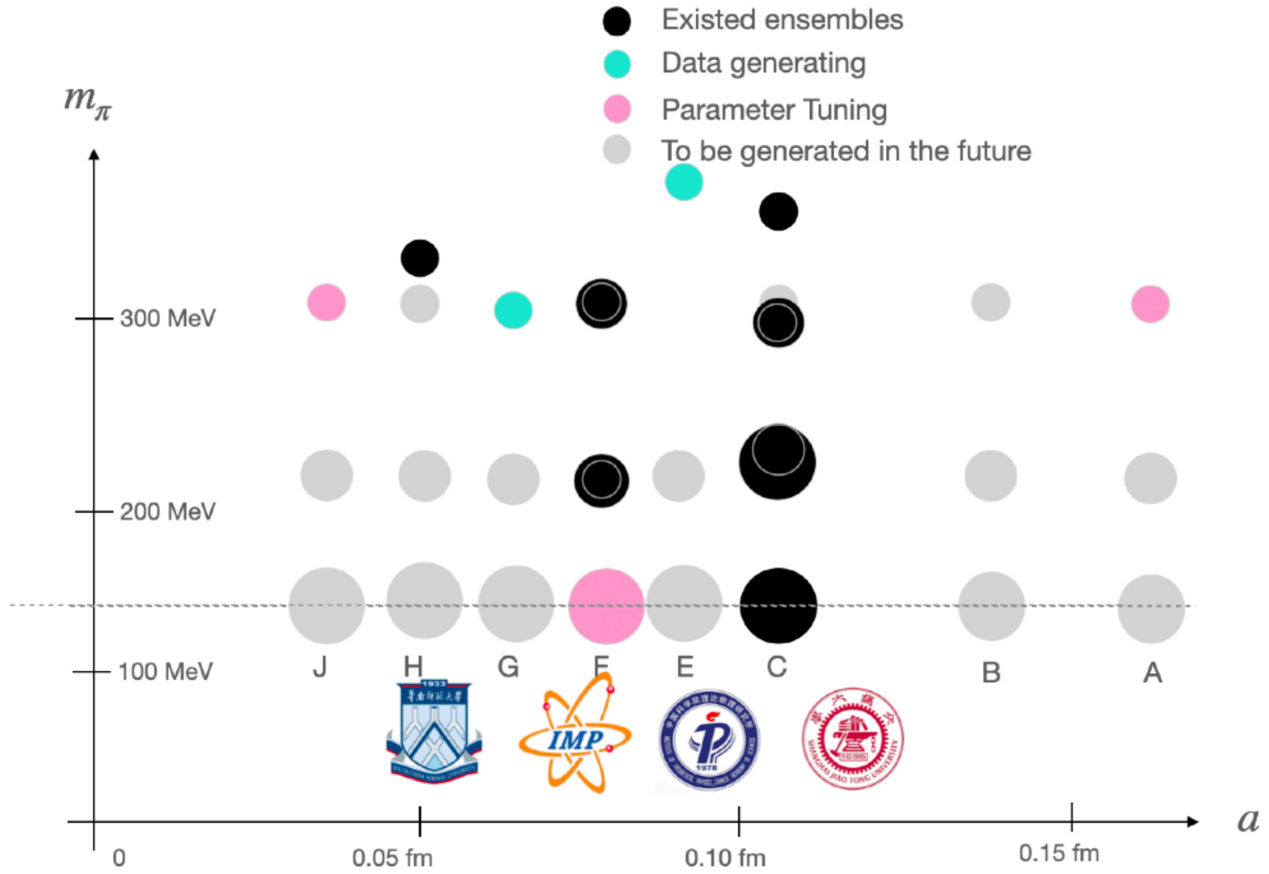
Outline

- Lattice QCD background

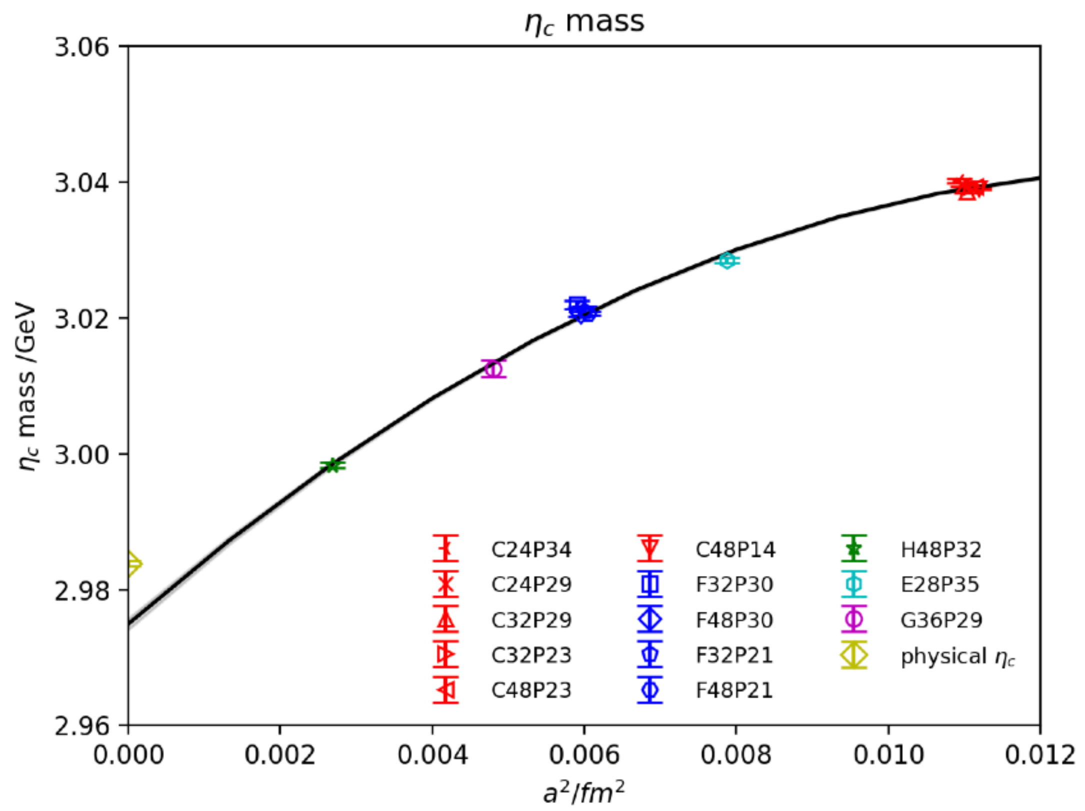


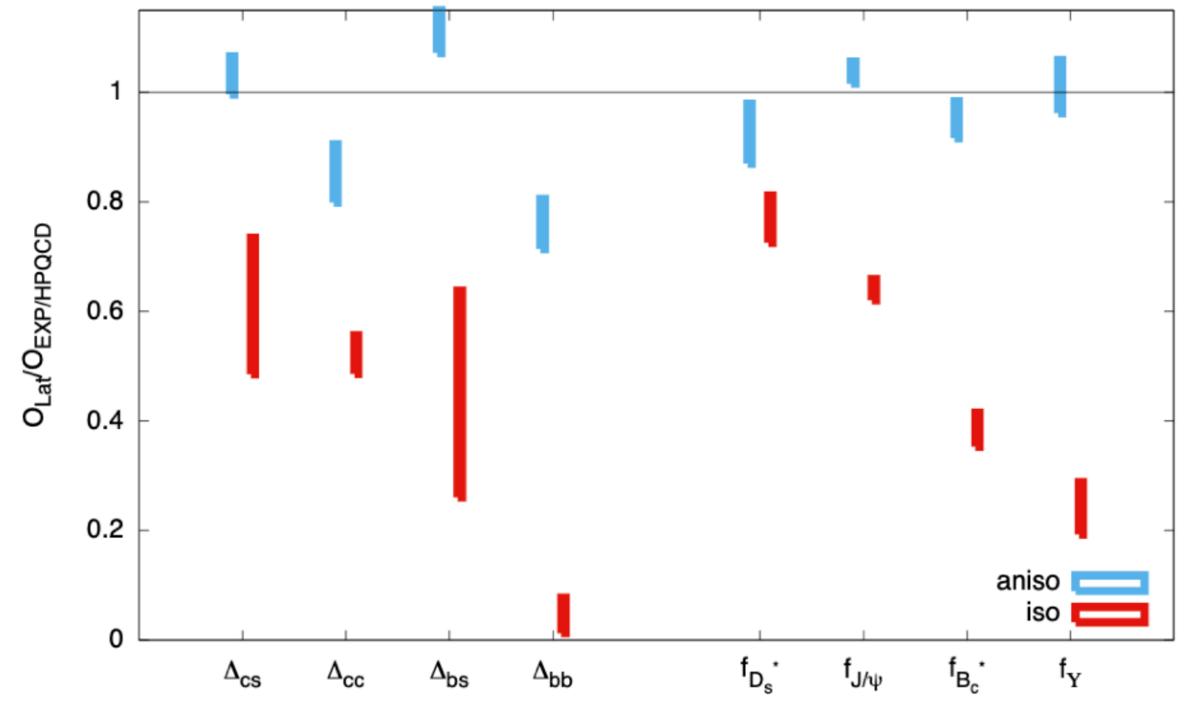






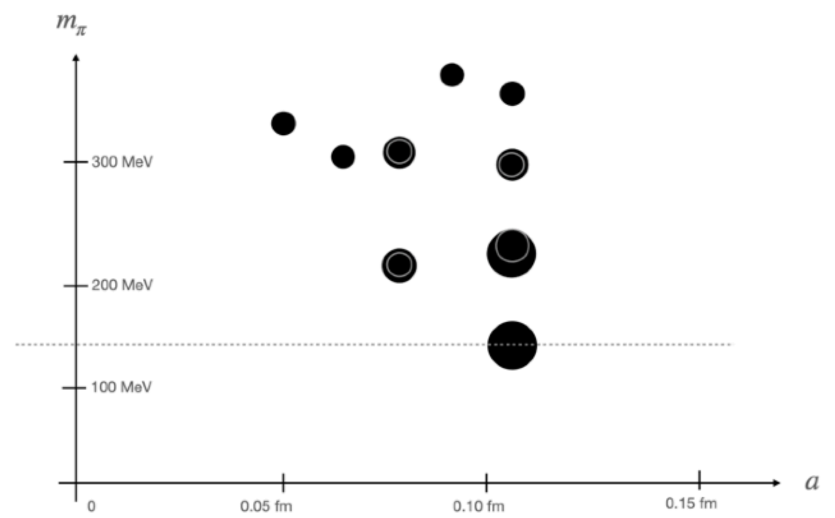
- Light quark and hadron masses

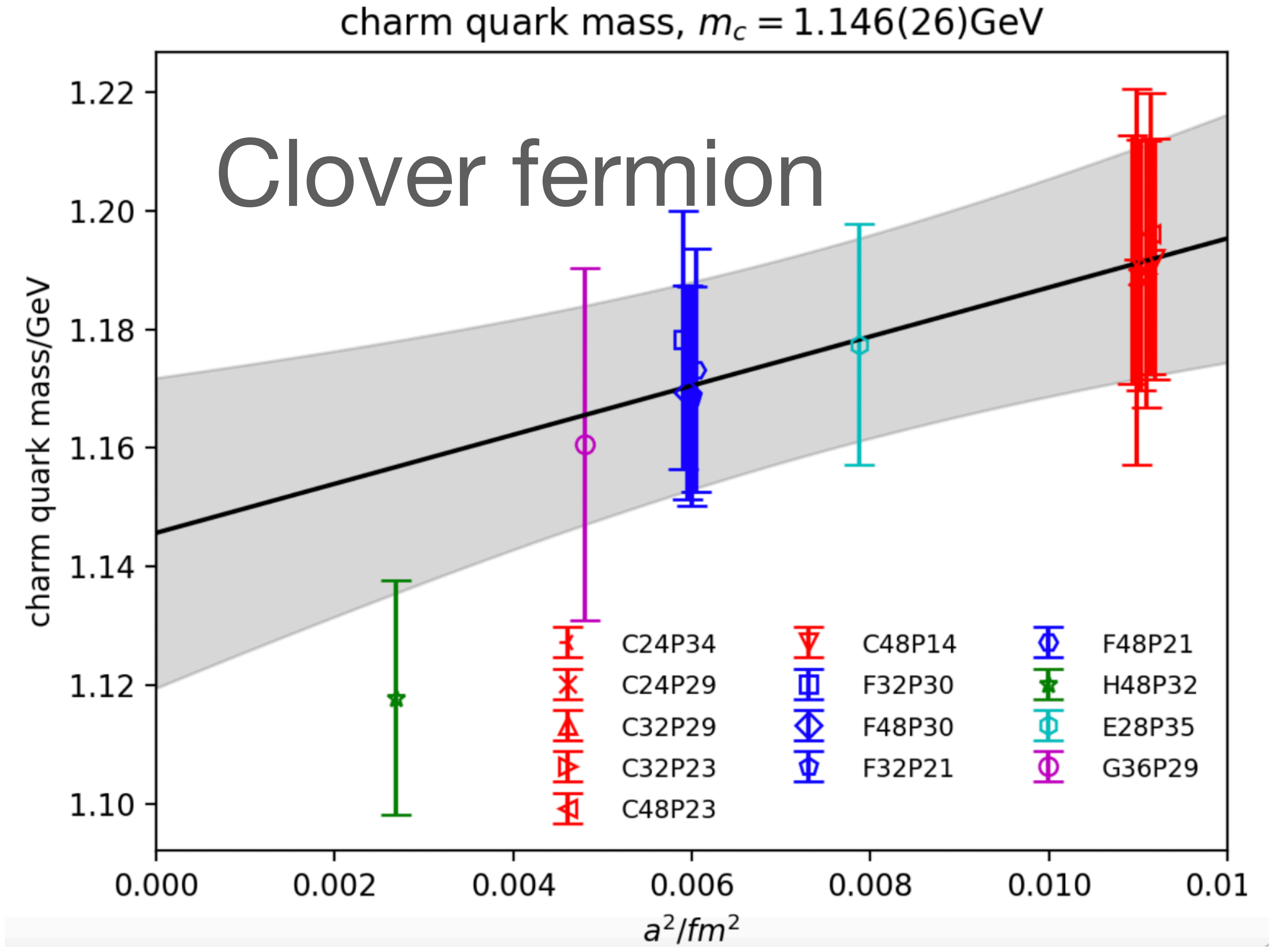




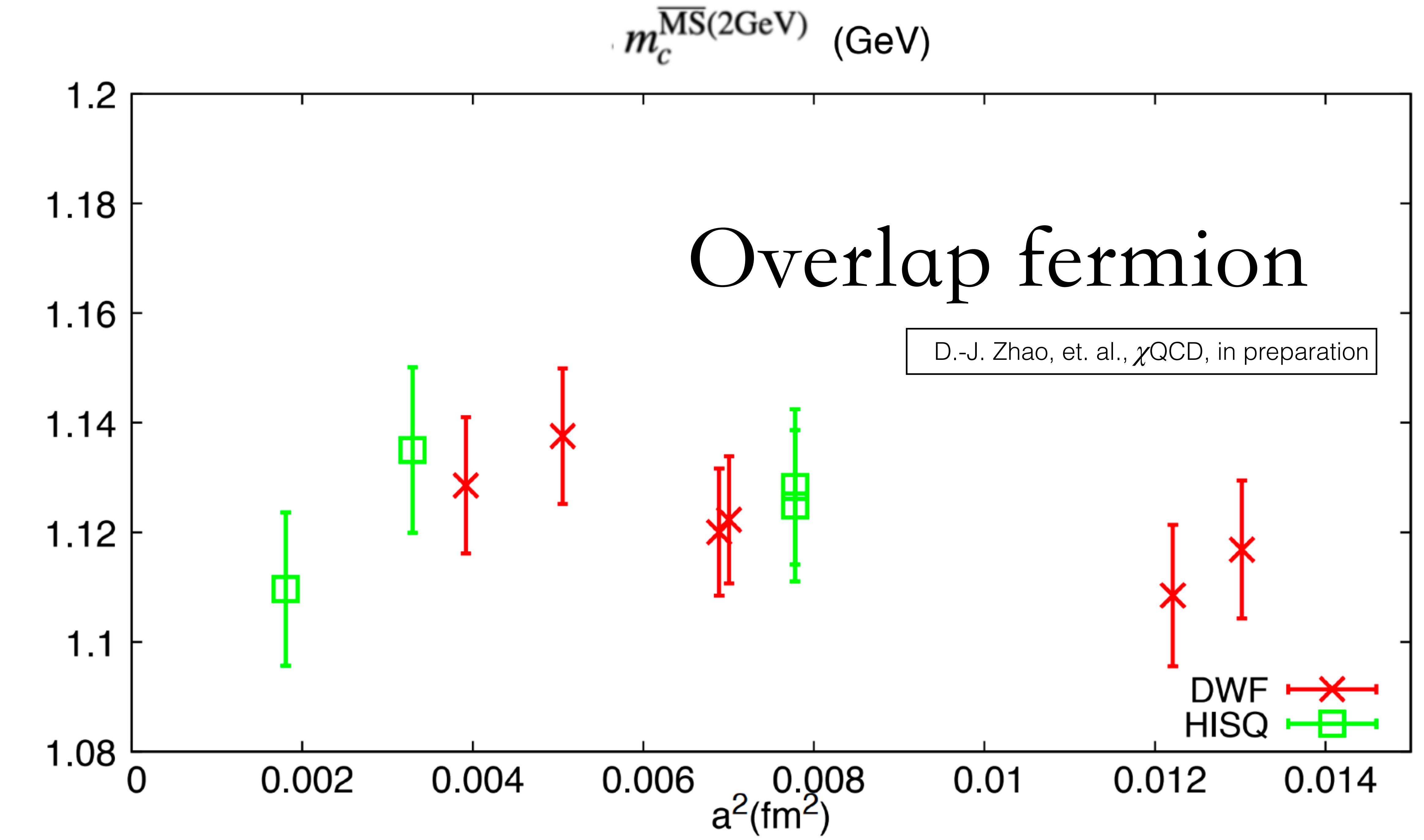
- **Investigation on the charm quark**

Charm quark mass determination



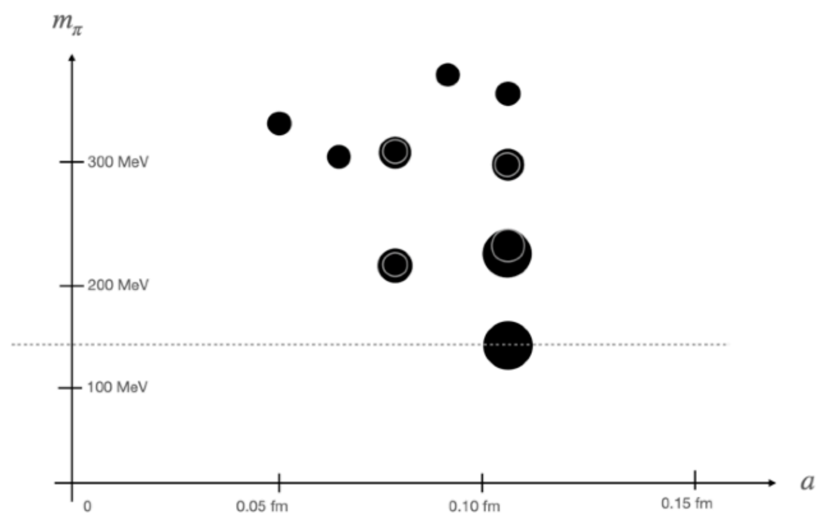


H.-Y. Du, et. al., CLQCD, in preparation



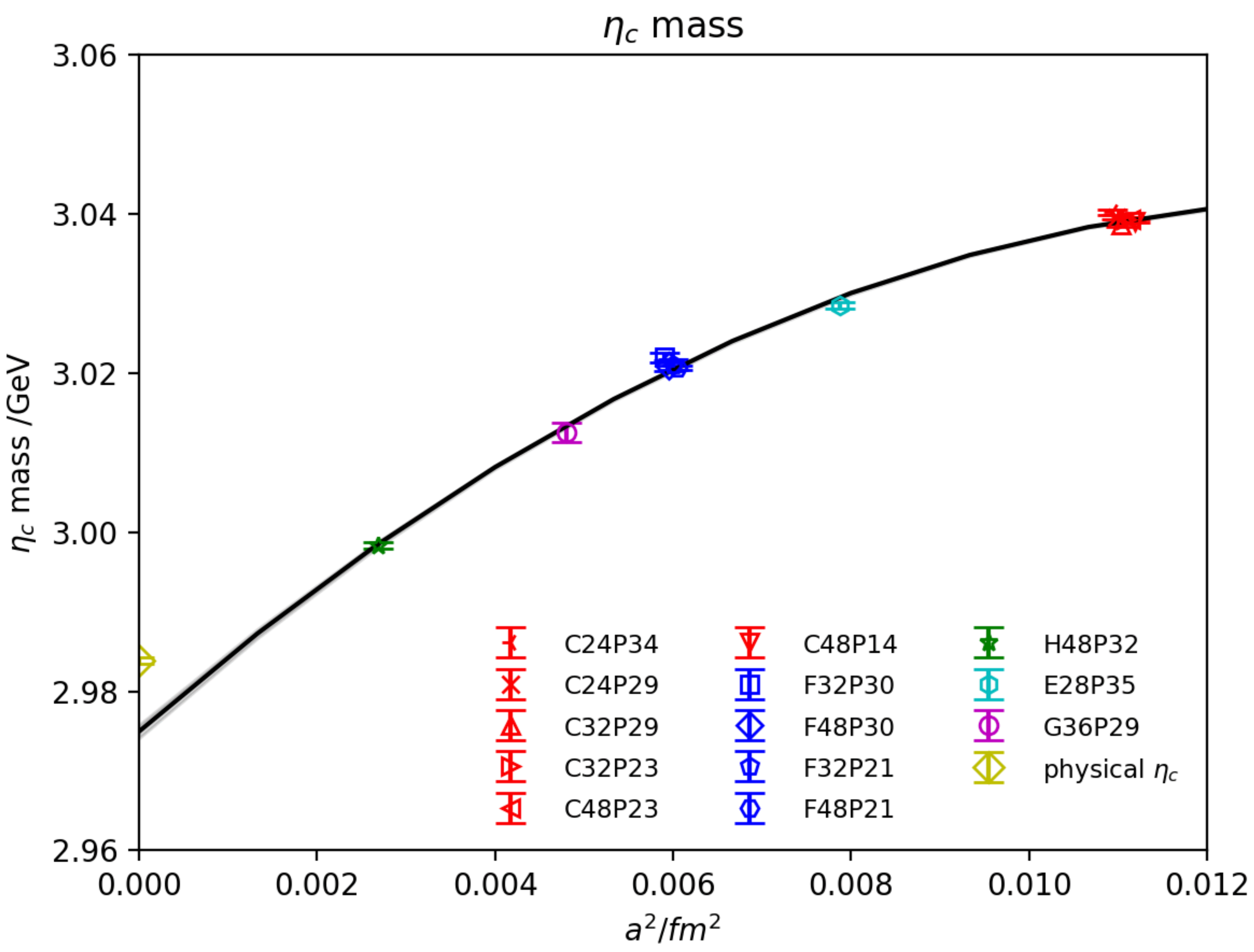
- The charm quark mass $m_c^{\overline{\rm MS}(2\rm GeV)}$ extracted using the physical J/ψ mass is 1.146(26) GeV.
- Consistent with the overlap fermion results while requiring further suppression on the systematic uncertainties.

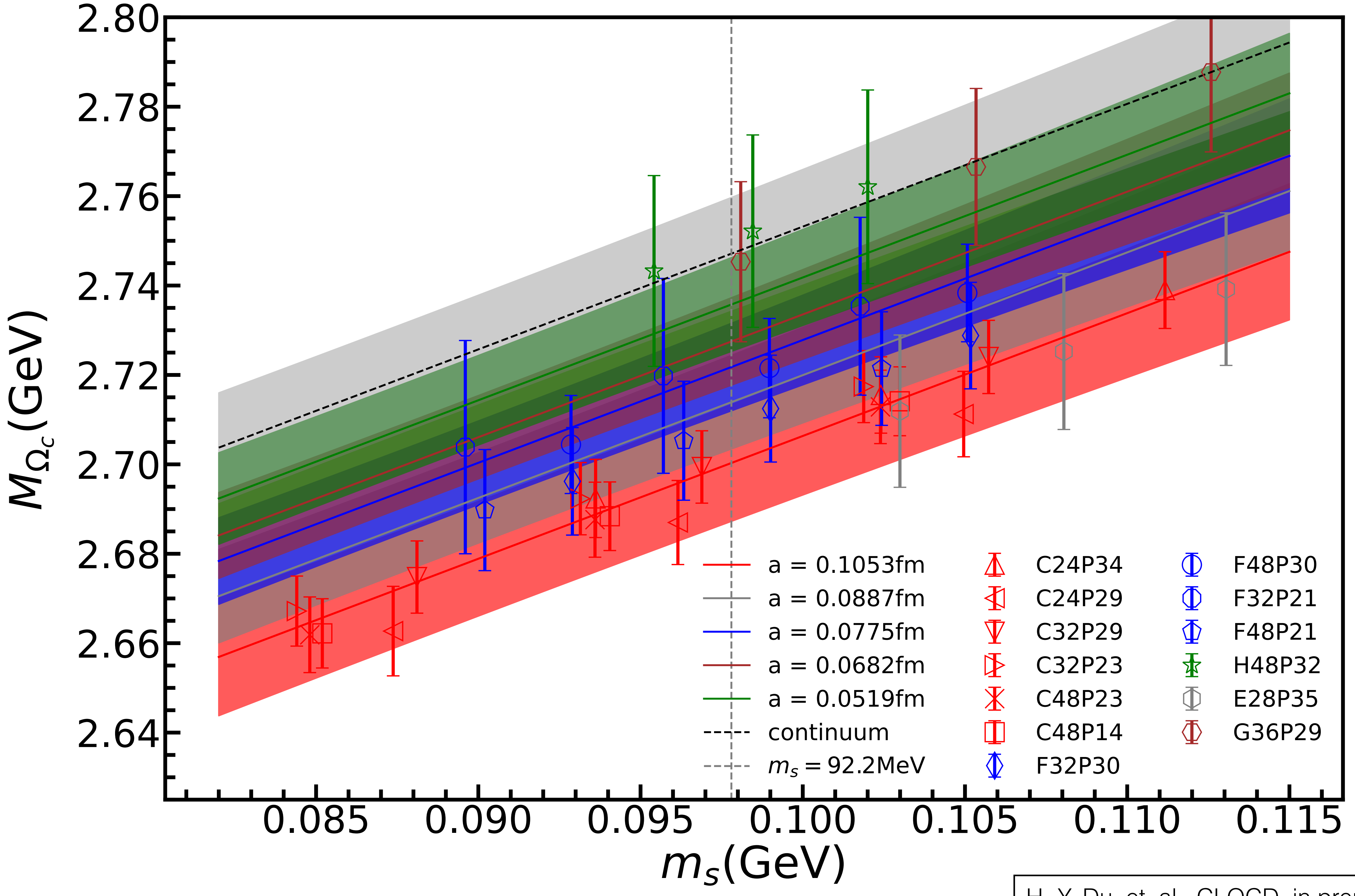
Stable hadron masses



- The η_c mass extrapolated to continuum is consistent with the previous HPQCD results while slightly lower than the experimental value due to the QED and disconnected charm sea effects.

- The Ω_c mass extrapolated to continuum is 2745(10)(20) MeV.

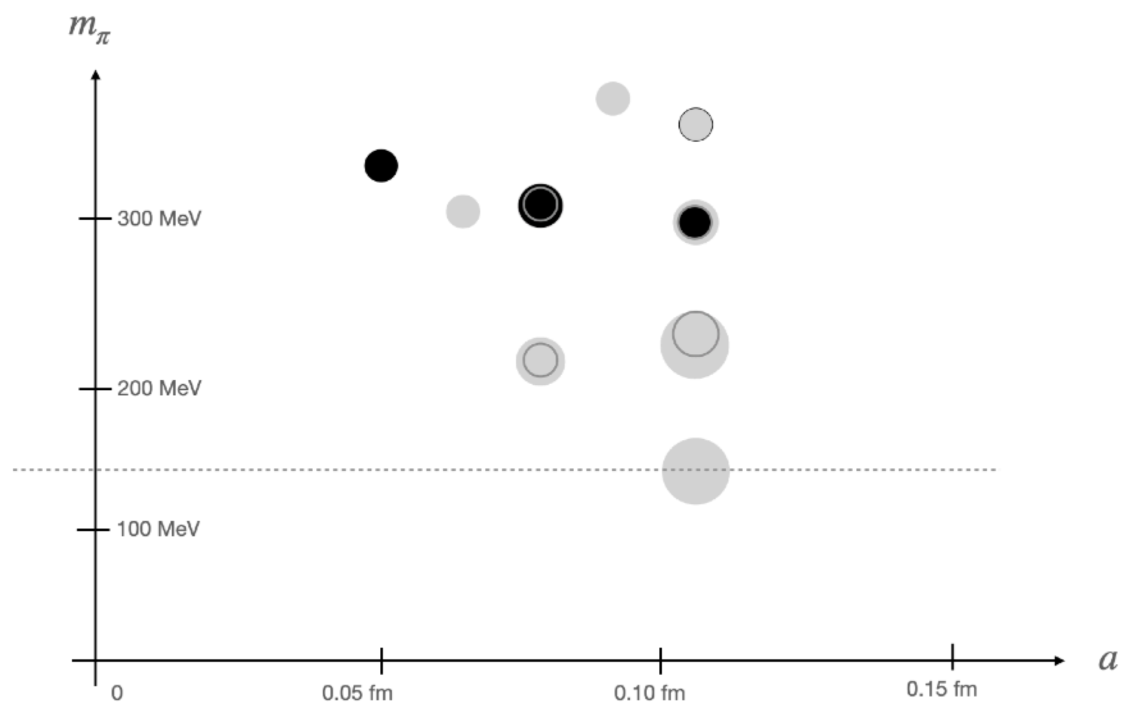




H.-Y. Du, et. al., CLQCD, in preparation

CLQCD ensemble applications

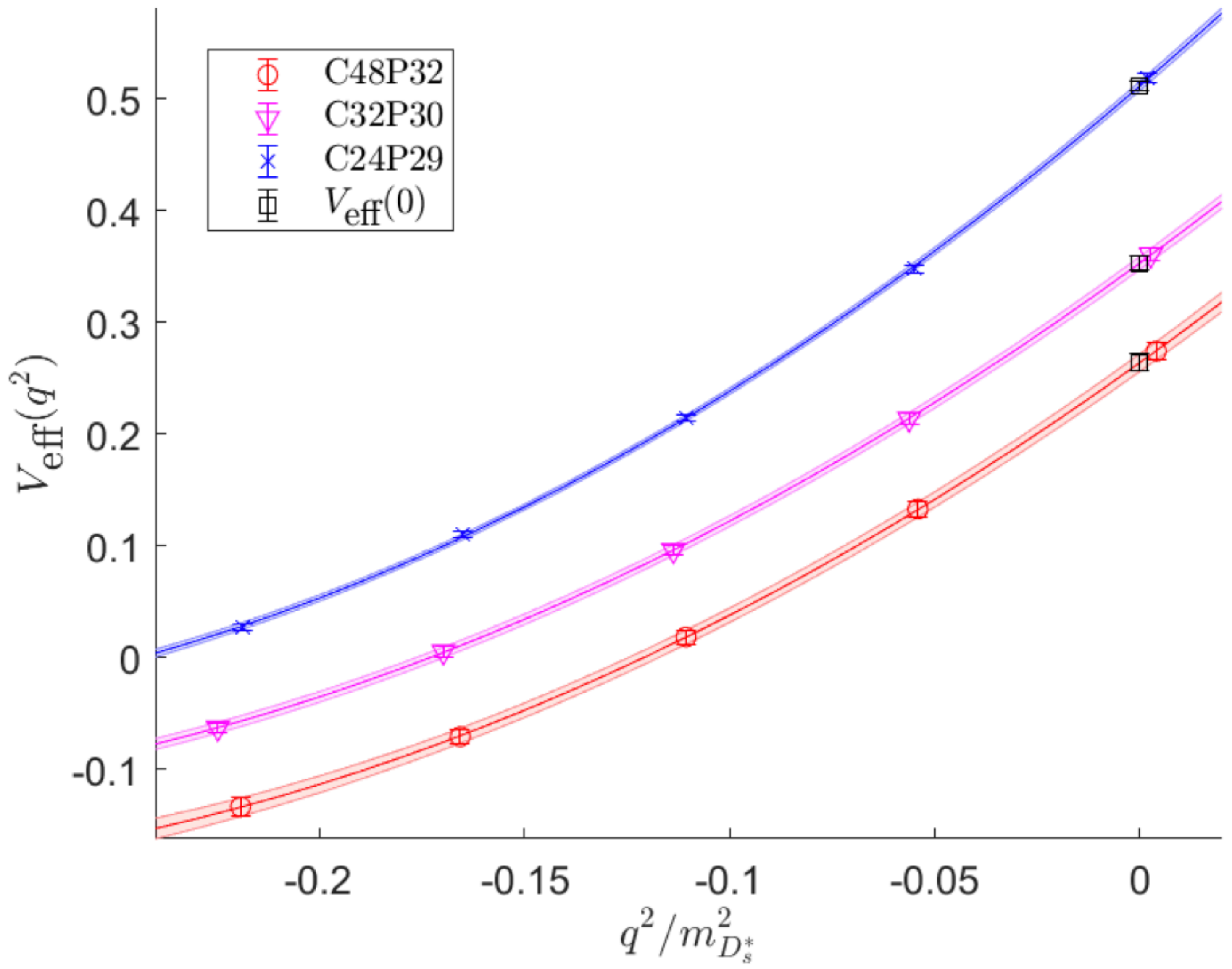
D_s^* radiative decay and V_{cs}



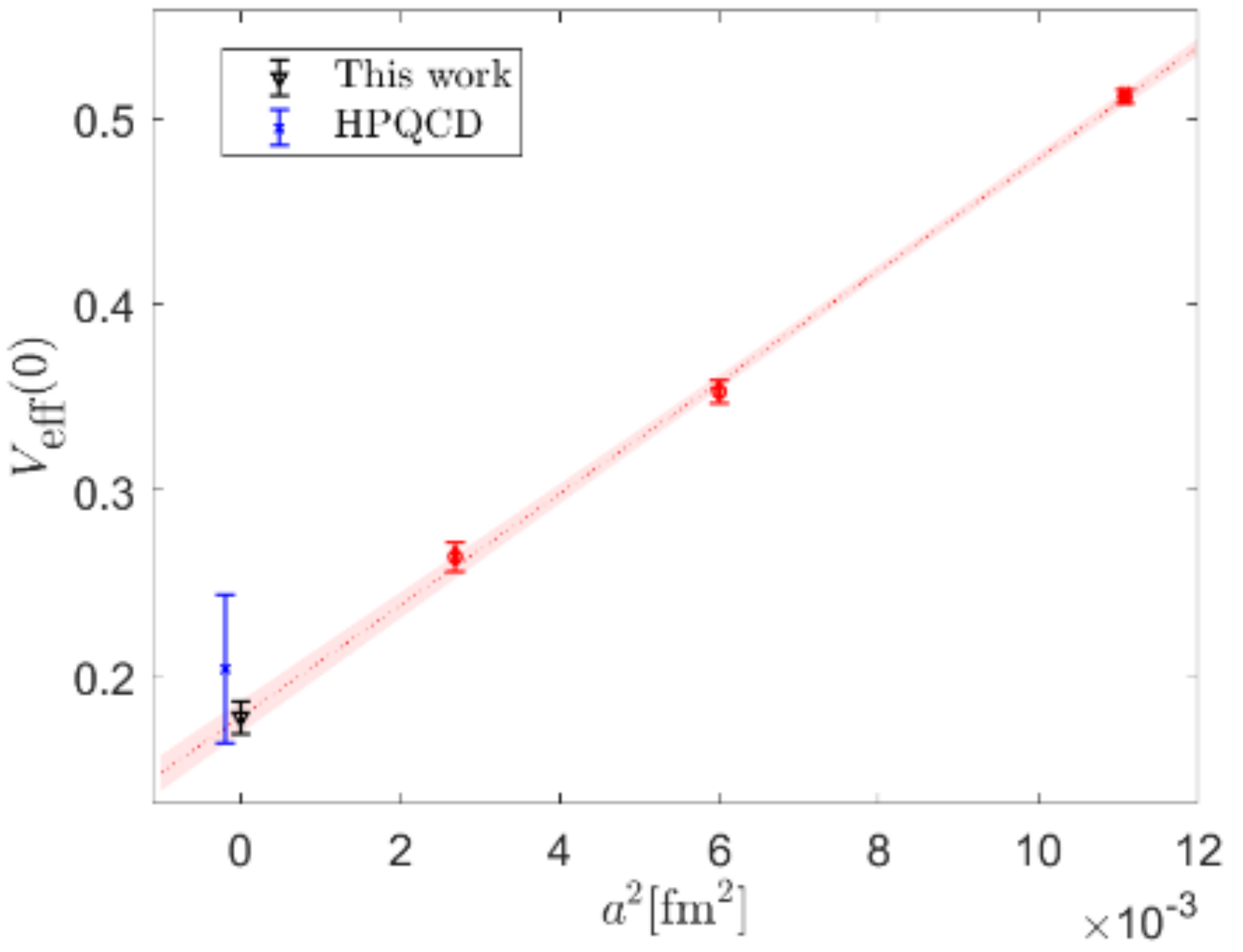
$$\langle 0|\mathcal{O}_{D_s}(0)|D_s(\vec{p})\rangle = Z_{D_s}$$

$$\langle D_s(p)|J_\nu^{\rm em}(0)|D_{s,\mu}^*(p')\rangle = \frac{2V_{\rm eff}(q^2)}{m_{D_s}+m_{D_s^*}}\epsilon_{\mu\nu\alpha\beta}p_\alpha p'_\beta$$

$$V_{\rm eff}(q^2) = \frac{-(m_{D_s}+m_{D_s^*})E_{D_s}}{2Z_{D_s}m_{D_s^*}}e^{E_{D_s}t} \times \int d^3\vec{x}\frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|}\epsilon_{\mu\nu\alpha 0}x_\alpha H_{\mu\nu}(\vec{x},t)$$



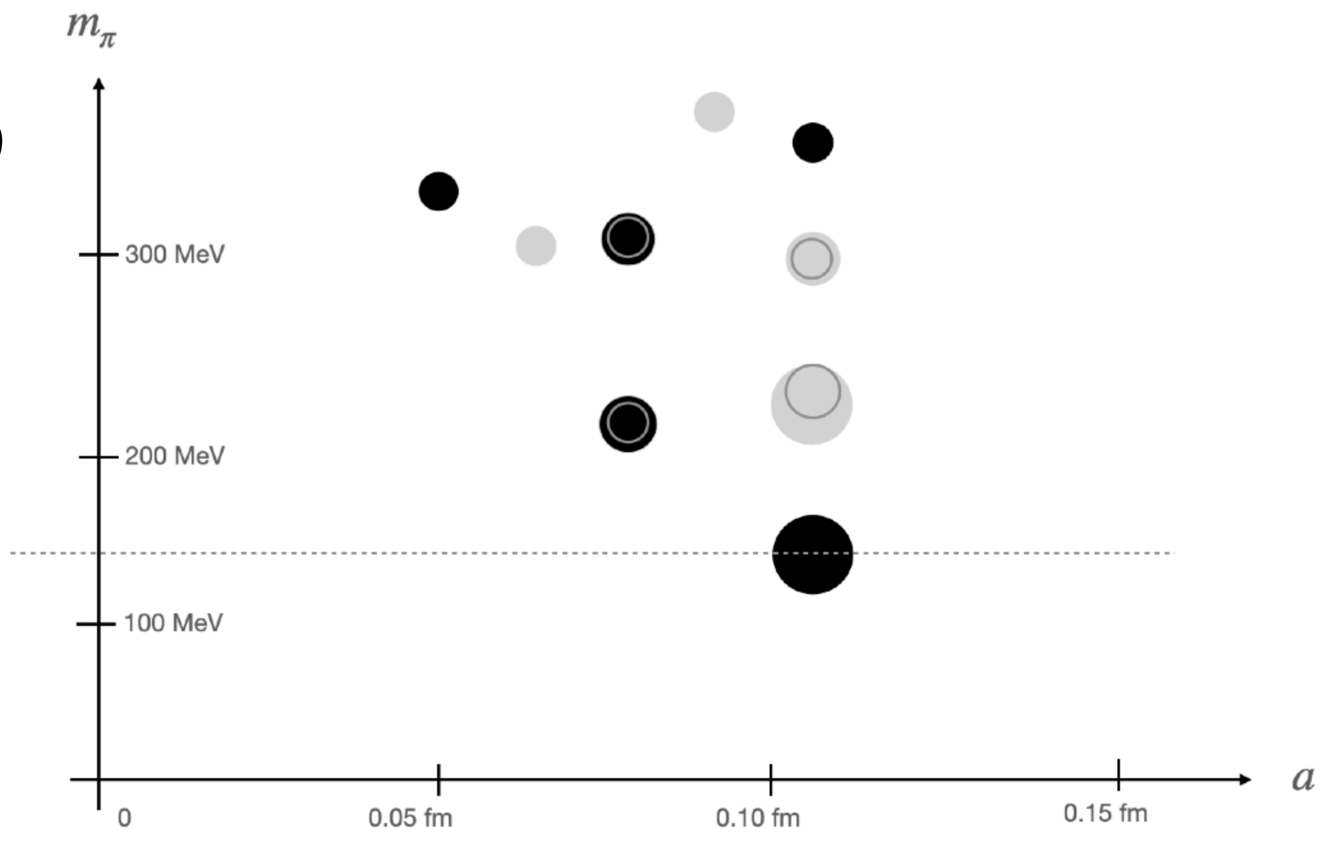
- Predict $\Gamma(D_s^* \to D_s\gamma)$ =0.0549(54) KeV and then suppress the uncertainty of the previous HPQCD calculation by a factor of 4;
- Combining the recent experiment, one can obtain $f_{D_s^*}|V_{cs}| = 190.5^{+55.1}_{-41.7} \pm 12.6$ MeV.



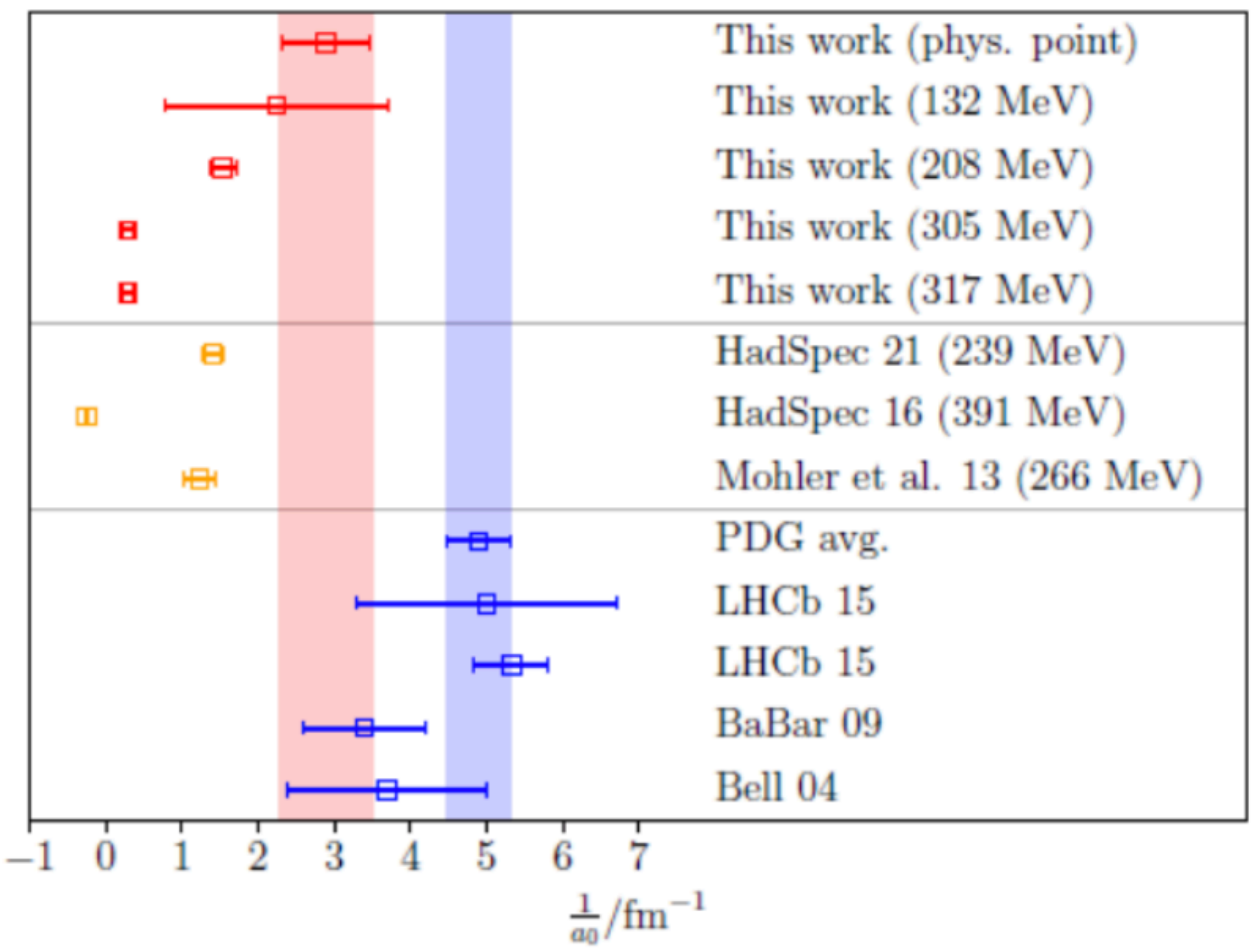
Y. Meng, et.al, 2401.13475, accepted by PRD

CLQCD ensemble applications

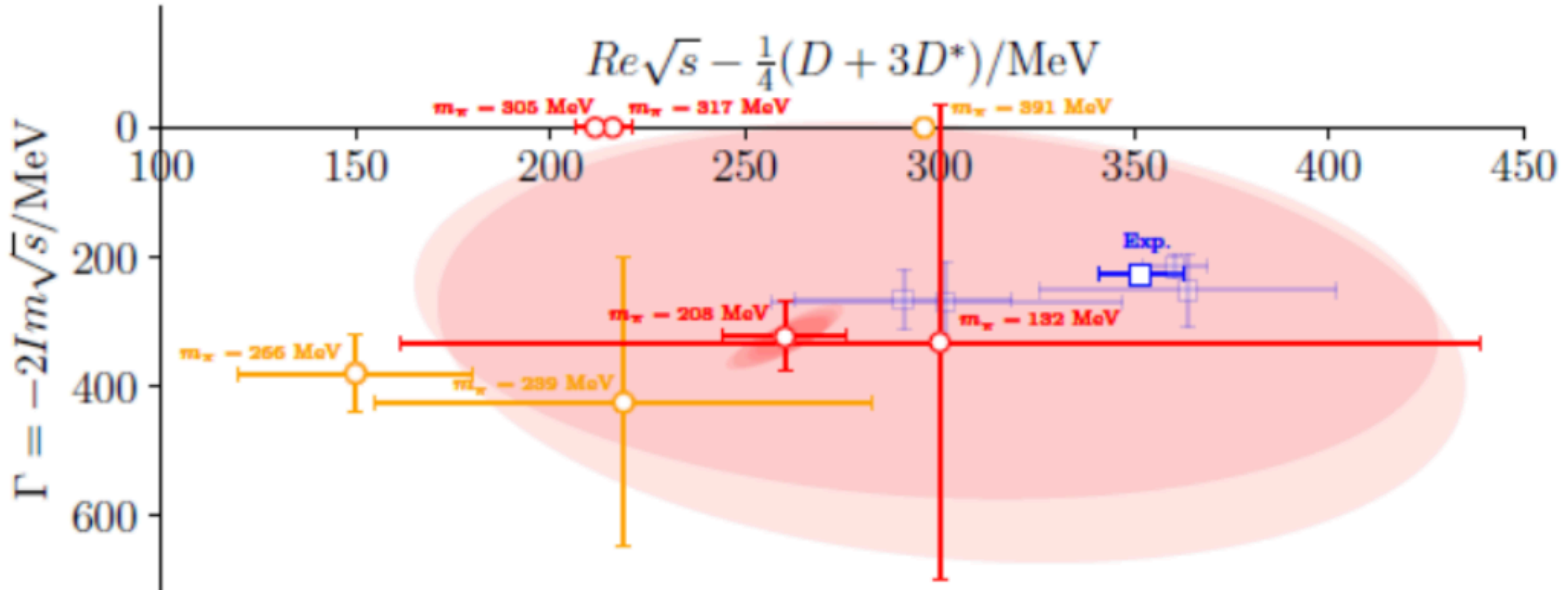
Study of $D_0^*(2300)$



Scattering length



Pole position



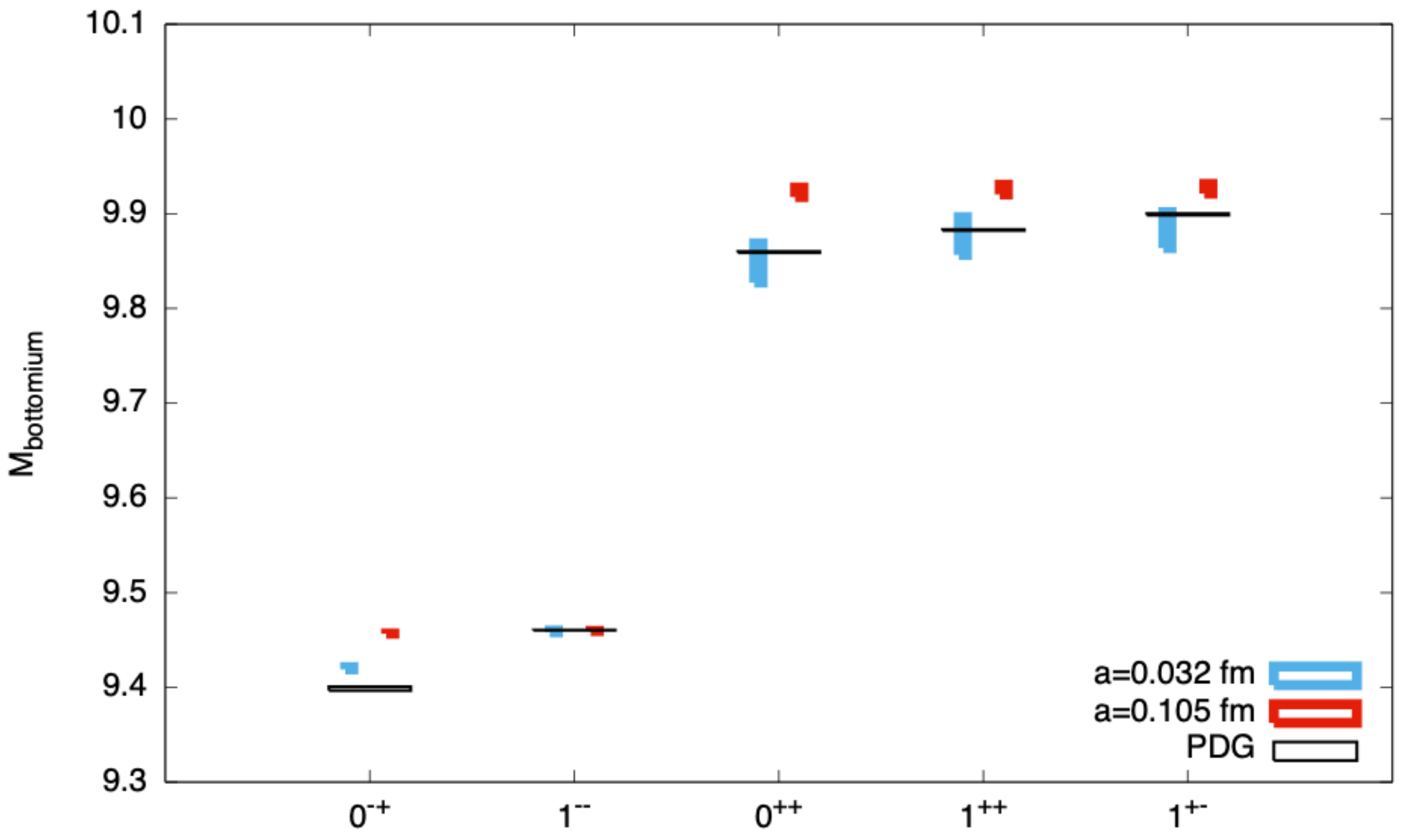
- At $m_\pi \sim$ 300 MeV, there is a virtual state pole;
- When pion mass decreases, it becomes a resonance and the pole position gets close to the experiment.

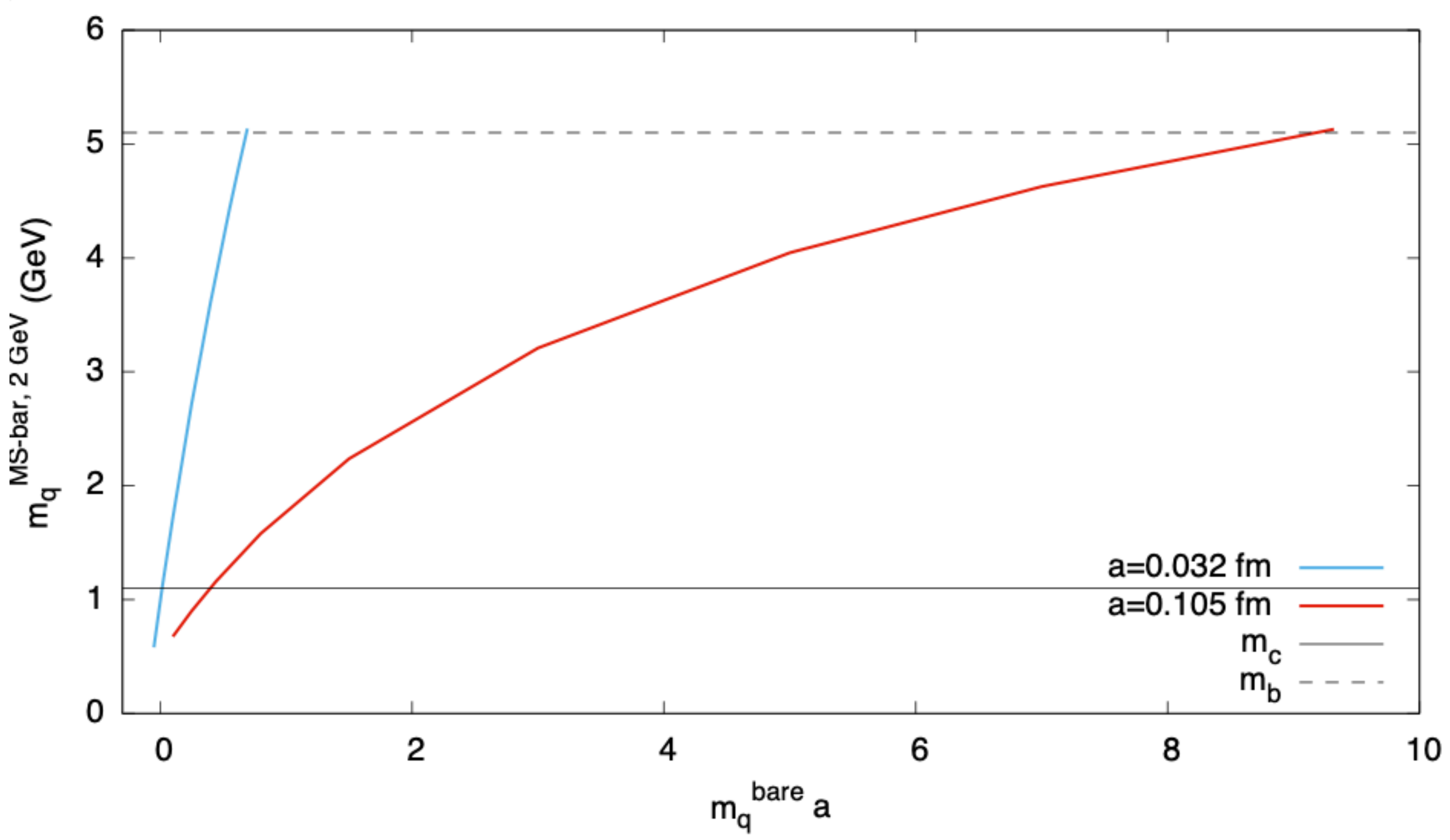
H.-B. Yan, et.al., 2404.13479

Bottom quark mass

Using clover fermion

- The $m_Q^2 a^2$ discretization error of the heavy quark can be huge for the bottom quark.
- For example, the bare quark mass at 0.1 fm with physical bottom quark mass (~5 GeV) will be 17 GeV.
- It is obvious that the renormalization constant deviates from 1 by $m_Q^2 a^2$ correction which is suppressed at a~0.03 fm.





If we use such a bottom quark mass at 0.1 fm:

- The fine splitting $m_{1P} - m_{1S}$ is close to the experimental value thanks to the heavy quark symmetry;
- But the hyperfine splitting $m_{\Upsilon} - m_{\eta_b}$ will be 5% of the experimental value, while the situation will improve significantly at a~0.03 fm.

Non-relativistic actions

of heavy quark

$$S_\psi = a^3 \sum_{\mathbf{x},t} \psi^\dagger(\mathbf{x},t)\left[\psi(\mathbf{x},t) - K(t)\,\psi(\mathbf{x},t-a)\right],$$

$$K(t) = \left(1 - \frac{a\,\delta H|_t}{2}\right)\left(1 - \frac{aH_0|_t}{2n}\right)^n U_0^\dagger(t-a) \times \left(1 - \frac{aH_0|_{t-a}}{2n}\right)^n \left(1 - \frac{a\,\delta H|_{t-a}}{2}\right).$$

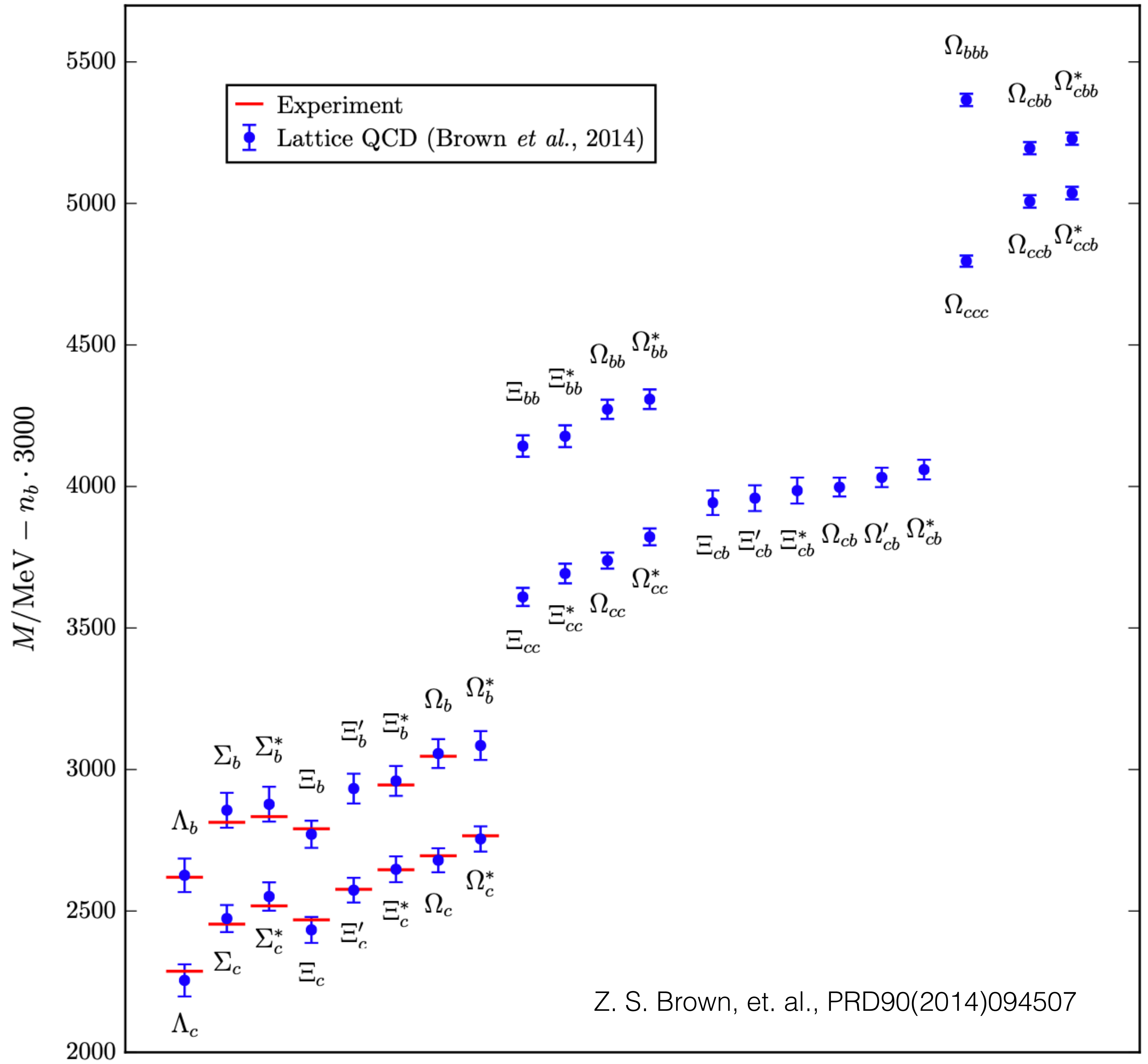
$$H_0 = -\frac{\Delta^{(2)}}{2m_b},$$

$$\delta H = -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8m_b^3} + c_2 \frac{ig}{8m_b^2}\left(\boldsymbol{\nabla}\cdot\widetilde{\mathbf{E}} - \widetilde{\mathbf{E}}\cdot\boldsymbol{\nabla}\right) - c_3 \frac{g}{8m_b^2}\,\boldsymbol{\sigma}\cdot\left(\widetilde{\boldsymbol{\nabla}}\times\widetilde{\mathbf{E}} - \widetilde{\mathbf{E}}\times\widetilde{\boldsymbol{\nabla}}\right) - c_4 \frac{g}{2m_b}\,\boldsymbol{\sigma}\cdot\widetilde{\mathbf{B}} + c_5 \frac{a^2\Delta^{(4)}}{24m_b} - c_6 \frac{a\left(\Delta^{(2)}\right)^2}{16n\,m_b^2}.$$

G. P. Lepage , et. al., PRD46(1992)4052

	Data sets	am_b	u_{0L}	c_4
$a = 0.111$ fm	C104, C14, C24, C54, C53	2.52	0.8439	1.09389
$a = 0.083$ fm	F23, F43, F63	1.85	0.8609	1.07887

$c_{1,2,3} = 1$



Z. S. Brown, et. al., PRD90(2014)094507

Relativistic actions

of heavy quark

"Fermilab action"

$$S_{\text{Fermilab}} = S_0 + S_B + S_E,$$

$$\begin{aligned} S_0 =& a^4 \sum_x \Big[m_0 \bar\psi(x)\psi(x) + \bar\psi(x)\gamma_4 D_{\text{lat},4}\psi(x) \\ &+ \zeta\bar\psi(x)\boldsymbol{\gamma}\cdot\boldsymbol{D}_{\text{lat}}\psi(x) - \frac{1}{2}a\bar\psi(x)\Delta_4\psi(x) \\ &- \frac{1}{2}r_s\zeta a\bar\psi(x)\Delta^{(3)}\psi(x)\Big], \end{aligned}$$

$$S_B = -\frac{1}{2}c_B\zeta a^5 \sum_x \bar\psi(x) i\boldsymbol{\Sigma}\cdot\boldsymbol{B}_{\text{lat}}\psi(x),$$

$$S_E = -\frac{1}{2}c_E\zeta a^5 \sum_x \bar\psi(x)\boldsymbol{\alpha}\cdot\boldsymbol{E}_{\text{lat}}\psi(x).$$

A. X. El-Khadra, A. S. Kronfeld., P. B. Mackenzie PRD55(1997)3933

"OK (Oktay-Kronfeld) action"

$$S_{\text{OK}} = S_0 + S_B + S_E + S_6 + S_7,$$

$$\begin{aligned} S_6 = a^6 \sum_x \bar\psi(x)\Big[& c_1 \sum_i \gamma_i D_{\text{lat},i}\Delta_{\text{lat},i} + c_2\{\boldsymbol{\gamma}\cdot\boldsymbol{D}_{\text{lat}}, \Delta^{(3)}\} \\ &+ c_3\{\boldsymbol{\gamma}\cdot\boldsymbol{D}_{\text{lat}}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}_{\text{lat}}\} + c_{EE}\{\gamma_4 D_{\text{lat},4}, \boldsymbol{\alpha}\cdot\boldsymbol{E}_{\text{lat}}\}\Big]\psi(x), \end{aligned}$$

$$S_7 = a^7 \sum_x \bar\psi(x) \sum_i \Big[c_4\Delta_i^2 + c_5 \sum_{j\neq i}\{i\Sigma_i B_{\text{lat},i}, \Delta_j\}\Big]\psi(x).$$

M. B. Oktay, A. S. Kronfeld., PRD78(2008)014504

$$S_Q = a^4 \sum_x \bar{Q}\mathscr{M}Q, \ \mathscr{M} = \Big[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\Big]$$

"tadpole improved anisotropic action"

L. Liu, et. al., PRD81(2010)094505

Z. S. Brown, et. al., PRD90(2014)094507

Why

the anisotropic action works?

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{1}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

Momentum space →

$$\sum_i \sin^2(p_i a) + (m_Q - m_{\rm res})^2 \simeq \sinh^2(E(\vec{p})a) \gg E^2(\vec{p})a^2$$

Effective dispersion relation →

$$E^2(\vec{p}) = E^2(0) + \frac{E^2(0)a^2}{\sinh^2(E(0)a)}\vec{p}^2 + \mathcal{O}(\frac{\vec{p}^4a^2}{E^2(0)})$$

“bare” speed of light →

$$\nu \sim \frac{\sinh(E(0)a)}{E(0)a}$$

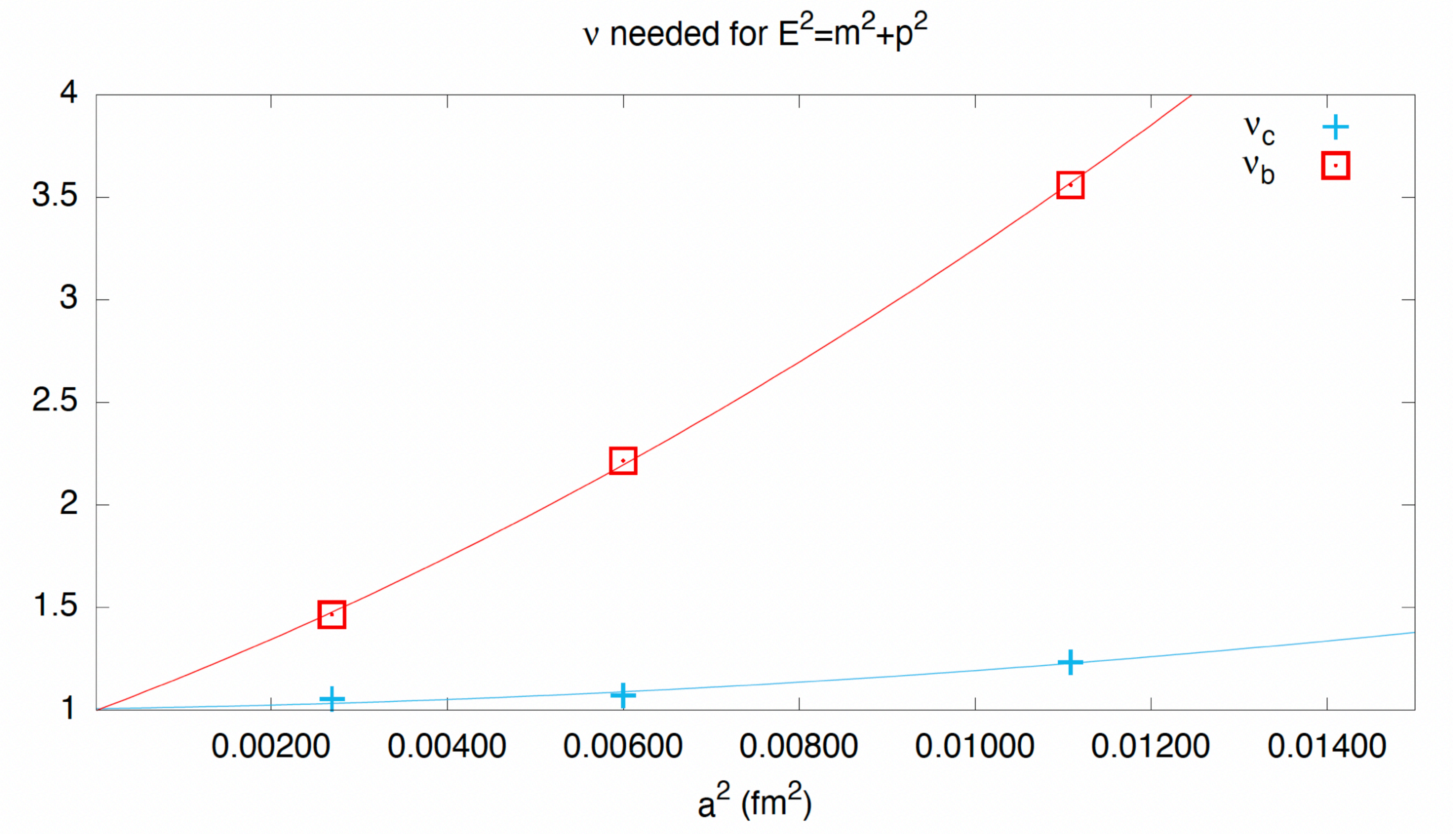
- When the quark mass and then hadron mass are large, the difference between $\sinh(ma)$ and ma will be large.
- Effectively it makes the speed of light to be different from 1 by $\mathcal{O}(m^2a^2)$.
- Such an effect can be compensated by a so-call “bare” speed of light factor in the action.

Parameter tuning

of the anisotropic action

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu \sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3} a \sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{1}{4u_0^3} a \sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507



★bare speed of light factor (anisotropic factor) $\nu(m_Q, a)$ approaches 1 with the form $1 + \mathcal{O}(a^2)$ when the lattice spacing approaches 0;

★$\nu \simeq \dfrac{\sinh(cm_H a)}{cm_H a}$, with $c = 0.621$ due to the impact from the Wilson term.

Precision problem

of the heavy quark action

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{1}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

$$\mathcal{M}^{-1}(x,y)Q(y) = \alpha(x_4)\mathcal{M}'^{-1}(x,y)Q'(y),$$

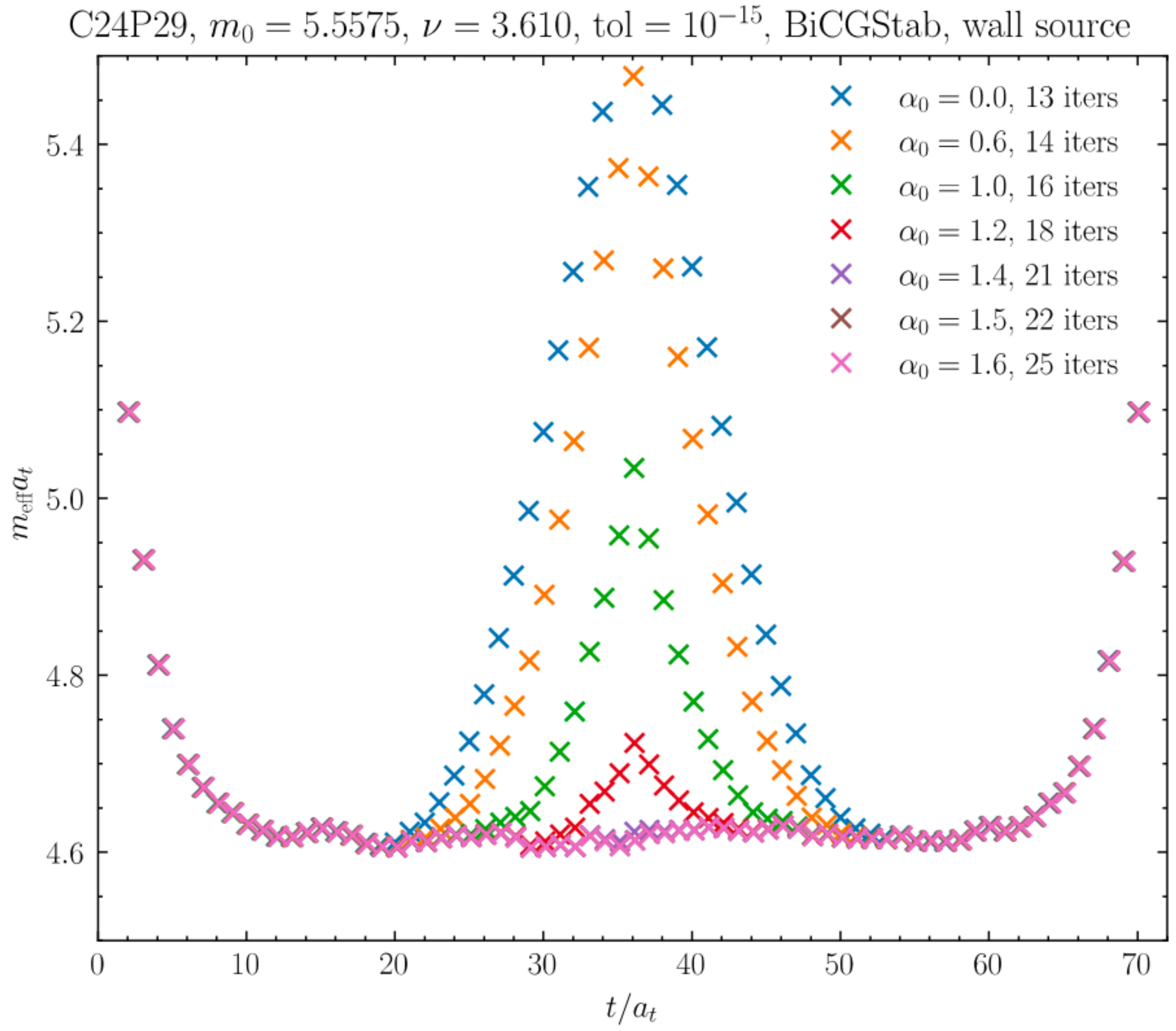
$$Q'(x) = \alpha^{-1}(x_4)Q(x),\ \mathcal{M}'(x,y) = \alpha^{-1}(x_4)\mathcal{M}(x,y)\alpha(y_4),\ \alpha(t) = \cosh\left[\alpha_0\left(t - L_t/2\right)\right]$$

★Distance pre-conditioning can resolve the problem of $\mathcal{M}^{-1}(x,0) \propto e^{-m_Q x_4}$ which will be smaller than 10^{-15} at large t:

- Before the pre-conditioning（$\alpha_0 = 0$）, η_b effective mass will not saturate at large t with double precision;

G.M. de Divitiis, et. al., PLB692(2010)157

- With large enough pre-conditioning factor α_0, η_b effective mass becomes a constant at large t.



Improvement

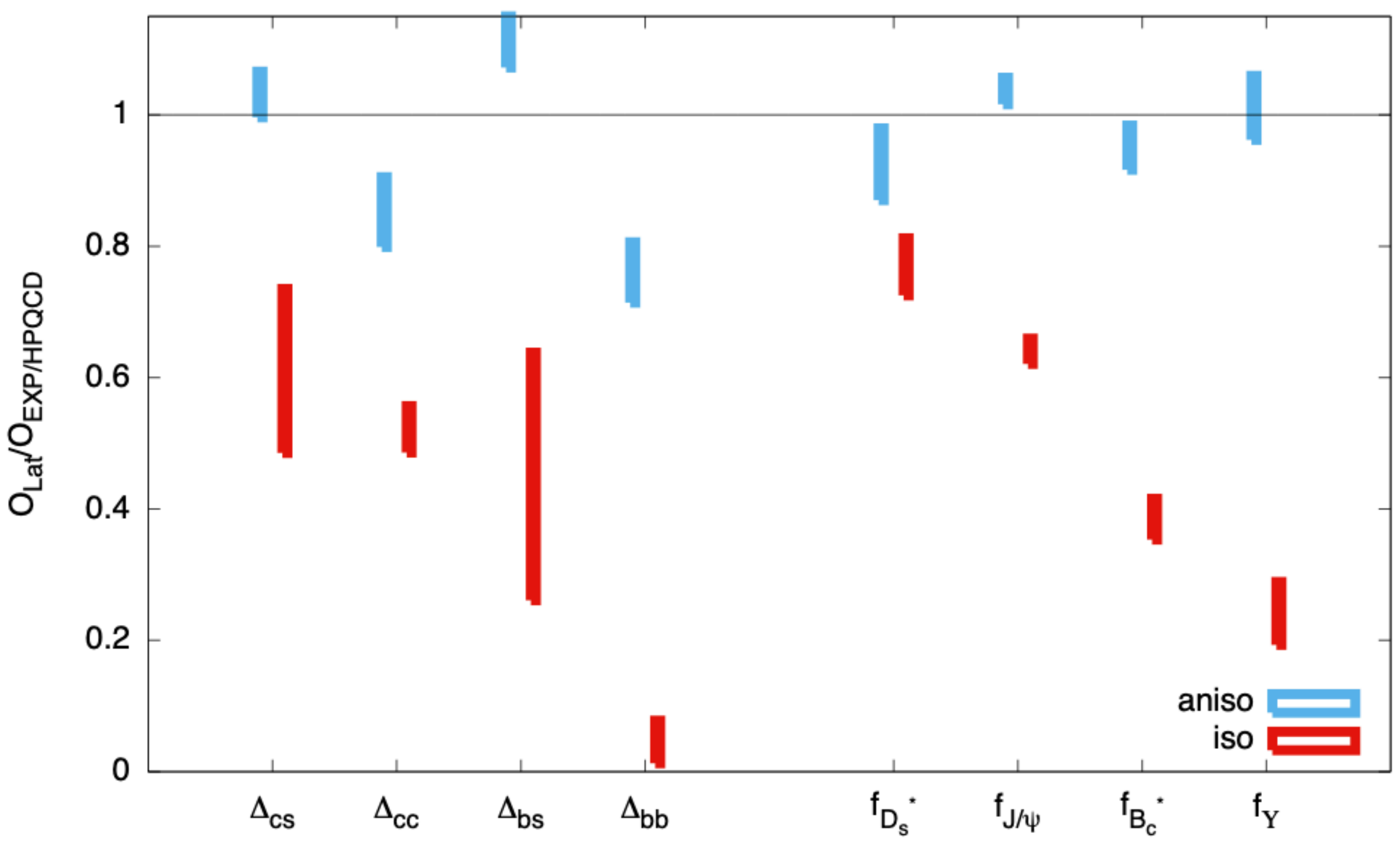
from the anisotropic action

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{1}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

★The anisotropic action can improve the hyperfine splitting significantly and consistently, especially the $\bar{b}b$ case.

★The predictions on the vector meson decay constants are also much better comparing to the isotropic action, using $Z_{\bar{q}_1\gamma_i q_2} \sim \sqrt{Z_{V_i}^{q_1} Z_{V_i}^{q_2}}$ with $Z_{V_i}^{q} = Z_{\bar{q}\gamma_i q}$.



Sumary

- Light quark mass and low energy constants have been properly extracted using the CLQCD ensembles, and study on the charm quark and also hadron spectrums are on going.
- Systematic hadron spectrum and structure studies can be carried out reliablely using the CLQCD ensembles.
- We expect more LQCD studies will use the CLQCD ensembles in the near future, based on the techniques we developed on the other ensembles.