Moduli Stabilization

in Type IIB Flux Compactifications

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Introduction

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String Compactification

The idea of compactification arises naturally to resolve the discrepancy between the critical dimensions D=10 and the observed dimensions d=4:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{K}_6$$

To have a minimal number of supersymmetries survived in $\mathcal{M}_4$, the manifold $\mathcal{K}_6$ is required to be a Calabi-Yau threefold $\mathcal{CY}_3$ with the properties:

- Compact
- Ricci-flat
- Complex
- Kähler with $SU(3)$ holonomy[1]

[1] Holonomy group $\mathcal{H}$: Upon parallel transport along a closed curve on $\mathcal{CY}_3$, a vector v is rotated into Uv. The set of matrices U forms $\mathcal{H}$ of $\mathcal{CY}_3$.

Moduli Space

Consider the zero modes of the metric decomposition,

$$g_{MN} = g_{\mu\nu} \oplus g_{\mu n} \oplus g_{mn}$$

The massless scalars g_{mn} determines the size and shape of $\mathcal{CY}_3$, of which the fluctuations are called Moduli.
The moduli of $\mathcal{CY}_3$ are usually divided into two classes:

1. Kähler moduli
2. Complex structure moduli

   To make the ideas transparent, we now describe the $\mathcal{CY}_3$ as a special kind of complex Kähler manifold.

- **Complex Manifold**:
  A complex manifold is an even-dimensional real manifold, d=2n, such that we can form n complex coordinates z^i $(i = 1,\ 2,\ \cdots, n)$ and the transition functions

$$z'^i = z'^i(z^j), \quad (i,\ j = 1,\ 2,\ \cdots\ n)$$

  are holomorphic between all pairs of patches.

Complex Manifold

On a complex manifold we can define the Hermitian metric,

$$ds^2 = g_{i\bar{j}}(dz^i \otimes d\bar{z}^{\bar{j}} + d\bar{z}^{\bar{j}} \otimes dz^i)$$

i.e., $g_{ij} = g_{\bar{i}\bar{j}} = 0$.

Moreover, we can define the so-called fundamental form on a complex manifold with a Hermitian metric,

$$-i\mathcal{J} = 2g_{i\bar{j}}dz^i \wedge d\bar{z}^{\bar{j}}$$

It is a $(1,1)$-form on the complex manifold.

- **Kähler Manifold**:
  A Kähler manifold is a special complex manifold on which the fundamental form is closed (**Kähler form**),

$$d\mathcal{J} = 0$$

Kähler Manifold

Properties:

1. The metric on a Kähler manifold is locally of the form

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(z, \bar{z})$$

where $K(z, \bar{z})$ is called Kähler potential.

2. The only non-vanishing connection coefficients are $\Gamma^i_{\ jk}$ and $\Gamma^{\bar{i}}_{\ \bar{j}\bar{k}}$, i.e., under parallel transport the holomorphic and anti-holomorphic indices do not mix.

3. Parallel transport takes holomorphic indices only into holomorphic indices. The holonomy group of a Kähler manifold is

$$\mathcal{H} \subset U(n) \subset SO(2n)$$

4. The only non-vanishing components of the Riemann curvature tensor are $R_{i\bar{j}k\bar{l}}$ and those related by symmetries. The Ricci tensor reads,

$$R_{i\bar{j}} = g^{k\bar{l}} R_{i\bar{j}k\bar{l}} = \partial_i \partial_{\bar{j}} (\ln \det g)$$

Calabi-Yau Manifold

- **Calabi-Yau manifold**:
  A n-dimensional Ricci-flat Kähler manifold is known as Calabi-Yau manifold $\mathcal{CY}_n$.
  1. The Holonomy group of $\mathcal{CY}_n$ is $SU(n)$.
  2. On a general $\mathcal{CY}_3$ the non-vanishing harmonic forms are: one $(3,0)$-form Ω and its $(0,3)$-form conjugate $\bar{\Omega}$; $h^{1,1}$ $(1,1)$-form $b^{\alpha}_{i\bar{j}}$; $h^{2,1}$ primitive $(2,1)$-form χ_α and their $(1,2)$-form conjugate $\bar{\chi}_{\bar{\alpha}}$. They satisfy the constraints:
  $$*_6\Omega = -i\Omega, \quad *_6\chi_\alpha = i\chi_\alpha$$
  3. The Euler number of $\mathcal{CY}_3$ is,
  $$\chi = 2(h^{1,1} - h^{2,1})$$

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Having the above preliminary knowledge, we now study the moduli space of $\mathcal{CY}_3$.

Moduli Space

The question is:

- Given a Ricci-flat Riemann metric g_{mn} on a $\mathcal{CY}_3$, what are the allowed infinitesimal variations δg_{mn} such that

$$R_{mn}(g) = 0 \;\Rightarrow\; R_{mn}(g + \delta g) = 0 \quad ?$$

There are two types of metric fluctuations:

$$\begin{cases} \delta g_{i\bar{j}} \\ \delta g_{ij} \; (\delta g_{\bar{i}\bar{j}}) \end{cases}$$

These fluctuations are independent of one another because they satisfy the decoupled equations,

$$\left.\begin{aligned} \nabla^a \nabla_a \delta g_{i\bar{j}} - 2R_{i\bar{l}k\bar{j}} \delta g^{k\bar{l}} &= 0 \\ \nabla^a \nabla_a \delta g_{ij} + 2R_{i\bar{k}j\bar{l}} \delta g^{\bar{k}\bar{l}} &= 0 \end{aligned}\right\} \quad \text{decoupled !}$$

Moduli Space

- **Kähler Muduli of $\mathcal{CY}_3$:**
  The harmonic $(1,1)$-forms $\delta g_{i\bar{j}}$ describe the fluctuations of the Kähler form

$$-i\mathcal{J} = g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$$

In general, we expand $\delta g_{i\bar{j}}$ in a basis of real $(1,1)$-forms,

$$\delta g_{i\bar{j}} = \sum_{\alpha=1}^{h^{1,1}} t^\alpha b^\alpha_{i\bar{j}}, \quad t^\alpha \in \mathbf{R}$$

t^α are called the Kähler moduli of $\mathcal{CY}_3$, whose number is the Hodge number $h^{1,1}$.

Moduli Space

- **Complexified Kähler Moduli of $\mathcal{CY}_3$:**
  In string theories compactified on $\mathcal{CY}_3$, we have the 0-th modes of the anti-symmetric tensor fields either in NS-NS sector or in RR sector,

$$\begin{cases} B_{mn} & \text{in NSNS sector} \\ C_{mnpq} & \text{in RR sector} \end{cases}$$

Since $h^{2,0} = h^{0,2} = 0$, the (complex) index structure of these antisymmetric tensors on $\mathcal{CY}_3$ should be $B_{i\bar{j}}$ and $C_{i\bar{j}k\bar{l}}$. Equivalently,

$$B_2 = B_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$$
$$C_4 = C_{i\bar{j}k\bar{l}} dz^i \wedge d\bar{z}^{\bar{j}} \wedge dz^k \wedge d\bar{z}^{\bar{l}}$$

and

$$*_6 C_4 = \epsilon^{i\bar{j}k\bar{l}m\bar{n}} g_{m\bar{q}} g_{p\bar{n}} C_{i\bar{j}k\bar{l}} dz^p \wedge d\bar{z}^{\bar{q}}$$

Moduli Space

We can combine these massless form fields with the Kähler deformations of the metric to form the so-called "Complexified Kähler Moduli". In heterotic string theories,

$$\mathcal{T}_{i\bar{j}} = \delta g_{i\bar{j}} + iB_{i\bar{j}}$$

In Type IIB string theory,

$$\mathcal{T}_{p\bar{q}} = \delta g_{p\bar{q}} + i\epsilon^{i\bar{j}k\bar{l}m\bar{n}} g_{m\bar{q}} g_{p\bar{n}} C_{i\bar{j}k\bar{l}}$$

- **Complex Structure Moduli of $\mathcal{CY}_3$:**
  With the help of the unique holomorphic $(3,0)$-form Ω on $\mathcal{CY}_3$, we can define the following complex $(2,1)$-forms

  $$\Omega_{ijk} g^{k\bar{m}} \delta g_{\bar{m}\bar{l}} dz^i \wedge dz^j \wedge d\bar{z}^{\bar{l}}$$

  and their complex conjugates. Hence, the total number of the moduli fields δg_{ij} $(\delta g_{\bar{i}\bar{j}})$ is $2h^{2,1}$.

Moduli Space

Question: What is the meaning of δg_{ij} ($\delta g_{\bar{i}\bar{j}}$) ?

1. For the new metric $g_{mn} + \delta g_{mn}$ to be again Kähler, there must be a coordinate system in which

$$\delta g_{ij} = \delta g_{\bar{i}\bar{j}} = 0.$$

2. Since holomorphic coordinate transformations do not change the structure of the complex indices, δg_{ij} can only be removed by a non-holomorphic coordinate transformation: Compared with the original one, the new metric is Kähler with respect to a different complex structure.

Hence, δg_{ij} ($\delta g_{\bar{i}\bar{j}}$) describe the complex structure deformations of $\mathcal{CY}_3$, which are known as "Complex structure moduli".

IIB Supergravity

The massless bosonic spectrum of IIB string theory in $D = 10$ consists of

- **NSNS sector**: dilaton $\hat{\phi}$, metric $\hat{g}_{MN}$, 2-form $\hat{B}_2$
- **RR sector**: axion $\hat{l}$, 2-form $\hat{C}_2$, 4-form $\hat{C}_4$

The IIB low-energy effective action in $D = 10$ Einstein frame is given by,

$$
\begin{aligned}
S_{IIB}^{10} = \; & -\int (\tfrac{1}{2}\hat{R} * 1 + \tfrac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} + \tfrac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3) \\
& -\frac{1}{4} \int (e^{2\hat{\phi}} d\hat{l} \wedge * d\hat{l} + e^{\hat{\phi}} \hat{F}_3 \wedge * \hat{F}_3 + \tfrac{1}{2} \hat{F}_5 \wedge * \hat{F}_5) \\
& -\frac{1}{4} \int \hat{C}_4 \wedge \hat{H}_3 \wedge \hat{F}_3
\end{aligned}
$$

The field strengths[2] are:

$$
\begin{aligned}
\hat{H}_3 &= d\hat{B}_2 \\
\hat{F}_3 &= d\hat{C}_2 - \hat{l} d\hat{B}_2 \\
\hat{F}_5 &= d\hat{C}_4 - \tfrac{1}{2} d\hat{B}_2 \wedge \hat{C}_2 + \tfrac{1}{2} \hat{B}_2 \wedge d\hat{C}_2
\end{aligned}
$$

[2] The self-duality condition $\hat{F}_5 = * \hat{F}_5$ has to be imposed by hand at the level of equations of motion.

IIB Orientifold Compactification

Compactification Scheme:

1. The standard Calabi-Yau compactification of IIB string on $\mathcal{CY}_3$ results in $\mathcal{N}=2$, $D=4$ effective supergravity.
2. To reduce supersymmetries from $\mathcal{N}=2$ to $\mathcal{N}=1$, we further carry out an orientifold projection,

$$\mathcal{O}=(-1)^{F_L}\Omega_p\sigma^*$$

where F_L is the "space-time fermion number" in the left moving sector, Ω_p is the worldsheet parity operator, σ that acts solely on $\mathcal{CY}_3$ is an isometric and holomorphic involution of $\mathcal{CY}_3$: $\sigma^2=1$, which leaves both the metric and the complex structure invariant[3]. However, on the holomorphic 3-form $\Omega \sim dy^1\wedge dy^2\wedge dy^3$,

$$\sigma^*\Omega=-\Omega$$

3. The orientifold action $\mathcal{O}$ introduces some spacetime filling $O3$- and/or $O7$-planes into the theory. The cancellation of induced tadpole divergences requires to further introduce some $D3$- and/or $D7$-branes (open string sectors).

[3] The pull-back of σ on forms is denoted by σ^*.

Orientifold Action

Note that $\mathcal{O} = (-1)^{F_L}\Omega_p\sigma^*$:

- $(-1)^{F_L}$ leaves the NSNS fields $\hat{\phi}$, $\hat{g}_{MN}$, $\hat{B}_2$ invariant but changes the sign of RR fields $\hat{l}$, $\hat{C}_2$ and $\hat{C}_4$;
- Under Ω_p the fields $\hat{\phi}$, $\hat{g}_{MN}$ and $\hat{C}_2$ are even while $\hat{B}_2$, $\hat{l}$ and $\hat{C}_4$ are odd;

The fields survived under the orientifold projection have to obey the constraints:

$$\begin{array}{ll} \sigma^*\hat{\phi} = \hat{\phi} & \sigma^*\hat{l} = \hat{l} \\ \sigma^*\hat{g}_{MN} = \hat{g}_{MN} & \sigma^*\hat{C}_2 = -\hat{C}_2 \\ \sigma^*\hat{B}_2 = -\hat{B}_2 & \sigma^*\hat{C}_4 = \hat{C}_4 \end{array}$$

In addition[4],

$$\sigma^*\Omega = -\Omega$$ [5]

[4]Under σ^*, the cohomology groups $H^{p,q}$ on $\mathcal{CY}_3$ split into two eigenspaces: $H^{p,q} = H^{p,q}_+ \oplus H^{p,q}_-$.

[5]The internal part of an orientifold plane is either a point ($O3$-plane) or a surface of complex dimension two ($O7$-plane).

Spectrum

By compactification, the 10-dimensional metric takes the form,

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{i\bar{j}}(x,y)dy^i d\bar{y}^{\bar{j}}$$

where $g_{\mu\nu}$ $(\mu,\nu = 0,\cdots,3)$ is a Minkowski metric and $g_{i\bar{j}}$ $(i,\bar{j} = 1,2,3)$ is the Kähler metric on $\mathcal{CY}_3$.

Spectrum:

- Deformations of this metric which respect to Calabi-Yau conditions form the scalar fields in $D = 4$. Relying on the invariance of metric under σ and $\sigma^*\Omega = -\Omega$, the deformations of the Kähler form $\mathcal{J} = ig_{i\bar{j}}dy^i \wedge d\bar{y}^{\bar{j}}$ give rise to $h_+^{1,1}$ real scalar fields $v^\alpha(x)$:

$$\delta\mathcal{J} = v^\alpha(x)\omega_\alpha, \quad (\alpha = 1,\cdots,h_+^{1,1})$$

  Besides, the following deformations of complex structure are kept in the spectrum:

$$\delta g_{ij} = \frac{i}{||\Omega||^2}\bar{z}^k(x)(\bar{\chi}_k)_{i\bar{p}\bar{q}}\Omega_j^{\bar{p}\bar{q}}, \quad (k = 1,\cdots,h_-^{1,2})$$

Spectrum

- The survived form fields are:

$$\hat{B}_2 = b^a(x)\omega_a, \quad \hat{C}_2 = c^a(x)\omega_a, \qquad (a = 1, \cdots, h_-^{1,1})$$
$$\hat{C}_4 = V^\kappa(x) \wedge \alpha_\kappa + \rho_\alpha(x)\tilde{\omega}^\alpha, \qquad (\kappa = 1, \cdots, h_+^{1,2}; \; \alpha = 1, \cdots, h_+^{1,1})$$

  where the self-duality of $\hat{F}_5$ has been taken into account. Remarkably, there are no $D = 4$ form fields $B_2(x)$ and $C_2(x)$ survived under σ^*. The non-vanishing of c^a, b^a and V^κ is related to the appearance of $O7$-planes.

- Both 10-dimensional scalars survive under σ^*:

$$\hat{\phi} = \phi(x), \quad \hat{l} = l(x)$$

The resulting $D = 4$, $\mathcal{N} = 1$ spectrum (bosonic part) reads:

gravity multiplet	1	$g_{\mu\nu}$
vector multiplet	$h_+^{2,1}$	V^κ
chiral multiplet	1	(ϕ, l)
	$h_-^{2,1}$	z^k
	$h_-^{1,1}$	(b^a, c^a)
	$h_+^{1,1}$	(v^α, ρ_α)

Background Flux

Background Fluxes:

1. In IIB string theory it is possible to allow background 3-form fluxes H_3 and F_3 on the $\mathcal{CY}_3$;
2. Both H_3 and F_3 have to be harmonic 3-forms;
3. Both H_3 and F_3 have to be odd under σ^*

For the presence of background fluxes, the strengths of the 10-dimensional form fields are found to be:

$$\hat{H}_3 = H_3 + db^a \wedge \omega_a$$
$$\hat{F}_3 = F_3 - lH_3 + (dc^a - ldb_a) \wedge \omega_a$$
$$\hat{F}_5 = dV^\kappa \wedge \alpha_\kappa + d\rho_\alpha \wedge \tilde{\omega}^\alpha - \frac{1}{2}(c^a db^b - b^a dc^b) \wedge \omega_a \wedge \omega_b$$

Besides, a nontrivial warp factor $e^{2A(y)}$ would have to be included into the metric decomposition:

$$ds^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y)} g_{i\bar{j}}(x,y) dy^i d\bar{y}^{\bar{j}}$$

However, in the (interested) large volume limit, the warp factor approaches *one* so that it can be ignored.

4-d Supergravity Action

By substituting the above expansions into S^{10}_{IIB} and performing a Weyl rescaling of the 4-dimensional metric $g_{\mu\nu} \to \mathrm{Vol}(\mathcal{CY}_3) g_{\mu\nu}$, we get the effective supergravity action in 4 dimensions. It can be cast as the standard $\mathcal{N} = 1$ supergravity form:

$$\begin{aligned} S^4_{O3/O7} = \quad & -\int \tfrac{1}{2} R * 1 + K_{I\bar{J}} DM^I \wedge * D\bar{M}^{\bar{J}} + \tfrac{1}{2}(\Re f_{\kappa\lambda}) F^\kappa \wedge * F^\lambda \\ & + \tfrac{1}{2}(\Im f_{\kappa\lambda}) F^\kappa \wedge F^\lambda + V * 1 \end{aligned}$$

with

$$V = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2) + \frac{1}{2}[(\Re f)^{-1}]^{\kappa\lambda} D_\kappa D_\lambda$$

in terms of a Kähler potential

$$K = -\ln\left[-i \int_{\mathcal{CY}_3} \Omega(z) \wedge \bar{\Omega}(\bar{z})\right] - \ln[-i(\tau - \bar{\tau})] - 2\ln\left[\mathrm{Vol}(\mathcal{CY}_3)\right]$$

a holomorphic (background flux dependent) superpotential

$$W(\tau, z^k) = \int_{\mathcal{CY}_3} \Omega(z) \wedge G_3$$

and a holomorphic gauge-kinetic coupling $f_{\kappa\lambda}$.

4-d Supergravity Action

where $F^\kappa = dV^\kappa$, $\tau = l + ie^{-\phi}$ and M^I collectively denote all complex scalars. $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K(M, \bar{M})$ is the Kähler metric on moduli space[6], $D_I W = \partial_I W + (\partial_I K) W$ is the Kähler derivative.

[6]

1. The complex structure deformations z^k are good Kähler coordinates. Hence,

$$K_{k\bar{l}} = \frac{\partial}{\partial z^k} \frac{\partial}{\partial \bar{z}^{\bar{l}}} K$$

2. $\tau = l + ie^{-\phi}$ and $G^a = c^a - \tau b^a$ are also Kähler coordinates on moduli space.
3. v^α are not Kähler coordinates. Instead, they are the implicit functions of the Kähler coordinates (τ, T^α, G^a), where,

$$T_\alpha = \frac{3i}{2}\rho_\alpha + \frac{3}{4}\mathcal{K}_\alpha(v) - \frac{3}{2}\zeta_\alpha(\tau, \bar{\tau}, G, \bar{G})$$

with $\mathcal{K}_\alpha(v) = \mathcal{K}_{\alpha\beta\gamma} v^\beta v^\gamma$, $\zeta_\alpha = -\frac{i}{2(\tau - \bar{\tau})}\mathcal{K}_{\alpha bc} G^b (G - \bar{G})^c$ and $\mathcal{K}_{ABC} = \int_{\mathcal{CY}_3} \omega_A \wedge \omega_B \wedge \omega_C$ are the intersection number of $\mathcal{CY}_3$. Particularly, $\mathrm{Vol}(\mathcal{CY}_3) = \frac{1}{6}\mathcal{K} = \frac{1}{6}\mathcal{K}_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma$.

4-d Supergravity Action

In the following, we only consider the simple orientifolds without $O7$-planes. For such cases, $b^a = c^a = G^a = 0$.

Properties:

- The Kähler moduli are decoupled of the complex dilaton τ,

$$T_\alpha = \frac{3i}{2}\rho_\alpha + \frac{3}{4}\mathcal{K}_\alpha(v)$$

  However, the superpotential W remains T_α-independent.

- The Kähler potential satisfies a no-scale type condition:

$$K^{T_\alpha \bar{T}_\beta} \frac{\partial K}{\partial T_\alpha} \frac{\partial K}{\partial \bar{T}_\beta} = 3$$

  The F-term potential becomes semi-definite positive,

$$V_F = e^K K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \geqslant 0$$

  where M^i stand for all scalar fields except the Kähler moduli T_α.

- The minimal of V_F is $V_F^{\text{min}} = 0$, at which $D_{z^k} W = D_\tau W = 0$ but

$$D_{T_\alpha} W = (\partial_{T_\alpha} K) W = -2\frac{v^\alpha}{\mathcal{K}} W \neq 0$$

  This minimal is a supersymmetric broken state.

4-d Supergravity Action

- Not an arbitrary 3-form background fluxes G_3 can yield the optimization conditions. From the identity $D_{z^k}\Omega(z) = i\chi_k$, we get

$$D_\tau W = \frac{i}{2} e^\phi \int_{\mathcal{CY}_3} \Omega \wedge \bar{G}_3$$
$$D_{z^k} W = i \int_{\mathcal{CY}_3} \chi_k \wedge G_3$$

Recall that ${}_{*6}\Omega = -i\Omega$, ${}_{*6}\chi_k = i\chi_k$ and

$$\int_{\mathcal{CY}_3} A \wedge B = \int_{\mathcal{CY}_3} {}_{*6}A \wedge_{*6} B$$

the above Kähler derivatives can be alternatively written as:

$$D_\tau W = \frac{1}{2} e^\phi \int_{\mathcal{CY}_3} \Omega \wedge_{*6} \bar{G}_3$$
$$D_{z^k} W = - \int_{\mathcal{CY}_3} \chi_k \wedge_{*6} G_3$$

Hence, the conditions $D_{z^k} W = D_\tau W = 0$ require the imaginary self-dual background fluxes: ${}_{*6}G_3 = iG_3$.

- Because $D_{T_\alpha} W \neq 0$, at the F-term vacuum the Kähler moduli are undetermined. So there is a degenerate family of vacua with arbitrary supersymmetry-breaking scale $\Re T_\alpha$.

KKLT Mechanism

The remedy for fixing all scalar fields including the Kähler moduli T_α is to spoil the no-scale structure of the 4-dimensional effective supergravity.

The known **KKLT mechanism** is:

- adding the effect of a Euclidean D3-brane wrapping a 4-cycle in $\mathcal{CY}_3$. [Witten, hep-th/9604030]
- adding the gaugino condensation effects coming from stacks of D7-branes with fluxes wrapping a 4-cycle in $\mathcal{CY}_3$. [Nilles et al, Phys. Lett. B155(1985)65]

Including either effect yields a nonperturbative (Kähler moduli dependent) correction to superpotential,

$$W_{np} \propto e^{-a^\alpha T_\alpha}$$

The revised superpotential becomes:

$$W = \int_{\mathcal{CY}_3} \Omega(z) \wedge G_3 + Ae^{-a^\alpha T_\alpha}$$

KKLT procedure

KKLT procedure:

1. The complex structure moduli and dilaton are assumed to be much heavier than T_α so that they can be fixed by sole flux superpotential $W_f = \int_{\mathcal{CY}_3} \Omega(z) \wedge G_3$ through supersymmetric optimization conditions $D_\tau W_f = D_{z^k} W_f = 0$.
2. After the complex structure moduli and dilaton are stabilized, the superpotential does only depend upon the Kähler moduli,
$$W = W_0 + A e^{-a^\alpha T_\alpha} \qquad (W_0 \neq 0)$$
Then, the Kähler moduli are fixed by the supersymmetric optimization condition $D_{T_\alpha} W = 0$.
3. The resulting supersymmetric vacuum (at F-term potential level) is an anti-de Sitter space,
$$V_F^{\text{min}} = -3e^K |W|^2 \leqslant 0$$
which can be lifted to a deSitter vacuum by some supersymmetric effects such as D3-anti-D3 interaction.

Doubts

Though KKLT mechanism is commonly accepted, KKLT procedure for implementing such a mechanism has received some criticisms.

Doubts:

1. D3-anti-D3 interaction is an effect of explicit supersymmetry breaking. Is it possible to replace it by a spontaneous SUSY breaking effect such as D-term potential ?
2. Is the KKLT two step decoupled procedure for stabilizing moduli a true approximation of the exact one step procedure based on the full superpotential ?

Uplifting Mechanisms

Difficulty for D-term Potential:

- In KKLT procedure, the resulting minimal of F-term potential is supersymmetric with vanishing F-terms.
- In supergravity, a model with vanishing F-terms must have also vanishing D-terms.

D-terms in N=1 Supergravity:

The gauge invariant action for $D = 1, \mathcal{N} = 1$ supergravity with chiral multiplets $\phi^i \sim (z^i, \psi^i)$ and vector multiplets $V^a \sim (\lambda^a, A^a_\mu)$ is completely fixed by 3 ingerdients:

1. real gauge invariant Kähler function $G = K + \ln |W|^2$;
2. holomorphic gauge kinetic function f_{ab};
3. holomorphic Killing vectors $X_a = X^i_a(z)\frac{\partial}{\partial z^i}$.

The scalar potential is:

$$V = V_F + V_D = e^G(G^{i\bar{j}}G_iG_{\bar{j}} - 3) + \frac{1}{2}[(\Re f)^{-1}]^{ab}D_aD_b$$

where $G_i = \partial G/\partial z^i$ are called F-terms,

$$G_i = K_i + W_i/W = D_iW/W$$

where the holomorphicity of the superpotential has been used.

Uplifting Mechanism

D_a are known as the D-terms:

$$D_a = iG_i X_a^i = i\frac{D_i W}{W} X_a^i$$

Question:

- *Can we have a supersymmetry broken vacuum at the level of F-term potential (during the moduli stabilization) ?*

Attempts:

1. Ignore the nonperturbative corrections of the superpotential (so that $D_{T_\alpha} W \neq 0$). The no-scale property of V_F is spoiled by α'-correction and string loop corrections to Kähler potential.
   [Parameswaran et al, hep-th/0602253]
2. Interpret the shift symmetry of Kähler potential as an anomalous $U(1)$ symmetry and construct gauge invariant nonperturbative superpotential. In this way some open string moduli M^a (massless squark condensation) are introduced which satisfy $D_{M^a} W \neq 0$.
   [Achúcarro et al, hep-th/0601190]
3. Directly search for the supersymmetry broken vacua (dS and/or AdS) of F-term potential. *Yang, Phys. Rev. D73, 066006(2006)*

my work

Goal:

- to examine the moduli stabilization of IIB orientifolds with one complex structure modulus in one-step procedure.
- to find conditions for having a supersymmetry broken deSitter vacuum at the level of F-term potential.

Models:

The models under consideration are IIB orientifolds at orbifold limits with orbifold groups $\Gamma = \mathbf{Z}_{6-II}, \mathbf{Z}_2 \times \mathbf{Z}_3$ or $\mathbf{Z}_2 \times \mathbf{Z}_6$, for which $h^{1,1} = 3, h^{2,1} = 1$ in untwisted sectors. We consider isotropic case where $T_1 = T_2 = T_3 = T$. The models are defined by

- Kähler potential:

$$K = -3\ln(T+\bar{T}) - \ln(S+\bar{S}) - \ln(U+\bar{U})$$

  where $S = e^{-\phi} + il$ is the complex dilaton-axion field ($i\bar{\tau}$).

- Superpotential whose flux part is defined as $\frac{1}{(2\pi)^2\alpha'}\int_{CY_3} \Omega \wedge G_3$ is explicitly expressed as:

$$W = \alpha_0 + \alpha_1 U + \alpha_2 S + \alpha_3 SU + 3ge^{-hT}$$

my work

The potential energy of the models is due to the presence of background 3-form flux G_3, which can be organized into the standard F-term potential of $\mathcal{N} = 1$ supergravity,

$$V_F = e^K \left(K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right)$$

Let $T = t + i\tau$, $S = s + i\sigma$ and $U = u + i\nu$. The points $\tau = \sigma = \nu = 0$ define some flat directions in moduli space on which $\partial_\tau V_F = \partial_\sigma V_F = \partial_\nu V_F = 0$, at which the potential takes a simple expression:

$$\begin{aligned} V_F = \quad & \frac{1}{16t^3 su}\Big[6ghte^{-ht}(\alpha_0 + u\alpha_1 + s\alpha_2 + us\alpha_3 + 3ge^{-ht}) \\ & + 6(ght)^2 e^{-2ht} + (\alpha_0 - us\alpha_3 + 3ge^{-ht})^2 \\ & + (u\alpha_1 - s\alpha_2)^2\Big] \end{aligned}$$

The remaining optimization conditions $\partial_t V_F = \partial_s V_F = \partial_u V_F = 0$ can be written as either:

$$\alpha_2 s = \alpha_1 u, \qquad u = -\frac{1}{\alpha_1}[\alpha_0 + (3 + ht)ge^{-ht}],$$

$$\alpha_3 = \frac{\alpha_1 \alpha_2 (\alpha_0 + 3ge^{-ht})}{\left[\alpha_0 + (3 + ht)ge^{-ht}\right]^2}.$$

my work

or (for simplicity here we only give the conditions in $\alpha_0 = 0$):

$$\alpha_2 s = \alpha_1 u, \qquad u = -\frac{g(72+177ht+109h^2t^2+16h^3t^3-2h^4t^4)e^{-ht}}{ht(7+2ht)^2\alpha_1},$$
$$\alpha_3 = -\frac{3\alpha_1\alpha_2(ht)^2(7+2ht)^3(1+4ht+2h^2t^2)e^{ht}}{g(72+177ht+109h^2t^2+16h^3t^3-2h^4t^4)^2}.$$

The former corresponds to a supersymmetry-restored extremum ($D_T W = D_S W = D_U W = 0$),

$$V_{F,\mathrm{c}}^{\mathrm{susy}} = -\frac{3\alpha_1\alpha_2 g^2 h^2 e^{-2ht}}{8t\left[\alpha_0 + (3+ht)ge^{-ht}\right]^2} \leqslant 0$$

but the latter corresponds to a supersymmetry-breaking extremum ($D_U W \neq 0, D_S W \neq 0$)[7]:

$$V_{F,\mathrm{c}}^{\mathrm{nonsusy}} = \frac{3\alpha_1\alpha_2 h^2(7+2ht)^2(-48-84ht-73h^2t^2-22h^3t^3+2h^4t^4)}{8t(-72-177ht-109h^2t^2-16h^3t^3+2h^4t^4)^2}$$

[7] Provided $\alpha_0 \geq 0$, the supersymmetric extremum is only accessible for models with $\alpha_1 < 0$, $\alpha_2 < 0$ and $\alpha_3 > 0$, the supersymmetry-broken extremum is only accessible for models with $\alpha_3 < 0$.

Hessian determinants indicate that $V_{F,\mathrm{c}}^{\mathrm{nonsusy}}$ is a positive minimum if the Kähler modulus is fixed at $\frac{13.86}{h} < t < \frac{14.70}{h}$. an illustrative model with,

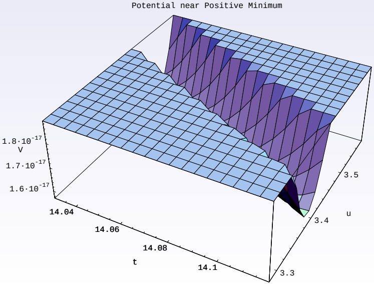
$$W = 1.25 \times 10^{-7} U + 2.5 \times 10^{-7} S - 5 \times 10^{-6} SU + 3e^{-T}$$

is in order. Its potential $V_F \approx \frac{A}{t^3 u^2}$ with

$$A = 0.125 \times [6e^{-2t} t^2 + 6e^{-t} t(3e^{-t} + 2.5 \times 10^{-7} u - 2.5 \times 10^{-6} u^2) + (3e^{-t} + 2.5 \times 10^{-6} u^2)^2]$$

has a local deSitter minimum $V_{F,\mathrm{min}}^{\mathrm{nonsusy}} \approx 1.62 \times 10^{-17}$ at $t \approx 14.08$, $u \approx 3.45$ and $s \approx 1.72$.

The potential curve near the founded deSitter vacuum is,



Thank you!

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