A Holographic Dual of CFT with Flavor on de Sitter Space

March 2006 @USTC

Takayuki Hirayama
@
USTC Shanghai Institute for Advanced Studies

☐ AdS/CFT correspondence (Maldacena '97) is an explicit example of Holography.

☐ The most understood example is

$$\text{supergravity on } AdS_5 \times S^5 \leftrightarrow \text{4d N=4 SYM}$$

(Gubser, Klebanov, Polyakov, Witten '98) give how to compute correlation functions in CFT from supergravity. (the AdS/CFT dictionary)

☐ Many deformations have been discussed since then.
Replacing AdS_5 by an asymptotically AdS_5 space, such as an extremal or non-extremal AdS black hole. Compactifying time direction with SUSY breaking boundary conditions. In CFT side, these modifications correspond to turning on couplings, temperature or etc.

☐ Holography ('t Hooft '93, Susskind '94) is a more broad conjecture. If there is a consistent quantum gravity on a spacetime background, we expect there is a holographic description by a theory on one dimension less spacetime.

□ There many other speculation of holography.

(Strominger '01), dS/CFT, dS_p gravity $\leftrightarrow$ CFT on $d = p - 1$

(de Boer, Solodukhin '03) gravity on $d = p$ Minkowski $\leftrightarrow$ CFT on $d = p - 1$

(Alishahiha, Karch, Silverstein and Tong '04, '05), (A)dS/dS

$$\text{gravity on } AdS_p \text{ or } dS_p \leftrightarrow \text{CFT on } dS_{p-1}$$

However since a string realization of these holography has been not known, their arguments are limited to be qualitative.

□ A special case of (A)dS/dS is

$$AdS_5 \text{ gravity} \leftrightarrow \text{CFT on } dS_4$$

This is unexpected since the original AdS/CFT tells

$$AdS_5 \text{ gravity} \leftrightarrow \text{N=4 SYM on 4d Minkowski}$$

These two descriptions are very different.

☐ What I will study in my talk is:

- I realize (A)dS/dS setup in string theory in a probe limit.
- Then I follow the AdS/CFT dictionary (Gubser, Klebanov, Polyakov, Witten ’98) which give how to compute correlation functions in CFT from supergravity.
- Then I check if resultant correlation functions are consistently understood as those on dS_4 space.
  Since dS_4 has a finite temperature proportional to inverse of curvature, I will focus on the temperature effects on correlation functions and will check if the temperature effects are properly reproduced in my calculations.
- Yes! Therefore my results support (A)dS/dS.
- Then we will have a question why two very different theories can be dual to a same AdS_5 gravity.
  I will speculate a certain conformal transformation relates AdS/CFT with (A)dS/dS.

Realizing (A)dS/dS setup in string theory

□ I use D3/D7 system. Since I will modify the supersymmetric D3/D7 system, I first give a brief review of supersymmetric D3/D7 system.

□ CFT side of supersymmetric D3/D7 system.
The low energy effective theory on N_c D3-brane is 4d N=4 SYM.
Adding N_f D7-branes, CFT becomes 4d N=2 SYM with N_f Hypermultiplets.
The D3-D7 open string introduce Hypermultiplets.

	$X_0 \cdots X_3$	$X_4 \cdots X_7$	X_8, X_9	
D3	o	x	x	D3 is located at $X_4 = \cdots = X_9 = 0$.
D7	o	o	x	D7 is located at $X_8 = m, X_9 = 0$.

The superpotential becomes $W_{N=4} + mHH_c$.

□ Take Large N_c and Large 't Hooft coupling limit.
Flavor physics is described by gauge invariant operators, i.e. Meson and Baryons, and the characteristic scale is the chiral condensate $\langle q\bar{q} \rangle$.

□ Gravity side of supersymmetric D3/D7 system.

The near horizon limit of D3-brane background is $AdS_5 \times S^5$ whose metric is

$$ds^2 \;=\; R^2 dx_{M4}^2 + \frac{1}{R^2}(\underline{dR^2 + R^2 d\Omega_5^2}), \qquad R^2 = (X_4)^2 + \cdots + (X_9)^2$$
$$dX_4^2 + \cdots dX_9^2 = \underline{dr^2 + r^2 d\Omega_3^2 + dX_8^2 + dX_9^2}, \qquad R^2 = r^2 + X_8^2 + X_9^2$$

□ Introduce N_f D7-branes into $AdS_5 \times S^5$ as a probe ($N_f \ll N_c$) (Karch, Katz '02). It is consistent with large N_c limit.

□ D7-brane world volume has 8 dimensions and extends along $M4$, S^3 and r and is localized at X_8 and X_9.

Because the metric is nontrivial in r, X_8 can depend on r, i.e. $X_8 = X_8(r)$. Because of rotational symmetry, D7 is placed at $X_9 = 0$.

□ The induced metric becomes

$$\begin{aligned} ds^2 &= R^2 dx_{M4}^2 + \frac{1}{R^2}(dr^2 + r^2 d\Omega_3^2 + (\frac{dX_8(r)}{dr})^2 dr^2) \\ &= h_{ab} dx^a dx^b, \qquad (R^2 = r^2 + X_8^2) \end{aligned}$$

□ The Dirac-Born-Infeld action of D7 is then

$$S_{DBI} = -T_7 \int d^8x \sqrt{-\det h_{ab}} \propto \int dr\, r^3 \sqrt{1 + X_8'(r)^2}$$

and this becomes an action for $X_8(r)$. The equation of motion for $X_8(r)$ is

$$\frac{d}{dr}\left(\frac{r^3 X_8'(r)}{\sqrt{1 + X_8'(r)^2}}\right) = 0$$

The asymptotic solution near AdS boundary ($r = \infty$) is

$$X_8(r) = m + \frac{v}{r^2}.$$

Since $v \neq 0$ does not give a regular solution, $v = 0$ and $X_8 = m$.

□ Now we have obtained D7 configuration. We can use the AdS/CFT dictionary, and the CFT operator $\mathscr{O}$ for $X_8(r)$ is known to be $\mathscr{O} \sim q\bar{q}$. ($q$ is a fermion in Hypermultiplet.)

$$\mathscr{L} = \mathscr{L}_{CFT} + m\mathscr{O}$$

This is consistent with superpotential in CFT side before.

□ The correlation function is calculated from AdS gravity,

$$\left\langle \exp \int d^4x\, m\mathscr{O}(x) \right\rangle_{CFT} \propto \lim_{r_\infty \to \infty} \exp\left\{S_{DBI}(X_8(r_\infty) = m)\right\}$$

$$\langle \mathscr{O}(x) \rangle_{CFT} \propto \lim_{r_\infty \to \infty} \frac{dS_{DBI}}{dm} \propto v$$

r_∞ is a cutoff near AdS boundary ($r = \infty$). In supersymmetric case the chiral condensate $\langle \mathscr{O} \rangle \sim \langle q\bar{q} \rangle \propto v = 0$

□ The Meson spectrum is computed from fluctuations of D7 configuration.

□ Now we explain how to realize (A)dS/dS setup. We introduce D7 into D3 geometry $AdS_5 \times S^5$ in a different way.

□ As 4d Minkowski is a subspace in AdS_5, dS_4 is also a subspace in AdS_5. Then AdS_5 metric is written with using dS_4 metric

$$ds^2 \;=\; r^2(1-\frac{1}{4r^2l_4^2})^2 dx_{dS4}^2 + \frac{1}{r^2}(\underline{dr^2+r^2d\Omega_5^2})$$

$$\underline{d\rho^2+\rho^2 d\Omega_3^2+dy^2+dz^2}, \qquad r^2=\rho^2+y^2+z^2$$

□ Here is an explanation how dS_4 is embedded in AdS_5. AdS_5 is a hypersurface in 6d flat space. The equation is

$$X_5^2 + \underline{X_0^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2} = l_5^2$$

Similarly dS_4 is a hypersurface in 5d flat space. The equation is

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 = l_4^2$$

We can easily see that 4d Minkowski is embedded along $X_0, \cdots X_3$.

□ Since I would like to realize dS_4 in CFT side, I embed D7-brane such that D7-brane extends dS_4, S^3 and ρ and is localized at y and z.
Since the metric is nontrivial in ρ, $y = y(\rho)$ and $z = 0$.

□ The induced metric becomes

$$\begin{aligned} ds^2 &= r^2(1 - \frac{1}{4r^2 l_4^2})^2 dx_{dS4} + \frac{1}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + (\frac{dy(\rho)}{d\rho})^2 d\rho^2) \\ &= h_{ab} dx^a dx^b, \qquad (r^2 = \rho^2 + y^2) \end{aligned}$$

Near AdS boundary $\rho \to \infty$, $1/l_4^2$ term becomes subleading. In CFT this is understood as dS_4 curvature is negligible in high energy limit.

□ The DBI action of D7 brane becomes

$$S_{DBI} = -T_7 \int d^8x \sqrt{-\det h_{ab}} \propto \int d\rho (1 - \frac{1}{4r^2 l_4^2})^4 \rho^3 \sqrt{1 + y'(\rho)^2}$$

Then the equation of motion for $y(\rho)$ is

$$y''(\rho) = -3\frac{(1+y'^2(\rho))}{\rho}y'(\rho) + \frac{2}{l_4^2 r^2 - \frac{1}{4}}\frac{y(\rho)-\rho y'(\rho)}{r^2}(1+y'^2(\rho))$$

□ Since the solution near AdS boundary determines the CFT operator and coupling constant, we first study the asymptotic solution near AdS boundary ($\rho = \infty$).

$$y(\rho) = m\left(1 - \frac{\ln(\rho^2+m^2)}{2l_4^2\rho^2}\right) + \frac{v}{\rho^2}$$

Using the AdS/CFT dictionary, the CFT operator $\mathcal{O}$ is same and is $\mathcal{O} \sim q\bar{q}$, and $y(\rho \to \infty) = m$ is the bare mass for q on dS_4.
(We later give an explanation why this is true.)

□ v is determined so that the solution is smooth everywhere and we obtain such v numerically by shooting method.

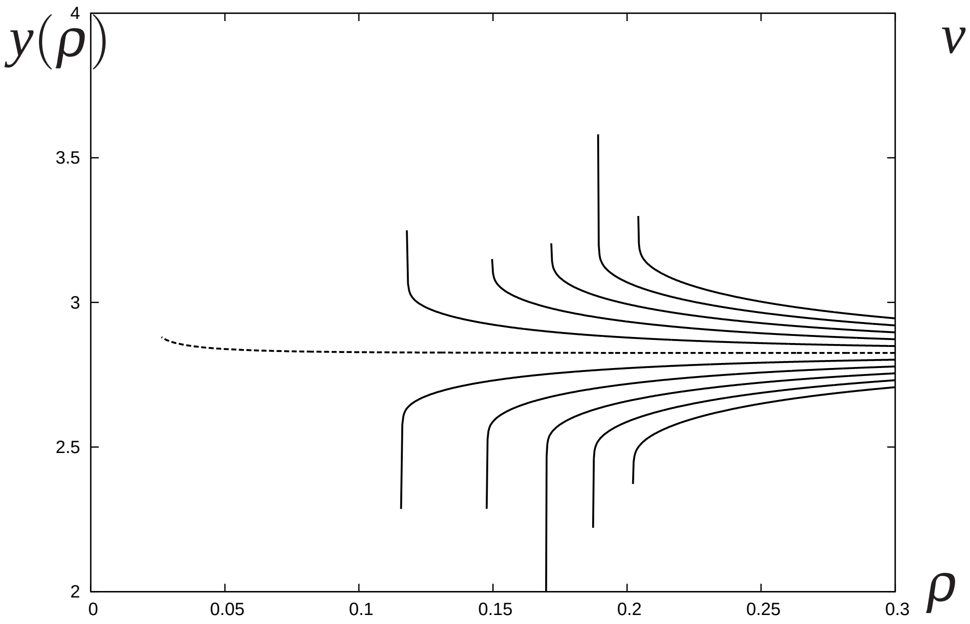

Figure 1: Shooting method. Change v fixing other parameters.

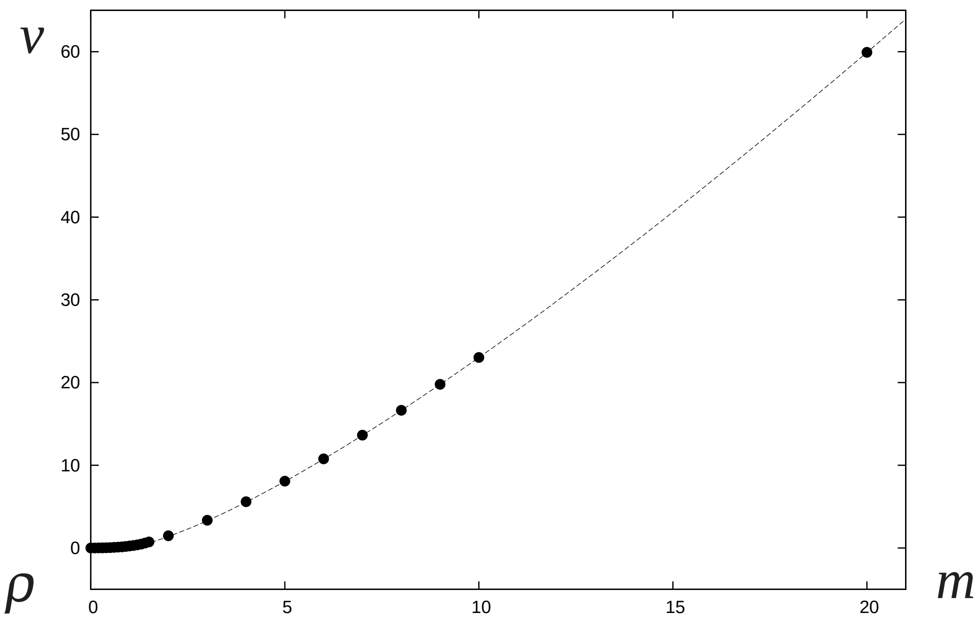

Figure 2: v of regular solutions vs m. The line is the best fit function $1.0 \times m \ln m$

□ We have D7-brane configuration. We use the AdS/CFT dictionary to compute $\langle \mathcal{O} \rangle$. $(y(\infty) = m)$

$$\langle \mathcal{O}(x) \rangle \propto \lim_{\rho_\infty \to \infty} \frac{d(S_{DBI} + S_C)}{dm}$$

where ρ_∞ is the cutoff near AdS boundary $\rho = \infty$ and S_c is the counter term defined at $\rho = \rho_\infty$ since S_{DBI} itself is divergent.

$$S_c = T_7\Omega_3 \int d^4x\sqrt{-\gamma}\left[\frac{1}{4} - \frac{y^2}{2\rho^2} - \frac{1}{4\rho^2 l_4^2} + \left(\frac{3}{8l_4^4\rho^4} + \frac{y^2}{l_4^2\rho^4}\right)\left\{(c_1+1)\ln(l_4\rho) + c_1\ln\frac{y}{\rho}\right\}\right.$$
$$\left.\left. + c_2\frac{y^4}{\rho^4} + c_3\frac{1}{\rho^4 l_4^4} + c_4\frac{y^2}{\rho^4 l_4^2}\right]\right|_{\rho=\rho_\infty, y=y(\rho_\infty)}$$

where γ is the induced metric in AdS_5 at constant ρ. $c_1 \cdots c_4$ are finite counter terms. (cf. Bianchi, Freedman, Skenderis, '01)

□ Then using AdS/CFT dictionary,

$$\langle\mathcal{O}(x)\rangle \propto v + \frac{m}{l_4^2}(A + B\ln m), \qquad A = 2(c_1+1)\ln l_4 + c_1 + 2c_4, \qquad B = 2c_1,$$

We fix A and B as follows. The curvature l_4 collections should be negligible as $m \gg 1/l_4$ since massive field should be insensitive to low energy $1/l_4$ collections. Therefore we fix A and B such that $\langle\mathcal{O}(x)\rangle \to 0$ as $m \gg 1/l_4$ limit.

□ Then we obtain $A = 0$ and $B = 1$ numerically and thus the final results of chiral condensate $\langle \mathscr{O}(x) \rangle$ are

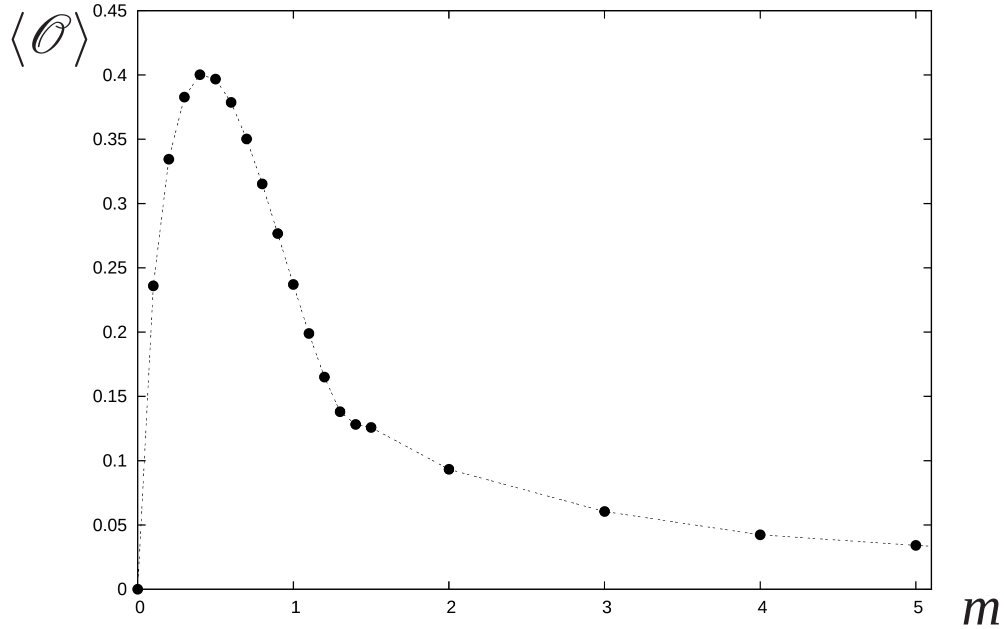

Figure 3: The chiral condensate $\langle \mathscr{O} \rangle$ vs m. $l_4 = 1$.

□ First it is actually nontrivial if $\langle \mathscr{O}(x) \rangle \to 0$ is realizable using A and B, since $v(m)$ can be an arbitrary function of m.

Thus it is a nontrivial check of AdS/CFT dictionary.

□ Second, this is totally different from supersymmetric case where $\langle \mathcal{O}(x) \rangle = 0$ always.

□ This behaviour is very similar to that studied from *AdS* black hole. (CFT is CFT at finite temperature.) (Babington, Erdmenger, Evans, Guralnik, Kirsch, '03)

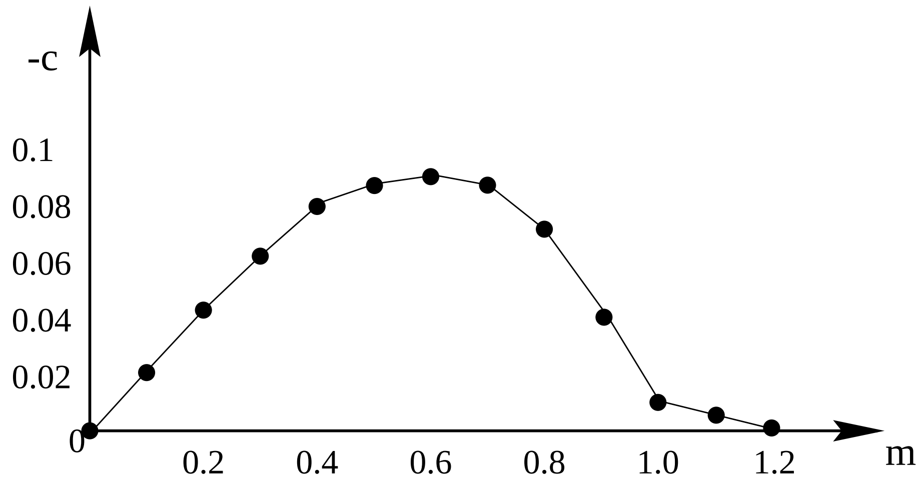

Figure 4: The chiral condensate from AdS Schwarzschild black hole. Copied from hep-th/0306018

□ Therefore we have seen that the chiral condensate shows a proper finite temperature dependence.

□ We have also studied the meson spectrum from fluctuation of D7-brane configuration, and the quark anti-quark static potential from Wilson line in my setup and all of them have expected temperature dependence.
In meson spectrum, we find confinement and deconfinement phases.
In quark anti-quark potential, we find the string tension is reduced by the dS_4 curvature.

□ Therefore our results give positive supports on (A)dS/dS.

Why succeeded?

□ Then the next question is why we have obtained reasonable results. Why can two different theories be dual to a same AdS_5 gravity? I will give a speculation.
I again study a field and operator matching in AdS/CFT dictionary of original AdS/CFT.

□ Let us study a scalar field in AdS_5 with mass M

$$S = \int d^5x\sqrt{-g_{AdS}}\left(-(\partial\phi)^2 - M^2\phi^2\right).$$

□ Consider a static solution along dS_4 directions. The asymptotic solution near AdS boundary is

$$\phi(u) = c_1 u^{\alpha_+} + c_2 u^{\alpha_-}, \qquad \alpha_\pm = -2 \pm \sqrt{4+M^2},$$

here I use the AdS_5 metric

$$ds^2_{AdS5} = \left(u^2 - \frac{1}{l_4^2}\right)dx^2_{dS4} + \left(u^2 - \frac{1}{l_4^2}\right)^{-1}du^2, \qquad dx^2_{dS4} = \frac{l_4^2}{s^2}(-ds^2 + dx_i^2)$$

The coordinate transformation

$$u = Rt/l_4, \qquad s = (t^2 - 1/R^2)^{1/2}$$

changes AdS_5 metric

$$ds^2_{AdS5} = R^2 dx^2_{M4} + \frac{1}{R^2} dR^2, \qquad dx^2_{M4} = -dt^2 + dx_i^2$$

Therefore in this coordinate system, the asymptotic solution becomes

$$\phi(u) = c_1 (Rt/l_4)^{\alpha_+} + c_2 (Rt/l_4)^{\alpha_-}$$

□ Then using the original AdS/CFT, since $\phi(u) \propto c_1 (t/l_4)^{\alpha_+}$ in $R \to \infty$ limit we obtain

$$\mathscr{L} \;=\; \mathscr{L}_{M4} + c_1 (t/l_4)^{\alpha_+} \mathscr{O}(x), \qquad S = \int \sqrt{g_{M4}} \mathscr{L}$$

$$\langle \mathscr{O}(x) \rangle \;\propto\; \lim_{R_\infty \to \infty} \frac{\delta S}{(Rt/l_4)^{\alpha_+} \delta c_1}\bigg|_{c_1 \to 0} \propto c_2 (t/l_4)^{\alpha_-}$$

The scaling dimension of $\mathscr{O}(x)$ is $4 + \alpha_+$.

□ Therefore we were studying time dependent mass term for quarks (thus SUSY is broken) and were studying the responses to it.

□ Since dS_4 is conformal to flat Minkowski, we act the conformal transformation (+ scale transformation to fields)

$$g_{M4} = \Omega^{-2} g_{dS4}, \qquad \Omega = \frac{l_4}{t}.$$

Then

$$\begin{aligned} c_1 (t/l_4)^{\alpha_+} \mathscr{O}(x) &\rightarrow \Omega^{-4} \Omega^{-\alpha_+} \Omega^{4+\alpha_+} \mathscr{O}(x) \\ \langle \mathscr{O}(x) \rangle \propto c_2 (t/l_4)^{\alpha_-} &\rightarrow \langle \Omega^{4+\alpha_+} \mathscr{O}(x) \rangle \propto c_2 \Omega^{-\alpha_-}, \qquad (\alpha_\pm = -2 \pm \sqrt{4+M^2}) \end{aligned}$$

and the action becomes,

$$\begin{aligned} \mathscr{L} &= \mathscr{L}_{dS4} + c_1 \mathscr{O}(x), \qquad S = \int \sqrt{g_{dS4}} \mathscr{L} \\ \langle \mathscr{O}(x) \rangle &\propto c_2 \end{aligned}$$

These coupling and the VEV are the exactly ones obtained from (A)dS/dS!

□ We studied D7 configuration which is static along dS_4.
In AdS/CFT picture, we were studying time dependent mass for quarks.
In (A)dS/dS picture, we were studying constant mass for quarks on dS_4.
And these two are related by a certain conformal transformation.

□ This is naive argument, but a conformal transformation from flat to dS_4 connect AdS/CFT with (A)dS/dS.

□ Summary

- We realize (A)dS/dS setup in a D3/D7 system where D7 is treated as a probe.
- We follow the AdS/CFT dictionary to compute the chiral condensate in CFT.
- The resultant chiral condensate is consistently understood as that on dS_4.
- Therefore our results give a support on (A)dS/dS.
- We speculated a conformal transformation relates AdS/CFT with (A)dS/dS.

□ Some future directions

- Construct a model much closer to QCD
- Introduce chemical potential and study phase diagram
  Apply to physics of early universe
- study quantum theory on dS_4 in more detail. (α vacua)

- phenomenological application.

  We realized dS_4 space from string theory and the localized gravity in D3/D7 system has been discussed (Fitzpatrick, Randall '05).

Thank you!