超对称粒子物理理论

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Standard Model :

$SU(3)_c \times SU(2)_L \times U(1)_Y$

$g \qquad W_\pm ,\ Z_0 ,\ \gamma$

$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad e_R \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R \quad d_R$

$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \mu_R \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad c_R \quad s_R$

$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \tau_R \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad t_R \quad b_R$

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h

Agree with experiments !

Questions :

1. fermion mass pattern ? mixings ? ~~CP~~ ?

2. Strong CP

I. Motivation

Standard Model:

Higgs mass:

$$m_h^2 = m_{h_0}^2 + \frac{\alpha_2}{4\pi}\Lambda^2$$

New physics scale

$\Downarrow$

unnaturalness (K. Wilson, 't Hooft 79's)

$\therefore$

GUT ? ?

SUSY can solve this problem (E. Witten, ...81's)

Extra dimensions (Arkani-Hamed et al., 98)

Assumption: matter — 4-dimension

gravity — $(4+n)$ D

Compactified (Newton's Law)

size-R

$r > R$: $G_N \dfrac{m_1 m_2}{r^2}$

$r < R$: $G_N' \dfrac{m_1 m_2}{r^{2+n}}$

$$\Downarrow$$

$$\frac{1}{M_{pl}^2} \sim G_N \sim \frac{G_N'}{R^n}$$

$$M_{pl}^2 = M^{2+n} R^n$$

O.K, if $M \sim$ TeV, $n \geq 2$, $R <$ milli-meter

— Large extra dimension

why? ($\Leftarrow$ String. M-theory)

II. SUSY

A symmetry which relates bosons and fermions;

A kind of space-time symmetry.

Poincare symmetry:

$$[P_m, P_n] = 0,$$

$$[P_m, J_{nr}] = i(P_n J_{rm} - P_r J_{nm}),$$

$$[J_{mn}, J_{rs}] = -i(\eta_{nr} J_{ms} - \eta_{mr} J_{ns} + \eta_{ns} J_{rm} - \eta_{ms} J_{rn}).$$

Introduce fermionic (new) generators Q and $\bar{Q}$,

$$[P_m, Q] = [P_m, \bar{Q}] = 0,$$

$$[J_{mn}, Q] = -i(\sigma)_{mn} Q, \quad [J, \bar{Q}] = i\bar{\sigma}\bar{Q},$$

$$\{Q \bar{Q}\} = 2\sigma_m P^m,$$

$$\{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0.$$

— $N=1$ SUSY (D. Volkov, ... 72's).

SUSY field theory:

Fields: $\{A(x), \psi(x)\}$ — a SUSY rep.

$$\mathcal{L} = i\,\partial\bar{\psi}\bar{\sigma}\psi + A^{*}\Box A - \tfrac{1}{2}m\psi\psi - \tfrac{1}{2}m\bar{\psi}\bar{\psi}$$

$$- g\psi\psi A - g\bar{\psi}\bar{\psi}A^{*} - V(A, A^{*})$$

scalar potential

In terms of superfield, w/ θ and $\bar{\theta}$ spinor like spacetime,

$$\phi \equiv A + \sqrt{2}\theta\psi + i\theta\sigma\bar{\theta}\,\partial_m A + \theta\theta F + \tfrac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^m\partial_m\psi$$

$$+ \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\Box A$$

$$\mathcal{L} = \phi^{\dagger}\phi\,\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \Big[\Big(\tfrac{1}{2}m\phi\phi + \tfrac{1}{3}g\phi\phi\phi\Big)\Big|_{\theta\theta} + \text{h.c.}\Big]$$

Important points

1. Perturbatively, $F\,(\theta\theta)$ terms have no renormalization.

2. Non-perturbative super-Yang-Mills can break SUSY.

$$m_{EW}^2 = M_{GUT}^2 \exp\left(-\frac{2\pi}{\alpha_{GUT}}\right)$$

(physics!)

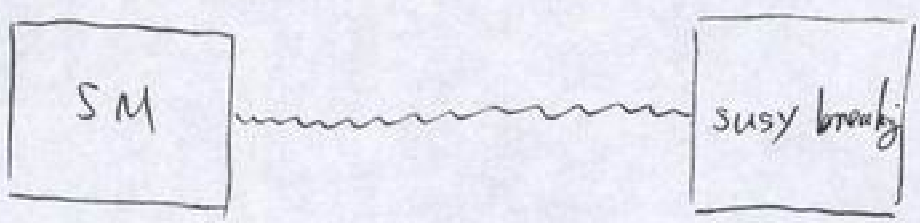
Things are not so simple.

susy extended SM models w/ spontaneous susy breaking:

$$m_{\tilde{q}} < m_q \quad \text{— ruled out}$$

$$\Downarrow$$

susy breaking occurs in a hidden sector, which is then mediated to SM (our) sector via loops or non-renormalizable interactions (like gravity).



Results of susy breaking:

masses of scalars and gauginos

— soft breaking

III. SUSY SM

1. There are SUSY partners of the SM particles,

$$(\gamma \quad \tilde{\gamma}), \quad (e, \tilde{e}), \cdots ;$$

2. Two Higgs doublets needed.

$$H_1, H_2 \quad (\text{anomaly cancellation and up-type quark mass});$$

3. Soft breaking masses assumption,

$$m_{\tilde{\gamma}} > 90 \text{ GeV}, \cdots (\text{low energy effective theory});$$

4. R-parity:

$$\theta \to -\theta .$$

$$(Q, u^c, d^c, L, e^c) \to -(Q, u^c, d^c, L, e^c).$$

$$(H_1, H_2) \to (H_1, H_2).$$

LSP and dark matter:

$$\tilde{\gamma}$$

Electroweak symmetry breaking:

$$\mathcal{L}^{susy} = \text{gauge} + \text{Yukawa} + \mu H_1 H_2$$

$$\mathcal{L}^{soft} = m_1^2 |h_1|^2 + m_2^2 |h_2|^2 - B\mu (h_1 h_2 + h.c.)$$

$$+ \text{other soft masses} + \cdots$$

$$\Downarrow$$

$$V^{Higgs} = (|\mu|^2 + m_1^2)|h_1|^2 + (|\mu|^2 + m_2^2)|h_2|^2 - \mu B (h_1 h_2 + h.c.)$$

$$+ \frac{g^2 + g'^2}{8}\left(|h_1|^2 - |h_2|^2\right)^2 + \frac{1}{2} g^2 \left|h_1^* h_2\right|^2$$

One requirement:

$$|\mu|^2 + m_1^2 < 0 \quad \text{and} \quad |\mu|^2 + m_2^2 > 0 .$$

Top quark mass $\Rightarrow$

$$m_1^2 = (m_1)_0^2 - \frac{6 y_t^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{m_{EW}^2}\right) m_{\tilde{t}}^2 .$$

(Note: top quark is not necessarily 175 GeV heavy.)

60 ok.

IV. Evidences

1. Gauge coupling unification
2. Neutrino masses
3. top mass
4. a_μ
5. ?

indirect

V. Problems

1. Why R-parity?

   Neutrino mass?

   Pati-Salam.

   SO(10) w/ $\underline{126}$ & w/o $\underline{16}$.

   Baryon parity.

2. μ problem

   Super-gravity?

3. SUSY flavor & CP problem

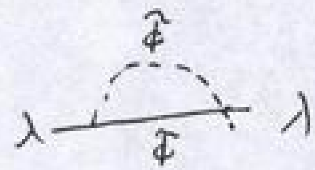
VI. How SUSY breaking is mediated.

via gravity (supergravity),

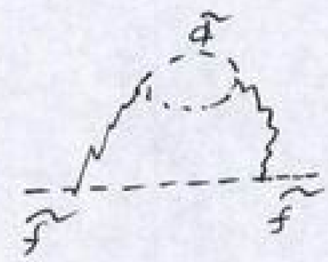
gauge interaction (GMSB).

e.g. GMSB: messengers: $\tilde{\Phi}$. $[\mathcal{L} = \tilde{\Phi}\tilde{\Phi}\bar{X}]$

gaugino masses from one-loop.



scalar masses from two-loop.



VII. How susy is broken?

$$\mathcal{L} = X^{+}X\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + c(X\Big|_{\theta\theta} + h.c.)$$

$\Rightarrow V = |c|^2 > 0$ — susy breaking.

Dynamical understanding:

super $SU(2)$ with one q in $\frac{3}{2}$.

VIII. Comments

1. Competitors :

$\left\{\begin{array}{l}\text{Technicolor} \\ \text{extra dimension}\end{array}\right.$ (Weinberg, L. Susskind, 79)
(98)

combination ?

2. String ?

3. Anthropic consideration

IX. Summary

Developments are driven by experiments and theoretical problems.

Electro-weak scale can be stablized by SUSY.

SUSY is beautiful, on the other hand, it has problems.

LHC as well as Tevatron experiments will verify low energy SUSY.

$\mathcal{N} = 1$ QCD

刘纪

Sept. 4, 2003. CCAST

Based on

[1] M.E. Peskin, Duality in Supersymmetric Yang-Mills Theory
hep-th/9702094.

[2] K. Intriligator and N. Seiberg,
Lectures on Supersymmetric Gauge Theories and Electromagnetic Duality
Nucl. Phys. B (Proc. Suppl.) 45B,C (1996)].

Chiral Superfields: Φ^i (quark)

Vector Superfields: V^a

Effective Lagrangian w/ at most two derivatives:

$$\mathcal{L} = \int d^4\theta \; K(\Phi^\dagger, e^{V\cdot t}\Phi) + \left(\frac{-i}{16\pi}\right)\int d^2\theta \; \tau(\Phi)\, W^{\alpha a} W_{\alpha a} + h.c.$$

$$+ \int d^2\theta \; W(\Phi) + h.c.$$

K — kinetic terms

W — superpotential

perturbative calculation $\Rightarrow$ no renormalization

SUSY $\Rightarrow$ $\left\{ \begin{array}{l} W \\ \tau \end{array} \right.$ holomorphic (解析) of Φ (nonperturbative)

$$\tau = \frac{\theta}{2\pi} + i\,\frac{4\pi}{g^2}$$

$\theta \int F\tilde{F}$ $\qquad$ $\frac{1}{4}\int FF$

τ obtains one-loop corrections only.

V:

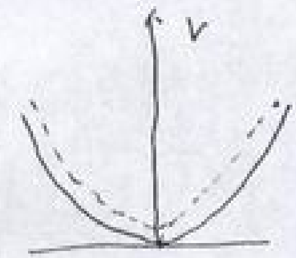
$$V^{+}(x,\theta,\bar\theta) = V(x,\theta,\bar\theta),$$

Wess-Zumino gauge:

$$V = -\theta\sigma^{m}\bar\theta\, V_m(x) + i\theta\theta\bar\theta\,\bar\lambda(x) - i\bar\theta\bar\theta\,\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\, D(x)$$

$$W_\alpha \equiv -\frac{1}{4}(\bar D\bar D)D_\alpha V$$

Vacuum :



$$V = F_i^\dagger F_i + \frac{1}{2} g^2 D^a D^a \geq 0$$

$$\cancel{\text{SUSY}} \iff \langle F \rangle \neq 0 \text{ or } \langle D^a \rangle \neq 0$$

R-Symmetry:

$$\theta \to e^{-i\alpha} \theta$$

$$\text{or: } \psi \to e^{-i\alpha} \psi, \quad \lambda \to e^{i\alpha} \lambda$$

$K,\ \mathcal{W}\mathcal{W}$ — no change

$$W \to e^{2i\alpha} W$$

Gaugino Condensate :

$\langle \lambda \lambda \rangle$ — scalar component of $W^{\alpha a} W_{\alpha a}$

$$\langle \lambda \lambda \rangle = 16\pi \frac{\partial}{\partial F_\tau} \log Z, \quad Z = \int e^{i \int \mathcal{L}}$$

$$= 16\pi i \frac{\partial}{\partial F_\tau} \int d^2\theta\, W_{eff}(\tau, \phi)$$

$$= 16\pi i \frac{\partial}{\partial \tau} W_{eff}(\tau, \phi)$$

SQCD:

$Q_i\ ,\ \bar{Q}_i \qquad i = 1 - N_f$

$$\mathcal{L} = \left(Q_i^+ e^V Q_i + \bar{Q}_i^+ e^V \bar{Q}_i \right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{-i}{16\pi} \tau\, W^{\alpha a} W_{\alpha}^{a} \Big|_{\theta\theta} + h.c.$$

perturbation:

$$\beta(g) = -\frac{b_0}{(4\pi)^2} g^3,$$

$$b_0 = 3N_c - N_f \qquad \left(QCD: \frac{11 N_c}{3} - \frac{2 N_f}{3} \right)$$

$$\frac{4\pi}{g^2}(Q) = \frac{4\pi}{g^2} - \frac{3N_c - N_f}{2\pi} \log \frac{M}{Q}$$

$$\Lambda^{b_0} = M^{b_0} e^{-8\pi^2/g^2} = M^{b_0} e^{2\pi i \tau}$$

global symmetry:

$$SU(N_f) \times SU(N_f) \times U_B(1) \times \underbrace{U(1)}_{\text{combination of } U(1)_A \text{ and } U(1)_R}$$

generally a chiral rotation

$$\psi \to e^{i\alpha} \psi$$

the θ parameter of YM: $\theta \to \theta - n\alpha$

n { 1 $\qquad N_c$ or $\bar{N}_c$ of $SU(N_c)$

1. $N_f = 0$

$$\mathcal{L} = -\frac{1}{4g^2}(F^a_{\mu\nu})^2 + \frac{1}{g^2}\bar{\lambda}^a i\not{D}\lambda_a + \frac{i\theta}{32\pi^2}F\tilde{F}$$

like QCD, λ can have a mass.

confinement and chiral symmetry breaking ✓

No anomaly-free $U(1)$, but a discrete symmetry remains:

$$\theta \to \theta + 2N_c\alpha, \quad \text{or} \quad \tau \to \tau + \frac{2N_c}{2\pi}\alpha$$

$$\alpha = 2\pi\frac{n}{2N_c}, \quad n = 0, 1, \cdots 2N_c - 1$$

$$\Downarrow$$

$$Z_{2N_c}$$

Effective theory: no gg, $g\lambda$, ...,

only τ as background field.

$$W \to e^{2i\alpha} W$$

$$W_{eff} = cM^3 e^{2\pi i\tau/N_c}$$

$$\langle\lambda\lambda\rangle = -\frac{32\pi^2}{N_c} cM^3 e^{-8\pi^2/N_c g^2}$$ — non-perturbative

$\downarrow$

$Z_{2N_c} \Rightarrow Z_2$ spontaneously

2.
$0 < N_f < N_c$ (Affleck-Dine-Seiberg)

$$A: \ \psi_Q \to e^{i\alpha}\psi_Q \ , \ \psi_{\bar{Q}} \to e^{i\alpha}\psi_{\bar{Q}} \ , \ \theta \to \theta + 2N_f\alpha$$

$$R: \ \psi_Q \to e^{-i\alpha}\psi_Q \ , \ \psi_{\bar{Q}} \to e^{-i\alpha}\psi_{\bar{Q}} \ , \ \lambda \to e^{i\alpha}\lambda \ , \ \theta \to \theta + (2N_c - 2N_f)\alpha$$

$$\Downarrow$$

$$R_{AF} = R + \frac{N_f - N_c}{N_f} A$$

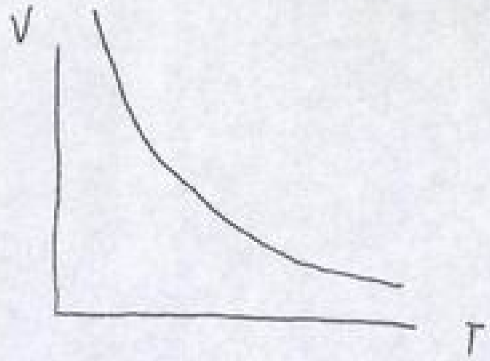
	B	A	R	R_{AF}
Q_i	+1	+1	0	$(N_f - N_c)/N_f$
ψ_{Q_i}	+1	+1	-1	$-N_c/N_f$
$\bar{Q}_i$	-1	+1	0	$(N_f - N_c)/N_f$
$\psi_{\bar{Q}_i}$	-1	+1	-1	$-N_c/N_f$
λ	0	0	+1	+1

Effective theory : superfield: $T_{ij} = Q_i \cdot \bar{Q}_j$ (— meson)

$SU(N_f) \times SU(N_f)$ invariance $\Rightarrow$ $\det T_{ij}$

	B	A	R	R_{AF}
$\det T$	0	$2N_f$	0	$2(N_f - N_c)$
Λ^{b_0}	0	$2N_f$	$2(N_c - N_f)$	0

$$W_{eff} = c\left(\frac{\Lambda^{b_0}}{\det T}\right)^{1/(N_c-N_f)}$$



A consistency check:

$$\Delta W = m\, Q_{N_f} \overline{Q_{N_f}} = m\, T_{N_f N_f}$$

$$W = c\left(\frac{\Lambda^{b_0}}{\det T}\right)^{\frac{1}{N_c-N_f}} + m\, T_{N_f N_f}$$

write $T = \begin{pmatrix} \tilde{T} & T_{iN_f} \\ T_{N_f i} & T_{N_f N_f} \end{pmatrix}$

F-flatness $\Rightarrow$ $T_{iN_f} = T_{N_f i} = 0$

$$-\frac{c}{N_c-N_f}\left(\frac{\Lambda^{b_0}}{\det \tilde{T}}\right)^{1/N_c-N_f}\left(\frac{1}{T_{N_f N_f}}\right)^{1+1/N_c-N_f} + m = 0$$

$$W \sim \left(\frac{m\Lambda^{b_0}}{\det \tilde{T}}\right)^{1/(N_c-N_f+1)}$$

3. $N_f = N_c - 1$

$$W_{eff} = c \frac{\Lambda^{b_0}}{\det T}$$ — checked by instanton calculation

A physical picture for quark condensate:

$$\Delta \mathcal{L} = -m^2 \left(|Q|^2 + |\bar{Q}|^2 \right)$$

$\langle T_{ij} \rangle = A \delta_{ij}$ — finite

$m^2 \nearrow$, $\langle T \rangle \searrow$

$\langle \psi_Q \psi_{\bar{Q}} \rangle = \langle F_T \rangle \nearrow$ (SUSY larger)

4. $N_f = N_c$

previous superpotential does not work

new feature: baryons

$$B = \epsilon_{a_1 \cdots a_{N_c}} Q_1^{a_1} \cdots Q_{N_c}^{a_{N_c}}$$

$$\bar{B} = \epsilon \cdots \bar{Q} \cdots \bar{Q}$$

classically,

$$\det T = B\bar{B}$$

Quantum effects:

$$\det T - B\bar{B} = \Lambda^{2N_c}$$ — a manifold of degenerate vacuum states

no $T = B = \bar{B} = 0$, Confinement & ~~Chiral symmetry~~

consistency check:

$$W = m\, T_{N_f N_f}$$

write $T = \begin{pmatrix} \tilde{T} & T_{N_f i} \\ T_{i N_f} & T_{N_f N_f} \end{pmatrix}$,

choose $B = \bar{B} = T_{N_f i} = 0$

The constraint $\Rightarrow$ $\det \tilde{T} \cdot t = \Lambda^{2N_c}$

$$W = \frac{m\,\Lambda^{2N_c}}{\det \tilde{T}}$$

$N_f = N_c + 1$

$$B_i = \epsilon_{i\, j_1 \cdots j_{N_c}} \epsilon_{a_1 \cdots a_{N_c}} Q^{a_1}_{j_1} \cdots Q^{a_{N_c}}_{j_{N_c}}$$

$$\bar{B}_i = \ldots$$

$$W = \frac{1}{\Lambda^{b_0}} \left(\det T - B_i T^{ij} \bar{B}_j \right)$$

$N_f > N_c + 1$

T^{ij} $\quad B_{ij\cdots k}$ $\quad \bar{B}_{ij\cdots k}$

$$W \sim \left(\det T - B_{ij\cdots k} T^{i\bar{i}} T^{j\bar{j}} \cdots T^{k\bar{k}} \bar{B}_{\bar{i}\bar{j}\cdots\bar{k}} \right)$$

does not work

Seiberg's dual QCD

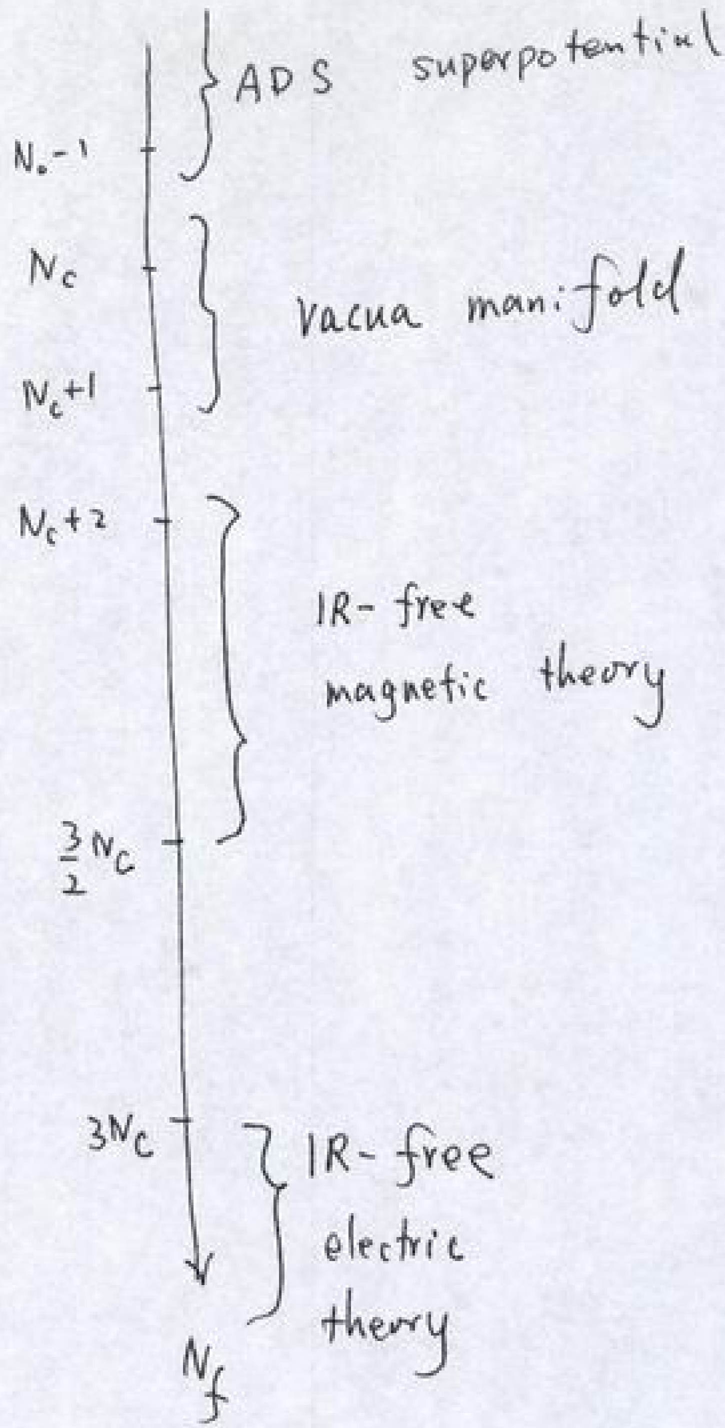
$\tilde{N}_c \equiv N_f - N_c$

$B_{ij\cdots k} = \epsilon_{a_1 \cdots a_{\tilde{N}_c}} q_i^{a_1} q_j^{a_2} \cdots q_k^{a_{\tilde{N}_c}}$

$\bar{B}$ $\quad \cdots$

$SU(\tilde{N}_c)$. q_i , $\bar{q}_i$ $\quad i = 1, \cdots, N_f$

$W = q T \bar{q}$


ADS superpotential
N_c-1
N_c
vacua manifold
N_c+1
N_c+2
IR-free
magnetic theory
$\frac{3}{2}N_c$
$3N_c$
IR-free
electric
theory
N_f