Topological Strings and Condensed Matter Physics

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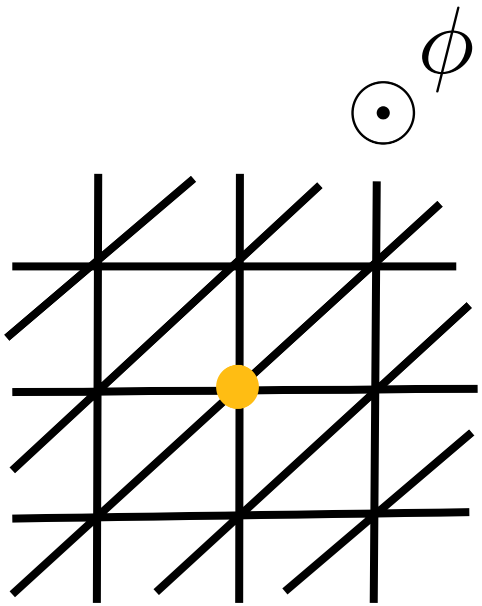
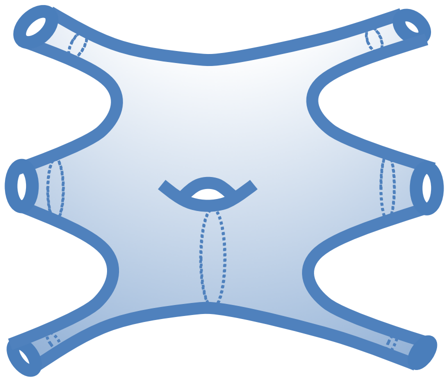
Message



Electron
on 2d lattice



Non-perturbative
Topological string

From Cond. Mat. Phys. to String

Contents

1. Introduction

Topological String (TS)

- Toy model of superstring

↑

capture some information of superstring on R4 × CY3

- Mirror symmetry

A-model TS on CY_A ⟺ B-model TS on CY_B

$$t = t(\mathcal{E})$$

Mirror map

t — Kahler moduli

$\mathcal{E}$ — Complex moduli

Topological String (TS)

- Toy model of superstring
- In non-compact toric CY,
  TS relates some fields

M theory
[Gopakumar, Vafa(1998)]

Quantum Mechanics
[Grassi, Hatsuda, Marino (2014)]

A model TS

B model TS

$$H(x, y; \mathcal{E}) = 0$$

Susy. gauge theory
[Hollowood, Iqbal, Vafa(2008)]

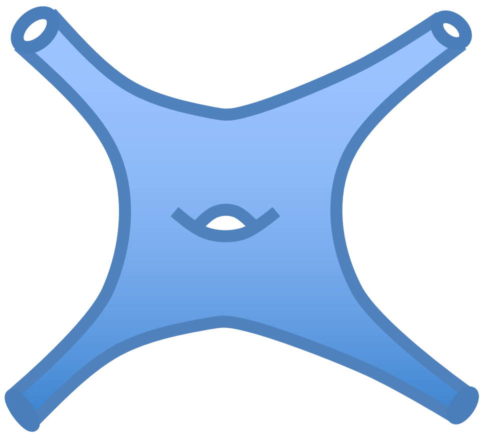
Cond. Mat. Phys.
My talk

2. (Non-perturbative) topological string

B-model TS

Toric CY : $uv + H(x, y; \mathcal{E}) = 0$

TS: encoded into the curve, $H(x, y; \mathcal{E}) = 0$



$$t(\mathcal{E}) = \oint_A \mathrm{d}x \; y(x; \mathcal{E})$$

$$\partial_t F_0(t) = \oint_B \mathrm{d}x \; y(x; \mathcal{E})$$

$$H = \mathrm{e}^{x} + \mathrm{e}^{y} + \mathrm{e}^{-x} + \mathrm{e}^{-y} - \mathcal{E}$$
$$= 0$$

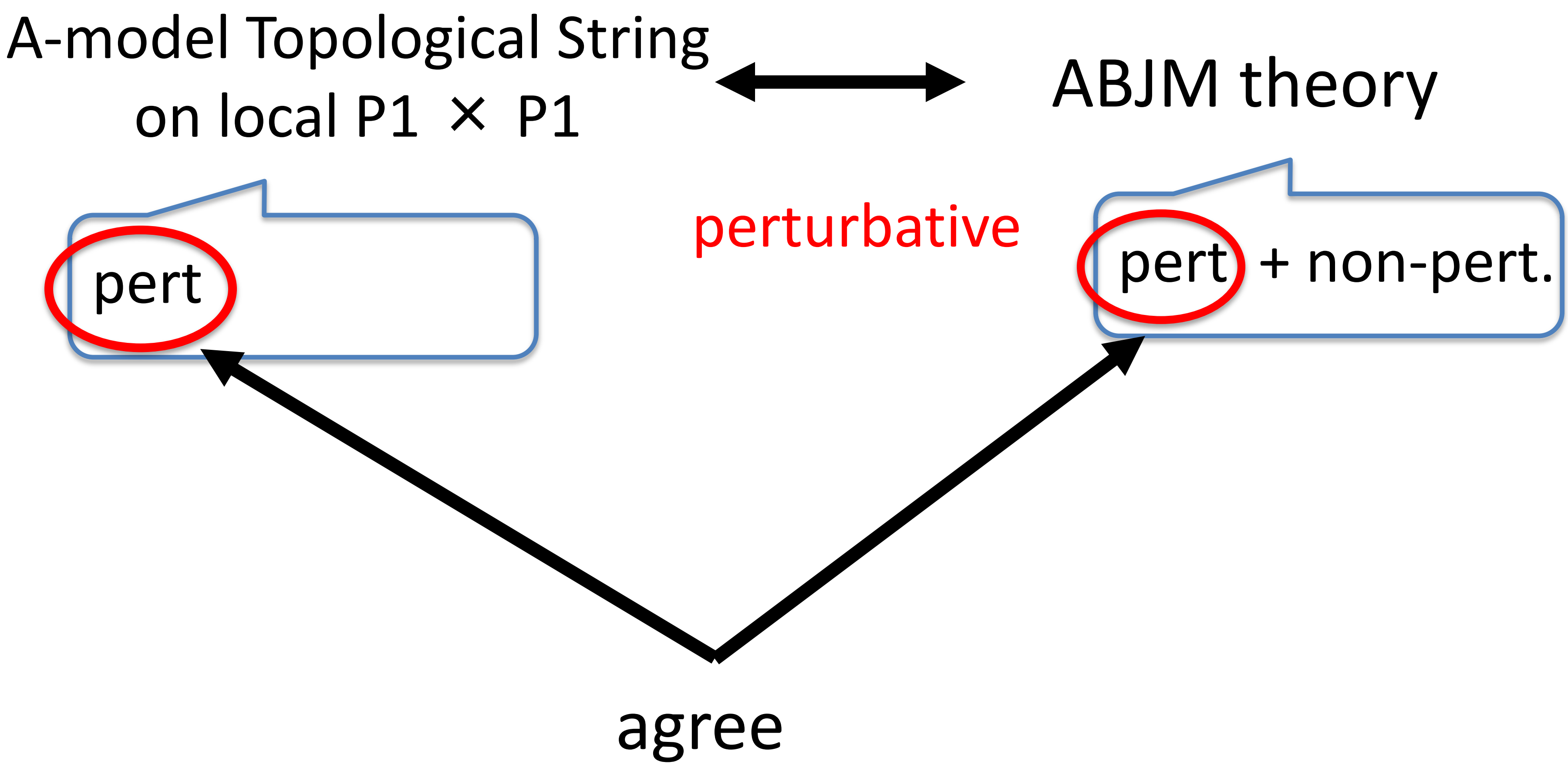
$$\left(F(t, g_s) = \sum_{g \geq 0} g_s^{2g-2} F_g(t) \right)$$

Local $P_1 \times P_1$

Non-perturbative ??

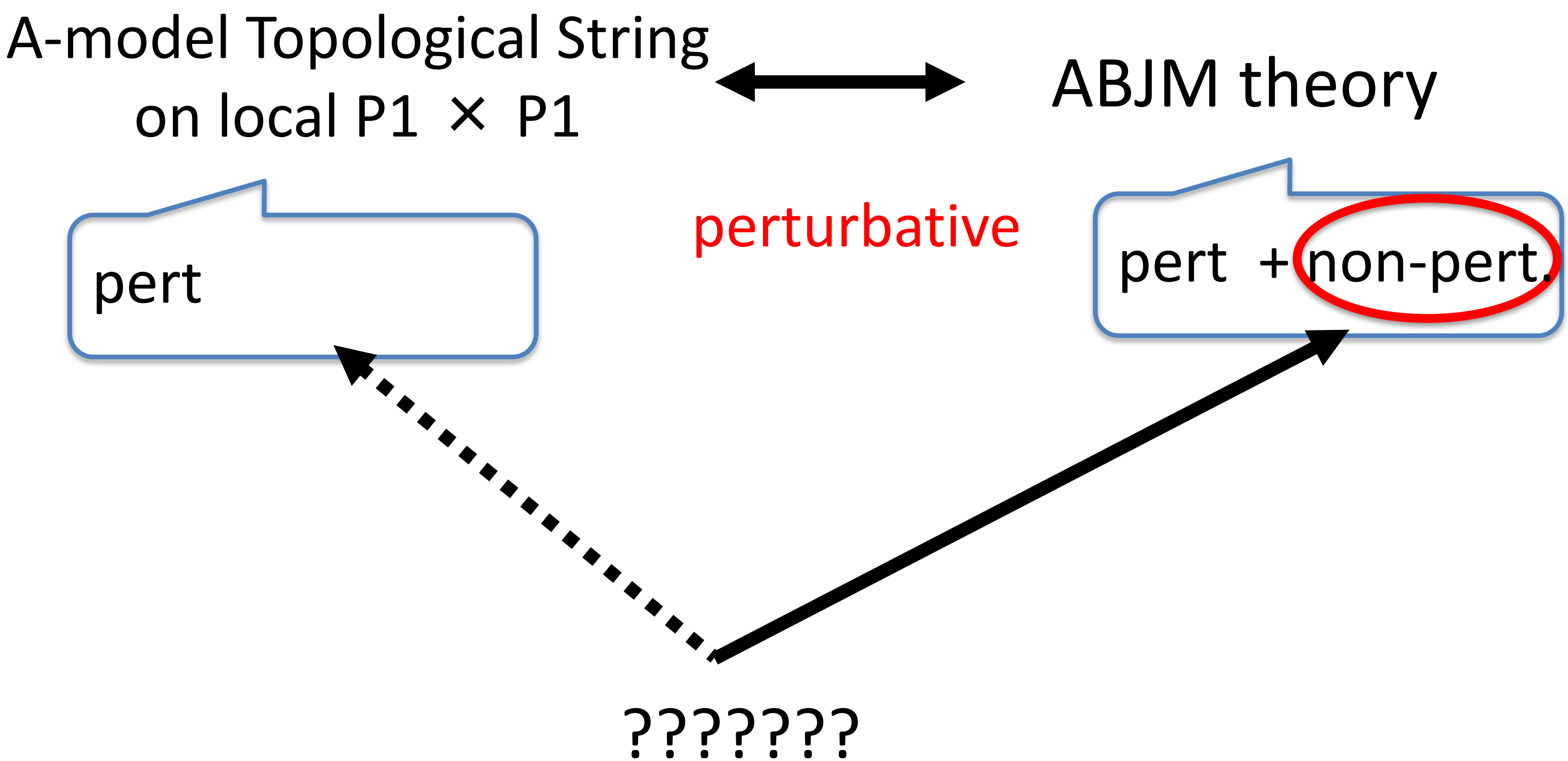
Non-perturbative ??

Hatsuda, Moriyama, Okuyama(2013)



Non-perturbative ??

Hatsuda, Moriyama, Okuyama(2013)



Non-perturbative ??

Hatsuda, Moriyama, Okuyama(2013)

A-model Topological String on local P1 × P1 ⟷ ABJM theory

pert

perturbative

pert + non-pert.

If agree non-perturbatively

$$F^{(\mathrm{n.p.})}(\boldsymbol{t}, g_s) = F(\boldsymbol{t}, g_s) + \frac{1}{2\pi \mathrm{i}} \frac{\partial}{\partial g_s}\left[g_s F_{\mathrm{NS}}\left(\frac{\boldsymbol{t}}{g_s}, \frac{1}{g_s} \right) \right]$$

$$F_{\mathrm{NS}}(\boldsymbol{t}, \hbar) = \sum_{j_L, j_R} \sum_{w, \boldsymbol{d}} N^{\boldsymbol{d}}_{j_L\ j_R} \frac{\sin\left(\frac{\hbar w}{2}(2j_L+1)\right)\sin\left(\frac{\hbar w}{2}(2j_R+1)\right)}{2w^2 \sin^3\left(\frac{\hbar w}{2}\right)} \mathrm{e}^{-w\boldsymbol{d}\cdot\boldsymbol{t}}$$

Non-perturbative completion of topological string

Non-perturbative ??

Hatsuda, Moriyama, Okuyama(2013)

A-model Topological String on local P1 × P1 ⟷ ABJM theory

$$Z(N,k)=\int \mathrm{d}^N x \sum_{\sigma\in S_N}\frac{(-1)^{\epsilon(\sigma)}}{N!}\prod_{i=1}^{N}\langle x_i|\rho(\mathsf{p},\mathsf{q})|x_{\sigma(i)}\rangle$$

$$\rho(\mathsf{p},\mathsf{q})=\frac{1}{\left(2\cosh\left[\frac{\mathsf{q}}{2}\right]\right)^{\frac{1}{2}}}\frac{1}{\left(2\cosh\left[\frac{\mathsf{p}}{2}\right]\right)}\frac{1}{\left(2\cosh\left[\frac{\mathsf{q}}{2}\right]\right)^{\frac{1}{2}}},\quad [\mathsf{q},\mathsf{p}]=2\pi\mathrm{i}k$$

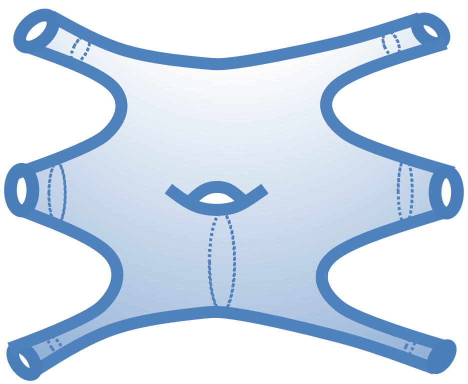
$$=\left(\mathrm{e}^{\mathsf{q}}+\mathrm{e}^{\mathsf{p}}+\mathrm{e}^{-\mathsf{q}}+\mathrm{e}^{-\mathsf{p}}\right)^{-1}$$

(Mirror curve)$^{-1}$ of local P1 × P1

➡ B-model

Non-perturbative from the curve ??

Aganagic, Cheng, Dijkgraaf, Krefl, Vafa(2012), Grassi, Hatsuda, Marino(2014)



$$H = \mathrm{e}^{x} + \mathrm{e}^{y} + \mathrm{e}^{-x-y} + \mathrm{e}^{-x} + \mathrm{e}^{-y} + \mathrm{e}^{-x-y} - \mathcal{E} = 0$$

Local B3

Quantization of the curve

$$\mathcal{E} = \mathrm{e}^{\mathsf{x}} + \mathrm{e}^{-\mathsf{x}} + \mathrm{e}^{\mathsf{y}} + \mathrm{e}^{-\mathsf{y}} + \mathrm{e}^{\mathsf{x}+\mathsf{y}} + \mathrm{e}^{-\mathsf{x}-\mathsf{y}}$$

$$[\mathsf{x}, \mathsf{y}] = \mathrm{i}\hbar \qquad \hbar = 4\pi^2/g_s$$

Quantum mirror map

$$t(\mathcal{E}, \hbar)$$

How to define $t(\mathcal{E},\hbar)$

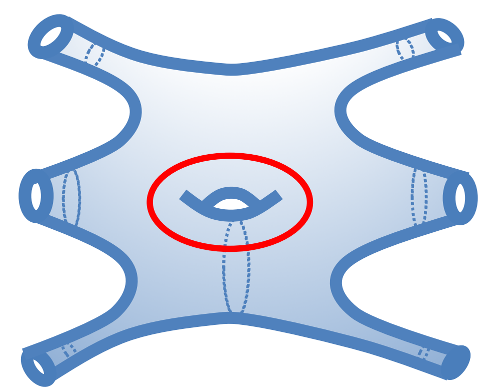
Aganagic, Cheng, Dijkgraaf, Krefl, Vafa(2012)

$$(\mathrm{e}^{\mathsf{x}}+\mathrm{e}^{\mathsf{y}}+\mathrm{e}^{-\mathsf{x}-\mathsf{y}}+\mathrm{e}^{-\mathsf{x}}+\mathrm{e}^{-\mathsf{y}}+\mathrm{e}^{-\mathsf{x}-\mathsf{y}}-\mathcal{E})\Psi(x)=0$$

$$\Psi(x,\hbar)=\exp\left[\frac{1}{\hbar}S(x,\hbar)\right],\quad S(x,\hbar)=\sum_{n=0}^{\infty}S_n(x)\hbar^n$$

$$t(\mathcal{E},\hbar)=\oint_A \partial S(x,\hbar)\mathrm{d}x$$

order by order

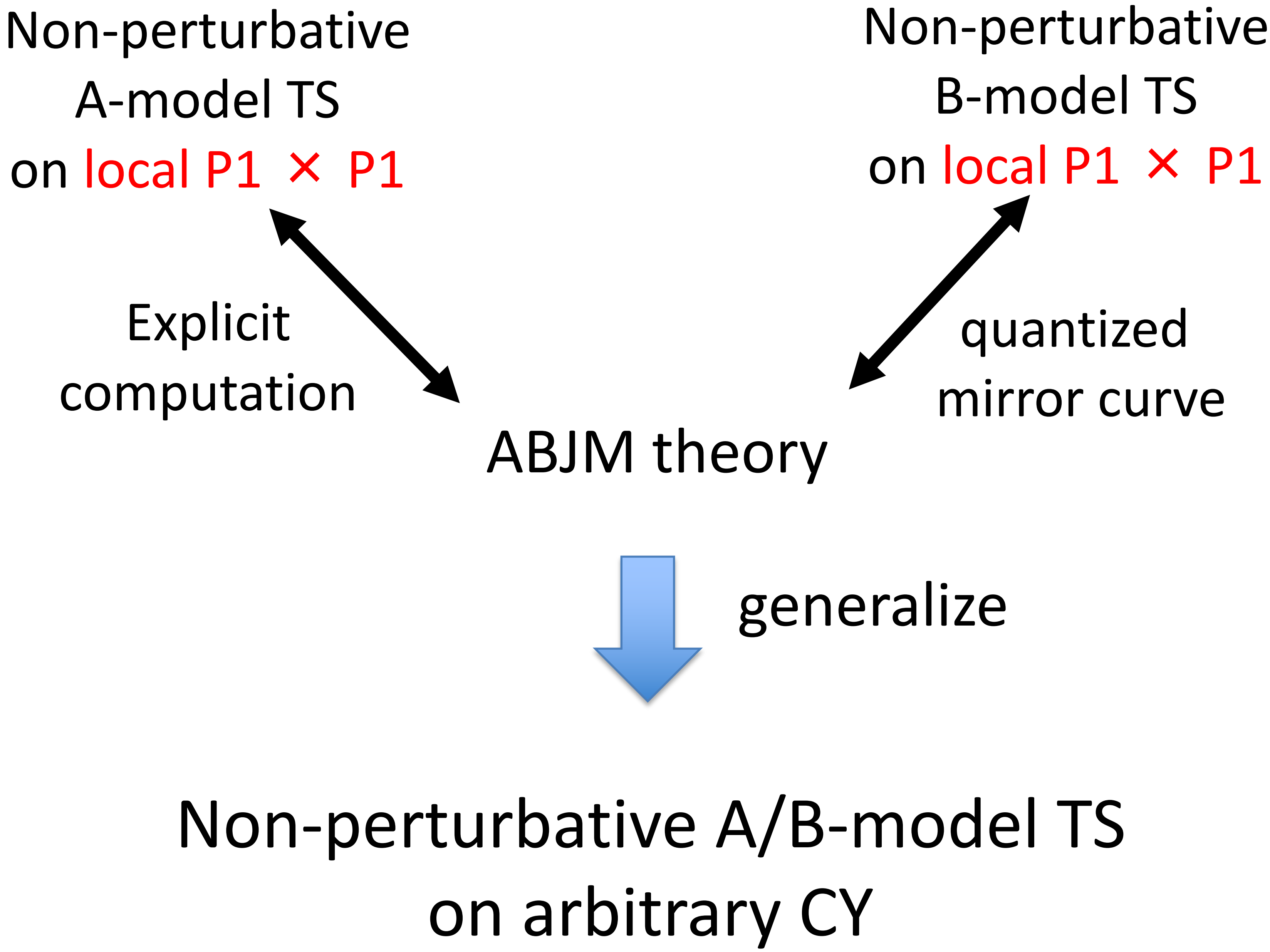


$$\mathrm{e}^{x}+\mathrm{e}^{y}+\mathrm{e}^{-x-y}+\mathrm{e}^{-x}$$
$$+\mathrm{e}^{-y}+\mathrm{e}^{-x-y}-\mathcal{E}=0$$

perturbative
A-model TS
on local P1 × P1

perturbatively

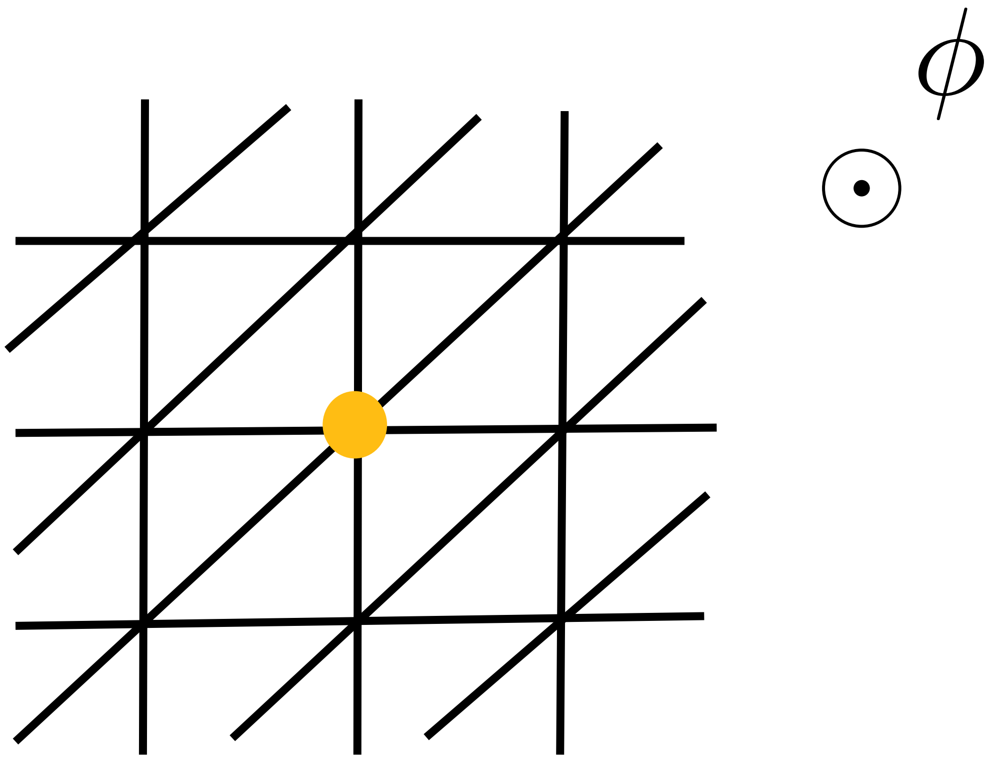
ABJM theory


Non-perturbative
A-model TS
on local P1 × P1
Non-perturbative
B-model TS
on local P1 × P1
Explicit
computation
quantized
mirror curve
ABJM theory
generalize
Non-perturbative A/B-model TS
on arbitrary CY

3. Relation

Cond. Mat. Phys.

Cond. Mat. Phys.



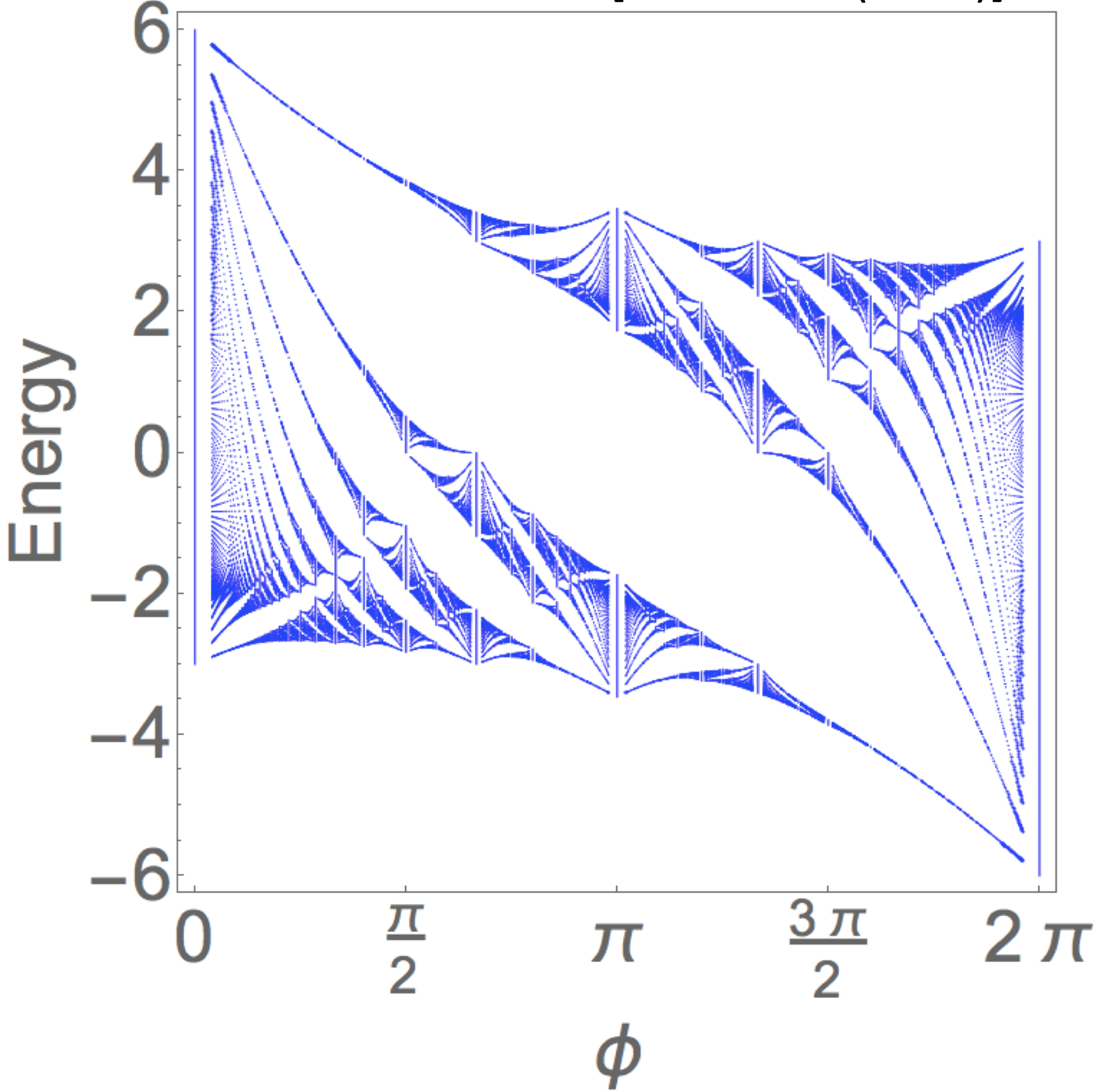
$$H_{\mathrm{tri.}} = T_x + T_x^\dagger + T_y + T_y^\dagger + \mathrm{e}^{-\frac{\mathrm{i}\phi}{2}} T_x T_y + \mathrm{e}^{\frac{\mathrm{i}\phi}{2}} T_y^\dagger T_x^\dagger$$

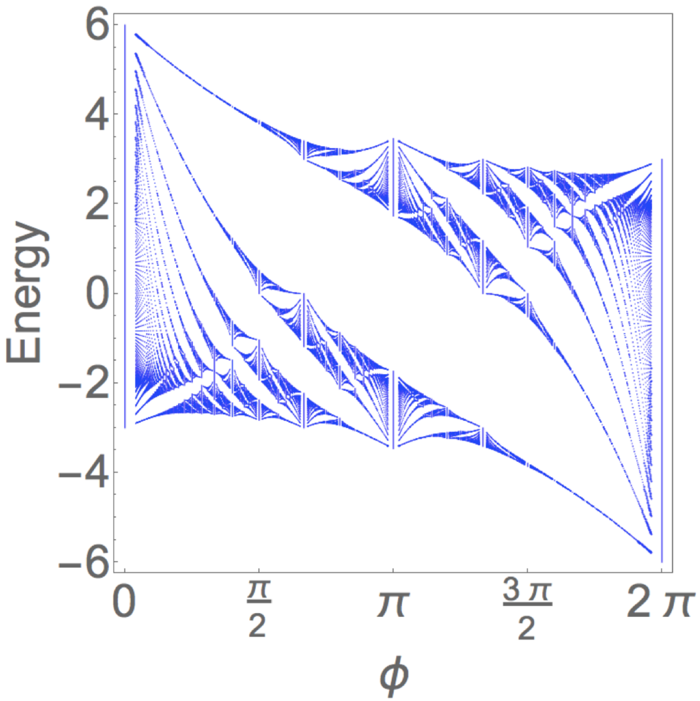
$$T_x T_y = \mathrm{e}^{\mathrm{i}\phi} T_y T_x$$

$$a, b \in \mathbb{Z}_{\geq 0}, b \neq 0$$

$$\phi = 2\pi \frac{a}{b}$$

[Hofstadter(1976)]





$$H_{\text{tri.}} = T_x + T_x^\dagger + T_y + T_y^\dagger + e^{-\frac{i\phi}{2}} T_x T_y + e^{\frac{i\phi}{2}} T_y^\dagger T_x^\dagger$$

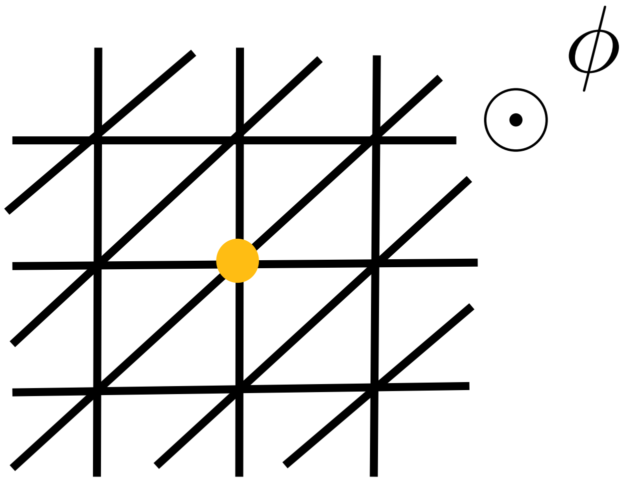
$$T_x T_y = e^{i\phi} T_y T_x$$

$$\phi = 2\pi \frac{a}{b} \qquad a, b \in \mathbb{Z}_{\geq 0}, b \neq 0$$

$$\det[D_{a/b}(E)] = 0$$

$$D_{a/b}(E) = \begin{pmatrix} A_1 - E & B_1 & 0 & \cdots & 0 & 0 & B_b^* e^{-ibk_y} \\ B_1^* & A_2 - E & B_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & B_{b-2}^* & A_{b-1} - E & B_{b-1} \\ B_b e^{ibk_y} & 0 & 0 & \cdots & 0 & B_{b-1}^* & A_b - E \end{pmatrix}$$

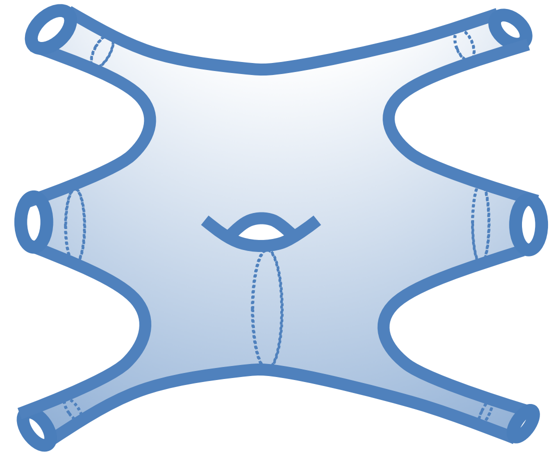
$$A_j = -2\cos(k_x + 2\pi\phi j), \ \tilde{B}_j = -e^{ik_y} B_j, \ B_j = 1 + e^{i(k_x + 2\pi\phi j)}$$



Electron
on 2d lattice

$$H_{\mathrm{tri.}} = T_x + T_x^\dagger + T_y + T_y^\dagger + \mathrm{e}^{-\frac{\mathrm{i}\phi}{2}} T_x T_y + \mathrm{e}^{\frac{\mathrm{i}\phi}{2}} T_y^\dagger T_x^\dagger$$

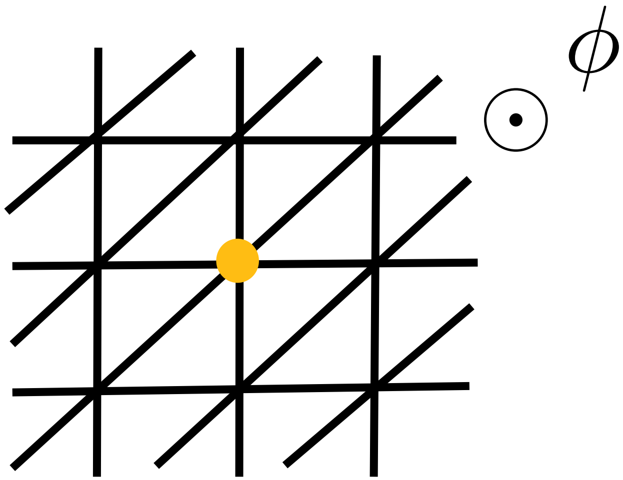
$$T_x T_y = \mathrm{e}^{\mathrm{i}\phi} T_y T_x$$



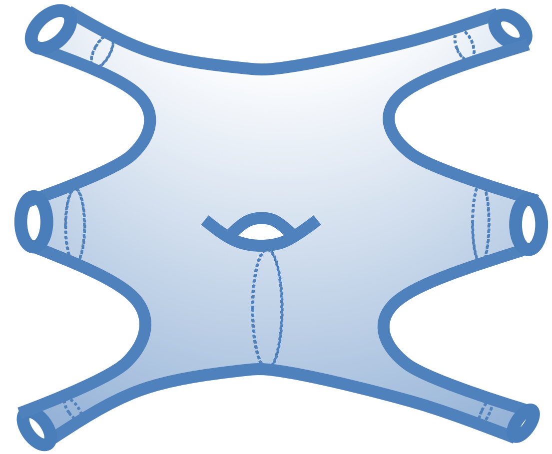
Non-perturbative
Topological string

$$\mathcal{E} = \mathrm{e}^{\mathsf{x}} + \mathrm{e}^{-\mathsf{x}} + \mathrm{e}^{\mathsf{y}} + \mathrm{e}^{-\mathsf{y}} + \mathrm{e}^{\mathsf{x}+\mathsf{y}} + \mathrm{e}^{-\mathsf{x}-\mathsf{y}}$$

$$[\mathsf{x}, \mathsf{y}] = \mathrm{i}\hbar$$



Electron
on 2d lattice



Non-perturbative
Topological string

$$H_{\text{tri.}} = T_x + T_x^\dagger + T_y + T_y^\dagger + \mathrm{e}^{-\frac{\mathrm{i}\phi}{2}} T_x T_y + \mathrm{e}^{\frac{\mathrm{i}\phi}{2}} T_y^\dagger T_x^\dagger$$

$$T_x T_y = \mathrm{e}^{\mathrm{i}\phi} T_y T_x$$

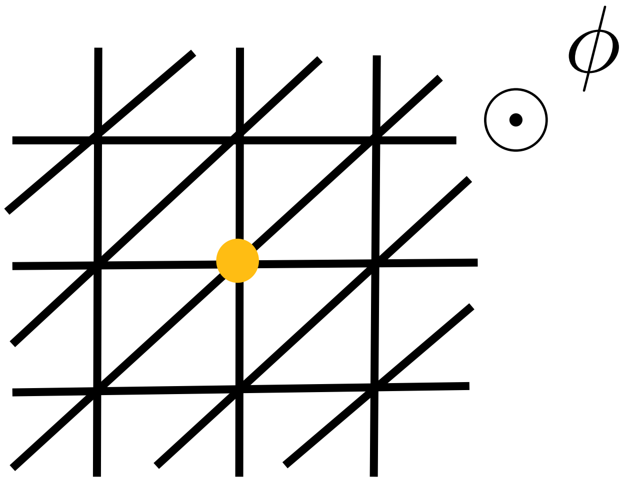
$$\phi = 2\pi \frac{a}{b} \qquad a, b \in \mathbb{Z}_{\geq 0}, b \neq 0$$

Known results

$$\mathcal{E} = \mathrm{e}^{\mathsf{x}} + \mathrm{e}^{-\mathsf{x}} + \mathrm{e}^{\mathsf{y}} + \mathrm{e}^{-\mathsf{y}} + \mathrm{e}^{\mathsf{x}+\mathsf{y}} + \mathrm{e}^{-\mathsf{x}-\mathsf{y}}$$

$$[\mathsf{x}, \mathsf{y}] = \mathrm{i}\hbar$$

Some implication ???



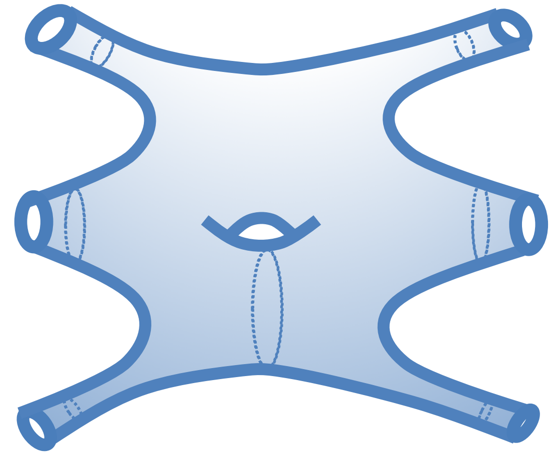
Electron
on 2d lattice

$$H_{\text{tri.}} = T_x + T_x^\dagger + T_y + T_y^\dagger + e^{-\frac{i\phi}{2}} T_x T_y + e^{\frac{i\phi}{2}} T_y^\dagger T_x^\dagger$$

$$T_x T_y = e^{i\phi} T_y T_x$$

$$\phi = 2\pi \frac{a}{b} \qquad a, b \in \mathbb{Z}_{\geq 0}, b \neq 0$$

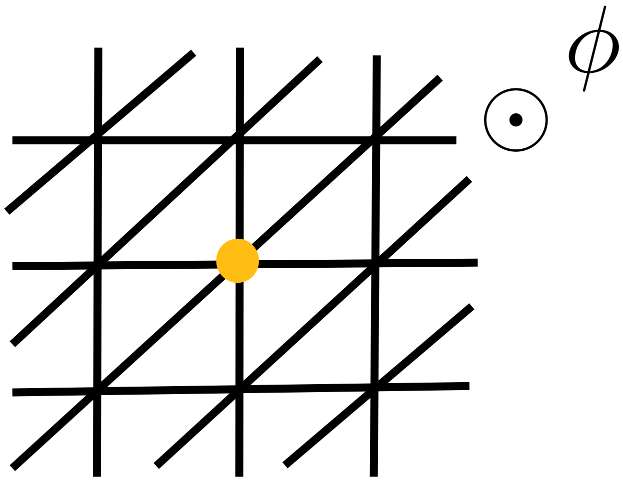
Known results



Non-perturbative
Topological string

when $\hbar = 2\pi \frac{a}{b}$

obtain $t(\mathcal{E}, \hbar)$
in closed form



Electron
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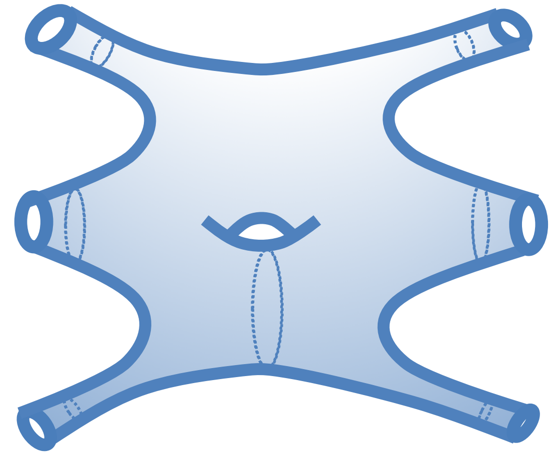
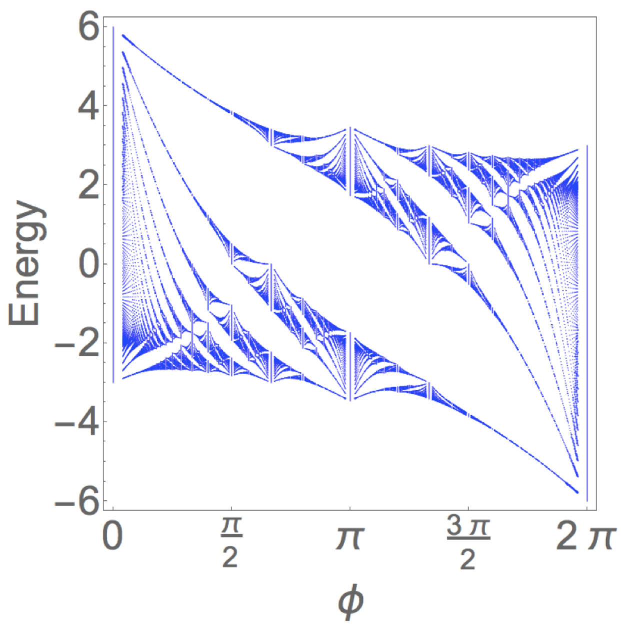


Non-perturbative
Topological string



⟷

$$\frac{1}{2\pi}\mathrm{Im}\left[\frac{\partial t(\mathcal{E},\hbar=2\pi a/b)}{\partial \mathcal{E}}\right]$$

4. Calculation detail

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

$$\frac{1}{\hbar}\oint_{B} \partial_x S(x,\hbar)\mathrm{d}x = 2\pi\left(n+\frac{1}{2}\right)$$

$$\Psi(x,\hbar) = \exp\left[\frac{1}{\hbar}S(x,\hbar)\right], \quad S(x,\hbar) = \sum_{n=0}^{\infty} S_n(x)\hbar^n$$

J. Dunham(1932)

Quantization condition

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

$$\underline{\partial_t F_{\mathrm{NS}}(t,\hbar)} = 2\pi\left(n + \frac{1}{2}\right)$$

Diverge at

Nekrasov-Shatashvili(2009)

$$\hbar = 2\pi a/b$$

$$F_{\mathrm{NS}}(\boldsymbol{t},\hbar) = \sum_{j_L, j_R} \sum_{w, \boldsymbol{d}} N^{\boldsymbol{d}}_{j_L\ j_R} \frac{\sin\left(\frac{\hbar w}{2}(2j_L+1)\right)\sin\left(\frac{\hbar w}{2}(2j_R+1)\right)}{2w^2 \sin^3\left(\frac{\hbar w}{2}\right)} \mathrm{e}^{-w\boldsymbol{d}\cdot\boldsymbol{t}}$$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

conjecture

$$\partial_t F_{\mathrm{NS}}(t;\hbar) + \partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar}) = 2\pi\left(n+\frac{1}{2}\right)$$

Wang, Zhang, Huang(2015)

$$\tilde{t} = \frac{2\pi}{\hbar}t,\ \ \tilde{\hbar} = \frac{4\pi^2}{\hbar}$$

$$F_{\mathrm{NS}}(\boldsymbol{t},\hbar) = \sum_{j_L,j_R}\sum_{w,\boldsymbol{d}} N^{\boldsymbol{d}}_{j_L\ j_R} \frac{\sin\left(\frac{\hbar w}{2}(2j_L+1)\right)\sin\left(\frac{\hbar w}{2}(2j_R+1)\right)}{2w^2\sin^3\left(\frac{\hbar w}{2}\right)}\mathrm{e}^{-w\boldsymbol{d}\cdot\boldsymbol{t}}$$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

conjecture

$$\partial_t F_{\mathrm{NS}}(t;\hbar) + \partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar}) = 2\pi\left(n+\frac{1}{2}\right)$$

Invariant under $t \leftrightarrow \tilde{t},\ \hbar \leftrightarrow \tilde{\hbar}$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

$$\underline{\partial_t F_{\mathrm{NS}}(t;\hbar)} + \partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar}) = 2\pi\left(n+\frac{1}{2}\right)$$

Mirror curve parametrized by $(\hbar, \mathcal{E})$

$$t(\mathcal{E},\hbar) = \oint_A \partial_x S(x,\hbar)\mathrm{d}x, \;\; \Psi(x,\hbar) = \exp\left[\frac{1}{\hbar} S(x,\hbar)\right]$$

mirror curve & mirror map

$$\tilde{\mathcal{E}} = \mathrm{e}^{\tilde{x}} + \mathrm{e}^{-\tilde{x}} + \mathrm{e}^{\tilde{y}} + \mathrm{e}^{-\tilde{y}} + \mathrm{e}^{\tilde{x}+\tilde{y}} + \mathrm{e}^{-\tilde{x}-\tilde{y}}$$

$$\partial_t F_{\mathrm{NS}}(t;\hbar) + \underline{\partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar})} = 2\pi\left(n + \frac{1}{2}\right)$$

Mirror curve parametrized by $(\tilde{\hbar}, \tilde{\mathcal{E}})$

$$t(\tilde{\mathcal{E}}, \tilde{\hbar}) = \oint_A \partial_x S(x, \tilde{\hbar})\mathrm{d}x, \quad \Psi(\tilde{x}, \tilde{\hbar}) = \exp\left[\frac{1}{\tilde{\hbar}} S(\tilde{x}, \tilde{\hbar})\right]$$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

$$\tilde{\mathcal{E}} = \mathrm{e}^{\tilde{x}} + \mathrm{e}^{-\tilde{x}} + \mathrm{e}^{\tilde{y}} + \mathrm{e}^{-\tilde{y}} + \mathrm{e}^{\tilde{x}+\tilde{y}} + \mathrm{e}^{-\tilde{x}-\tilde{y}}$$

$$\underline{\partial_t F_{\mathrm{NS}}(t;\hbar)} + \underline{\partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar})} = 2\pi\left(n + \frac{1}{2}\right)$$

$(\hbar, \mathcal{E})$ -mirror curve

$$t = t(\mathcal{E}, \hbar)$$

$(\tilde{\hbar}, \tilde{\mathcal{E}})$ -mirror curve

$$\tilde{t} = t(\tilde{\mathcal{E}}, \tilde{\hbar})$$

$$\tilde{t} = \frac{2\pi}{\hbar} t$$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$

$$\tilde{\mathcal{E}} = \mathrm{e}^{\tilde{x}} + \mathrm{e}^{-\tilde{x}} + \mathrm{e}^{\tilde{y}} + \mathrm{e}^{-\tilde{y}} + \mathrm{e}^{\tilde{x}+\tilde{y}} + \mathrm{e}^{-\tilde{x}-\tilde{y}}$$

$$\partial_t F_{\mathrm{NS}}(t;\hbar) + \partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar}) = 2\pi\left(n + \frac{1}{2}\right)$$

$$t(\tilde{\mathcal{E}},\tilde{\hbar}) = \frac{2\pi}{\hbar} t(\mathcal{E},\hbar), \;\; \tilde{\hbar} = \frac{4\pi^2}{\hbar}$$

mirror curve & mirror map

$$\mathcal{E} = \mathrm{e}^{x} + \mathrm{e}^{-x} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{x+y} + \mathrm{e}^{-x-y}$$
$$\tilde{\mathcal{E}} = \mathrm{e}^{\tilde{x}} + \mathrm{e}^{-\tilde{x}} + \mathrm{e}^{\tilde{y}} + \mathrm{e}^{-\tilde{y}} + \mathrm{e}^{\tilde{x}+\tilde{y}} + \mathrm{e}^{-\tilde{x}-\tilde{y}}$$

$$\partial_t F_{\mathrm{NS}}(t;\hbar) + \partial_{\tilde{t}} F_{\mathrm{NS}}(\tilde{t};\tilde{\hbar}) = 2\pi\left(n + \frac{1}{2}\right)$$

$$t(\tilde{\mathcal{E}},\tilde{\hbar}) = \frac{2\pi}{\hbar} t(\mathcal{E},\hbar), \ \ \tilde{\hbar} = \frac{4\pi^2}{\hbar}$$
$$t(\mathcal{E},\hbar) = t(\mathcal{E},\hbar \pm 4\pi)$$

mirror curve & mirror map

$$t(\tilde{\mathcal{E}}, \tilde{\hbar}) = \frac{2\pi}{\hbar} t(\mathcal{E}, \hbar), \ \tilde{\hbar} = \frac{4\pi^2}{\hbar}$$

$$t(\mathcal{E}, \hbar) = t(\mathcal{E}, \hbar \pm 4\pi)$$

mirror curve & mirror map

$$t(\mathcal{E}, 2\pi a/b) = \frac{a}{b} t(\tilde{\mathcal{E}}, 2\pi b/a) \quad \text{S}$$

$$t(\mathcal{E}, 2\pi a/b) = t(\mathcal{E}, 2\pi(a/b \pm 2)) \quad \text{T}$$

$$\hbar = 2\pi \frac{a}{b}$$

e.g.)a=2, b=5

classical

$$2\pi \cdot \frac{2}{5} \to 2\pi \cdot \frac{5}{2} \to 2\pi \cdot \frac{1}{2} \to 2\pi \cdot 2 \to 0$$

S T S T

mirror curve & mirror map

$$t(\mathcal{E}, 2\pi a/b) = \frac{a}{b} t(\tilde{\mathcal{E}}, 2\pi b/a)$$ S

$$t(\mathcal{E}, 2\pi a/b) = t(\mathcal{E}, 2\pi(a/b \pm 2))$$ T

$$\hbar = 2\pi \frac{a}{b}$$

$$t(\mathcal{E}, 2\pi a/b) = \frac{1}{b} t(\tilde{\mathcal{E}}, 0) \quad ab \in 2\mathbb{Z}$$

$\tilde{\mathcal{E}} = \mathcal{F}_{a/b}(\mathcal{E})$: polynomial of degree-b

Mirror map

$$t(\mathcal{E}, 2\pi a/b) = \frac{1}{b}\tilde{t}(\tilde{\mathcal{E}}, 0) \qquad ab \in 2\mathbb{Z}$$

$$\tilde{\mathcal{E}} = \mathcal{F}_{a/b}(\mathcal{E})$$

$$\mathcal{F}_{a/b}(\mathcal{E}) = \mathcal{D}_{a/b}(\mathcal{E}) + 2\left\{1 + (-1)^{(a-1)b} + (-1)^b\right\}$$

$$\mathcal{D}_{a/b}(\mathcal{E}) = \det \begin{pmatrix} \mathcal{A}_1 + \mathcal{E} & B_1 & 0 & \cdots & 0 & 0 & \mathcal{B}_b^* \\ \mathcal{B}_1^* & \mathcal{A}_2 + \mathcal{E} & \mathcal{B}_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathcal{B}_{b-2}^* & \mathcal{A}_{b-1} + \mathcal{E} & \mathcal{B}_{b-1} \\ \mathcal{B}_b & 0 & 0 & \cdots & 0 & \mathcal{B}_{b-1}^* & \mathcal{A}_b + \mathcal{E}' \end{pmatrix}$$

$$\mathcal{A}_j = 2\cos\left(\frac{2\pi a j}{b}\right), \ \mathcal{B}_j = -1 + \mathrm{e}^{\frac{2\pi \mathrm{i} a}{b}}$$

Mirror map

$$\frac{\partial t(\mathcal{E};2\pi a/b)}{\partial \mathcal{E}} =$$

$$\frac{2\mathcal{F}'(\mathcal{E})}{\pi b\sqrt{\mathcal{F}^2(\mathcal{E})-12+8\sqrt{3+\mathcal{F}(\mathcal{E})}}}\mathbb{K}\left(\frac{16\sqrt{3+\mathcal{F}(\mathcal{E})}}{\mathcal{F}^2(\mathcal{E})-12+8\sqrt{3+\mathcal{F}(\mathcal{E})}}\right),$$

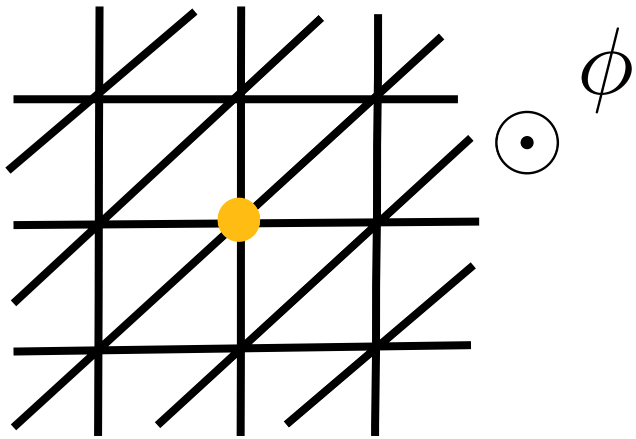
$$\mathcal{F}(\mathcal{E}) = (-1)^{ab}\mathcal{F}_{a/b}(\mathcal{E})$$

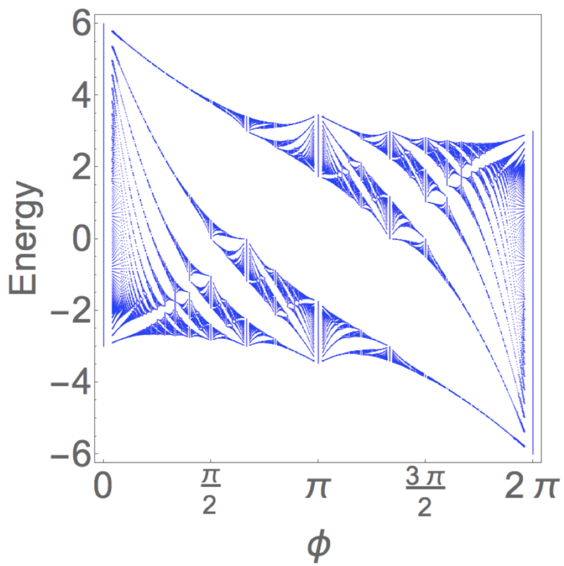
exact form

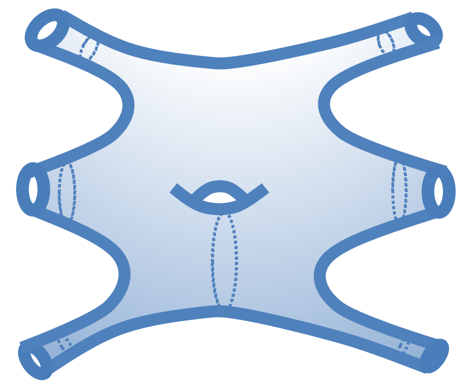
Hatsuda-YS-Xu(2017)

obtain Hofstadter butterfly from it

Summary







$$\frac{1}{2\pi}\mathrm{Im}\left[\frac{\partial t(\mathcal{E},\hbar=2\pi a/b)}{\partial\mathcal{E}}\right]$$

From Cond. Mat. Phys. to String

Future work

Higher genus mirror curve ??

Non-Hermitian ??

From string to cond.mat.phys ??

…

End.