Positivity Bounds in Effective Field Theories

Shuang-Yong Zhou (USTC)

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de Rham, Melville, Tolley & **SYZ**, arXiv:1702.06134 [hep-th]
de Rham, Melville, Tolley & **SYZ**, arXiv:1702.08577 [hep-th]
de Rham, Melville, Tolley & **SYZ**, arXiv:1706.02712 [hep-th]
de Rham, Melville, Tolley & **SYZ**, arXiv:1804.10624 [hep-th]
Cen Zhang & **SYZ**, arXiv:1808.00010 [hep-ph]

Effective field theories (EFTs)

- EFTs are widely used in modern physics

GR, inflation, dark energy, BSM physics,…

- Separation of physics at different scales

write down all local operators consistent
with symmetries suppressed by cut-off scale

$$\mathscr{L} = \sum_i \Lambda^4 f_i \, \mathcal{O}_i \left(\frac{\text{boson}}{\Lambda}, \frac{\text{fermion}}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right)$$

f_i : Wilson coefficients

Are all EFTs allowed?

Answer: No!

Not every effective field theory can be UV completed!
(Not every set of Wilson coefficients are allowed!)

$$e^{\frac{i}{\hbar}S_W[\text{light}]} = \int D[\text{heavy}]\ e^{\frac{i}{\hbar}S_{\text{UV}}[\text{light,heavy}]}$$

UV completion satisfies: Lorentz invariance, unitarity, locality, causality, analyticity, crossing symmetry,…

↓

Positivity bounds on Wilson coefficients

Simplest example: P(X)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \cdots$$

Positivity bound: $\lambda > 0$

Theories with $\lambda < 0$ do not have a local and Lorentz invariant UV completion

Outline

- Positivity Bounds for Spin 0: Main idea
- Positivity Bounds for Spin >0: Subtleties
- Applications in Some low energy EFTs:
    - Galileon (spin 0), SMEFT (spin 1), Massive gravity (spin 2)
- Summary

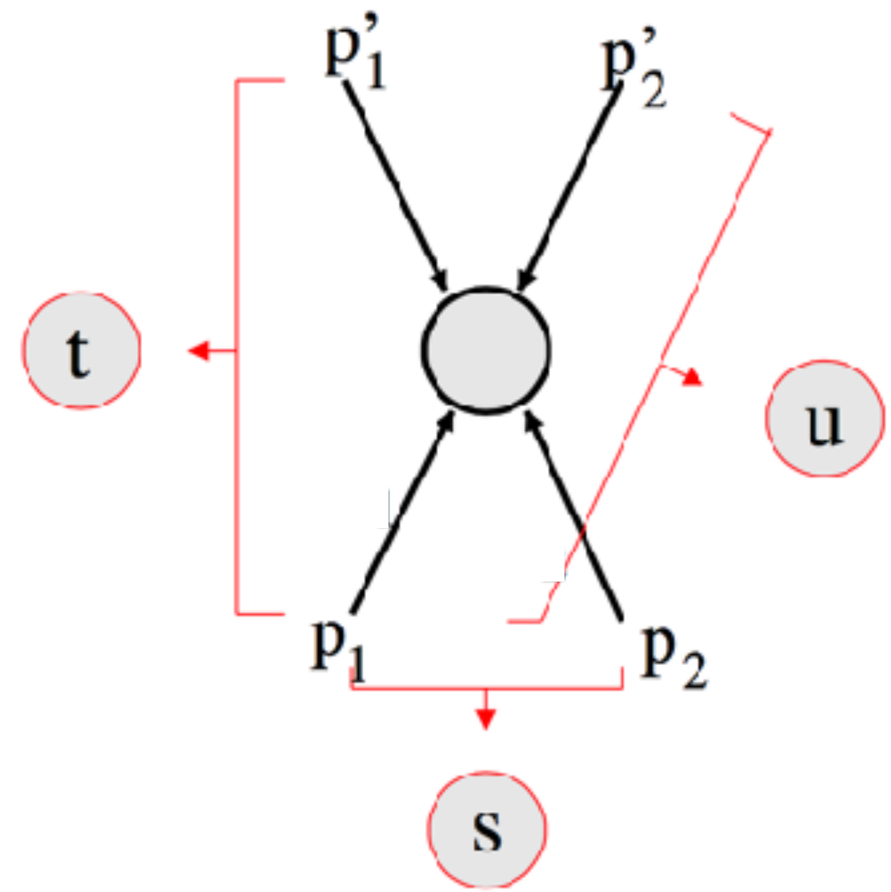
2 to 2 scattering

Mandelstam variables

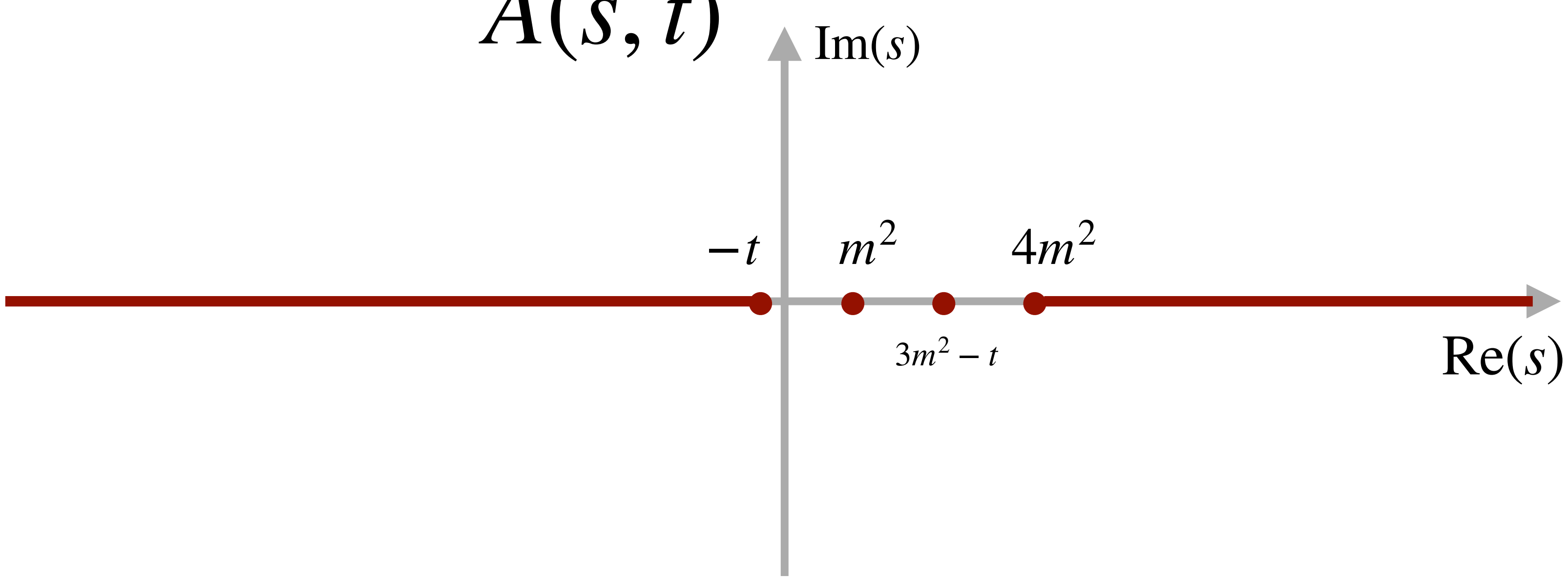
$$s = -(p_1 + p_2)^2 = E_{\text{cm}}^2$$

$$t = -(p_1 + p_1')^2 = -\frac{s - 4m^2}{2}(1 - \cos\theta)$$

$$u = -(p_1 + p_2')^2 = 4m^2 - s - t$$



$A(s, t)$



Crossing symmetry

$$A(s, t) = A(u, t)$$

Optical theorem

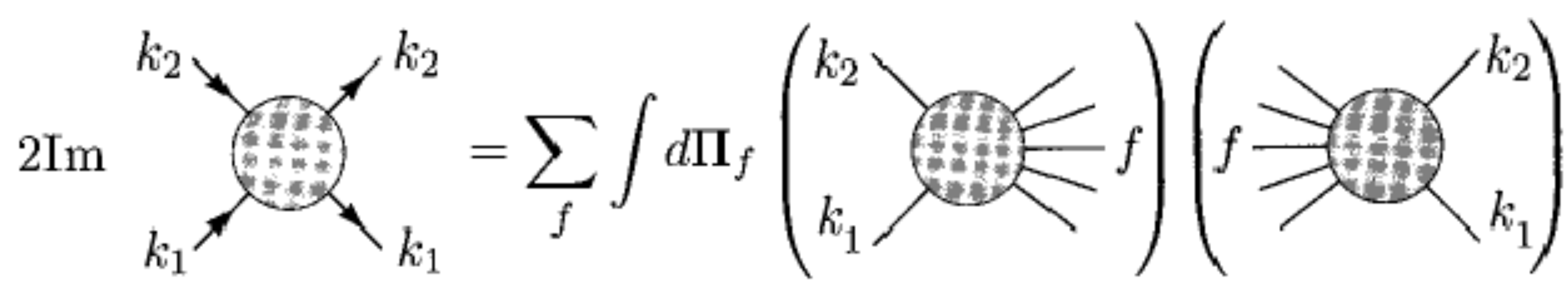
Unitarity:

$$S^\dagger S = 1, \qquad S = 1 + iT \quad \longrightarrow \quad (T - T^\dagger) = iT^\dagger T$$

Acting initial and final states:

$$\langle F|T|I\rangle - \langle I|T|F\rangle^* = \sum_f \int \mathrm{d}\Pi_f \langle f|T|F\rangle^* \langle f|T|I\rangle$$

Optical theorem:



$$\mathrm{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\,\sigma(s) > 0$$

Partial wave unitarity

Partial wave expansion:

$$A(s,t) = 16\pi\sqrt{\frac{s}{s-4m^2}}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l(s)$$

Partial wave unitary bounds:

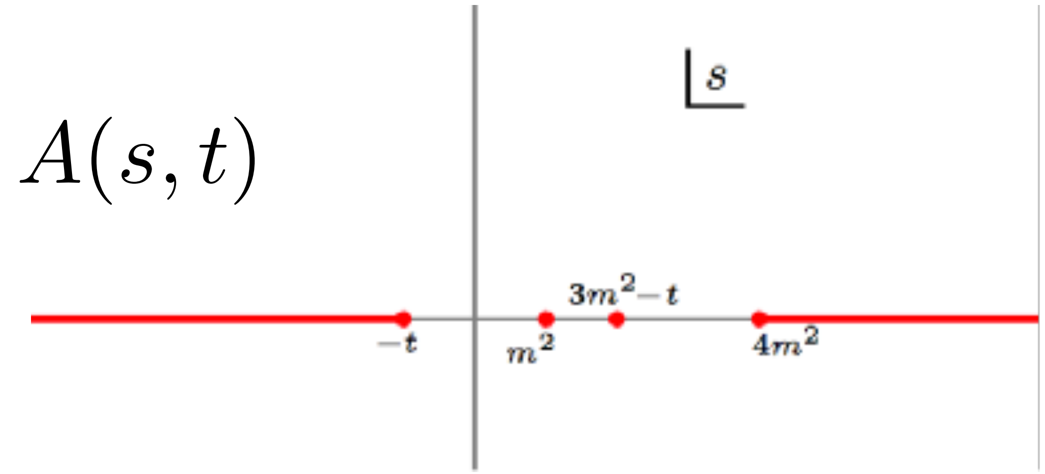
$$0 \leq |a_l(s)|^2 \leq \mathrm{Im}\, a_l(s) \leq 1$$

Also $\partial_t^n P_l(1+t)|_{t=0} \geq 0$, we get

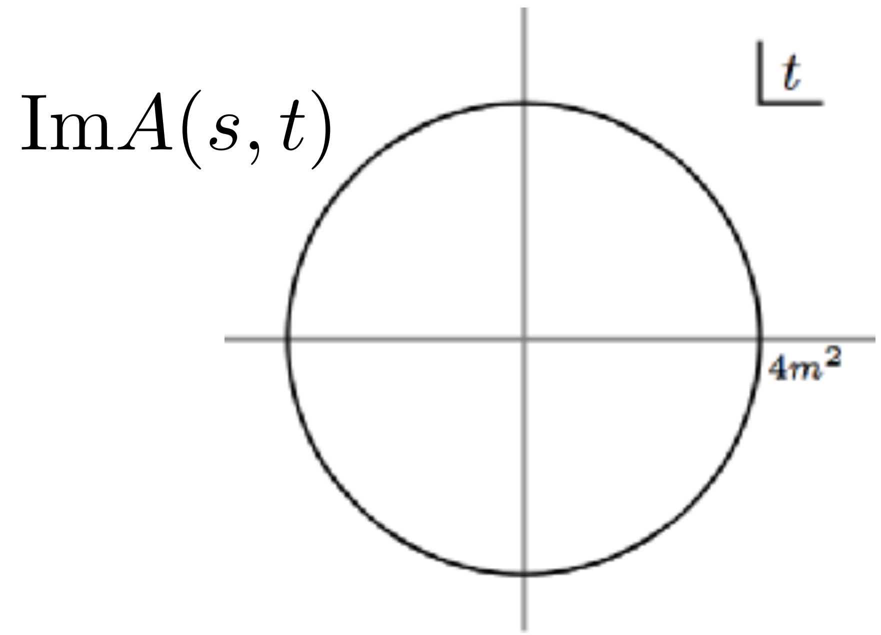
$$\left.\frac{\partial^n}{\partial t^n}\mathrm{Im}[A(s,t)]\right|_{t=0} > 0 \quad \forall\, n \geq 0 \quad \text{and} \quad s \geq 4m^2$$

Analyticity

Fixed *t*:



Fixed *s*:



Martin, 1965

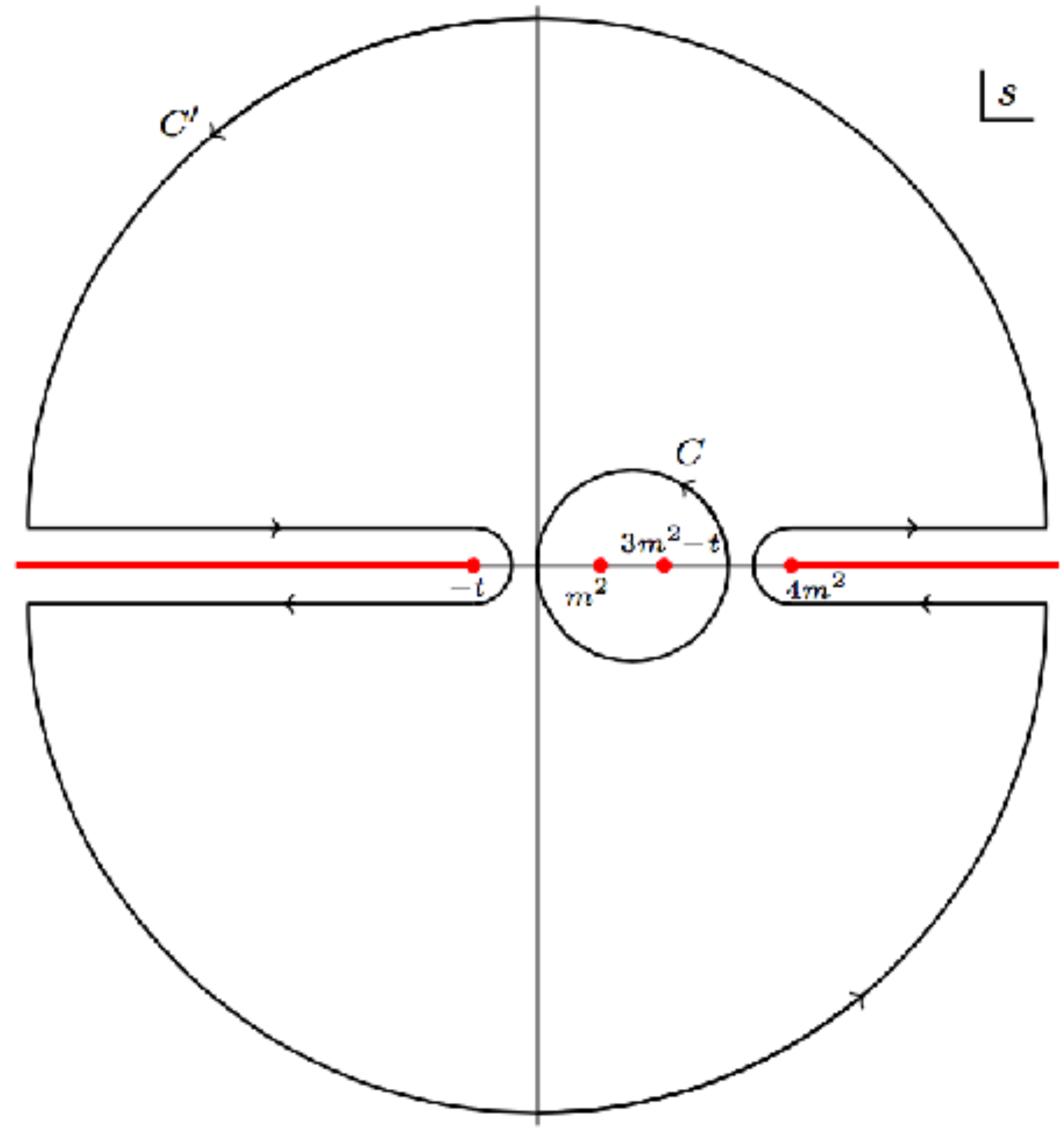
Key ingredient 1:

$$\frac{\partial^n}{\partial t^n}\mathrm{Im}[A(s,t)] > 0 \quad \forall \quad s \geq 4m^2\,,\, 0 \leq t < 4m^2.$$

Dispersion relation (1)

$$A(s,t) = \frac{1}{2\pi i}\oint_C ds' \frac{A(s',t)}{s'-s},$$
$$= \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \int_{C_\infty^\pm} ds' \frac{A(s',t)}{s'-s}$$
$$+ \int_{4m^2}^{\infty} \frac{d\mu}{\pi}\left(\frac{\mathrm{Im}A(\mu,t)}{\mu-s} + \frac{\mathrm{Im}A(\mu,t)}{\mu-u}\right)$$

$$A(s,t) = A(u,t)$$



Froissart-Martin bound:

$$\lim_{s\to\infty} |A(s,t)| < Cs^{1+\epsilon(t)}, \qquad 0 \le t < 4m^2.$$
$$\epsilon(t) \;<\; 1$$

Froissart, 1961
Martin, 1962

Dispersion relation (2)

Identity: $$\frac{1}{\mu - s} = \frac{(s-\mu_p)^2}{(\mu-\mu_p)^2}\frac{1}{\mu - s} + 2\frac{(s-\mu_p)}{(\mu-\mu_p)^2} + \frac{(\mu - s)}{(\mu-\mu_p)^2}$$

Twice subtracted dispersion relation:

$$A(s,t) = a(t) + \frac{\lambda}{m^2 - s} + \frac{\lambda}{m^2 - u} + \int_{4m^2}^{\infty} \frac{d\mu}{\pi} \left(\frac{(s-\mu_p)^2 \mathrm{Im} A(\mu,t)}{(\mu-\mu_p)^2(\mu - s)} + \frac{(u-\mu_p)^2 \mathrm{Im} A(\mu,t)}{(\mu-\mu_p)^2(\mu - u)} \right)$$

Define: $\tilde{B}(v,t) = A(s,t) - \dfrac{\lambda}{m^2 - s} - \dfrac{\lambda}{m^2 - u}$, $v = s + \dfrac{t}{2} - 2m^2$

$$\bar{s} = s - \frac{4m^2}{3}$$

Key ingredient 2: $$\tilde{B}(v,t) = a(t) + \int_{4m^2}^{\infty} \frac{d\mu}{\pi(\bar{\mu} + \bar{t}/2)} \frac{2v^2 \,\mathrm{Im} A(\mu,t)}{(\bar{\mu} + \bar{t}/2)^2 - v^2}$$

Key ingredients: Recap

Key ingredient 1: $$\frac{\partial^n}{\partial t^n}\mathrm{Im}[A(s,t)] > 0 \quad \forall \quad s \geq 4m^2\,,\, 0 \leq t < 4m^2.$$

Key ingredient 2: $$\tilde{B}(v,t) = a(t) + \int_{4m^2}^{\infty} \frac{d\mu}{\pi(\bar{\mu}+\bar{t}/2)} \frac{2v^2\,\mathrm{Im}\,A(\mu,t)}{(\bar{\mu}+\bar{t}/2)^2 - v^2}$$

Define: $$B^{(2N,M)}(t) = \frac{1}{M!}\partial_v^{2N}\partial_t^M \tilde{B}(v,t)\Big|_{v=0} = \sum_{k=0}^{M} \frac{(-1)^k}{k!2^k} I^{(2N+k,M-k)}$$

$$I^{(q,p)}(t) = \frac{q!}{p!}\frac{2}{\pi}\int_{4m^2}^{\infty} \frac{d\mu \;\; \partial_t^p \mathrm{Im}\, A(\mu,t)}{(\bar{\mu}+\bar{t}/2)^{q+1}} > 0$$

$$I^{(q,p)} < \frac{q}{\mathcal{M}^2} I^{(q-1,p)} \qquad \mathcal{M}^2 = (t+4m^2)/2$$

An infinite tower of positivity bounds

Recurrence relation:

de Rham, Melville, Tolley & **SYZ**, arXiv:1702.06134

$$Y^{(2N,M)} = \sum_{r=0}^{M/2} c_r B^{(2N+2r,M-2r)} + \frac{1}{\mathcal{M}^2} \sum_{k \text{ even}}^{(M-1)/2} (2(N+k)+1)\beta_k Y^{(2(N+k),M-2k-1)} > 0$$

$$\text{sech}(x/2) = \sum_{k=0}^{\infty} c_k x^{2k} \quad \text{and} \quad \tan(x/2) = \sum_{k=0}^{\infty} \beta_k x^{2k+1}$$

Forward limit positivity bound

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

Forward limit positivity bound

$$f(s_p) = \frac{1}{2\pi i}\oint_{C'} ds' \, \frac{A(s',0)}{(s'-s_p)^3} \qquad 0 \le s_p < 4m^2$$

$$= \sum_{N=1}^{\infty} \frac{s_p^{2N-2}}{2(2N-2)!} Y^{(2N,0)}(0) > 0$$

$$\mathrm{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\sigma(s) > 0$$

Our generalizations:

1. an infinite number of derivative bounds
2. away from the forward limit $0 \le t < 4m^2$
3. applicable to general spins

Positivity Bounds for Spin >0: Subtleties

Subtleties with nonzero spins (1)

$$m_1 = m_2 = m_3 = m_4 = m, \quad S_3 = S_1, \quad S_4 = S_2$$

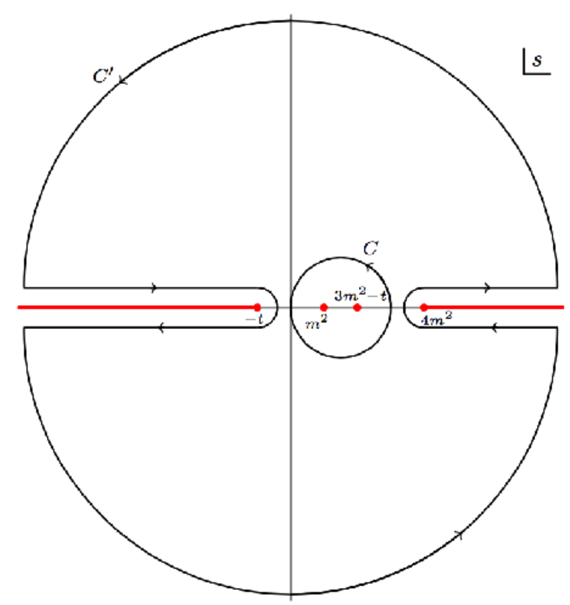
$$A(s,t) = A(u,t)$$

Crossing is nontrivial in helicity formalism:

$$\mathcal{H}^s_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t,u) = (-1)^{2S_2} \sum_{\lambda_i'} e^{i\pi(\lambda_1'-\lambda_3')} d^{S_1}_{\lambda_1'\lambda_1}(\chi_u) d^{S_2}_{\lambda_2'\lambda_2}(-\pi+\chi_u) \cdot d^{S_1}_{\lambda_3'\lambda_3}(-\chi_u) d^{S_2}_{\lambda_4'\lambda_4}(\pi-\chi_u) \mathcal{H}^u_{\lambda_1'\lambda_4'\lambda_3'\lambda_2'}(u,t,s),$$

$$d^J_{ab}(\beta) = \langle Ja|e^{-i\beta J_y}|Jb\rangle \qquad e^{\pm i\chi_u} = \frac{-su \mp 2im\sqrt{stu}}{\sqrt{su}}$$

Positivity of LH cut can not be established!



$$\frac{\partial^n}{\partial t^n} \mathrm{Im}\mathcal{H}^s_{\lambda_1\lambda_2\lambda_3\lambda_4} > 0, \quad s > 4m^2$$

Subtleties with nonzero spins (2)

Extra kinematical singularities:

Pole at threshold $s = 4m^2$

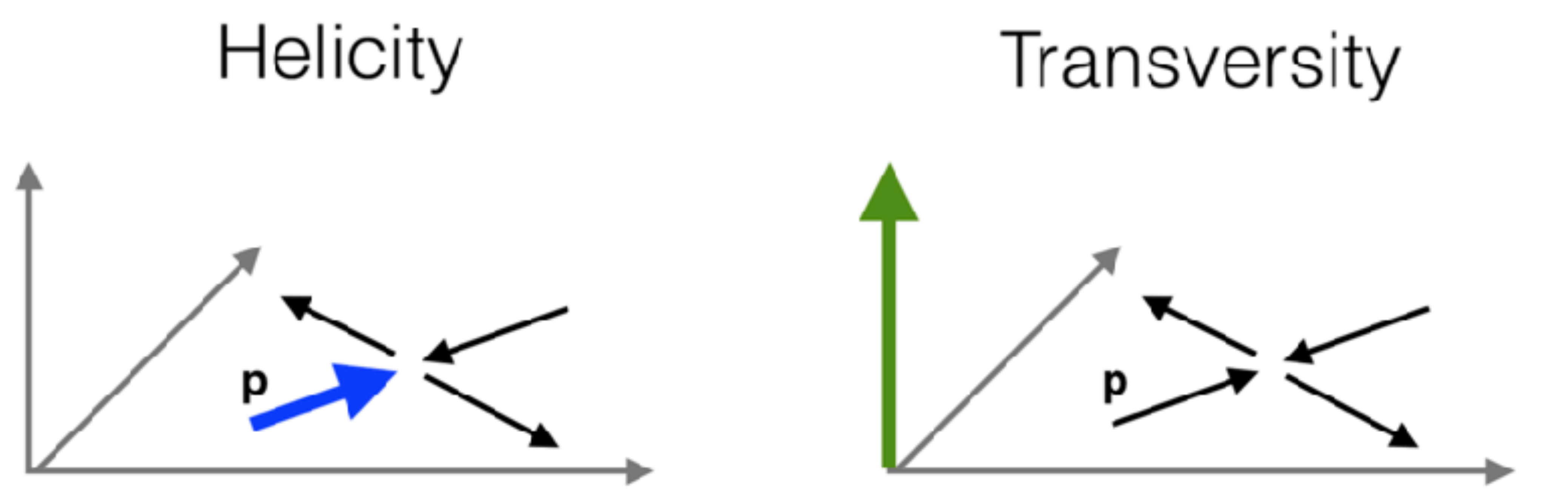
Branch point at $stu = 0$

Branch point at $u = 0$ for boson-fermion scattering

$$\cos\theta = 1 + \frac{2t}{s-4m^2}, \quad \sin\theta = \frac{2\sqrt{tu}}{s-4m^2}.$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{-u}{s-4m^2}}$$

Helicity vs **Transversity**



Regularized transversity amplitude

For singularities: $u = 0 \quad s = 4m^2$

$$(\sqrt{-su})^{\xi}(s(s-4m^2))^{S_1+S_2}\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}$$

$\xi = 1$ for BF scattering, $\xi = 0$ otherwise

For singularities: $\sqrt{stu} = 0$

$\sqrt{stu} \leftrightarrow -\sqrt{stu} \longleftrightarrow \theta \leftrightarrow -\theta$

$$\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(\theta) + \mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(-\theta)$$

Regularized transversely amplitude:

$$\mathcal{T}^{+}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) = (\sqrt{-su})^{\xi}\mathcal{S}^{S_1+S_2}\left(\mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,\theta) + \mathcal{T}_{\tau_1\tau_2\tau_3\tau_4}(s,-\theta)\right),$$

Positivity bounds

Then it reduces to formally the same as the spin 0 case.

$$Y_{\tau_1\tau_2}^{(2N,M)}(t) = \sum_{r=0}^{M/2} c_r B_{\tau_1\tau_2}^{(2N+2r,M-2r)}(t) + \frac{1}{\mathcal{M}^2} \sum_{\text{even } k=0}^{(M-1)/2} (2N+2k+1)\beta_k Y_{\tau_1\tau_2}^{(2N+2k,M-2k-1)}(t) > 0$$

de Rham, Melville, Tolley & **SYZ**, arXiv:1706.02712 [hep-th]

Applications in some low energy EFTs

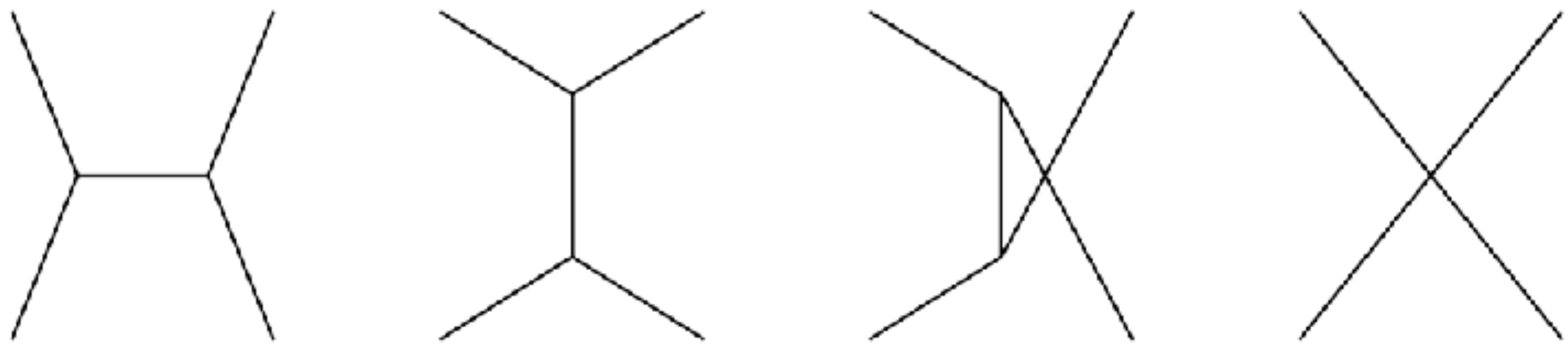
Application 1: Massive Galileon

$$\mathcal{L}_{\rm mGal} = -\frac{1}{2}\partial^\mu\pi\partial_\mu\pi - \frac{1}{2}m^2\pi^2 + \sum_{n=3}^{5}\frac{g_n}{n!\Lambda^{3(n-2)}}\pi\partial_{\mu_1}\partial^{[\mu_1}\pi...\partial_{\mu_{n-1}}\partial^{\mu_{n-1}]}\pi$$

Arises from DGP braneworlds and dRGT massive gravity

Non-renormalization theorem

Leading positivity bounds (1)



Pole subtracted amplitude:

$$B(s,t) = a_{00} - a_{10}\left(\bar{s}\bar{t} + \bar{t}\bar{u} + \bar{u}\bar{s}\right) - a_{01}\bar{s}\bar{t}\bar{u}$$

$$a_{00} = \frac{m^6}{\Lambda^6}\left[\frac{16 g_4}{27} - \frac{295 g_3^2}{144}\right], \quad a_{10} = \frac{m^2}{\Lambda^6}\left[-\frac{g_4}{3} + \frac{3 g_3^2}{8}\right], \quad a_{01} = \frac{1}{\Lambda^6}\left[-\frac{g_4}{4} + \frac{3 g_3^2}{16}\right]$$

Positivity bounds:

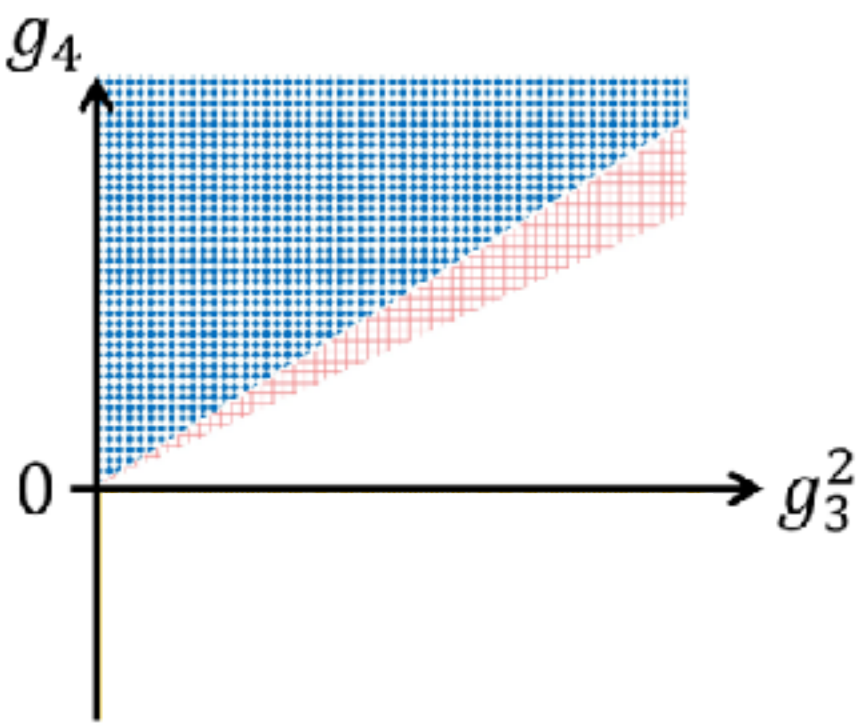
$$Y^{(2,0)}: \quad a_{10} + a_{01}\bar{t} > 0$$

$$Y^{(2,1)}: \quad a_{01} + \frac{3}{2\Lambda_{\text{th}}^2}\left(a_{10} + a_{01}\bar{t}\right) > 0$$

$$0 \le t < 4m^2$$

Leading positivity bounds (2)

strongest bound: $t = 4m^2$



- No analytic UV completion $\Rightarrow$ $g_4/g_3^2 > 7/8$
- Potential UV analytic completion but at low cutoff $\Rightarrow$ $3/4 < g_4/g_3^2 \leq 7/8$
- No direct obstruction to potential existence of analytic UV completion $\Rightarrow$ $g_4/g_3^2 \leq 3/4$

$3/4 < g_4/g_3^2 \leq 7/8$

$$\Lambda_{\rm th}^2 < \frac{1}{2} m^2 g_3^2 \quad \Rightarrow \quad \Lambda_{\rm th} \sim m$$

de Rham, Melville, Tolley & **SYZ**, arXiv:1702.06134

Application 2: Standard Model Effective Field Theory

- Write down all possible operators with
  - SM particle contents
  - SM gauge group structure $SU(3)_c \times SU(2)_L \times U(1)$

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_j \frac{f_j^{(6)} O_j^{(6)}}{\Lambda^2} + \sum_i \frac{f_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Vector boson scattering (VBS)

VBS: a sensitive probe to new physics

$$V_1 + V_2 \to V_3 + V_4, \quad V_i \in \{Z, W^+, W^-, \gamma\}$$

$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times [(D_\beta\Phi)^\dagger D^\beta\Phi] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi] \\
O_{M,2} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times [(D_\beta\Phi)^\dagger D^\beta\Phi] \\
O_{M,3} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi] \\
O_{M,4} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right] \times \hat{B}^{\beta\nu} \\
O_{M,5} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right] \times \hat{B}^{\beta\mu} \\
O_{M,7} &= \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]
\end{aligned}
$$

$$
\begin{aligned}
O_{T,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{T,2} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
O_{T,5} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,6} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
O_{T,7} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
O_{T,8} &= \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,9} &= \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},
\end{aligned}
$$

Leading order bounds

If dim-6 ops alone, all positivity bounds violated

$$\mathcal{O}(\Lambda^{-4}) : \qquad \sum_i (-C_i)\left(\sum_j D_j f_j^{(6)}\right)^2 \geq 0, \quad C_i > 0$$

Positivity bounds require the existence of higher-D ops!

(dim-8 part) - (dim-6 operators) > 0

↓ weak but simpler bounds

(dim-8 part) > 0 → $\sum_i E_i f_i^{(8)} \geq 0$

Cen Zhang & **SYZ**, arXiv:1808.00010

Leading order bounds (2)

Dim-6:

WZ scattering: $a_3^2 b_3^2 \left[e^2 C_{DW} - s_W^2 c_W^2 C_{\varphi D} - 4 s_W^3 c_W C_{\varphi WB}\right]^2 + 36(a_1 b_1 + a_2 b_2)^2 e^2 s_W^2 c_W^2 C_W^2$

WW scattering: $a_3^2 b_3^2 s_W^2 \left(e^2 C_{DB} + c_W^2 C_{\varphi D}\right)^2 + e^2 c_W^2 \left[6(a_1 b_1 + a_2 b_2) s_W C_W + a_3 b_3 e C_{DW}\right]^2$

$$\epsilon_1^\mu = \left(a_3 p_1/m_1, a_1, a_2, a_3 E_1/m_1\right)$$
$$\epsilon_2^\mu = \left(b_3 p_2/m_2, b_1, b_2, b_3 E_2/m_2\right)$$

Dim-8:

$ZZ:$

$$8At_W^4\left(F_{S,0}+F_{S,1}+F_{S,2}\right)+Dt_W^2\left(-t_W^4F_{M,3}+t_W^2F_{M,5}-2F_{M,1}+F_{M,7}\right)$$
$$+(B+C)\left(2t_W^8F_{T,9}+4t_W^4F_{T,7}+8F_{T,2}\right)+8B\left[t_W^4\left(t_W^4F_{T,8}+2F_{T,5}+2F_{T,6}\right)+4F_{T,0}+4F_{T,1}\right]\geq 0$$

$W^\pm W^\pm:$

$$4As_W^4\left(2F_{S,0}+F_{S,1}+F_{S,2}\right)-8Es_W^2F_{M,0}-2(E+F)s_W^2F_{M,1}+Fs_W^2F_{M,7}$$
$$+(4B+6C)F_{T,2}+16BF_{T,0}+24BF_{T,1}\geq 0$$

Leading order bounds (3)

$W^\pm W^\mp$:

$$4As_W^4\left(2F_{S,0}+F_{S,1}+F_{S,2}\right)-2(G-E)s_W^2F_{M,1}+8Es_W^2F_{M,0}+Gs_W^2F_{M,7}$$
$$+(4B+6C)F_{T,2}+16BF_{T,0}+24BF_{T,1}\geq 0$$

$W^\pm Z$:

$$4Ac_W^2t_W^4\left(F_{S,0}+F_{S,2}\right)+t_W^2\left(D-Hs_W^2\right)\left(F_{M,7}-2F_{M,1}\right)-Hc_W^2t_W^4\left(t_W^2F_{M,3}+F_{M,5}\right)$$
$$+4B\left(t_W^4F_{T,6}+4F_{T,1}\right)+C\left(t_W^4F_{T,7}+4F_{T,2}\right)\geq 0$$

$Z\gamma$:

$$B\left[32c_W^4\left(F_{T,0}+F_{T,1}\right)-16c_W^2s_W^2F_{T,5}+4(c_W^2-s_W^2)^2F_{T,6}-F_{T,7}+8s_W^4F_{T,8}\right]$$
$$+(B+C)\left[(c_W^2-s_W^2)^2F_{T,7}+8c_W^4F_{T,2}+2s_W^4F_{T,9}\right]-Hc_W^2s_W^2\left(2F_{M,1}+F_{M,3}+F_{M,5}-F_{M,7}\right)\geq 0$$

$W^\pm\gamma$:

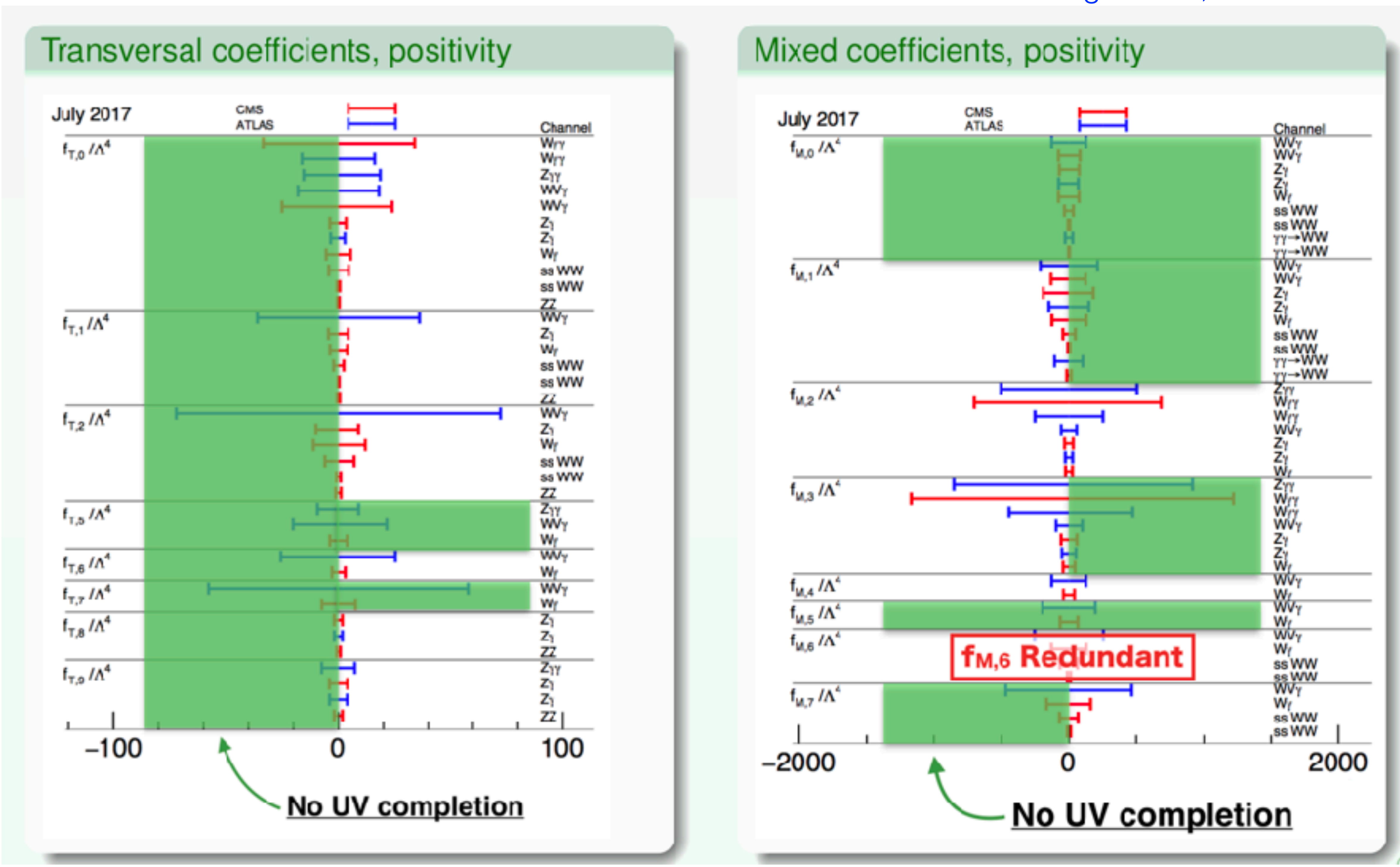
$$4B\left(4F_{T,1}+F_{T,6}\right)+C\left(4F_{T,2}+F_{T,7}\right)-Hs_W^2\left(2F_{M,1}+F_{M,3}-F_{M,5}-F_{M,7}\right)\geq 0$$

$\gamma\gamma$:

$$(B+C)\left(4F_{T,2}+2F_{T,7}+F_{T,9}\right)+4B\left(4F_{T,0}+4F_{T,1}+2F_{T,5}+2F_{T,6}+F_{T,8}\right)\geq 0,$$

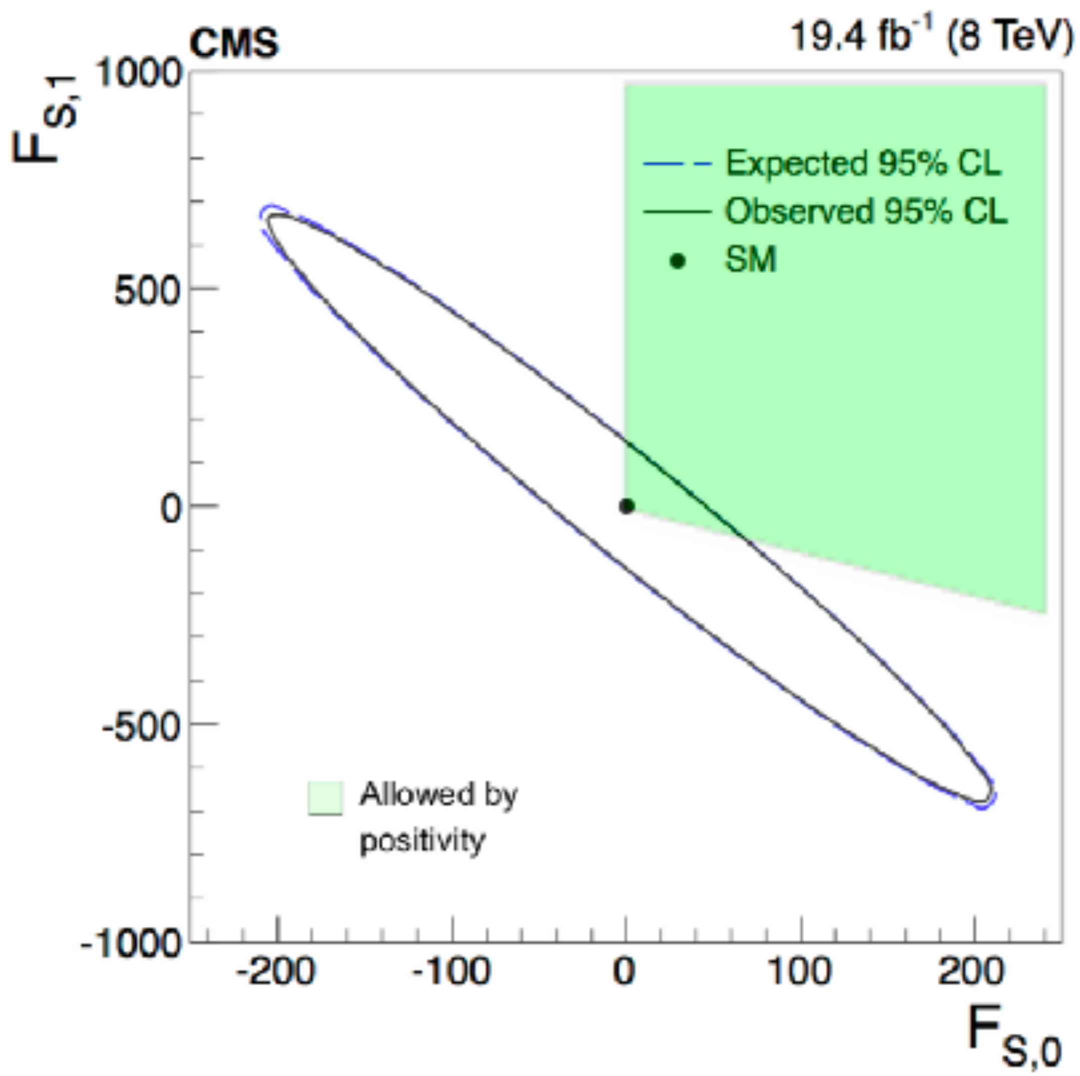
1D bounds

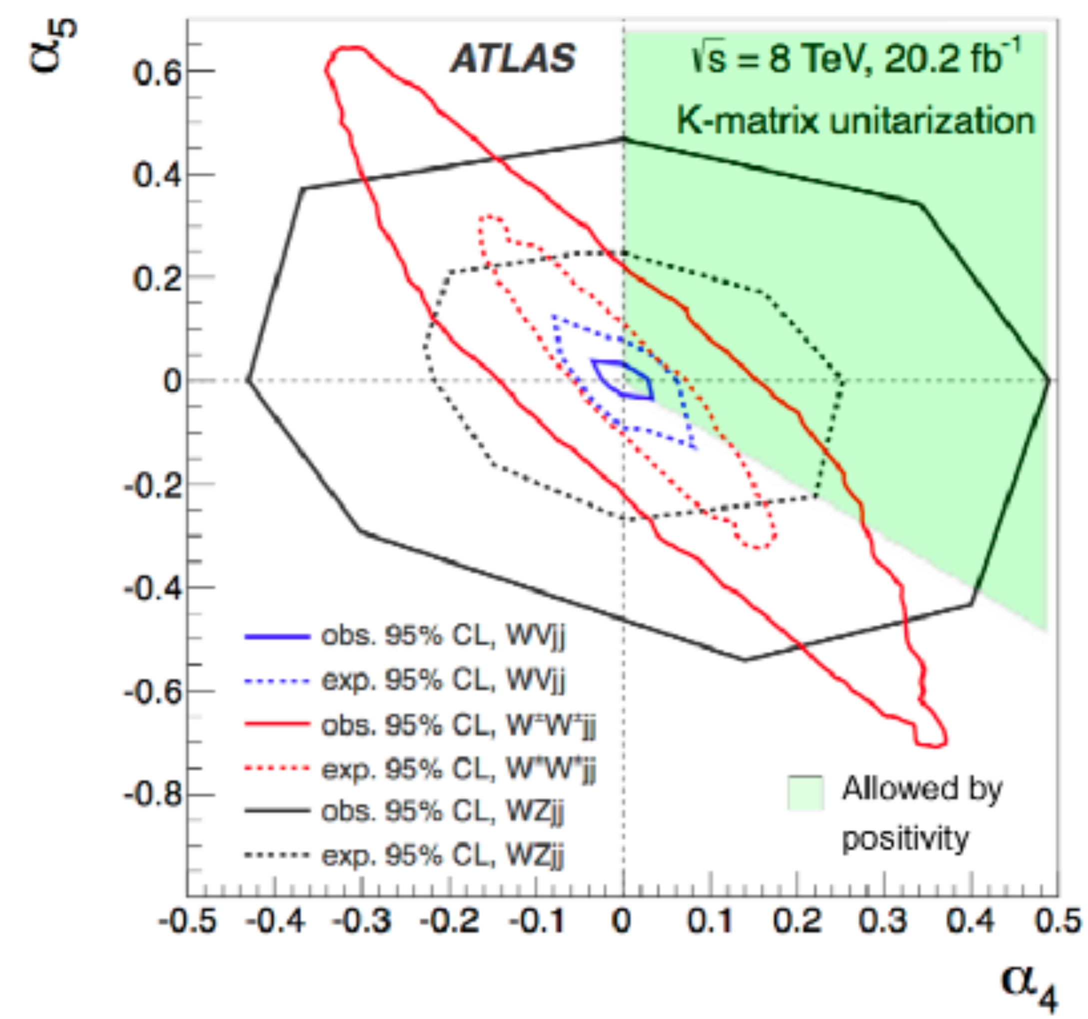
Cen Zhang & **SYZ**, arXiv:1808.00010



2D bounds: an example

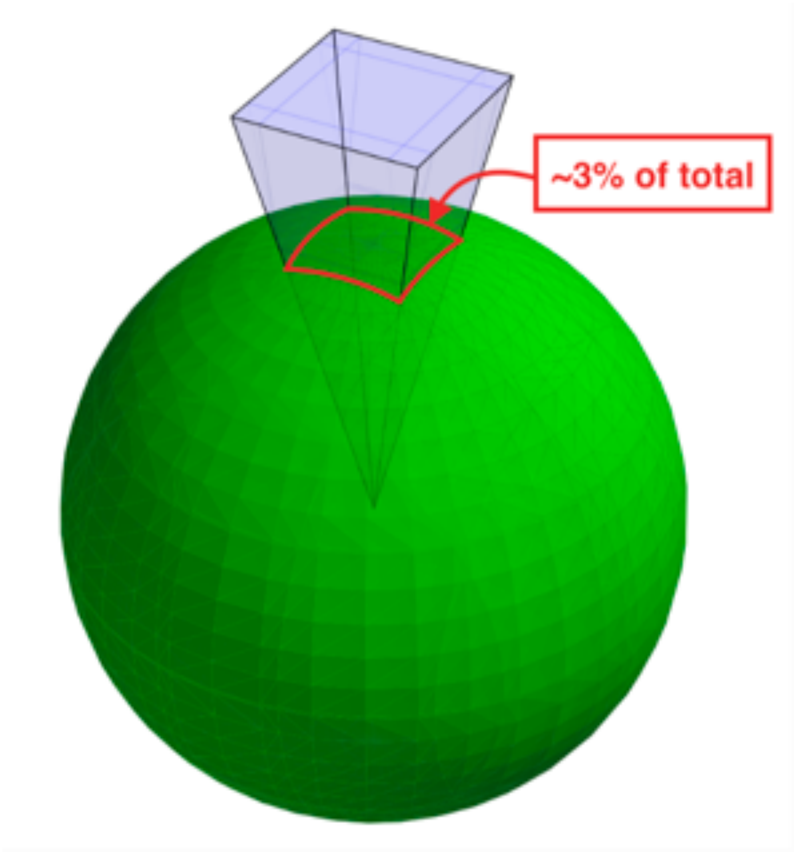
The case of O_{S0} and O_{S1}





General bounds

All 18 parameters:



Randomly take points on the 18D sphere, uniformly distributed, and count how many of the them satisfy all positivity constraints for all polarizations.

Only ~3% of the total parameter space admit a local/Lorentz invariant UV completion!

Application 3: Massive gravity

Generic massive gravity $S = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}V(g,h)\right]$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + \cdots$$
$$V_2(g,h) = +b_1\langle h^2\rangle + b_2\langle h\rangle^2$$
$$V_3(g,h) = +c_1\langle h^3\rangle + c_2\langle h\rangle^2\langle h\rangle + c_3\langle h\rangle^3$$
$$V_4(g,h) = +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4$$

$b_1 = -b_2 = 1$ Fierz-Pauli tuning

Fierz & Pauli, 1930s

Boulware-Deser ghost

There is a nonlinear ghost/6th mode!

Boulware & Deser, 1970s

dRGT Massive Gravity

de Rham, Gabadzdze & Tolley, 2010

$$\mathcal{L} = M_P^2\sqrt{-g}\left(\frac{R}{2} + m^2\left(\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_{\nu]} + \alpha_3\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_{\nu}\mathcal{K}^\rho_{\rho]} + \alpha_4\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_{\nu}\mathcal{K}^\rho_{\rho}\mathcal{K}^\sigma_{\sigma]}\right) + \mathcal{L}_m\right)$$

where $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \mathcal{X}^\mu_\nu \qquad \mathcal{X} = \sqrt{g^{-1}\eta},\ g^{-1} = (g^{\mu\nu}),\ \eta = (\eta_{\mu\nu})$

The **unique** graviton potential to eliminate the BD ghost!

de Rham & Gabadzdze, 2010
de Rham, Gabadzdze & Tolley, 2010
Hassan & Rosen, 2011

dRGT in Vierbein form

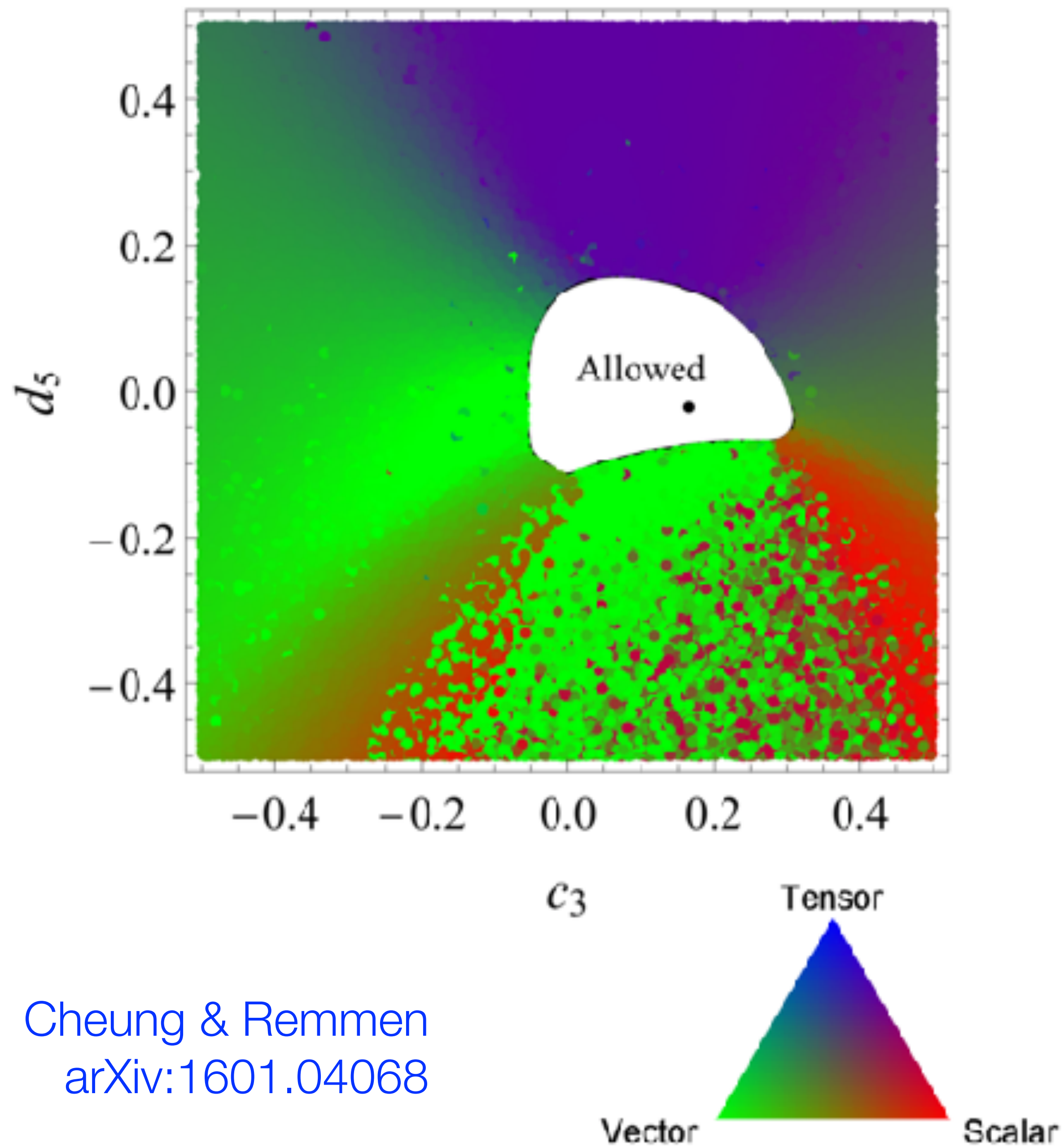
Hinterbichler & Rosen, 2012

$$S_{\mathrm{dRGT}} = \frac{M_P^2}{4} \int \epsilon_{abcd} R^{ab} \wedge e^a \wedge e^b + m^2 V(e, I)$$

$$\begin{aligned} V(e, I) = \epsilon_{abcd}\big(&c_0 e^a \wedge e^b \wedge e^c \wedge e^d + c_1 e^a \wedge e^b \wedge e^c \wedge I^d \\ &+ c_2 e^a \wedge e^b \wedge I^c \wedge I^d + c_3 e^a \wedge I^b \wedge I^c \wedge I^d \\ &+ c_4 I^a \wedge I^b \wedge I^c \wedge I^d\big) \end{aligned}$$

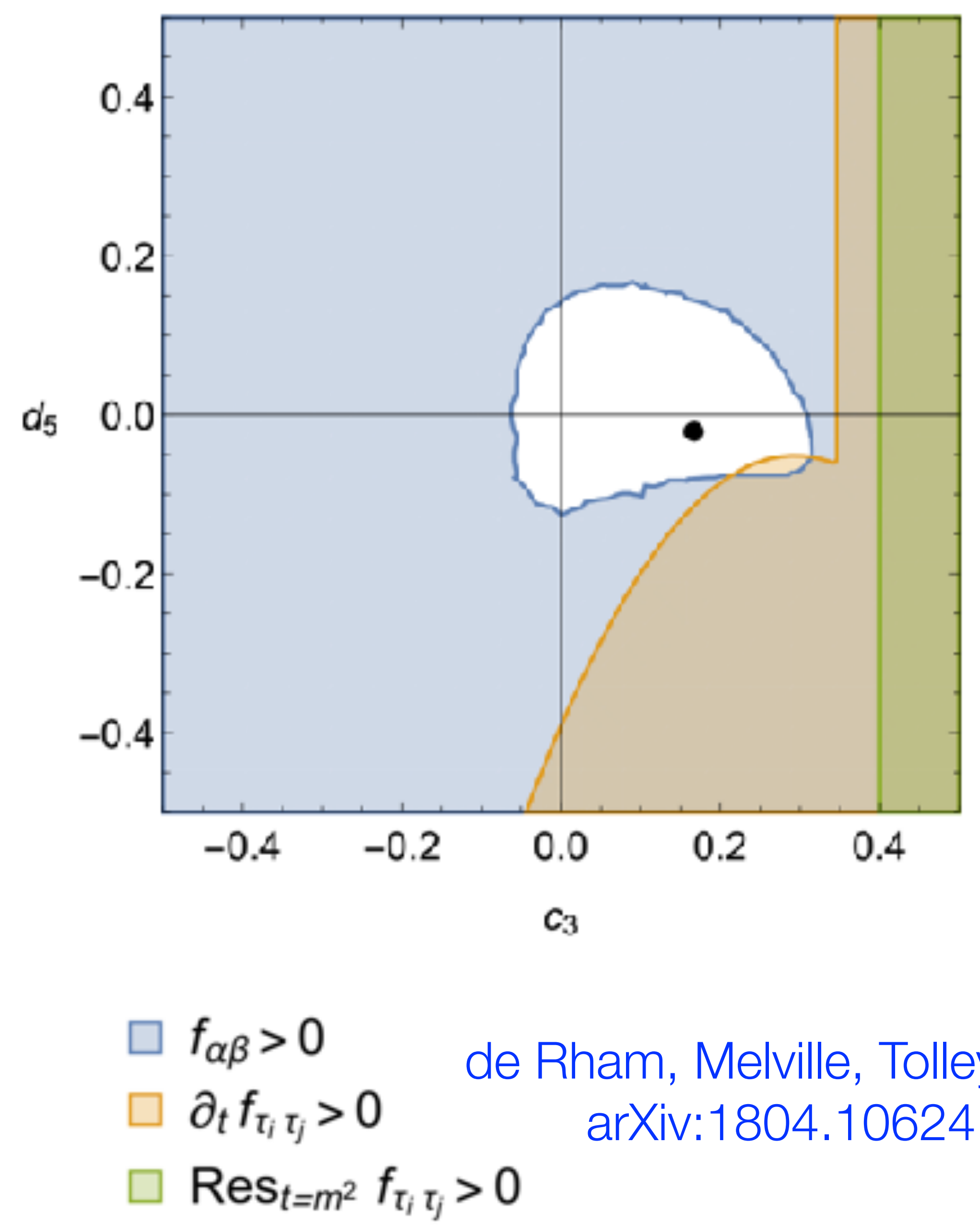
Positivity bounds on dRGT

“Old” bound



Cheung & Remmen
arXiv:1601.04068

Generalized bounds



de Rham, Melville, Tolley & **SYZ**
arXiv:1804.10624 [hep-th]

Generalized bounds on generic massive gravity

Generic massive gravity

relavant parameters $\{c_1, d_1, \Delta c, \Delta d\}$

dRGT massive gravity: $\Delta c = \Delta d = 0$

generalized bounds → $\Delta c = \Delta d = 0$

↓

dRGT massive gravity

de Rham, Melville, Tolley & **SYZ**, arXiv:1804.10624 [hep-th]

Take-home messages

Not all low energy EFTs have a UV completion!

Positivity bounds: constraints on Wilson coefficients

Generalized positivity bounds can often improve the bounds!

Massive gravity may be UV completed in the standard way.

Generalized bounds → dRGT model

Most of the parameter space of the SMEFT do not admit a UV completion.

Low energy? Think positive!

(pun intended)