Higher Spin and Yangian

Wei Li

Strings-QFT-Cosmology Workshop

Huzhou, 2018/11/24

Reference

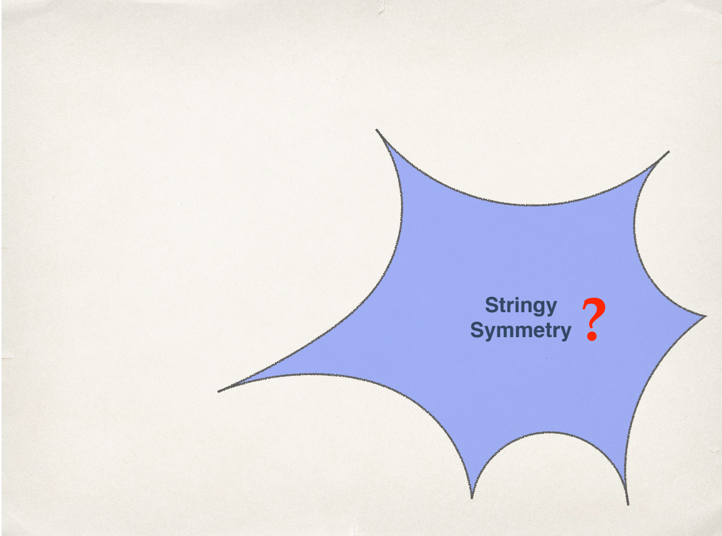
1. **Higher Spins and Yangian Symmetries**
   JHEP **1704**, 152 (2017), [arXiv:1702.05100]
   with *Matthias Gaberdiel, Rajesh Gopakumar, and Cheng Peng*

2. **Twisted sectors from plane partitions**
   JHEP **1609**, 138 (2016), [arXiv:1606.07070]
   with *Shouvik Datta, Matthias Gaberdiel, and Cheng Peng*

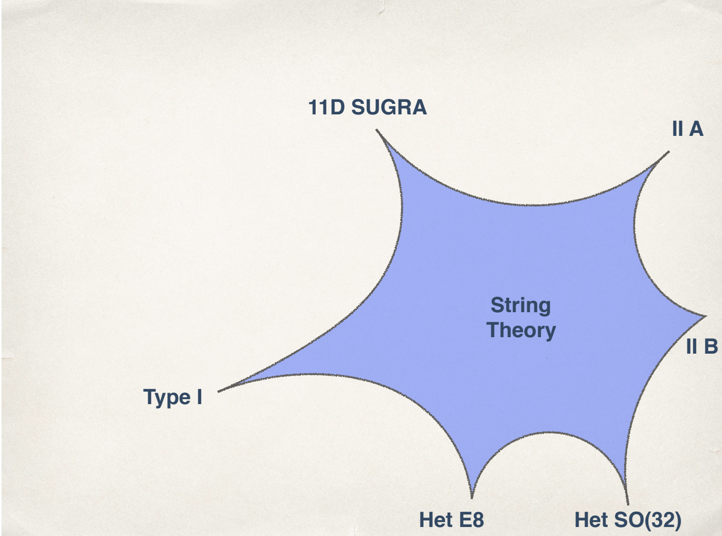
3. **The supersymmetric affine yangian**
   JHEP **1805**, 200 (2018), [arXiv:1711.07449]
   with *Matthias Gaberdiel, Cheng Peng, and Hong Zhang*

4. **Twin plane partitions and $\mathcal{N}=2$ affine yangian**
   JHEP **1812**, xxx (2018), [arXiv:1807.11304]
   with *Matthias Gaberdiel and Cheng Peng*

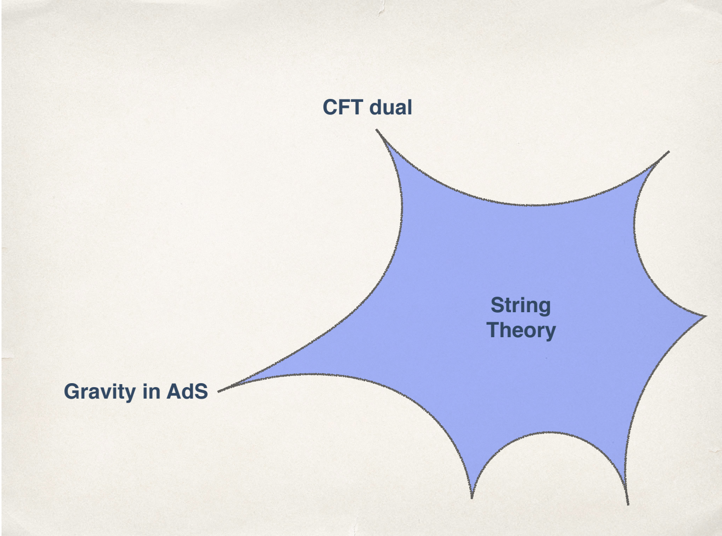
Main question: what is the hidden stringy symmetry?



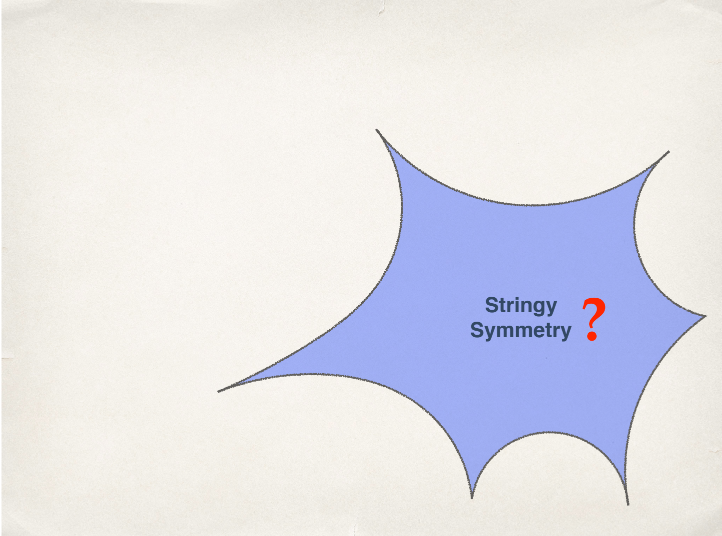
Dualites in string theory: M



Dualites in string theory: AdS/CFT

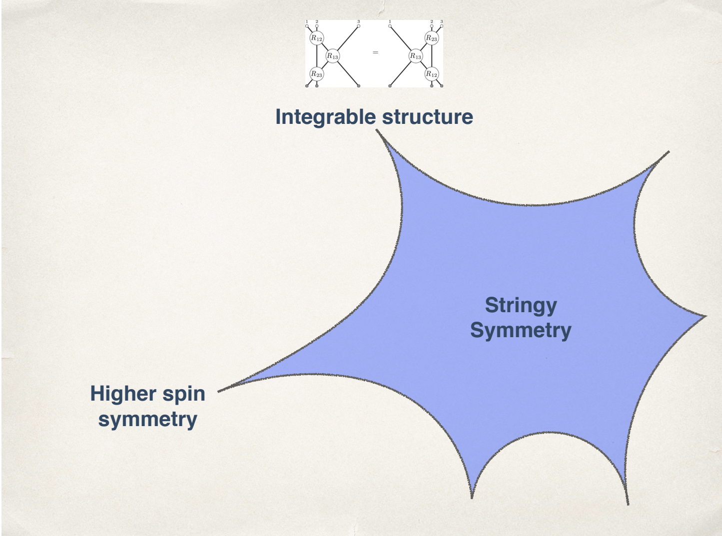


There is a large hidden symmetry in string theory



How to see these symmetries explicitly?

Different manifestation of stringy symmetry



Higher spin symmetry and stringy symmetry

- String theory has infinite number of massive higher spin particles
- Tensionless limit:
  massive higher spin particle $\Longrightarrow$ massless $\Longrightarrow$ stringy symmetry
  - subalgebra: Vasiliev higher spin symmetry (one per spin)
    (from Leading Regge trajectory)
    Vasiliev '91
    Sundborg '01, Witten '01, Mikhailov '02, Klebanov-Polyakov '02
- Tensionless String in AdS_3 $\Longrightarrow$ maximal stringy symmetry?
  Gaberdiel Gopakumar '15
  - higher spin symmetry $\Longrightarrow$ $\mathcal{W}$ symmetry
    (Virasoro + higher spin currents)
    Campoleoni Fredenhagen Pfenninger Theisen '10, *Henneaux Rey '10*

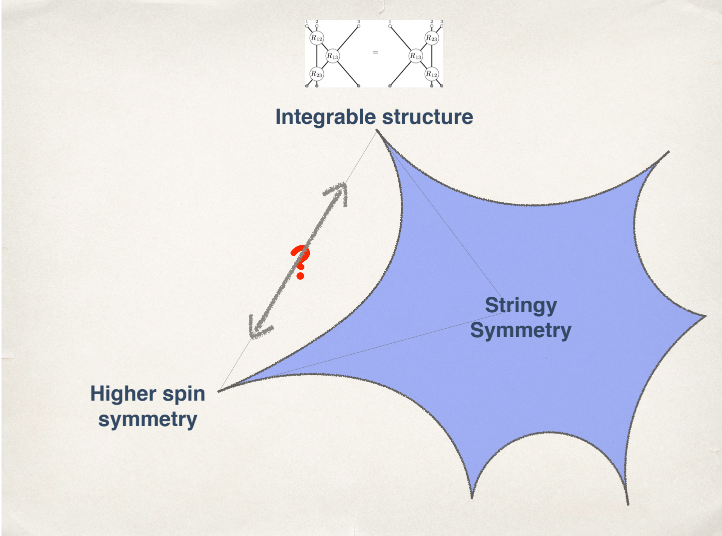
Integrability and string theory

Integrability $\Longrightarrow$ ∞ number of conserved charges

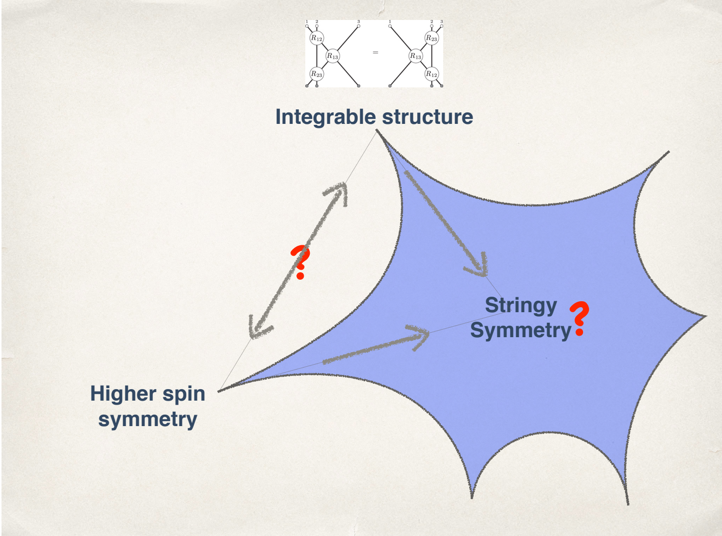
Also appear in string theory

- CFT dual of string in AdS
  - string in $\mathrm{AdS}_5 \times \mathrm{S}^5$
    Minahan Zarembo '02, Beisert Kristjansen Staudacher '03
    Bena Polchinsk Roiban '03...
  - string in $\mathrm{AdS}_4 \times \mathbb{CP}^3$
    Minahan Zarembo '08, Gromov Vieira '08...
  - string in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathcal{M}_4$
    Babichenko Stefanski Zarembo '09, Ohlson-Sax Stefanski '11
    Borsato Ohlsson-Sax Sfondrini Stefanski Torrielli '12.....
- supersymmetric gauge theories fromg string
  - AGT and its extensions....

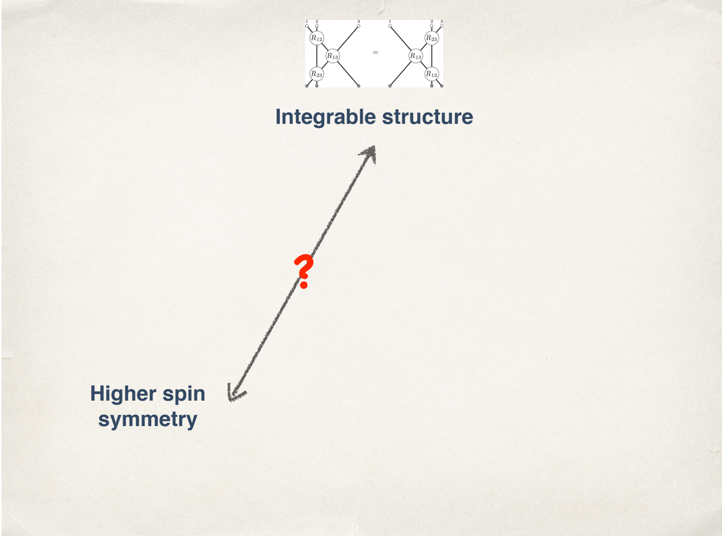
Different manifestation of stringy symmetry



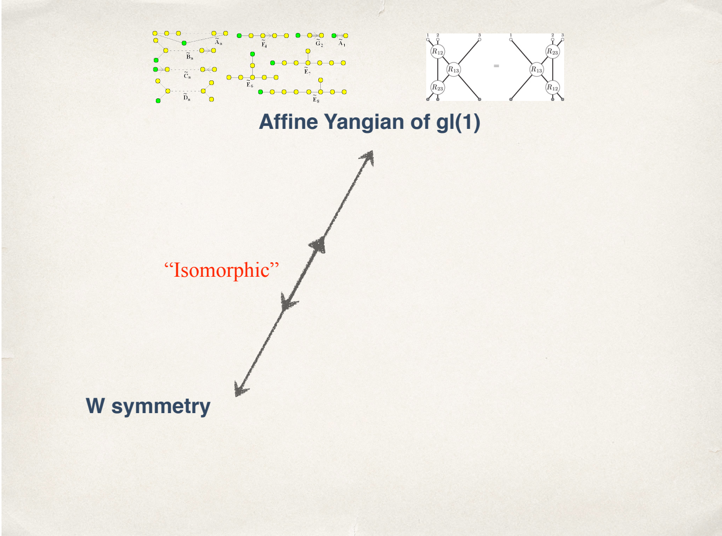
Different manifestation of stringy symmetry



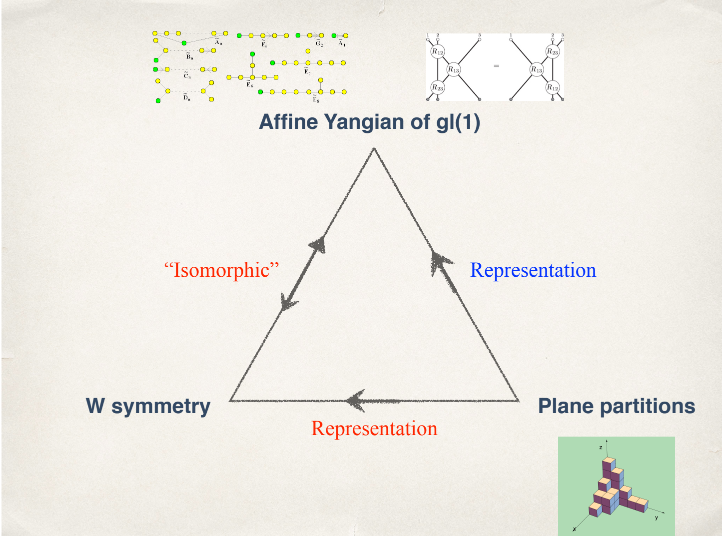
Today



A concrete relation between HS and integrability



Application: plane partition as representations of $\mathcal{W}_\infty$



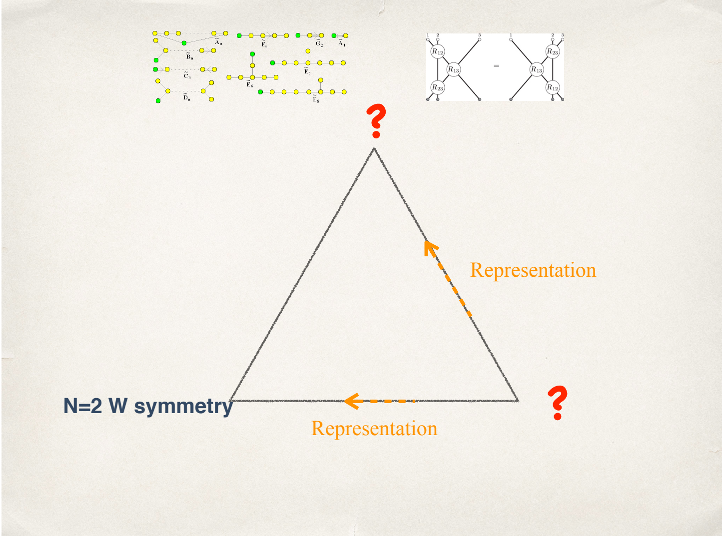
Two questions

1. Supersymmetrize $\triangle$?
2. $\triangle$ as lego pieces for new VOA/affine Yangian?

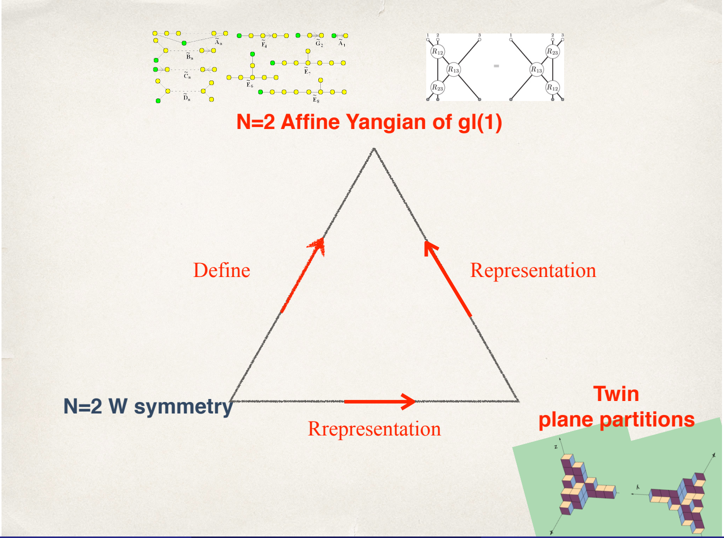
A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N} = 2$ version of $\triangle$

$\mathcal{N}=2$ version?



New Yangian algebra from W algebra



Finite truncation of affine Yangian of $\mathfrak{gl}_1$

Fukuda Matsuo Nakamura Zhu '15

Prochazka '15

- gives chiral algebra of Y-junction

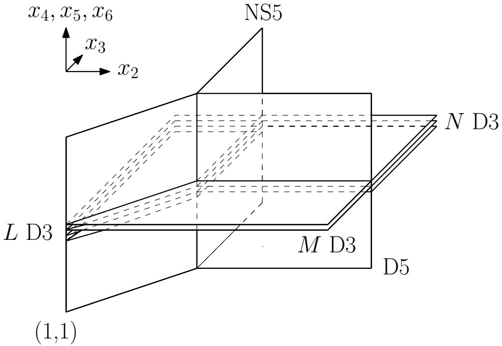
Gaiotto Rapcak '17

- Gluing of these finite truncations should give chiral algebra of Y-junction webs

Rapcak Prochazka'17

5-brane junction with D3 brane interfaces

Gaiotto Rapcak '17



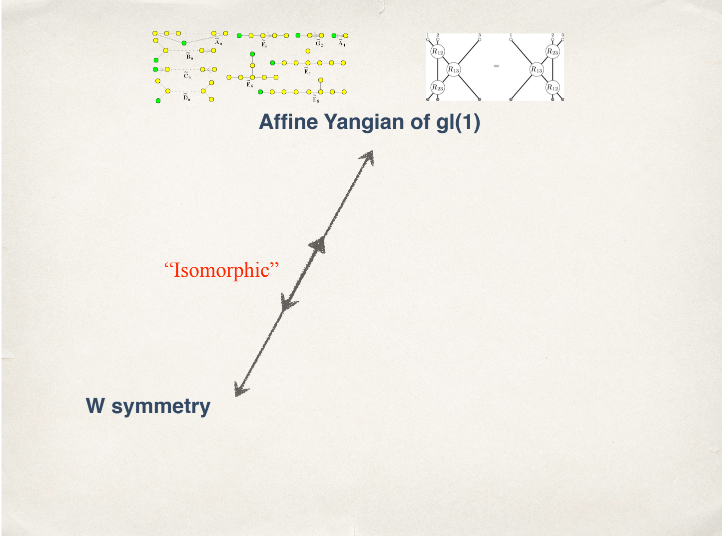
$\times \underset{x_0, x_1}{C} \times \underset{x_7, x_8, x_9}{R^3}$

picture: Gaiotto Rapcak '17

conjecture: VOA on the 2D junction of 4D QFT

Outline

Relation between W algebra and affine Yangian



Modes of $\mathcal{W}_{1+\infty}$

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W_n^{(s)}}{z^{n+s}} \qquad s = 1, 2, 3, \ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
spin-5	$\ldots$	X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	X_4	$\ldots$
spin-4	$\ldots$	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	U_4	$\ldots$
spin-3	$\ldots$	W_{-4}	W_{-3}	W_{-2}	W_{-1}	W_0	W_1	W_2	W_3	W_4	$\ldots$
spin-2	$\ldots$	L_{-4}	L_{-3}	L_{-2}	L_{-1}	L_0	L_1	L_2	L_3	L_4	$\ldots$
spin-1	$\ldots$	J_{-4}	J_{-3}	J_{-2}	J_{-1}	J_0	J_1	J_2	J_3	J_4	$\ldots$

Regrouping the modes

$$W^{(s)}(z)=\sum_{n\in\mathbb{Z}}\frac{W_n^{(s)}}{z^{n+s}}\qquad s=1,2,3,\ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
spin-5	$\ldots$	X_{-3}	X_{-2}	$X_{-1}\sim e_4$	$X_0\sim\psi_5$	$X_1\sim f_4$	X_2	X_3	X_4
spin-4	$\ldots$	U_{-3}	U_{-2}	$U_{-1}\sim e_3$	$U_0\sim\psi_4$	$U_1\sim f_3$	U_2	U_3	U_4
spin-3	$\ldots$	W_{-3}	W_{-2}	$W_{-1}\sim e_2$	$W_0\sim\psi_3$	$W_1\sim f_2$	W_2	W_3	W_4
spin-2	$\ldots$	L_{-3}	L_{-2}	$L_{-1}\sim e_1$	$L_0\sim\psi_2$	$L_1\sim f_1$	L_2	L_3	L_4
spin-1	$\ldots$	J_{-3}	J_{-2}	$J_{-1}\sim e_0$	$J_0\sim\psi_1$	$J_1\sim f_0$	J_2	J_3	J_4

affine Yangian generators

$$e(z)=\sum_{j=0}^{\infty}\frac{e_j}{z^{j+1}}\qquad \psi(z)=1+\sigma_3\sum_{j=0}^{\infty}\frac{\psi_j}{z^{j+1}}\qquad f(z)=\sum_{j=0}^{\infty}\frac{f_j}{z^{j+1}}$$

Affine Yangian of $\mathfrak{gl}_1$

Def: Associative algebra with generators e_j, f_j and ψ_j, $j = 0, 1, \ldots$

- Generators

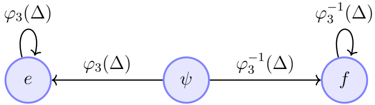
$$\psi(z) = 1 + (h_1 h_2 h_3) \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

- Parameters (h_1, h_2, h_3) with $h_1 + h_2 + h_3 = 0$
- One $\mathcal{S}_3$ invariant function $\varphi(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}$
- Defining relations

$$[e(z)\,, f(w)] = -\frac{1}{h_1 h_2 h_3} \frac{\psi(z) - \psi(w)}{z - w}$$

$$\psi(z)\, e(w) \sim \varphi(z-w)\, e(w)\, \psi(z) \qquad \psi(z)\, f(w) \sim \varphi(w-z)\, f(w)\, \psi(z)$$

$$e(z)\, e(w) \sim \varphi(z-w)\, e(w)\, e(z) \qquad f(z)\, f(w) \sim \varphi(w-z)\, f(w)\, f(z)$$



W algebra and affine Yangian

$$\mathcal{Y}[\widehat{\mathfrak{gl}_1}] \cong \text{UEA}[\mathcal{W}_{1+\infty}[\lambda]]$$

Procházka '15

Gaberdiel Gopakumar Li Peng '17

for q-version $\mathcal{U}[\widehat{\widehat{\mathfrak{gl}_1}}] \cong \text{UEA}[\text{q-}\mathcal{W}_{1+\infty}[\lambda]]$

Miki '07

Feigin Jimbo Miwa Mukhin '10-11

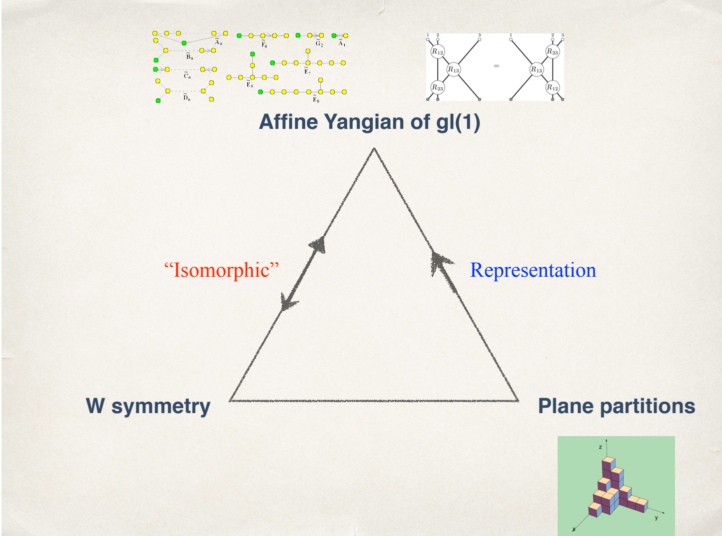
Advantages of affine Yangian over $\mathcal{W}_\infty$

1. number of generators
   - $\mathcal{W}_\infty$: ∞
     $$J(z)\,,T(z)\,,W^{(3)}(z)\,,W^{(4)}(z)\,\ldots$$
   - affine Yangian of $\mathfrak{gl}_1$: only 3
     $$\psi(z)\,,e(z)\,,f(z)$$
2. Defining relations
   - $\mathcal{W}_\infty$:

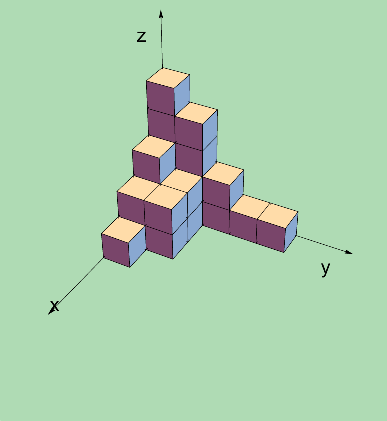
     non-linear, fixed order by order by Jacobi-identities
   - affine Yangian of $\mathfrak{gl}_1$:

     linear, given explicitly
3. $\mathcal{S}_3$ invariance
   - $\mathcal{W}_\infty$: Hidden
   - affine Yangian of $\mathfrak{gl}_1$: manifest

Plane partition as representations of affine Yangian

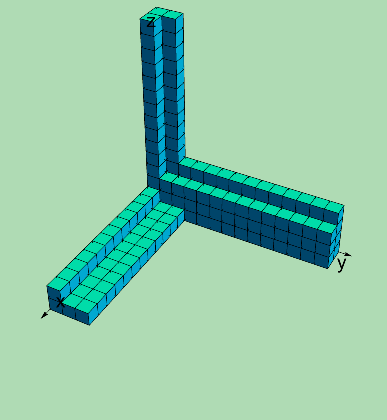


Plane partition via box stacking



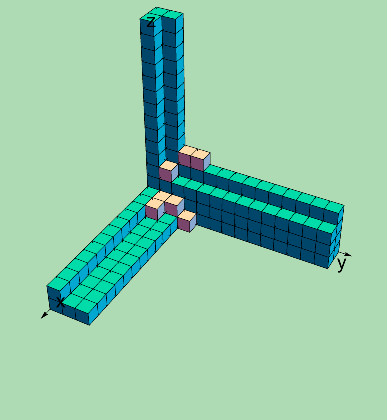
Plane partition with non-trivial asymptotics

Ground state of $(\Lambda_x, \Lambda_y, \Lambda_z)$

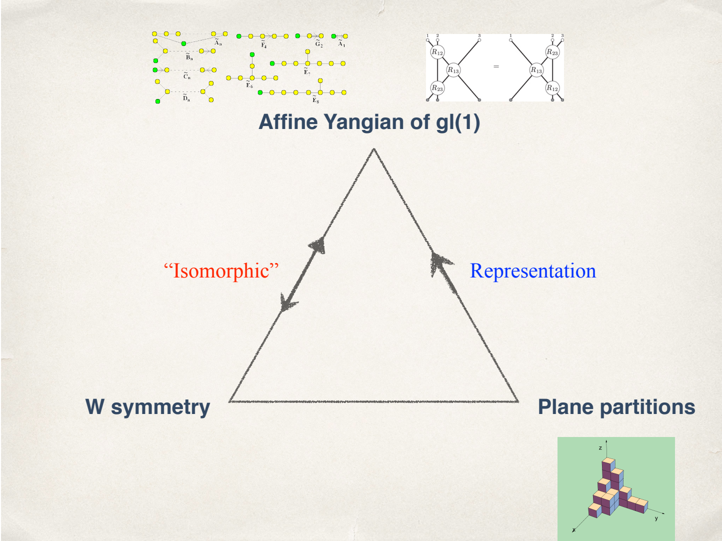


Plane partition with non-trivial asymptotics

a level-7 excited states of $(\Lambda_x, \Lambda_y, \Lambda_z)$



Plane partitions are faithful representations of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$



Action of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$ on a plane partition

- $\psi(z)$ acts diagonally *Tsymbaliuk '14, Prochazka '15*

$$\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$$

$$\psi_\Lambda(z) \equiv \left(1+\frac{\psi_0\sigma_3}{z}\right) \prod_{\square\in(\Lambda)} \varphi(z-h(\square))$$

$$h(\square) = h_1 x(\square) + h_2 y(\square) + h_3 z(\square)$$

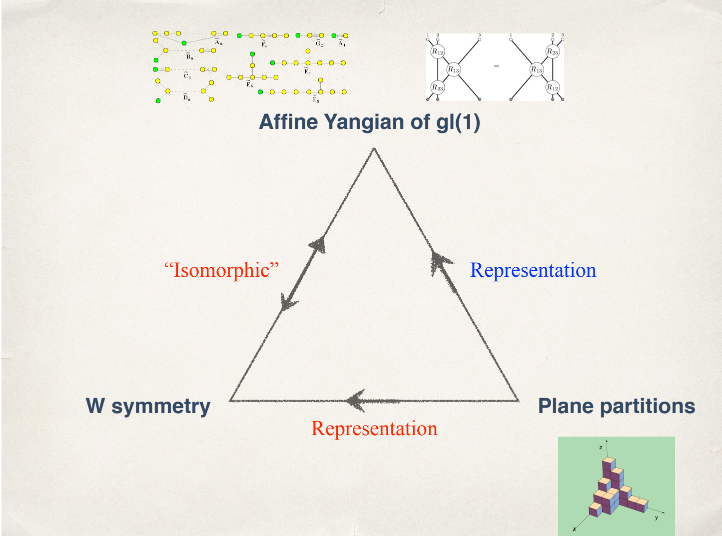
- $e(z)$ adds one box

$$e(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Add}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda+\square\rangle$$

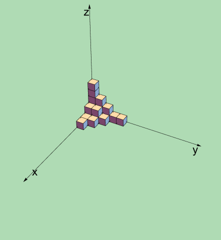
- $f(z)$ removes one box

$$f(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Rem}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda-\square\rangle$$

plane partition as representations

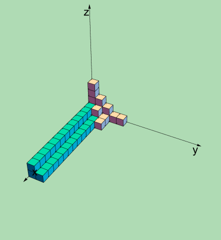


Plane partition as representations of W



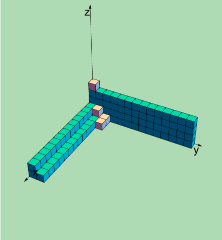
Trivial b.c.

vacuum



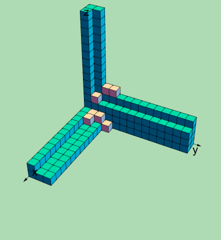
$(\Lambda_x;0)=(\Lambda;0)$

perturbative
in Vasiliev



$(\Lambda_x;\Lambda_y)=(\Lambda_+;\Lambda_-)$

non-perturbative
in Vasiliev

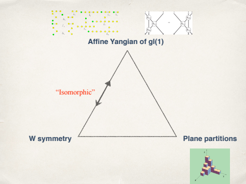


$(\Lambda_x;\Lambda_y;\Lambda_z)$

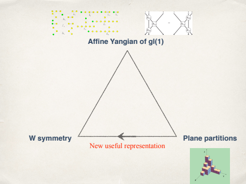
new representation

character of $\mathcal{W}_{1+\infty}$= generating function of plane partition

Application



- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



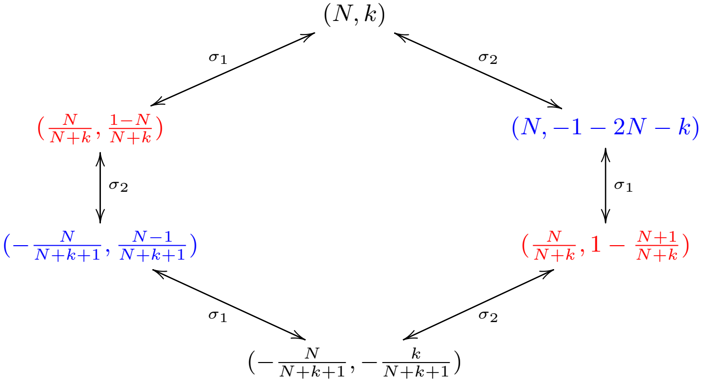
- Character computation more transparent

$\mathcal{S}_3$ action on $\mathcal{W}_{N,k}$ coset

$\mathcal{W}_{N,k}$ coset

$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

had hidden $\mathcal{S}_3$

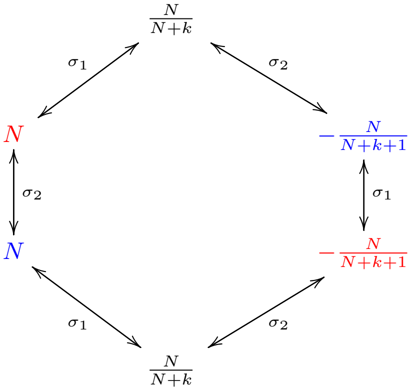


$\mathcal{S}_3$ action on 't Hooft coupling

$\mathcal{W}_{N,k}$ coset

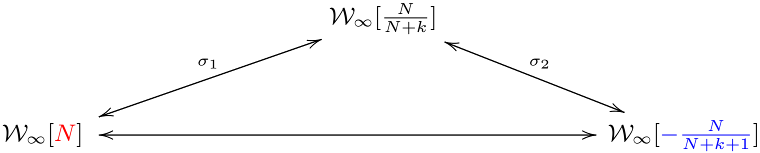
$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

't Hooft coupling $\lambda = \frac{N}{N+k}$ transform under $\mathcal{S}_3$



Triality symmetry for higher spin holography

For fixed c, three $\mathcal{W}_\infty[\lambda]$ are isomorphic *Gaberdiel Gopakumar '12*



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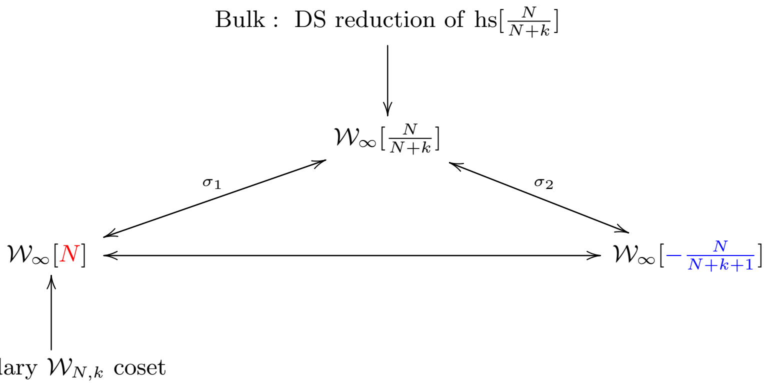


Crucial in Higher spin $\mathrm{AdS}_3/\mathrm{CFT}_2$ (Vasiliev theory in $\mathrm{AdS}_3 = \mathcal{W}_{N,k}$ coset)

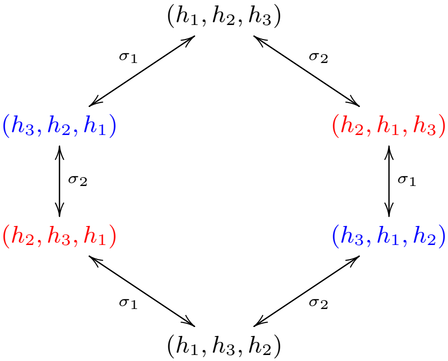
- $\mathcal{S}_3$ symmetry in $\mathcal{W}_\infty$CFT is highly non-trivial
  - hard to check/prove *Gaberdiel Gopakumar '12, Linshaw '17*
  - UV — IR
- Manifest in $\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$

$\mathcal{Y}[\widehat{\mathfrak{gl}}_1]$ depends on (h_1, h_2, h_3) symmetrically

$$h_1 = -\sqrt{\frac{N+k+1}{N+k}} \qquad h_2 = \sqrt{\frac{N+k}{N+k+1}} \qquad h_3 = \frac{1}{\sqrt{(N+k)(N+k+1)}}$$

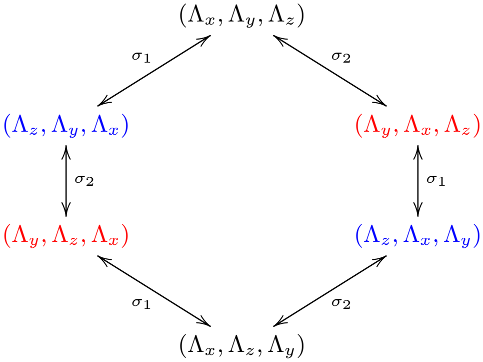
Procházka '15, *Gaberdiel Gopakumar Li Peng '17*

Under $\mathcal{S}_3$ transformation on (N, k)

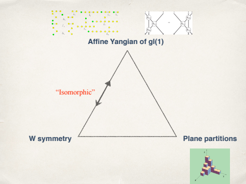


$\mathcal{S}_3$ symmetry of plane partition

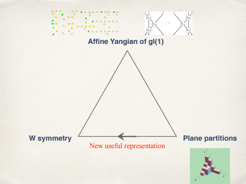
The representations of $\mathcal{W}_\infty$ comes in $\mathcal{S}_3$ family



Application



- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

Outline

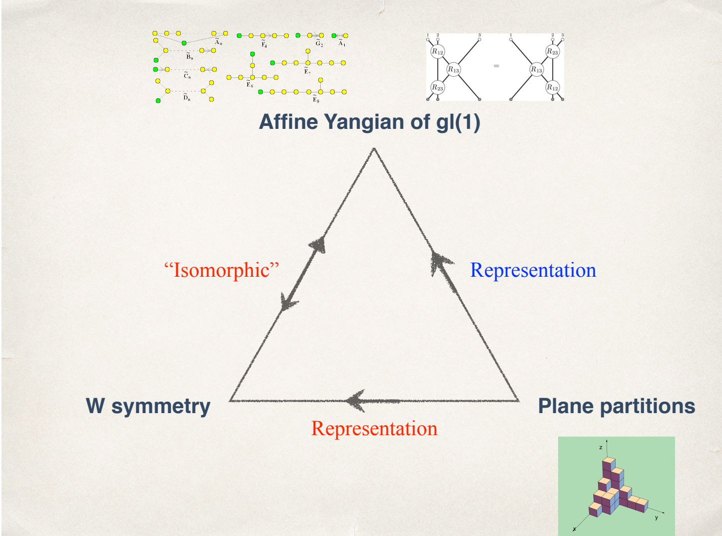
Intro

W—Affine Yangian—Plane Partition

Gluing and $\mathcal{N} = 2$ affine Yangian

Summary

Bosonic W and affine Yangian



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Rapcak Prochazka '17, Gaberdiel Li Peng Zhang'17

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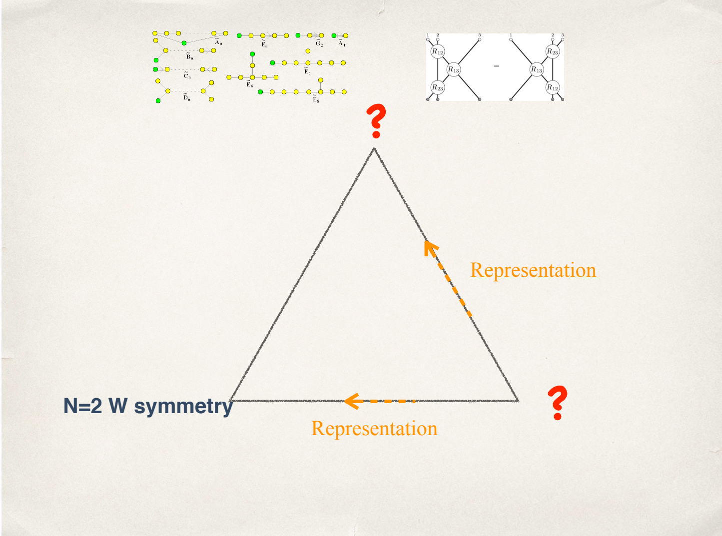
Gaberdiel Li Peng Zhang'17

$\mathcal{N}=2$ version?



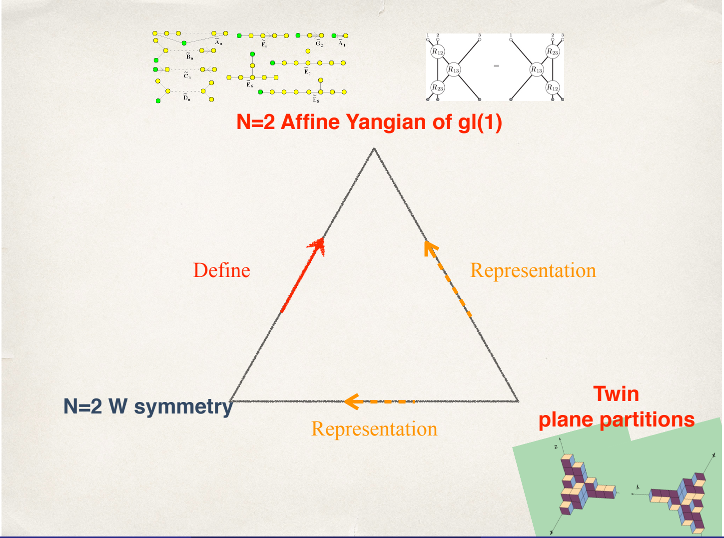
Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2$ $\mathcal{W}_\infty$ in terms of (some version) of plane partitions

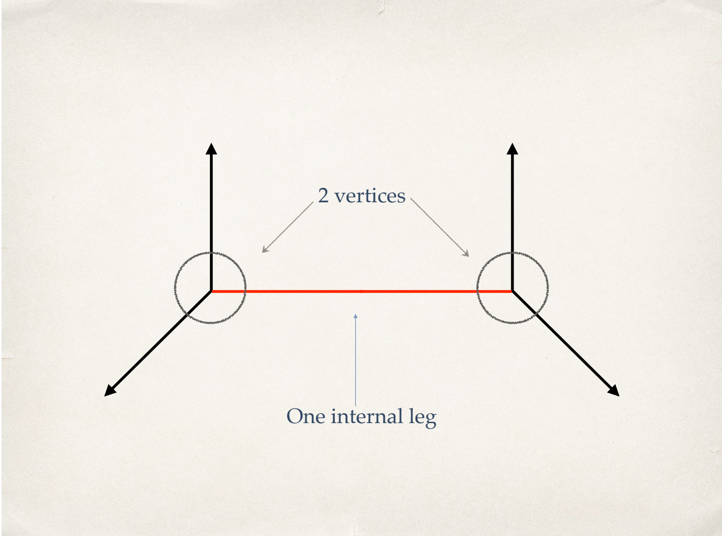
Twin plane partition

2. Define $\mathcal{N}=2$ affine Yangian such that
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2$ $\mathcal{W}_\infty$ charges

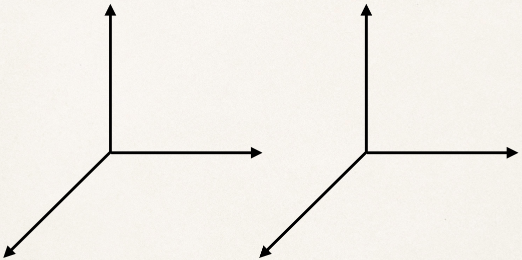
$\mathcal{N}=2$ version



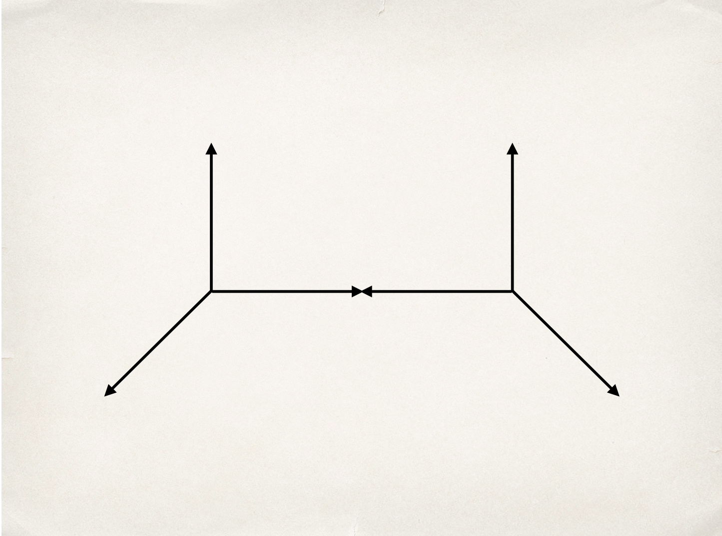
Simplest gluing: 2 vertices and 1 internal leg



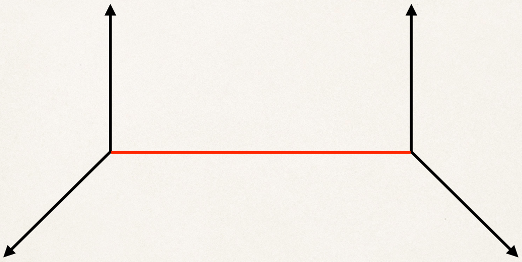
Two copies: left and right



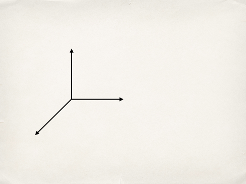
Gluing: two external legs facing opposite directions



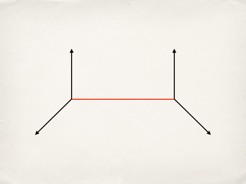
Gluing: two external legs fuse and become internal leg



Building blocks and gluing

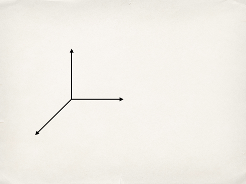


1. Algebra: $\mathcal{W}_{1+\infty} \Rightarrow$ affine Yangian of $\mathfrak{gl}_1$
2. Representation: plane partitions

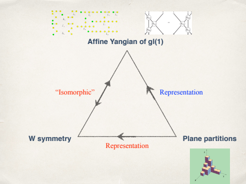


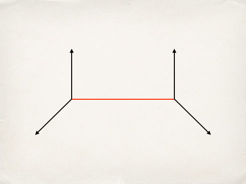
1. Algebra: internal leg $\Rightarrow$ additional operators
2. Representation:
   bi-module: change b.c. for both vertices

Building blocks and gluing

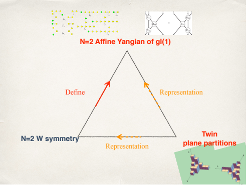


$\Longrightarrow$





$\Longrightarrow$



$\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ algebra

One $\mathcal{N}=2$ multiplet per spin

Creutzig, Hikida, Ronne '11

Candu Gaberdiel '12

$$\begin{pmatrix} & T & \\ G^- & & G^+ \\ & J & \end{pmatrix} \quad \begin{pmatrix} & W^{(2)1} & \\ W^{(2)-} & & W^{(2)+} \\ & W^{(2)0} & \end{pmatrix} \quad \begin{pmatrix} & W^{(3)1} & \\ W^{(3)-} & & W^{(3)+} \\ & W^{(3)0} & \end{pmatrix} \dots$$

Rearrange by spin

spin									
4								$W^{(3)1}$	
$\frac{7}{2}$							$W^{(3)-}$		$W^{(3)+}$
3					$W^{(2)1}$			$W^{(3)0}$	
$\frac{5}{2}$				$W^{(2)-}$		$W^{(2)+}$			
2		T			$W^{(2)0}$				
$\frac{3}{2}$	G^-		G^+						
1		J							

$\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ algebra

One $\mathcal{N}=2$ multiplet per spin

Creutzig, Hikida, Ronne '11

Candu Gaberdiel '12

$$\begin{pmatrix} & T & \\ G^- & & G^+ \\ & J & \end{pmatrix} \quad \begin{pmatrix} & W^{(2)1} & \\ W^{(2)-} & & W^{(2)+} \\ & W^{(2)0} & \end{pmatrix} \quad \begin{pmatrix} & W^{(3)1} & \\ W^{(3)-} & & W^{(3)+} \\ & W^{(3)0} & \end{pmatrix} \dots$$

Rearrange by spin

spin											
4										$W^{(3)1}$	
$\frac{7}{2}$									$W^{(3)-}$		$W^{(3)+}$
3							$W^{(2)1}$			$W^{(3)0}$	
$\frac{5}{2}$						$W^{(2)-}$		$W^{(2)+}$			
2			T				$W^{(2)0}$				
$\frac{3}{2}$		G^-		G^+							
1	U		J								

$\mathfrak{u}(1) \oplus \mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has 2 fields per spin

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

$$
\begin{array}{ccccl}
\text{Vasiliev shs}[\lambda] & \supset & \text{hs}[\lambda]\oplus\text{hs}[1-\lambda] & & \textit{Prokushkin Vasiliev '98} \\
\text{wedge subalgbra}\uparrow & & \downarrow & & \\
\mathcal{N}=2 \quad \mathcal{W}_\infty[\lambda] & \supset & \mathcal{W}_\infty[\lambda]\oplus\mathcal{W}_\infty[1-\lambda] & & \\
\downarrow\text{Truncation} & & \uparrow & & \\
\mathcal{N}=2 \quad \mathcal{W}_3 & \supset & \text{Virasoro}\oplus\text{Virasoro} & & \textit{Romans '92}
\end{array}
$$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

$$\text{Vasiliev shs}[\lambda] \supset \text{hs}[\lambda] \oplus \text{hs}[1-\lambda]$$ *Prokushkin Vasiliev '98*

wedge subalgbra ↑ ↓

$$\mathcal{N}=2 \quad \mathcal{W}_\infty[\lambda] \supset \mathcal{W}_\infty[\lambda] \oplus \mathcal{W}_\infty[1-\lambda]$$

↓ Truncation ↑

$$\mathcal{N}=2 \quad \mathcal{W}_3 \supset \text{Virasoro} \oplus \text{Virasoro}$$ *Romans '92*

- Checked up to $s=3$ (non-trivial!) *Gaberdiel Li Peng Zhang '17*

to appear

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — fermionic part

4									$W^{(3)1}$	
$\frac{7}{2}$								$W^{(3)-}$		$W^{(3)+}$
3						$W^{(2)1}$			$W^{(3)0}$	
$\frac{5}{2}$					$W^{(2)-}$		$W^{(2)+}$			
2			T			$W^{(2)0}$				
$\frac{3}{2}$		G^-		G^+						
1	U		J							

- Bosonic sub-algebra

$$\boxed{\mathfrak{u}(1) \oplus \mathcal{W}_\infty^{\mathcal{N}=2}[\lambda] \supset \mathcal{W}_{1+\infty}[\lambda] \oplus \mathcal{W}_{1+\infty}[1-\lambda]}$$

- How do fermions fit in?

Decomposing $\mathcal{W}_{\infty}^{\mathcal{N}=2}[\lambda]$ vacuum character

- Vacuum character of $\mathfrak{u}(1) \oplus \mathcal{W}_{\infty}^{\mathcal{N}=2}[\lambda]$

$$\chi_0^{\text{Full}}(q,y) = \prod_{n=1}^{\infty} \frac{(1+yq^{n+\frac{1}{2}})^n(1+\frac{1}{y}q^{n+\frac{1}{2}})^n}{(1-q^n)^{2n}}$$

$$= \chi_{\text{pp}}(q)\left(\sum_{\text{R}} y^{|\text{R}|}\chi_{\text{R}}^{(\text{wedge})\,[\lambda]}(q)\cdot\chi_{\bar{\text{R}}^T}^{(\text{wedge})\,[1-\lambda]}(q)\right)$$

$$\cdot\left(\sum_{\text{S}} \frac{1}{y^{|\text{S}|}}\chi_{\bar{\text{S}}}^{(\text{wedge})\,[\lambda]}(q)\cdot\chi_{\text{S}^T}^{(\text{wedge})\,[1-\lambda]}(q)\right)\chi_{\text{pp}}(q)$$

- Fermions transform as

$$(\rho, \overline{\rho^t}) \qquad\qquad (\overline{\rho^t}, \rho)$$

of $\mathcal{W}_{1+\infty}[\lambda] \oplus \mathcal{W}_{1+\infty}[1-\lambda]$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$

Gaberdiel Li Peng Zhang '17

1. Bosonic sub-algebra

$$\mathcal{W}_{1+\infty}[\lambda] \qquad \oplus \qquad \mathcal{W}_{1+\infty}[1-\lambda]$$

2. Fermions:

$$(\rho, \overline{\rho^t}) \qquad\qquad (\overline{\rho^t}, \rho)$$

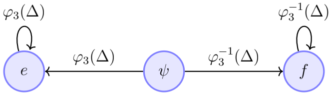
How to translate these into affine Yangian ?

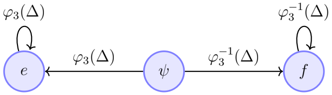
Decomposing $\mathcal{N}=2\ \mathcal{W}_\infty[\lambda]$

Gaberdiel Li Peng Zhang '17

1. Bosonic sub-algebra

$$\begin{array}{ccc} \mathcal{W}_{1+\infty}[\lambda] & \oplus & \mathcal{W}_{1+\infty}[1-\lambda] \\ \Downarrow & & \Downarrow \\ \widehat{\mathcal{Y}(\mathfrak{gl}_1)} & \oplus & \widehat{\mathcal{Y}(\mathfrak{gl}_1)} \end{array}$$





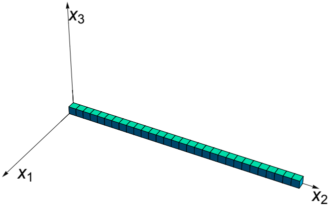
Left plane partition | right plane partition

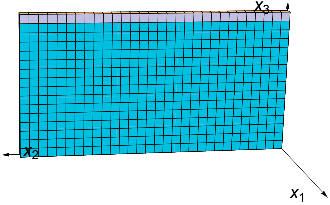
2. Fermions: internal legs $\Longrightarrow$ additional operators

$(\rho, \overline{\rho^t})$ $(\overline{\rho^t}, \rho)$

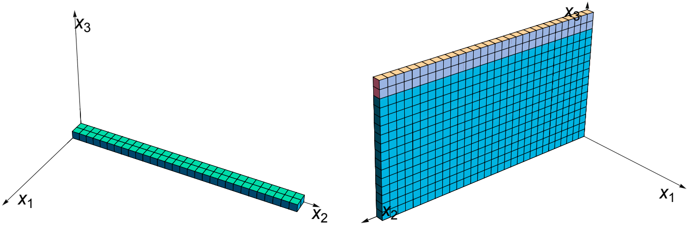
How to translate these into affine Yangian ?

$(\square, \bar{\square})$ connecting two plane partitions

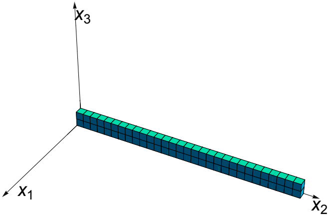


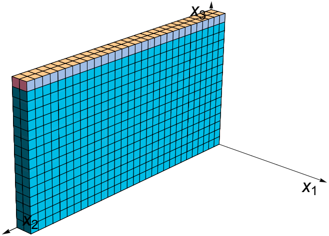


$(\square\square, \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}})$ connecting two plane partitions

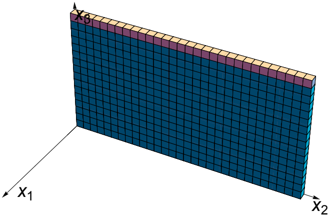


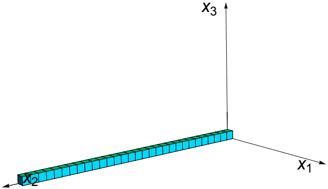
$(\yng(1,1), \overline{\yng(2)})$ connecting two plane partitions





$(\bar{\Box}, \Box)$ connecting two plane partitions

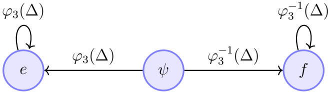


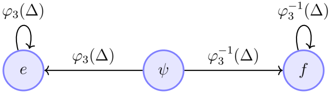


What are the generators of internal leg?

From plane partition building blocks to yangian generators

Bosonic sub-algebra $\quad \widehat{\mathcal{Y}(\mathfrak{gl}_1)} \quad \oplus \quad \widehat{\mathcal{Y}(\mathfrak{gl}_1)}$





- ψ: Cartan of left $\widehat{\mathcal{Y}(\mathfrak{gl}_1)}$
- e/f: adds/removes $\square$

- $\hat{\psi}$: Cartan of right $\widehat{\mathcal{Y}(\mathfrak{gl}_1)}$
- $\hat{e}/\hat{f}$: adds/removes $\widehat{\square}$

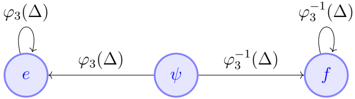
Fermions = internal legs = additional operators

- x/y: adds/removes $\quad \blacksquare \equiv (\square, \overline{\square})$
- $\bar{x}/\bar{y}$: adds/removes $\quad \overline{\blacksquare} \equiv (\square, \overline{\square})$

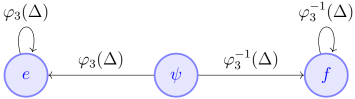
Gaberdiel Li Peng Zhang '17

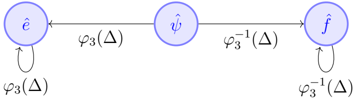
Building blocks of bosonic affine Yangian of $\mathfrak{gl}_1$

Building blocks of bosonic affine Yangian of $\mathfrak{gl}_1$

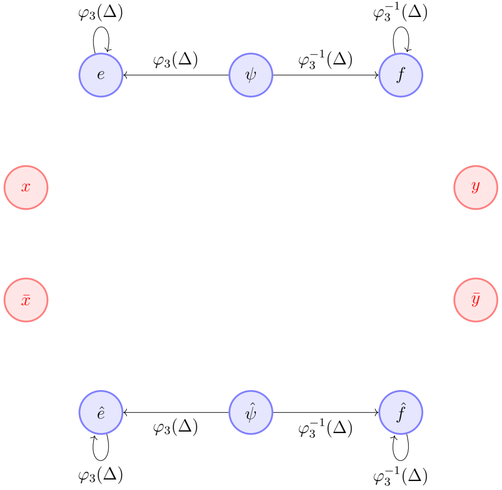


A pair of bosonic affine Yangian of $\mathfrak{gl}_1$





Building blocks of $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$



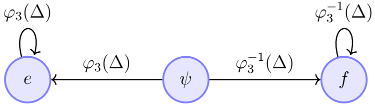
Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2$ $\mathcal{W}_\infty$ in terms of (some version) of plane partitions

   Twin plane partition

2. Define $\mathcal{N}=2$ affine Yangian such that
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2$ $\mathcal{W}_\infty$ charges

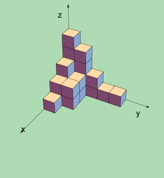
Bosonic affine Yangian: $\varphi_3(z)$ plays central role



$$\psi(z)\,e(w) \sim \varphi_3(z-w)\,e(w)\,\psi(z) \qquad \psi(z)\,f(w) \sim \varphi_3(w-z)\,f(w)\,\psi(z)$$

$$e(z)\,e(w) \sim \varphi_3(z-w)\,e(w)\,e(z) \qquad f(z)\,f(w) \sim \varphi_3(w-z)\,f(w)\,f(z)$$

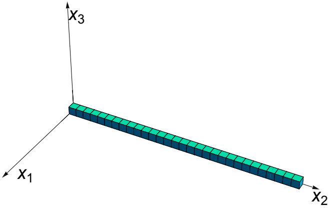
$$\boxed{\varphi_3(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}}$$

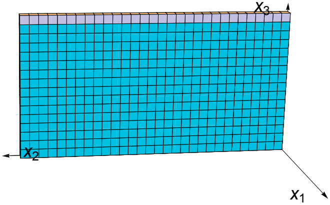


- $\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$

$$\boxed{\psi_\Lambda(z) \equiv \left(1+\frac{\psi_0\sigma_3}{z}\right)\prod_{\Box\in\Lambda}\varphi_3(z-h(\Box))}$$

Internal leg: $\varphi_2(z)$ build directly from $\varphi_2(z)$





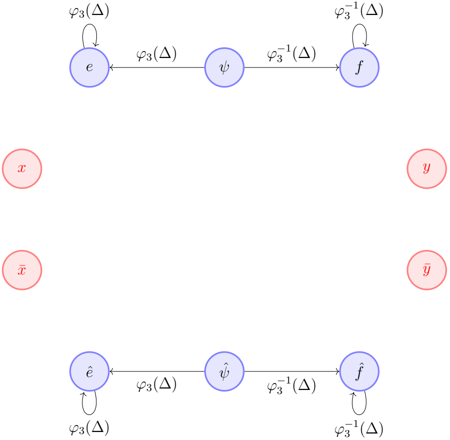
$$\begin{cases} \psi(z) &= \left(1+\frac{\psi_0\sigma_3}{z}\right)\prod_{n=0}^{\infty}\varphi_3(z-nh_2) = \left(1+\frac{\psi_0\sigma_3}{z}\right)\varphi_2(z) \\ \hat{\psi}(z) &= \left(1+\frac{\psi_0\sigma_3}{z}\right)\varphi_2^{-1}(-z-\sigma_3\hat{\psi}_0) \end{cases}$$

$$\boxed{\varphi_2(z) = \frac{z(z+h_2)}{(z-h_1)(z-h_3)}}$$

Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

Gaberdiel Li Peng Zhang'17

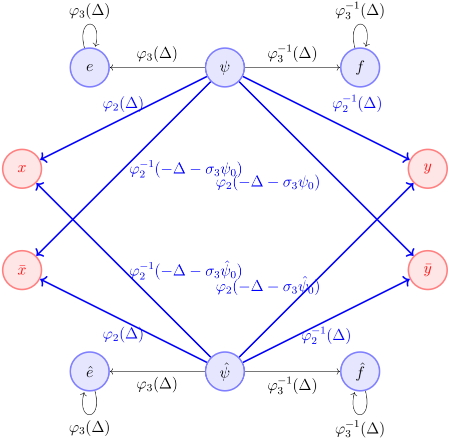
Gaberdiel Li Peng '18



Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

Gaberdiel Li Peng Zhang'17

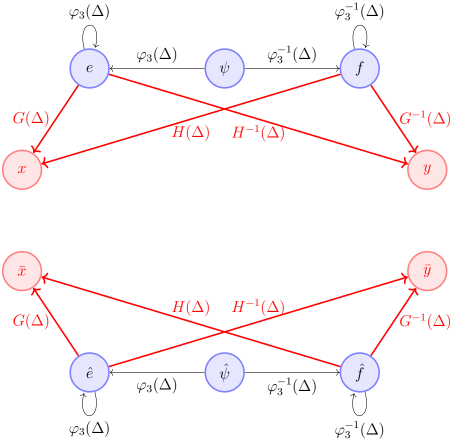
Gaberdiel Li Peng '18



Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

Gaberdiel Li Peng Zhang'17

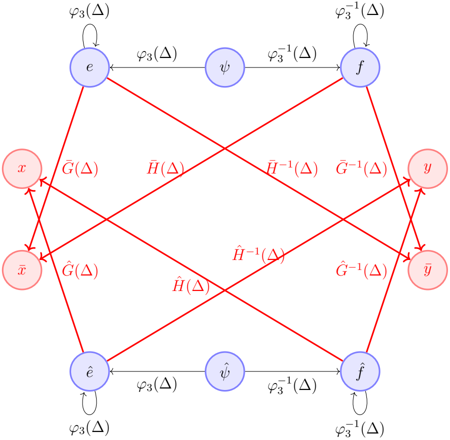
Gaberdiel Li Peng '18

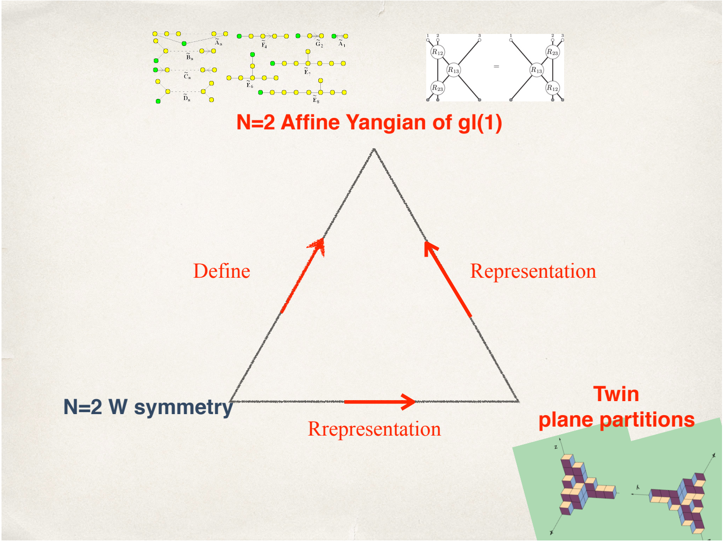


Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

Gaberdiel Li Peng Zhang'17

Gaberdiel Li Peng '18




N=2 Affine Yangian of gl(1)
Define
Representation
N=2 W symmetry
Rrepresentation
Twin
plane partitions

Lessons

- plane partition is also very useful in the gluing process
  - visualize Fock space
  - Define algebra by faithful representation

Outline

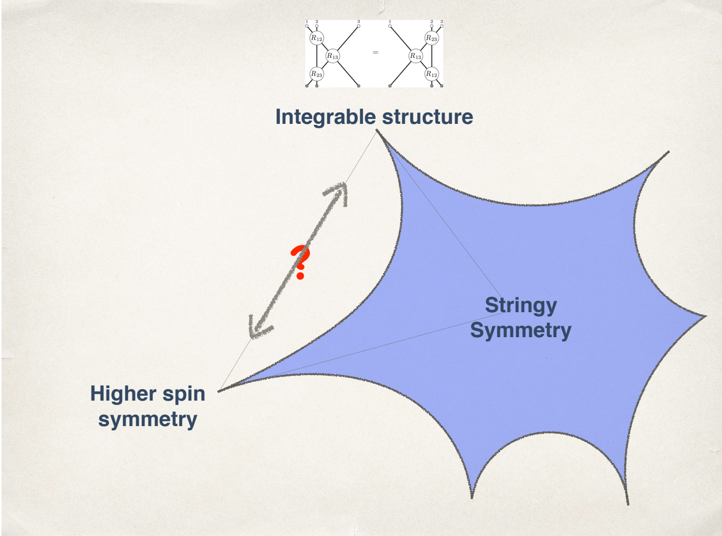
Intro

W—Affine Yangian—Plane Partition

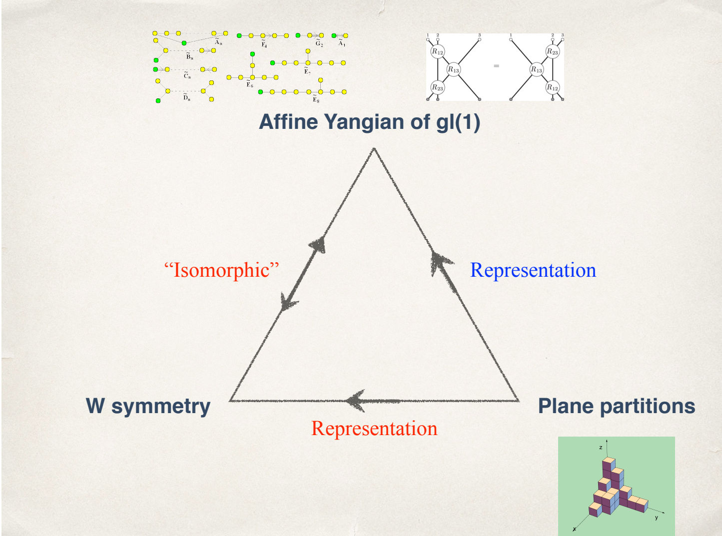
Gluing and $\mathcal{N}=2$ affine Yangian

Summary

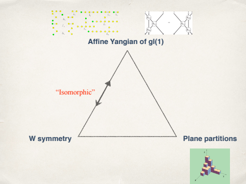
HS and integrability within stringy symmetry



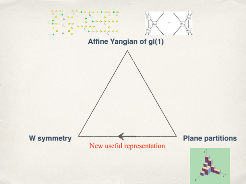
W — affine Yangian — Plane partition



Applications of bosonic triangle

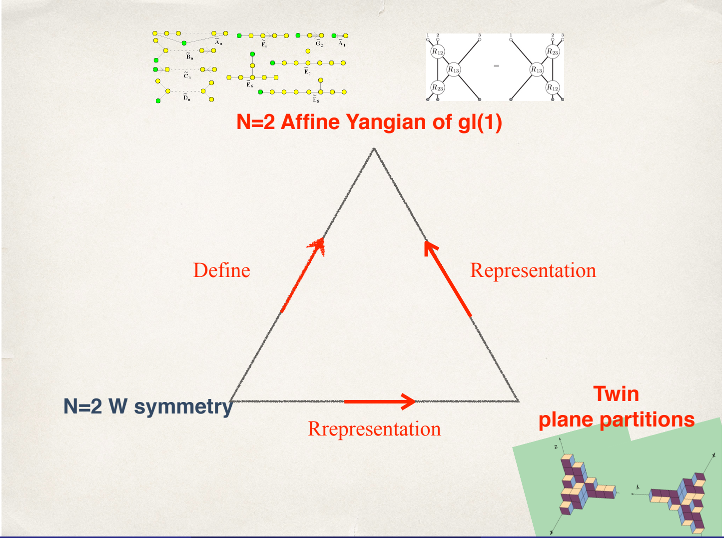


- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

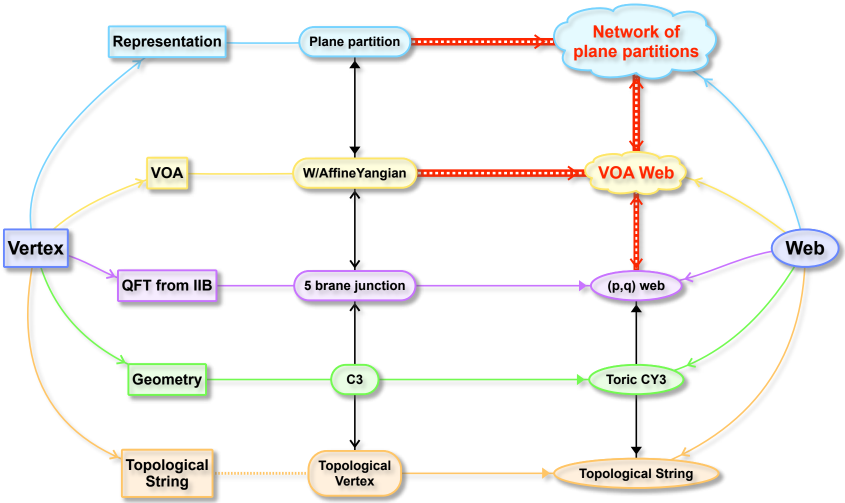
New affine Yangian via gluing



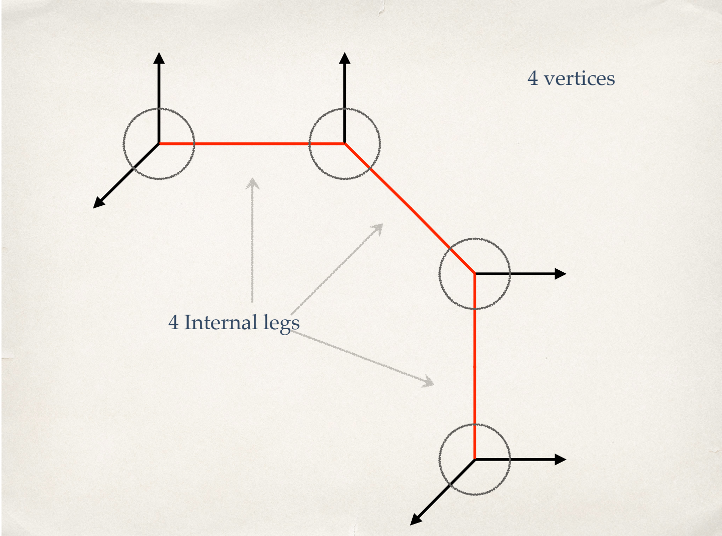
Open problems

1. large $\mathcal{N}=4$ $\mathcal{W}_\infty[\lambda]$
2. Classification of affine Yangians from gluing
3. Gluing of finite truncations

chiral algebra of the (p, q) web



Gluing example: 4 vertices and 3 internal legs



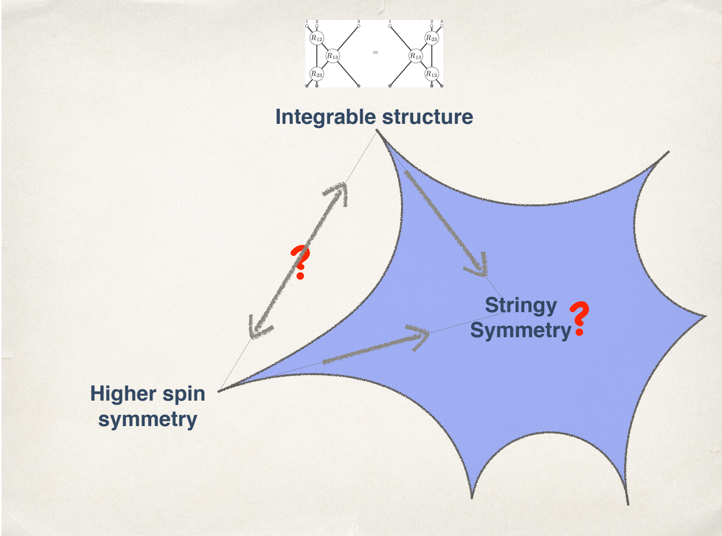
More open problems

1. Deeper relation between higher spin symmetry and integrable structure ?

2. What is stringy symmetry?

3. Application of stringy symmetry?

Different manifestation of stringy symmetry



Thank you very much !