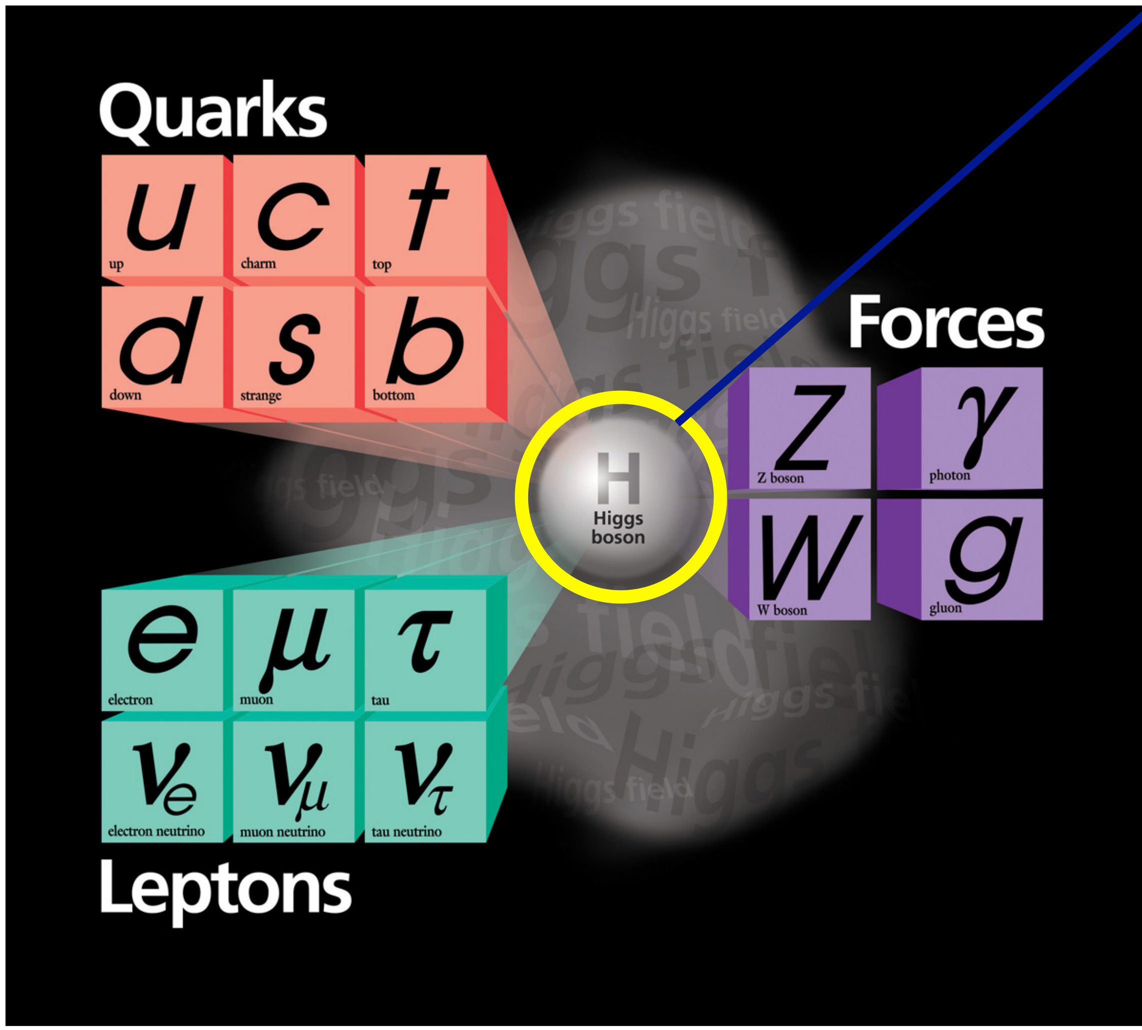
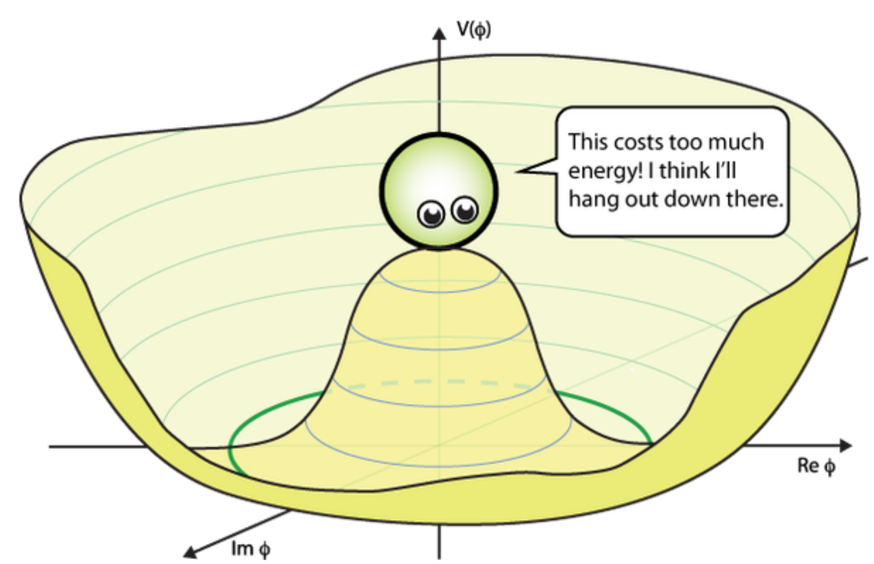
Precision Higgs Physics at LHC and future Higgs Factories

Li Lin Yang
Peking University

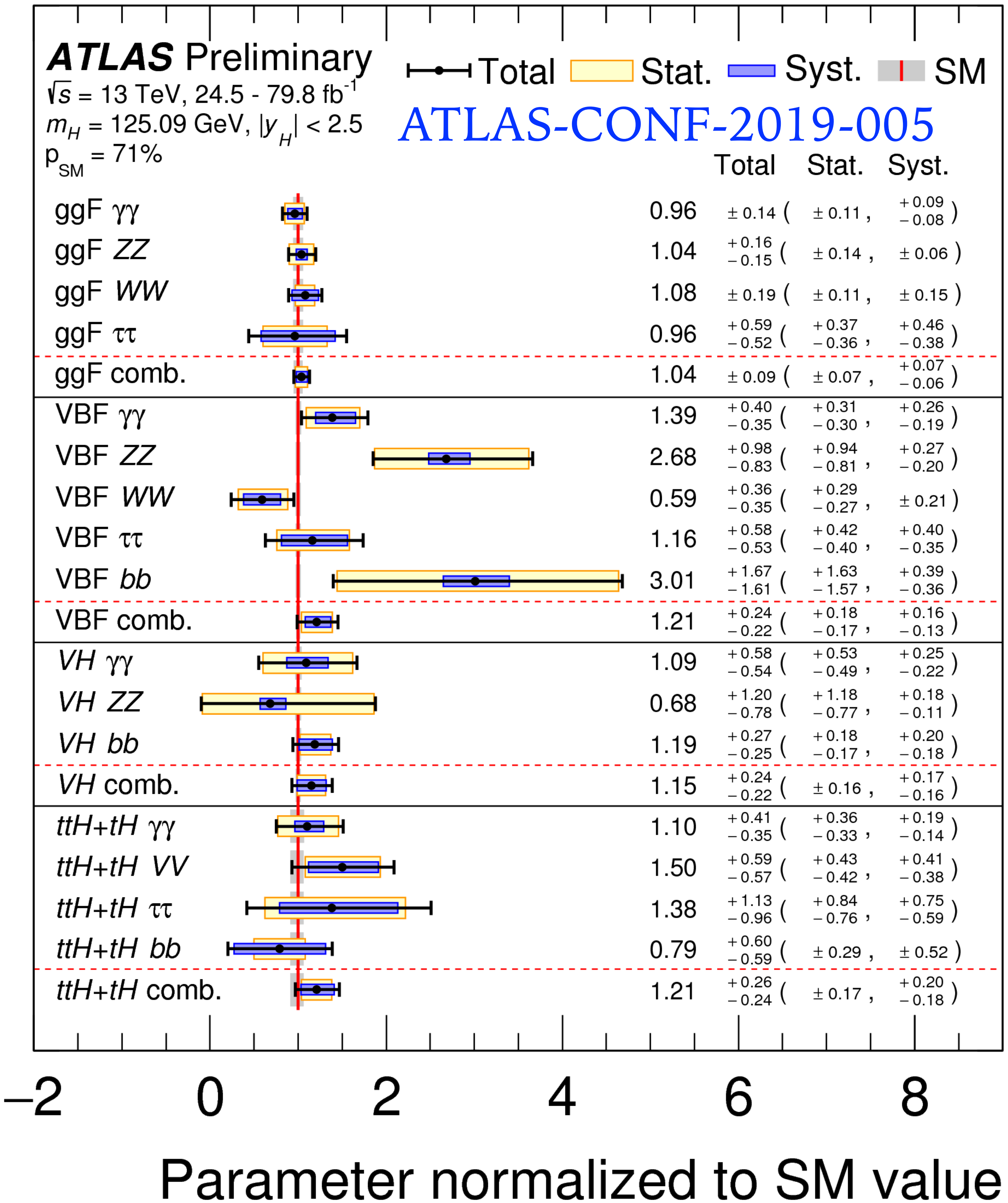
The topic

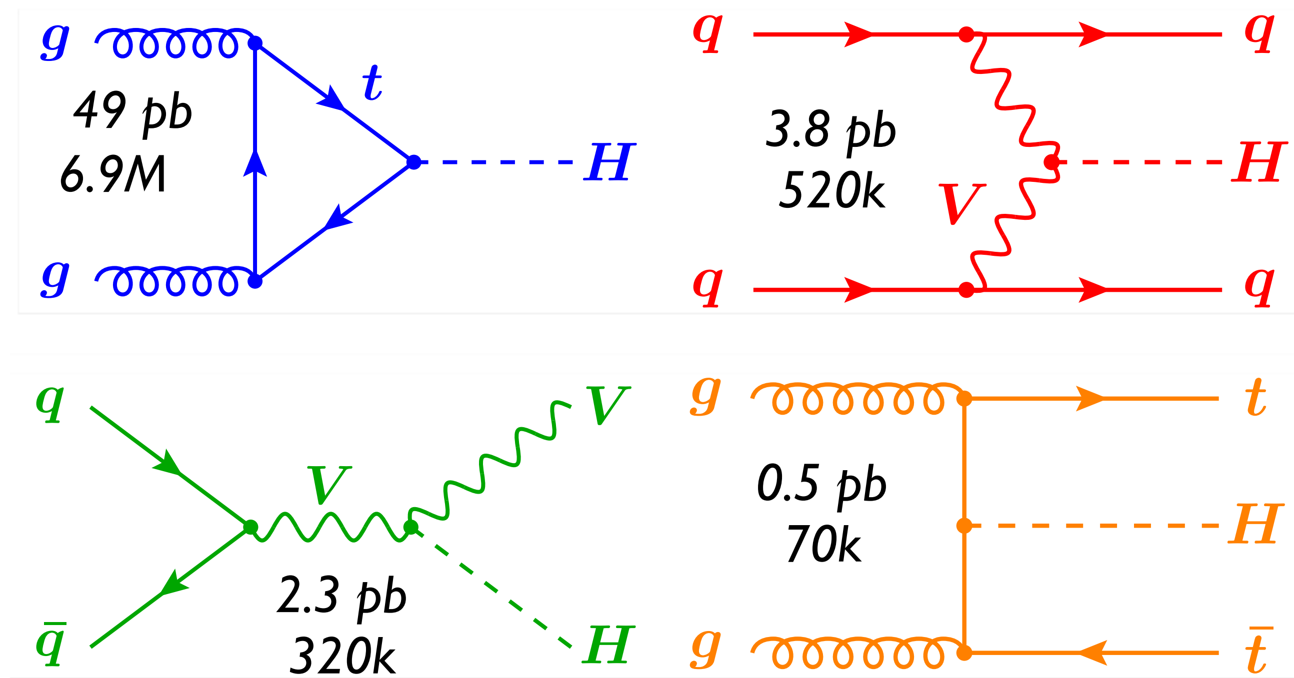


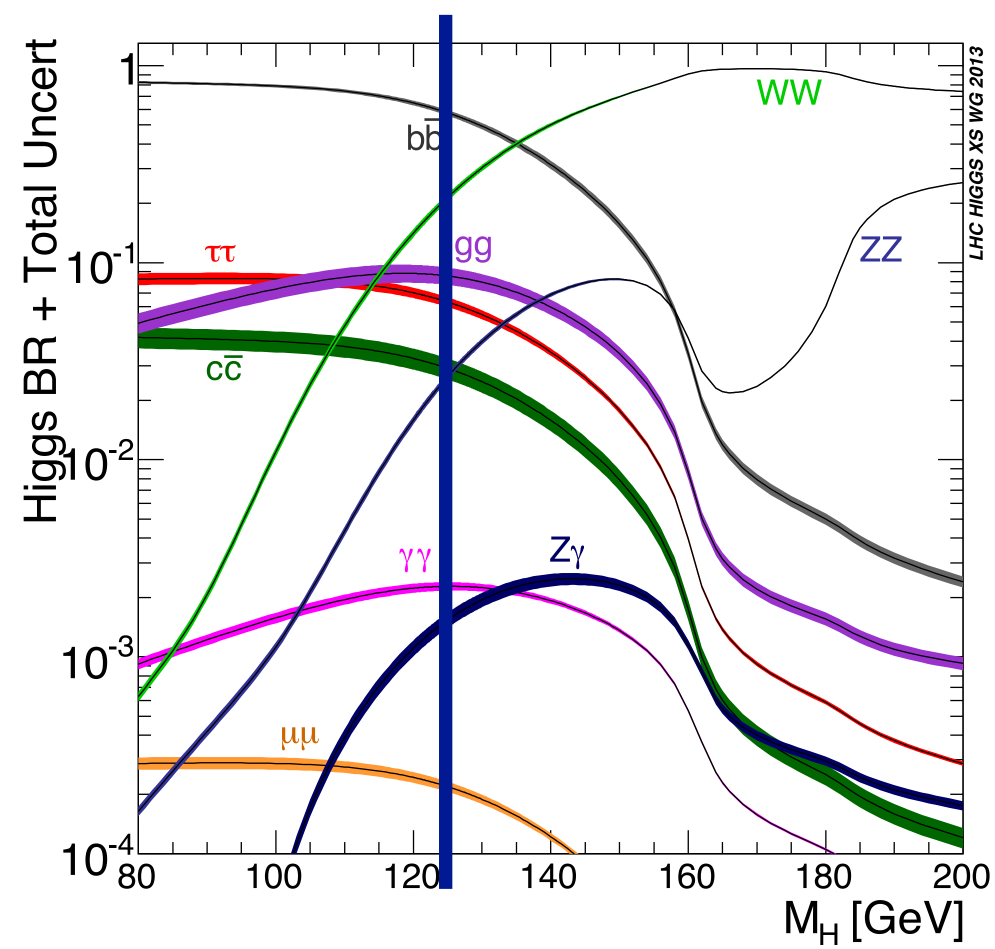
- The "God" particle
- Discovered in 2012 at LHC
- Electroweak symmetry breaking
- Quark masses and lepton masses



A key quest of LHC: Higgs properties

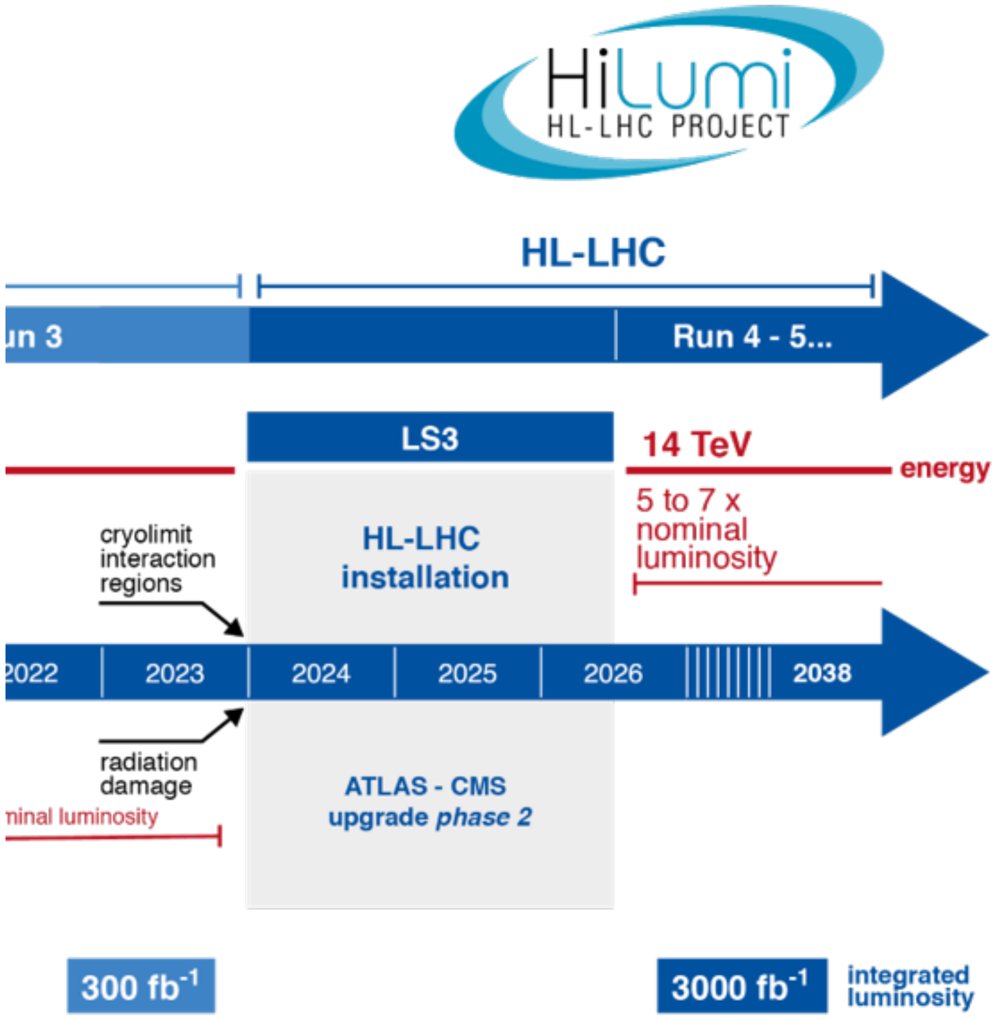






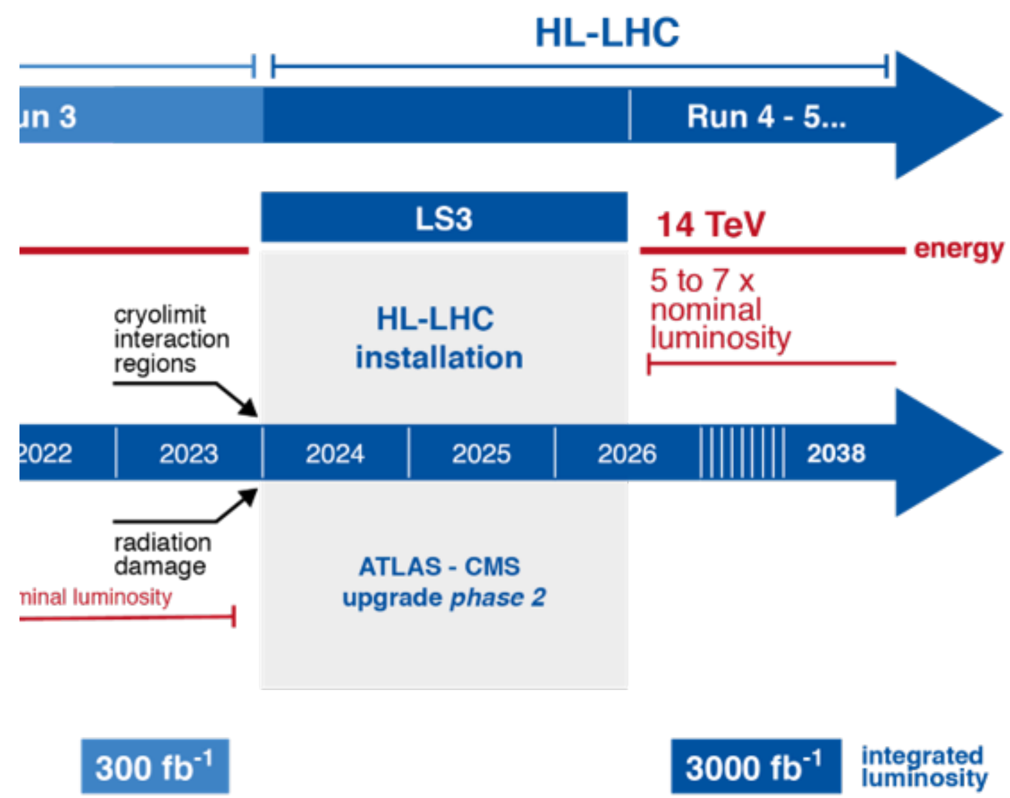
Future plans

Future plans



Future plans

HiLumi
HL-LHC PROJECT



What do we expect to gain?

The Higgs potential

Key to electroweak phase transition, vacuum stability, etc.

But: LHC (currently) can only tell us very limited information!

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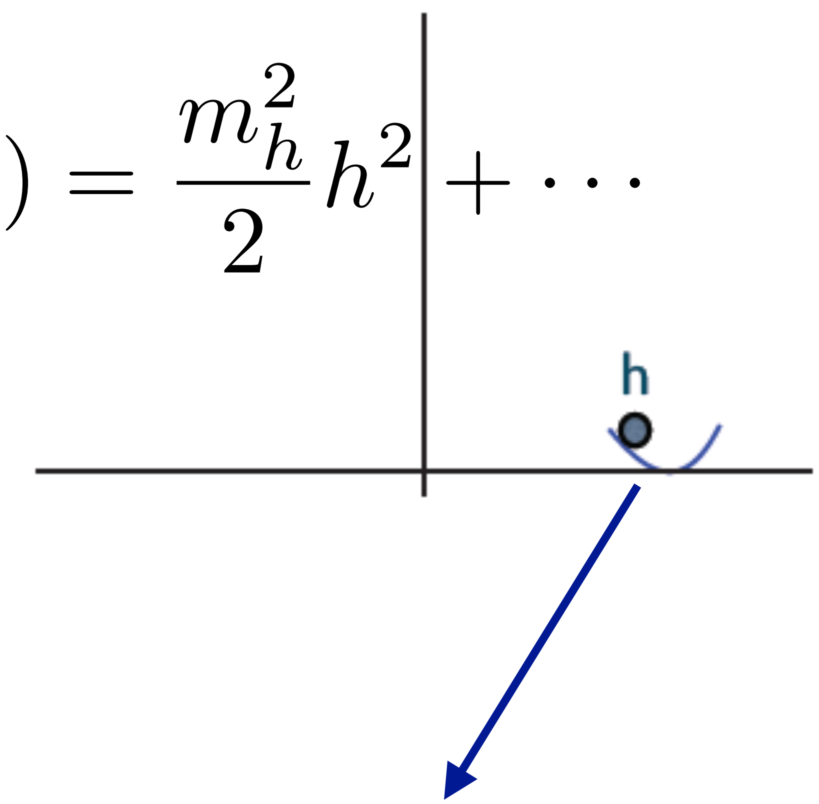
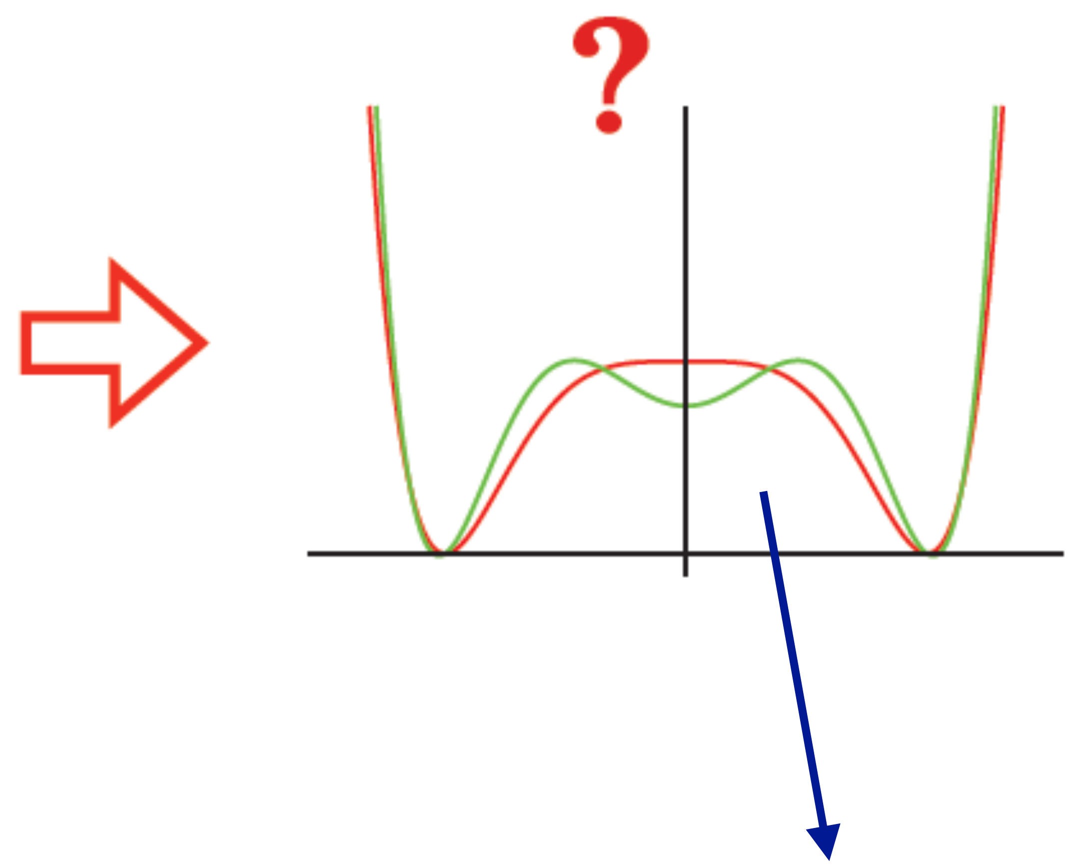
$$V(h) = \frac{m_h^2}{2} h^2 + \cdots$$



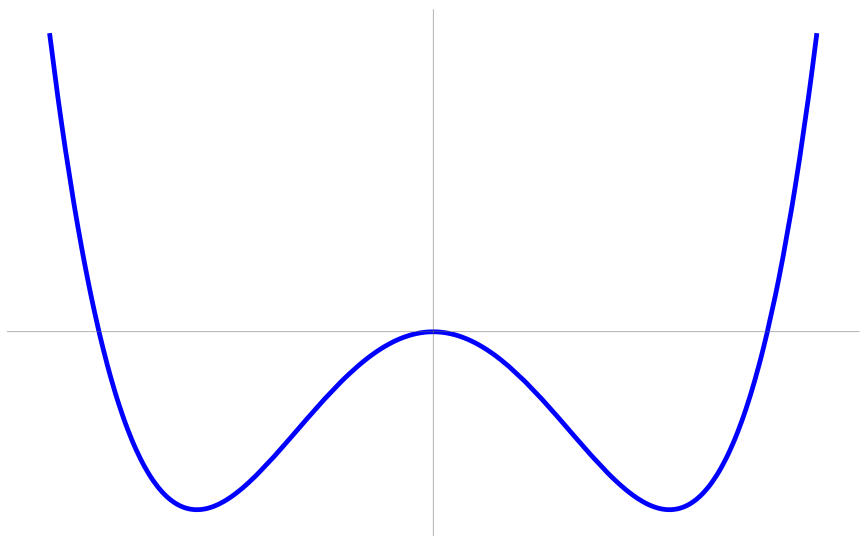


What LHC can tell us now

What's the global picture?

Higgs potential: alternatives

The SM **assumes** $V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}\left(\Phi^\dagger\Phi\right)^2$



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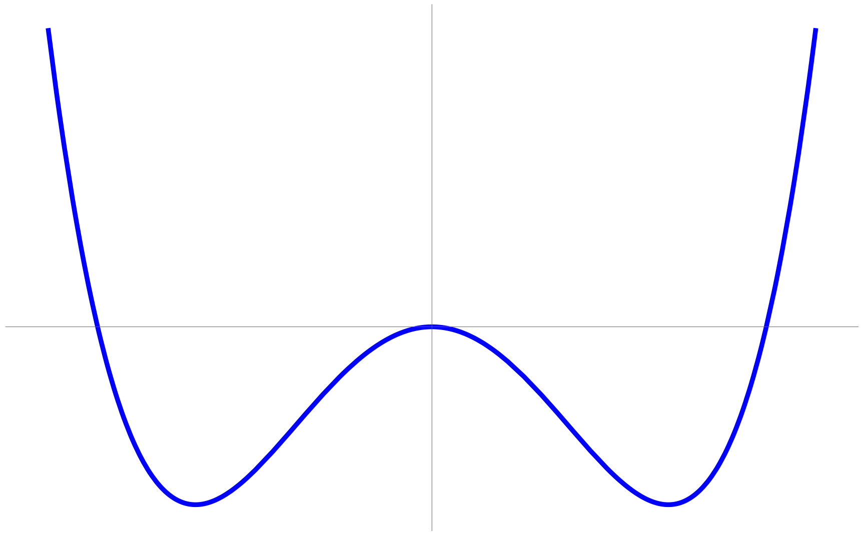

Not the only option!

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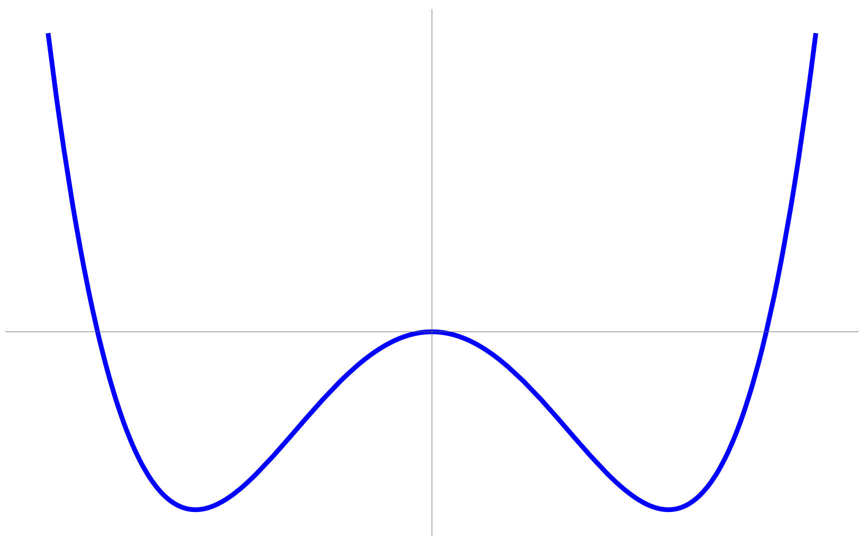
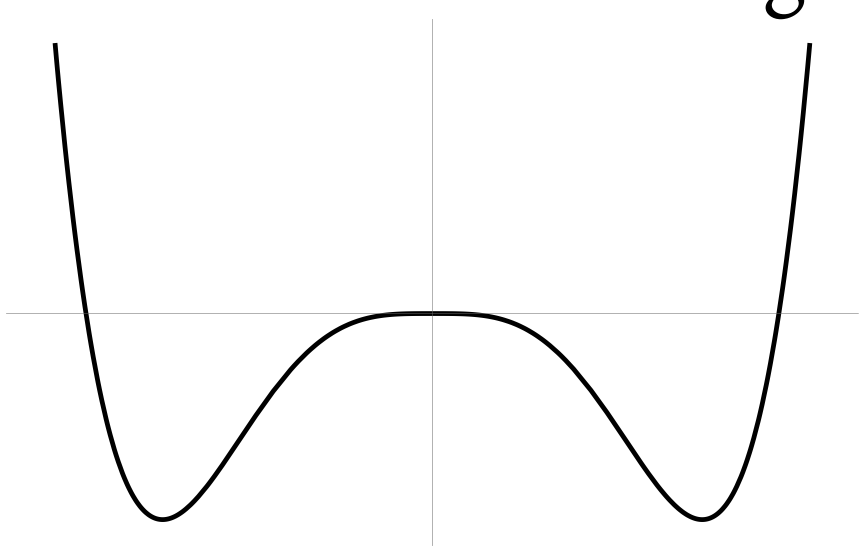


Coleman-Weinberg?



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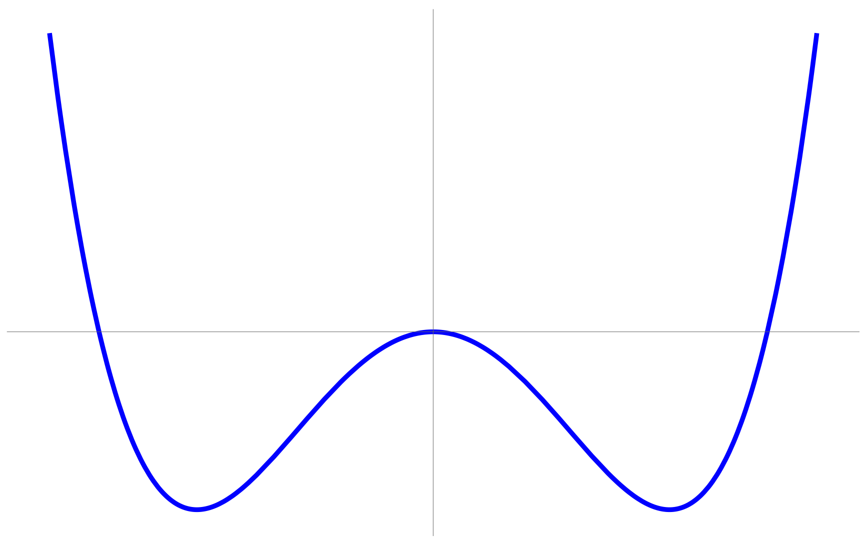

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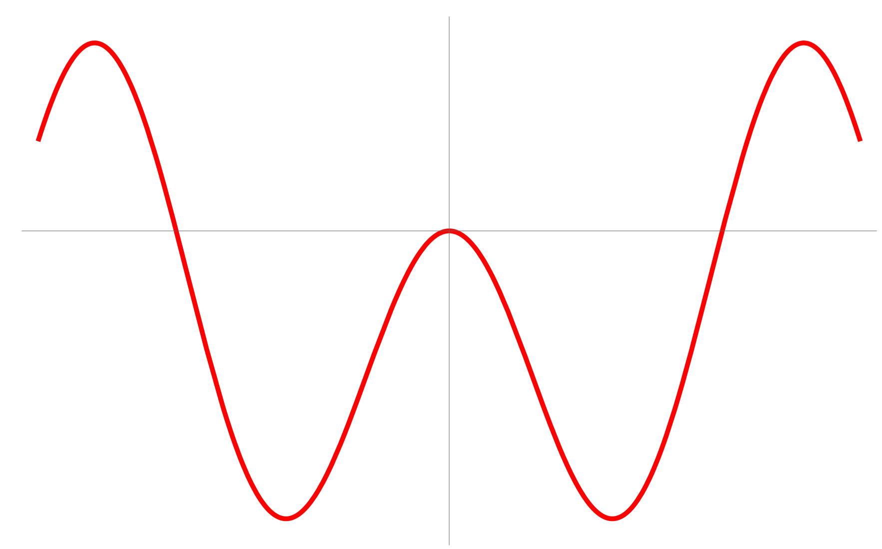
Not fundamental?



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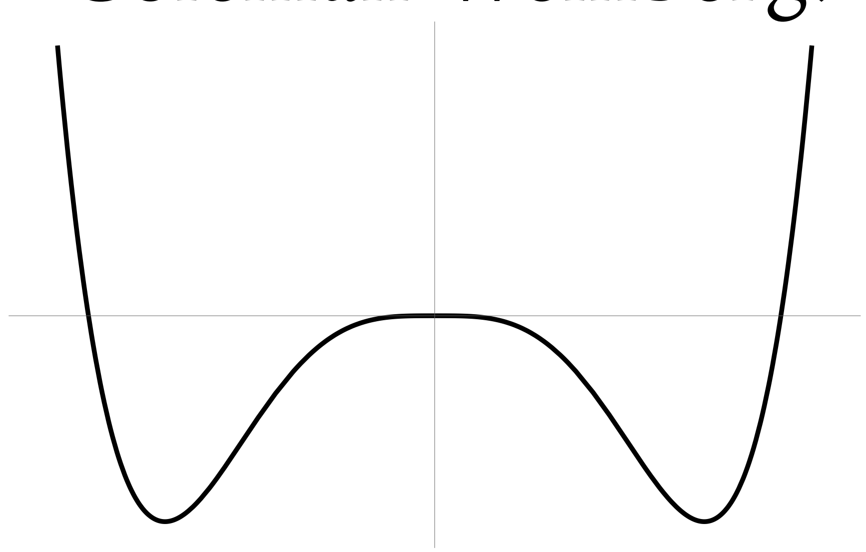

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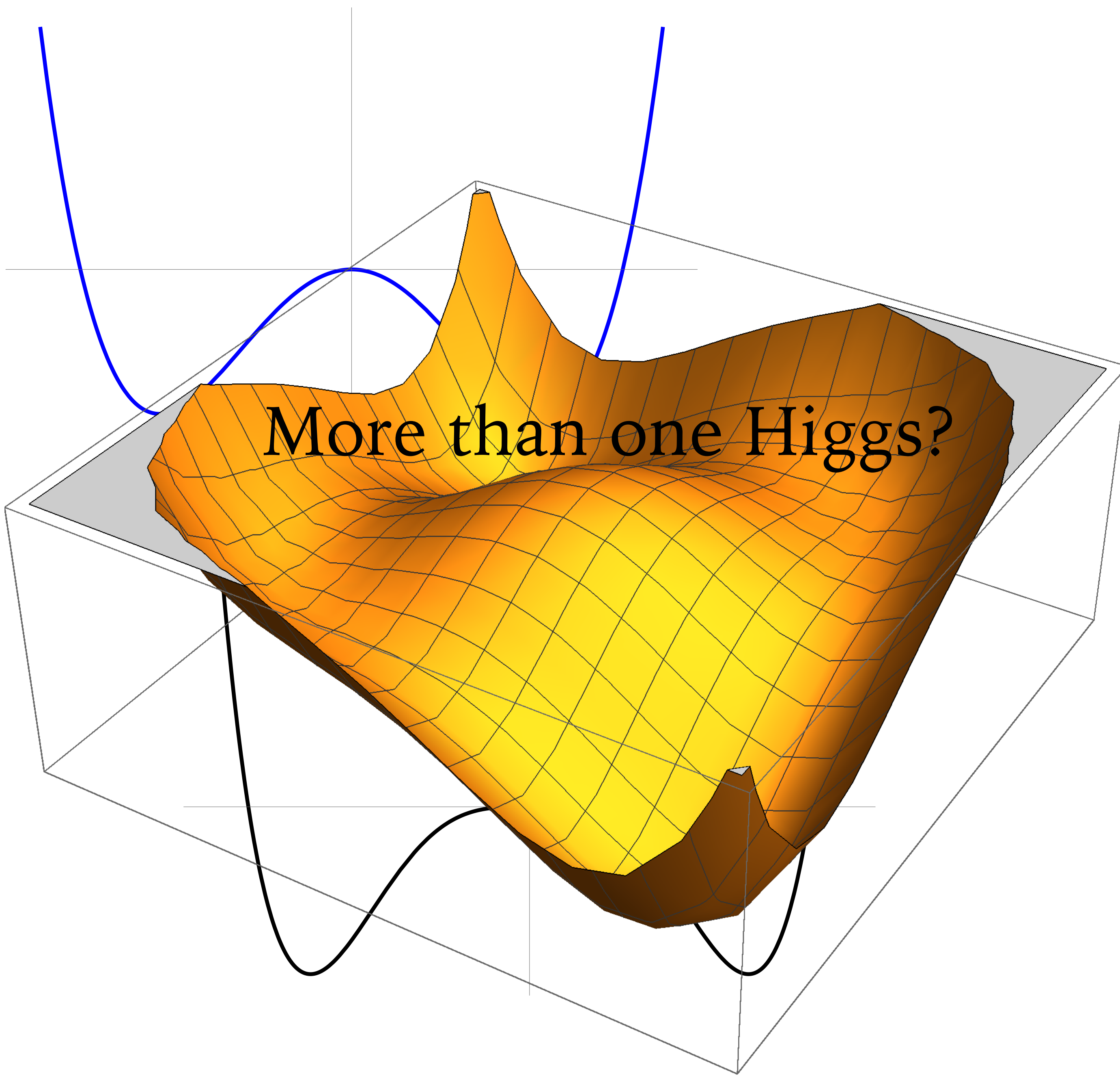

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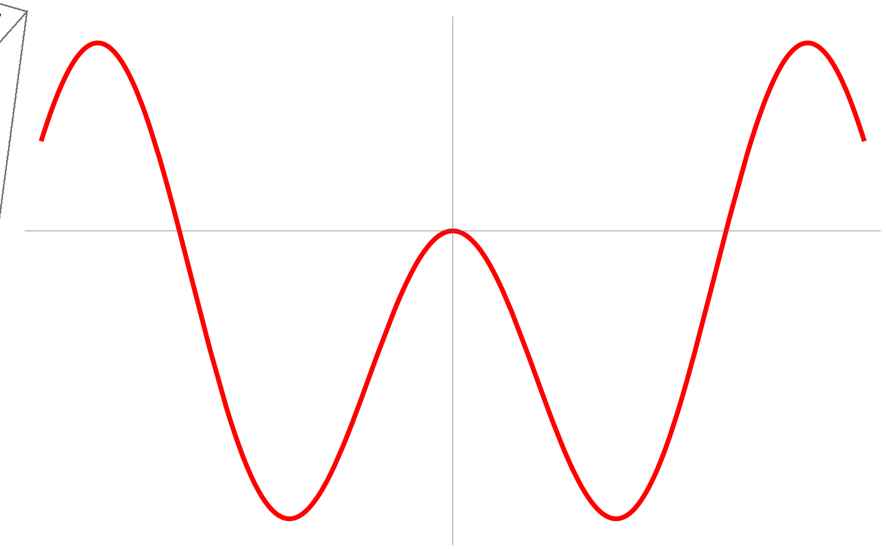
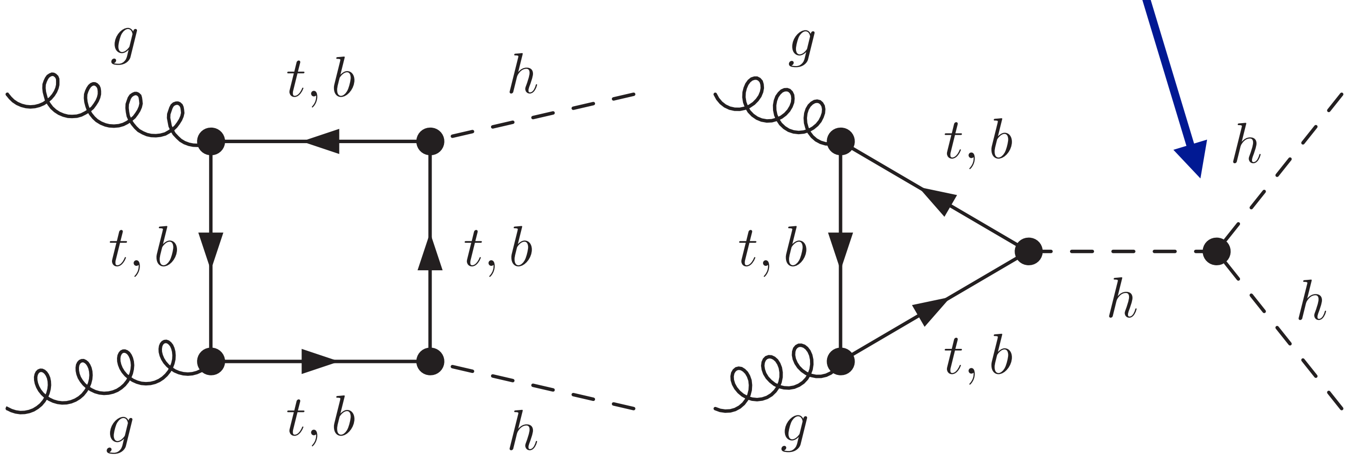


Higgs self-coupling

May test the next term in the Taylor expansion of the potential around our vacuum

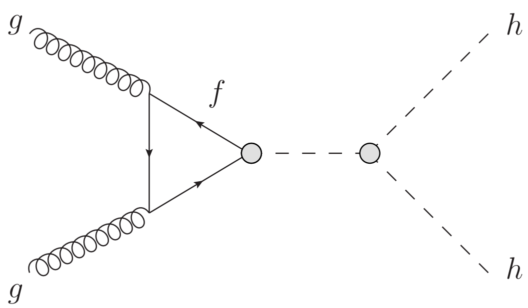
$$V(h) = \frac{m_h^2}{2}h^2 + \lambda_3 h^3 + \lambda_4 h^4 + \cdots$$



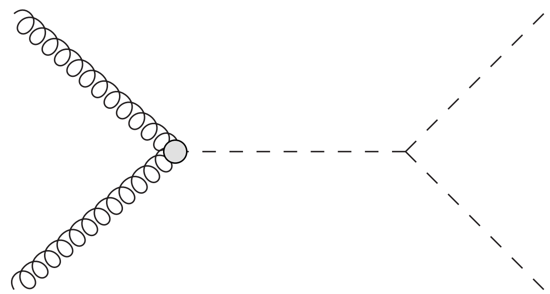
Higgs pair production

Sounds good…

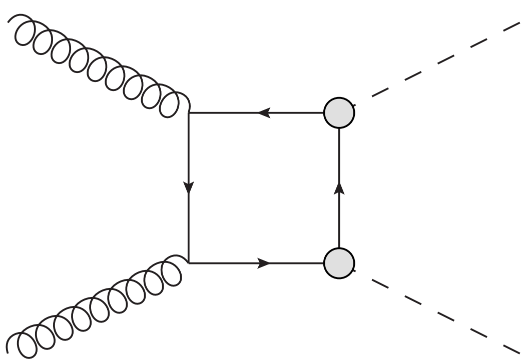
Not that simple!



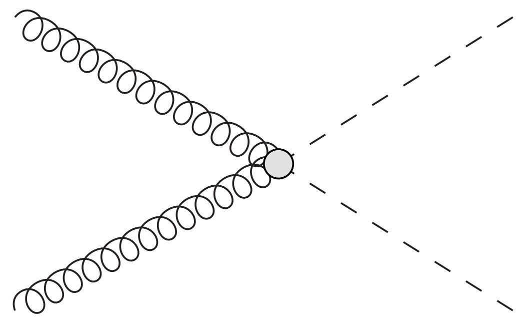
(1A)



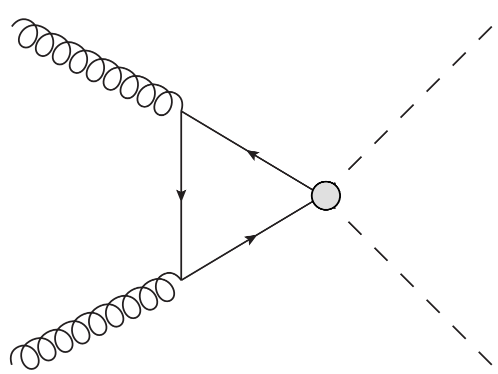
(1B)



(1C)



(1D)



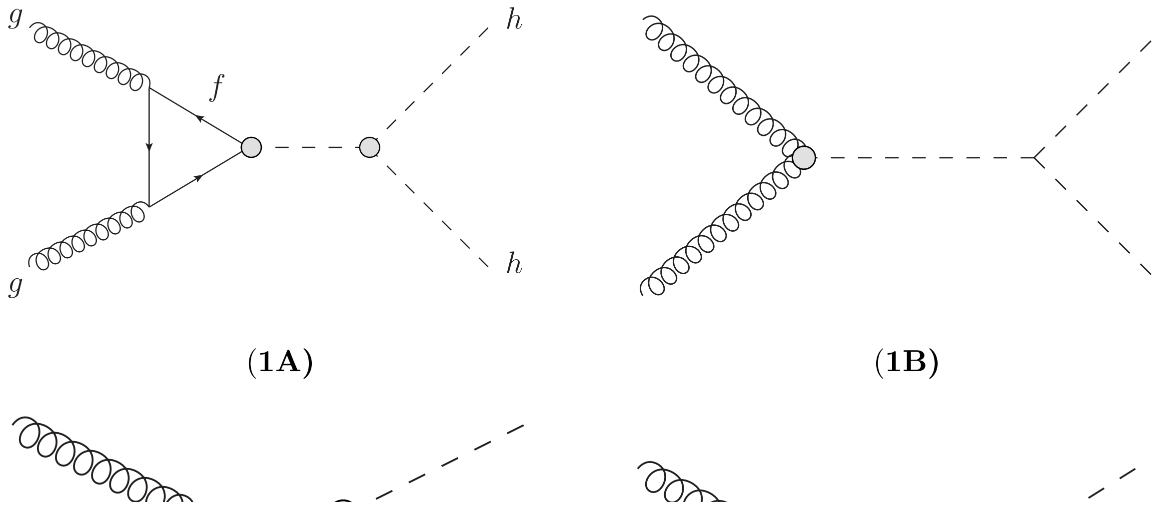
(1E)

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm SM} &+ \frac{c_H}{2\Lambda^2}(\partial^\mu |H|^2)^2 - \frac{c_6}{\Lambda^2}\lambda|H|^6 \\ &- \left(\frac{c_t}{\Lambda^2} y_t |H|^2 \bar{Q}_L H^c t_R + \frac{c_b}{\Lambda^2} y_b |H|^2 \bar{Q}_L H b_R + \frac{c_\tau}{\Lambda^2} y_\tau |H|^2 \bar{L}_L H \tau_R + \text{h.c.}\right) \\ &+ \frac{\alpha_s c_g}{4\pi\Lambda^2}|H|^2 G^a_{\mu\nu} G^{\mu\nu}_a + \frac{\alpha' \, c_\gamma}{4\pi\Lambda^2}|H|^2 B_{\mu\nu} B^{\mu\nu} \\ &+ \frac{ig\, c_{HW}}{16\pi^2\Lambda^2}(D^\mu H)^\dagger \sigma_k (D^\nu H) W^k_{\mu\nu} + \frac{ig'\, c_{HB}}{16\pi^2\Lambda^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\ &+ \frac{ig\, c_W}{2\Lambda^2}(H^\dagger \sigma_k \overleftrightarrow{D}^\mu H) D^\nu W^k_{\mu\nu} + \frac{ig'\, c_B}{2\Lambda^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu} \\ &+ \mathcal{L}_{\rm CP} + \mathcal{L}_{\rm 4f}\,,\end{aligned}$$

Goertz, Papaefstathiou, **LLY**, Zurita: 1410.3471

New physics effects may enter through multiple effective operators

Not that simple!



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\end{aligned}
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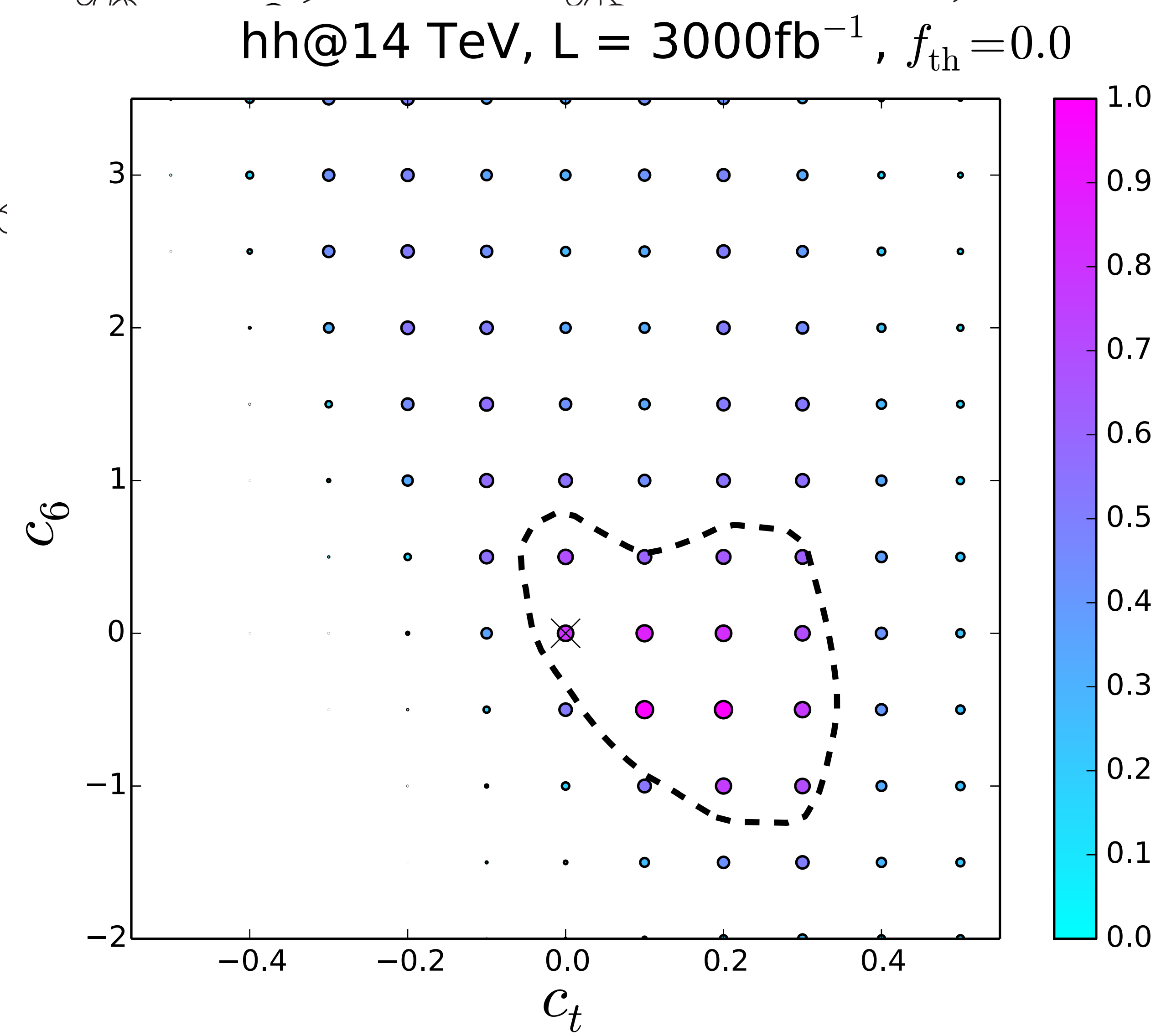

rtz, Papaefstathiou, **LLY**, Zurita: 1410.3471

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Requires a global analysis…

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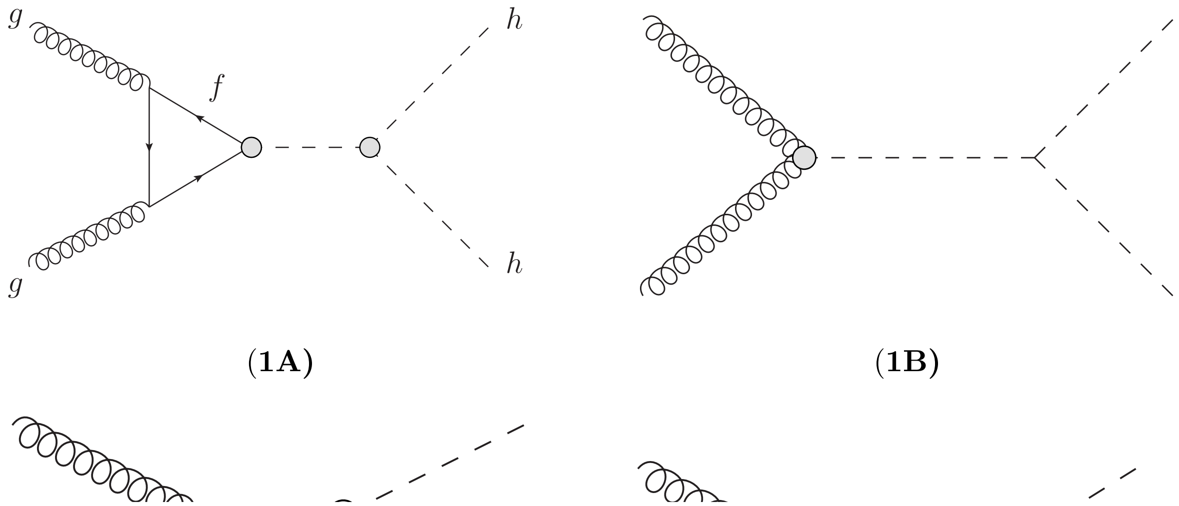

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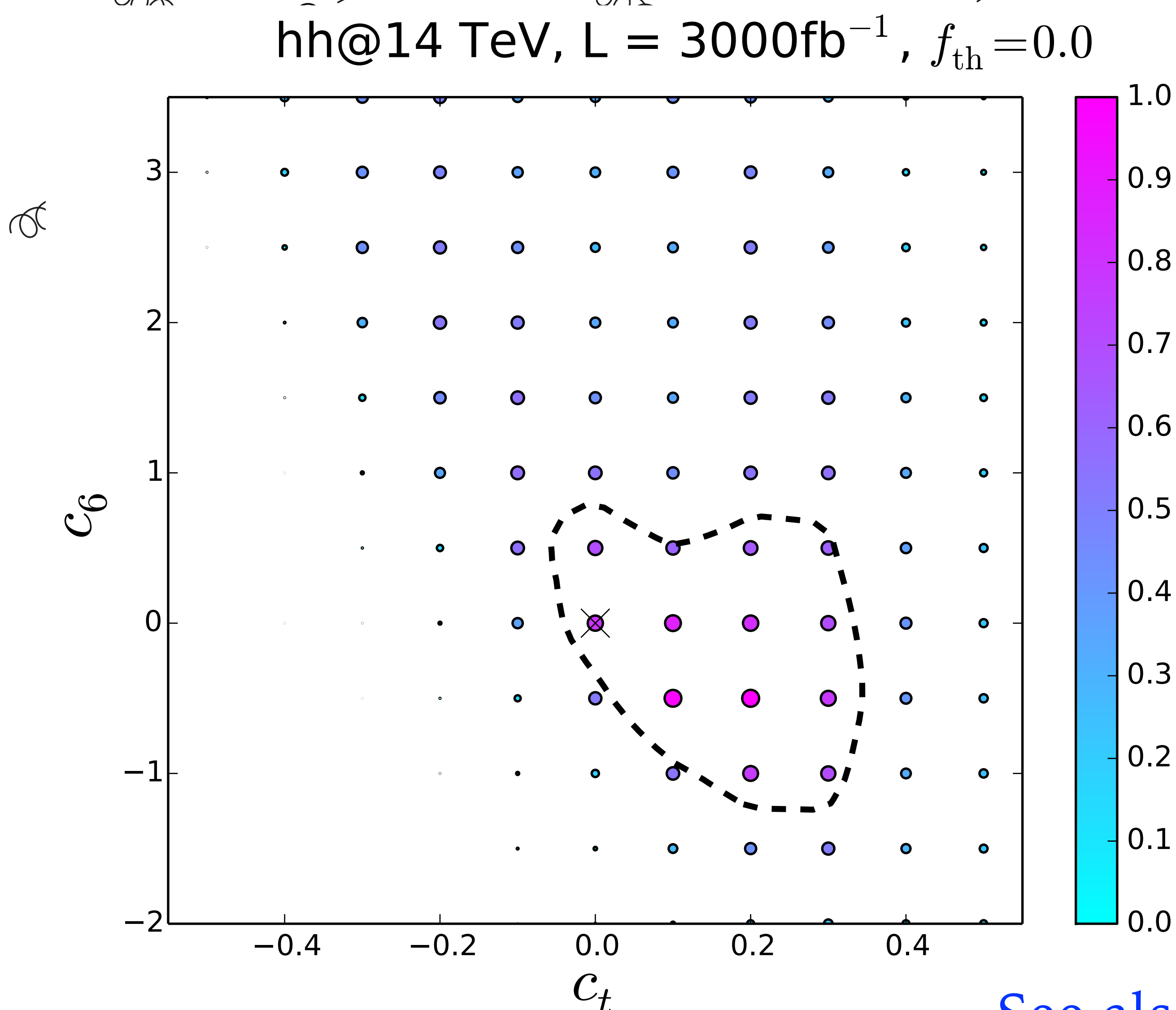


rtz, Papaefstathiou, **LLY**, Zurita: 1410.3471

hysics effects may enter through
le effective operators

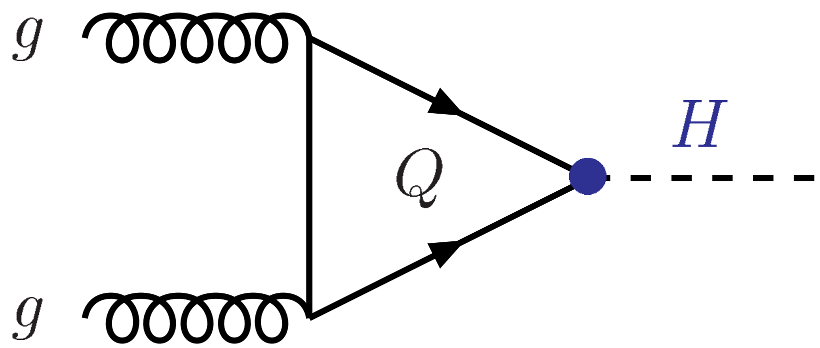
Requires a global analysis…

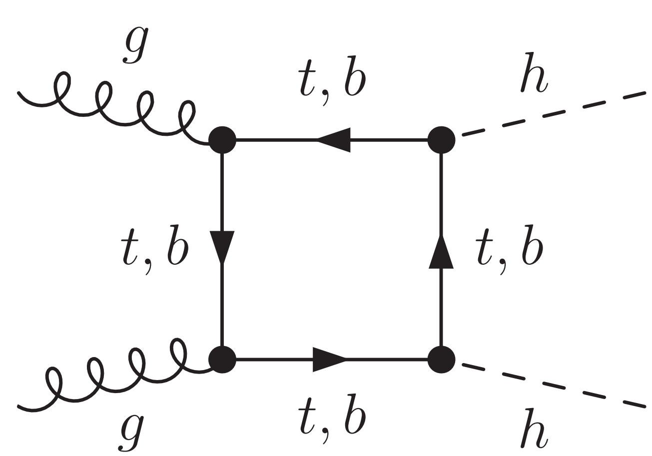
See also: Azatov, Contino, Panico, Son: 1502.00539

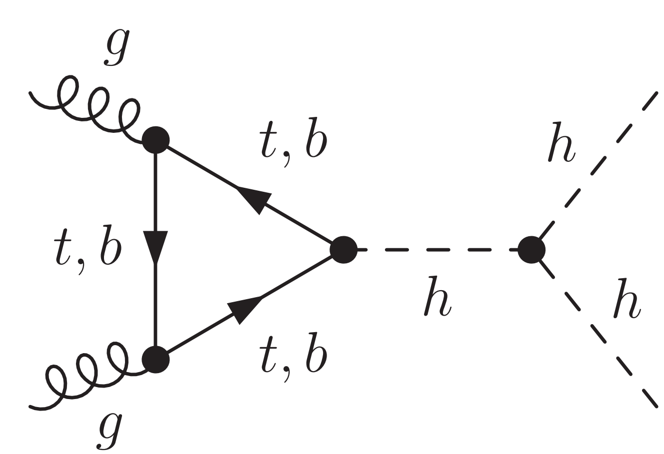
Yukawa couplings

Origin of quark masses and lepton masses

Heavy quark Yukawa couplings: everywhere in Higgs physics







Precise knowledge highly-wanted!

Yukawa couplings

Origin of quark masses and lepton masses

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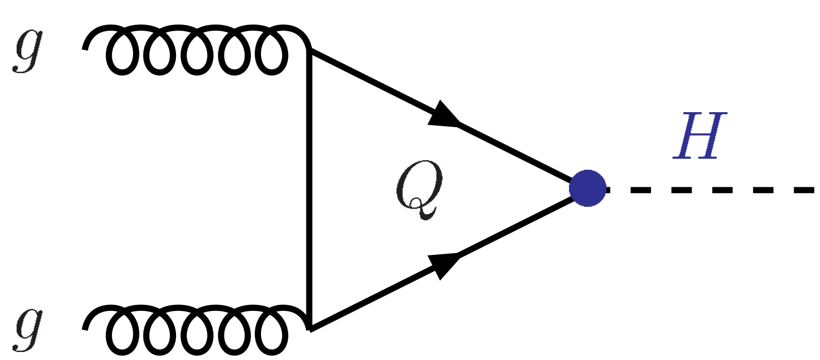
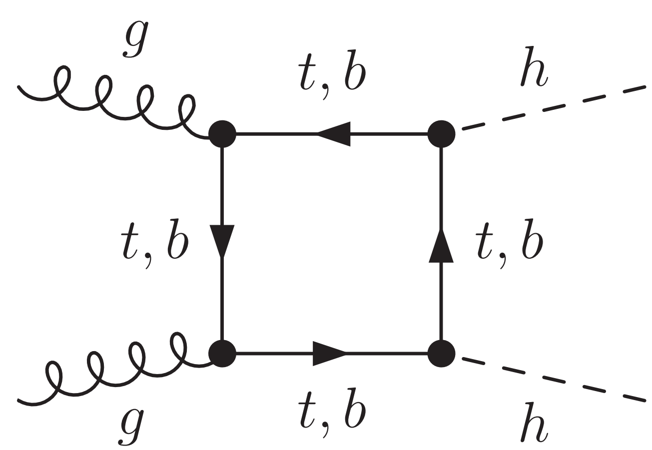
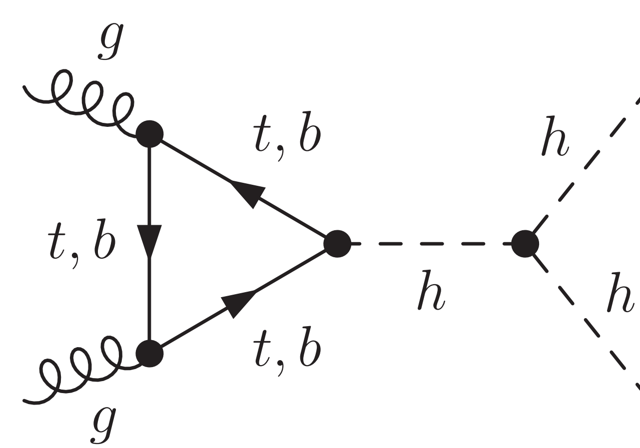
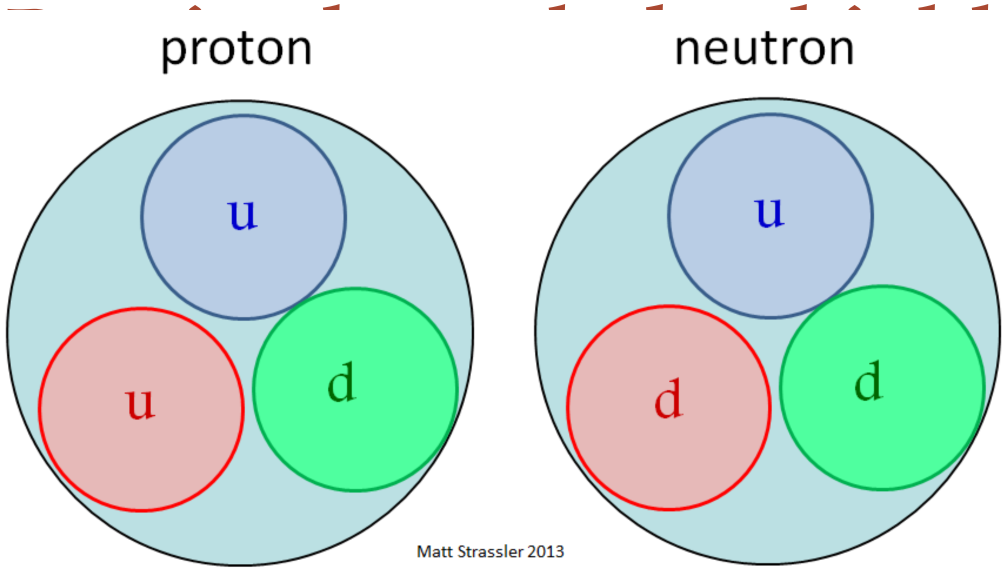






y-wanted!

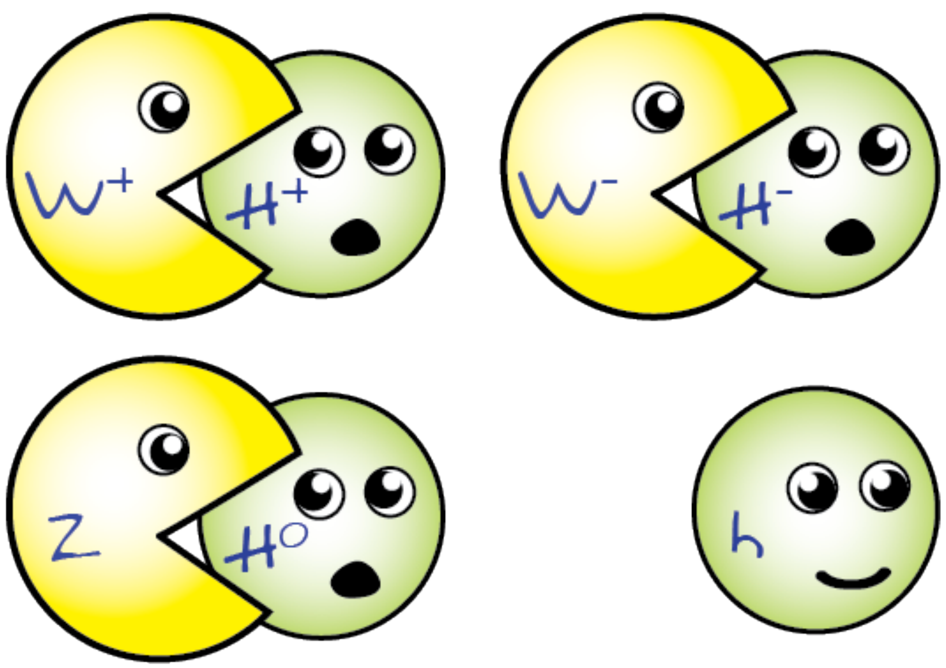


Light quark Yukawa couplings:
are they really **that** small?

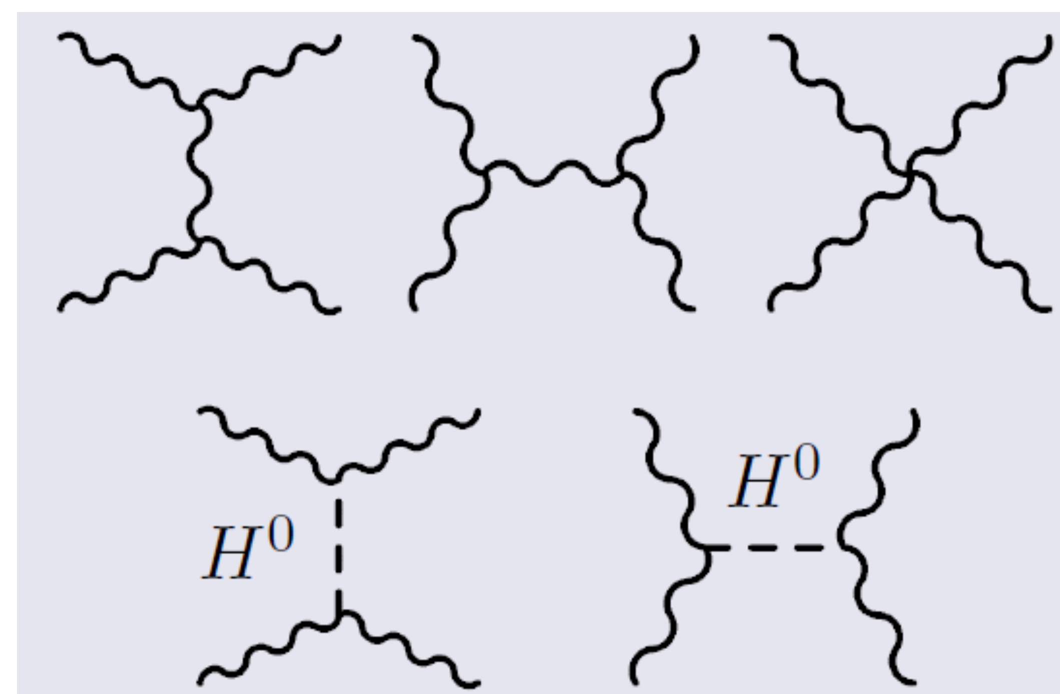
Is the mass difference between proton and neutron a pure accident?

Gauge coupling

Key quantity for electroweak symmetry breaking



Relevant for unitarity

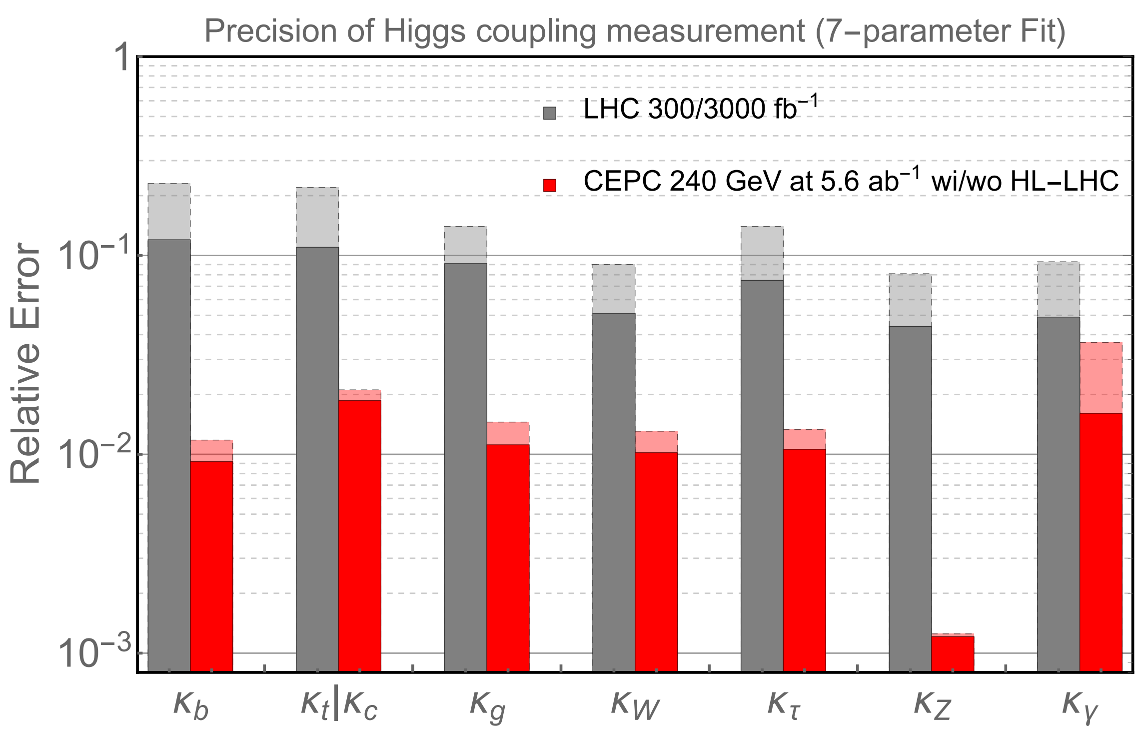


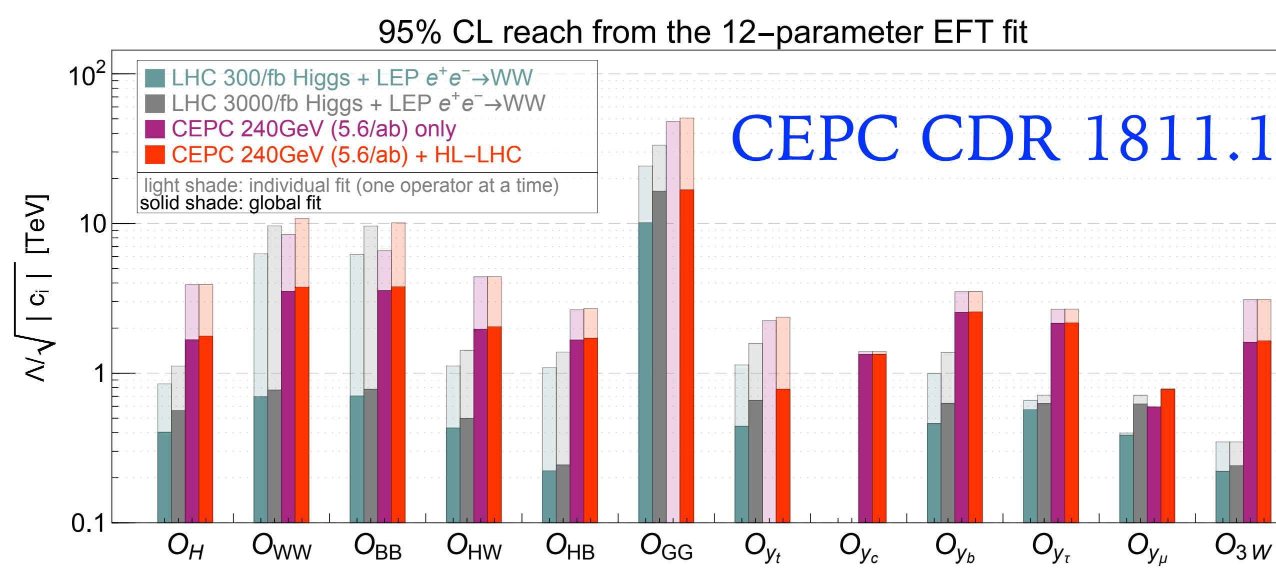
Modified by ~v^2/Λ^2 if, e.g., Higgs has inner structure

Sub-percent effect if new physics enters at a few TeV!

Precision experiments meet precision calculations

Future facilities will dramatically improve the experimental precisions of various observables





CEPC CDR 1811.10545

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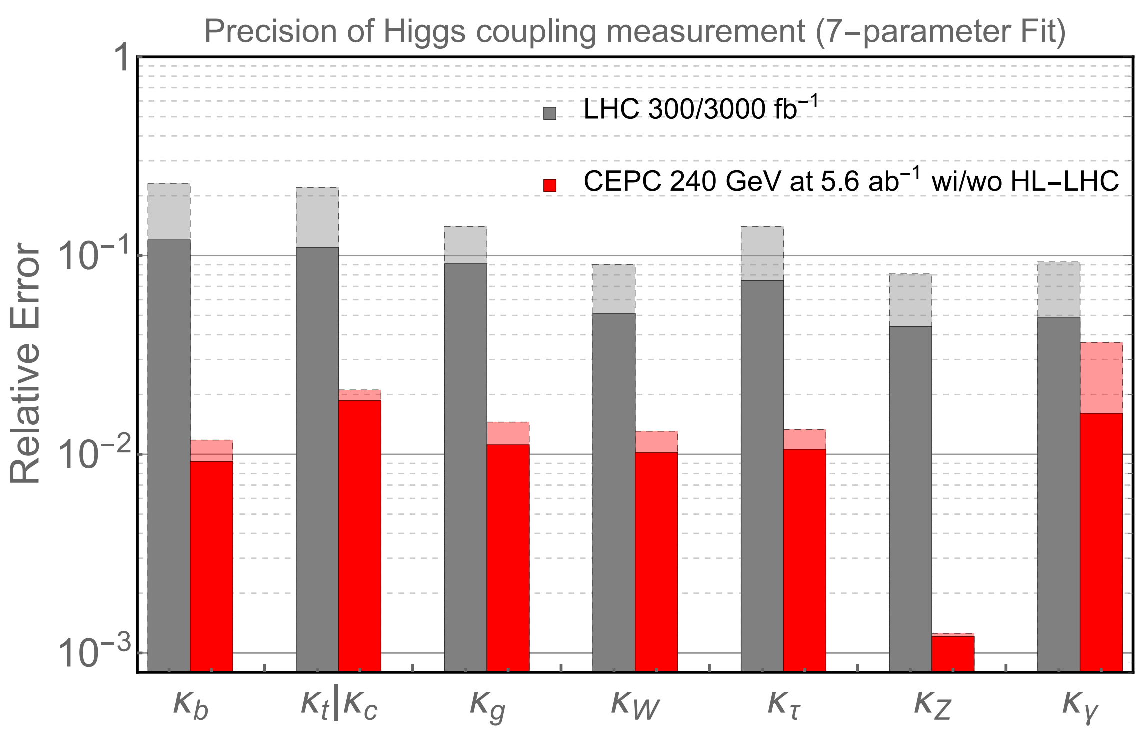
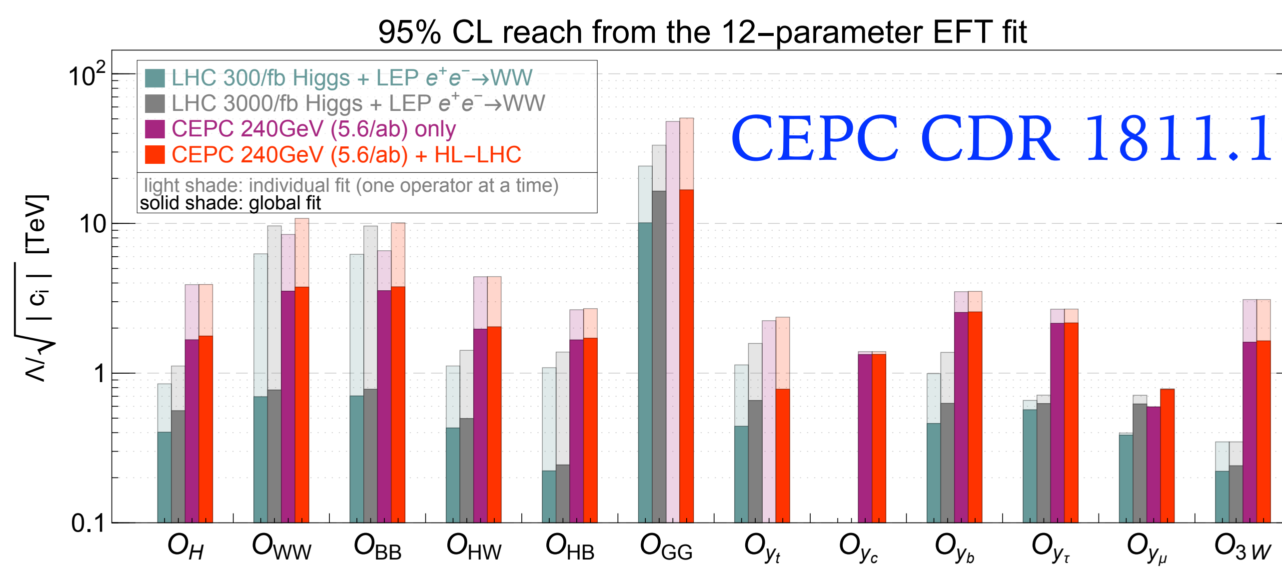


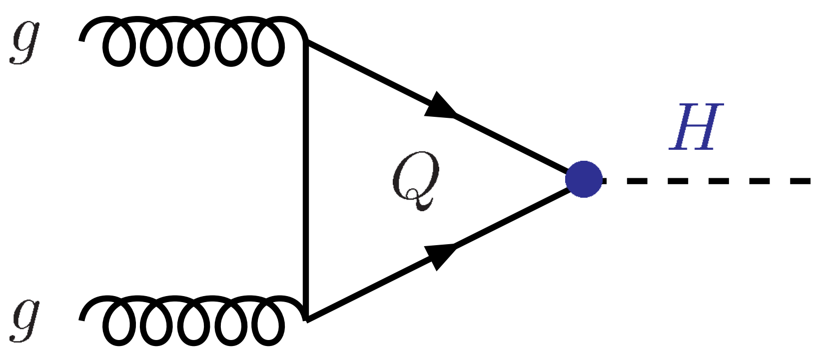


CEPC CDR 1811.10545

In order to extract the Higgs properties from these precision measurements, we need equally precision theoretical calculations!

An example: gluon fusion

LO already loop-induced



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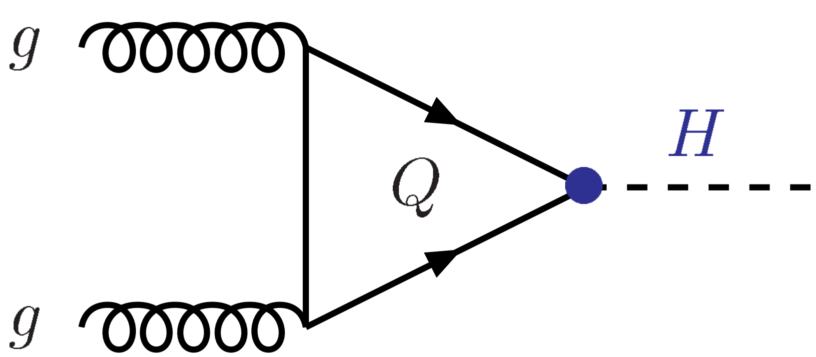
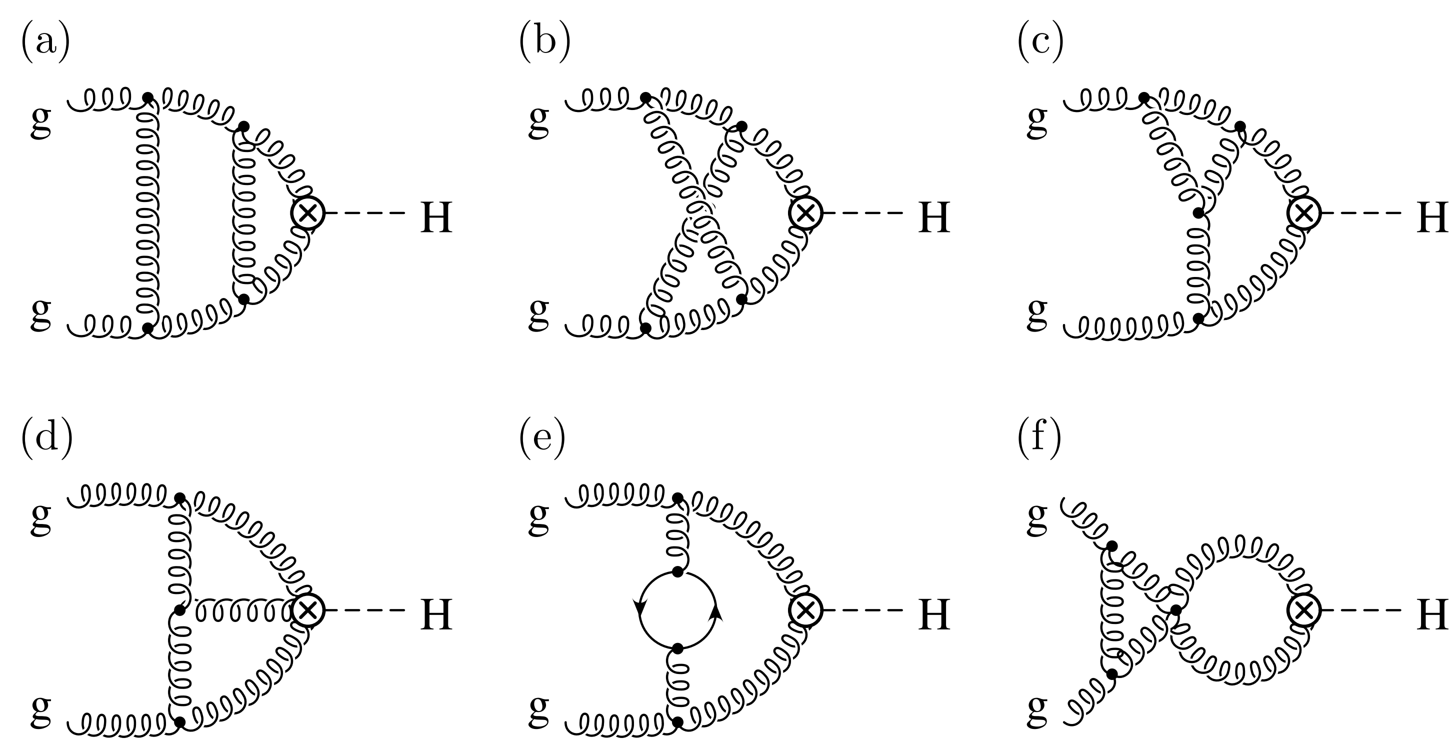


For better precision, need to calculate higher-order quantum effects

NNLO QCD in the heavy top limit



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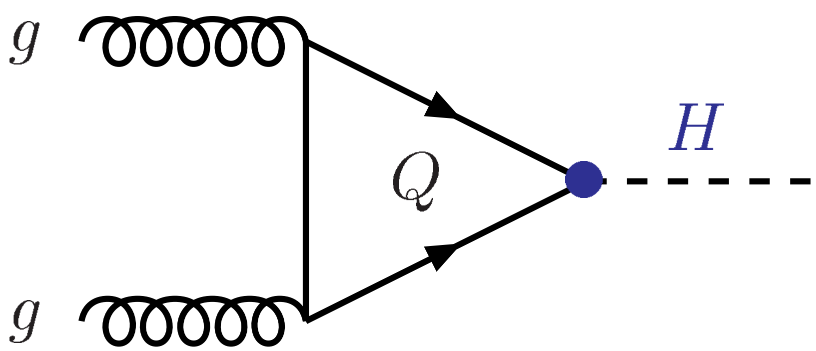

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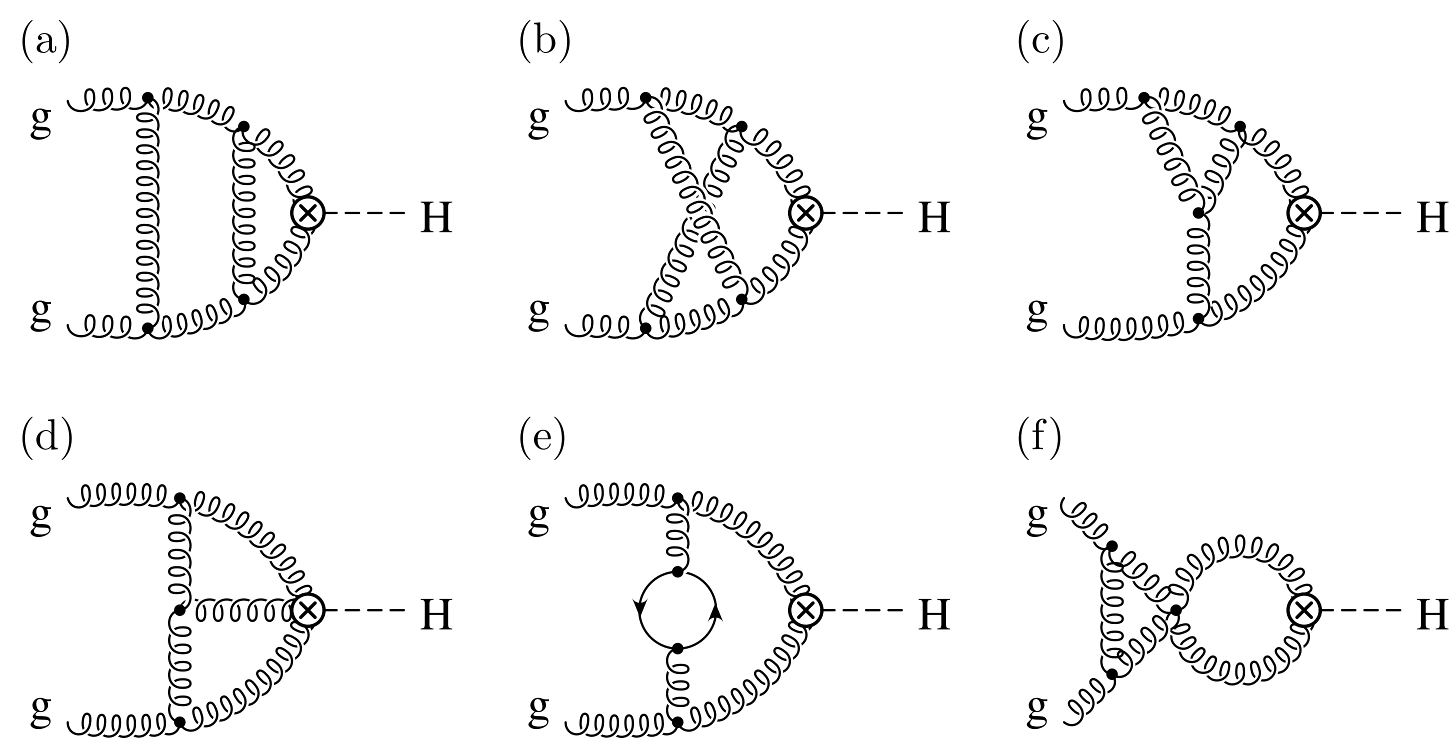
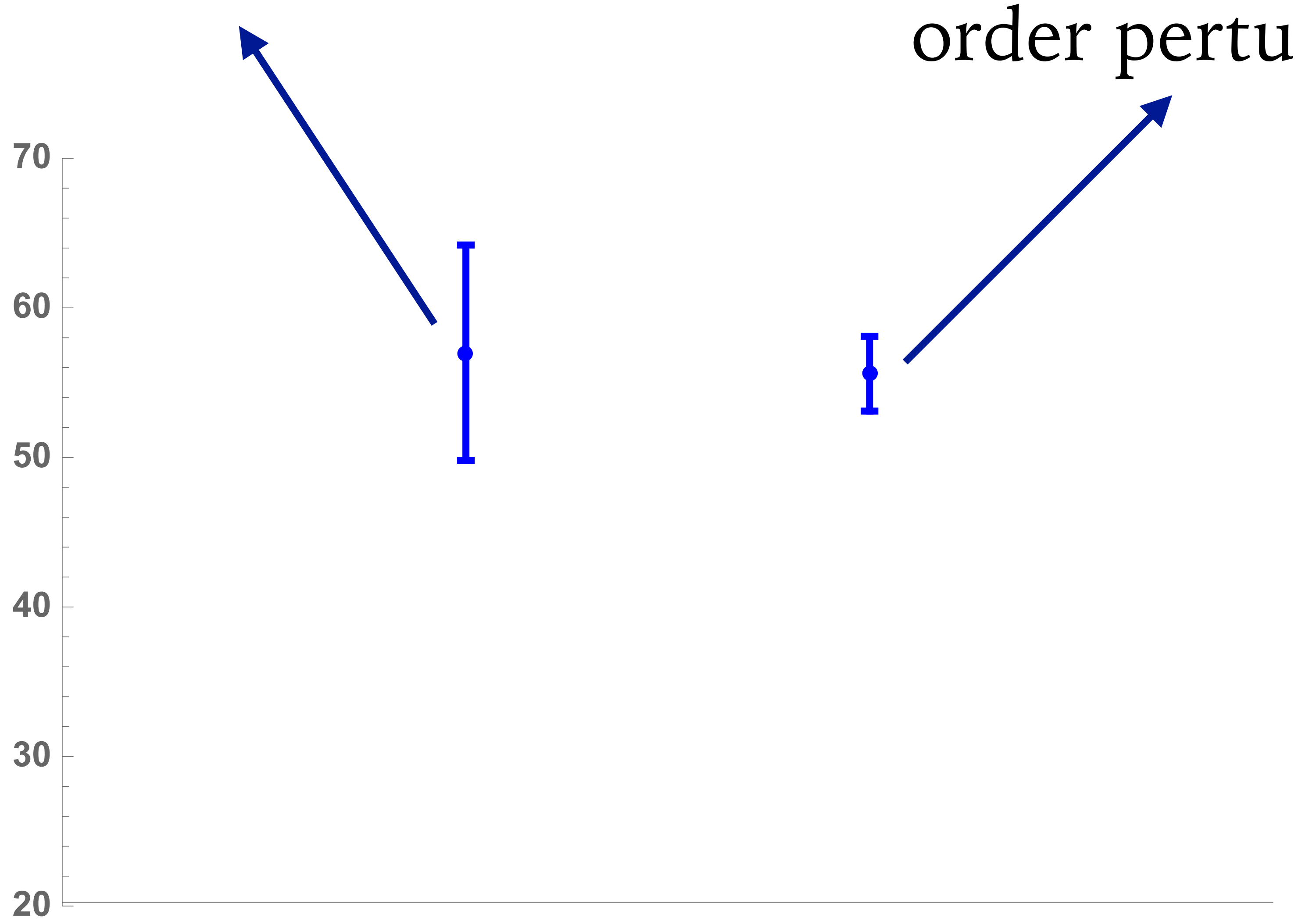


Do we really need such high precision? The answer is yes!

LO is not enough!

Measured cross section

Theoretical value (**with** higher-order perturbative calculations)



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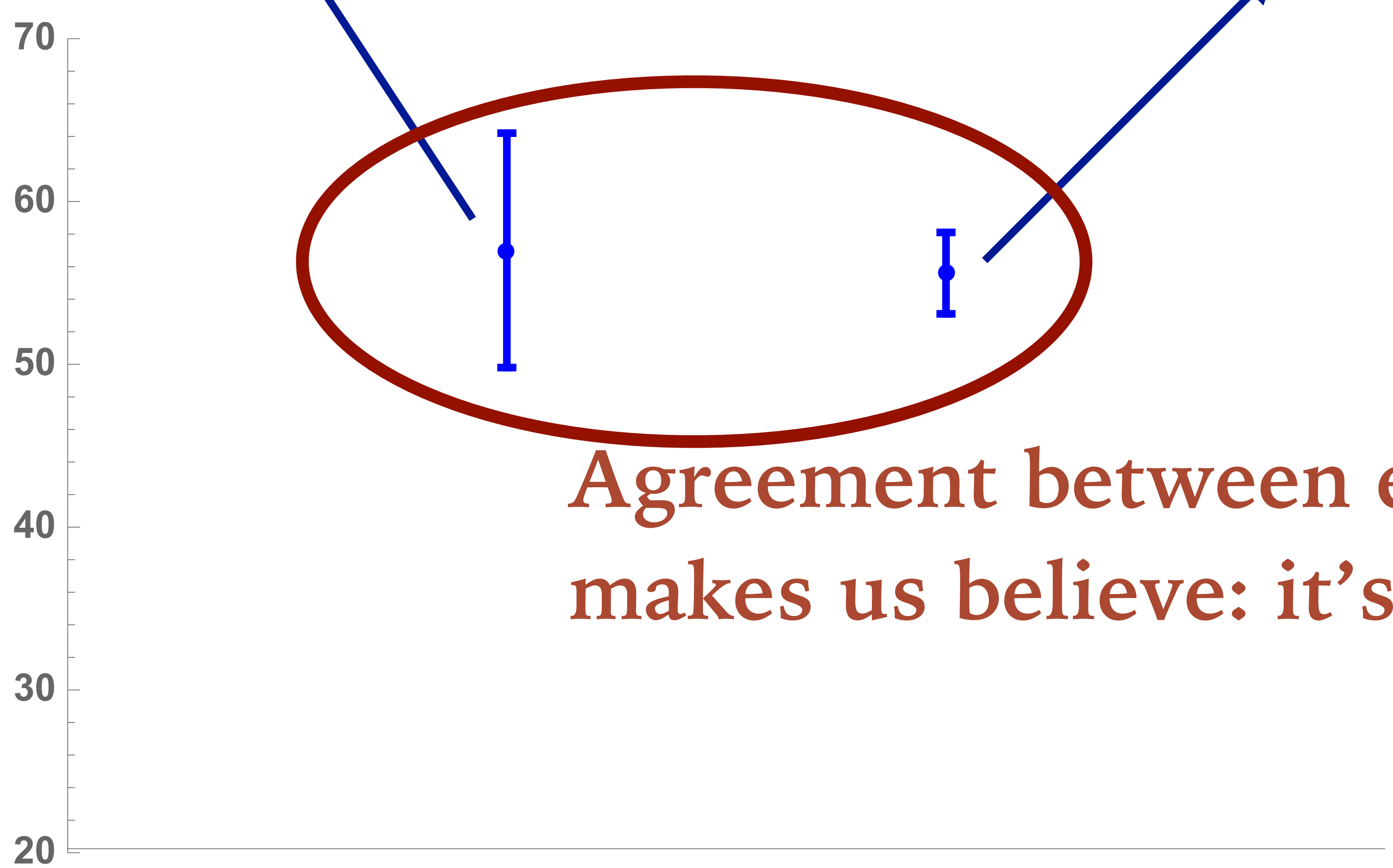

Agreement between experiment and theory makes us believe: it's the Higgs!

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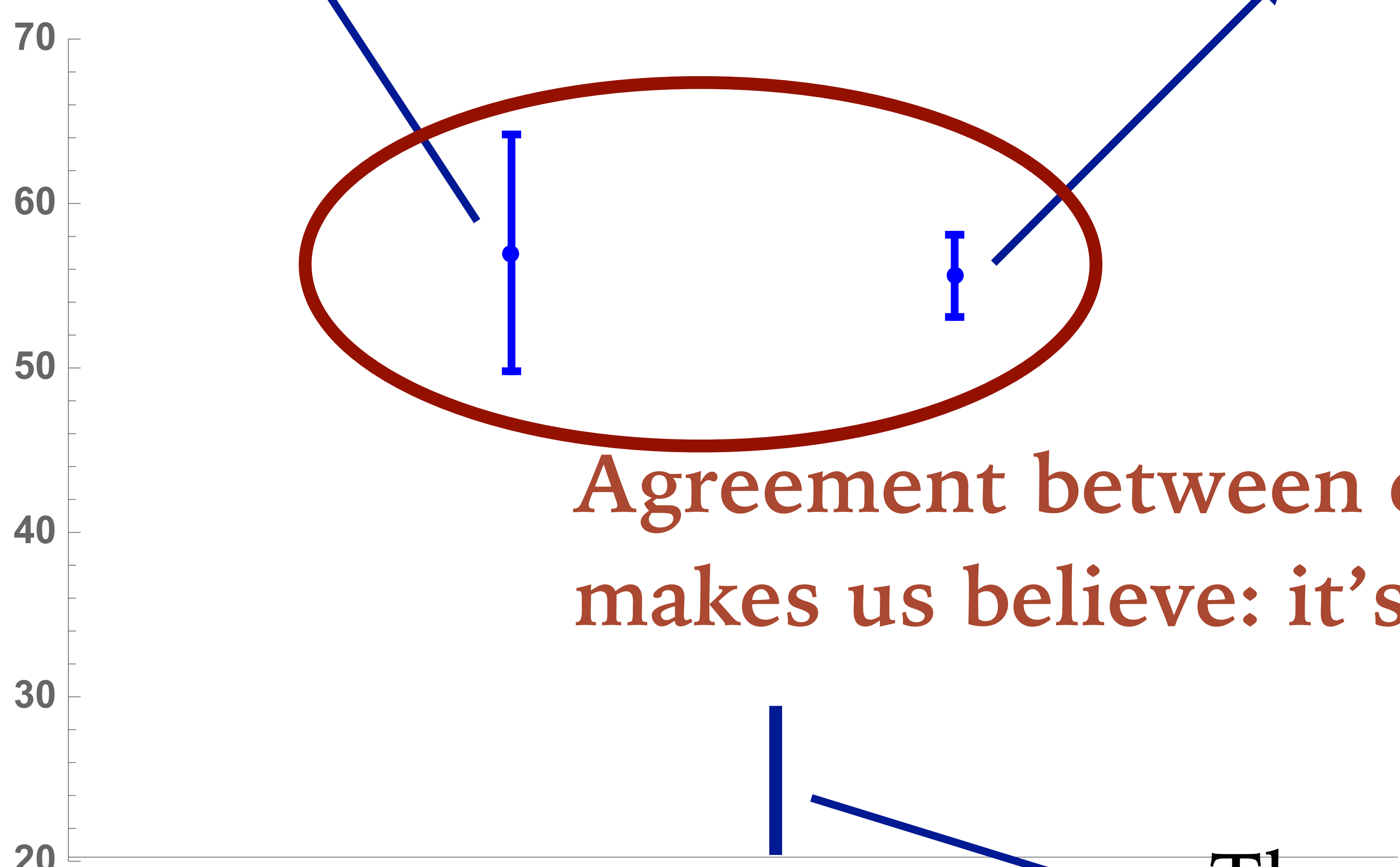

Agreement between experiment and theory makes us believe: it's the Higgs!

Theoretical value **without** higher-order calculations

LO is not enough!

Measured cross section

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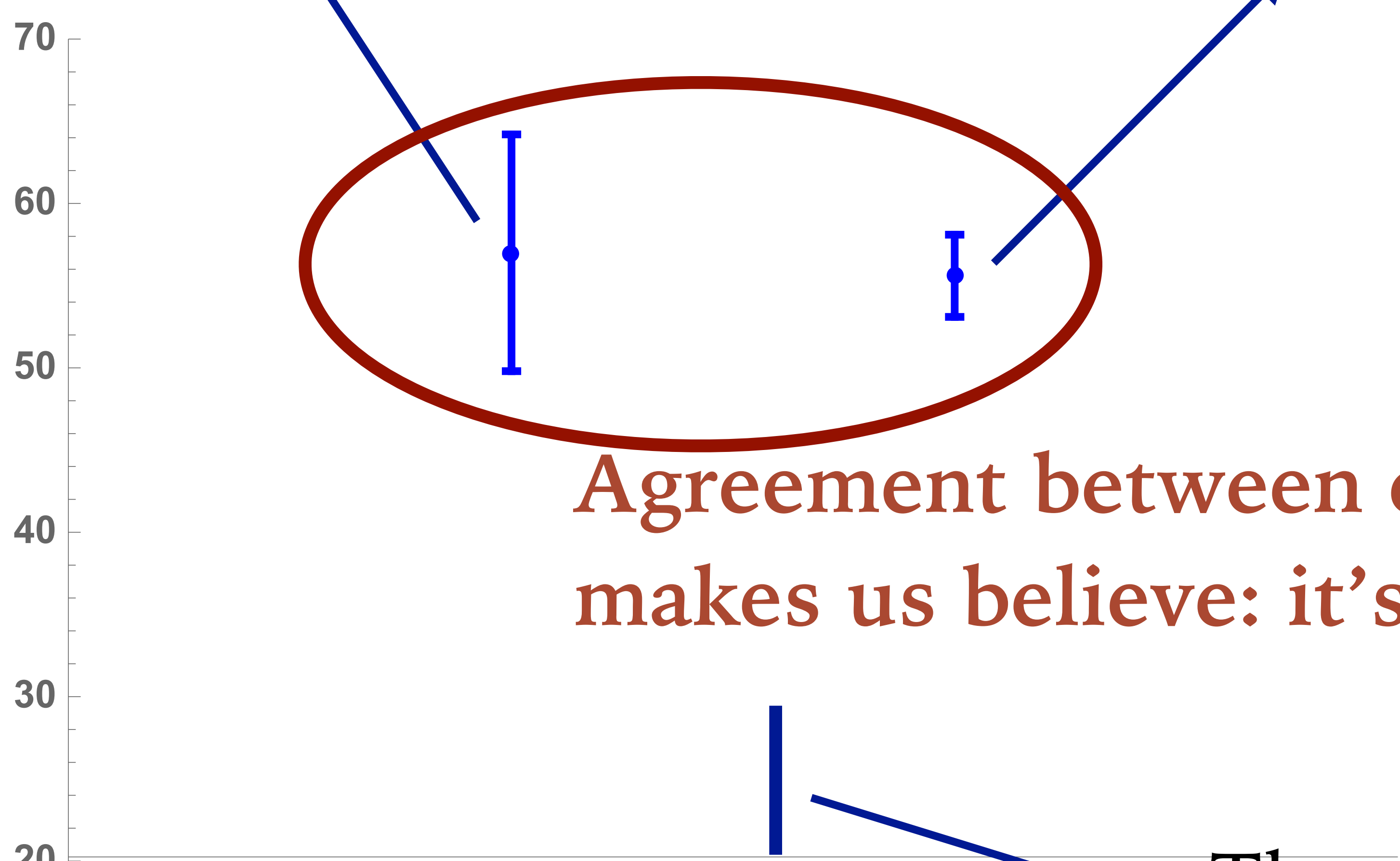


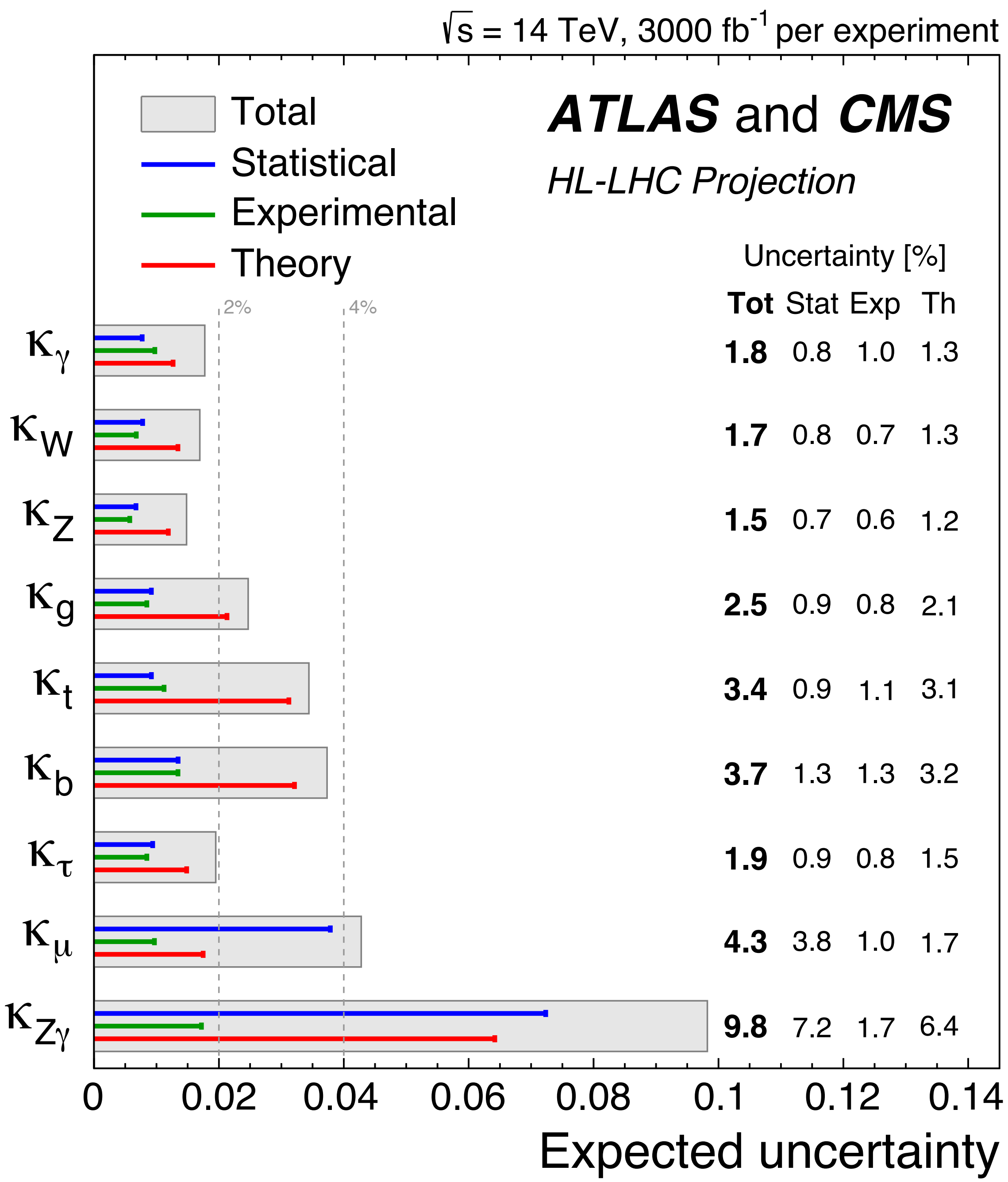
Agreement between experiment and theory makes us believe: it's the Higgs!

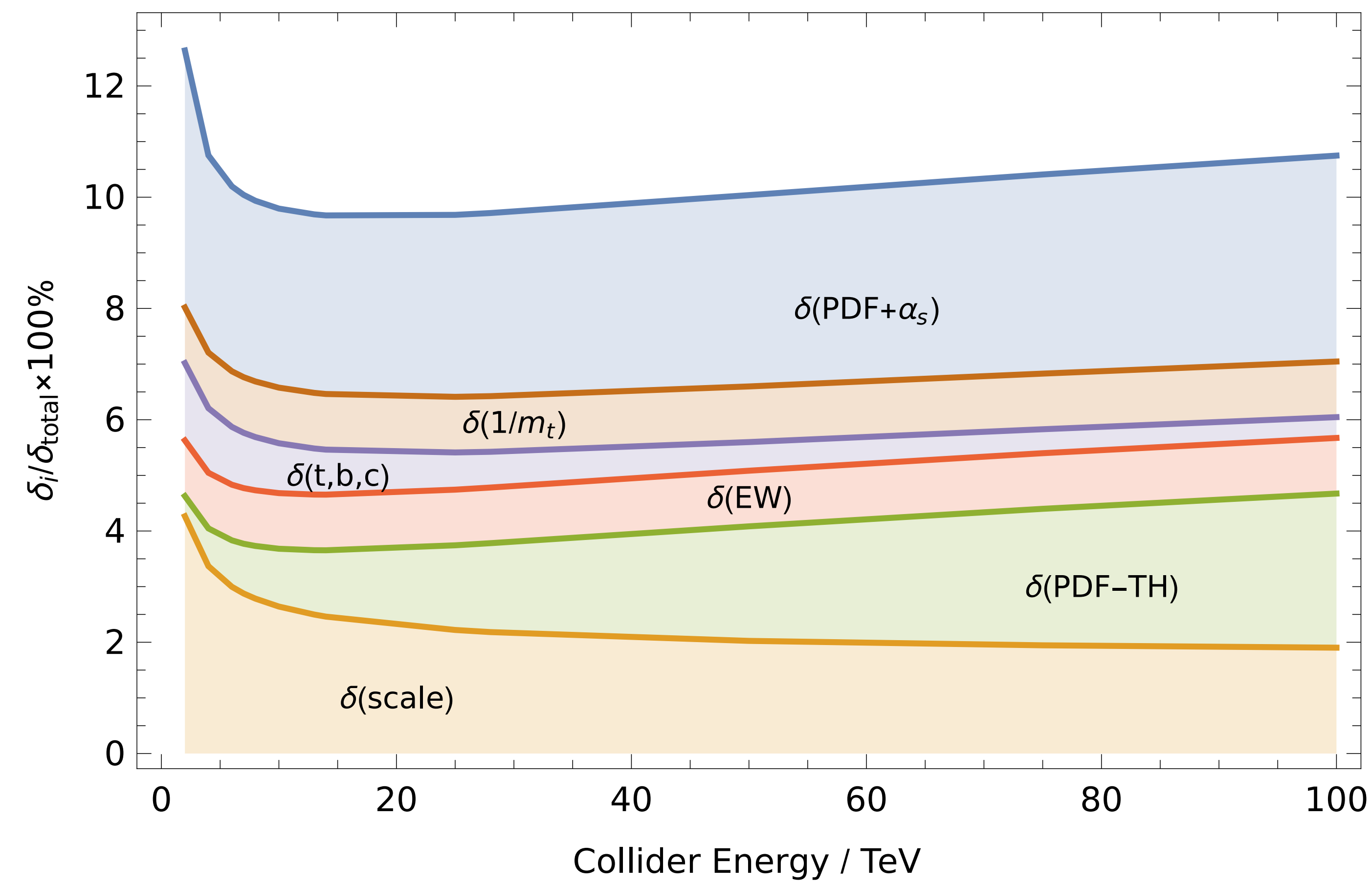
Theoretical value **without** higher-order calculations

Note: origin of large higher-order corrections explained in

Ahrens, Becher, Neubert, **LLY**: 0808.3008

HL-LHC demands more theoretical inputs



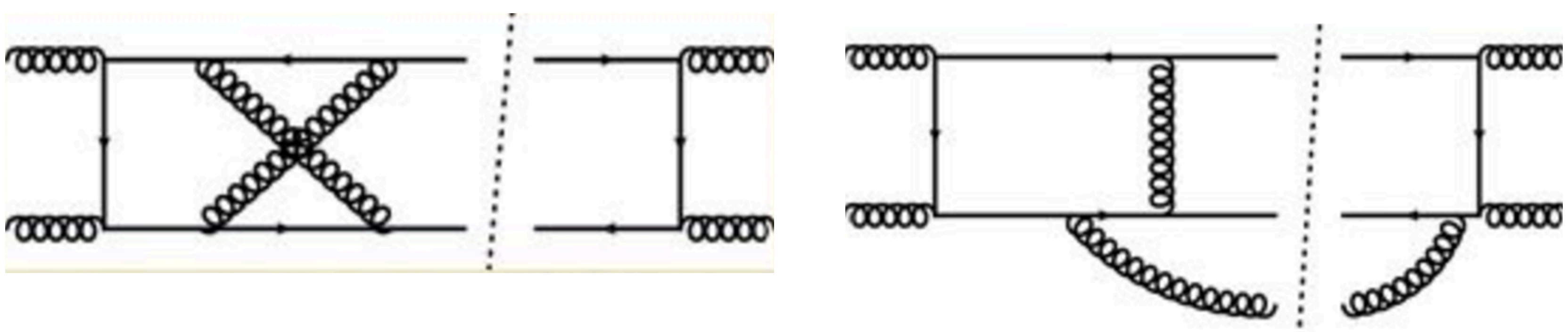


Challenges from experiments: reducing the theoretical uncertainties!

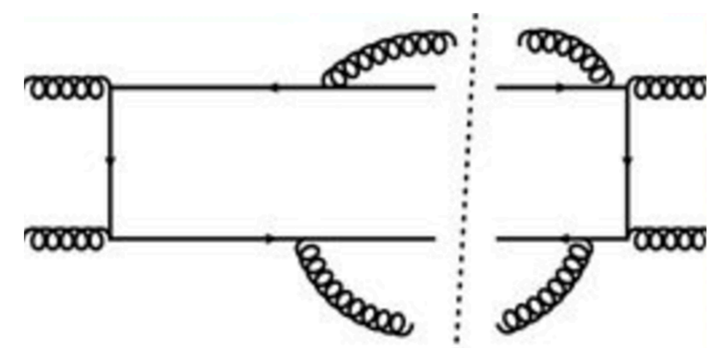
Perturbative calculations

Generic procedure for a perturbative calculation

loop amplitudes



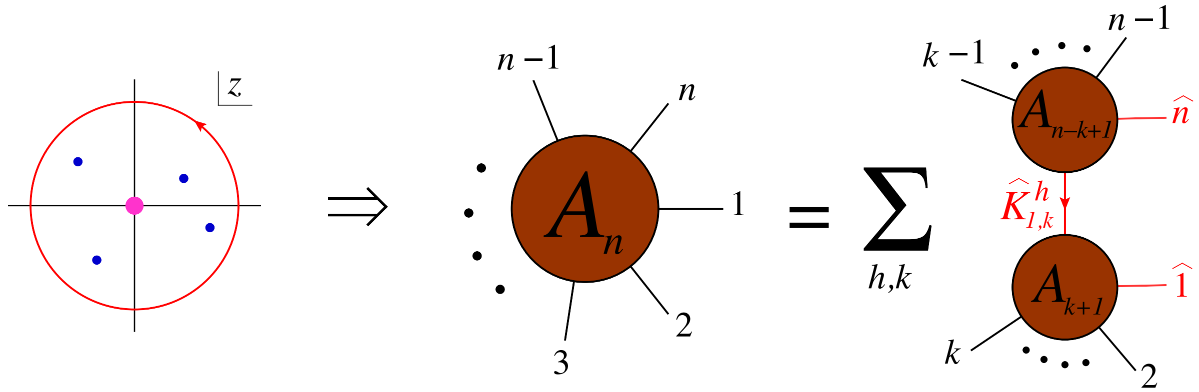
tree amplitudes

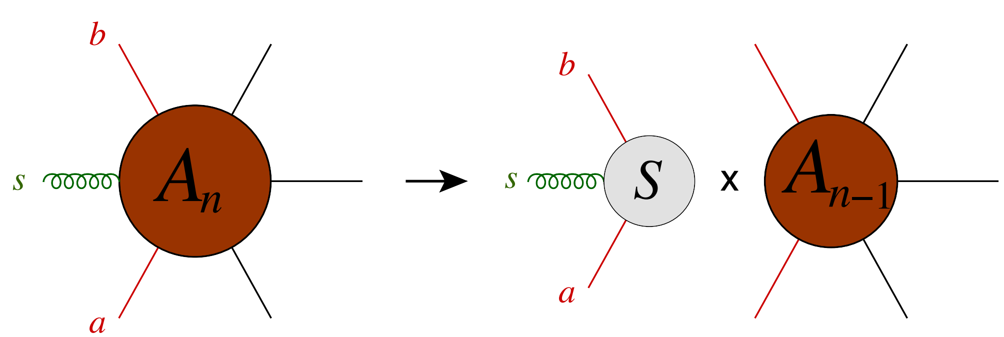


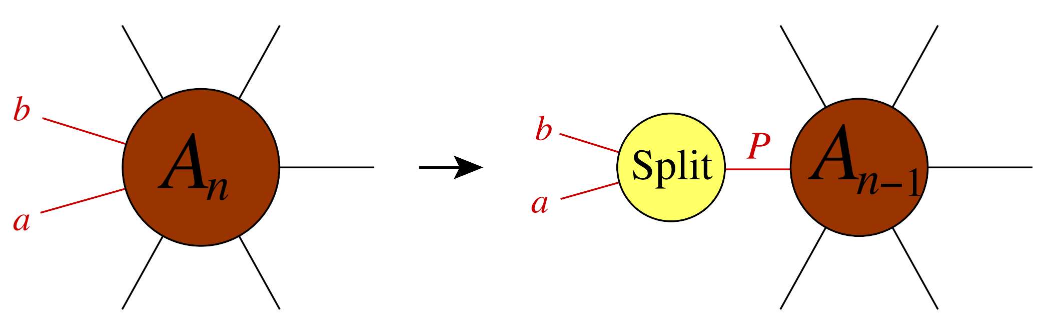
Need to combine them to get rid of infrared divergences

Tree-level amplitudes

We already have rather good understanding of tree-level amplitudes in gauge theories





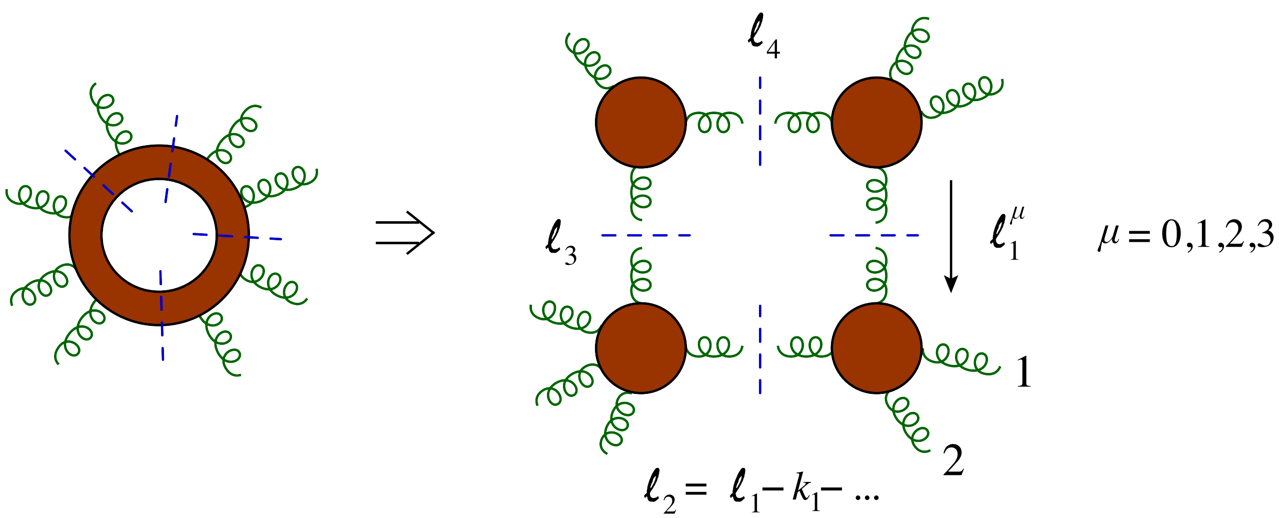


- Spinor helicity
- Little group scaling
- On-shell recursion
- Hints from N=4 SUSY
- Collinear limit
- Soft limit
- …

See, e.g.,
Dixon: hep-ph/9601359
Elvang, Huang: 1308.1697
Dixon: 1310.5353
…

Loop integrands

We also have some techniques to simplify the integrands for loop-level amplitudes



- Unitarity cuts
- Integration-by-parts
- Color-kinematics duality
- …

$$0 = \int \frac{d^D k}{i\pi^{D/2}} \frac{\partial}{\partial k^\mu}\left(k^\mu \frac{1}{(-k^2+m^2)^{a_1}(-(k+p)^2+m^2)^{a_2}}\right)$$

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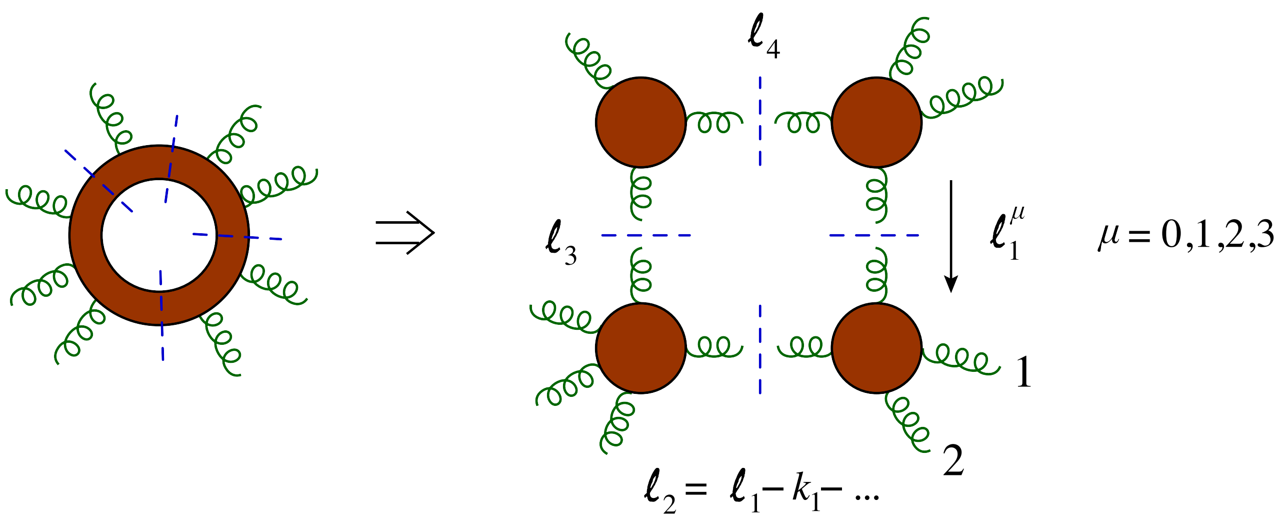

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But: we have much less information about the results of these loop integrals!

Loop integrals

What we really need are results of integrals

$$I = \int \left(\prod_{j=1}^{L} \mu^{2\epsilon} e^{\epsilon \gamma_E} \frac{d^{4-2\epsilon} k_j}{i\pi^{2-\epsilon}} \right) \prod_{i=1}^{n} \frac{1}{\left(q_i^2 - m_i^2\right)^{a_i}}$$

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Usually interested in the coefficients of its Laurent expansion

$$I = \sum_{j=-2L}^{\infty} \epsilon^j c_j \longrightarrow$$

(Often complicated) functions of kinematic invariants

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Usually interested in the coefficients of its Laurent expansion

$$I = \sum_{j=-2L}^{\infty} \epsilon^j c_j$$

→ (Often complicated) functions of kinematic invariants

→ How complicated can they be?

Loop integrals

From experience, one encounters logarithms, polylogarithms and Riemann zeta values in the results for loop integrals

$$\mathrm{Li}_1(z) = -\log(1-z)$$

$$\mathrm{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} = \int_0^z \frac{dt}{t} \mathrm{Li}_{n-1}(t)$$

$$\zeta_n = \sum_{k=1}^{\infty} \frac{1}{k^n} = \mathrm{Li}_n(1) \quad (n > 1)$$

There are more!

Multiple polylogarithms

Goncharov (1998)

Generalizations of polylogarithms

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t)$$

$$G(; z) = 1 \qquad G(\vec{0}_n; z) = \frac{1}{n!} \log^n z$$

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Naturally arise as solutions of differential equations

$$\frac{\partial}{\partial x} \vec{f}(\epsilon, x) = \epsilon\, A(x)\, \vec{f}(\epsilon, x)$$

→ Matrix of rational functions

Multiple polylogarithms

A good set of functions

Multiple polylogarithms

A good set of functions

Fast numerics

Numerical evaluation of multiple polylogarithms

Jens Vollinga and Stefan Weinzierl

Institut für Physik, Universität Mainz,
D - 55099 Mainz, Germany

hep-ph/0410259

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Good analytic and algebraic properties, e.g.,

$$G(a,b;z)G(c;z) = G(a,b,c;z) + G(a,c,b;z) + G(c,a,b;z)$$

More: Hopf algebra

Iterated integrals and symbols

MPLs are iterated integrals

$$G(a_1, \ldots, a_n; z) = \int_0^z d\log(t_1 - a_1) \int_0^{t_1} d\log(t_2 - a_2) \cdots \int_0^{t_{n-1}} d\log(t_n - a_n)$$

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Symbol representation

$$(z - a_n) \otimes \cdots \otimes (z - a_2) \otimes (z - a_1)$$

Encodes algebraic properties of MPLs!

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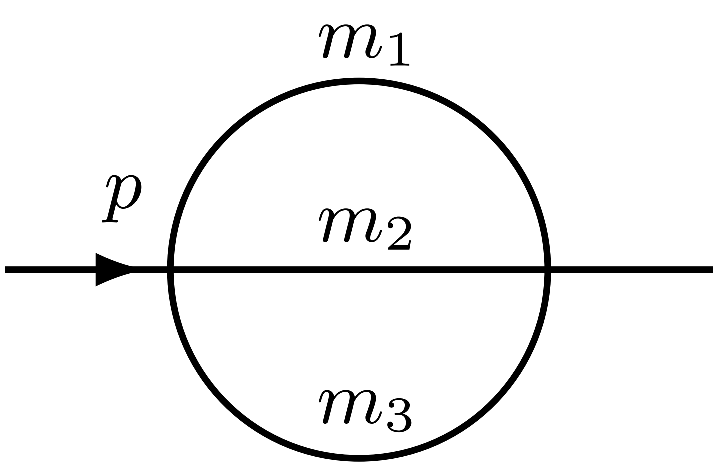
Iterated integrals can be more generic and complicated

$$\int_0^z d\log R_1(t_1) \cdots \int_0^{t_{n-1}} d\log R_n(t_n)$$

May contain algebraic functions, e.g., square roots

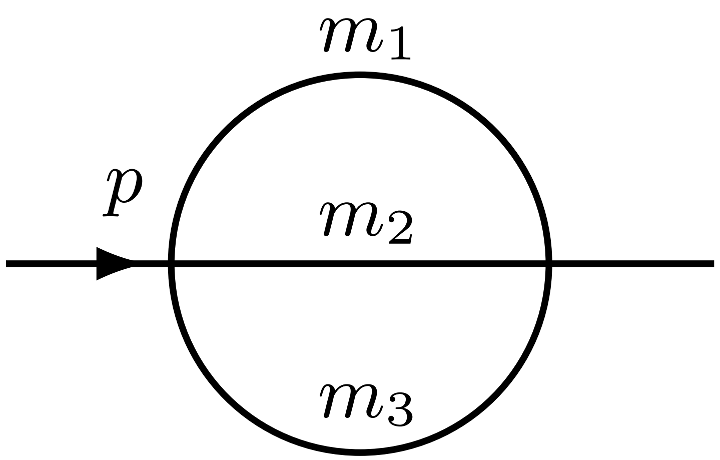
There are more!



$$\int dx \frac{\log(R(x))}{\sqrt{Q(x)}}$$

Elliptic integrals

There are more!



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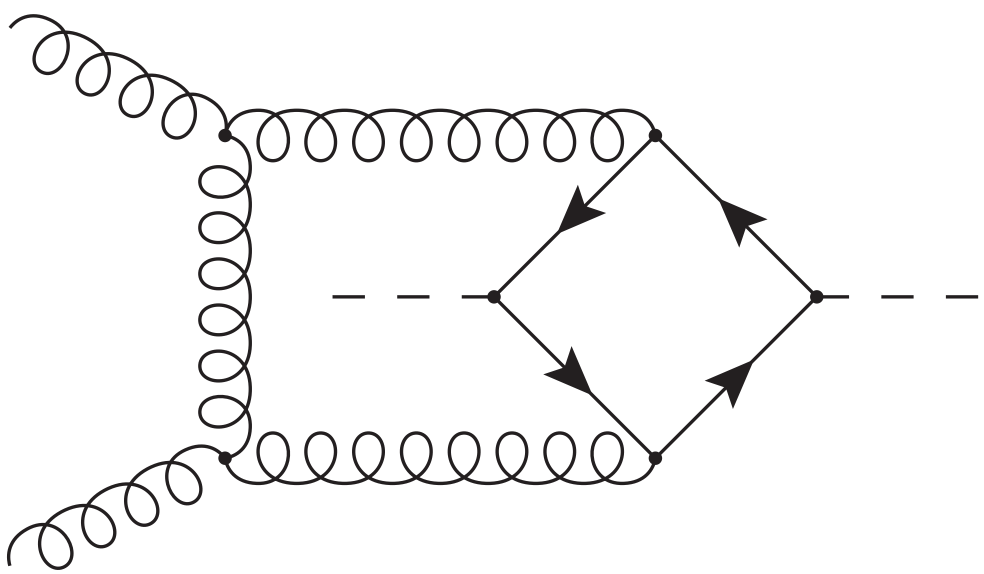

More than elliptic integrals?

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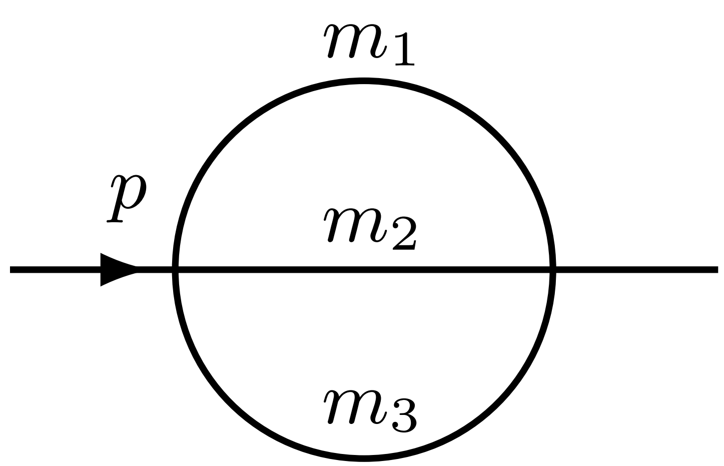
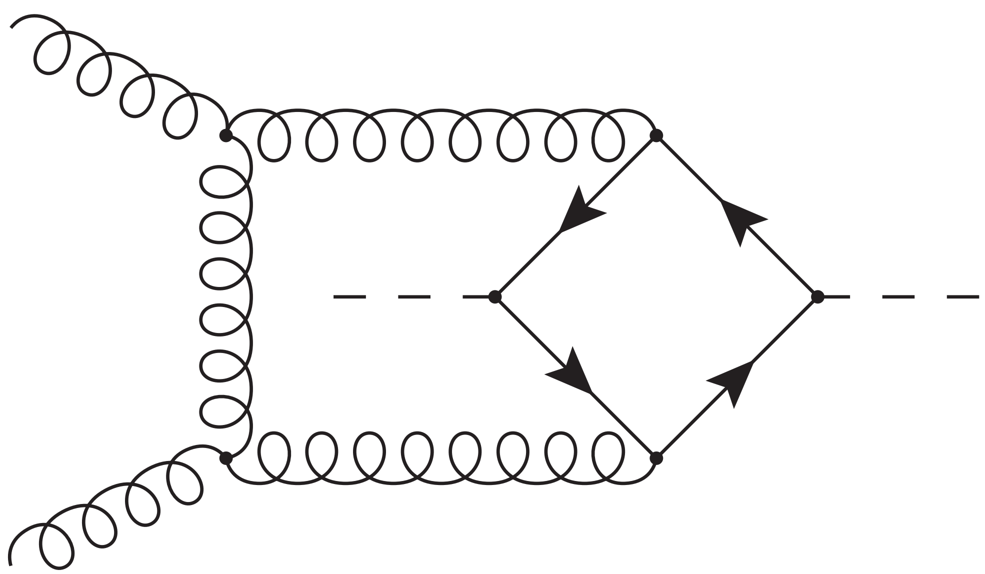


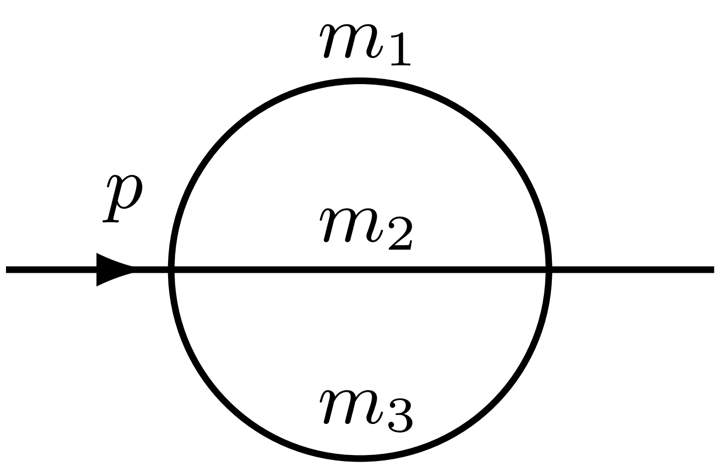
$$\int dx \frac{\log(R(x))}{\sqrt{Q(x)}}$$ Elliptic integrals



More than elliptic integrals?

Extremely difficult integrals in Higgs physics:
massive particles flowing around!

There are more!



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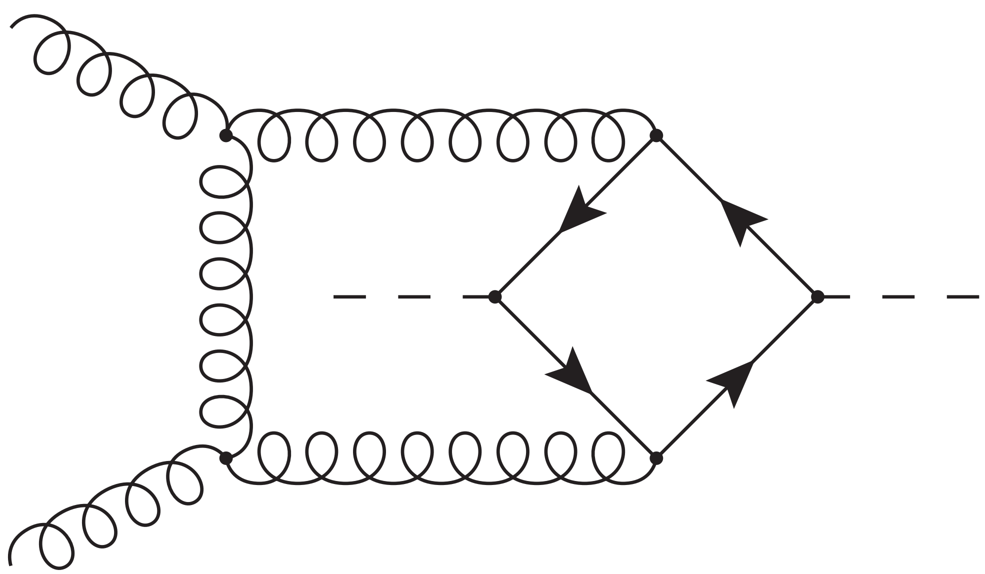

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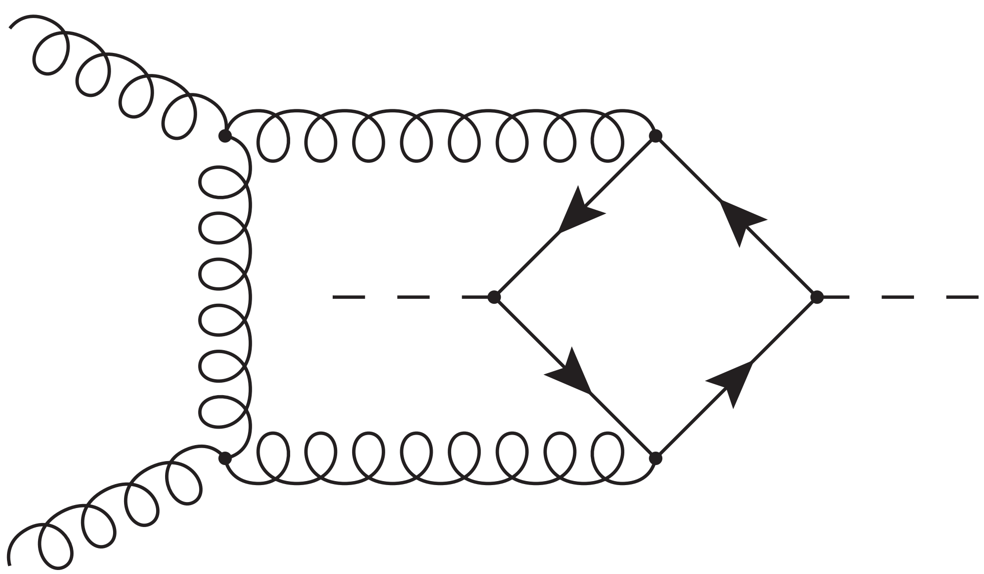
We either spend time with purely numeric methods, or
we need clever approximations…

Outline

- A new approximation for Higgs pair production at NLO
- Approximate and exact NNLO results for HZV vertex
- Approximate result for ttH production beyond NLO
- Thrust distribution in Higgs hadronic decays

Higgs pair production at NLO (two loops)



4 scales: s, t, m_t, m_h

Purely numerical computation using sector decomposition (resource demanding)

Borowka et al.: 1604.06447, 1608.04798

Approximations

1/mt expansion (only valid for low energy region)

$$m_t^2 \gg |s|, |t|, m_h^2$$

Grigo, Hoff, Melnikov, Steinhauser: 1305.7340

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Large energy expansion

$$|s|, |t| \gg m_t^2 \gg m_h^2$$

Davies, Mishima, Steinhauser, Wellmann: 1801.09696

Tricky: singular behavior for $m_t \to 0$

Small Higgs mass expansion

Xu, **LLY**: 1810.12002

A novel approximation method

$$I(s,t,m_t^2,m_h^2,\epsilon) = \sum_{n=0}^{\infty} \frac{m_h^{2n}}{n!} I^{(n)}(s,t,m_t^2,\epsilon)$$

Taylor expansion: no singularity in the $m_h \to 0$ limit

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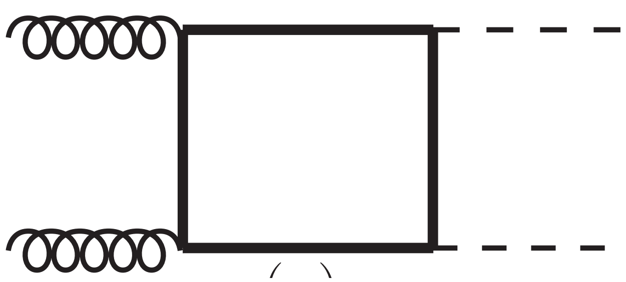


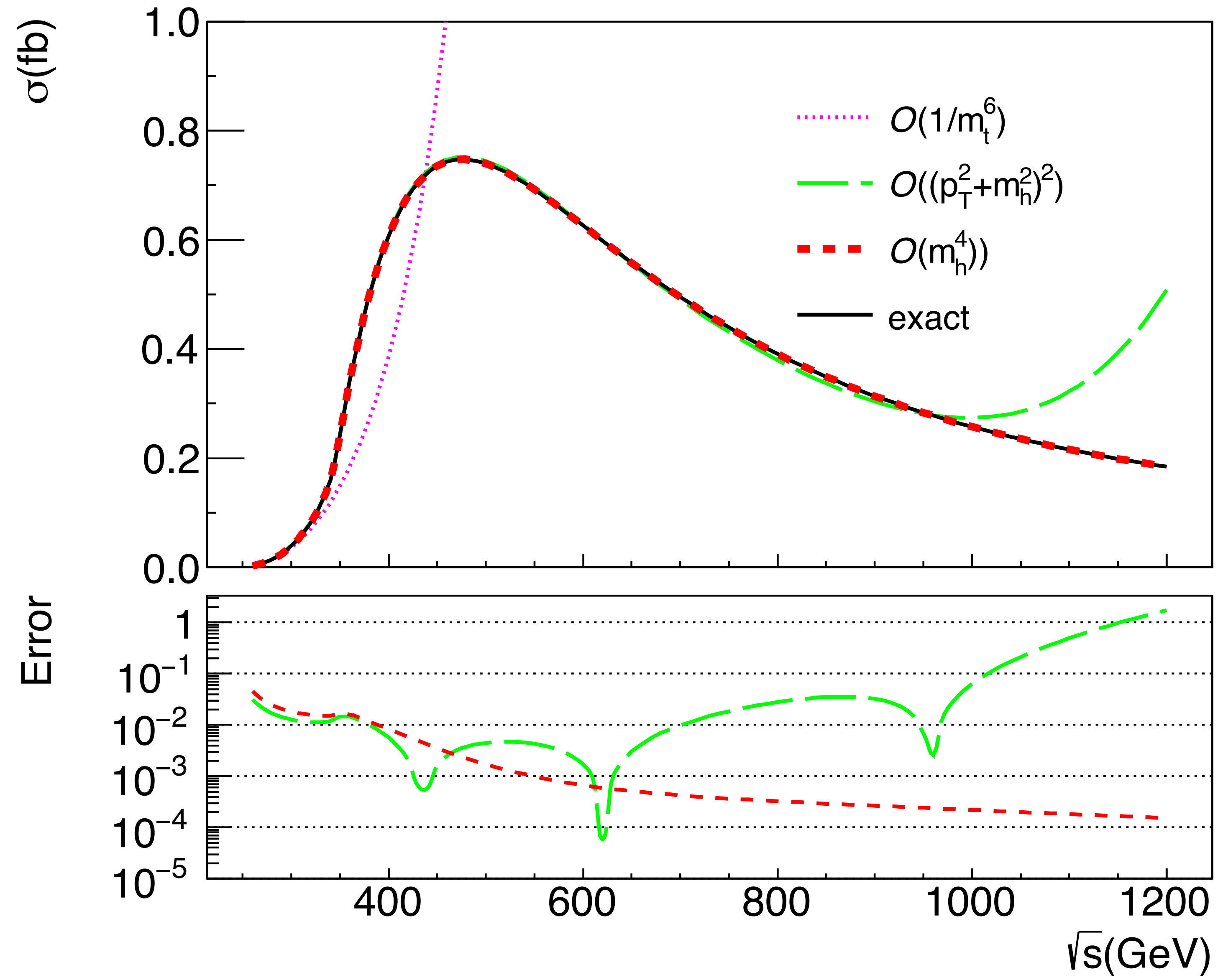
One loop example:

$$I_{1,1,1,1} = \tilde{I}_{1,1,1,1} + \frac{m_h^2}{s+t}\left[-t\tilde{I}_{1,1,1,2} - \left(2\tilde{I}_{1,1,0,2} - \tilde{I}_{1,0,1,2} - \tilde{I}_{1,1,1,1}\right)\right] + \mathcal{O}(m_h^4)$$

$$\tilde{I}_{a_1,a_2,a_3,a_4}(s,t,m_t^2,\epsilon) = \lim_{m_h^2\to 0} I_{a_1,a_2,a_3,a_4}(s,t,m_t^2,m_h^2,\epsilon)$$

Comparing approximations at one-loop

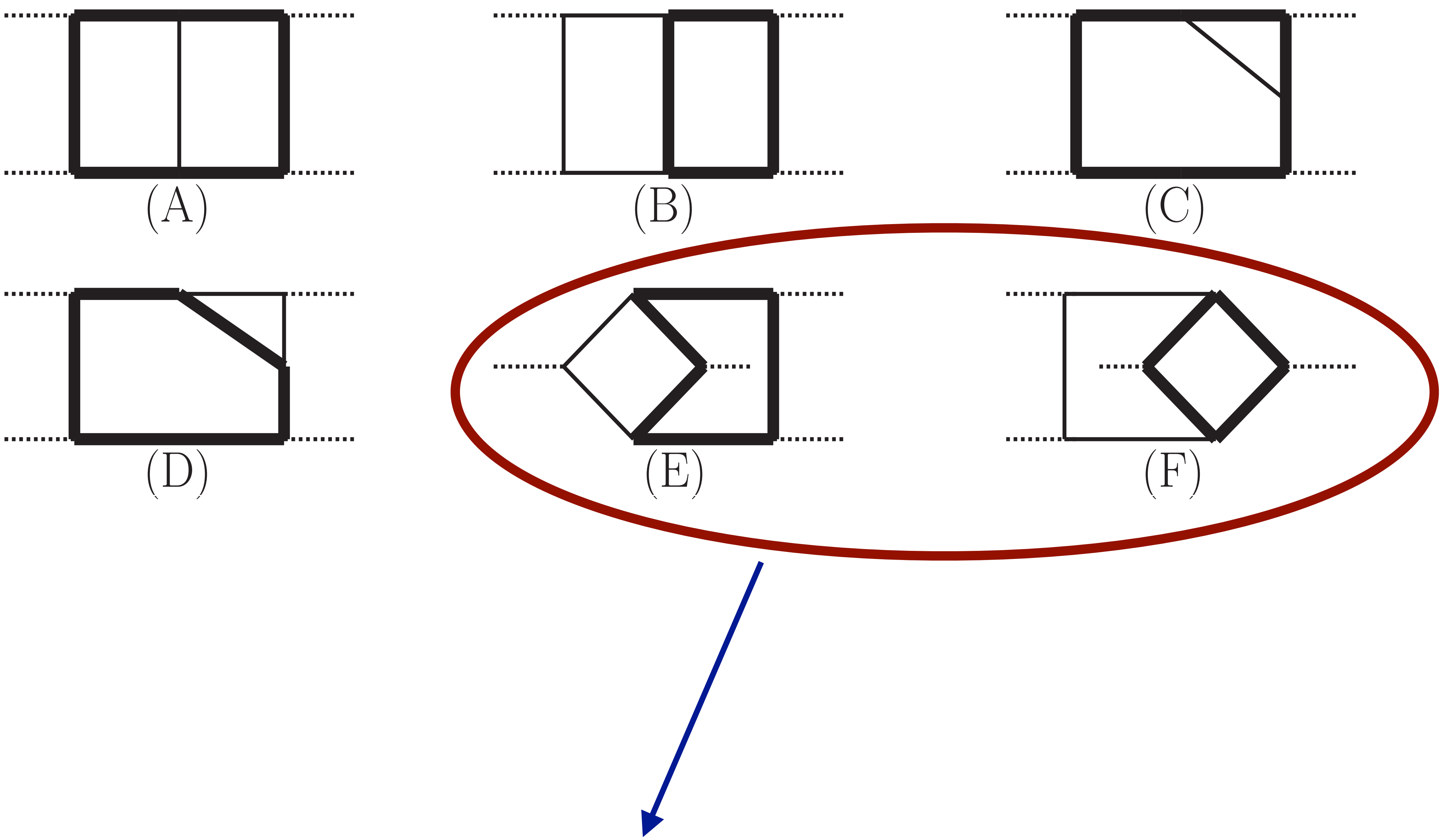
Xu, **LLY**: 1810.12002



Our method is valid in the entire phase space

Expansion at two-loop

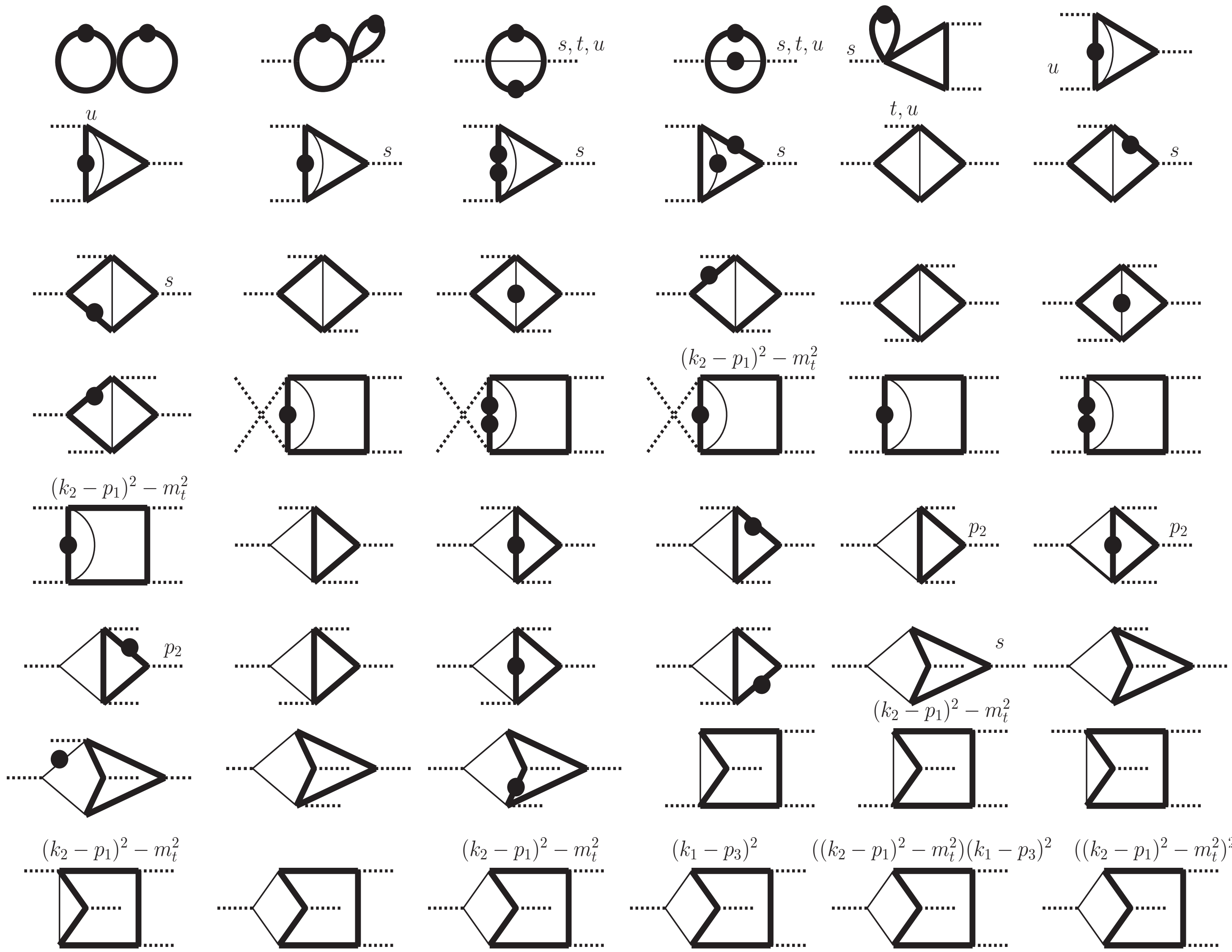
Xu, **LLY**: 1810.12002



Difficult part: two non-planar topologies

Master integrals

Xu, **LLY**: 1810.12002



Solve the master integrals using the method of differential equations

Analytic results

Xu, **LLY**: 1810.12002

Weight-2 functions reconstructed from symbols

$$\frac{\sqrt{\beta_i+1}-1}{\sqrt{\beta_i+1}+1}\otimes\beta_i \to 2\mathrm{Li}_2(1-z_i)+\frac{1}{2}\log^2(z_i)\,,$$

$$\frac{\sqrt{\beta_i+1}-1}{\sqrt{\beta_i+1}+1}\otimes(\beta_i+1) \to 2\mathrm{Li}_2(1-z_i)+2\mathrm{Li}_2(-z_i)+2\log(z_i)\log(z_i+1)+\frac{\pi^2}{6}\,,$$

$$\frac{\sqrt{\beta_i+1}-1}{\sqrt{\beta_i+1}+1}\otimes\frac{\sqrt{\beta_i+1}-\sqrt{\beta_i+\beta_j+1}}{\sqrt{\beta_i+1}+\sqrt{\beta_i+\beta_j+1}}+(\beta_i\leftrightarrow\beta_j)$$

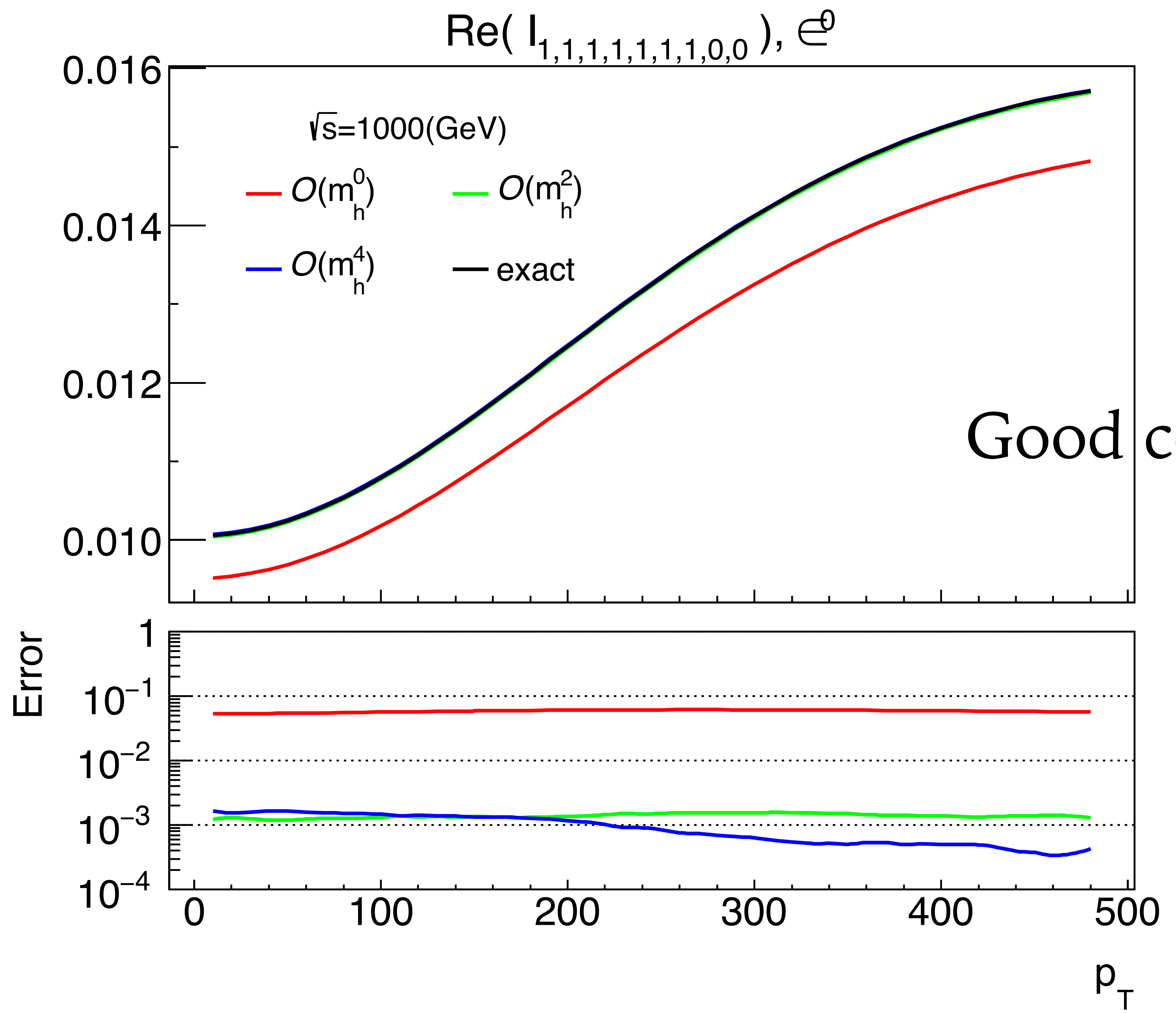
$$\to 2\mathrm{Li}_2(-x_{ij})-2\mathrm{Li}_2(x_{ij})-\log(x_{ij})\log\frac{1-y_{ij}}{1+y_{ij}}-\log(x_{ij})\log\frac{1-x_{ij}}{1+x_{ij}}$$

$$-2\mathrm{Li}_2(-y_{ij})+2\mathrm{Li}_2(y_{ij})+\log(y_{ij})\log\frac{1-y_{ij}}{1+y_{ij}}+\log(y_{ij})\log\frac{1-x_{ij}}{1+x_{ij}}\,.$$

More complicated functions at higher transcendental weights!

Numeric results

Xu, **LLY**: 1810.12002

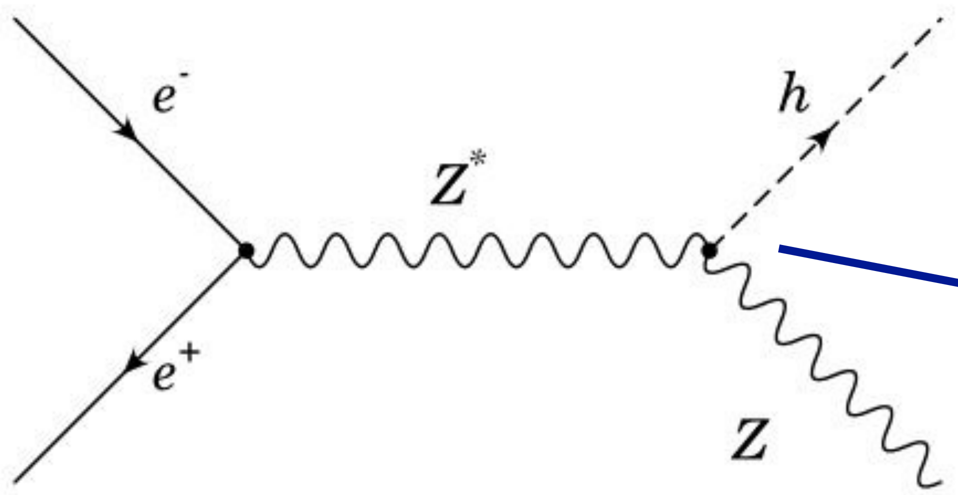


Good convergence at two-loop

Phenomenological applications upcoming!

Can also be applied to other processes (ZH, Hj, etc.)

Higgs production at CEPC



Expected to be measured with an uncertainty less than 0.5%

Probing Higgs gauge-coupling

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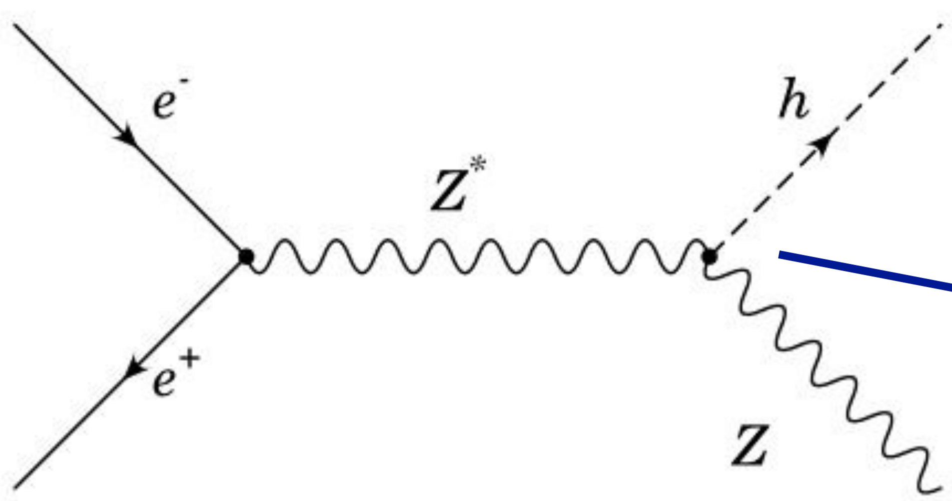

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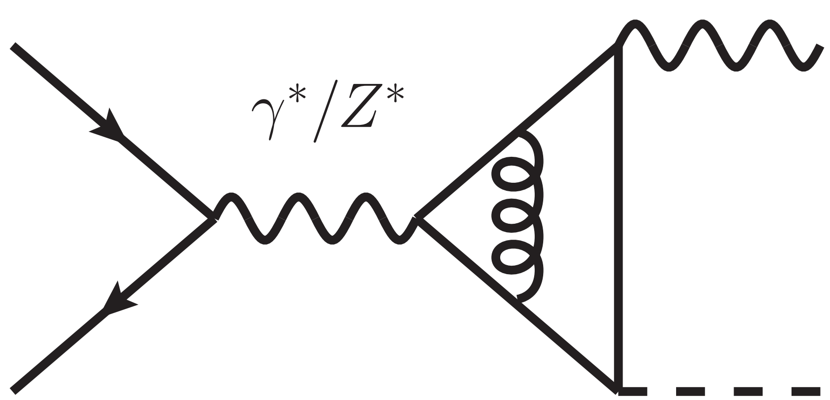
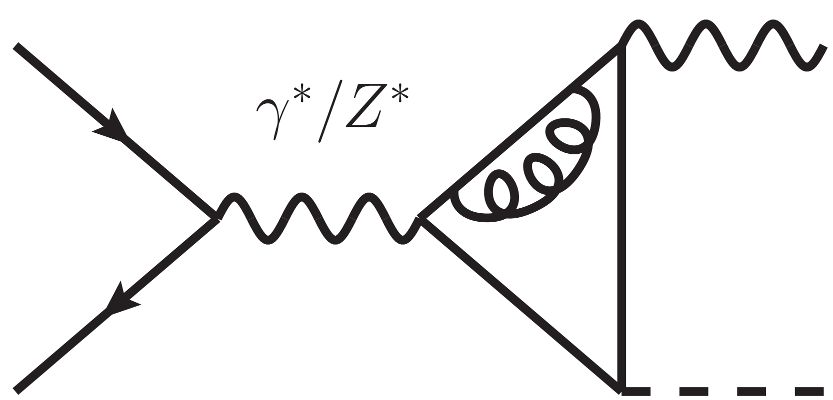
NNLO calculations demanded!





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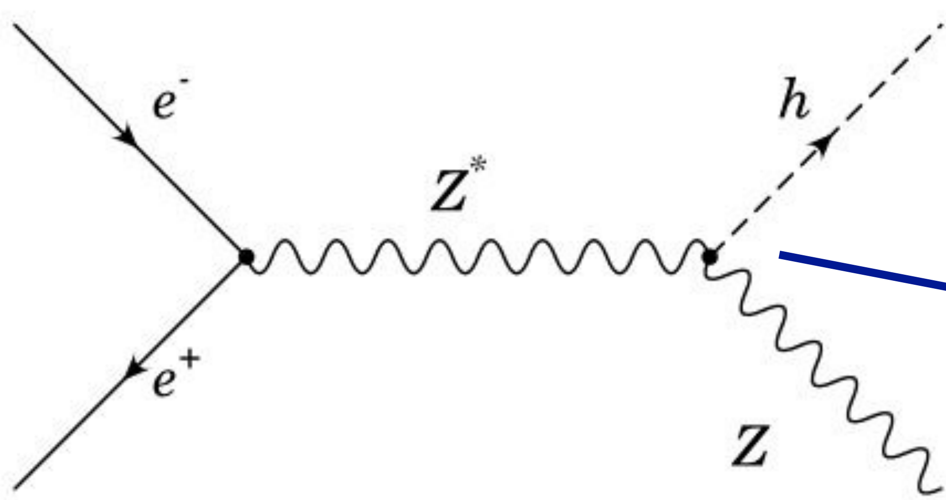

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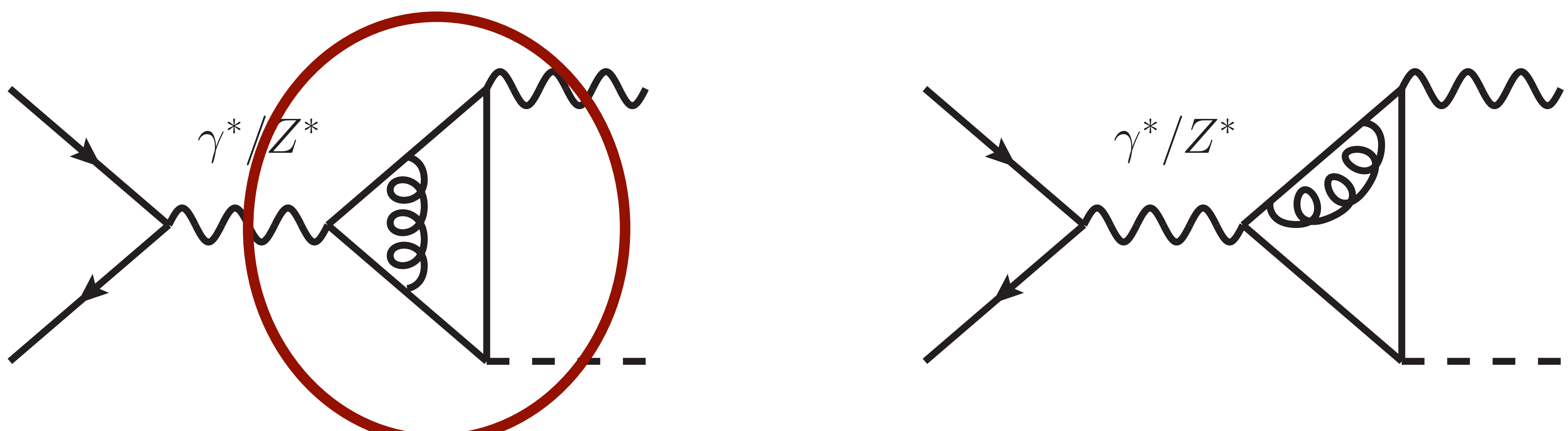
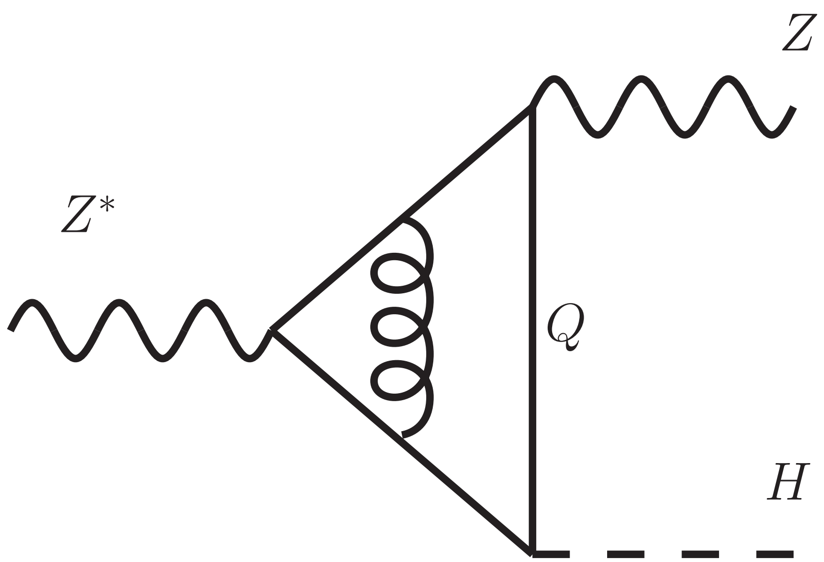


HZV vertex



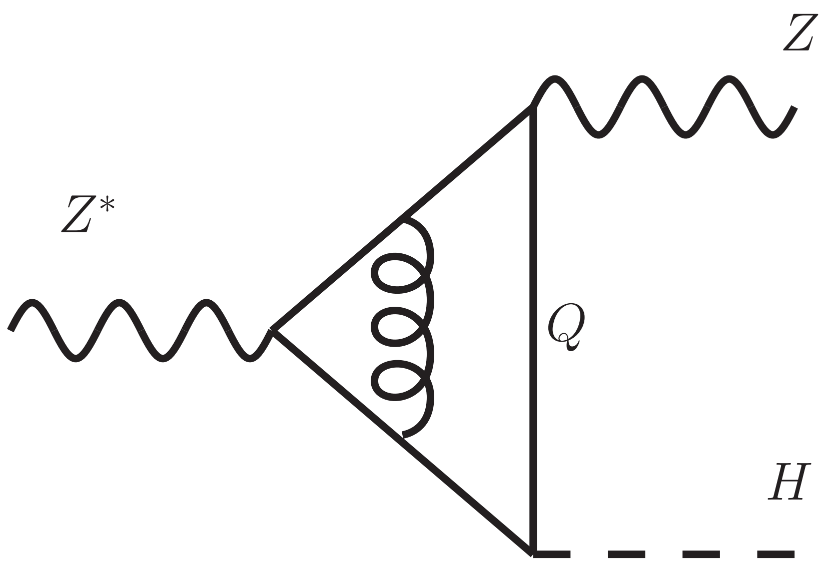
Relevant to $H \to 4l$

$e^+e^- \to ZH$

Looks simple, difficult to calculate

Involves 4 scales: m_t, m_H, m_Z, Q

HZV vertex



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Numeric integration using sector decomposition

Gong, Li, Xu, **LLY**, Zhao: 1609.03955
Sun, Feng, Jia, Sang: 1609.03995

Time-consuming, especially for bottom quark loops and for high energies (above the top quark pair threshold)

HZV vertex: $1/m_t$ expansion

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

Taylor series in $\frac{s, m_H^2, m_Z^2}{m_t^2}$

Can be done at the level of integrands (with top quark loop only)

$$\begin{aligned}\sigma^{\alpha\alpha_s}(\sqrt{s}, m_H, m_Z, m_t) &= m_t^2\, c_2(\sqrt{s}, m_H, m_Z) \\ &+ m_t^0\, c_0(\sqrt{s}, m_H, m_Z) \\ &+ m_t^{-2}\, c_{-2}(\sqrt{s}, m_H, m_Z) \\ &+ \cdots\end{aligned}$$

Simple analytic expressions

HZV vertex: $1/m_t$ expansion

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

Good convergence for optimal energies of Higgs factories

$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%

$$\frac{m_t^2\, c_2}{\sigma^{\alpha\alpha_s}}$$

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No difference between exact and expanded results (4 digits)

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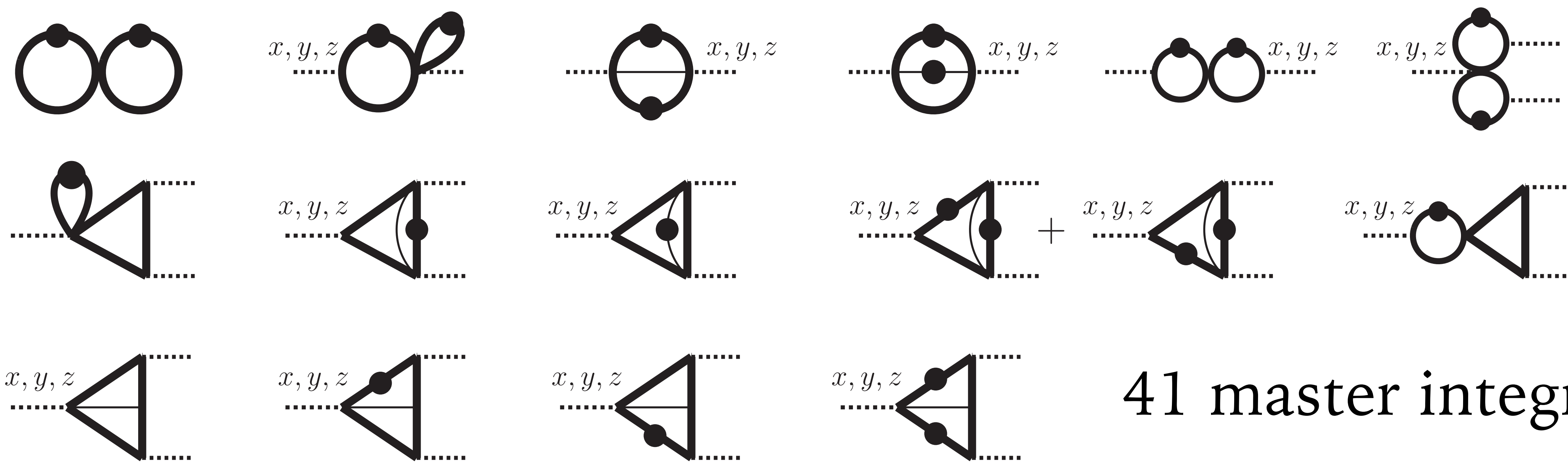
No difference between exact and expanded results (4 digits)

But note: expansion not working at high energies (neither for bottom quark loops)

HZV vertex: analytic result

Wang, Xu, **LLY**: 1905.11463

To deal with the difficulties, exact analytic result necessary!



41 master integrals

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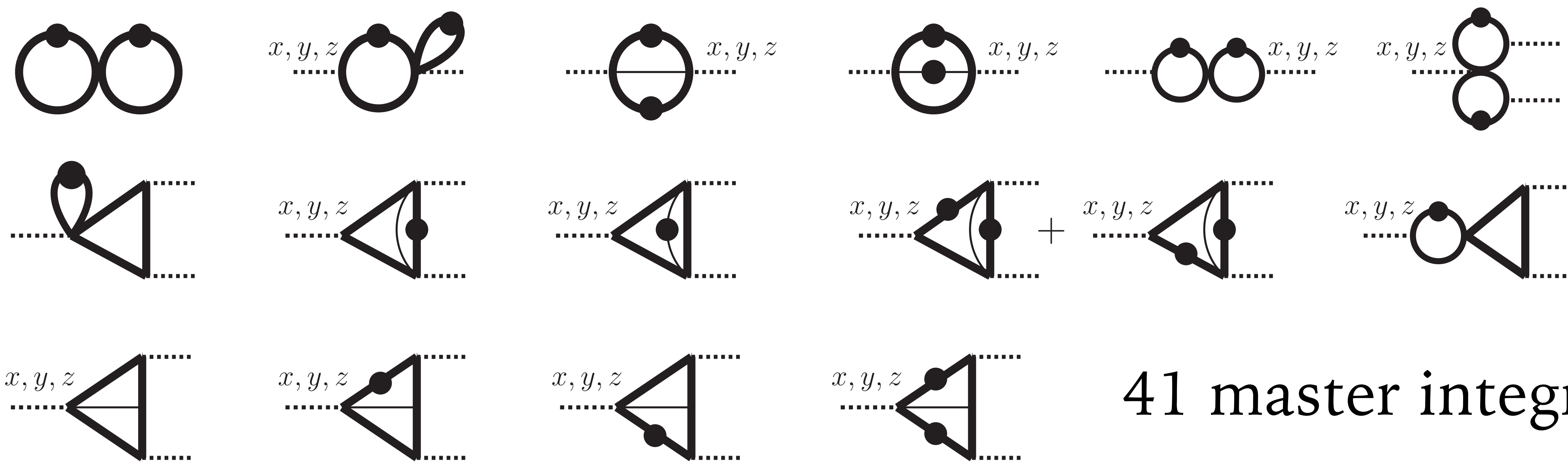
Wang, Xu, **LLY**: 1905.11463

To deal with the difficulties, exact analytic result necessary!



41 master integrals

4 kinds of square roots appear in the differential equations

$$R_1(x) = \sqrt{x(x+1)}\,, \quad R_1(y) = \sqrt{y(y+1)}\,, \quad R_1(z) = \sqrt{z(z+1)}$$

$$R_2(x,y,z) = \sqrt{x^2+y^2+z^2-2xy-2yz-2zx}$$

$$x = -\frac{Q^2}{4m_t^2}\,, \quad y = -\frac{m_Z^2}{4m_t^2}\,, \quad z = -\frac{m_H^2}{4m_t^2}$$

HZV vertex: analytic result

Wang, Xu, **LLY**: 1905.11463

Explicit analytic expressions can be reconstructed from the symbol representation (up to weight 3)

Tricky: rationalization of square roots via change of variables

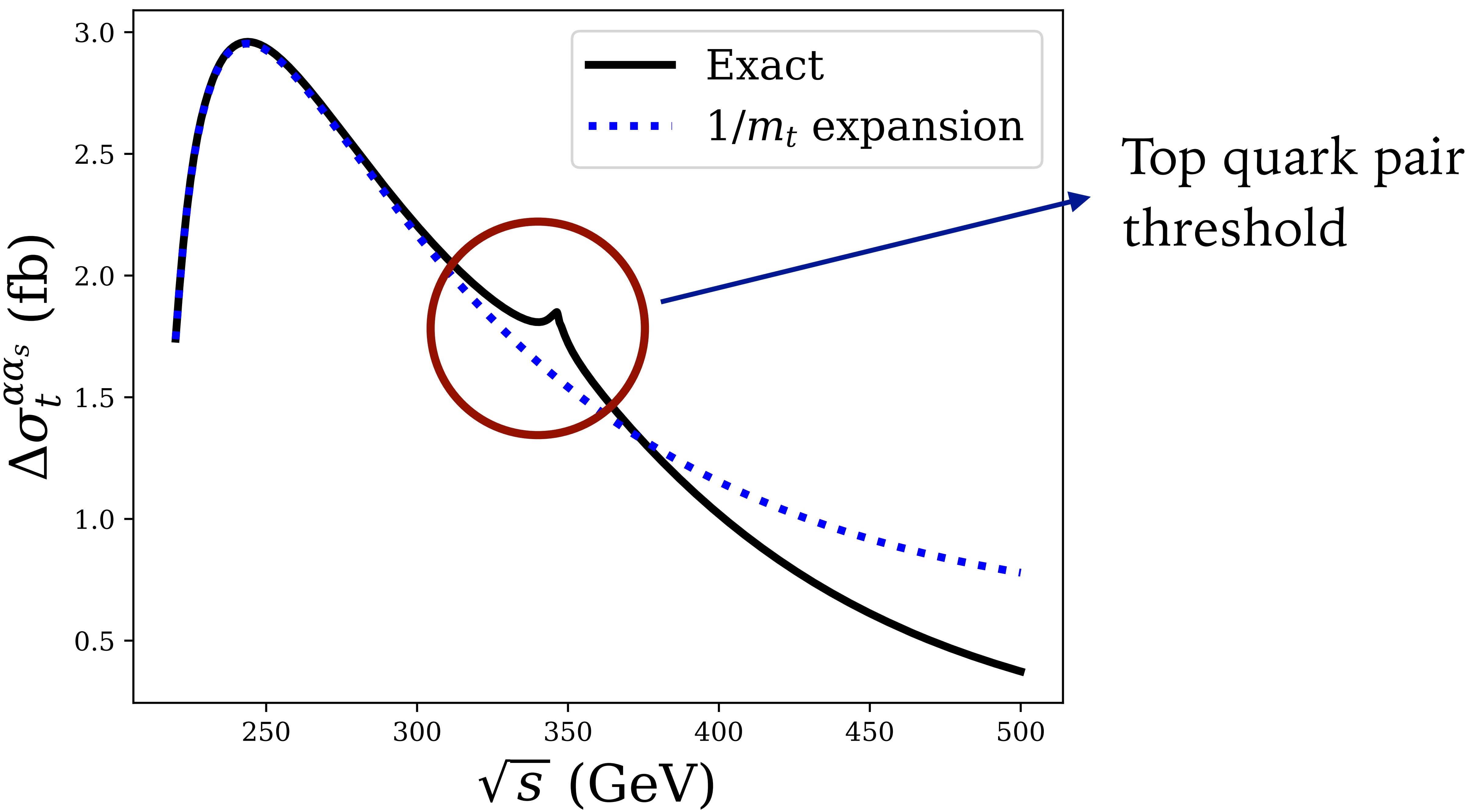
$$
\begin{aligned}
&\beta(x) \otimes \frac{x(x-y-z)-R_1(x)R_2}{x(x-y-z)+R_1(x)R_2} + (x \leftrightarrow y) + (x \leftrightarrow z) \\
&\to G\left(\frac{2R_2}{R_2+x-y-z}, 1; 1-\beta(x)\right) - G\left(\frac{2R_2}{R_2-x+y+z}, 1; 1-\beta(x)\right) \\
&+ G\left(\frac{2R_2}{R_2+y-x-z}, 1; 1-\beta(y)\right) - G\left(\frac{2R_2}{R_2-y+x+z}, 1; 1-\beta(y)\right) \\
&+ G\left(\frac{2R_2}{R_2+z-y-x}, 1; 1-\beta(z)\right) - G\left(\frac{2R_2}{R_2-z+y+x}, 1; 1-\beta(z)\right).
\end{aligned}
$$

Allows fast numerics for all phase-space points!

HZV vertex: numeric results

Wang, Xu, **LLY**: 1905.11463

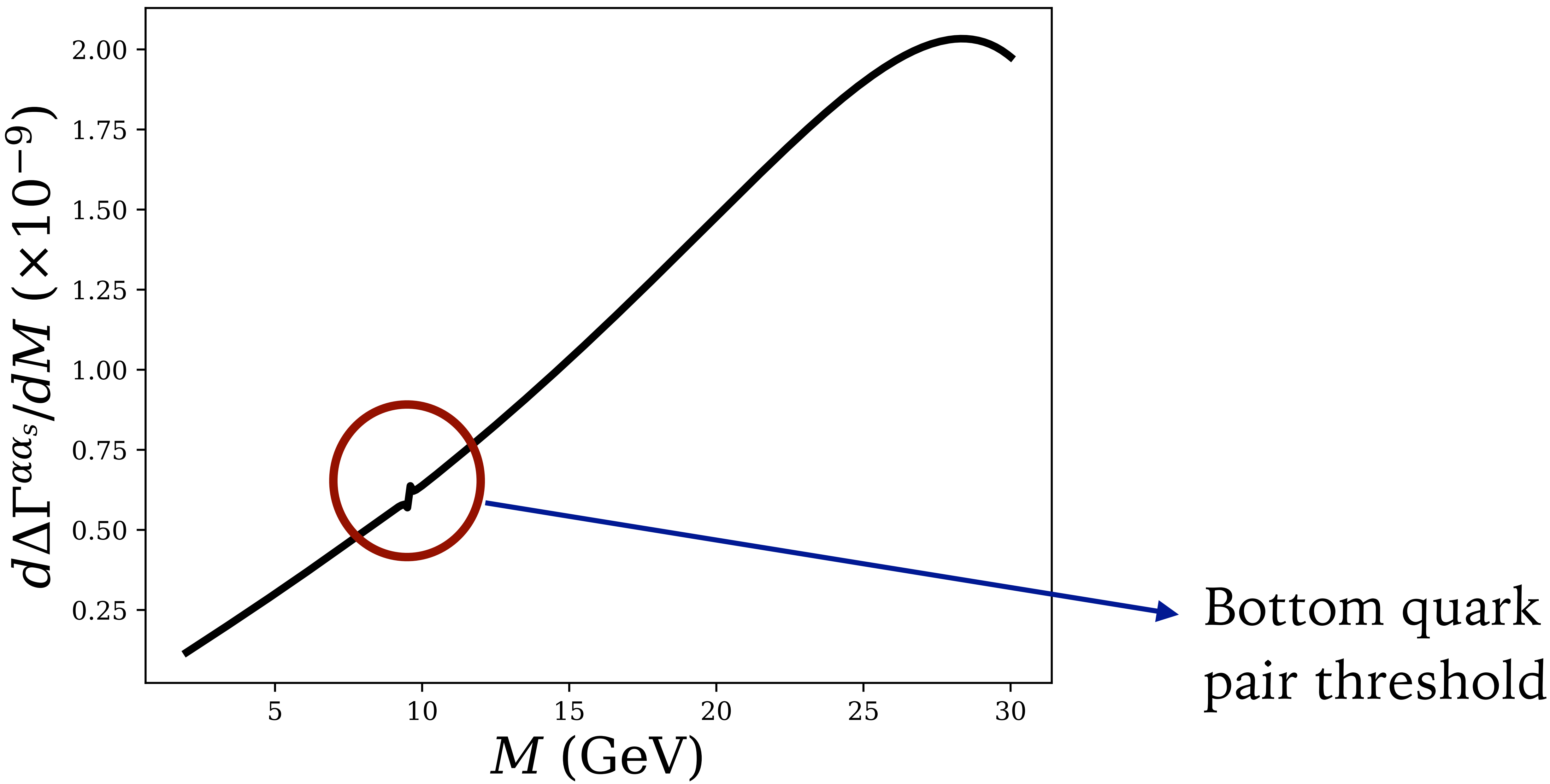
Applied to $e^+e^- \to ZH$



HZV vertex: numeric results

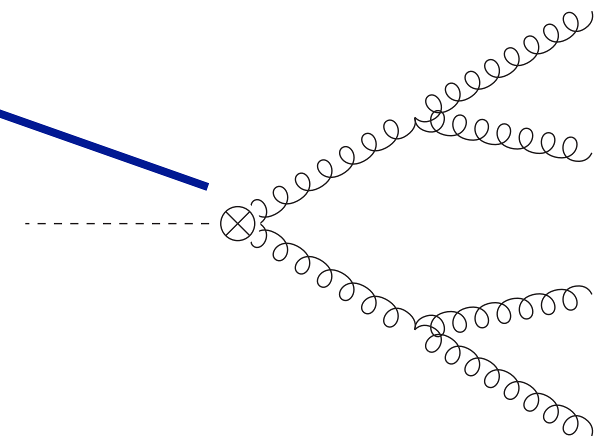
Wang, Xu, **LLY**: 1905.11463

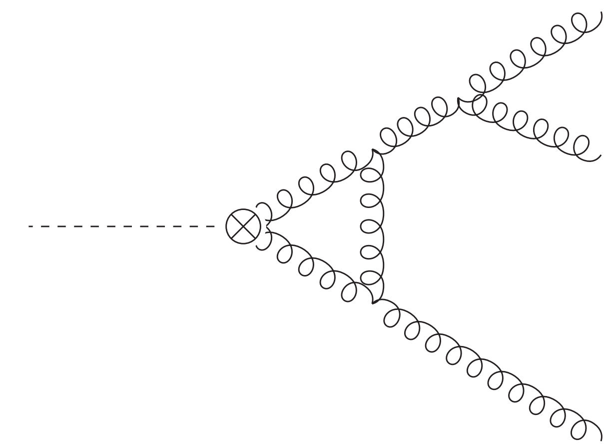
Can also be applied to $H \to ZZ^*$



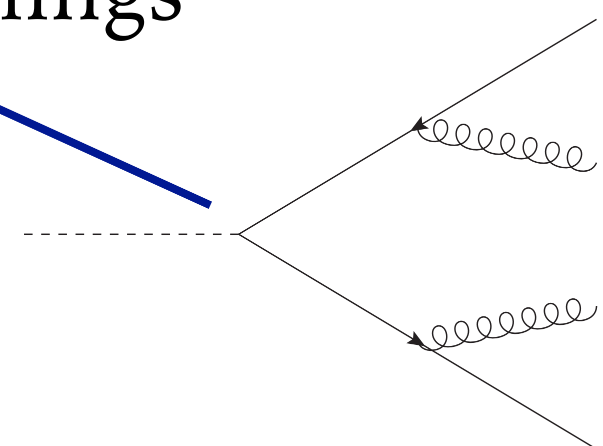
Higgs hadronic decay

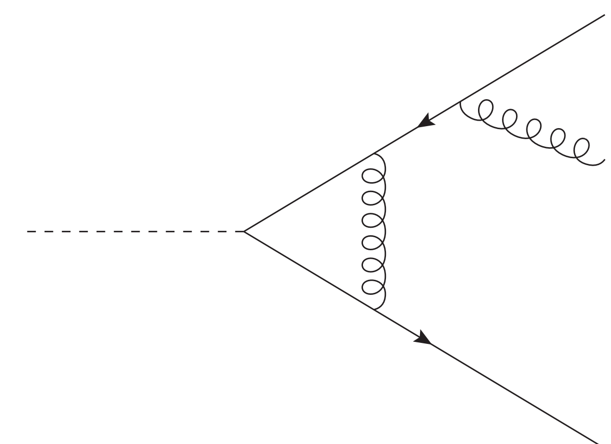
Probing Hgg coupling…





and anomalous Yukawa couplings



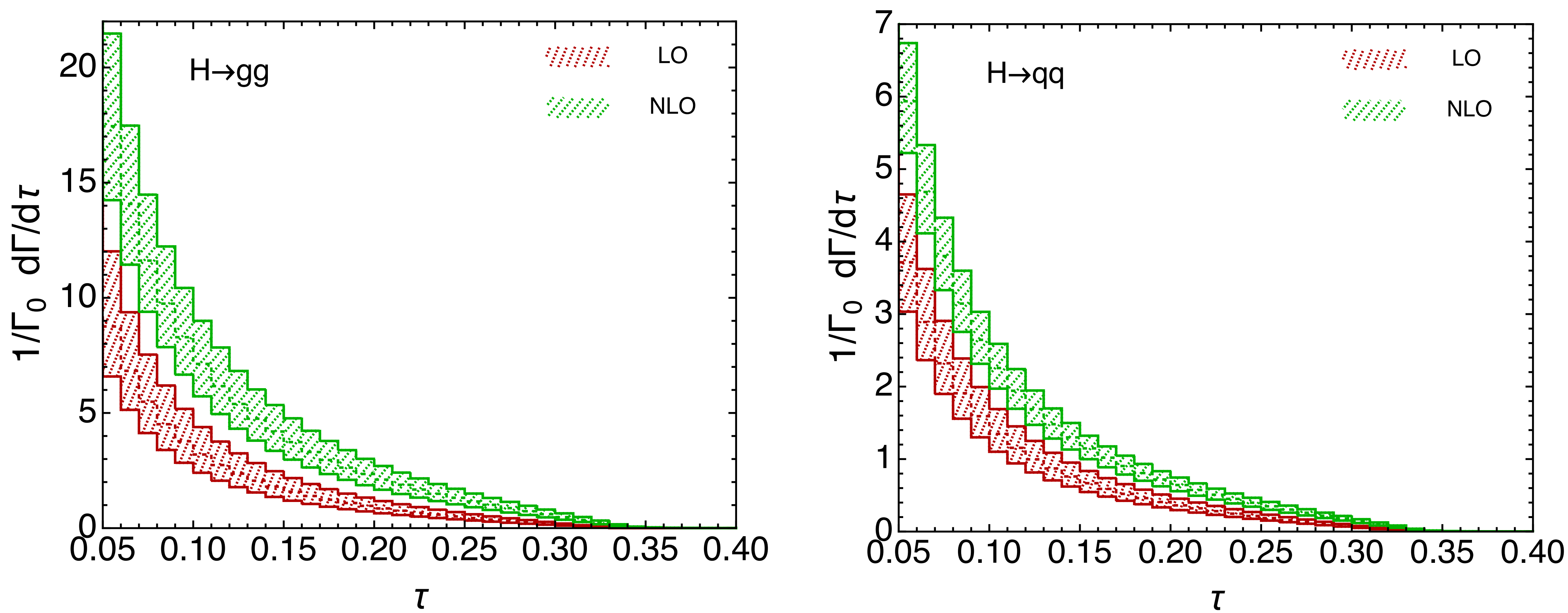


An important observable: thrust

$$T \equiv 1 - \tau \equiv \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

Thrust distribution at NLO

Gao, Gong, Ju, **LLY**: 1901.02253



Thrust distribution at NLO

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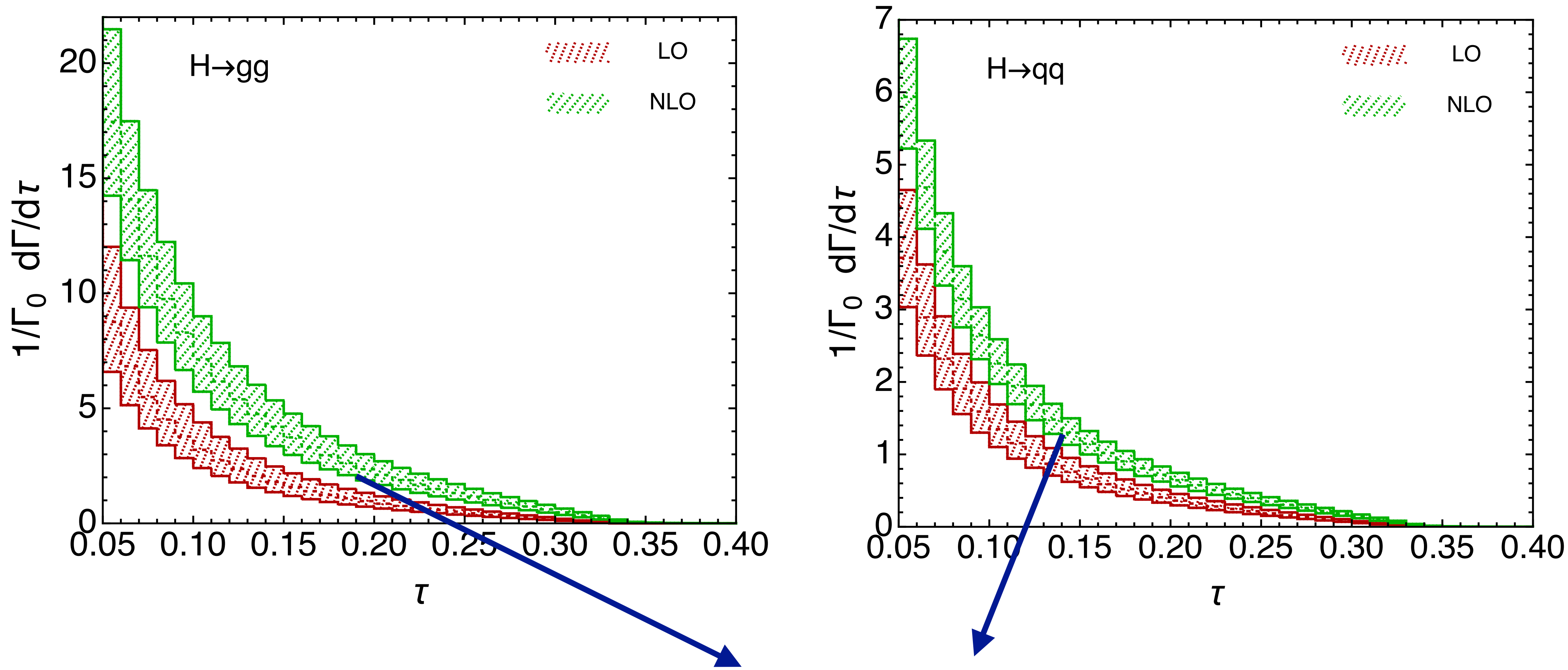
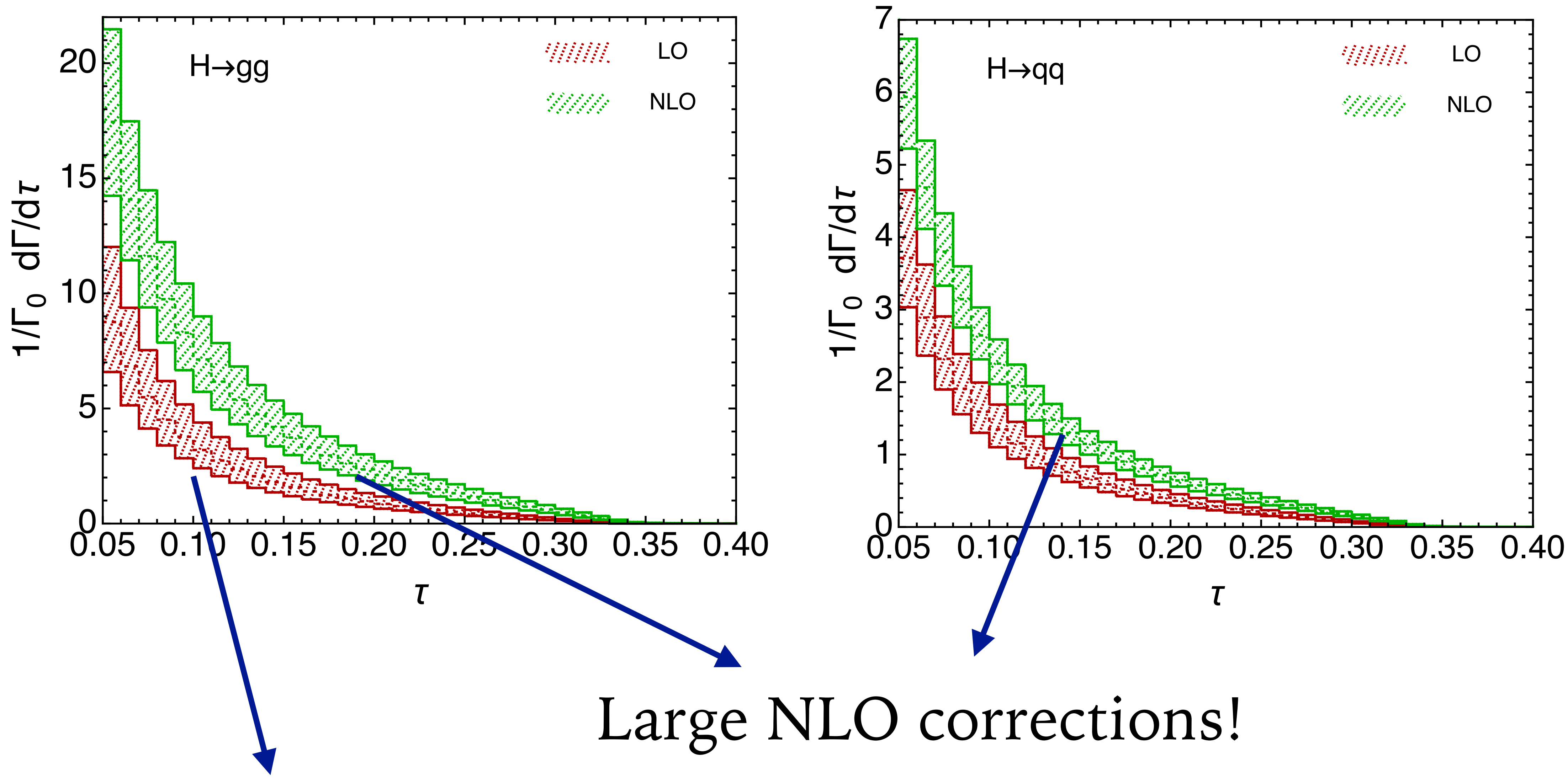


Large NLO corrections!

Thrust distribution at NLO

Gao, Gong, Ju, **LLY**: 1901.02253



Large NLO corrections!

LO bands underestimate the uncertainties

Thrust distribution at NLO

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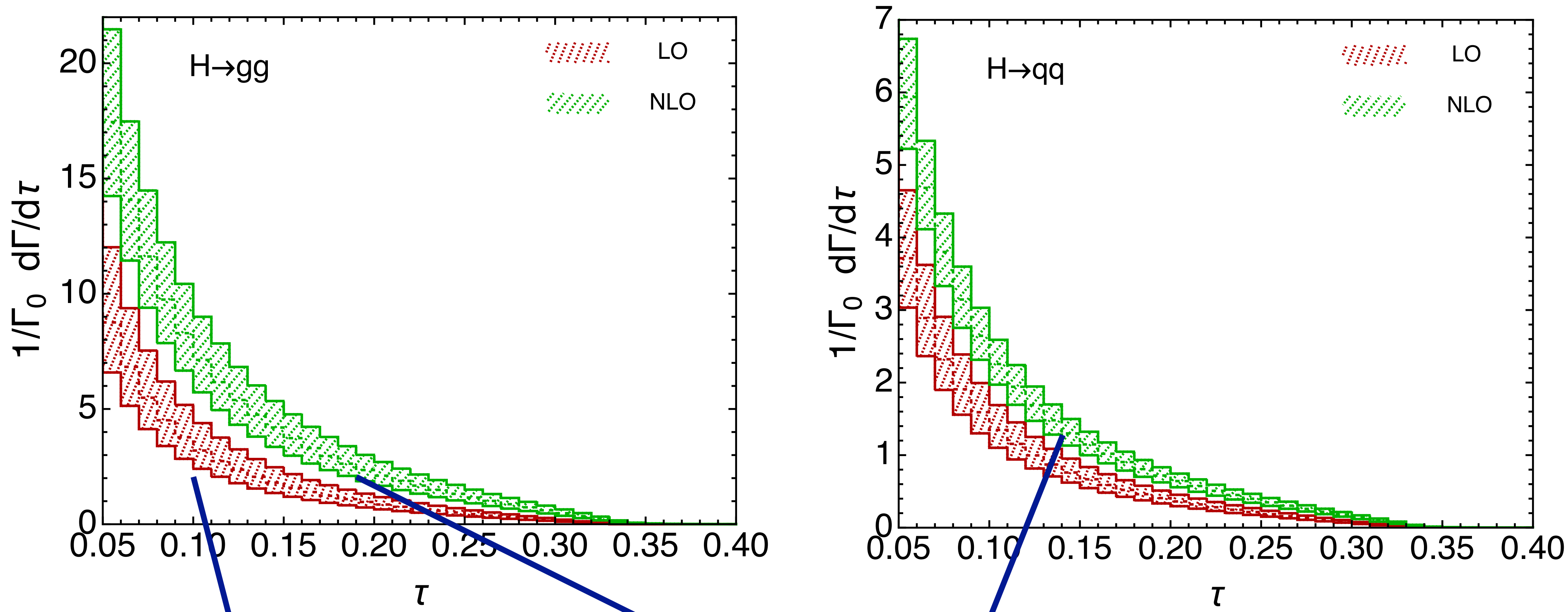


Large NLO corrections!

LO bands underestimate the uncertainties

NNLO calculation necessary!

Approximate NNLO

Gao, Gong, Ju, **LLY**: 1901.02253

It is possible to reconstruct the logarithmically enhanced terms at NNLO and beyond

$$
\begin{aligned}
\Delta_g^{(3)}(\tau, m_H) = \Bigg[& \left(\frac{256}{9} n_f^2 - 368 n_f - 1672\right) L_{HT} + \left(\frac{800}{81} - \frac{80\pi^2}{81}\right) n_f^3 \\
& + \left(\frac{1304\pi^2}{27} - \frac{992\zeta_3}{3} - \frac{31081}{27}\right) n_f^2 + \left(\frac{742121}{27} - \frac{4276\pi^2}{9} + 7552\zeta_3 - \frac{176\pi^4}{15}\right) n_f \\
& - 37152\zeta_5 + 3456\pi^2\zeta_3 - 20904\zeta_3 + \frac{968\pi^4}{5} - \frac{698\pi^2}{3} - \frac{1610351}{9} \Bigg] \frac{1}{\tau} \\
& + \Bigg[- (512 n_f + 1824) L_{HT} - \frac{320}{27} n_f^3 + \left(\frac{352\pi^2}{9} + \frac{5512}{9}\right) n_f^2 \\
& + \left(7072\zeta_3 - 896\pi^2 - \frac{2044}{3}\right) n_f - 90288\zeta_3 - \frac{72\pi^4}{5} - 568\pi^2 - \frac{205012}{3} \Bigg] \frac{\ln(\tau)}{\tau} \\
& + \left[\frac{32}{9} n_f^3 + 144 n_f^2 - (624\pi^2 + 11616) n_f - 26784\zeta_3 + 10296\pi^2 + 126876\right] \frac{\ln^2(\tau)}{\tau} \\
& + \left[- \frac{1184}{9} n_f^2 + \frac{9184}{3} n_f + 2304\pi^2 - 3752\right] \frac{\ln^3(\tau)}{\tau} + (960 n_f - 15840) \frac{\ln^4(\tau)}{\tau} \\
& - 1728 \frac{\ln^5(\tau)}{\tau} \,, \qquad \text{(C.3)}
\end{aligned}
$$

The theoretical tool is factorization

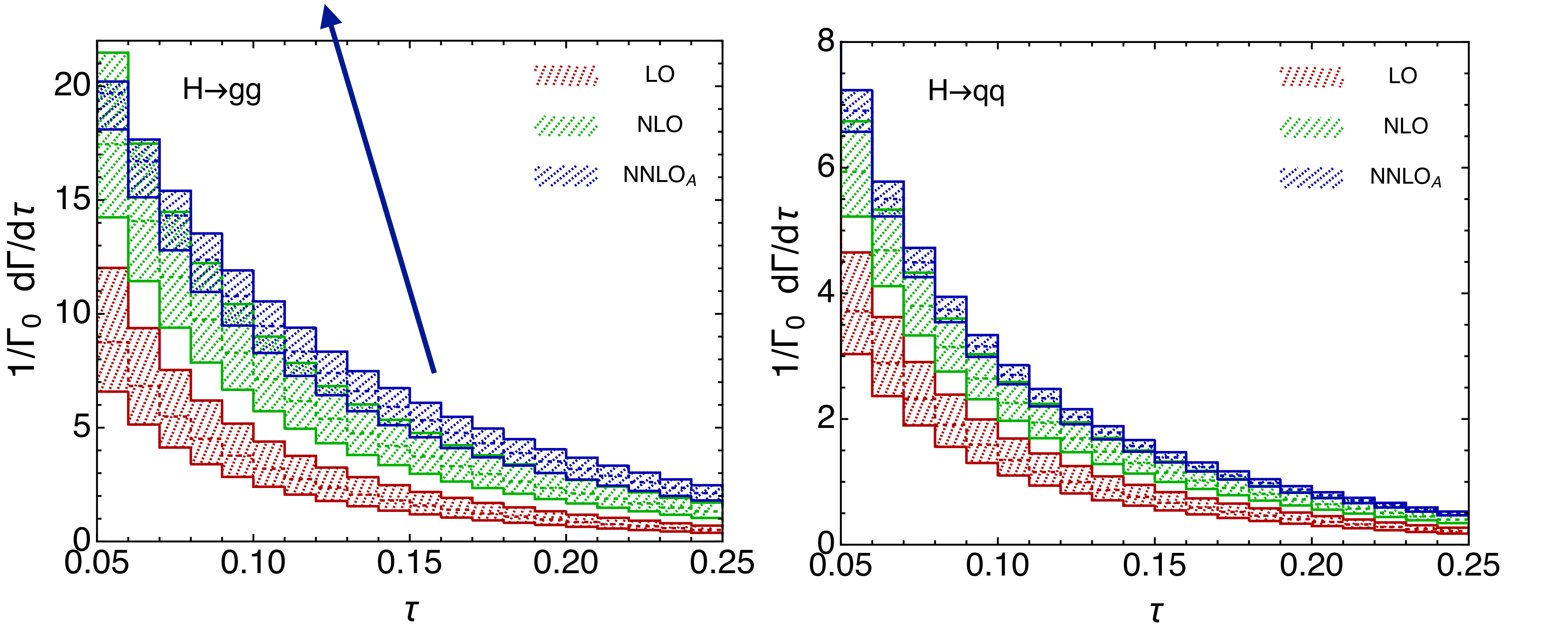
$$
\frac{d\Gamma^i}{d\tau} = \Gamma_0^i(\mu)\, |C_t^i(m_t, \mu)|^2\, |C_S^i(m_H, \mu)|^2 \int dp_n^2\, dp_{\bar{n}}^2\, dk\, \delta\!\left(\tau - \frac{p_n^2 + p_{\bar{n}}^2}{m_H^2} - \frac{k}{m_H}\right) \times J_n^i(p_n^2, \mu)\, J_{\bar{n}}^i(p_{\bar{n}}^2, \mu)\, S^i(k, \mu)
$$

valid in the limit T→1

Approximate NNLO

Gao, Gong, Ju, **LLY**: 1901.02253

NNLO corrections still large, but overlap
with the NLO bands (finally)



Approximate NNLO

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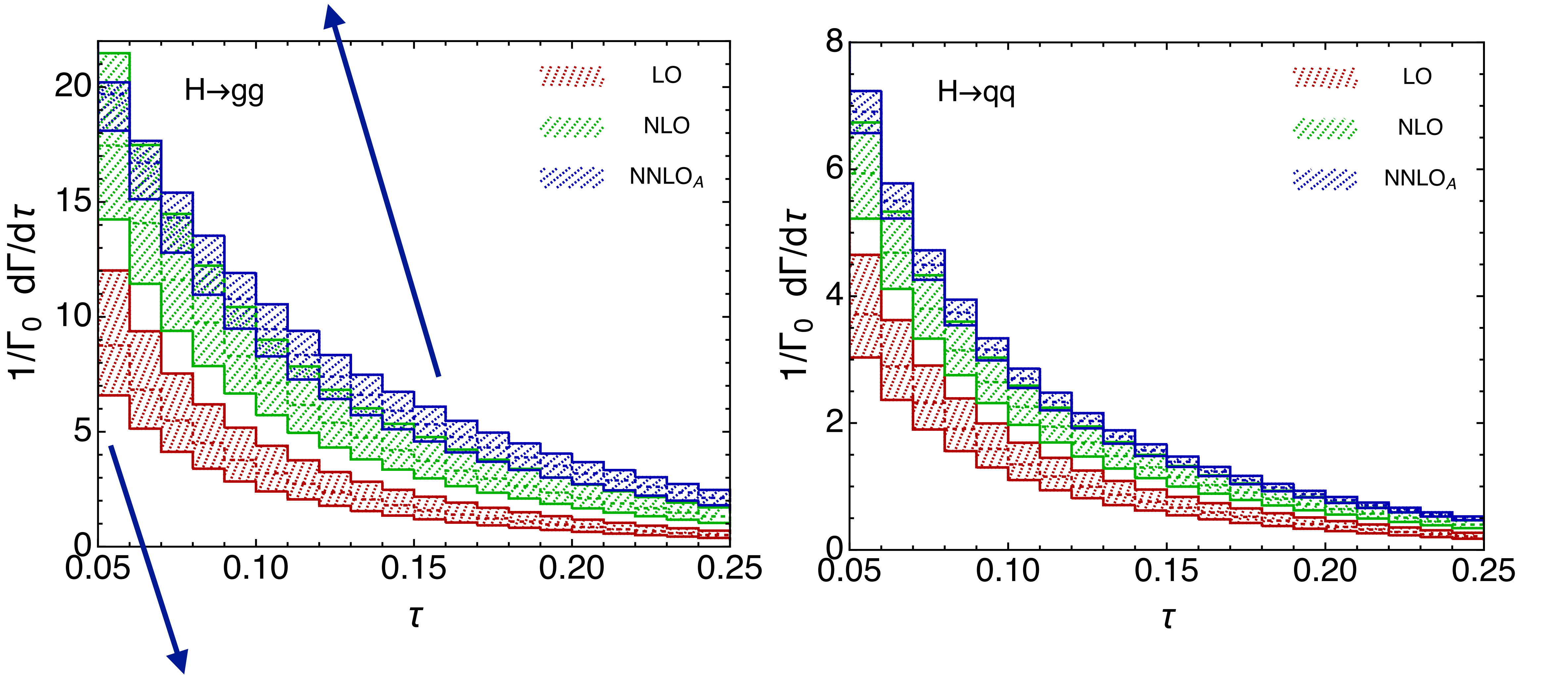
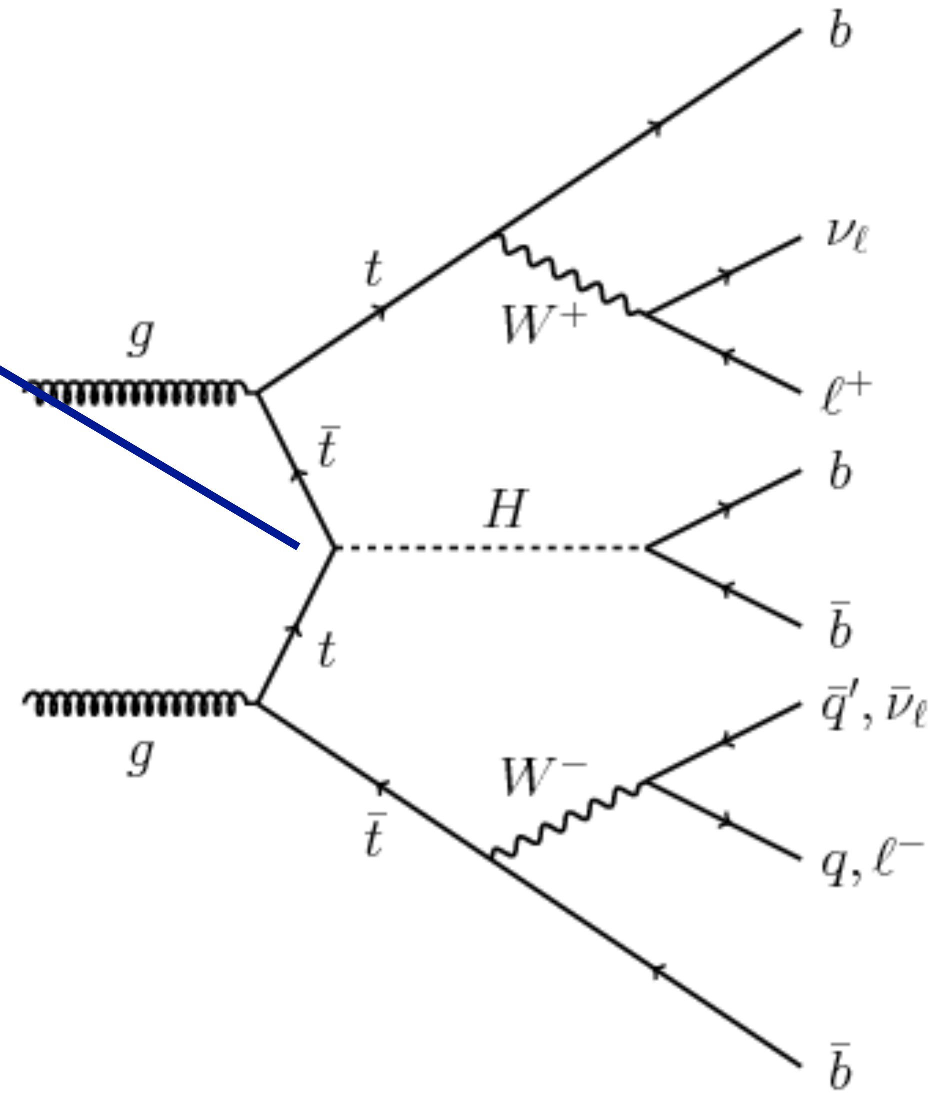


Back-to-back region, requires resummation (to appear)

Higgs and top quark pair

Probing the Yukawa coupling of the top quark



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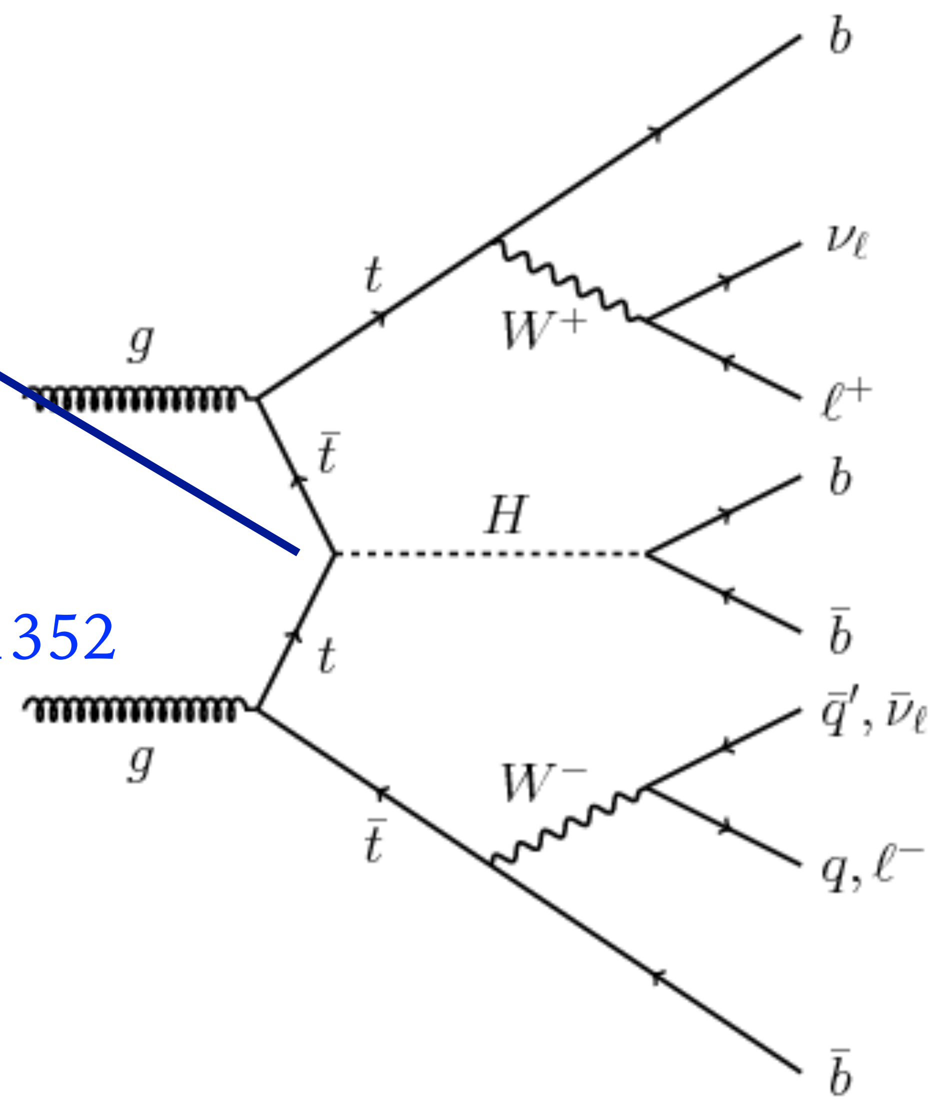

NLO QCD known since 2001

Beenakker et al.: hep-ph/0107081, hep-ph/0211352
Reina, Dawson: hep-ph/0107101
Reina, Dawson, Wackeroth: hep-ph/0109066

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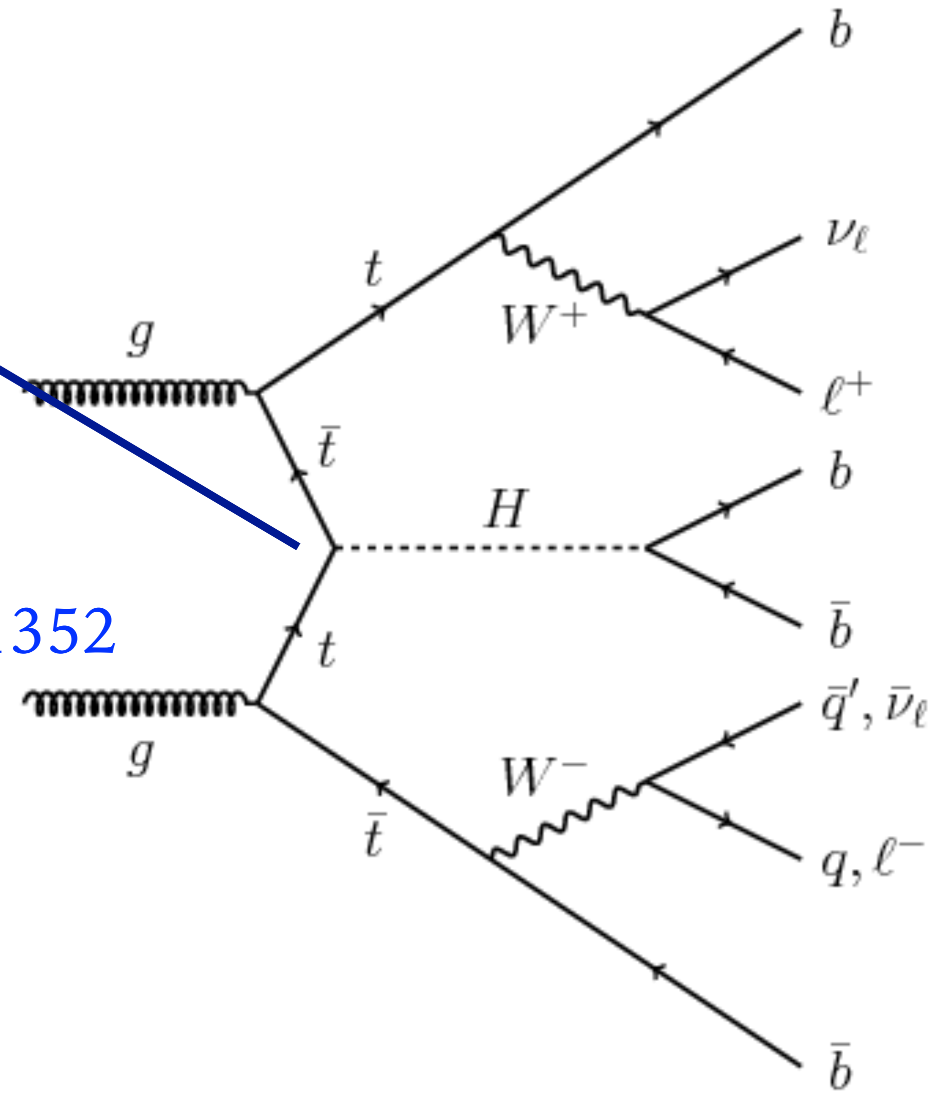

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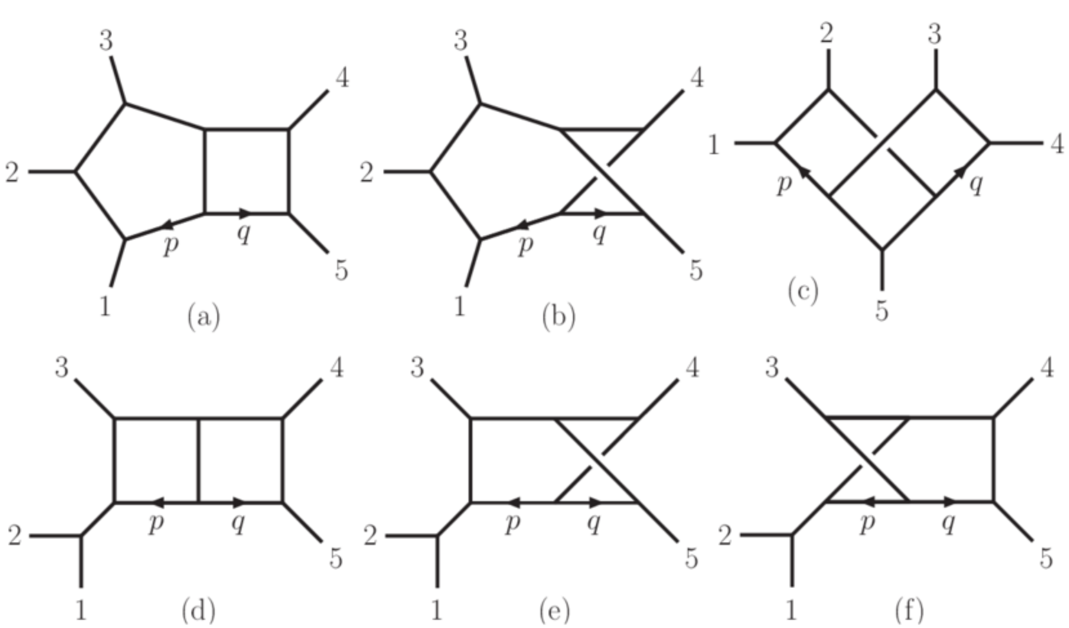


NNLO extremely difficult

(two-loop integrals with 7 scales)

Beyond NLO

Broggio, Ferroglia, Pecjak, **LLY**: 1611.00049

Construct logarithmically enhanced terms beyond NLO for the differential cross section

$$\left[\frac{1}{(1-z)}\ln^n\left(\frac{M^2(1-z)^2}{\mu^2 z}\right)\right]_+$$

Using factorization

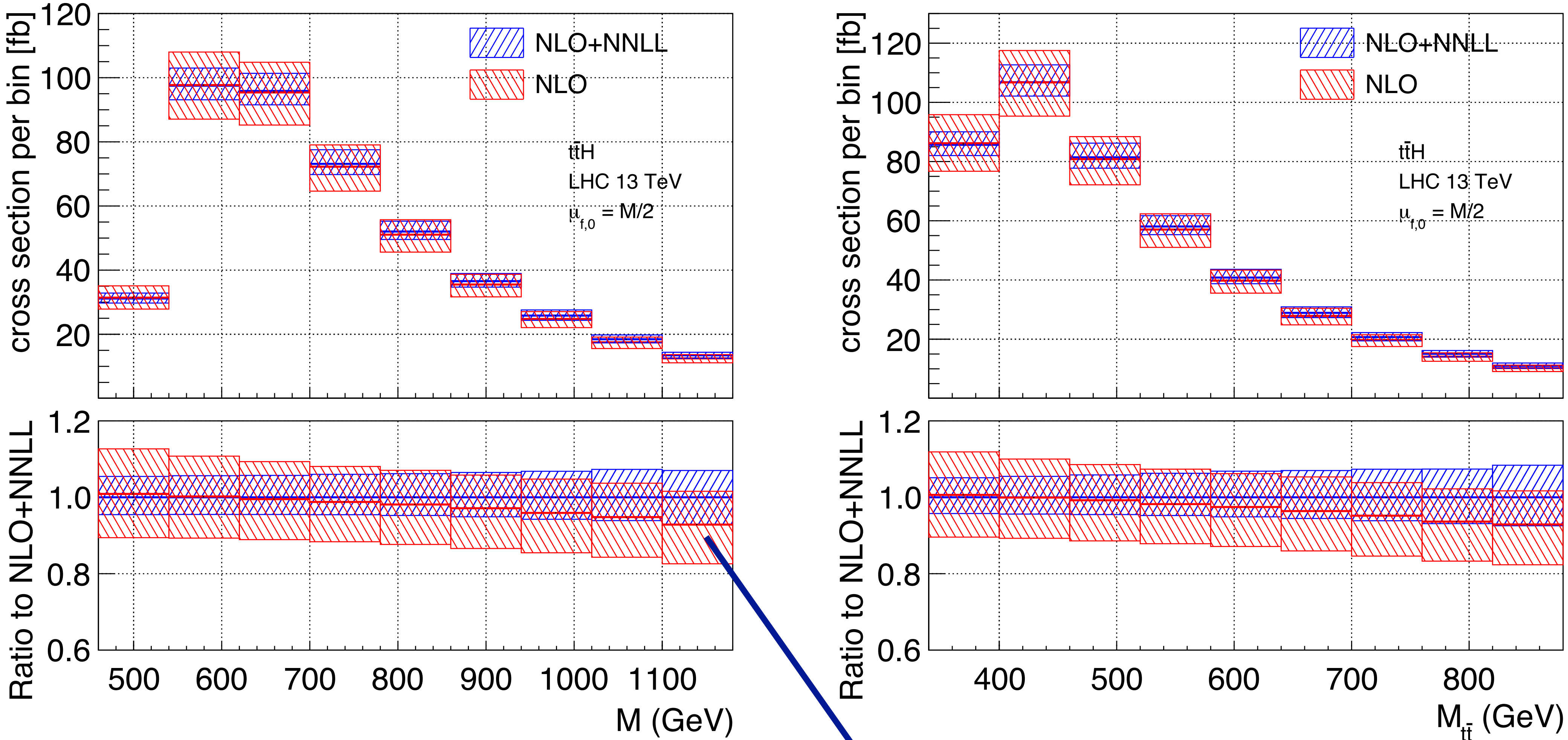
$$\sigma(s, m_t, m_H) = \frac{1}{2s}\int_{\tau_{\min}}^1 d\tau \int_\tau^1 \frac{dz}{\sqrt{z}} \sum_{ij} ff_{ij}\left(\frac{\tau}{z}, \mu\right) \times \int d\mathrm{PS}_{t\bar{t}H}\,\mathrm{Tr}\left[\mathbf{H}_{ij}(\{p\}, \mu)\,\mathbf{S}_{ij}\left(\frac{M(1-z)}{\sqrt{z}}, \{p\}, \mu\right)\right]$$

valid in the limit $z \equiv \frac{M^2_{t\bar{t}h}}{\hat{s}} \to 1$

Beyond NLO

Broggio, Ferroglia, Pecjak, **LLY**: 1611.00049

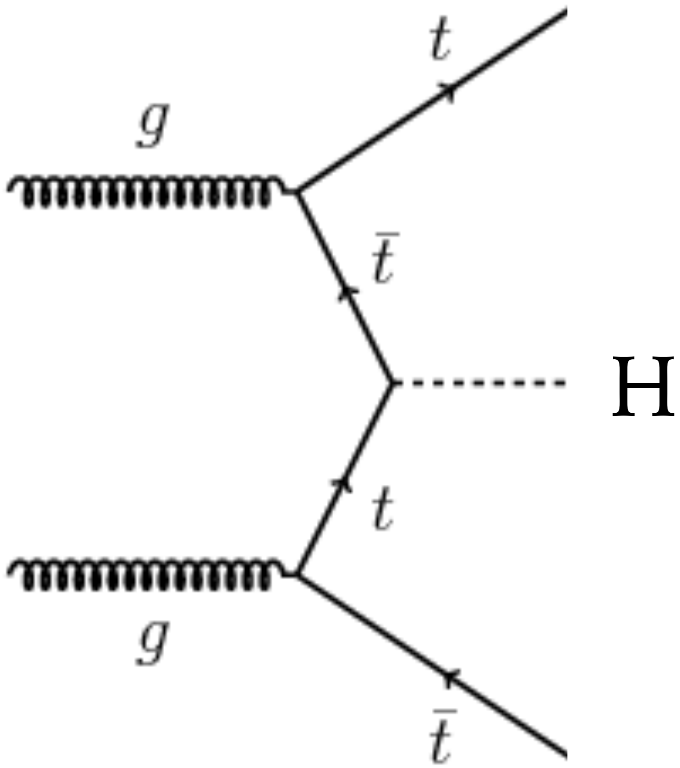
State-of-the-art QCD predictions for this process



Higher order effect important at high energies

Threshold for the total cross section

Ju, **LLY**: 1904.08744



Consider the threshold region

$$\sqrt{\hat{s}} \to 2m_t + m_H$$

$$\beta = \sqrt{1 - \frac{(2m_t + m_H)^2}{s}} \to 0$$

Sudakov and Sommerfeld corrections

Threshold for the total cross section

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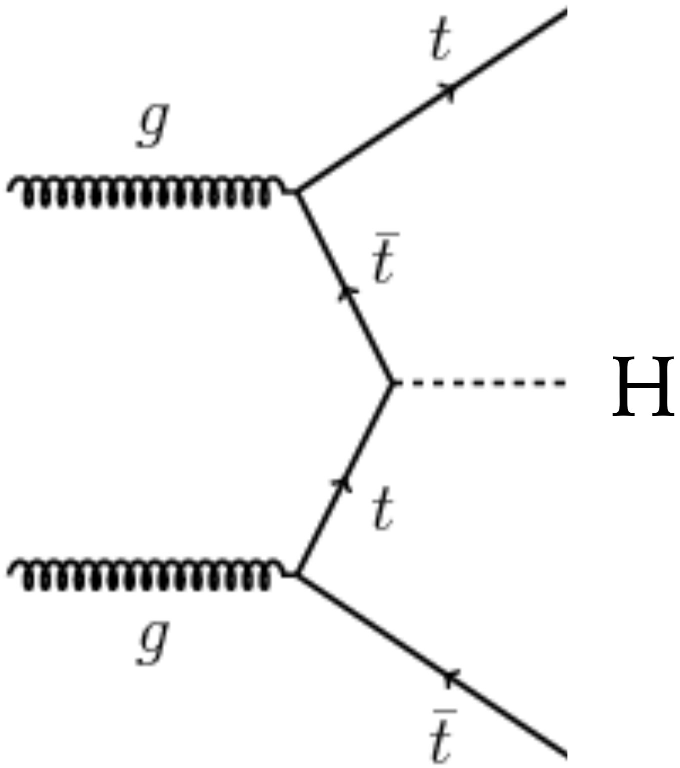

Consider the threshold region

$$\sqrt{\hat{s}} \to 2m_t + m_H$$

$$\beta = \sqrt{1 - \frac{(2m_t + m_H)^2}{s}} \to 0$$

Sudakov and Sommerfeld corrections

Combination of SCET and pNRQCD

$$\begin{aligned}
&\text{hard}: && k^\mu \sim \sqrt{\hat{s}}\,, \\
&\text{soft}: && k^\mu \sim \sqrt{\hat{s}}\,\beta\,, \\
&\text{potential}: && k^0 \sim \sqrt{\hat{s}}\,\beta^2\,, \quad \vec{k} \sim \sqrt{\hat{s}}\,\beta\,, \\
&\text{ultrasoft}: && k^\mu \sim \sqrt{\hat{s}}\,\beta^2\,, \\
&\text{collinear}: && (k_+, k_-, k_\perp) \sim \sqrt{\hat{s}}\,(1, \beta^2, \beta)\,, \\
&\text{anticollinear}: && (k_+, k_-, k_\perp) \sim \sqrt{\hat{s}}\,(\beta^2, 1, \beta)\,.
\end{aligned}$$

Threshold for the total cross section

Ju, **LLY**: 1904.08744

Factorization up to next-to-leading power (NLP)

$$\hat{\sigma}_{ij} = \sum_{\alpha} \frac{1}{2\hat{s}} \int d\Phi_h d\omega \, H_{ij}^{\alpha}(\mu) \, J^{\alpha}\left(E_J - \frac{\omega}{2}, \vec{p}_J\right) S_{ij}^{\alpha}(\omega, \mu)$$

hard modes

potential modes

ultrasoft modes

Threshold for the total cross section

Ju, **LLY**: 1904.08744

Factorization up to next-to-leading power (NLP)

$$\hat{\sigma}_{ij} = \sum_{\alpha} \frac{1}{2\hat{s}} \int d\Phi_h d\omega \, H_{ij}^{\alpha}(\mu) \, J^{\alpha}\left(E_J - \frac{\omega}{2}, \vec{p}_J\right) S_{ij}^{\alpha}(\omega, \mu)$$

hard modes → $H_{ij}^{\alpha}(\mu)$

potential modes → J^{α}

ultrasoft modes → $S_{ij}^{\alpha}(\omega, \mu)$

Resummation at NLL' accuracy

$$\hat{\sigma}_{ij}^{\mathrm{NLL}'} \sim \alpha_s^0 \Big\{1, \beta\Big\} + \alpha_s \Big\{ \ln^2\beta, \ln\beta, 1, \frac{1}{\beta}, \beta\ln^2\beta, \beta\ln\beta \Big\}$$
$$+ \alpha_s^2 \Big\{ \ln^4\beta, \ln^3\beta, \ln^2\beta, \frac{1}{\beta^2}, \frac{1}{\beta}, \frac{\ln^2\beta}{\beta}, \frac{\ln\beta}{\beta}, \beta\ln^4\beta, \beta\ln^3\beta \Big\} + \cdots$$

	13 TeV LHC (pb)	14 TeV LHC (pb)
NLO	$0.493^{+5.8\%}_{-9.2\%}$	$0.597^{+6.1\%}_{-9.2\%}$
NLL'+NLO	$0.521^{+1.9\%}_{-2.6\%}$	$0.630^{+2.3\%}_{-2.6\%}$
K-factor	1.06	1.06

Summary

- To fully exploit the capability of future experimental facilities, we need precision theoretical calculations
- For that purpose, we need better understanding of multi-loop integrals
- Talked about several examples in Higgs physics
  - Higgs boson pair production at LHC
  - HZV vertex and ZH production at Higgs factories
  - Thrust distribution in Higgs hadronic decays
  - Higgs production associated with a top quark pair

Summary

- To fully exploit the capability of future experimental facilities, we need precision theoretical calculations
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Thank you!