Equivalence theorem evasion from MHV Lagrangian

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Outline

Problems in using MHV vertices

S-matrix equivalence theorem (ET)

Tree level evasion of ET: $(++-)$

General evasion of ET at tree level and One-loop

- Considerations
- Canonical transformations with dimensional regularization
- Evasion of ET at tree level revisit
- Evasion of ET at one-loop

Conclusion and discussion

Problems in using MHV vertices

- 3-point Pure Y-M MHV amplitude$(++-)$:

$$A(1^-, 2^+, 3^+) = ig\frac{[2\,3]^3}{[3\,1][1\,2]},$$

- One-loop amplitudes $(+\cdots++)$ and $(-+\cdots++)$ are not zero. They have only rational part.
- How to construct this amplitude from CSW rule? The MHV vertices have at least two "-" helicity.
- Solution: In MHV lagrangian, the fields are B but in amplitudes the external legs are A fields. There are canonical transformations between them.

LSZ Formalism

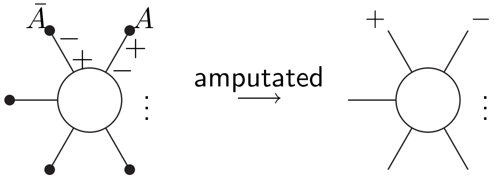
- LSZ formalism, from correlation function to amplitude: For outgoing momenta $\{p_i\}$ and helicities $\{h_i\}$,

$$\langle p_1^{h_1}, \ldots, p_n^{h_n} | S | 0 \rangle = (-i)^n \lim_{p_i^2 \to 0} p_1^2 \cdots p_n^2 \langle E_{h_1}^{\mu_1} A_{\mu_1} \cdots E_{h_n}^{\mu_n} A_{\mu_n} \rangle .$$

The $E_{h_i}^{\mu_i}$: polarisation vectors

$$E_+ = \sqrt{2} \frac{\eta \, \tilde{\lambda}}{\langle \eta \, \lambda \rangle} \quad \text{and} \quad E_- = \sqrt{2} \frac{\lambda \, \tilde{\eta}}{[\eta \, \lambda]}$$

$E_+ = \bar{E}_- = -1$,

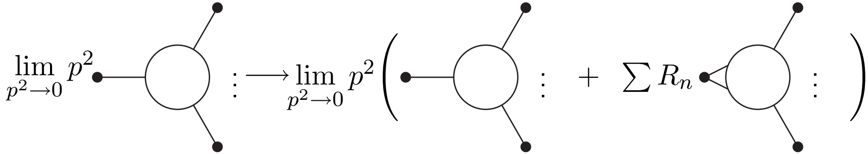


After amputated $A \sim -$ and $\bar{A} \sim +$

Equivalence theorem of S-matrix

Scalar fields as an example:

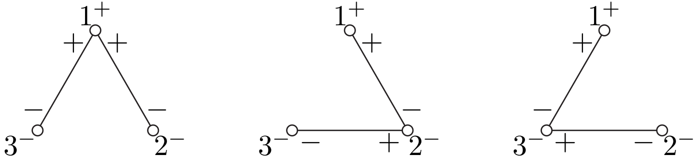
- Change $\phi(x) \to \phi(x) \sim \phi' + \sum R_n(x)\phi'^n(x)$
- LSZ:



If R is local, i.e. no poles to cancel p^2, the second term is canceled in the limit $p^2 \to 0$. No difference using ϕ or ϕ' in calculating the S-matrix.

Tree level evasion of equivalence theorem: 3-point $(-++)$ amplitude

Amplitude $A(-++) \sim \lim_{p^2\to 0} E_1^- E_2^+ E_3^+ p_1^2 p_2^2 p_3^2 \langle A_1 \bar{A}_2 \bar{A}_3 \rangle$



$$
\begin{aligned}
&A(1^-, 2^+, 3^+) \\
&= -\frac{g}{\sqrt{2}} p_1^2 p_2^2 p_3^2 \left\{ \frac{1}{p_2^2}\frac{1}{p_3^2}\Upsilon(123) - \frac{1}{p_3^2}\frac{1}{p_1^2}\frac{\hat{1}}{\hat{2}}\Xi^2(231) - \frac{1}{p_1^2}\frac{1}{p_2^2}\frac{\hat{1}}{\hat{3}}\Xi^1(312) \right\} \\
&= \frac{ig}{\sqrt{2}} \frac{\hat{1}^2}{(2\,3)} \left(\frac{p_1^2}{\hat{1}} + \frac{p_2^2}{\hat{2}} + \frac{p_3^2}{\hat{3}} \right) \\
&= ig\sqrt{2}\frac{\hat{1}}{\hat{2}\hat{3}}\{2\,3\} = ig\frac{[2\,3]^3}{[3\,1][1\,2]},
\end{aligned}
$$

Considerations in one-loop level

To study this evasion of the Equivalence theorem in one-loop, we need to consider

- Dimensional regularization, FDH: external polarizations – Four dimensinoal, internal momentum D-dimensional A_μ still $(\hat{A}, \check{A}, A, \bar{A})$. All the ∂_μ contracted with A_μ are still four-component vector. But the contraction $\partial^\mu \partial_\mu$ are D-dimensional.
- All the canonical transformation should be rederived in D-dimensional and may not have closed form.

Integral equations and Recursion relations with dimensional regularization

- Fixing the gauge and obtaining the light-cone YM

$$L_{LCYM} = L_{YM}^{-+} + L_{YM}^{++-} + L_{YM}^{--+} + L_{YM}^{--++}$$

$$\mathcal{L}^{-+} = \operatorname{tr} \bar{\mathcal{A}} \left(\check{\partial}\hat{\partial} - \sum_{i=1}^{D/2-1} \partial_{(i)}\bar{\partial}_{(i)} \right) \mathcal{A} \, .$$

- Canonical transformation $(A, \hat{\partial}\bar{A}) \to (B, \hat{\partial}\bar{B})$, s.t.:

$$L_{YM}^{-+}[A, \bar{A}] + L_{YM}^{++-}[A, \bar{A}] = L_{YM}^{-+}[B, \bar{B}]$$

we arrive at the same integral equation:

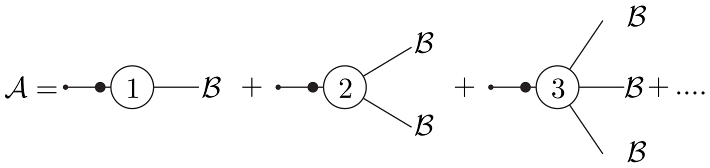
$$\sum_{i=1}^{D/2-1} \frac{\partial_{(i)}\bar{\partial}_{(i)}}{\hat{\partial}} \mathcal{A}(x) + \mathcal{A}(x) \left(\frac{\bar{\partial}}{\hat{\partial}} \mathcal{A}(x) \right) - \left(\frac{\bar{\partial}}{\hat{\partial}} \mathcal{A}(x) \right) \mathcal{A}(x)$$

$$= \int_{\hat{x}=const.} \left(\sum_{i=1}^{D/2-1} \frac{\partial_{(i)}\bar{\partial}_{(i)}}{\hat{\partial}} \mathcal{B}(x') \right) \frac{\delta \mathcal{A}(x)}{\delta \mathcal{B}(x')} \, d^{D-1}x'$$

Integral equations and Recursion relations with dimensional regularization: $\mathcal{A}$

- Canonical transformation in momentum space:

$$\mathcal{A}_p = \sum_{n=1}^{\infty} \int \Upsilon(p, p_1, \ldots, p_n)\, \delta(p + \sum_{i=1}^{n} p_i)\, \mathcal{B}_{\bar{1}} \ldots \mathcal{B}_{\bar{n}}\, \mathrm{d}^D p_1 \ldots \mathrm{d}^D p_n \,,$$



Integral equations and Recursion relations with dimensional regularization: $\mathcal{A}$

1)The integral equation gives a recursion relation for the coefficients:

$$\Upsilon(\bar{1}\cdots\bar{n}) = \frac{1}{\hat{1}(\Omega_1+\cdots+\Omega_n)} \times \sum_{j=2}^{n-1} \bar{V}_2(P_{2j}, P_{j+1,n}, 1)\Upsilon(-,\bar{2},\ldots,\bar{j})\Upsilon(-,\overline{j+1},\ldots,\bar{n}),$$

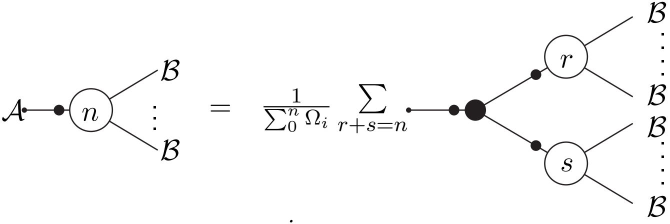
$$\Omega_i = \frac{\sum_{j=1}^{D/2-1}\tilde{p}_{i(j)}\bar{p}_{i(j)}}{\hat{p}_i}$$

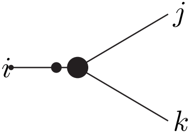
$\bar{V}_2(p_1,p_2,p_3) = i(\bar{1}/\hat{1}-\bar{2}/\hat{2})\hat{3}$ is the factor from the three-point $(++-)$ vertex of the Light-cone lagrangian

2)Relation: for $\sum p_j = 0$

$$-\sum_j \Omega(p_j) = -\sum_j \sum_{i=1}^{D/2-1} \tilde{p}_{j(i)}\bar{p}_{j(i)}/\hat{p}_j = \sum_j \frac{p_j^2+i\epsilon}{\hat{p}_j}$$

IN DIAGRAM



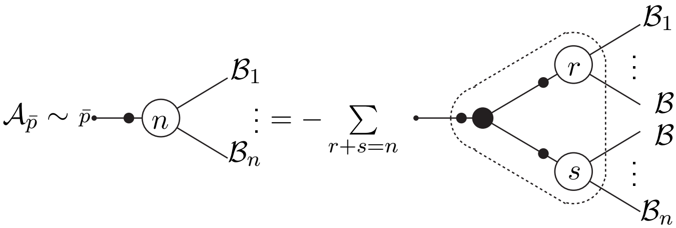


$= \bar{V}_2(p_j, p_k, p_i)/\hat{p}_i$,

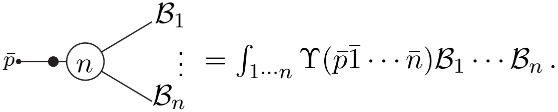
$V_2(p_1, p_2, p_3)$ is the $(+ + -)$ vertice of the LCYM.

In diagram

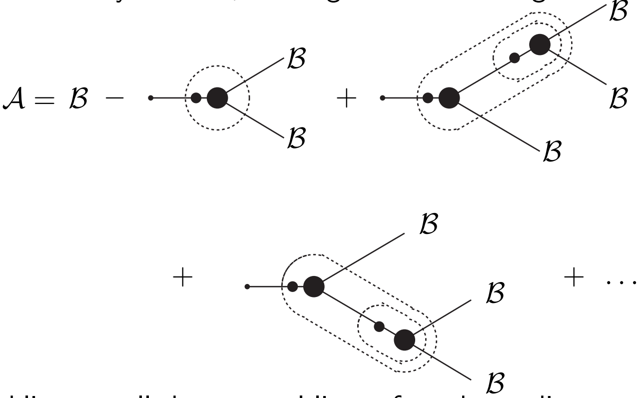
We use the dashed line to denote the $-\frac{1}{\sum \Omega}$ factor. The momenta summed over are denoted by the lines cut by the dashed line.



where



This can be easily iterated, starting with the leading term $\mathcal{A} = \mathcal{B}$:



The dashline cut all the external lines of a subtree diagram.

Construct the transformation coefficients from Light-cone Feynmann diagram

1)The light-cone Feynman rule for vertex $(++-)$ is

$$\bar{V}(1,2,3) = i\frac{4}{g^2}\bar{V}_2(\bar{1},\bar{2},\bar{3}) = -i\frac{4}{g^2}\bar{V}_2(1,2,3)$$

and the light-cone propagator is

$$\langle \mathcal{A}_p \mathcal{A}_{\bar{p}} \rangle = -i\frac{g^2}{2p^2}\,.$$

2)make the replacement $-2/p_3^2 \rightarrow 1/(\hat{p}_3(\Omega_3+\Omega_1+\Omega_2))$

$$\langle \mathcal{A}_3 \mathcal{A}_{\bar{3}} \rangle \bar{V}(1,2,3) = -\frac{2}{p_3^2}\bar{V}_2(1,2,3) \rightarrow \frac{1}{\hat{p}_3(\Omega_3+\Omega_1+\Omega_2)}\bar{V}_2(1,2,3)$$

is consistent with the coefficient of each term in the recursion relation.

Construct the transformation coefficients from Light-cone Feynmann diagram

3)As a result, we can reconstruct the terms of $\mathcal{A}$ from light-cone tree-level calculations by

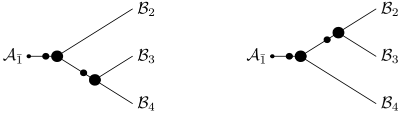
a) drawing the tree level diagram using only $(++-)$ vertices

b) replacing the light-cone propagators using

$$\frac{1}{P_{ij}^2} \to -\frac{1}{2\hat{P}_{j+1,i-1}(\Omega_{j+1,i-1} + \Omega_i + \Omega_{i+1} + \cdots + \Omega_j)}.$$

Cut one propagator whose outgoing momentum is $P_{j+1,i-1}$. Sum over Ω of all legs of the subtree diagram.

Example



For terms with $\mathcal{B}_2\mathcal{B}_3\mathcal{B}_4$:

$$\begin{aligned}\mathcal{A}_{\bar{1}} &\sim \Upsilon(\bar{1}\bar{2}\bar{3}\bar{4})\mathcal{B}_2\mathcal{B}_3\mathcal{B}_4 \\ &= \frac{1}{\hat{1}(\Omega_1+\cdots+\Omega_4)}\left(\frac{\bar{V}_2(2,34,1)\bar{V}_2(3,4,12)}{\hat{P}_{12}(\Omega_{12}+\Omega_3+\Omega_4)}\right. \\ &\quad \left.+\frac{\bar{V}_2(23,4,1)\bar{V}_2(2,3,41)}{\hat{P}_{41}(\Omega_{41}+\Omega_2+\Omega_3)}\right)\mathcal{B}_2\mathcal{B}_3\mathcal{B}_4\,.\end{aligned}$$

Light-cone calculation:

$$2^2\frac{1}{p_1^2}\left(\frac{\bar{V}_2(2,34,1)\bar{V}_2(3,4,12)}{P_{12}^2}+\frac{\bar{V}_2(23,4,1)\bar{V}_2(2,3,41)}{P_{41}^2}\right)$$

Replacements:

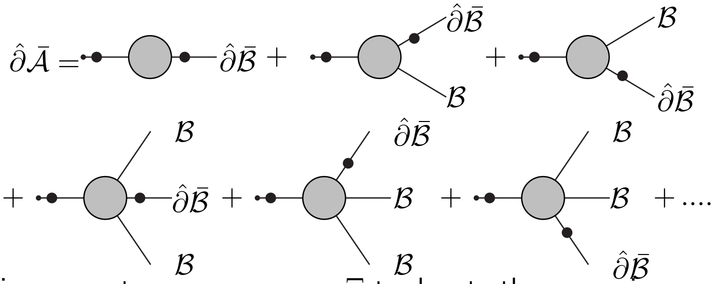
$$\begin{aligned}1/p_1^2 &\to -1/(2\hat{1}(\Omega_1+\cdots+\Omega_4))\,, \\ 1/P_{12}^2 &\to -1/(2\hat{P}_{12}(\Omega_{12}+\Omega_3+\Omega_4))\,, \\ 1/P_{41}^2 &\to -1/(2\hat{P}_{41}(\Omega_{41}+\Omega_2+\Omega_3)).\end{aligned}$$

Expansion of $\hat{\partial}\bar{\mathcal{A}}$

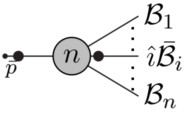
Expansion of $\hat{\partial}\bar{\mathcal{A}}$

$$\bar{\mathcal{A}}(\hat{x},\vec{p}) = \bar{\mathcal{B}}(\hat{x},\vec{p}) + \sum_{m=3}^{\infty}\sum_{s=2}^{m}\int \frac{d^3k_1}{(2\pi)^3}\dots\frac{d^3k_n}{(2\pi)^3}\frac{\hat{k}_s}{\hat{p}}\,\Xi^{s-1}(\vec{p},-\vec{k}_1,\dots,-\vec{k}_m)\times (2\pi)^3\,\delta^3(\vec{p}-\sum\vec{k}_i)\mathcal{B}(\hat{x},\vec{k}_1)\dots\bar{\mathcal{B}}(\hat{x},\vec{k}_s)\dots\mathcal{B}(\hat{x},\vec{k}_m)$$

Expansion of $\hat{\partial}\bar{\mathcal{A}}$ in diagrams



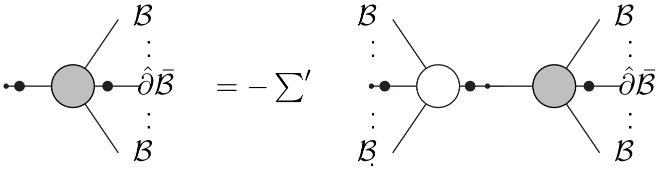
and in momentum space we use Ξ to denote the expansion coefficients



$$= \int_{1\cdots n} \hat{\imath}\, \Xi^i(\bar{p}\bar{1}\cdots\bar{n})\mathcal{B}_1\cdots\bar{\mathcal{B}}_i\cdots\mathcal{B}_n\,.$$

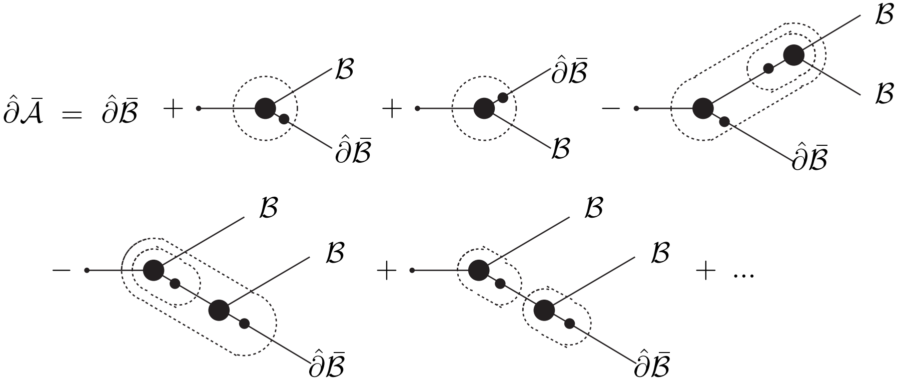
Recursion relation for $\hat{\partial}\bar{\mathcal{A}}$

$$\hat{\partial}\bar{\mathcal{B}}^a(\hat{x},\vec{x}) = \int_\Sigma d^3\vec{y}\,\frac{\delta\mathcal{A}^b(\hat{x},\vec{y})}{\delta\mathcal{B}^a(\hat{x},\vec{x})}\hat{\partial}\bar{\mathcal{A}}^b(\hat{x},\vec{y}).$$



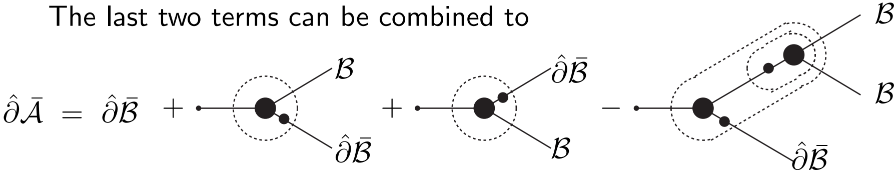
At least two left legs on the white blob.

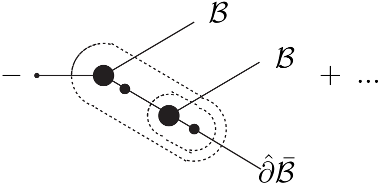
Iteration : $\hat{\partial}\bar{\mathcal{A}}$



ITERATION : $\hat{\partial}\bar{\mathcal{A}}$

The last two terms can be combined to





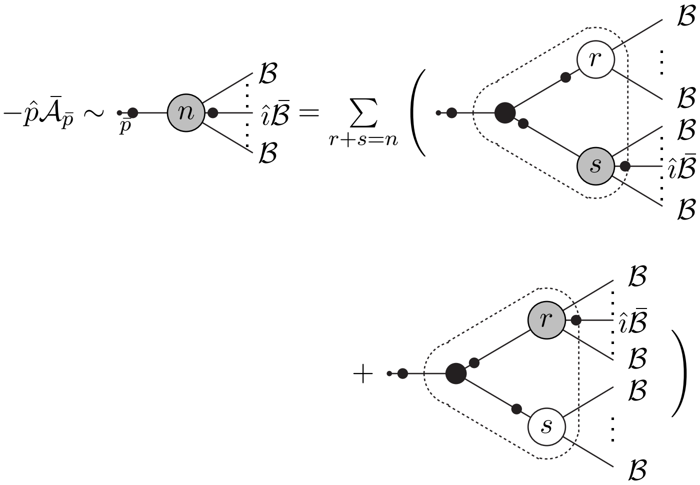
New recursion relation : $\hat{\partial}\bar{\mathcal{A}}$

In fact, by induction, one can prove another recursion relation of Ξ^s:

$$
\begin{aligned}
& \Xi^{i-1}_{\bar{1},\ldots,\bar{n}} \\
= \; & -\frac{1}{\sum_{i=1}^{n}\Omega_i} \\
\times \; & \Big(\sum_{l=2}^{i-1} \frac{1}{\hat{P}_{l+1,n}} \bar{V}_2(p_1, P_{2,l}, P_{l+1,n}) \Upsilon(-,\bar{2},\ldots,\bar{l}) \Xi^{i-l}(-,\overline{l+1},\ldots,\bar{n}) \\
+ \; & \sum_{l=i}^{n-1} \frac{1}{\hat{P}_{2,l}} \bar{V}_2(P_{l+1,n}, p_1, P_{2,l}) \Xi^{i-1}(-,\bar{2},\ldots,\bar{l}) \Upsilon(-,\overline{l+1},\ldots,\bar{n}) \Big) .
\end{aligned}
$$

Diagram representation

Using this we can represent each order of the expansion of $\hat{\partial}\bar{\mathcal{A}}$ by diagrams:



Construct the transformation coefficients from light-cone Feynmann diagrams for $\hat{\partial}\bar{\mathcal{A}}$

The same rule as in $\mathcal{A}$ can be applied here:

1) One needs to first draw the tree-level diagrams with one $\bar{\mathcal{A}}$ as an external propagator, all the $\mathcal{B}$, $\bar{\mathcal{B}}$ as amputated legs, using only $(++-)$ vertices.

2)Then calculate the diagram using the light-cone Feynman rules with the replacement:

$$\frac{1}{P_{ij}^2} \to -\frac{1}{2\hat{P}_{j+1,i-1}(\Omega_{j+1,i-1} + \Omega_i + \Omega_{i+1} + \cdots + \Omega_j)}.$$

Cut one propagator whose outgoing momentum is $P_{j+1,i-1}$. Sum over Ω of all legs of the subtree diagram.

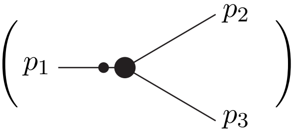
3) Relation: for $\sum p_j = 0$

$$-\sum_j \Omega(p_j) = -\sum_j \sum_{i=1}^{D/2-1} p_{j(i)}\bar{p}_{j(i)}/\hat{p}_j = \sum_j \frac{p_j^2 + i\epsilon}{\hat{p}_j}$$

Evasion of equivalence theorem in tree level revisit

1) 3-point $(+\,+\,-)$: No tree-level MHV vertices. Only from the translation kernels. Equals the $(+\,+\,-)$ vertex from LC-YM

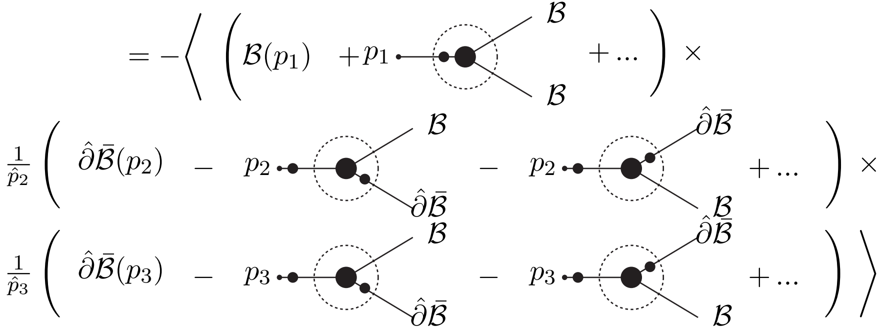
$$\lim_{p_1^2, p_2^2, p_3^2 \to 0} p_1^2\, p_2^2\, p_3^2 \,\langle\, \mathcal{A}(p_1)\, \bar{\mathcal{A}}(p_2)\, \bar{\mathcal{A}}(p_3)\,\rangle \;=$$



$\hat{p}_1,$

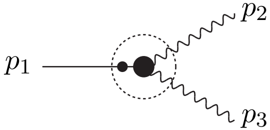
Evasion of equivalence theorem in tree level revisit

$\langle \mathcal{A}(p_1)\, \bar{\mathcal{A}}(p_2)\, \bar{\mathcal{A}}(p_3) \rangle$



Evasion of equivalence theorem in tree level revisit

$\langle \mathcal{A}(p_1)\, \bar{\mathcal{A}}(p_2)\, \bar{\mathcal{A}}(p_3) \rangle \;=$



$+$

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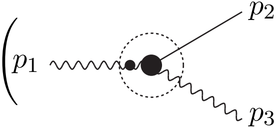
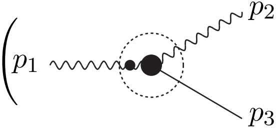
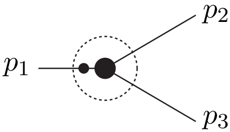


$\Big)\,\frac{\hat{p}_1}{\hat{p}_2} \;+\; \Big($



$\Big)\,\frac{\hat{p}_1}{\hat{p}_3}$

$= \Big($



$\Big)\,\frac{\hat{p}_1}{(p_1^2+i\epsilon)\,(p_2^2+i\epsilon)\,(p_3^2+i\epsilon)}\left(\frac{p_1^2+i\epsilon}{\hat{p}_1}+\frac{p_2^2+i\epsilon}{\hat{p}_2}+\frac{p_3^2+i\epsilon}{\hat{p}_3}\right),$

$$-\sum_j \Omega(p_j) = -\sum_j \sum_{i=1}^{D/2-1} p_{j(i)}\bar{p}_{j(i)}/\hat{p}_j = \sum_j \frac{p_j^2+i\epsilon}{\hat{p}_j}$$

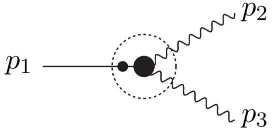
The dashed circle cancels with the last factor the denominator cancels with the LSZ factor.

Changing the limiting order

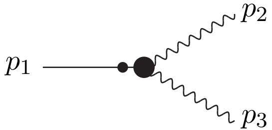
Since the whole limit $p_1^2, p_2^2, p_3^2 \to 0$ exist we can do the limit in any order: $p_2^2, p_3^2 \to 0$ first and then $p_1^2 \to 0$

$$\lim_{p_2^2+i\epsilon\to 0} \; \lim_{p_3^2+i\epsilon\to 0} (p_2^2+i\epsilon)\,(p_3^2+i\epsilon)\,\langle\, \mathcal{A}(p_1)\,\bar{\mathcal{A}}(p_2)\,\bar{\mathcal{A}}(p_3)\,\rangle \quad =$$

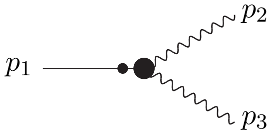
$$\lim_{p_2^2+i\epsilon\to 0} \; \lim_{p_3^2+i\epsilon\to 0} (p_2^2+i\epsilon)\,(p_3^2+i\epsilon) \left(\right) =$$



$$\lim_{p_2^2+i\epsilon\to 0} \; \lim_{p_3^2+i\epsilon\to 0} \frac{1}{\frac{p_1^2+i\epsilon}{\hat{p}_1}+\frac{p_2^2+i\epsilon}{\hat{p}_2}+\frac{p_3^2+i\epsilon}{\hat{p}_3}} \left(\right) =$$



$$\frac{\hat{p}_1}{p_1^2+i\epsilon} \left(\right) .$$



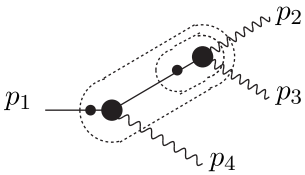
From this we learn how the equivalence theorem is violated:

$$\frac{1}{\sum \Omega} = \frac{1}{\frac{p^2+i\epsilon}{\hat{p}} + \sum_{j=1}^{n} \frac{p_j^2+i\epsilon}{\hat{p}_j}},$$

$\sum_{j=1}^{n} (p_j^2 + i\epsilon)/\hat{p}_j \to 0$ limit first and then $p^2 \to 0$ in the LSZ procedure.

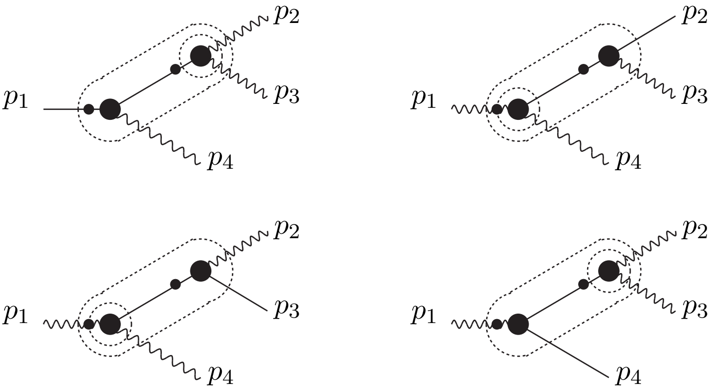
Tree level evasion of equivalence theory: General

1) All the external on-shell legs are cut by a dashline: $(-++\cdots+)$. For example $(-+++)$,



Tree level evasion of equivalence theory: General

LCYM calulations can be reproduced using translation kernel:



Reproducing LCYM Feynmann diagram

Numerators are the same. We only examine the denominators

$$\lim_{p_1^2,p_2^2,p_3^2,p_4^2\to 0} p_1^2\, p_2^2\, p_3^2\, p_4^2 \left\{ \frac{1}{\hat{1}\left(\frac{p_1^2}{\hat{1}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}+\frac{p_4^2}{\hat{4}}\right)(\hat{1}+\hat{4})\left(\frac{p_{41}^2}{\hat{1}+\hat{4}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}\right)p_2^2p_3^2p_4^2} \right.$$

$$+\frac{1}{\hat{2}\left(\frac{p_1^2}{\hat{1}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}+\frac{p_4^2}{\hat{4}}\right)(\hat{2}+\hat{3})\left(\frac{p_{23}^2}{\hat{2}+\hat{3}}+\frac{p_1^2}{\hat{1}}+\frac{p_4^2}{\hat{4}}\right)p_1^2p_3^2p_4^2}$$

$$+\frac{1}{\hat{3}\left(\frac{p_1^2}{\hat{1}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}+\frac{p_4^2}{\hat{4}}\right)(\hat{2}+\hat{3})\left(\frac{p_{23}^2}{\hat{2}+\hat{3}}+\frac{p_1^2}{\hat{1}}+\frac{p_4^2}{\hat{4}}\right)p_1^2p_2^2p_4^2}$$

$$\left. +\frac{1}{\hat{4}\left(\frac{p_1^2}{\hat{1}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}+\frac{p_4^2}{\hat{4}}\right)(\hat{1}+\hat{4})\left(\frac{p_{41}^2}{\hat{1}+\hat{4}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}\right)p_1^2p_2^2p_3^2} \right\}$$

$$=\lim_{p_1^2,p_2^2,p_3^2,p_4^2\to 0} \frac{-\frac{p_{41}^2}{\hat{1}+\hat{4}}+\frac{p_1^2}{\hat{1}}+\frac{p_4^2}{\hat{4}}-\frac{p_2^2}{\hat{2}}-\frac{p_3^2}{\hat{3}}}{(\hat{1}+\hat{4})\left(\frac{p_{41}^2}{\hat{1}+\hat{4}}+\frac{p_2^2}{\hat{2}}+\frac{p_3^2}{\hat{3}}\right)\left(-\frac{p_{41}^2}{\hat{1}+\hat{4}}+\frac{p_1^2}{\hat{1}}+\frac{p_4^2}{\hat{4}}\right)}$$

$$=\frac{1}{p_{41}^2}\,,$$

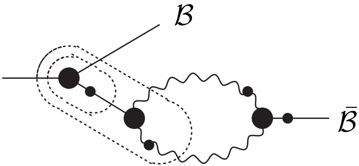
Tree level evasion of equivalence theory: General

1) We could change the limit order, put $p_i \to 0$, $i = 2, \ldots n$ first and then $p_1^2 \to 0$ last. The outermost dashed circle b cancels with the inverse propagator. We recover light-cone calculation.

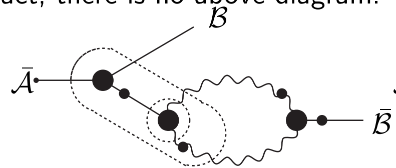
2) Since all the $(- + + \cdots +)$ amplitudes are zero except three-point $(- + +)$, the translation kernel contributions should be cancelled. So there is no tree level evasion except the 3-point case.

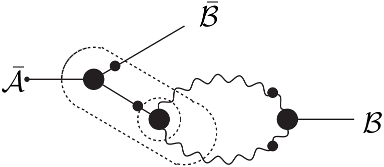
One-loop evasion of equivalence theorem: Dressing propagators

1) Dressing propagators : Connecting MHV vertices and translation kernel, like



This will contribute to the Green function $\langle\, \mathcal{A}(p_1)\, \bar{\mathcal{A}}(p_2)\, \bar{\mathcal{A}}(p_3)\,\rangle$, In fact, there is no above diagram.

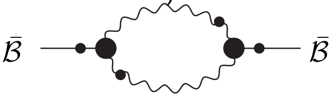




$\sum p_i^2/\hat{p}_i$ can not be zero. No contribution.

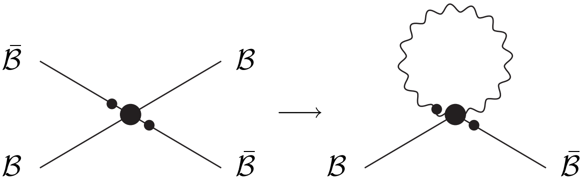
One-loop evasion of equivalence theorem: Dressing propagators

Other cases:a)



vanishes.

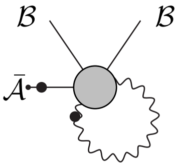
b)From $\mathcal{L}^{--++}$:

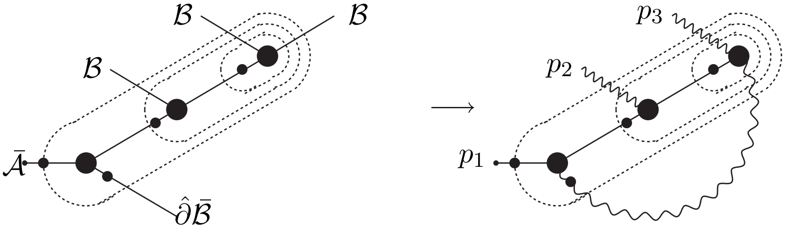


This is nonzero. In this case, only when tree-level evasion of Equivalence theorem happens, the same thing in one-loop happens. So nothing new.

One-loop evasion of equivalence theorem: Tadpoles

2) Tadpoles constructed from translation kernel. These provide $(+ + \cdots +)$ amplitudes:

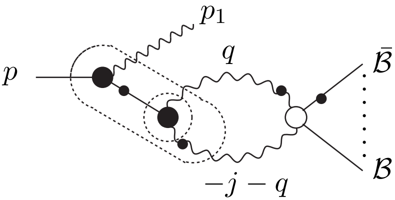




The outmost dashline: $\frac{1}{\sum \Omega} = \frac{1}{\sum p_j^2/\hat{j}}$, $p_2^2, p_3^2 \to 0$, $p_l^2/\hat{l} - p_{\bar{l}}^2/\hat{l} = 0$, $\frac{1}{\sum \Omega} \to 1/p_1^2$.

Infra-red divergent loop integration

3)Infra-red divergent loop integration:



$$\int d^D q \quad \frac{1}{\frac{p^2+i\epsilon}{\hat{p}} + \frac{q^2+i\epsilon}{\hat{q}} - \frac{(q+j)^2+i\epsilon}{\hat{q}+\hat{j}}} \frac{1}{\frac{j^2+i\epsilon}{\hat{j}} + \frac{q^2+i\epsilon}{\hat{q}} - \frac{(q+j)^2+i\epsilon}{\hat{q}+\hat{j}}}$$

$$\times \quad \frac{1}{q^2+i\epsilon} \frac{1}{(j+q)^2+i\epsilon} f(j,q)$$

with $j = p + p_1$.

Infra-red divergent loop integration

Integrate $\check{q}$ first:

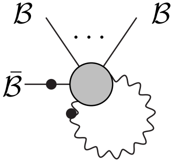
$$-\frac{2\pi i\,\hat{p}}{p^2+i\epsilon}\int\left(\prod_{i=1}^{D/2-1} dq_{(i)}d\bar{q}_{(i)}\right) d\hat{q}\,\frac{\theta(\hat{q})\theta(-\hat{q}-\hat{j})-\theta(-\hat{q})\theta(\hat{q}+\hat{j})}{\hat{q}\,(\hat{q}+\hat{j})}$$

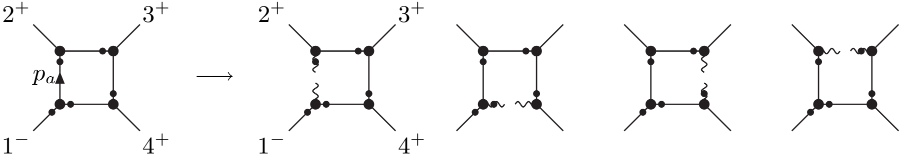
$$\times\left[\frac{1}{\frac{p^2+i\epsilon}{\hat{p}}+\frac{q^2+i\epsilon}{\hat{q}}-\frac{(q+j)^2+i\epsilon}{\hat{q}+\hat{j}}}-\frac{1}{\frac{q^2+i\epsilon}{\hat{q}}-\frac{(q+j)^2+i\epsilon}{\hat{q}+\hat{j}}}\right]f_1(j,q)$$

This could be a source for evasion of equivalence theorem. We can prove the integral vanishes in the on-shell limit. So in this case there is no contribution.

One-loop $(- + ++)$ from tadpole of MHV vertices

We have understood that $(+ + \cdots +)$ is from the tadpole of the translation kernel. The only possibility to obtain $(- + \cdots +)$ amplitude is the tadpole of MHV vertices.





We can explicitly calculate the diagram from both sides giving the same result.

Conclusion and discussion

- We considered the canonical transformation under the FDH dimensional regularization. We have understood how the transformation coefficient generated from light-cone Feynmann rules.
- We discussed the cases when equivalence theorem is evaded in tree level and one-loop.
  Tree-level: only in the $(- + +)$ amplitude.
  One-loop: only in $(+ \cdots +)$ amplitude—tadpole of the translation kernel.
- The $(- + \cdots +)$ amplitude comes from the tadpole of the MHV vertices.
- To extend the MHV method to one-loop is still difficult: the vertices themselves need regularization, not in 4-dimension , may not have simple closed forms.