Chiral transport phenomena: From QCD to Quantum Simulators

Niklas Mueller
Brookhaven National Laboratory

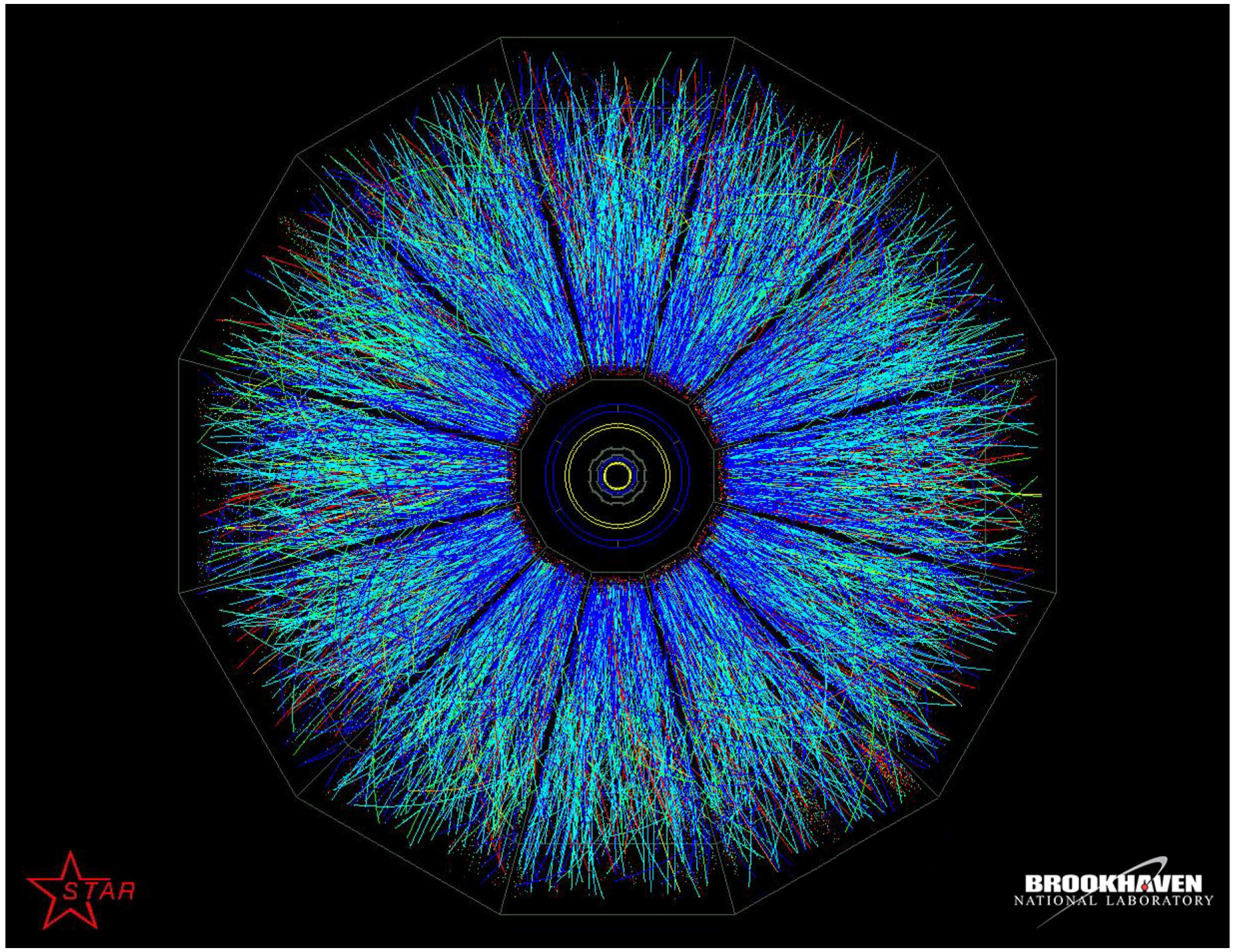
with J. Berges, P. Hauke, F. Jendrzejewski, J. Schneider, R. Ott, R. Venugopalan, T. Zache

PRD 99, 056003 (arXiv:1901.10492);
PRD 97, 051901 (arXiv:1701.03331)
PRD 96, 016023 (arXiv:1702.01233)

PRL 122 (2019) 050403 (arXiv:1808.07885)
arXiv:1903.11109

USTC Hefei - April 2019

Motivation



particle tracks at STAR detector at Relativistic Heavy Ion Collider, Brookhaven National Lab

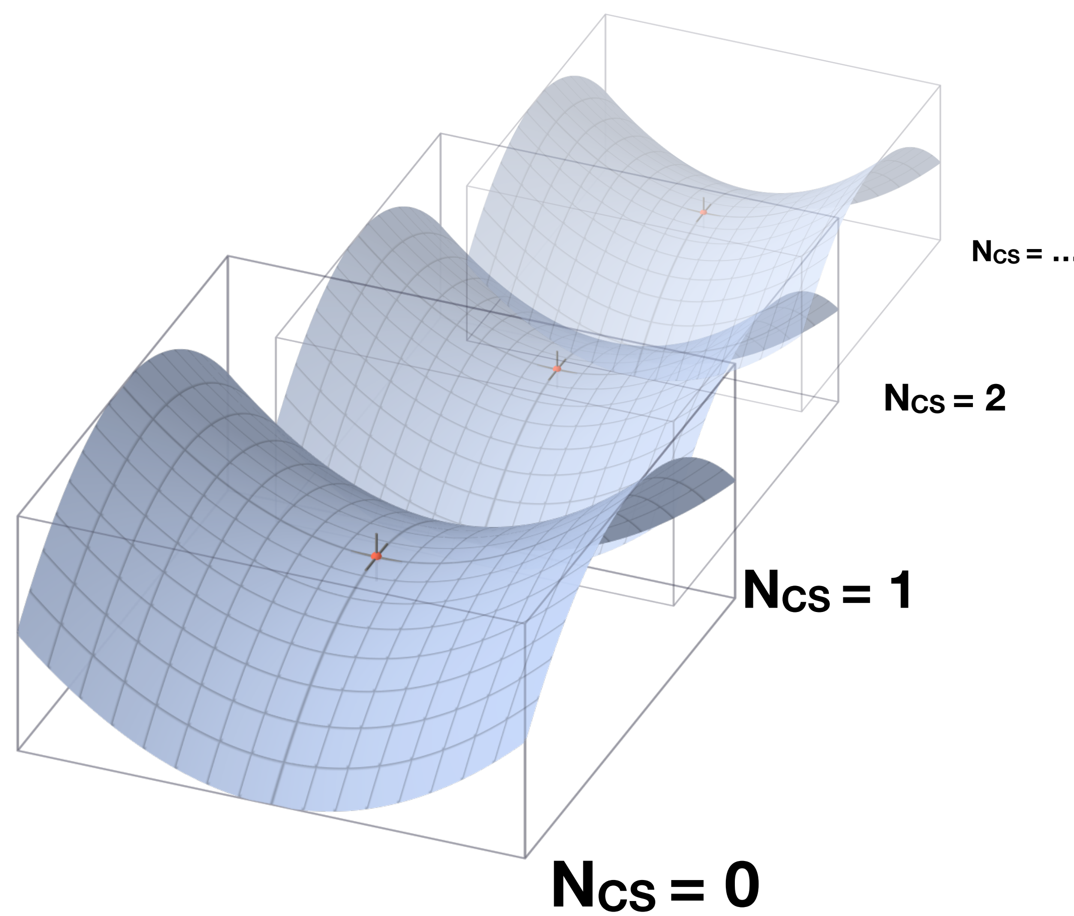
- **strong CP problem**

$$\frac{g^2\theta}{16\pi^2}\tilde{F}^{\mu\nu}F_{\mu\nu} \qquad \theta < 10^{-9}$$

- **Topological structure of QCD**

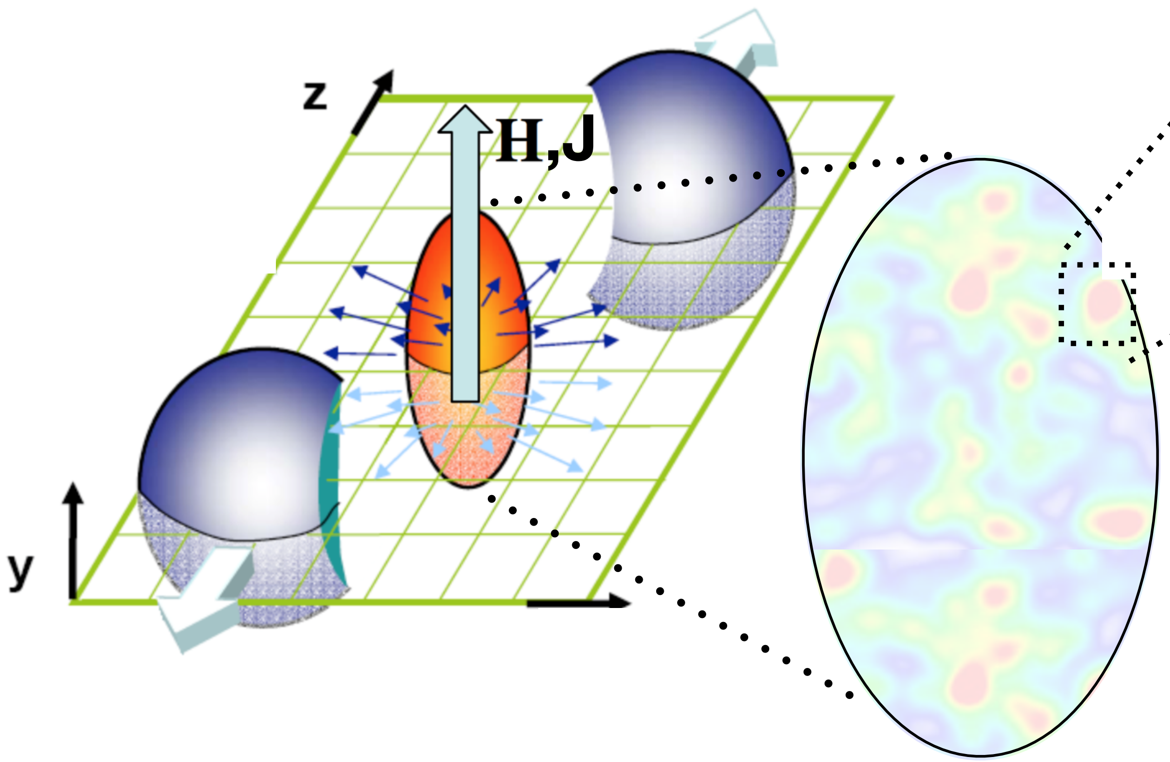
$$n_{CS}(t,\mathbf{x}) \equiv \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}(t,\mathbf{x})$$

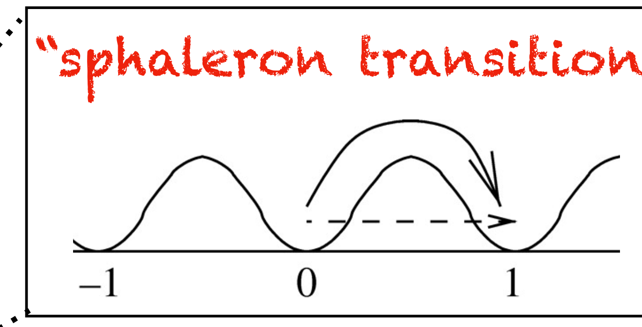
$$N_{CS}(t) \equiv \int d^3x\, n_{CS}(t,\mathbf{x})$$



Motivation

- **Macroscopic Chiral Effects**

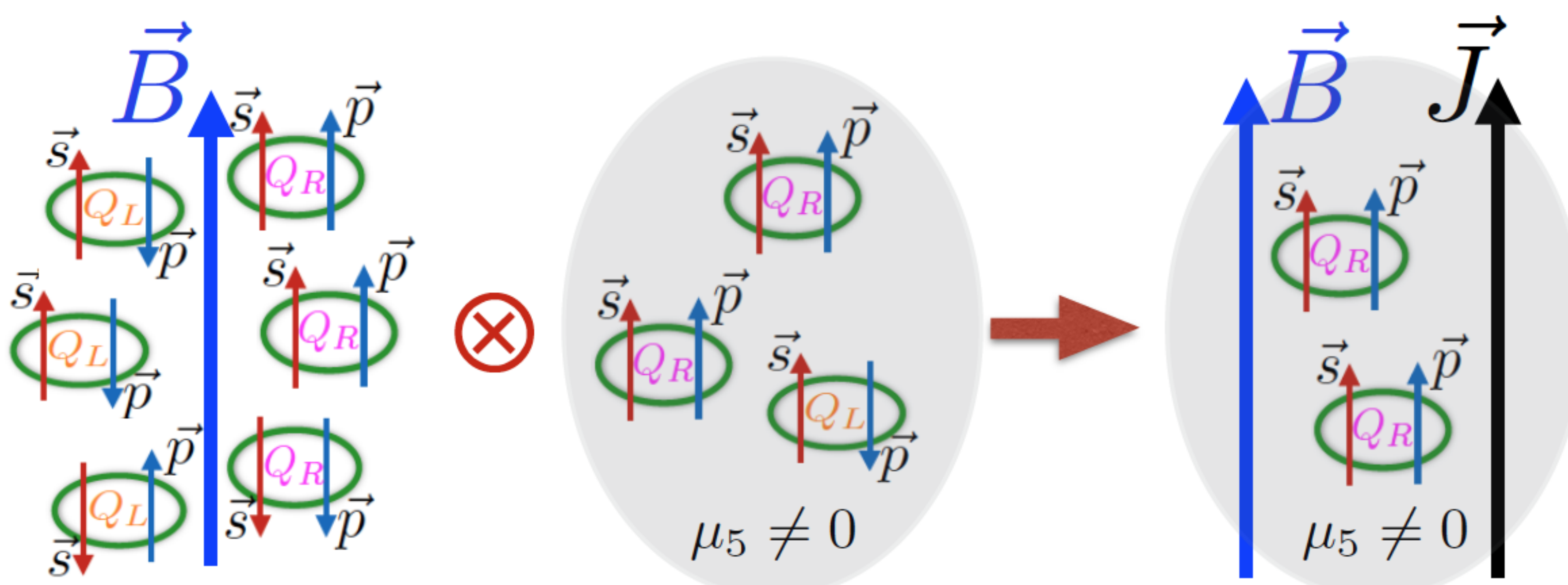




+

"chiral anomaly"

$$\langle \partial_\mu J_5^\mu(t,\mathbf{x}) \rangle = -\frac{g^2 N_f}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}(t,\mathbf{x})$$



Fukushima, Kharzeev, McLerran, Warringa ~ 2008

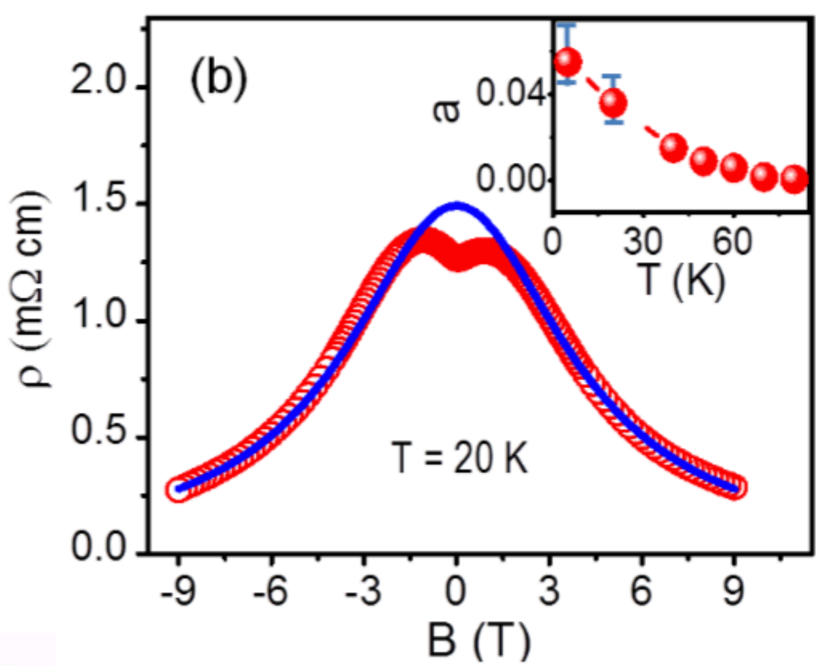
sources: (top) wikipedia O. Alexndrov; (bottom) Kharzeev, Warringa, Fukushima

Motivation

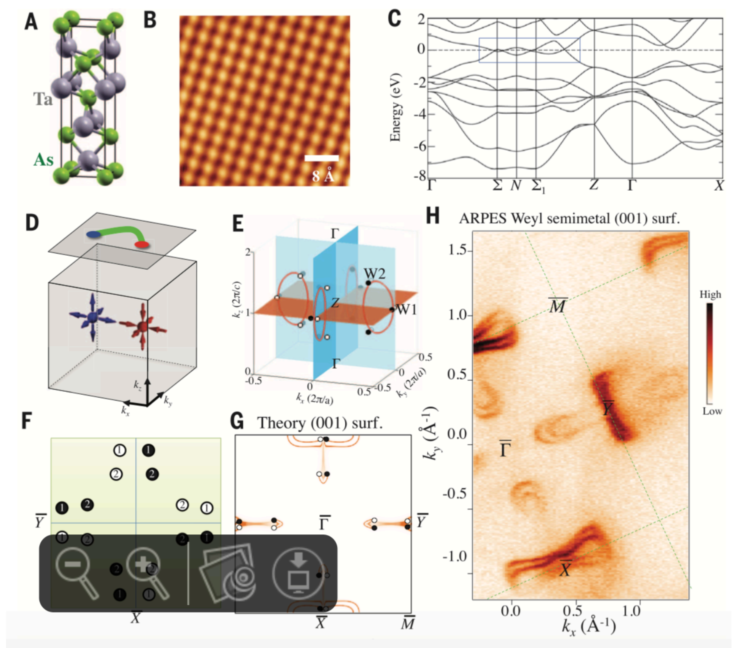
- **More Chiral Effects:** *novel electronic properties*

Dirac and Weyl Semi-metals **pseudo-chiral effects**

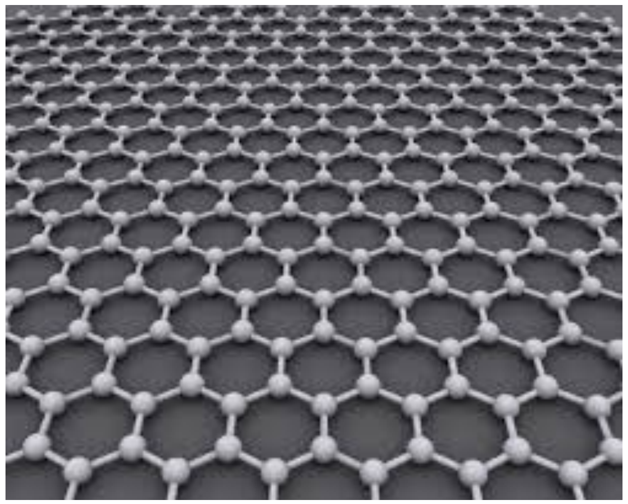
ZrTe *$Mo_xW_{1-x}Te_2$* *TaA* *Graphene*



Li et al, Nature Physics volume 12, pages 550–554 (2016)



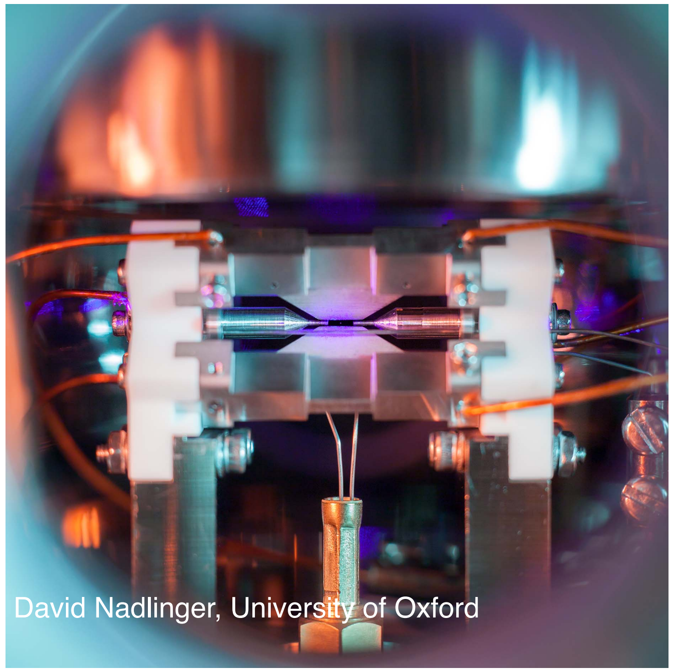
Xu et al., *Science* 07 Aug



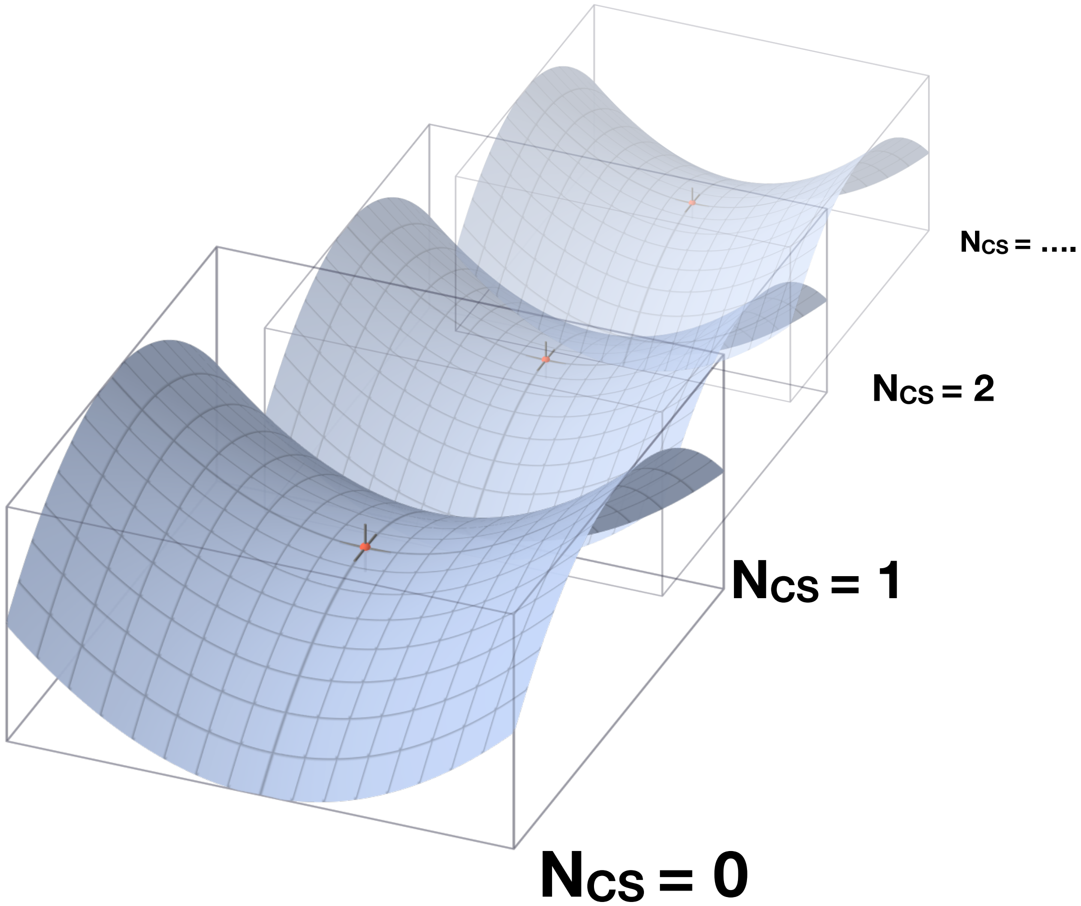
source: wikipedia

Mizher, Raya, Villavicenci 2016

Motivation



- **Manipulating nature on the quantum level**
- **Can high energy / nuclear physics benefit from quantum simulation / computation?**

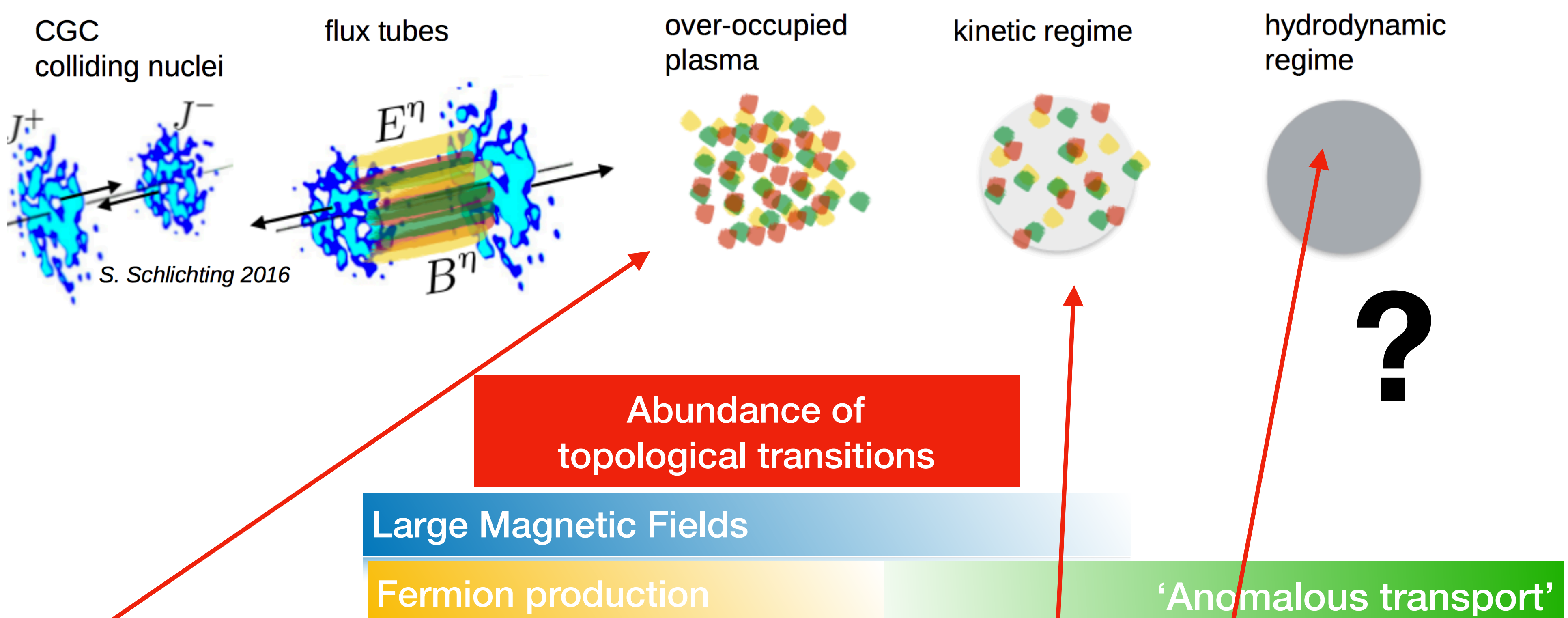


Outline

1. Chiral Fluids in QCD
2. World-line approach
3. Quantum Phase space with internal symmetries
4. Quantum Simulation - Overview
5. Quantum simulating CP violation

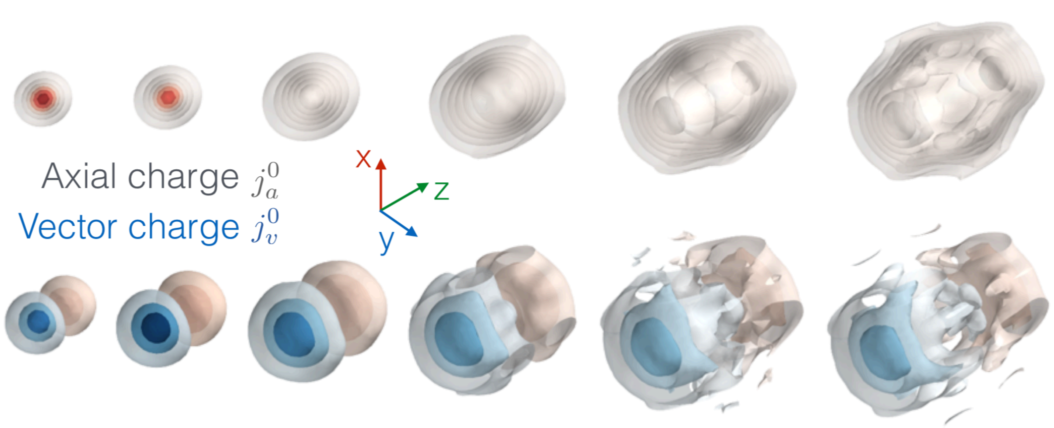
1. Chiral Fluids: Theory

Chiral fluids in QCD: difficult because non-equilibrium



real-time QFT
simulations work here

but not here!



NM, Schlichting, Sharma PRL 117 (2016) 142301

need effective transport descriptions!

1. Chiral Fluids: Theory

Anomalous Hydrodynamics

$$\nu^\mu = -\sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi\omega^\mu + \xi_B B^\mu$$
$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu,$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda\,, \quad \partial_\mu j^\mu = C E^\mu B_\mu$$

- **Son & Surowka (2009)**
- **entropy conserving contributions (from symmetries)**
- **Landau and Lifshitz would have allowed one to write this down**

<u>Some challenges:</u>

- **<u>QCD is CP even</u> "No mean Chiral Effects"**
- **Fluctuations in Hydrodynamics?**

  Crossley, Glorioso, Liu JHEP 1709 (2017) 095; Glorioso, Son (2018), arXiv:1811.04879

- **Dissipative corrections and anomalies**

1. Chiral Fluids: Theory

Chiral Kinetic Theory

- **Phase space and internal symmetries?**

$$f(x,p) \quad \rightarrow \quad ???$$

  - **Berry CKT: Son, Yamamoto; Stephanov, Yi (2012)**
  - **Xiao, Shi, Niu (2005)**

$$\dot{\mathbf{x}} = \frac{1}{\hbar}\frac{\epsilon_n(\mathbf{p})}{\partial \mathbf{p}} - \dot{\mathbf{k}} \times \mathbf{\Omega}_n(\mathbf{p})\,,$$
$$\hbar\dot{\mathbf{p}} = e\mathbf{E}(\mathbf{x}) - e\dot{\mathbf{r}} \times \mathbf{B}(\mathbf{x})\,,$$
$$\mathbf{\Omega}_n(\boldsymbol{k}) = \mathrm{i}\langle \nabla_{\boldsymbol{k}} u_n(\boldsymbol{k})| \times |\nabla_{\boldsymbol{k}} u_n(\boldsymbol{k})\rangle$$

- **Berry origin of the anomaly?**

  Fujikawa & Deguchi 2005, arXiv:1709.08181,
  NM & R. Venugopalan, arXiv:1701.03331, arXiv:1702.01233

- **Microscopic to macroscopic:**
  — fluctuations and collision terms

2. Worldline approach

- **Strassler 1992: worldline representation of QFT**

$$\Gamma[A] = -\mathrm{Tr}\,\log(-iD^2[A]) = \int_0^\infty \frac{dT}{T}\mathcal{N}\,Dx\,\mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^T d\tau\left(\frac{\dot{x}^2}{2\epsilon}+gA_\mu[x]\dot{x}^\mu\right)\right]$$

- **internal symmetries: Grassmann variables**

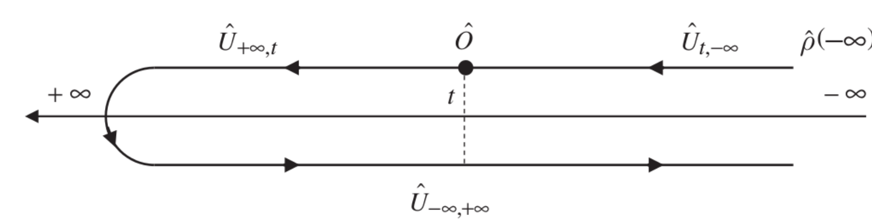
$$\mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^T d\tau M(\tau)\right] = \int\mathcal{D}\phi\int\mathcal{D}\lambda^\dagger\mathcal{D}\lambda\, e^{i\phi(\lambda^\dagger\lambda+\frac{n}{2}-1)}\exp\left[i\int_0^T d\tau\left(i\lambda^\dagger\frac{d\lambda}{d\tau}+\lambda^\dagger M\lambda\right)\right]$$

here: color

D'Hoker & Gagne

- **Non-equilibrium generalization: Schwinger-Keldysh**

$$\Gamma_{\mathcal{C}}[A;\chi] = \int d^4x_i^+ d^4x_i^- \int d\lambda_i^+ d\lambda_i^- \int d\lambda^{\dagger}{}_i^{+} d\lambda^{\dagger}{}_i^{-}$$
$$\times\, \chi_A(x_i^+, x_i^-, \lambda_i^+, \lambda_i^-, \lambda^{\dagger}{}_i^{+}, \lambda^{\dagger}{}_i^{-})$$
$$\times \int_{\mathcal{C}}\mathcal{D}\epsilon\mathcal{D}\phi\int_{\mathcal{C}}\mathcal{D}x\int_{\mathcal{C}}\mathcal{D}\lambda^\dagger\mathcal{D}\lambda\, e^{iS_{\mathcal{C}}[A]}\,.$$



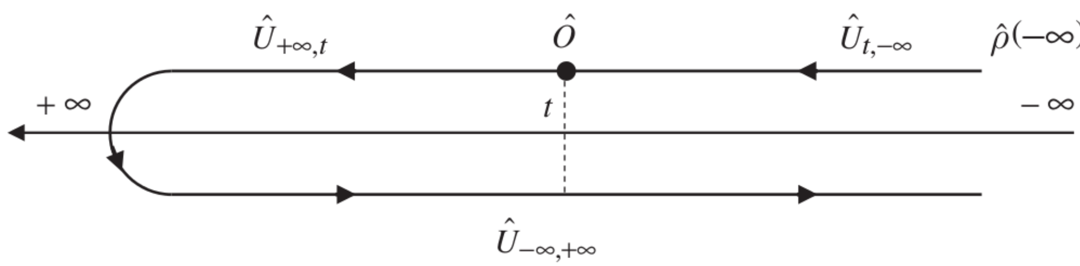
"Wigner distribution"

$$\chi_A(x_i^+, x_i^-, \lambda_i^+, \lambda_i^-, \lambda^{\dagger}{}_i^{+}, \lambda^{\dagger}{}_i^{-})$$
$$\equiv \int \frac{d^4\bar{p}_i}{(2\pi)^4} W_A^\chi(\bar{x}_i, \bar{p}_i, \bar{\lambda}_i, \bar{\lambda}_i^\dagger)\, e^{i(\bar{p}_i\cdot\tilde{x}_i+\frac{1}{2}\bar{\lambda}_i^\dagger\cdot\tilde{\lambda}_i+\frac{1}{2}\bar{\lambda}_i\cdot\tilde{\lambda}_i^\dagger)}$$

2. Worldline approach

- **semi-classical phase space: Truncated Wigner Approximation**

review: Polkovnikov 2009



$$\bar{x} = \frac{x^+ + x^-}{2} \quad \text{“classical”}$$

$$\tilde{x} = x^+ - x^- \quad \text{“quantum”}$$

$$\Gamma_{\mathcal{C}} \approx \int d^4\bar{x}_i d^4\bar{p}_i d\bar{\lambda}_i d\bar{\lambda}_i^\dagger \; W_A^\chi(\bar{x}_i, \bar{p}_i, \bar{\lambda}_i, \bar{\lambda}_i^\dagger)$$
$$\times \int_{\mathcal{C}} \mathcal{D}x\mathcal{D}p\mathcal{D}\lambda\mathcal{D}\lambda^\dagger\mathcal{D}\epsilon D\phi \; \exp\left\{iS_0 + i\int d\tau \left(\left[\dot{\bar{p}} - \frac{\partial H}{\partial \tilde{x}}\right]\tilde{x} - \left[\dot{\bar{x}} + \frac{\partial H}{\partial \tilde{p}}\right]\tilde{p} + \left[i\dot{\bar{\lambda}} - \frac{\partial H}{\partial \tilde{\lambda}^\dagger}\right]\tilde{\lambda}^\dagger - \left[i\dot{\bar{\lambda}}^\dagger + \frac{\partial H}{\partial \tilde{\lambda}}\right]\tilde{\lambda}\right)\right\}$$

- **yields (quantum-) Liouville equation**

$$\frac{d}{d\tau}W_A^\chi = \left(\dot{\bar{x}}_\mu \frac{\partial}{\partial \bar{x}_\mu} + \dot{\bar{P}}_\mu \frac{\partial}{\partial \bar{P}_\mu} + \dot{\bar{\lambda}}_a \frac{\partial}{\partial \bar{\lambda}_a} + \dot{\bar{\lambda}}_a^\dagger \frac{\partial}{\partial \bar{\lambda}_a^\dagger}\right) W_A^\chi(x, P, \underline{\lambda, \lambda^\dagger})$$

Grassmann coordinates for color

2. Worldline approach

Spin ?

- **Spin via anti-commuting variables** (Berezin and Marinov 1976)

$$\boxed{W_A^\chi(x, P, \lambda, \lambda^\dagger) \longrightarrow W_A^\chi(x, P, \lambda, \lambda^\dagger, \underline{\psi})}$$

- **bilinear form**

$$S_{\mu\nu} = -i\psi_\mu\psi_\nu$$

$$Q^a \equiv \lambda_c^\dagger t_{cd}^a \lambda_d$$

Not a classical representation!
Valid for any (!) representation

$$\begin{aligned}
\dot{x}^\mu &= \epsilon P^\mu\,, \\
\dot{P}^\mu &= \epsilon g F^{a,\mu\nu} Q^a P_\nu - \frac{i\epsilon g}{2}\psi^\alpha (D^\mu F_{\alpha\beta})^a Q^a \psi^\beta\,, \\
\dot{\psi}^\mu &= \epsilon g F^{a,\mu\nu} Q^a\, \psi_\nu\,, \\
\dot{\lambda}_a^\dagger &= -igv^\mu t_{ab}^c A_\mu^c \lambda_b^\dagger - \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c^\dagger \psi^\nu\,, \\
\dot{\lambda}_a &= igv^\mu t_{ab}^c A_\mu^c \lambda_b^\dagger + \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c \psi^\nu\,,
\end{aligned}$$

$$\boxed{\dot{Q}^a = -igv^\mu f^{abc} A_\mu^b Q^c - \frac{g\epsilon}{2} f^{abc}\psi^\mu F_{\mu\nu}^b \psi^\nu Q^c}$$

= Wong's equation (1970)

SK derivation in worldline formalism, see *PRD 99, 056003 (arXiv:1901.10492)*

3. Quantum Phase Space

Closer look: phase space for spin and chirality

- **Worldline path integral defines phase space measure in semi-classical limit**

$$\int dS \equiv -i \int d\psi_0 d\psi_1 d\psi_2 d\psi_3$$

- **Grassmann algebra fixes the form of the distribution function uniquely!**

polarized part

$$W_A^\chi(x, P, \lambda, \lambda^\dagger, \psi) = W_A^\chi(x, P, \lambda, \lambda^\dagger)\Big[\Sigma_\mu(x, P, \lambda, \lambda^\dagger) \times v_\lambda\, \psi^\mu \psi^\lambda - \frac{i}{6}\epsilon_{\mu\nu\alpha\beta} v^\mu v_\lambda \psi^\nu \psi^\alpha \psi^\beta \psi^\lambda\Big]$$

unpolarized part

3. Quantum Phase Space

- **practical approach: color and spin, via moments**

$$f(x,P),\, f^a(x,P)\, \Sigma_\mu(x,P),\, \tilde{\Sigma}_\mu(x,P), \text{ and } \tilde{\Sigma}^a_\mu(x,P)$$

exact spin structure

$$f_A(x,P,Q,S) = f_A(x,P,Q)\left[\underbrace{i\Sigma_\mu(x,P,Q)S^{\mu\nu}v_\nu}_{\text{polarized}} - \underbrace{\frac{i}{6}\epsilon_{\mu\nu\alpha\beta}v^\mu S^{\nu\alpha}S^{\beta\lambda}v_\lambda}_{\text{unpolarized}}\right]$$

exact color structure

$$f(x,P,Q) = \underbrace{f(x,P)\left[1+\frac{2}{A_R d^2}d^{abc}Q^aQ^bQ^c\right]}_{\text{singlet}} + \underbrace{2f^a(x,P)Q^a}_{\text{octet}}$$

- **e.g. Pauli-Lubanski vector (BMT equation)**

$$\dot{\Sigma}_\mu(x,P,Q) = \frac{g}{P^0}F^a_{\mu\nu}Q^a\,\Sigma^\nu(x,P,Q) + \frac{2g}{P^0}\Sigma_\alpha(x,P,Q)F^{a,\alpha\beta}Q^a v_\beta\, v_\mu$$

- **currents etc generalized phase space averages**

$$\langle J^\mu_{L/R}(x)\rangle \equiv e\int d^4P\, dS\, \epsilon\, [P^\mu + S^{\mu\nu}\partial_\nu]\, f(x,P,S)$$

- **anomaly: axial current requires ‘proper derivation’ from worldlines in TWA**

3. Quantum Phase Space

Closer look: chiral anomaly

- **Anomaly from phase of fermion determinant**

  Alvarez-Gaume & Witten, Nucl. Phys B234 (1984) 269

- **Can be explicitly computed in worldline formulation**

$$\Gamma[A,B] = \Gamma[A] + \int d^4y \, \frac{\delta\Gamma[A,B]}{\delta B_\mu(y)}\Big|_{B=0} B_\mu(y) \qquad\qquad \delta\Gamma/i\delta B_\mu(y) \equiv \langle J_{5,\mu}(y)\rangle$$

- **spectrum contains fermionic zero modes (contribution to initial density matrix)**

$$\langle \partial_\mu J_5^\mu(y)\rangle = -\frac{e^2}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(y)$$

detailed derivation: arxiv:1702.01233 or arxiv:1901.10492 in real-time formulation

Summary Part 1

- Worldline approach ab-initio:
  Compute (!) kinetic theory from QFT

- Closed Grassmann for internal symmetries

- Generalized Quantum Phase Space, measure,
  Wigner distribution, Liouville equation

- Chiral anomaly manifest

- May be useful to constrain
  anomalous hydrodynamics

Quantum Simulating CP odd phenomena in QCD

Heidelberg

Philipp Hauke

Jürgen Berges

Fred Jendrzejewski

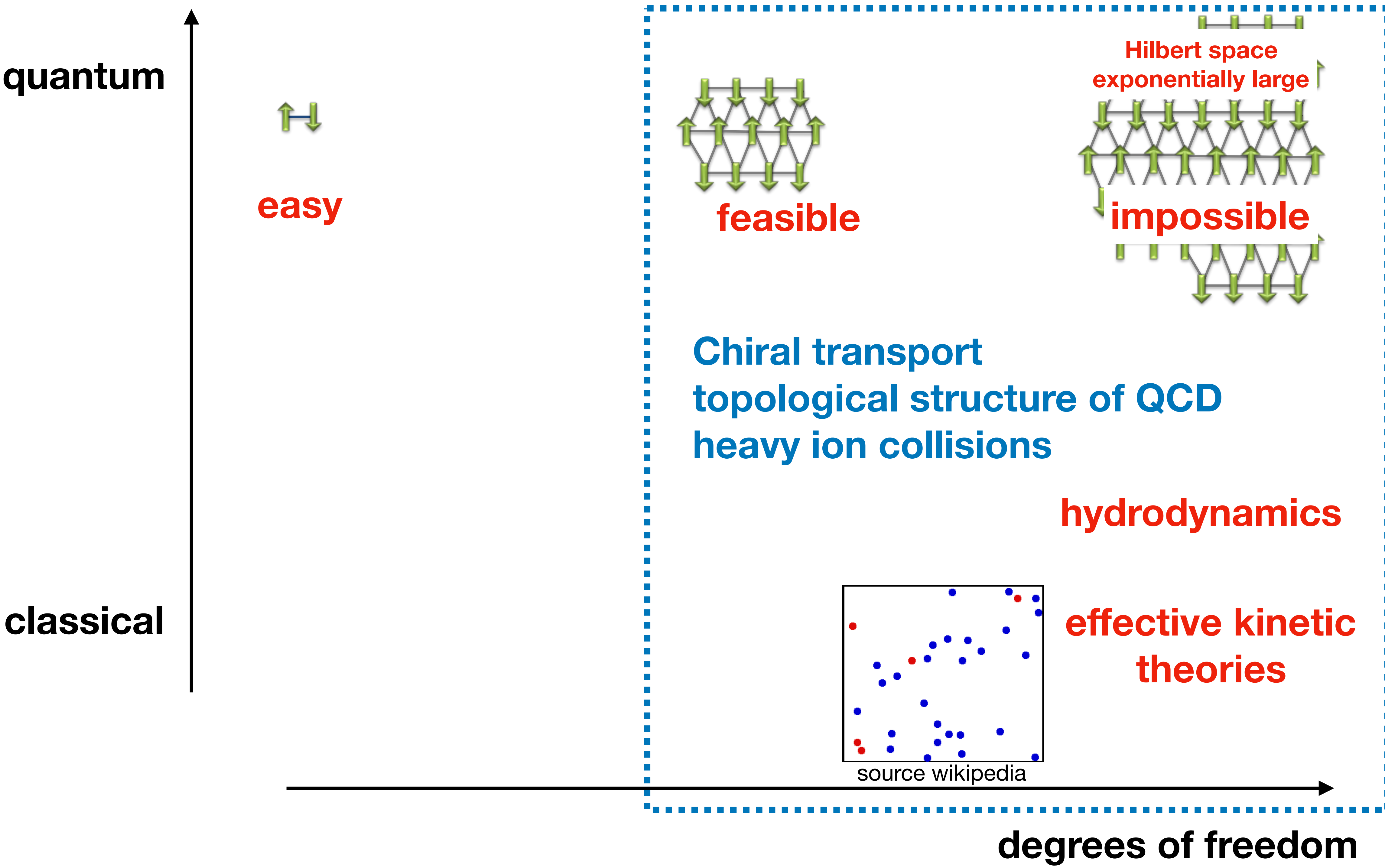
Robert Ott

Torsten Zache

Jan Schneider

PRL 122 (2019) 050403 (arXiv:1808.07885)
arXiv:1903.11109

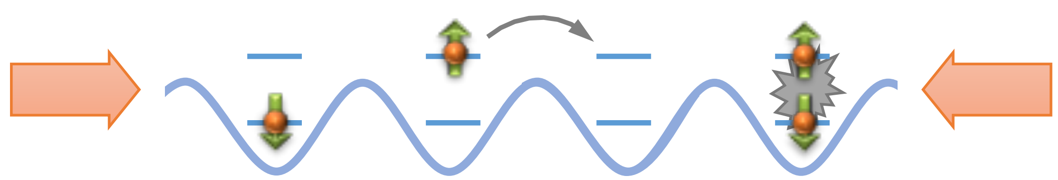
Why Quantum Simulation / Computation?



slide from P. Hauke, talk @ Lausanne Nov 23, 2018

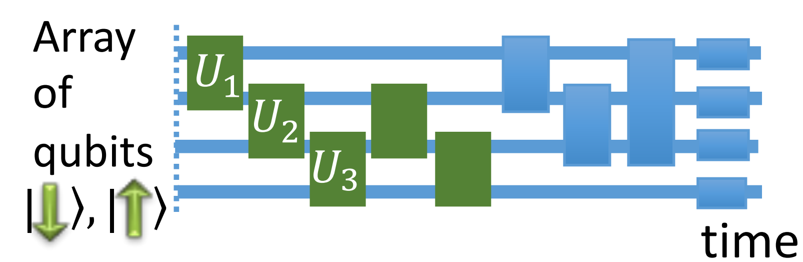
4. Quantum Simulation

Analog
e.g. ultra-cold atoms



- **bottom up engineering of specific model**
- **magnetic hyperfine states**

Digital
e.g. SC qubits

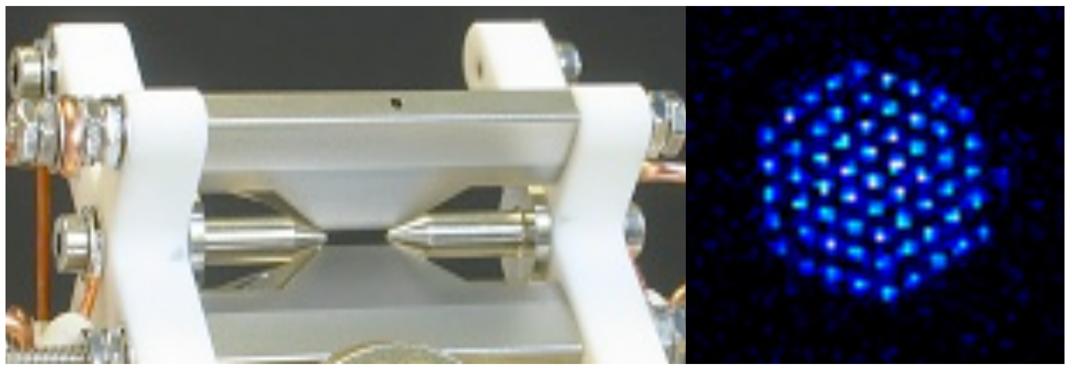


$$e^{-i\,H\,t} = U = U_M \dots U_2 U_1$$

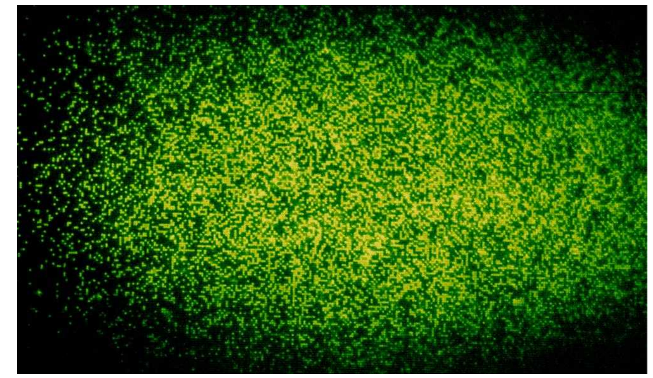
- **universal**
- **qubits**
- **few gates**
- **trotterized time evolution**

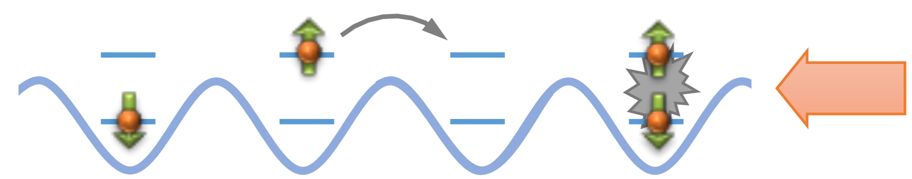
4. Quantum Simulation

trapped ions

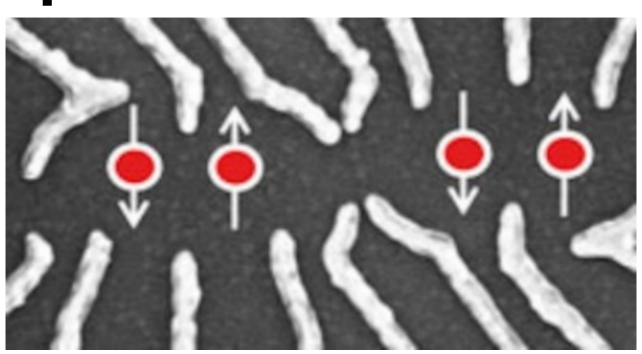


ultra-cold atoms

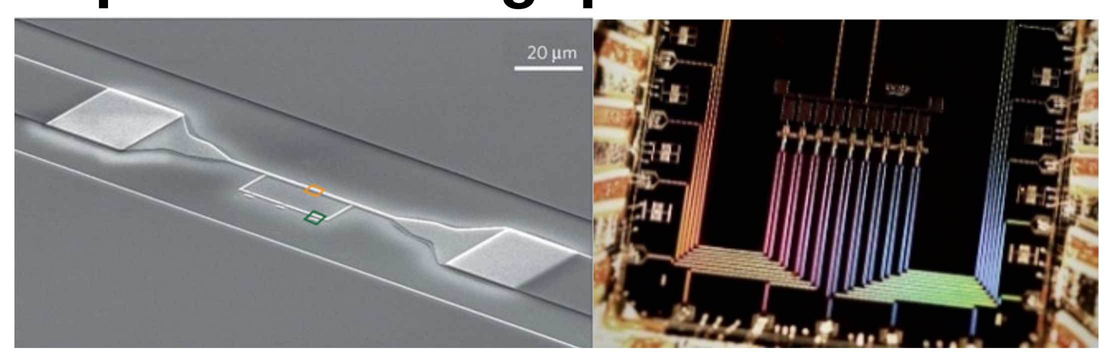


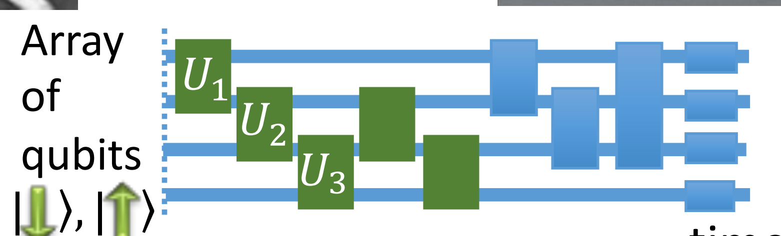


quantum dots



superconducting qubits





$e^{-i\,H\,t} = U = U_M \ldots U_2 U_1$

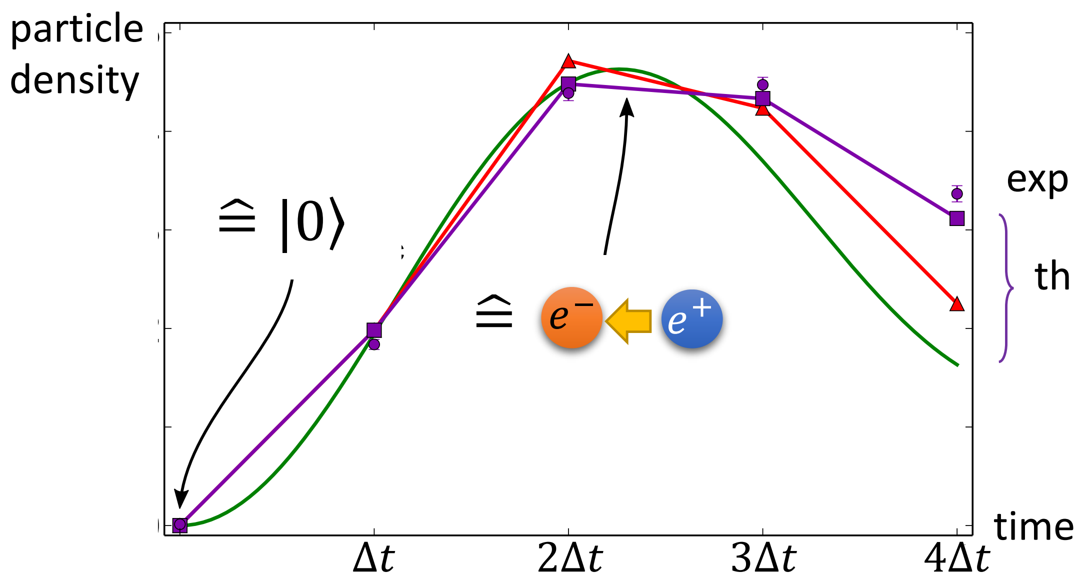
Photons
Lu et al., arXiv:1810.03959

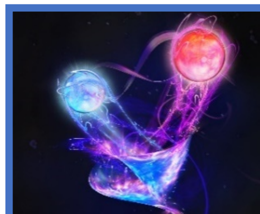
see reviews: Hauke, Cucchietti, Tagliacozzo, Deutsch, Lewenstein, Rep. Prog. Phys. 2012
Cirac, Zoller, Nat. Phys. 2012, . . .

slide from P. Hauke, talk @ Lausanne Nov 23, 2018

4. Quantum Simulation

- **First ever quantum simulation of a lattice gauge theory on trapped ion computer**
  Schwinger effect in 1+1D Schwinger model



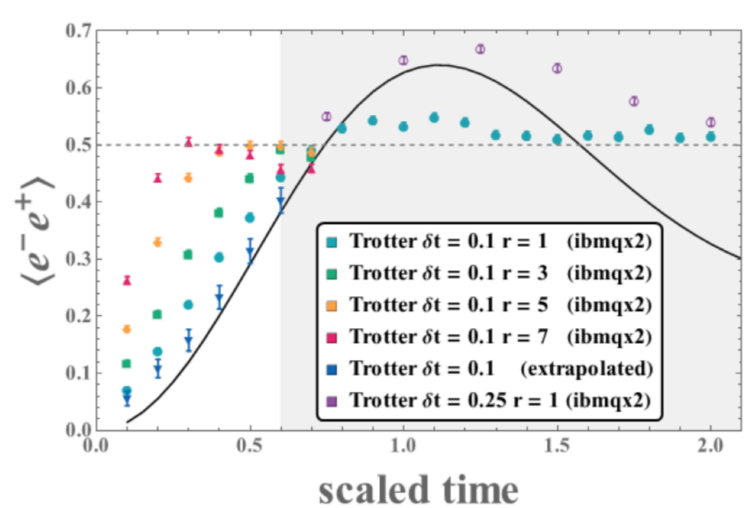


Physics world breakthrough of the year

Martinez, Muschik, Schindler, Nigg, Erhard, Heyl,
Hauke, Dalmonte, Monz, Zoller, Blatt, *Nature* 2016

- **Schwinger model on IBM-Q (2018)**

Klco, Dumutrescu, McCaskey, Morris,
Pooser, Sanz, Solano, Lougovski, Savage



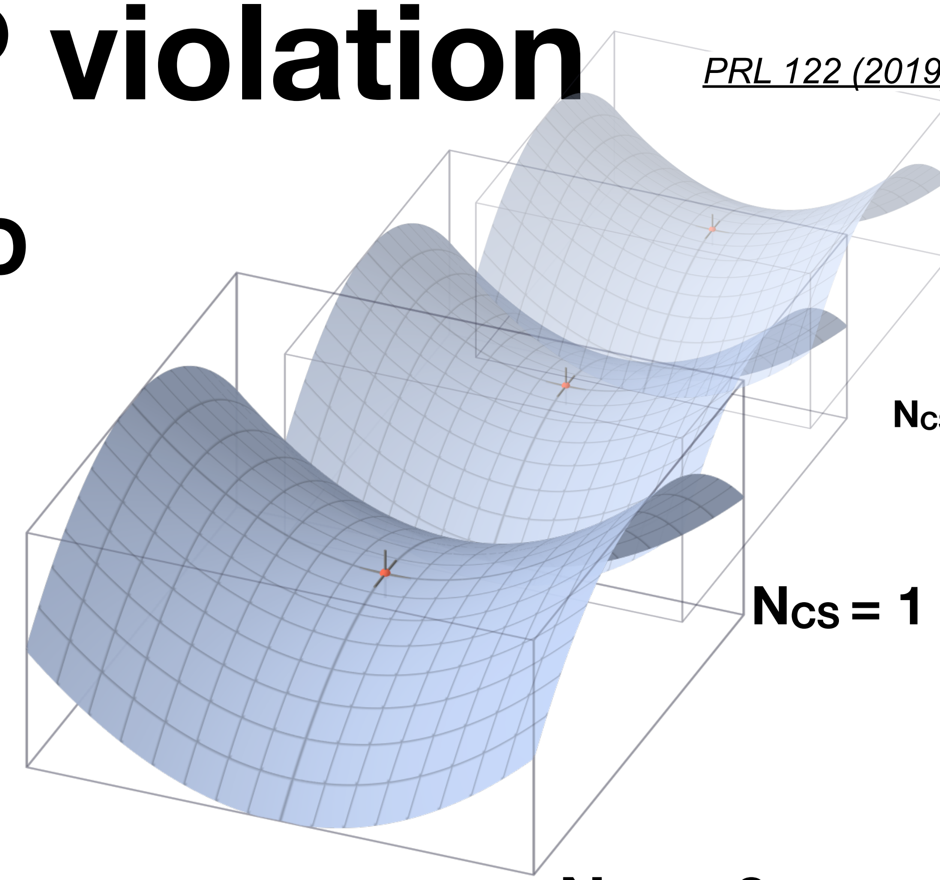
5. Q-simulating CP violation

PRL 122 (2019) 050403 (arXiv:1808.07885)

- **Topological structure of QCD**

$$n_{CS}(t,\mathbf{x}) \equiv \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}(t,\mathbf{x})$$

$$N_{CS}(t) \equiv \int d^3x \, n_{CS}(t,\mathbf{x})$$



- **strong CP problem**

$$\frac{g^2\theta}{16\pi^2}\,\tilde{F}^{\mu\nu}F_{\mu\nu} \qquad \theta < 10^{-9}$$

- **why zero? Could θ be dynamical (axion) ?**

$$\frac{g^2(\theta + c\,\hat{a})}{16\pi^2}\,\tilde{F}^{\mu\nu}F_{\mu\nu}$$

Weinberg, PRL 1978,
Wilczek, PRL 1978
Peccei, Quinn, PRL 1977

5. Q-simulating CP violation

PRL 122 (2019) 050403 (arXiv:1808.07885)

- ☑ Theta vacua
- ☑ Chiral Anomaly
- ☑ “Chiral Symmetry Breaking”
- ☑ “Confinement”

- **Toy model for QCD: 1+1D QED**

$$H_{\mathrm{QED}} = \int \mathrm{d}x \left[(\bar{\psi}(x)\, i\gamma_1 D_x\, \psi(x) + \mathrm{h.c.}) + m\bar{\psi}(x)\psi(x) + \frac{1}{2}E(x)^2 + \boxed{\frac{e\,\theta}{2\pi}\,E(x)} \right]$$

$$\psi \to e^{-i\theta\gamma^5/2}\psi$$

$$H_{\mathrm{QED}} = \int \mathrm{d}x \left[(\bar{\psi}(x)\, i\gamma_1 D_x\, \psi(x) + \mathrm{h.c.}) + \boxed{m\,\bar{\psi}(x)e^{i\,\gamma_5\theta}\psi(x)} + \frac{1}{2}E(x)^2 \right]$$

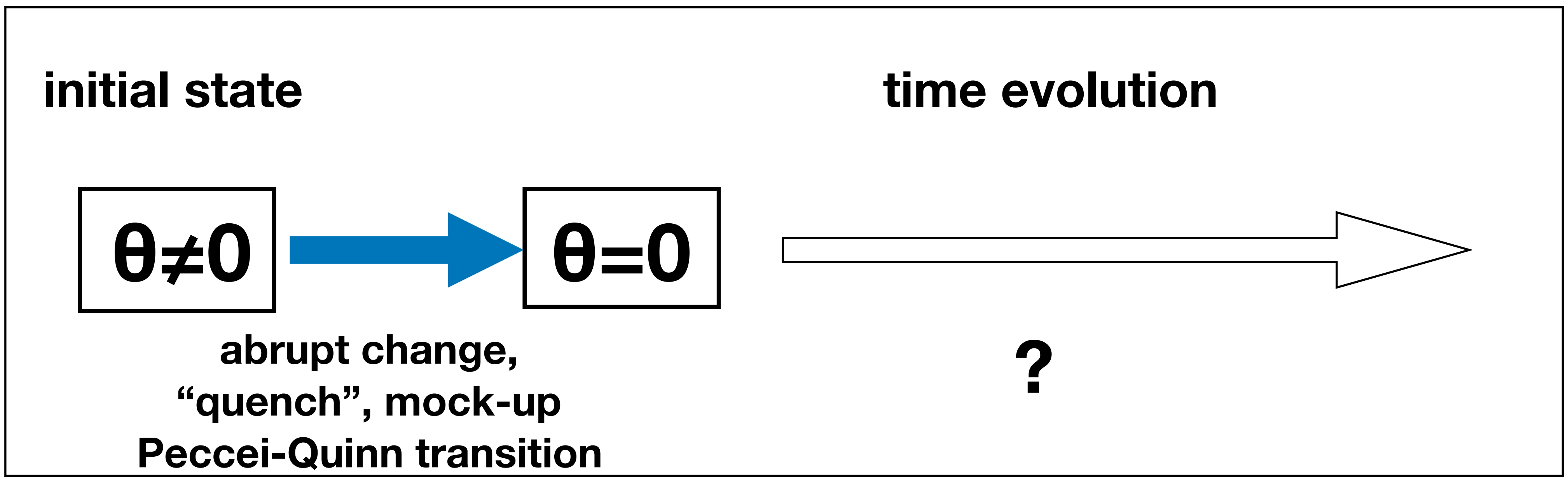
- **θ has physical consequences: dispersion relation independent of θ, but not the Hilbert state vectors!**

5. Q-simulating CP violation

PRL 122 (2019) 050403 (arXiv:1808.07885)

- **Our study:**

What are the consequences of a change in θ ?



5. Q-simulating CP violation

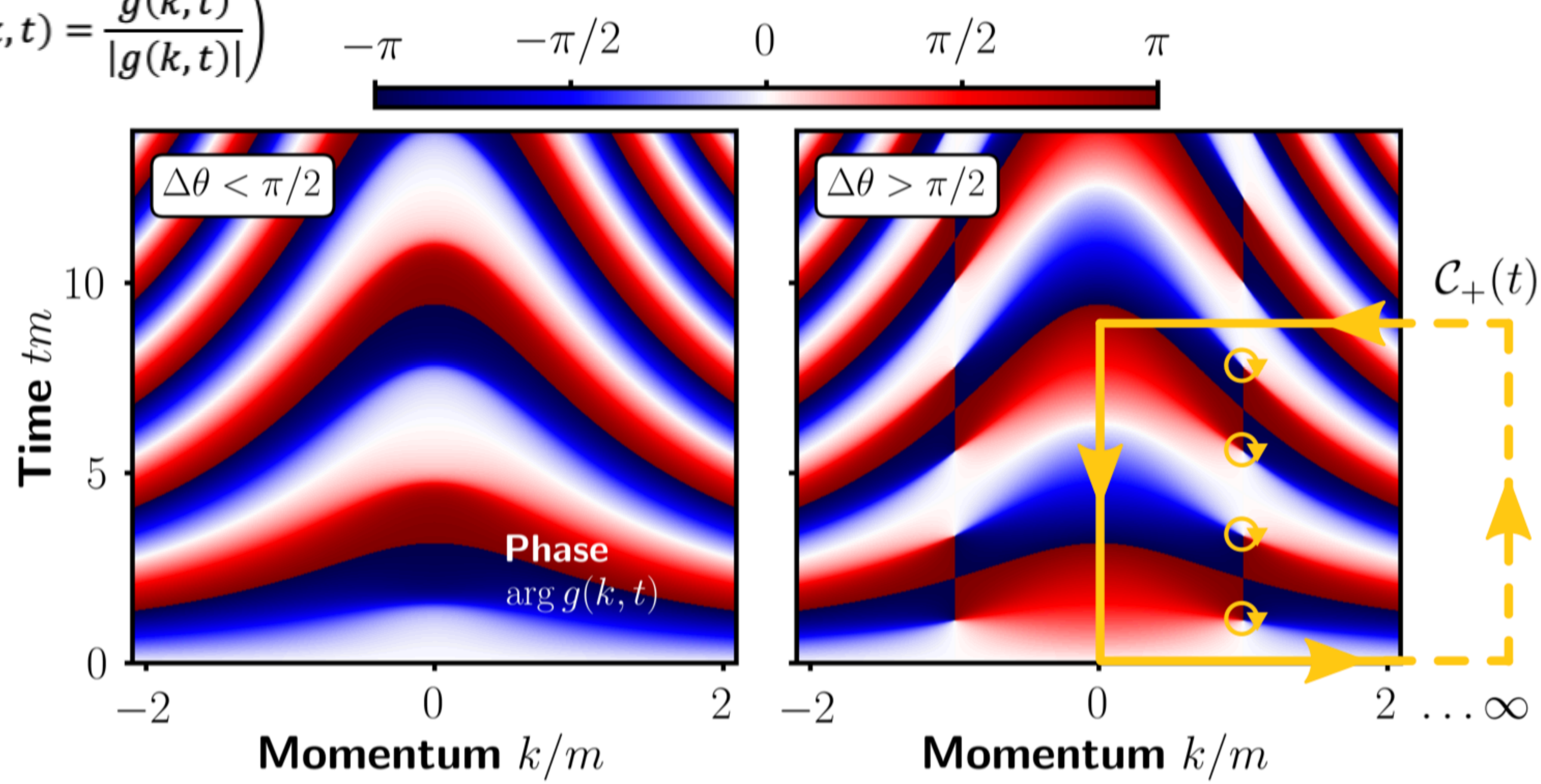
PRL 122 (2019) 050403 (arXiv:1808.07885)

- **single-particle propagator**

$$g(k,t) = \int \mathrm{d}x e^{-i\,k\,x} \left\langle \psi^\dagger(x,t)\, e^{-ie\int_0^x \mathrm{d}x' A(x',t)}\, \psi(0,0) \right\rangle$$

- **first: zero coupling**

$$\text{phase}\left(\tilde{g}(k,t) = \frac{g(k,t)}{|g(k,t)|}\right)$$



For $|\Delta\theta| > \frac{\pi}{2}$ **vortices appear!**

5. Q-simulating CP violation

PRL 122 (2019) 050403 (arXiv:1808.07885)

This transition is topological!

- **dynamical topological invariant**

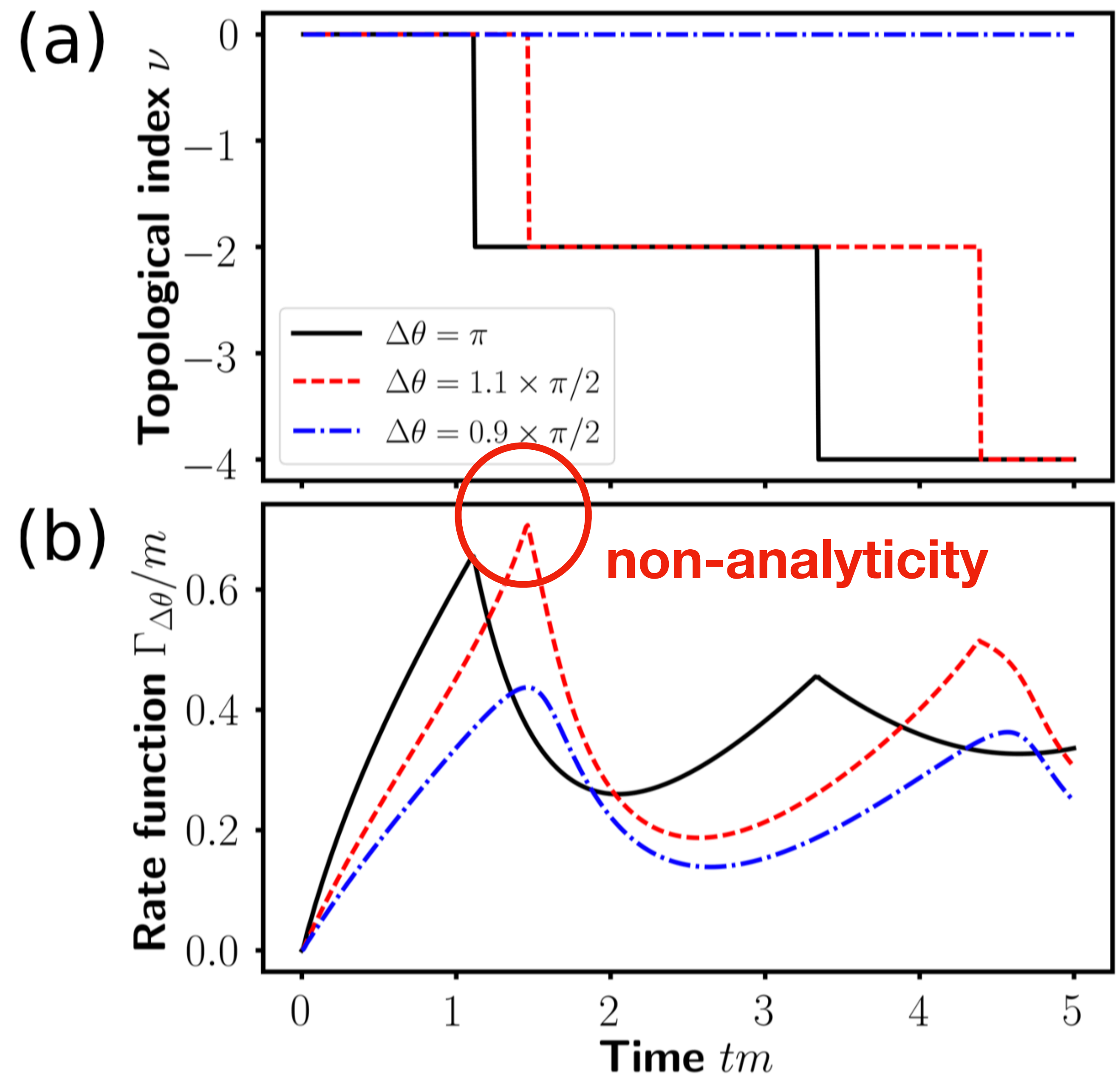
$$\nu \equiv n_+ - n_-$$

$$n_\pm(t) \equiv \frac{1}{2\pi} \oint_{\mathcal{C}_\pm(t)} d\mathbf{z}\, \{\tilde{g}^\dagger(\mathbf{z}) \nabla_{\mathbf{z}} \tilde{g}(\mathbf{z})\}$$

- **Lochschmidt echo**

$$\mathcal{L}(t) = \langle \mathrm{vac}(\theta)|\, e^{-i\,H_{\theta'}\,t}\, |\mathrm{vac}(\theta)\rangle$$

$$|L_{\theta\to\theta'}(t)| = |\langle\psi(0)|\psi(t)\rangle| = \mathrm{e}^{-V\Gamma_{\theta\to\theta'}(t)}$$



Identifies this transition as a Dynamical Quantum Phase Transition

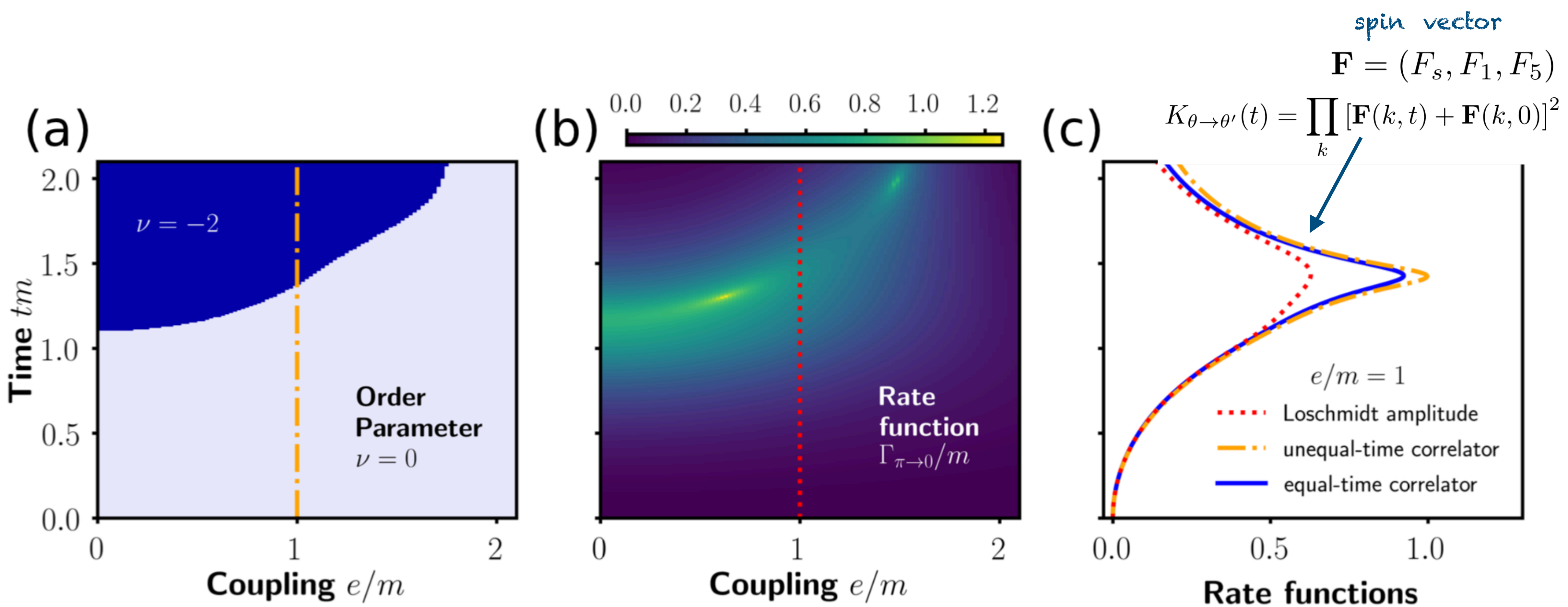
Heyl, Polkovnikov, Kehrein, PRL 2013
Recent review: Heyl, arXiv:1811.02575

5. Q-simulating CP violation

PRL 122 (2019) 050403 (arXiv:1808.07885)

- **Numerical lattice computation** **at arbitrary coupling**

- **"Classically simulating ideal quantum simulator"**
- **8 (!) lattice sites**



- **accessible with present day QC (coherence times and # sites)**

Summary Part 2

- **Quantum Simulation of gauge theories advances fast**

- **Interesting High Energy applications**

- **Dynamical Quantum Phase Transitions**
  observable for 8 sites, short times

- **Similar mechanisms in QCD?**

Extra: QED in 2+1 D

arXiv:1903.11109

- **no chiral, but <u>parity anomaly</u>**

$$j^{\mu}_{\rm an}(t) = \frac{e}{8\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}(t)$$

- **mass term violates parity**

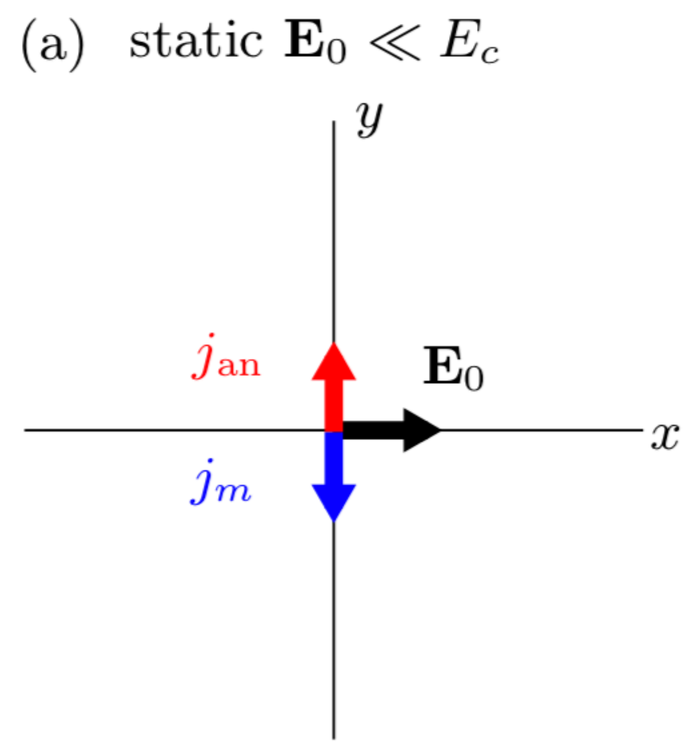
$$j^{\mu}_{\rm m}(t) \overset{\text{weak field}}{=} -\frac{m}{|m|}\frac{e}{8\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}(t)$$

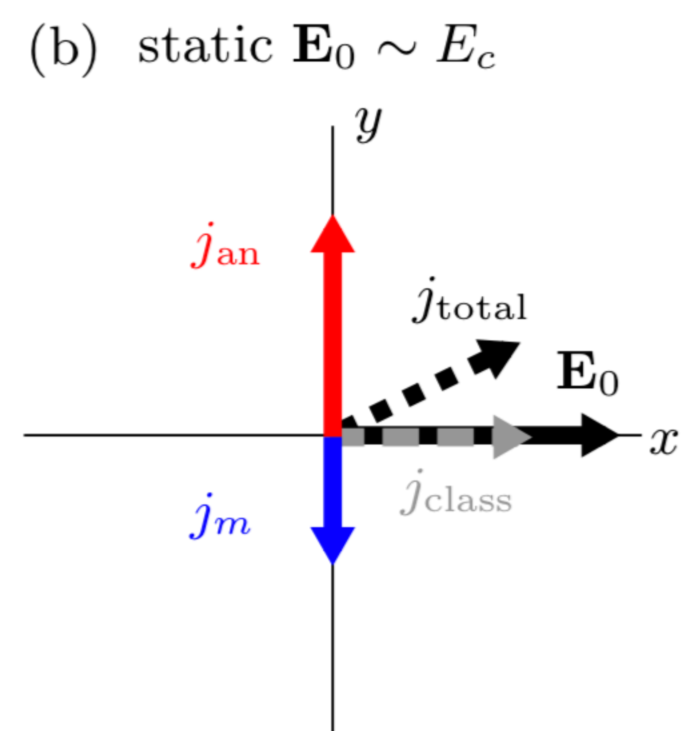
- **anomaly cancellation in weak fields**

  see also Copinger, Fukushima, Pu
  PRL 121 (2018) no.26, 261602

- **This is not true in strong fields! in-in vs. in-out**

$$j^{y}_{\rm m} = -\frac{m}{|m|}\frac{eE_x}{4\pi}\,\mathrm{Erf}\left(\sqrt{\frac{\pi m^2}{eE_x}}\right)$$

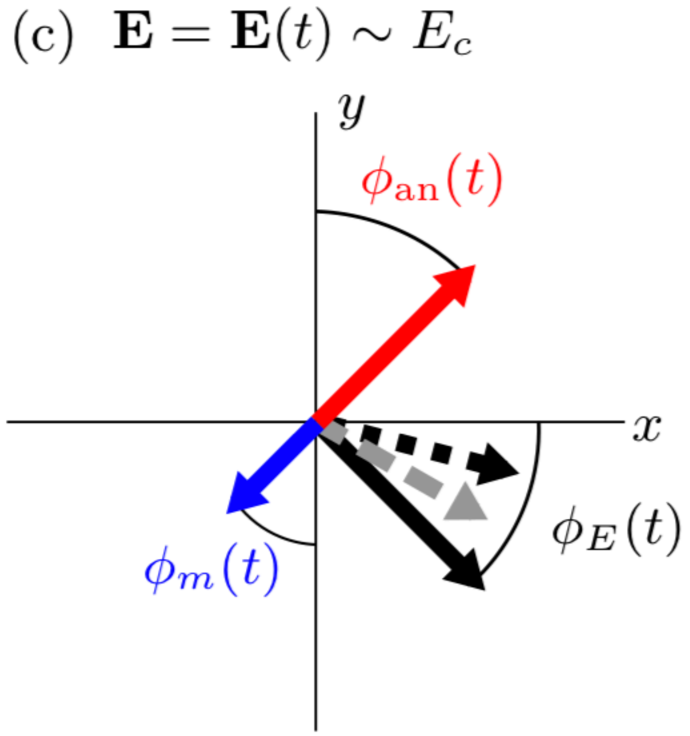


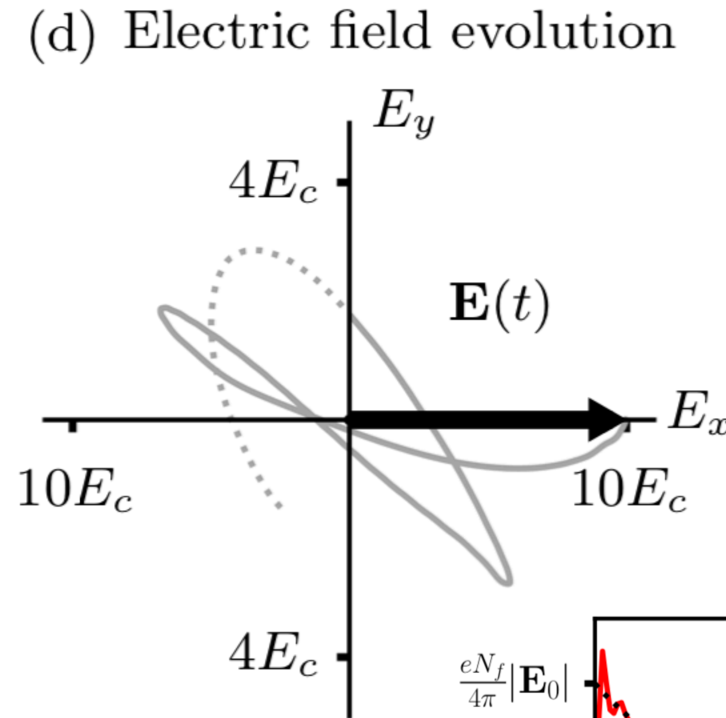


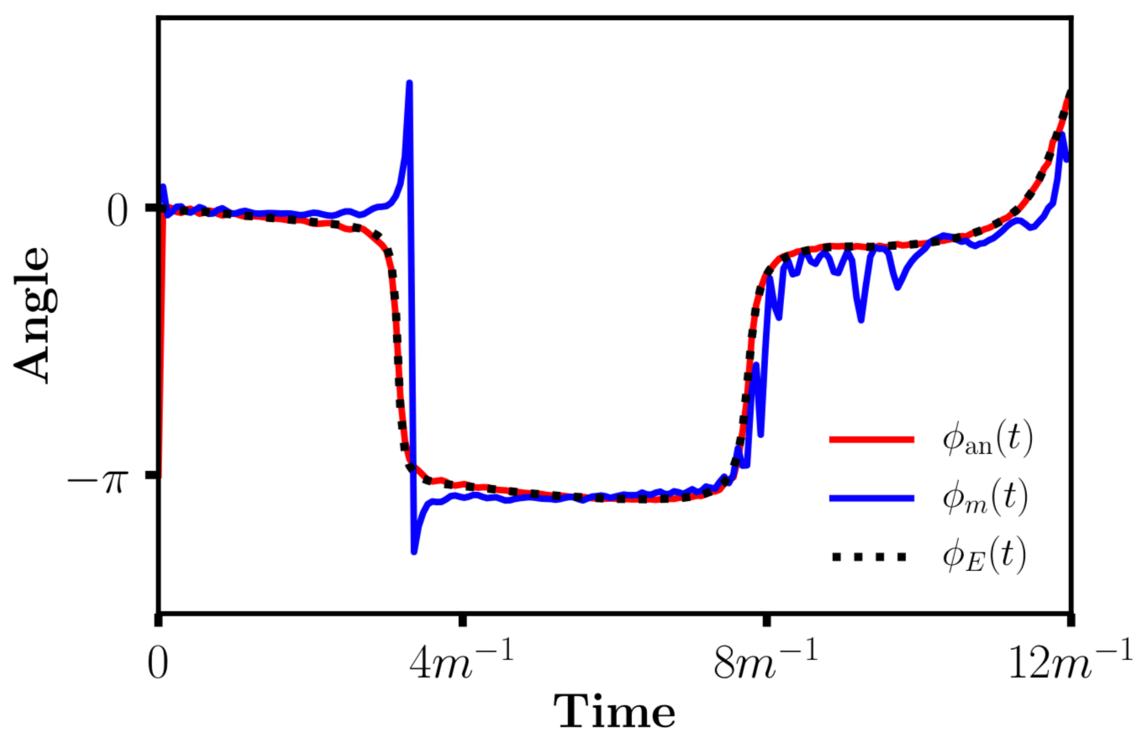
see e.g. Redlich 1984

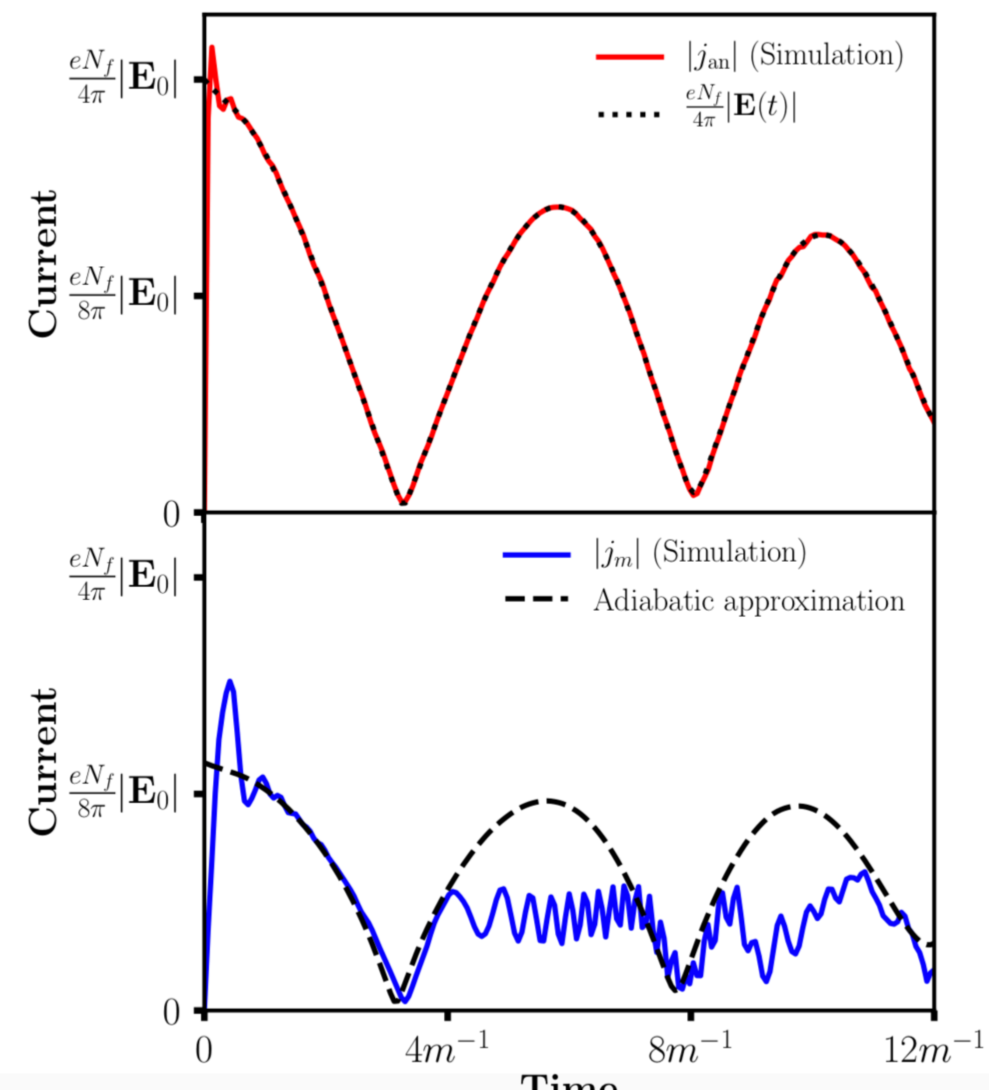
Extra: QED in 2+1 D

arXiv:1903.11109









Backup: worldline Hamiltonian

$$\Gamma[A,B] \equiv \mathrm{tr} \int d^4x_i^+ d^4x_i^- d^4\psi_i^+ d^4\psi_i^- \, \zeta^{A,B}(x_i^+, x_i^-, \psi_i^+, \psi_i^-)$$
$$\times \int_{x_i^+}^{x_i^-} \mathcal{D}x\mathcal{D}p \int_{\psi_i^+}^{\psi_i^-} \mathcal{D}\psi \int \mathcal{D}\epsilon\mathcal{D}\chi e^{iS_{\mathcal{C}}[A,B]} \,,$$

no approximations!

$$S[A \pm B] = \int d\tau_{\mathcal{C}} \left[p_\mu \dot{x}^\mu + \frac{i}{2}\psi_\mu \dot{\psi}^\mu - H[A \pm B] \right]$$

$$H[A \pm B] \equiv \frac{\epsilon}{2}\left(P^2 + ie\psi^\mu F_{\mu\nu}[A \pm B]\psi^\nu\right) + \frac{i}{4}\left(P_\mu\psi^\mu \pm \frac{i}{3}\epsilon_{\mu\nu\alpha\beta}P^\mu\psi^\nu\psi^\alpha\psi^\beta\right)\chi$$

Backup: structure of phase space: color

- **Color bilinears**

$$\boxed{\begin{aligned} &Q^a \equiv \lambda_c^\dagger t^a_{cd}\lambda_d \\ &\{Q^a, Q^b\} = \lambda^\dagger [t^a, t^b]\lambda = i f^{abc} Q^c \end{aligned}}$$

- **Color measure**

$$\int dQ = 0\,, \qquad f(x,P) \equiv \int dQ\, f(x,P,Q)\,,$$

$$\int dQ\, Q^a = 0\,, \qquad f^a(x,P) \equiv \int dQ\, Q^a\, f(x,P,Q)\,,$$

$$\int dQ\, Q^a Q^b = \frac{1}{2}\delta^{ab}\,, \qquad f^{ab}(x,P) \equiv \int dQ\, Q^a Q^b f(x,P,Q)\,,$$

$$\int dQ\, Q^a Q^b Q^c = \frac{A_R}{2} d^{abc} \qquad f^{abc}(x,P) \equiv \int dQ\, Q^a Q^b Q^c f(x,P,Q)\,.$$

- **One unique form of phase space distribution**

$$\boxed{f(x,P,Q) = f(x,P)\left[1 + \frac{2}{A_R d^2} d^{abc} Q^a Q^b Q^c\right] + 2 f^a(x,P) Q^a}$$

$$d^2 \equiv d^{abc} d^{abc} = N_c^2 - 4$$

Backup: anomaly

- **naive approach: phase space average**

$$\partial S / i\partial A_\mu$$

$$\boxed{\langle J^\mu_{L/R}(x)\rangle \equiv e \int d^4P\, dS\, \epsilon\, [P^\mu + S^{\mu\nu}\partial_\nu]\, f(x,P,S)}$$

$$\Sigma^\mu_{L/R} = \pm P^\mu / 2P^0$$

- **chiral current …**

$$\boxed{\langle J^\mu_5(x)\rangle = \langle J^\mu_R(x)\rangle - \langle J^\mu_L(x)\rangle = e \int d^4P\, \epsilon\, \epsilon^{\mu\nu\alpha\beta} P_\beta \partial_\nu [\Sigma_\alpha(x,P) f(x,P)]}$$

… is classically conserved. What is missing?

Backup: anomaly

- **derivation from worldline SK path integral**

$$\boxed{\Gamma[A,B] \equiv \mathrm{tr}\int d^4x_i^+ d^4x_i^- d^4\psi_i^+ d^4\psi_i^- \, \zeta^{A,B}(x_i^+,x_i^-,\psi_i^+,\psi_i^-)\int_{x_i^+}^{x_i^-}\mathcal{D}x\mathcal{D}p\int_{\psi_i^+}^{\psi_i^-}\mathcal{D}\psi\int\mathcal{D}\epsilon\mathcal{D}\chi e^{iS_C[A,B]}}$$

variational axial-vector gauge field

- **linear order in axial-vector field B**

$$\boxed{\Gamma[A,B] = \Gamma[A] + \int d^4y \left.\frac{\delta\Gamma[A,B]}{\delta B_\mu(y)}\right|_{B=0} B_\mu(y)}$$

- **Linear term: chiral current**

$$\begin{aligned}\left.\frac{\delta\Gamma[A,B]}{\delta B_\mu(y)}\right|_{B=0} = \mathrm{tr}\int d^4x_i^+ d^4x_i^- d^4\psi_i^+ d^4\psi_i^- \Bigg[&\zeta^{A,B}(x_i^+,x_i^-,\psi_i^+,\psi_i^-)\int_{x_i^+}^{x_i^-}\mathcal{D}x\mathcal{D}p\int_{\psi_i^+}^{\psi_i^-}\mathcal{D}\psi\int\mathcal{D}\epsilon\mathcal{D}\chi\,\frac{i\delta S_C[A,B]}{\delta B_\mu(y)}e^{iS_C[A]}\\ &+\frac{\delta\zeta^{A,B}(x_i^+,x_i^-,\psi_i^+,\psi_i^-)}{\delta B_\mu(y)}\int_{x_i^+}^{x_i^-}\mathcal{D}x\mathcal{D}p\int_{\psi_i^+}^{\psi_i^-}\mathcal{D}\psi\int\mathcal{D}\epsilon\mathcal{D}\chi\,e^{iS_C[A,B]}\Bigg]\Bigg|_{B=0}\end{aligned}$$

initial density matrix (“spectrum”)

Backup: anomaly

- **we computed second term already in arxiv:1702.01233 (*)**

$$\boxed{\zeta \equiv \zeta^{(0)} + \zeta^{(1)}}$$

$$\zeta^{(0)} \equiv \begin{pmatrix} \zeta_R^A[x_i^+, x_i^-, \psi_i^+, \psi_i^-] & 0 \\ 0 & \zeta_L^A[x_i^+, x_i^-, \psi_i^+, \psi_i^-] \end{pmatrix}$$

$$\zeta^{(1)} \equiv 2\,\mathbb{I}_{2\times 2}\left[\partial_\mu B_\mu(\bar{x}_i) - \{\partial_\mu, B_\nu(\bar{x}_i)\}\bar{\psi}^\nu \bar{\psi}^\mu\right] \times \delta(x_i^+ - x_i^-)\,\delta(\psi_i^+ - \psi_i^-)\,,$$

- **it gives the well known anomaly relation**

$$\boxed{\langle \partial_\mu J_5^\mu(y) \rangle = -\frac{e^2}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(y)}$$

(*) by analytic continuation. We did not realize then it could be written in SK form / density matrix

Backup: anomaly and (in-)compressibility of semi-classical phase space

- **Liouville's equation implies incompressibility of (semi-classical) phase space**

$$0=\frac{d}{d\tau}W(x,P,\psi,\lambda,\lambda^\dagger)=\Big(\dot{x}_\mu\frac{\partial}{\partial\bar{x}_\mu}+\dot{P}_\mu\frac{\partial}{\partial P}+\dot{\psi}_\mu\frac{\partial}{\partial\psi_\mu}+\dot{\lambda}_a\frac{\partial}{\partial\lambda_a}+\dot{\lambda}^\dagger_a\frac{\partial}{\partial\lambda^\dagger_a}\Big)W(x,P,\psi,\lambda,\lambda^\dagger)$$

- **canonical phase space variables: phase space incompressible at this order (reverse not true)**

- **higher orders: Moyal equation, quantum phase space compressible**

$$\frac{dW^\chi_A}{d\tau}=-2H_W\sin\left[\frac{\Lambda}{2}\right]W^\chi_A=\{W^\chi_A,H_W\}\;+O(\hbar^2)$$

- **compressibility on semi-classical level: understand as Jacobian to semi-classical phase space measure**

Does this have to do anything with the anomaly?

Backup: anomaly and (in-)compressibility of semi-classical phase space

Does this have to do anything with the anomaly?

- **Xiao, Shi, Niu make this semi-classical effective theory “many body”**

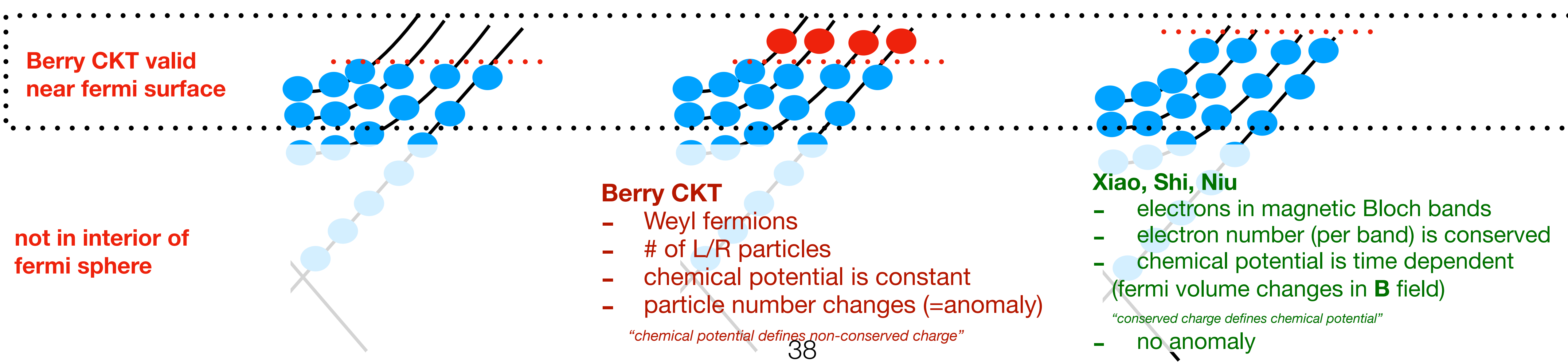
$$\dot{\mathbf{x}} = \frac{1}{\hbar}\frac{\epsilon_n(\mathbf{p})}{\partial \mathbf{p}} - \dot{\mathbf{k}} \times \mathbf{\Omega}_n(\mathbf{p}) \,,$$
$$\hbar\dot{\mathbf{p}} = e\mathbf{E}(\mathbf{x}) - e\dot{\mathbf{r}} \times \mathbf{B}(\mathbf{x}) \,,$$

- **compressibility of classical phase space**

$$\Delta V \equiv \frac{\Delta V_0}{1 + e\mathbf{B}\cdot\mathbf{\Omega}}$$

- **different interpretations of the same equations**

$$n_e = \int^{\mu} \frac{d^3p}{(2\pi)^3}\left[1 + \frac{e\mathbf{B}\cdot\mathbf{\Omega}}{\hbar}\right]$$



Backup: quantum simulation of LGT

see e.g. Berges, Hebenstreit, Kasper, Oberthaler 2016

$$H_{\rm QED} = \sum_n \left\{ \frac{a}{2} E_n^2 + M(-1)^n \psi_n^\dagger \psi_n - \frac{i}{2a} \left[\psi_n^\dagger U_n \psi_{n+1} - \psi_{n+1}^\dagger U_n^\dagger \psi_n \right] \right\}$$

$$H_{\rm CA} = \sum_n \Big\{ \frac{g^2 a}{4} [b_n^\dagger b_n^\dagger b_n b_n + d_n^\dagger d_n^\dagger d_n d_n] + M(-1)^n \psi_n^\dagger \psi_n - \frac{i}{2a\sqrt{\ell(\ell+1)}} \left[\psi_n^\dagger b_n^\dagger d_n \psi_{n+1} - \psi_{n+1}^\dagger d_n^\dagger b_n \psi_n \right] \Big\}$$

$$U_n \to [\ell(\ell+1)]^{-1/2} L_{+,n}$$

$$E_n \to g L_z$$

$$[E_n, U_m] = g \delta_{nm} U_m$$

$$[L_{i,n}, L_{j,m}] = i \delta_{nm} \epsilon_{ijk} L_{k,n}$$

$$G_n = E_n - E_{n-1} - g \psi_n^\dagger \psi_n$$

$$[L_{+,n}, L_{-,m}] = 2 \delta_{nm} L_{z,m}$$

$$L_{+,n} = b_n^\dagger d_n, \; L_{-,n} = d_n^\dagger b_n \text{ and } L_{z,n} = (b_n^\dagger b_n - d_n^\dagger d_n)/2.$$