DARK ENERGY (DE) PERTURBATIONS

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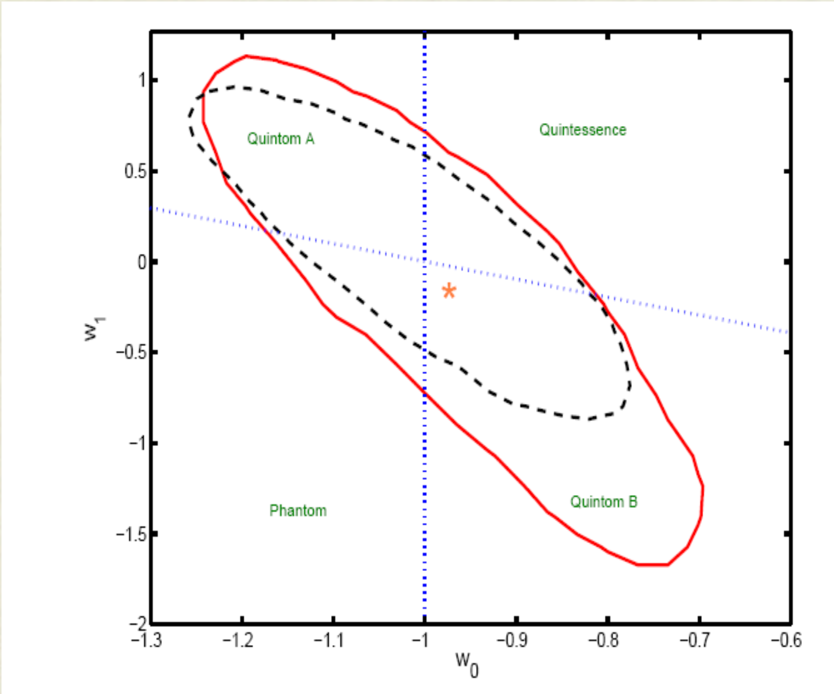
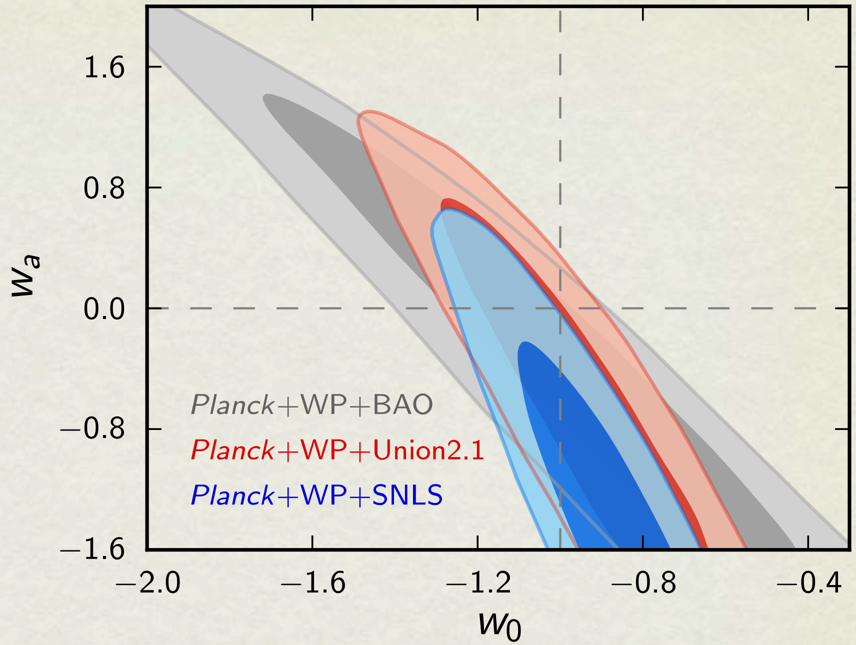
DE models: classified by w





$$w_0 = -1.04^{+0.72}_{-0.69} \quad (95\%; \textit{Planck}+\mathrm{WP}+\mathrm{BAO}),$$
$$w_a < 1.32 \quad (95\%; \textit{Planck}+\mathrm{WP}+\mathrm{BAO}).$$

$$w = w_0 + w_a(1-a)$$

Background evolution $w_0, \quad w_a$

Except for Cosmological constant, perturbations should be considered in consistent analyses

Cosmological constant: $w = -1, \ \dot{w} = 0$

Quintessence: $\mathcal{L} = (1/2)(\partial\phi)^2 - V(\phi)$

Phantom: $\mathcal{L} = -(1/2)(\partial\phi)^2 - V(\phi)$

K-essence: $\mathcal{L} = \mathcal{L}(\phi, \ X), \quad X = (1/2)\partial_\mu\phi\partial^\mu\phi$

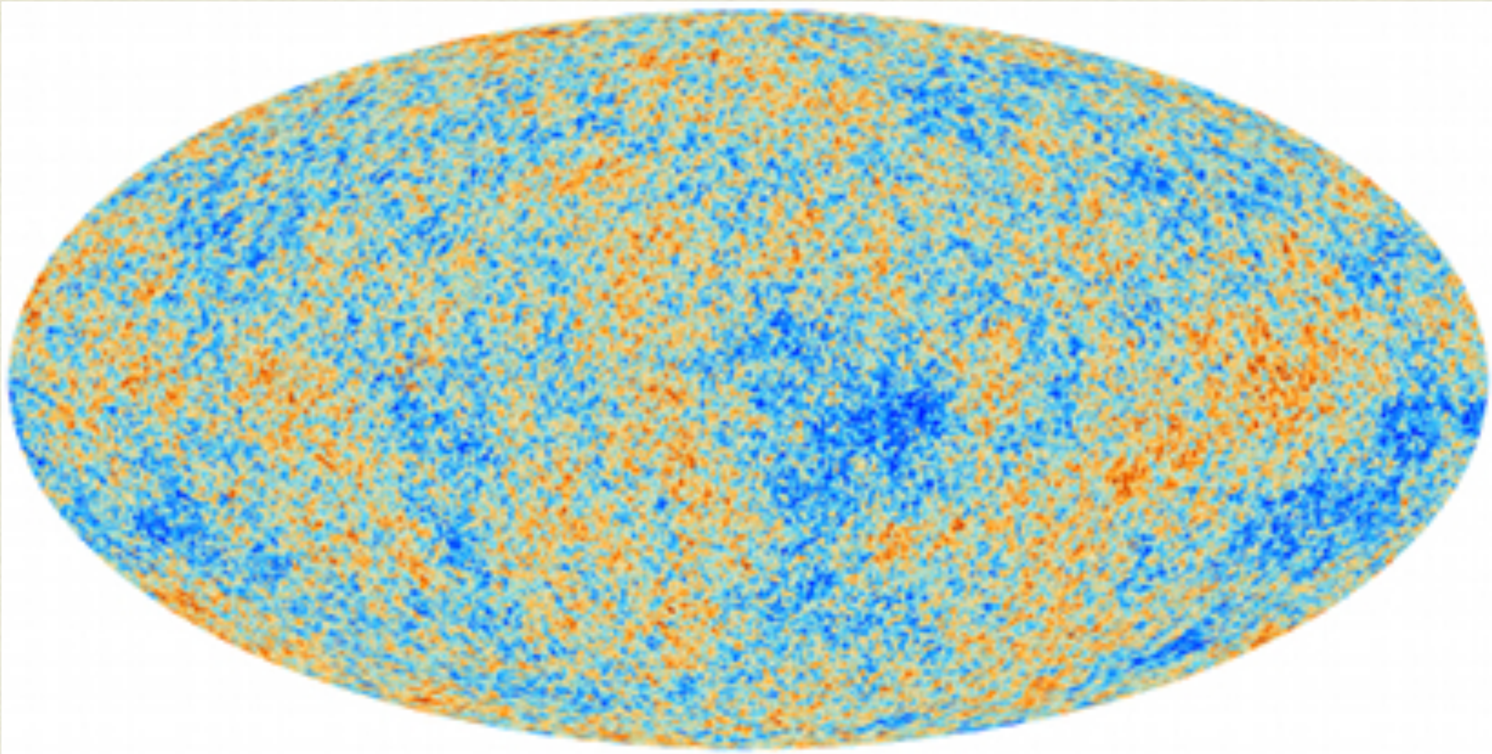
$$\begin{aligned} T^{\mu\nu} &= -\mathcal{L}g^{\mu\nu} + \frac{\partial\mathcal{L}}{\partial X}\partial^\mu\phi\partial^\nu\phi \\ &= -pg^{\mu\nu} + (p+\rho)u^\mu u^\nu, \quad u^\mu u_\mu = 1 \end{aligned}$$

$$p = \mathcal{L}, \ \rho = 2Xp_X - p, \ u^\mu = \frac{\partial^\mu\phi}{\sqrt{2X}}$$

OUTLINE

- Effects of DE perturbations on CMB and LSS
- Methods to deal with DE perturbations with parameterization
- Initial conditions of DE perturbations
- Anisotropic rotation angle induced by DE coupling

EFFECTS OF DE PERTURBATIONS ON CMB AND LSS



$$\frac{\Delta T(\hat{\mathbf{n}})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{\mathbf{n}})$$

$$\langle a^*_{l'm'} a_{lm} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

Generation of CMB anisotropies

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = [\frac{1}{4}\delta_\gamma + \Psi + \vec{v}\cdot\hat{\mathbf{n}}]_* + \int_*^{\eta_0} d\eta(\Phi' + \Psi')(\eta, \Delta\eta\hat{\mathbf{n}})$$

Sachs-Wolf

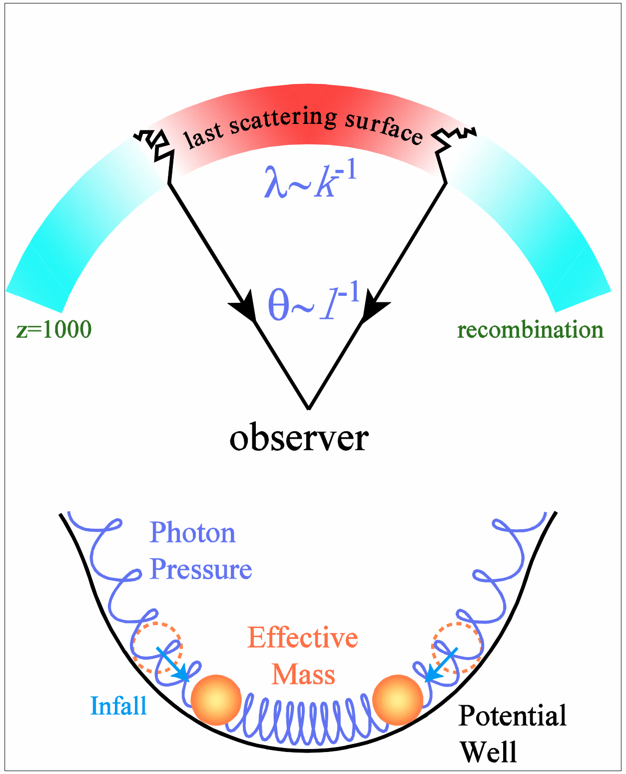
Doppler

Integrated Sachs-Wolf
(ISW)

$$\Delta\eta = \eta_0 - \eta_*$$

DE dominates the universe at very late time and contributes to CMB anisotropy through ISW effect.

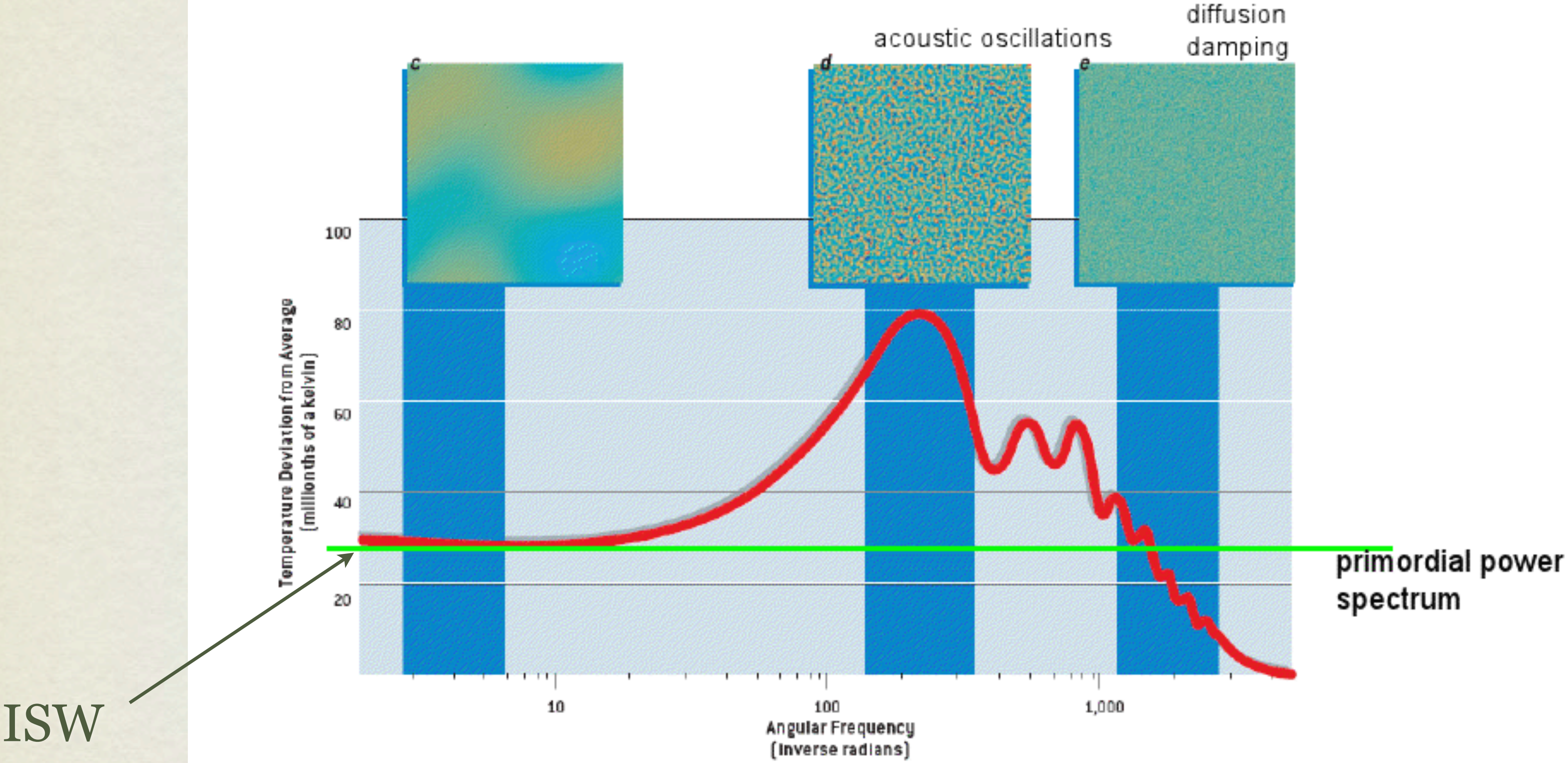
Acoustic physics



Other secondary effects: weak lensing, SZ, reionization, ...

CMB temperature power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)

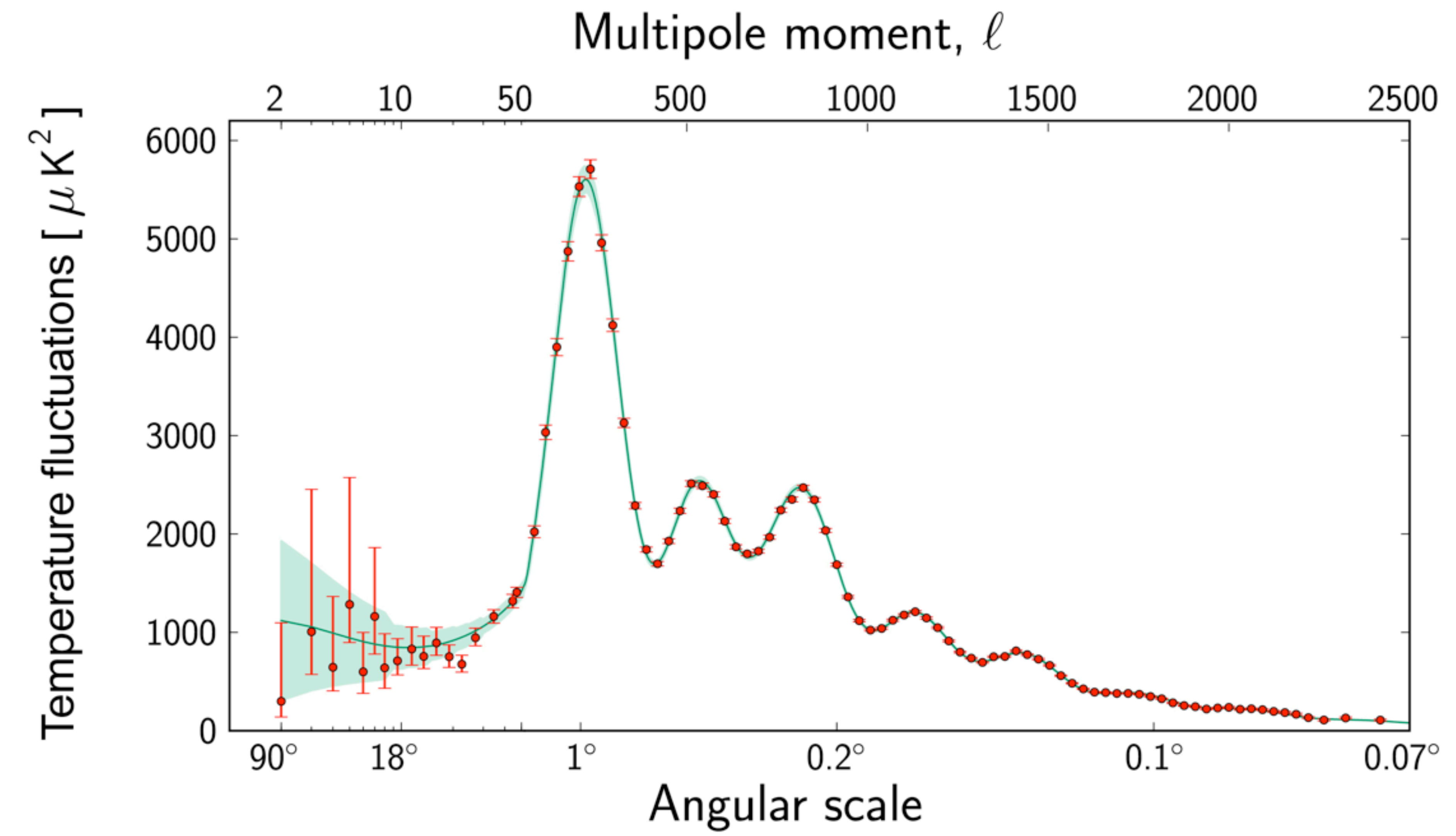
Without anisotropic stress $\Psi = \Phi$

$$\Phi'' + 3\mathcal{H}\Phi' + (-8\pi G a^2 p_D)\Phi = 4\pi G a^2 \delta p_D$$

Contributions from DE background

Contribution from DE perturbations

$$(\frac{\Delta T}{T})^{ISW} = 2\int_*^{\eta_0} d\eta \Phi'$$



fitted by ΛCDM

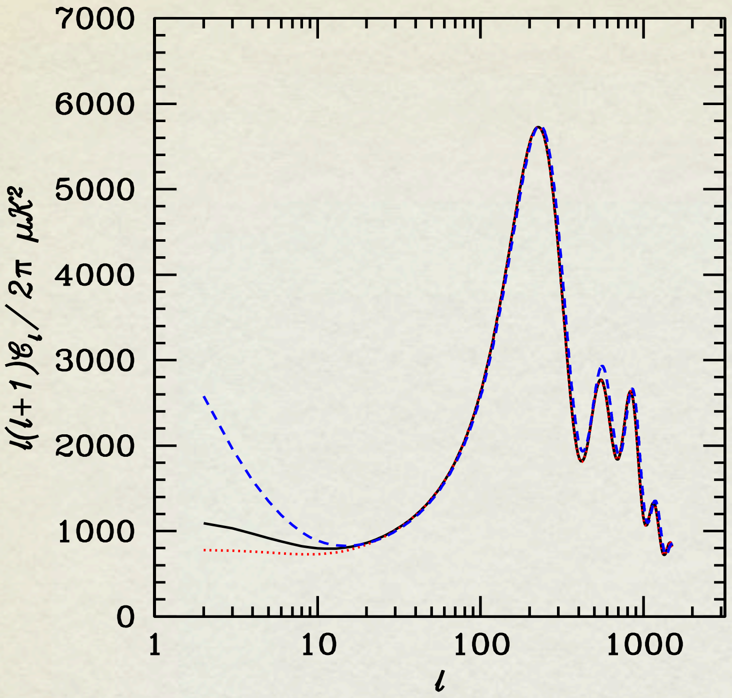


Figure 2. CMB angular power spectra for different dark energy models with *no* perturbations. The solid line is for a ΛCDM model, the dotted line for a model with $w = -0.6$ and dashed line $w = -2.0$. The parameters Ω_c, Ω_b and H_0 are adjusted to

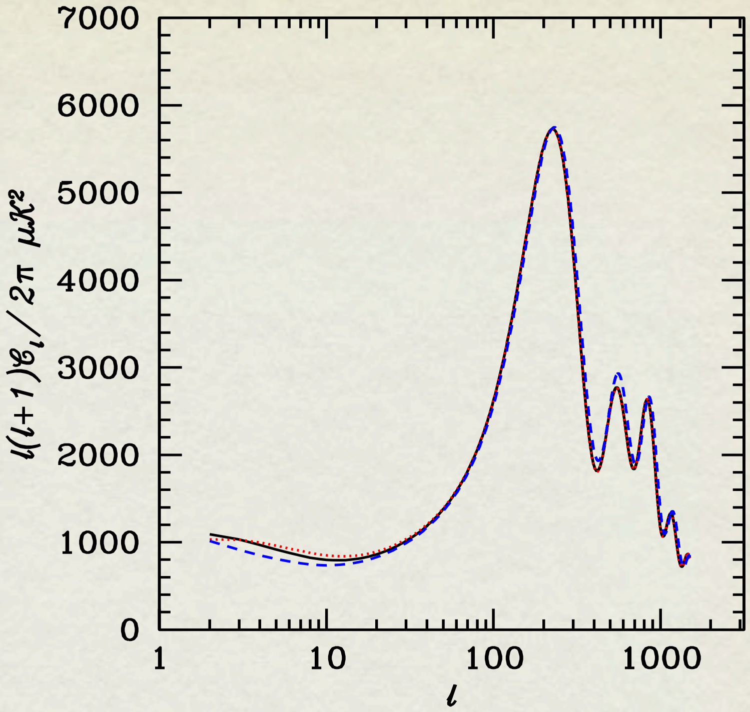


Figure 3. CMB angular power spectra for them dark energy models as in Fig. 2, but *with* dark energy perturbations.

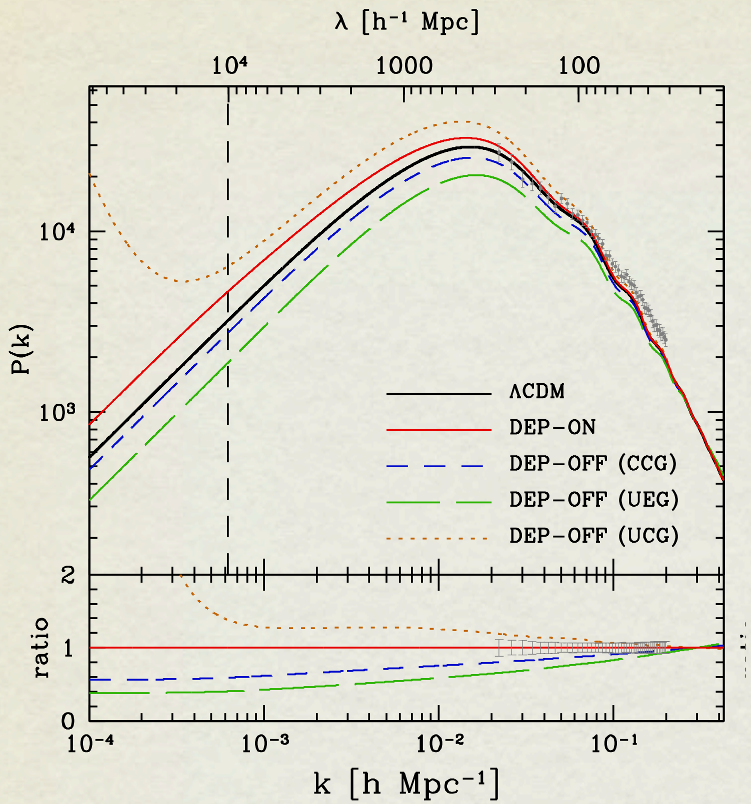
DE perturbations are ignored

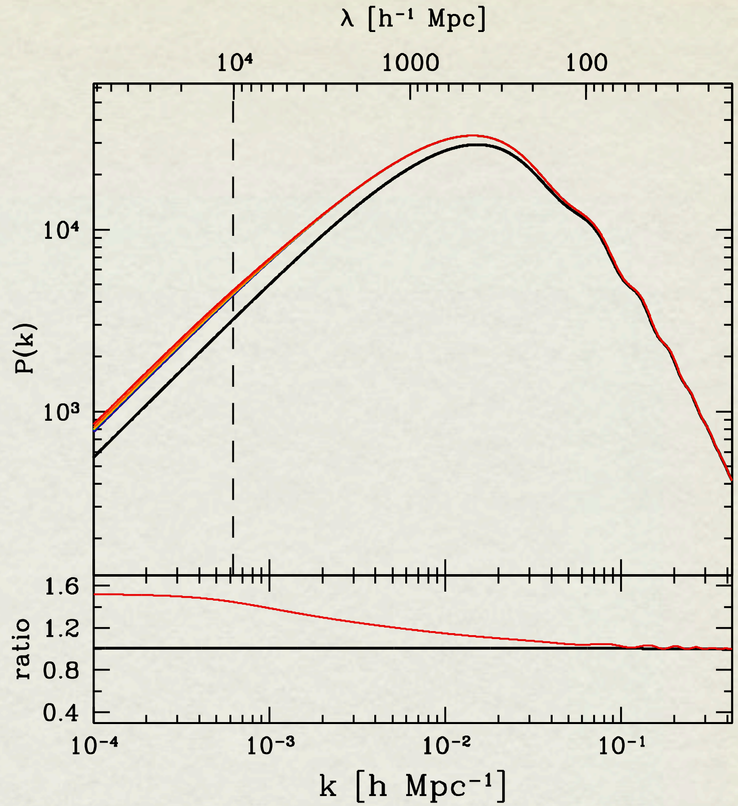
DE perturbations are included

DE perturbations increase parameter degeneracies

Constant w and based on quintessence and phantom models

J.Weller & A.M.Lewis, astro-ph/0307104





Without DE perturbations

With DE perturbations

Park, Hwang, Lee & Noh, PRL (2009)

METHODS TO DEAL WITH DE PERTURBATIONS WITH PARAMETERIZATION

The background contribution of DE can be described by two parameters w_0 and w_a

Q: How to include DE perturbations consistently in data analyses without referring to specific models? And with as few as possible extra parameters beyond w_0 and w_a?

Zhao,Xia,Li,Feng,Zhang, astro-ph/0507482, PRD(2005)

Perturbation Equations

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)\delta_{ij}dx^i dx^j]$$

$$\begin{aligned}
-k^2\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) &= 4\pi G a^2 \delta\rho \ , \\
k^2(\Psi' + \mathcal{H}\Phi) &= 4\pi G a^2 (\rho + p)\theta \ , \\
\Psi'' + \mathcal{H}(2\Psi' + \Phi') + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{k^2}{3}(\Psi - \Phi) &= 4\pi G a^2 \delta p \ , \\
k^2(\Psi - \Phi) &= 12\pi G a^2 (\rho + p)\sigma
\end{aligned}$$

Some definitions of the notations C.P. Ma & E. Bertschinger, ApJ (1995)

$$(\rho + p)\theta = ik^i \delta T^0_i \, .$$

σ relates to the anisotropic stress through the relation

$$(\rho + p)\sigma = \hat{k}_i \hat{k}_j (\delta T^i_j - 1/3 \delta^i_j \delta T^l_l) \, ,$$

$$\begin{aligned}
\rho\delta &= \sum_i \rho_i \delta_i \ , \\
(\rho + p)\theta &= \sum_i (\rho_i + p_i)\theta_i \ , \\
\delta p &= \sum_i \delta p_i \ .
\end{aligned}$$

$$\sigma = 0 \rightarrow \Psi = \Phi$$

$$\delta[T^{\mu\nu}_{(i);\nu}] = 0 \longrightarrow \begin{aligned} \delta_i' &= -(1+w_i)(\theta_i - 3\Phi') - 3\mathcal{H}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right), \\ \theta_i' &= -\mathcal{H}(1-3w_i)\theta_i - \frac{w_i'}{1+w_i}\theta_i + k^2\left(\frac{\delta p_i/\rho_i}{1+w_i} + \Phi\right) \end{aligned}$$

To close the equations, we need the form of $\delta p_i = f(\delta_i, \theta_i, \Phi)$
For barotropic fluid $p = p(\rho)$

$$\delta p = \frac{p'}{\rho'}\delta\rho = c_a^2 \delta\rho$$

For k-essence field $p = p(\phi, X)$, $\rho = \rho(\phi, X)$

Model dependent

$$\begin{aligned} \theta &= \frac{k^2}{\phi'}\delta\phi \\ \delta p &= \frac{p_X}{\rho_X}\delta\rho + 3\mathcal{H}(1+w)\frac{\rho}{k^2}(\frac{p_X}{\rho_X} - c_a^2)\theta \end{aligned}$$

Model independent perturbation equations

General form of scalar perturbations of the metric

$$ds^2 = a^2(\eta)\{(1+2A)d\eta^2 - 2B_{,i}dx^i d\eta - [(1-2\psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j\}$$

Gauge transformation

$$\begin{aligned} \eta &\rightarrow \tilde{\eta} = \eta + \xi^0 \quad \text{and} \\ x^i &\rightarrow \tilde{x}^i = x^i + \xi^{,i}\,, \end{aligned}$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - L_\xi g_{\mu\nu}$$

$$\begin{aligned} &A \rightarrow \tilde{A} = A - \mathcal{H}\xi^0 - \xi^{0'}\,, \quad B \rightarrow \tilde{B} = B + \xi^0 - \xi' \\ &\psi \rightarrow \tilde{\psi} = \psi + \mathcal{H}\xi^0\,, \qquad E \rightarrow \tilde{E} = E - \xi\,, \end{aligned}$$

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} - L_\xi T^{\mu\nu}$$

$$\begin{aligned} \widetilde{\delta\rho} &= \delta\rho - \rho'\xi^0 \\ \widetilde{\delta p} &= \delta p - p'\xi^0 \\ \tilde{\theta} &= \theta - k^2\xi^0 \end{aligned}$$

Gauge invariant $\quad \Phi = A + \frac{1}{a}[a(B-E')]',\ \Psi = \psi - \mathcal{H}(B-E')$

Conformal Newtonian gauge $\quad B = E = 0,\ \Phi = A,\ \Psi = \psi$

Definition of sound speed

Adiabatic sound speed $$c_a^2 = \frac{p'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Comoving gauge $$\tilde{\theta} = 0,\ c_s^2 \equiv \frac{\widetilde{\delta p}}{\widetilde{\delta \rho}}$$ True sound speed

Transform to arbitrary gauge

$$\xi^0 = \frac{\theta}{k^2},\ c_s^2 = \frac{\delta p - p'\xi^0}{\delta\rho - \rho'\xi^0} = \frac{\delta p - p'\theta/k^2}{\delta\rho - \rho'\theta/k^2}$$

$$\delta p = c_s^2\delta\rho + \frac{\rho'}{k^2}(c_a^2 - c_s^2)\theta = c_s^2\delta\rho + \frac{3\mathcal{H}(\rho+p)}{k^2}(c_s^2 - c_a^2)\theta$$

Barotropic fluid $$c_s^2 = c_a^2$$

Scalar field $$\tilde{\theta} = 0 \to \widetilde{\delta\phi} = 0$$

$$p = p(\phi, X),\ \rho = \rho(\phi, X)$$

$$\theta = \frac{k^2}{\phi'}\delta\phi$$

$$\widetilde{\delta p} = p_X \widetilde{\delta X},\ \widetilde{\delta\rho} = \rho_X \widetilde{\delta X}$$

$$c_s^2 = \frac{\widetilde{\delta p}}{\widetilde{\delta\rho}} = \frac{p_X}{\rho_X} \neq c_a^2$$

$$\delta p = c_s^2 \delta\rho + \frac{3\mathcal{H}(\rho + p)}{k^2}(c_s^2 - c_a^2)\theta$$

Normal quintessence and phantom

$$p = \pm X - V,\ \rho = \pm X + V,\ c_s^2 = 1$$

$$\delta_i' = -(1+w_i)[1+\frac{9\mathcal{H}^2}{k^2}(c_{si}^2 - c_{ai}^2)]\theta_i - 3\mathcal{H}(c_{si}^2 - w_i)\delta_i + 3(1+w_i)\Phi'$$

$$\theta_i' = \mathcal{H}(3c_{si}^2 - 1)\theta_i + k^2(\frac{c_{si}^2}{1+w_i}\delta_i + \Phi)$$

$$\Big\downarrow \quad \mathcal{H} = 0,\ w_i' \simeq 0$$

$$\delta_i'' + c_{si}^2 k^2 \delta_i = 3(1+w_i)(\Phi'' + k^2/3\Phi)$$

$c_{si}^2 \geq 0$ to guarantee the stability

$$\delta_i' = -(1+w_i)[1+\frac{9\mathcal{H}^2}{k^2}(c_{si}^2 - c_{ai}^2)]\theta_i - 3\mathcal{H}(c_{si}^2 - w_i)\delta_i + 3(1+w_i)\Phi'$$

$$\theta_i' = \mathcal{H}(3c_{si}^2 - 1)\theta_i + k^2(\frac{c_{si}^2}{1+w_i}\delta_i + \Phi)$$

$$\frac{k^2}{a^2}\Phi = -4\pi G\sum_i \rho_i[\delta_i + \frac{3\mathcal{H}}{k^2}(1+w_i)\theta_i]$$

$$\begin{aligned} -k^2\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) &= 4\pi G a^2 \delta\rho\ , \\ k^2(\Psi' + \mathcal{H}\Phi) &= 4\pi G a^2 (\rho + p)\theta \end{aligned}$$

Effective energy density perturbation

Simple universe model: i=m, DE

For DE with single degree of

freedom , only one extra parameter: c_{sD}^2

However......

$$\delta_i' = -(1+w_i)[1+\frac{9\mathcal{H}^2}{k^2}(c_{si}^2 - c_{ai}^2)]\theta_i - 3\mathcal{H}(c_{si}^2 - w_i)\delta_i + 3(1+w_i)\Phi'$$

$$\theta_i' = \mathcal{H}(3c_{si}^2 - 1)\theta_i + k^2(\frac{c_{si}^2}{1+w_i}\delta_i + \Phi)$$

$$c_{aD}^2 = w_D - \frac{w_D'}{3\mathcal{H}(1+w_D)}$$

$$1 + w_D = 0,\ \ w_D' \neq 0,\ \ \text{is a singularity}$$

1, Forbid crossing -1 a priori, leads to biased results;

2, Models allowing crossing require more degrees of freedom. Constraining multi components DE is cumbersome and model dependent.

Method: divide the parameter space into three regions

Zhao,Xia,Li,Feng,Zhang, PRD(2005)

$1 + w_D > \epsilon \; or \; 1 + w_D < -\epsilon, \;$ regular, can be considered as single component

$-\epsilon < 1 + w_D < \epsilon, \; \delta_D|_+ = \delta_D|_-, \; \theta_D|_+ = \theta_D|_-$

$\epsilon > 0$ is a small parameter

c_{sD}^2 is not constrained significantly by data

This approach approximated the results obtained in two-field quintom models accurately for $\epsilon \sim 10^{-5}$

The way to get matching conditions

Mingzhe Li, Y.Cai, H.Li, R.Brandenberger, X.Zhang, PLB (2011)

In a perturbed universe $w_D = w_D(\eta) + \delta w_D(\eta,\ x^i)$

$$\delta w_D \to \widetilde{\delta w}_D = \delta w_D - w'_D \xi^0 \ .$$

We will use the temporal gauge to obtain the matching conditions. In this gauge, the matching hypersurfa coincides with $\tilde{\eta} =$ const. and the equation of this hypersurface

$$\widetilde{w}_D(\tilde{\eta},\ \tilde{x}^i) \ = \ \text{const.}$$

implies

$$\widetilde{\delta w}_D \ = \ 0 \, .$$

Hence the time shift is

$$\xi^0 \ = \ \frac{\delta w_D}{w'_D} \ ,$$

but ξ remains arbitrary. The induced 3-metric of this hypersurface and its extrinsic curvature are expressed as

$$\begin{aligned} q_{ij} &= a^2[(1-2\tilde{\psi})\delta_{ij} + 2\tilde{E}_{,ij}] \ , \\ K_{ij} &= \frac{q_{ij}}{a}(\mathcal{H} - \mathcal{H}\tilde{A} - \tilde{\psi}') + \frac{1}{a}(\tilde{E}' - \tilde{B})_{,ij} \ , \end{aligned}$$

$$[\tilde{\psi}]_\pm = [\tilde{E}]_\pm = 0$$
$$[\mathcal{H}\tilde{A} + \tilde{\psi}']_\pm = [\tilde{E}' - \tilde{B}]_\pm = 0$$
$$[\tilde{\psi}]_\pm \equiv \tilde{\psi}_+ - \tilde{\psi}_- \text{ etc.}$$

To arbitrary gauge:

$$[\psi + \mathcal{H}\frac{\delta w_D}{w'_D}]_\pm = 0 \ ,$$
$$[E - \xi]_\pm = 0 \ ,$$
$$[\mathcal{H}A + \psi' + (\mathcal{H}' - \mathcal{H}^2)\frac{\delta w_D}{w'_D}]_\pm = 0 \ ,$$
$$[E' - B + \frac{\delta w_D}{w'_D}]_\pm = 0 \ .$$

To conformal Newtonian gauge:

$$[\Psi]_\pm = 0 \ ,$$
$$[\frac{\delta w_D}{w'_D}]_\pm = 0 \ ,$$
$$[\mathcal{H}\Phi + \Psi' + (\mathcal{H}' - \mathcal{H}^2)\frac{\delta w_D}{w'_D}]_\pm = 0 \ .$$

$$\frac{k^2}{a^2}\Phi = -4\pi G\{\rho_D[\delta_D + (1+w_D)\frac{\mathcal{H}}{k^2}\theta_D] + \rho_m[\delta_m + (1+w_m)\frac{\mathcal{H}}{k^2}\theta_m]\}$$

$$w_D \simeq -1$$

$$[\delta_D]_\pm = 0$$

$$\frac{\delta w_D}{w'_D} = \frac{1}{w'_D}(\frac{\delta p_D}{\rho_D} - w_D\delta_D) = \frac{c_{sD}^2 - w_D}{w'_D}[\delta_D + \frac{3\mathcal{H}(1+w_D)}{k^2}\theta_D] + \frac{\theta_D}{k^2}$$

$$\frac{\delta w_D}{w'_D} = \frac{c_{sD}^2 + 1}{w'_D}\delta_D + \frac{1}{k^2}\theta_D$$

$$[\theta_D]_\pm = 0$$

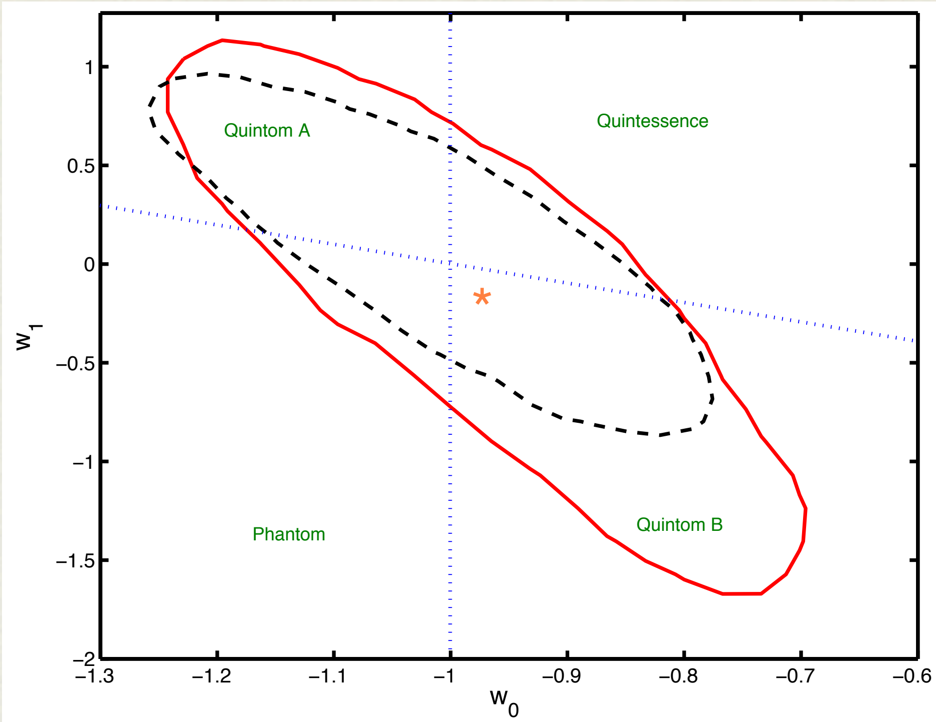


FIG. 1: Two dimensional constraints on (w_0, w_1) from current observations of CMB + SNIa + BAO. The red solid and black dash lines represent the 2 σ limits for the two cases with and without dark energy perturbations, respectively. The star represents the best fit value.

Parametrized Post-Friedmann (PPF) approach

W.Fang, W.Hu, A.Lewis, arXiv:0808.3125

The same motivation.

Replace $\delta p = c_s^2 \delta\rho + \frac{3\mathcal{H}(\rho + p)}{k^2}(c_s^2 - c_a^2)\theta$

with an assumed relationship between $(\rho_D + p_D)\theta_D$ and $\rho_m \theta_m$ on large scales and a transition scale under which the effective energy density perturbation of dark energy can be neglected. Match the evolution of the metric for scales much larger and much smaller than the transition scale.

Mimic multi fields. Assuming smoothness of DE on small scales and adiabatic initial condition.

One exception: models correspond to imperfect fluids

Deffayet, Pujolas, Sawicki & Vikman, JCAP (2010)

Kobayashi, Yamaguchi &Yokoyama, PRL (2010)
Kamada, Kobayashi, Yamaguchi &Yokoyama, PRD (2011)

Li, Qiu, Cai & Zhang, JCAP (2012)

$$\mathcal{L} = K(\phi,\ X) + F(\phi,\ X)\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi = K + F\nabla_\mu X\nabla^\mu\phi\ .$$

EOM

$$K_\phi - 2XK_{X\phi} - K_X\Box\phi - K_{XX}\nabla_\mu X\nabla^\mu\phi + F[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi + R_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi]$$
$$+\ 2F_\phi(\nabla_\mu X\nabla^\mu\phi + 2X\Box\phi) + 2XF_{\phi X}\nabla_\mu X\nabla^\mu\phi + 4X^2F_{\phi\phi} + F_X(\Box\phi\nabla_\mu X\nabla^\mu\phi - \nabla_\mu X\nabla^\mu X) = 0$$

$$T^{\mu\nu} = -(K + F\nabla_\rho X\nabla^\rho\phi)g^{\mu\nu} + (K_X - 2XF_\phi - F\Box\phi)\nabla^\mu\phi\nabla^\nu\phi + F(\nabla^\mu X\nabla^\nu\phi + \nabla^\nu X\nabla^\mu\phi)$$

$$T^{\mu\nu} = -pg^{\mu\nu} + (\rho + p)u^\mu u^\nu + (2X)^{3/2}F(a^\mu u^\nu + a^\nu u^\mu)$$

$$p = K + F\nabla_\mu X\nabla^\mu\phi\ ,$$
$$\rho = -K + F\nabla_\mu X\nabla^\mu\phi + 2X(K_X - 2XF_\phi - F\Box\phi)$$

$$u^\mu = \nabla^\mu\phi/\sqrt{2X},\ a^\mu = u^\rho\nabla_\rho u^\mu$$

$$\phi(t,\vec{x}) = \phi(t) + \pi(t,\vec{x})$$

$$S^{(2)}(\pi) \; = \; \frac{1}{2}\int d^3x dt a^3 (A\dot{\pi}^2 - \frac{B}{a^2}\partial_i\pi\partial_i\pi + C\pi^2)$$

$$\begin{aligned}
A &= K_X + 2XK_{XX} - 6H\dot{\phi}(F + XF_X) - 8XF_\phi - 4X^2F_{X\phi} \;, \\
B &= K_X - 2(\ddot{\phi} + 2H\dot{\phi})F - 4XF_\phi - 2X\ddot{\phi}F_X \;, \\
C &= \frac{d}{dt}(6HXF_\phi + 2X\dot{\phi}F_{\phi\phi} - \dot{\phi}K_{X\phi}) + 3H(6HXF_\phi + 2X\dot{\phi}F_{\phi\phi} - \dot{\phi}K_{X\phi}) + 2X\ddot{\phi}F_{\phi\phi} + K_{\phi\phi}
\end{aligned}$$

$$c_s^2 = \frac{B}{A} = \frac{K_X - 2(\ddot{\phi} + 2H\dot{\phi})F - 4XF_\phi - 2X\ddot{\phi}F_X}{K_X + 2XK_{XX} - 6H\dot{\phi}(F + XF_X) - 8XF_\phi - 4X^2F_{X\phi}}$$

INITIAL CONDITIONS FOR DE PERTURBATIONS

Gauge invariant perturbations

$$\begin{aligned}\zeta_\alpha &= \frac{\delta_\alpha}{3(1+w_\alpha)} - \Phi \ , \\ \Delta_\alpha &= \frac{\rho_\alpha \delta_\alpha}{3} + \frac{\mathcal{H}}{k^2}(\rho_\alpha + p_\alpha)\theta_\alpha\end{aligned}$$

Equations

$$\zeta'_\alpha + 3\mathcal{H}(c^2_{s\alpha} - c^2_{a\alpha})\frac{\Delta_\alpha}{\rho_\alpha + p_\alpha} + \frac{k^2}{3\mathcal{H}}(\frac{\Delta_\alpha}{\rho_\alpha + p_\alpha} - \zeta_\alpha) = \frac{k^2}{3\mathcal{H}}\Phi \ ,$$

$$\Delta'_\alpha + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}})\Delta_\alpha - (\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}})(\rho_\alpha + p_\alpha)\zeta_\alpha = (\rho_\alpha + p_\alpha)[\Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}})\Phi]$$

$$\frac{k^2}{a^2}\Phi = -12\pi G \sum_\alpha \Delta_\alpha$$

On large scales $k << \mathcal{H}$

$$\zeta'_\alpha + 3\mathcal{H}(c^2_{s\alpha} - c^2_{a\alpha})\frac{\Delta_\alpha}{\rho_\alpha + p_\alpha} = 0 \ ,$$

$$\Delta'_\alpha + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_\alpha - (\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})(\rho_\alpha + p_\alpha)\zeta_\alpha = (\rho_\alpha + p_\alpha)[\Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Phi] \ ,$$

$$\sum_\alpha \Delta_\alpha = 0 \ .$$

Conserved \zeta

(1) barotropic fluids, e.g., cdm, radiation $c^2_{s\alpha} = c^2_{a\alpha} = p'_\alpha/\rho'_\alpha$

(2) single component universe, e.g., single field inflation $\sum_\alpha \Delta_\alpha = \Delta = 0$

Adiabatic perturbation $\zeta_\alpha = \zeta_r = \text{constant., for all } \alpha$

$$\zeta = \frac{1}{\rho + p} \sum_\alpha (\rho_\alpha + p_\alpha)\zeta_\alpha = \zeta_r$$

decaying mode

$$(\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})\zeta = \Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Phi \longrightarrow \Phi^{\rm adi} = C\frac{\mathcal{H}}{a^2} - \zeta_r(1 - \frac{\mathcal{H}}{a^2}\int \frac{a da}{\mathcal{H}(a)})$$

Isocurvature (entropic) perturbation

$$S_\alpha \equiv 3(\zeta_\alpha - \zeta_r) = \frac{\delta_\alpha}{1 + w_\alpha} - \frac{3}{4}\delta_r$$

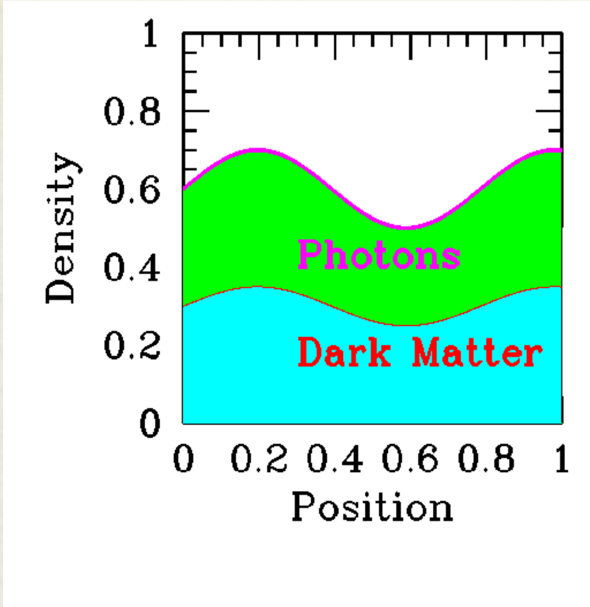
$$\zeta = \zeta_r + \frac{\rho_\alpha + p_\alpha}{3(\rho + p)} S_\alpha \;\; = \zeta_r + \frac{\xi_\alpha}{\rho + p}$$

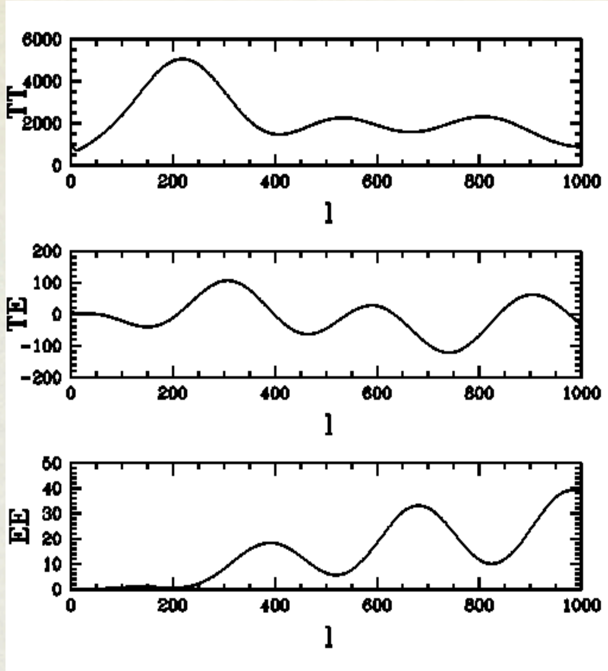
$$\xi_\alpha \equiv (\rho_\alpha + p_\alpha) S_\alpha / 3$$

$$\xi'_\alpha + 3\mathcal{H}(1 + c^2_{a\alpha})\xi_\alpha + 3\mathcal{H}(c^2_{s\alpha} - c^2_{a\alpha})\Delta_\alpha = 0 \;,$$
$$\Delta'_\alpha + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_\alpha + (\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})(1 - \frac{\rho_\alpha + p_\alpha}{\rho + p})\xi_\alpha = 0$$

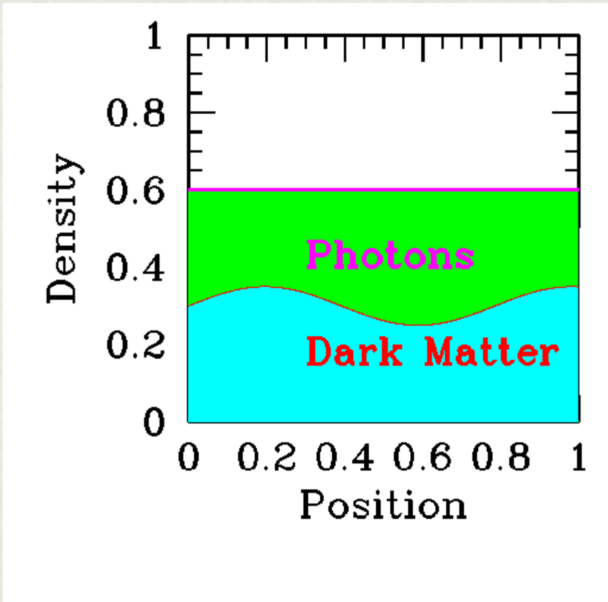
$$\Phi = \Phi^{\rm adi} - 4\pi G \frac{\mathcal{H}}{a^2} \int \xi_\alpha \frac{a^3}{\mathcal{H}(a)^3} da$$

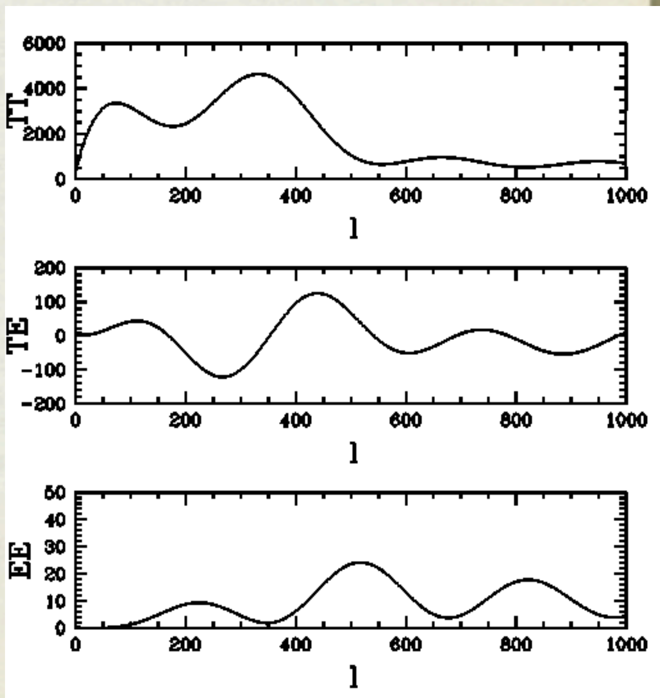
Adiabatic perturbation





DM isocurvature perturbation





$$\zeta = \zeta_r + \frac{\xi_\alpha}{\rho + p}$$

If \alpha-species is subdominant $\quad (\rho_\alpha + p_\alpha)/(\rho + p) \to 0$

$$\Phi \simeq \Phi^{\rm adi}$$

$$\xi'_\alpha + 3\mathcal{H}(1 + c^2_{a\alpha})\xi_\alpha + 3\mathcal{H}(c^2_{s\alpha} - c^2_{a\alpha})\Delta_\alpha = 0$$

$$\Delta'_\alpha + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_\alpha + (\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})\xi_\alpha = 0 \ .$$

General initial conditions for DE perturbations:

$$\mathcal{X}_i = \begin{cases} \zeta_r & \text{adiabatic} \ , \\ S_D & \text{DE isocurvature} \ . \end{cases}$$

mixed adiabatic and DE isocurvature

Primordial spectra $\langle \mathcal{X}_i(\mathbf{k}) \mathcal{X}_j^*(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{ij}(k) \delta(\mathbf{k} - \mathbf{k}').$ $\mathcal{P}_{ij} = A_{ij} (\frac{k}{k_0})^{n_{ij}-1}$

$$A_{ij} = \begin{pmatrix} A_{\rm adi} & \sqrt{A_{\rm adi} A_{\rm iso}} \cos\Delta \\ \sqrt{A_{\rm adi} A_{\rm iso}} \cos\Delta & A_{\rm iso} \end{pmatrix}$$

$$\cos\Delta = \frac{A_{\rm adi,iso}}{\sqrt{A_{\rm adi} A_{\rm iso}}}$$ correlation between adiabatic and isocurvature

CMB spectrum $\frac{\delta T}{T} = (\frac{\delta T}{T})_{\rm adi} + (\frac{\delta T}{T})_{\rm iso},$

$$C_l = A_{\rm adi} \hat{C}_l^{\rm adi} + A_{\rm iso} \hat{C}_l^{\rm iso} + 2\sqrt{A_{\rm adi} A_{\rm iso}} \cos\Delta \hat{C}_l^{\rm adi,iso} \qquad \hat{C}_l^{ij} = \frac{4\pi}{2l+1} \int d\ln k (\frac{k}{k_0})^{n_{ij}-1} \Theta_l^i(k) \Theta_l^j(k)$$

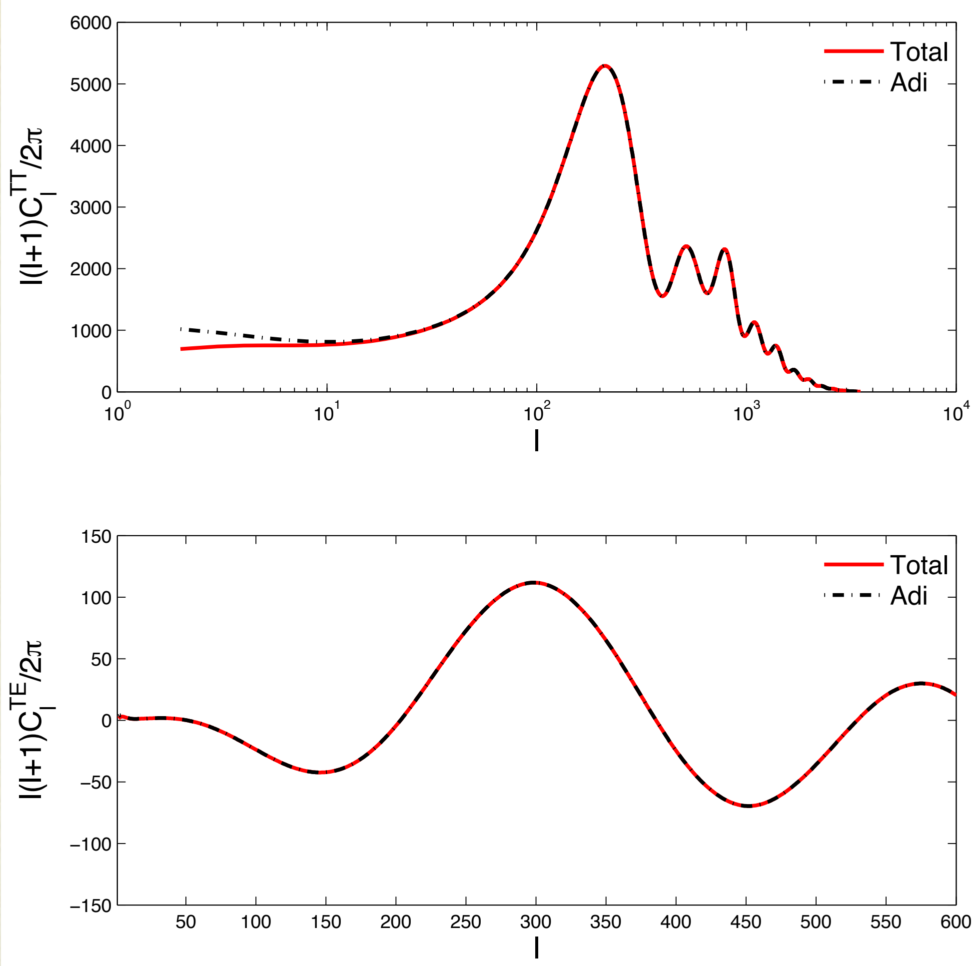
Matter power spectrum

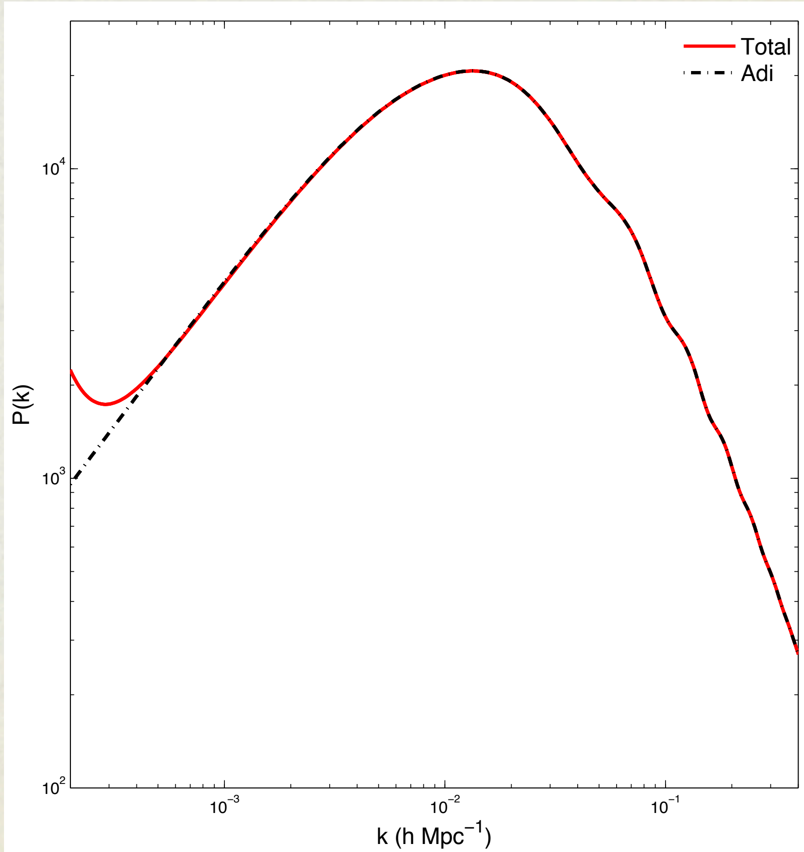
$$P(k) = A_{\rm adi} \hat{P}^{\rm adi}(k) + A_{\rm iso} \hat{P}^{\rm iso}(k) + 2\sqrt{A_{\rm adi} A_{\rm iso}} \cos\Delta \hat{P}^{\rm adi,iso}(k)$$

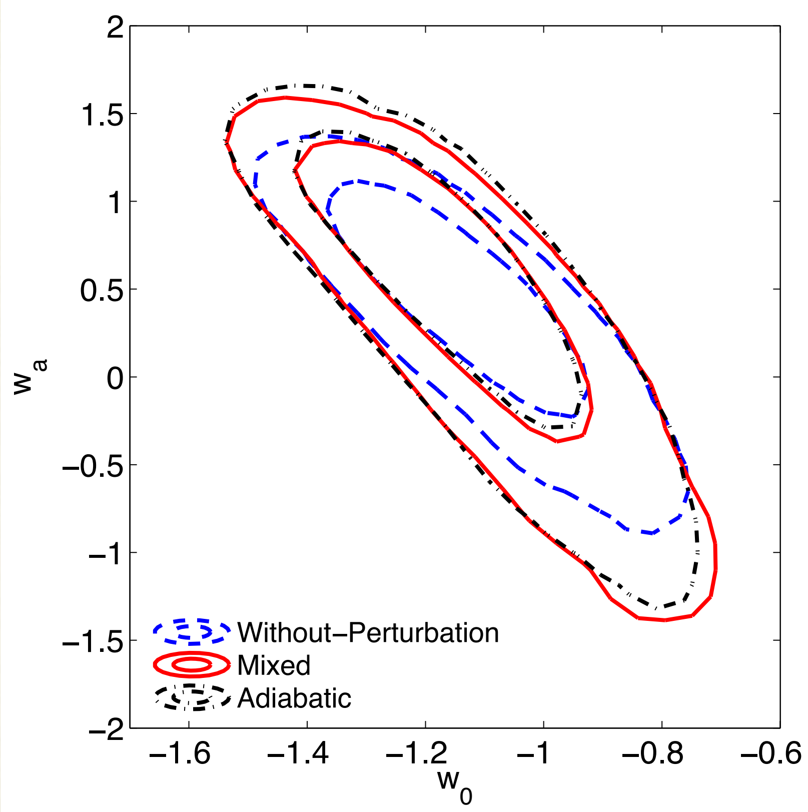
$$\hat{P}^{ij}(k) = (\frac{k}{k_0})^{n_{ij}-1} T^i(k) T^j(k),$$

Fully anti-correlation

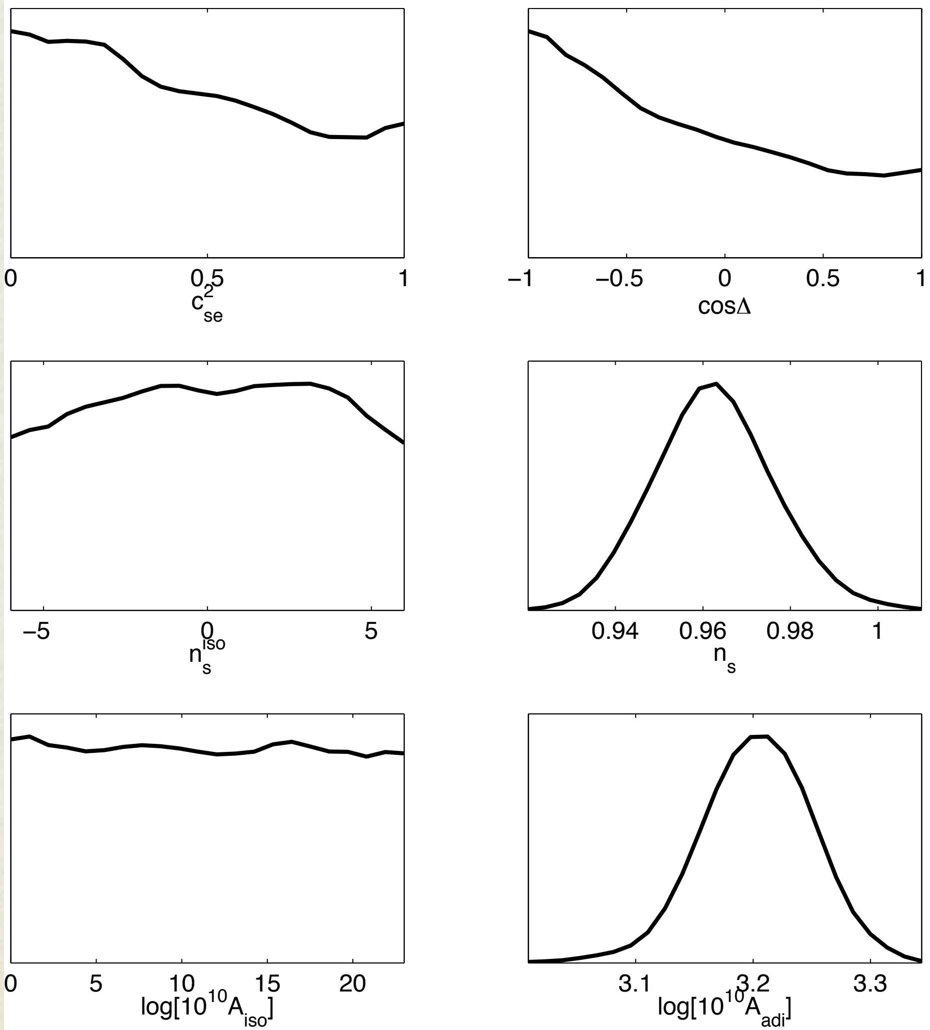
$$\cos\Delta = -1$$







Data are not sensitive to DE isocurvature perturbations



SUMMARY (I)

- DE perturbations have important effects and usually increase the parameter degeneracies
- Singularity w_D=-1 needs to be handled carefully
- Current data are not sensitive to the sound speed and initial perturbations of DE
- Models corresponds to imperfect fluid need more studies

ANISOTROPIC ROTATION ANGLE INDUCED BY DE COUPLING

“Quintessence and the rest of the world”, S. Carroll, PRL (1998)

Quintessence $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

Nearly massless $m_\phi \sim \sqrt{V_{\phi\phi}} < H_0 \sim 10^{-33}\text{eV}$

Hypothetical couplings (besides the gravity) to SM particles:
1, direct coupling

$$c\frac{\phi}{M}\mathcal{L}(\bar{\psi}\psi, F_{\rho\sigma}F^{\rho\sigma}, G_{\rho\sigma}G^{\rho\sigma}, ...)$$

A. Long range force, violates equivalence principle, constrained to $c \leq 10^{-4}(M/M_{pl})$;

B. Instability under quantum corrections, $\delta m_\phi \simeq \frac{\Lambda^2}{4\pi M} \sim 10^{-7}\text{eV} >> m_\phi$, $M \sim M_{pl},\ \Lambda \sim \Lambda_{ew}$.

2, derivative coupling, pseudo-Goldstone originated from $U(1)$ symmetry breaking

$$\frac{c}{M}\partial_\mu\phi\mathcal{O}^\mu(\psi, F_{\rho\sigma}, G_{\rho\sigma}, ...)$$

A. shift symmetry $\phi \to \phi + const.$, guarantees the flatness of the potential;

B. propagates spin-dependent force, short range, much weaker constraint from astrophysics $M \geq 10^{10}\mathrm{Gev}, PDG$.

Derivative coupling to photons

$$\frac{c}{M}\partial_\mu\phi A_\nu\widetilde{F}^{\mu\nu} \to -\frac{c}{2M}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

$$\Big\downarrow \text{ Rotation of polarization}$$

Rotation angle $\quad \chi = \frac{c}{M}\Delta\phi,\ \Delta\phi = \phi(\eta,\ \vec{x})|_{source} - \phi(\eta,\ \vec{x})|_{receiver}$

Change of Stokes parameters $\quad (\tilde{Q} \pm i\tilde{U}) = \exp{(\pm i2\chi)}(Q \pm iU)$

Applied to CMB, source=last scattering surface

$$\begin{aligned} T(\hat{\boldsymbol{n}}) &= \sum_{lm} a_{T,lm} Y_{lm}(\hat{\boldsymbol{n}}) \\ (Q \pm iU)(\hat{\boldsymbol{n}}) &= \sum_{lm} a_{\pm 2,lm} \; {}_{\pm 2}Y_{lm}(\hat{\boldsymbol{n}}) \; . \end{aligned}$$

E/B decomposition

$$(Q \pm iU)(\hat{\mathbf{n}}) = -\sum_{lm} (a_{E,lm} \pm i a_{B,lm}) \; {}_{\pm 2}Y_{lm}(\hat{\mathbf{n}})$$

$$a_{E,lm} \pm i a_{B,lm} = -\int d\Omega \; {}_{\pm 2}Y^*_{lm}(\hat{\mathbf{n}})(Q \pm iU)(\hat{\mathbf{n}})$$

Power spectra $\langle a^*_{X',l'm'} a_{X,lm} \rangle = C_l^{X'X} \delta_{l'l} \delta_{m'm} \qquad X, X' = T, E, B$

Rotated polarization

$$\begin{aligned} \tilde{a}_{E,lm} \pm i \tilde{a}_{B,lm} &= -\int d\Omega \; {}_{\pm 2}Y^*_{lm}(\hat{\mathbf{n}})(\tilde{Q} \pm i\tilde{U})(\hat{\mathbf{n}}) \\ &= -\int d\Omega \; {}_{\pm 2}Y^*_{lm}(\hat{\mathbf{n}}) e^{\pm 2i\chi(\hat{\mathbf{n}})} (Q \pm iU)(\hat{\mathbf{n}}) \end{aligned}$$

Isotropic rotation angle $\chi(\hat{\mathbf{n}}) = \bar{\chi} = \frac{c}{M}[\phi(\eta_*) - \phi(\eta_0)]$

$$\tilde{a}_{E,lm} \pm i\tilde{a}_{B,lm} = e^{\pm 2i\bar{\chi}}(a_{E,lm} \pm i a_{B,lm})$$

$$\begin{aligned}
\tilde{C}_l^{TT} &= C_l^{TT} \ , \\
\tilde{C}_l^{TE} &= C_l^{TE}\cos(2\bar{\chi}) \ , \\
\tilde{C}_l^{TB} &= C_l^{TE}\sin(2\bar{\chi}) \ , \\
\tilde{C}_l^{EE} &= C_l^{EE}\cos^2(2\bar{\chi}) + C_l^{BB}\sin^2(2\bar{\chi}) \ , \\
\tilde{C}_l^{BB} &= C_l^{EE}\sin^2(2\bar{\chi}) + C_l^{BB}\cos^2(2\bar{\chi}) \ , \\
\tilde{C}_l^{EB} &= \frac{1}{2}\sin(4\bar{\chi})(C_l^{EE} - C_l^{BB})
\end{aligned}$$

Cosmic Birefringence, CPT Violation

Feng,Li,Xia,Chen,Zhang, PRL (2006)

Li & Zhang, PRD (2008)

$$\chi(\hat{\mathbf{n}}) = \frac{c}{M}[\phi(\eta_*, \vec{x}_*) - \phi(\eta_0)] = \frac{c}{M}[\phi(\eta_*, (\eta_0 - \eta_*)\hat{\mathbf{n}}) - \phi(\eta_0)] = \bar{\chi} + \delta\chi(\hat{\mathbf{n}})$$

$$\phi(\eta_*, \Delta\eta\hat{\mathbf{n}}) = \phi(\eta_*) + \delta\phi(\eta_*, \hat{\mathbf{n}})$$

DE perturbation at last scattering surface

induced anisotropic rotation angle

$$\delta\chi(\hat{\mathbf{n}}) = \frac{c}{M}[\phi(\eta_*, \Delta\eta\hat{\mathbf{n}}) - \phi(\eta_*)] = \frac{c}{M}\delta\phi(\eta_*, \hat{\mathbf{n}})$$

Probing anisotropic rotation angle is helpful to investigate the dynamics of \phi in more detail

$$\delta\chi(\hat{\boldsymbol{n}}) = \sum_{lm} b_{lm} Y_{lm}(\hat{\boldsymbol{n}}) \qquad \langle b^*_{l'm'} b_{lm} \rangle = C_l^\chi \delta_{l'l}\delta_{m'm}$$

$$C_l^\chi = \frac{4\pi c^2}{M^2}\int d\ln k \mathcal{P}_\phi(k,\ \eta_*) j_l^2(k\Delta\eta)$$

Angular spectrum of rotation angle

New distortions due to anisotropic rotation

(up to second order)

$$
\begin{aligned}
(\tilde{Q} \pm i\tilde{Q}) &= e^{\pm 2i(\bar{\chi}+\delta\chi)}(Q \pm iU) \\
&= e^{\pm 2i\bar{\chi}}(1 \pm 2i\delta\chi - 2\delta\chi^2 + ...)(Q \pm iU)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{C}_l^{TE} &= C_l^{TE}\cos(2\bar{\chi})(1-2\langle\delta\chi^2\rangle)\ , \\
\tilde{C}_l^{TB} &= C_l^{TE}\sin(2\bar{\chi})(1-2\langle\delta\chi^2\rangle)\ , \\
\tilde{C}_l^{EE} &= [C_l^{EE}\cos^2(2\bar{\chi}) + C_l^{BB}\sin^2(2\bar{\chi})](1-4\langle\delta\chi^2\rangle) \\
&+ \sum_{l_1 l_2}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi}\{[1+(-1)^{L+1}\cos(4\bar{\chi})]C_{l_1}^{EE} + [1+(-1)^{L}\cos(4\bar{\chi})]C_{l_1}^{BB}\} \\
\tilde{C}_l^{BB} &= [C_l^{EE}\sin^2(2\bar{\chi}) + C_l^{BB}\cos^2(2\bar{\chi})](1-4\langle\delta\chi^2\rangle) \\
&+ \sum_{l_1 l_2}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi}\{[1+(-1)^{L}\cos(4\bar{\chi})]C_{l_1}^{EE} + [1+(-1)^{L+1}\cos(4\bar{\chi})]C_{l_1}^{BB}\} \\
\tilde{C}_l^{EB} &= \frac{1}{2}\sin(4\bar{\chi})(C_l^{EE} - C_l^{BB})(1-4\langle\delta\chi^2\rangle) \\
&+ \sin(4\bar{\chi})\sum_{l_1 l_2}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi}(-1)^{L+1}(C_{l_1}^{EE} - C_{l_1}^{BB})
\end{aligned}
$$

$$L = l_1 + l_2 + l$$

Similar to weak lensing

If $\bar{\chi} = 0,\ \tilde{C}_l^{TB} = 0,\ \tilde{C}_l^{EB} = 0$

Background evolution breaks Lorentz and CPT, perturbations are stochastic

Power conservation

$$
\begin{aligned}
& \sum_l (2l+1)(\tilde{C}_l^{EE} + \tilde{C}_l^{BB}) \\
= \quad & \sum_l (2l+1)(C_l^{EE} + C_l^{BB})(1 - 4\langle \delta\chi^2 \rangle) + \sum_{l l_1 l_2} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l+1)(2l_1+1)(2l_2+1)}{\pi} C_{l_2}^{\chi} (C_{l_1}^{EE} + C_{l_1}^{BB} \\
= \quad & \sum_l (2l+1)(C_l^{EE} + C_l^{BB})(1 - 4\langle \delta\chi^2 \rangle) + 4\langle \delta\chi^2 \rangle \sum_{l_1} (2l_1+1)(C_{l_1}^{EE} + C_{l_1}^{BB}) \\
= \quad & \sum_l (2l+1)(C_l^{EE} + C_l^{BB})
\end{aligned}
$$

Consistent with $\tilde{Q}^2 + \tilde{U}^2 = Q^2 + U^2$

arXiv:1206.5546

First CMB Constraints on Direction-Dependent Cosmological Birefringence from WMAP-7

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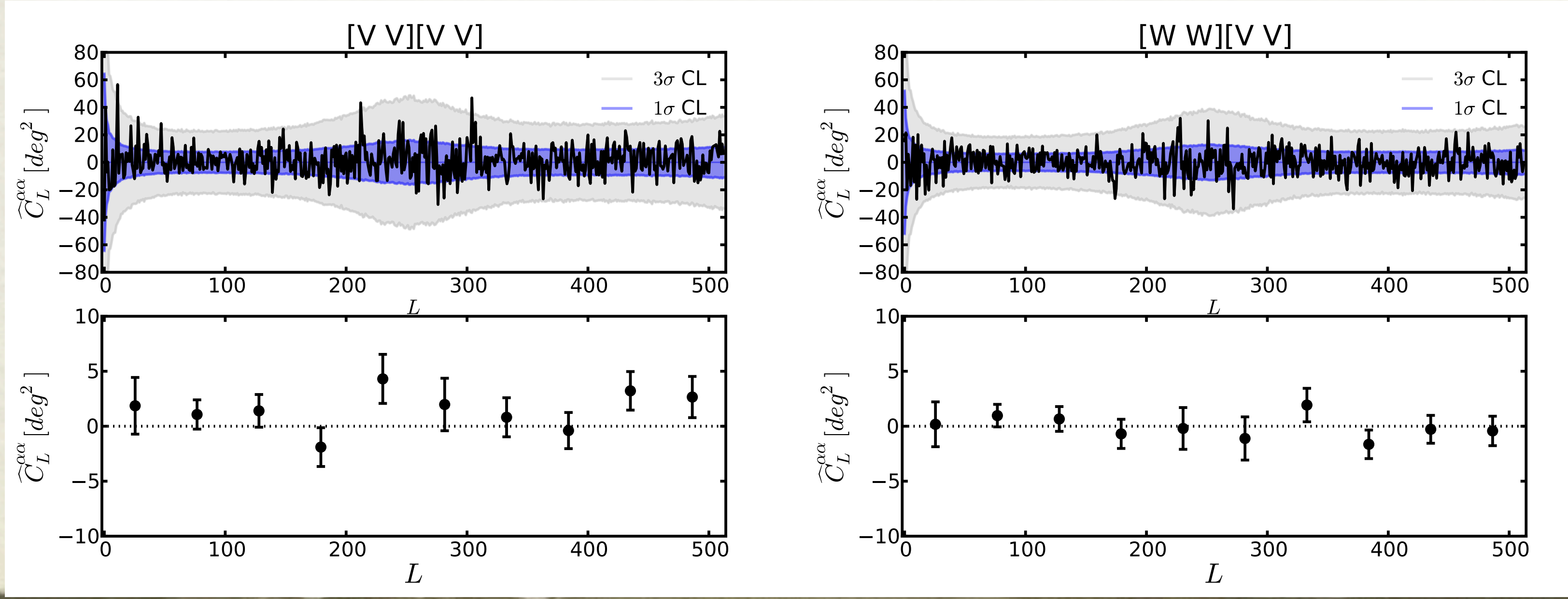
(Dated: October 9, 2012)

A Chern-Simons coupling of a new scalar field to electromagnetism may give rise to cosmological birefringence, a rotation of the linear polarization of electromagnetic waves as they propagate over cosmological distances. Prior work has sought this rotation, assuming the rotation angle to be uniform across the sky, by looking for the parity-violating TB and EB correlations a uniform rotation produces in the CMB temperature/polarization. However, if the scalar field that gives rise to cosmological birefringence has spatial fluctuations, then the rotation angle may vary across the sky. Here we search for direction-dependent cosmological birefringence in the WMAP-7 data. We report the first CMB constraint on the rotation-angle power spectrum $C_L^{\alpha\alpha}$ for multipoles between $L = 0$ and $L = 512$. We also obtain a 68% confidence-level upper limit of $\sqrt{C_2^{\alpha\alpha}/(4\pi)} \lesssim 1^\circ$ on the quadrupole of a scale-invariant rotation-angle power spectrum.



Two-point correlations

Li & Yu, arXiv:1303.1881

WMAP9+QUaD+BICEP

$$\tilde{\xi}_+ \equiv \langle (\tilde{Q}+i\tilde{U})^*(\hat{\boldsymbol{n}})(\tilde{Q}+i\tilde{U})(\hat{\boldsymbol{n}'})\rangle$$
$$\tilde{\xi}_- \equiv \langle (\tilde{Q}+i\tilde{U})(\hat{\boldsymbol{n}})(\tilde{Q}+i\tilde{U})(\hat{\boldsymbol{n}'})\rangle$$
$$\tilde{\xi}_X \equiv \langle T(\hat{\boldsymbol{n}})(\tilde{Q}+i\tilde{U})(\hat{\boldsymbol{n}'})\rangle \ .$$

$$\begin{aligned}\tilde{\xi}_+(\beta) &= \sum_{lm,l'm'} \langle (\tilde{a}^*_{E,lm} - i\tilde{a}^*_{B,lm})(\tilde{a}_{E,l'm'} + i\tilde{a}_{B,l'm'})\rangle {}_2Y^*_{lm}(\hat{\boldsymbol{n}}) {}_2Y_{l'm'}(\hat{\boldsymbol{n}'}) \\ &= \sum_{lm} (\tilde{C}_l^{EE} + \tilde{C}_l^{BB}) {}_2Y^*_{lm}(\hat{\boldsymbol{n}}) {}_2Y_{lm}(\hat{\boldsymbol{n}'}) \\ &= \sum_l \frac{2l+1}{4\pi}(\tilde{C}_l^{EE} + \tilde{C}_l^{BB}) d^l_{22}(\beta) \ ,\end{aligned}$$

$$\begin{aligned}\tilde{\xi}_-(\beta) &= \sum_l \frac{2l+1}{4\pi}(\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + 2i\tilde{C}_l^{EB}) d^l_{-22}(\beta) \ , \\ \tilde{\xi}_X(\beta) &= -\sum_l \frac{2l+1}{4\pi}(\tilde{C}_l^{TE} + i\tilde{C}_l^{TB}) d^l_{02}(\beta) \ ,\end{aligned}$$

$$\begin{aligned}\tilde{\xi}_+(\beta) &= \langle \exp\left[2i(\chi(\hat{\boldsymbol{n}'}) - \chi(\hat{\boldsymbol{n}}))\right](Q+iU)^*(\hat{\boldsymbol{n}})(Q+iU)(\hat{\boldsymbol{n}'})\rangle \\ &= \langle \exp\left[2i(\chi(\hat{\boldsymbol{n}'}) - \chi(\hat{\boldsymbol{n}}))\right]\rangle \xi_+(\beta) \\ &= \exp\left[-2\langle[\delta\chi(\hat{\boldsymbol{n}'}) - \delta\chi(\hat{\boldsymbol{n}})]^2\rangle\right]\xi_+(\beta) \\ &= \exp\left[-4C^\chi(0) + 4C^\chi(\beta)\right]\xi_+(\beta) \\ &= \exp\left[-4C^\chi(0) + 4C^\chi(\beta)\right]\sum_l \frac{2l+1}{4\pi}(C_l^{EE} + C_l^{BB})d^l_{22}(\beta)\end{aligned}$$

Gaussian variable $\langle e^{ix}\rangle = e^{-\langle x^2\rangle/2}$

$$C^\chi(\beta) = \langle \delta\chi(\hat{\boldsymbol{n}})\delta\chi(\hat{\boldsymbol{n}'})\rangle = \sum_l \frac{2l+1}{4\pi} C_l^\chi P_l(\cos\beta)$$

Variance $C^\chi(0) = \sum_l \frac{2l+1}{4\pi} C_l^\chi$

$$\tilde{C}_l^{EE} + \tilde{C}_l^{BB} = e^{-4C^\chi(0)} \sum_{l'} \frac{2l'+1}{2}(C_{l'}^{EE} + C_{l'}^{BB}) \int_{-1}^1 d^{l'}_{22}(\beta) d^l_{22}(\beta) e^{4C^\chi(\beta)} d\cos\beta\ .$$

$$\tilde{C}_l^{EE} - \tilde{C}_l^{BB} = \cos(4\bar{\chi}) e^{-4C^\chi(0)} \sum_{l'} \frac{2l'+1}{2}(C_{l'}^{EE} - C_{l'}^{BB}) \int_{-1}^1 d^{l'}_{-22}(\beta) d^l_{-22}(\beta) e^{-4C^\chi(\beta)} d\cos\beta$$

$$\tilde{C}_l^{EB} = \sin(4\bar{\chi}) e^{-4C^\chi(0)} \sum_{l'} \frac{2l'+1}{4}(C_{l'}^{EE} - C_{l'}^{BB}) \int_{-1}^1 d^{l'}_{-22}(\beta) d^l_{-22}(\beta) e^{-4C^\chi(\beta)} d\cos\beta$$

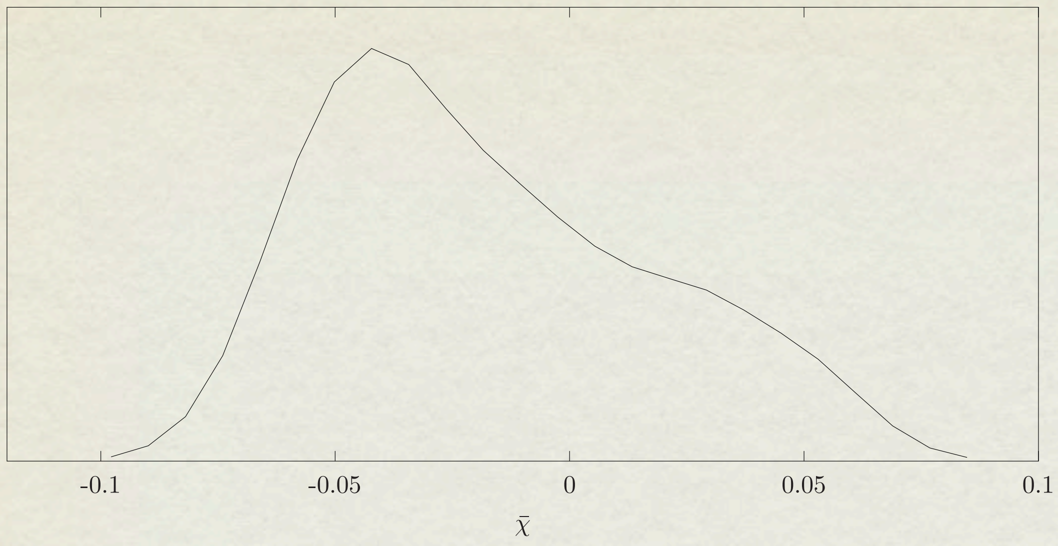
$$\tilde{C}_l^{TE} = \cos(2\bar{\chi}) e^{-2C^\chi(0)} \sum_{l'} \frac{2l'+1}{2} C_{l'}^{TE} \int_{-1}^1 d^{l'}_{02}(\beta) d^l_{20}(\beta) d\cos\beta = C_l^{TE}\cos(2\bar{\chi}) e^{-2C^\chi(0)}$$

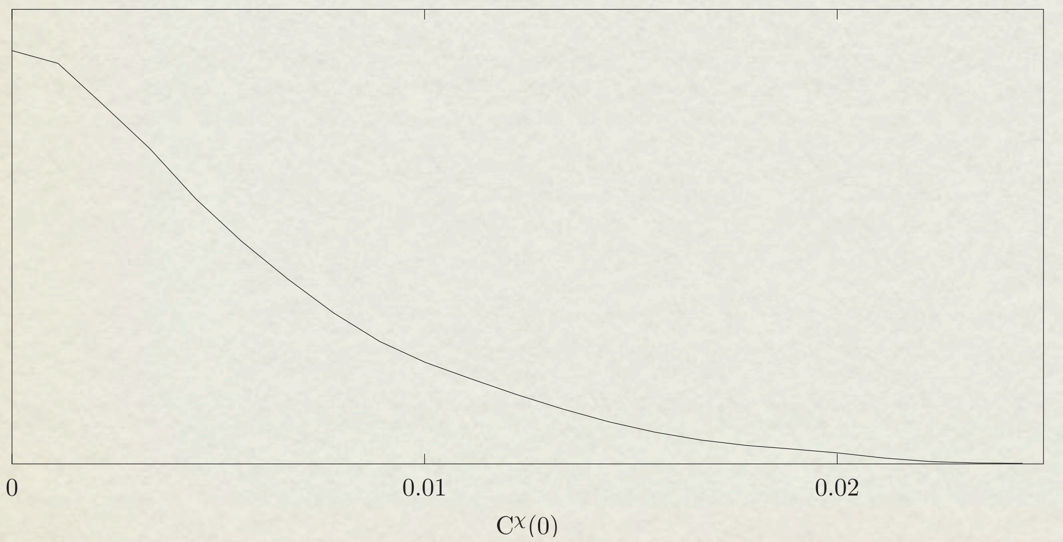
$$\tilde{C}_l^{TB} = \sin(2\bar{\chi}) e^{-2C^\chi(0)} \sum_{l'} \frac{2l'+1}{2} C_{l'}^{TE} \int_{-1}^1 d^{l'}_{02}(\beta) d^l_{20}(\beta) d\cos\beta = C_l^{TE}\sin(2\bar{\chi}) e^{-2C^\chi(0)}\ .$$

$$\tilde{C}_l^{EE} = \tilde{C}_{l,0}^{EE} + \Delta\tilde{C}_l^{EE} \ , \quad \tilde{C}_l^{BB} = \tilde{C}_{l,0}^{BB} + \Delta\tilde{C}_l^{BB} \ , \quad \tilde{C}_l^{EB} = \tilde{C}_{l,0}^{EB} + \Delta\tilde{C}_l^{EB}$$

$$\begin{aligned}
\tilde{C}_{l,0}^{EE} &= [C_l^{EE}\cos^2(2\bar{\chi}) + C_l^{BB}\sin^2(2\bar{\chi})]e^{-4C^{\chi}(0)} \\
\tilde{C}_{l,0}^{BB} &= [C_l^{EE}\sin^2(2\bar{\chi}) + C_l^{BB}\cos^2(2\bar{\chi})]e^{-4C^{\chi}(0)} \\
\tilde{C}_{l,0}^{EB} &= \frac{1}{2}\sin(4\bar{\chi})(C_l^{EE} - C_l^{BB})e^{-4C^{\chi}(0)} \\
\tilde{C}_l^{TE} &= C_l^{TE}\cos(2\bar{\chi})e^{-2C^{\chi}(0)} \\
\tilde{C}_l^{TB} &= C_l^{TE}\sin(2\bar{\chi})e^{-2C^{\chi}(0)} \ .
\end{aligned}$$

Only depend on variance which dominates the distortion





$$0.071 < \bar{\chi} < 0.057,\ C^{\chi}(0) < 0.014$$

SUMMARY (II)

- DE has natural derivative couplings to SM particles
- Chern-Simons coupling induces anisotropic rotation angle which distorts CMB polarization spectra similar to weak lensing effect
- Current data showed no evidence for the non-zero spectra of rotation angle
- It predicts non-Gaussianities in CMB polarization field

THANKS!