DARK ENERGY (DE) PERTURBATIONS

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2.0 1.6 1.2 0.8 0.4 DE models: classified by w

Planck Collaboration: Cosmological parameters

Padmanabhan & Finkbeiner 2005; Zhang et al. 2006; $\mathcal{L} = \mathcal{L}$ $\mathcal{L} = \mathcal{L} = \mathcal$ 2011; Natarajan 2012) CMB anisotropies o↵er an opportunity

 $\frac{1}{\sqrt{2}}$ gas by DM annihilation are typically conscale by high energy processes; once the shower has reached this energy scale, the secondary particles produced can ionize, excite or heat the thermal gas (Shull & van Steenberg 1985; $\mathcal{L}=\mathcal{L}$ of the free electron fraction *x*e, while the third a↵ects the tem-

The rate of energy release, *dE*/*dt*, per unit volume by a relic

where *p*ann is, in principle, a function of redshift *z*, defined as

where h_{vi} is the theory m is the mass of the m the Universe today, *g* is a degeneracy factor equal to 1/2 for Majorana particles and 1/4 for Dirac particles (in the following, constraints will refer to Majorana particles), and the parameter *f*(*z*) indicates the fraction of energy which is absorbed *overall* by the gas at redshift *z*. We note that the presence of the brackets in h*v*i denote a thermal average over the velocity distribution

In Eq. (98), the factor *f*(*z*) depends on the details of the annihilation process, such as the mass of the DM particle and the annihilation channel (see e.g., Slatyer et al. 2009). The func-

to constrain DM annihilation models.

annihilating DM particle is given by

perature of the baryons.

dE

dt (*z*) ⁼ ² *^g* ⇢²

*p*ann(*z*) ⌘ *f*(*z*)

of particles.

[20, 100] km s¹ Mpc¹ on *H*0. The dashed grey line indicates the cosmological constant $w = w_0$ -

sumed and, importantly for the CMB-only constraints, the prior

preference for *w* , 1 when combining BAO with *Planck*. In contrast, the addition of the addition of the HO measurement, or SNE data, or SNE data, or SNE data, or SNE da
SNE data, or SNE da $w_0 = -1.04^{+0.72}_{-0.69}$ (95%; *Planck*+WP+BAO), $P = \frac{122}{12}$ (050. Planck+WP+BAC $w_a < 1.32$ (95%; *Planck*+WP+BAO).

The most precise bounds on EDE arise from the analysis of CMB anisotropies (Doran et al. 2001; Caldwell et al. 2003;

sults from the *Planck*+WP+Union2.1 data combination are in

better agreement with a cosmological constant than the cosmological constant than the constant than the cosmological constant of the cosmological constant of the cosmological constant of the cosmological constant of the co

 $F = 3.04 \pm 0.72$ $(0.5 \text{eV} - \text{N}^2)$ w¹ w¹ N

Background evolution w_0, w_a which is in tension with the 2 level. Including the *H*⁰ measurement in place of the BAO data moves

 $t = \frac{1}{2}$ ant. perturbations should be 2013). The presence or absence of dark energy at the epoch of \mathcal{L}_max scattering is the dominant e \mathcal{L}_max considered in consistent analyses redshift supplementary data such as BAO. **Except for Cosmological constant, perturbations should be Except for cosmological constants** with the **Planck** base is no significant model, so the so that α is no significant model, so the so that α is no significant model. \blacksquare adding Sne data to **Planck. As discussed in detail in Sect. 5, 1990** stent analyses choice of SNE data set, and the reflection in Fig. 36. The reflection in Fig. 36. The reflection in Fig. 36. The reCosmological constant: $w = -1$, $\dot{w} = 0$ Quintessence: $\mathcal{L} = (1/2)(\partial \phi)^2 - V(\phi)$ Phantom: $\mathcal{L} = -(1/2)(\partial \phi)^2 - V(\phi)$ K-essence: $\mathcal{L} = \mathcal{L}(\phi, X), X = (1/2) \partial_{\mu} \phi \partial^{\mu} \phi$ $T^{\mu\nu}$ = $-\mathcal{L}g^{\mu\nu}$ + @*L* $\frac{\partial \mathcal{L}}{\partial X} \partial^{\mu} \phi \partial^{\nu} \phi$ $=$ $-pq^{\mu\nu} + (p+\rho)u^{\mu}u^{\nu}, u^{\mu}u_{\mu} = 1$ Ω _{*µ*} θ

$$
p = \mathcal{L}, \ \rho = 2Xp_X - p, \ u^{\mu} = \frac{\partial^{\mu} \phi}{\sqrt{2X}}
$$

OUTLINE

- Effects of DE perturbations on CMB and LSS
- Methods to deal with DE perturbations with parameterization
- Initial conditions of DE perturbations
- Anisotropic rotation angle induced by DE coupling

EFFECTS OF DE PERTURBATIONS ON CMB AND LSS

Generation of CMB anisotropies\n
$$
\frac{\Delta T}{T}(\hat{\mathbf{n}}) = [\frac{1}{4}\delta_{\gamma} + \Psi + \vec{v} \cdot \hat{\mathbf{n}}]_{*} + \int_{*}^{\eta_{0}} d\eta (\Phi' + \Psi')(\eta, \Delta \eta \hat{\mathbf{n}})
$$
\n
$$
\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow
$$
\nSachs-Wolf Doppler\nIntegrated Sachs-Wolf (ISW)

 $\Delta \eta = \eta_0 - \eta_*$

DE dominates the universe at very late time and contributes to CMB anisotropy through ISW effect.

Without anisotropic stress $\Psi = \Phi$

$$
\Phi'' + 3\mathcal{H}\Phi' + (-8\pi Ga^2p_D)\Phi = 4\pi Ga^2\delta p_D
$$

Contributions from DE background Contribution from DE perturbations

$$
(\frac{\Delta T}{T})^{ISW} = 2 \int_{*}^{\eta_0} d\eta \Phi'
$$

CMB and Dark Energy 3

6000 5000 μ K² 2π 4000 $\frac{1}{2}$ 3000
 $\frac{1}{4}$ 3000 1000 Ω 100 1000 10

7000

Figure 2. CMB angular power spectra for different dark energy models with no perturbations. The solid line is for a ΛCDM model, the dotted line for a model with $w = -0.6$ and dashed line $w = -2.0$. The parameters Ω_c , Ω_b and H_0 are adjusted to

w = −2, Ω^m = 0.17, Ω^b = 0.027, H⁰ = 84 km s[−]¹Mpc[−]¹.

 $\frac{1}{2}$ \mathcal{S} $d\bar{z}$ $f^{\prime\prime}$ The bold lines are including perturbations in the dark energy

Clearly a constant equation of state makes a very unnatural quintessence model. However a large class of models are $\frac{d\mathbf{r}}{d\mathbf{r}}$ anisotropy is concerned to an effective constant equation of \mathbb{R}^n state parameter. In this paper we do not explicitly consider

 $\frac{1}{2}$ parameter of the dark energy component on the cosmic microwave background anisotropies we will first look into primary degeneracies originating from smaller scales in the temperature anisotropy power spectrum. As discussed in

Figure 3. CMB angular power spectra for them dark energy models as in Fig. 2, but with dark energy perturbations.

¹⁰² ¹⁰³ ¹⁰⁴ ¹⁰[−]⁸

 \mathbf{S} \mathbf{z} with the data matter perturbation (dotted line), for dotted lines, for dotted lines, for dotted lines, \mathbf{z}

 \mathbb{R} \mathbb{R}

rameter estimation. Here we generalise the dark energy parameterisation by introducing a constant sound speed ˆc²

 $s = t$ the other perturbations if $w = t$ via the time ϵ An over density causes a decrease in the local expansion rate $\frac{1}{2}$ $\frac{d}{dt}$ subsequent evolution depends on the sound speed, as shown in Fig. 4. Consider the frame comoving with the dark mat-

10[−]⁶

 \overline{a}

a free parameter.

curvature perturbation.

10[−]⁴

10[−]²

100

102

have vanished. This is because for what we are written to be a more than \sim

DE perturbations are ignored DE perturbations are included between matter in the matter of the matter and data the matter and later, but more and more rapidly, and with a larger over- α for values of α is the ISW for values of α Constant w and based on quintessen to the ISW. The solid line is for a ΛCDM universe with J Weller $\&$ A M Lewis $\frac{1}{2}$ 0.073, H⁰ = 54 km s[−]¹Mpc[−]¹ and the thin dot-dashed for \mathbf{p} matter perturbations as they are sourced. E perturbations are included bations are exactly zero. We see how the bold dot-dashed DE perturbations increase parameter degeneracies \mathcal{L}_c is the contribution of the perturbation of the perturbation \mathcal{L}_c Constant w and based on quintessence and phantom models $s_n = \frac{1}{2} \log \frac{1}{2}$ J.Weller & A.M.Lewis, astro-ph/0307104 large scales when we did not include perturbations in Fig. 2

Without DE perturbations With DE perturbations

Park, Hwang, Lee & Noh, PRL (2009)

METHODS TO DEAL WITH DE PERTURBATIONS WITH PARAMETERIZATION

The background contribution of DE can be described by two parameters w_0 and w_a

Q: How to include DE perturbations consistently in data analyses without referring to specific models? And with as few as possible extra parameters beyond w_o and w_a?

Zhao,Xia,Li,Feng,Zhang, astro-ph/0507482, PRD(2005)

T perturbation T relation \mathcal{S} relation \mathcal{S} relation \mathcal{S} through the relation \mathcal{S} Perturbation Equations

$$
ds^{2} = a^{2}(\eta)[(1+2\Phi)d\eta^{2} - (1-2\Psi)\delta_{ij}dx^{i}dx^{j}]
$$

determined by the matter perturbations through the Einstein equations which take the following form when \mathbb{H}

 α and denotes the momentum denotes the momentum density perturbation, which is defined by α

$$
-k^2\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) = 4\pi Ga^2\delta\rho,
$$

\n
$$
k^2(\Psi' + \mathcal{H}\Phi) = 4\pi Ga^2(\rho + p)\theta,
$$

\n
$$
\Psi'' + \mathcal{H}(2\Psi' + \Phi') + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{k^2}{3}(\Psi - \Phi) = 4\pi Ga^2\delta p,
$$

\n
$$
k^2(\Psi - \Phi) = 12\pi Ga^2(\rho + p)\sigma
$$

 \blacksquare Some definitions of the notations C P Ma $\&$ E Bertschinger An I (1005) \mathbf{r} bonne deminitons of the notations C_{1} , ma α E_{2} , bert Some definitions of the notations C.P. Ma & E. Bertschinger, ApJ (1995) variable denotes the momentum denotes the momentum density perturbation, which is defined by the momentum density per If there are many components of matter, then each species has its own perturbation variables δi, θⁱ and δpi. The Some definitions of the notations C.P. Ma & E. Bertschinger, ApJ (1995)

where H and the conformal Hubble parameter and the prime denotes the prime denotes the derivative with respect to \mathbb{R}^n

where H and the conformal Hubble parameter and the prime denotes the prime denotes the derivative with respect to \mathcal{C}

 \overline{a} $(\rho + p)\theta = ik^i \delta T_i^0$

The shear perturbation of the shear perturbation of the anisotropic stress through the relationship of σ relates to the anisotropic stress through the relation

^a² ^Φ ⁼ [−]4πGρ[^δ ⁺

k2

from the Einstein equations (2):

from the Einstein equations (2):

$$
(\rho + p)\sigma = \hat{k}_i \hat{k}_j (\delta T^i_j - 1/3 \delta^i_j \delta T^l_l),
$$

 \mathbb{F}_q = $\mathbb{$

3H

δ!

$$
(\rho + p)\theta = ik^{i}\delta T_{i}^{0}.
$$
\n
$$
\sigma
$$
 relates to the anisotropic stress through the relation\n
$$
(\rho + p)\theta = \sum_{i} (\rho_{i} + p_{i})\theta_{i},
$$
\n
$$
(\rho + p)\sigma = \hat{k}_{i}\hat{k}_{j}(\delta T_{j}^{i} - 1/3\delta_{j}^{i}\delta T_{l}^{l}),
$$
\n
$$
\delta p = \sum_{i} \delta p_{i}.
$$

If there are no interactions beyond gravitations between \mathcal{L}_{max} and \mathcal{L}_{max}

) − 3H

"δpⁱ

ρi

− wiδⁱ

#

k2 (1 + w) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5) , (5)

i = + will also a state of a

3

ⁱ . (3)

k2 (1 + w)

 λ

δT ⁰

ⁱ , respectively. The

 $\sigma = 0 \rightarrow \Psi = \Phi$

where

$$
\delta_i' = -(1+w_i)(\theta_i - 3\Phi') - 3\mathcal{H}\left(\frac{\delta p_i}{\rho_i} - w_i \delta_i\right),
$$

$$
\delta[T_{(i);\nu}^{\mu\nu}] = 0 \longrightarrow \theta_i' = -\mathcal{H}(1 - 3w_i)\theta_i - \frac{w_i'}{1 + w_i}\theta_i + k^2\left(\frac{\delta p_i/\rho_i}{1 + w_i} + \Phi\right)
$$

If there are no interactions beyond gravitational ones among these components, the perturbations for each species

To close the equations, we need the form of $\delta p_i = f(\delta_i, \theta_i, \Phi)$ For barotropic fluid $p = p(\rho)$

$$
\delta p = \frac{p'}{\rho'} \delta \rho = c_a^2 \delta \rho
$$

 α α α β For k-essence field $p = p(\phi, X), \ \rho = \rho(\phi, X)$

Model dependent

. (7)

ai)θⁱ , (8)

i/holds in with the set of the set

$$
\theta = \frac{k^2}{\phi'} \delta \phi
$$

\n
$$
\delta p = \frac{p_X}{\rho_X} \delta \rho + 3\mathcal{H} (1+w) \frac{\rho}{k^2} (\frac{p_X}{\rho_X} - c_a^2) \theta
$$

Model independent perturbation equations

w.
W = wD(n) + 6 wD(n)

General form of scalar perturbations of the metric

 $ds^2 = a^2(\eta)\{(1+2A)d\eta^2 - 2B_{,i}dx^id\eta - [(1-2\psi)\delta_{ij} + 2E_{,ij}]dx^idx^j\}$ physical. Under the coordinate transformation

wD = wD(n) + 5 wD(n) + 6 wD(n) + 7 wD(n) +

of small amplitude inhomogeneities the EoS can be decomposed into a homogeneous part and a small perturbation:

To obtain the matching conditions on this hypersurface, it is better for us to consider the general form of the perturbed \mathbb{R}^n

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, (16) and (16

dxj

dxje (15) ,

Commas denote derivatives with respect to spatial conditions and the four variables A, e are physical. The four variables A, μ, ε αναφέρει το προσ Gauge transformation

metric

metric

 C

implies in the contract of the

$$
\begin{array}{rcl} \text{Gauge transformation} & & \\ & \eta & \rightarrow & \tilde{\eta} = \eta + \xi^0 \ \ \text{and} \\ & x^i & \rightarrow & \tilde{x}^i = x^i + \xi^{,i} \,, \end{array}
$$

$$
\tilde{g}_{\mu\nu} = g_{\mu\nu} - L_{\xi} g_{\mu\nu} \qquad\n\begin{aligned}\nA &\to \tilde{A} = A - \mathcal{H}\xi^{0} - \xi^{0'} , \quad B \to \tilde{B} = B + \xi^{0} - \xi' \\
\psi &\to \tilde{\psi} = \psi + \mathcal{H}\xi^{0} , \qquad E \to \tilde{E} = E - \xi ,\n\end{aligned}
$$
\n
$$
\tilde{T}^{\mu\nu} = T^{\mu\nu} - L_{\xi} T^{\mu\nu} \qquad\n\begin{aligned}\n\tilde{\delta \rho} &= \delta \rho - \rho' \xi^{0} \\
\tilde{\delta \rho} &= \delta \rho - p' \xi^{0} \\
\tilde{\theta} &= \theta - k^{2} \xi^{0}\n\end{aligned}
$$
\nGauge invariant

\n
$$
\Phi = A + \frac{1}{a} [a(B - E')]', \quad \Psi = \psi - \mathcal{H}(B - E')
$$
\nconformal Newtonian gauge

\n
$$
B = E = 0, \quad \Phi = A, \quad \Psi = \psi
$$

Definition of sound speed

Adiabatic sound speed
$$
c_a^2 = \frac{p'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}
$$

Comoving gauge
$$
\tilde{\theta} = 0
$$
, $c_s^2 \equiv \frac{\delta p}{\delta \rho}$ True sound speed

 $\tilde{\cdot}$

Transform to arbitrary gauge

putrary gauge

\n
$$
\xi^0 = \frac{\theta}{k^2}, \ c_s^2 = \frac{\delta p - p'\xi^0}{\delta \rho - \rho'\xi^0} = \frac{\delta p - p'\theta/k^2}{\delta \rho - \rho'\theta/k^2}
$$
\n
$$
\phi = c_s^2 \delta \rho + \frac{\rho'}{k^2} (c_a^2 - c_s^2) \theta = c_s^2 \delta \rho + \frac{3\mathcal{H}(\rho + p)}{k^2} (c_s^2 - c_a^2) \theta
$$

Barotropic fluid

$$
c_s^2 = c_a^2
$$

Scalar field

$$
\tilde{\theta} = 0 \rightarrow \widetilde{\delta \phi} = 0 \qquad p = p(\phi, X), \ \rho = \rho(\phi, X)
$$

$$
\theta = \frac{k^2}{\phi} \delta \phi
$$

$$
\theta \;\;=\;\; \frac{k^2}{\phi'} \delta \phi
$$

⇢ + 3*H*(1 + *^w*) ⇢

⇢ + 3*H*(1 + *^w*) ⇢

*k*2 (

^a)✓

*k*2 (

^a⇢

$$
\widetilde{\delta p}=p_X\widetilde{\delta X},\,\,\widetilde{\delta \rho}=\rho_X\widetilde{\delta X}
$$

 $\sqrt{ }$

$$
c_s^2 = \frac{\widetilde{\delta p}}{\widetilde{\delta \rho}} = \frac{p_X}{\rho_X} \neq c_a^2
$$

$$
\delta p = c_s^2 \delta \rho + \frac{3\mathcal{H}(\rho + p)}{k^2} (c_s^2 - c_a^2) \theta
$$

Normal quintessence and phantom

p = *c*²

$$
p = \pm X - V, \ \rho = \pm X + V, \ c_s^2 = 1
$$

For barotropic fluid *p* = *p*(⇢)

For barotropic fluid *p* = *p*(⇢)

$$
\delta'_{i} = -(1+w_{i})[1 + \frac{9\mathcal{H}^{2}}{k^{2}}(c_{si}^{2} - c_{ai}^{2})]\theta_{i} - 3\mathcal{H}(c_{si}^{2} - w_{i})\delta_{i} + 3(1+w_{i})\Phi'
$$

\n
$$
\theta'_{i} = \mathcal{H}(3c_{si}^{2} - 1)\theta_{i} + k^{2}(\frac{c_{si}^{2}}{1+w_{i}}\delta_{i} + \Phi)
$$

\n
$$
\mathcal{H} = 0, \ w'_{i} \simeq 0
$$

\n
$$
\delta''_{i} + c_{si}^{2}k^{2}\delta_{i} = 3(1+w_{i})(\Phi'' + k^{2}/3\Phi)
$$

\n
$$
c_{si}^{2} \geq 0 \text{ to guarantee the stability}
$$

$$
\delta'_{i} = -(1+w_{i})[1 + \frac{9\mathcal{H}^{2}}{k^{2}}(c_{si}^{2} - c_{ai}^{2})]\theta_{i} - 3\mathcal{H}(c_{si}^{2} - w_{i})\delta_{i} + 3(1+w_{i})\Phi'
$$
\n
$$
\theta'_{i} = \mathcal{H}(3c_{si}^{2} - 1)\theta_{i} + k^{2}(\frac{c_{si}^{2}}{1+w_{i}}\delta_{i} + \Phi)
$$
\n
$$
\frac{k^{2}}{a^{2}}\Phi = -4\pi G \sum_{i} \rho_{i}[\delta_{i} + \frac{3\mathcal{H}}{k^{2}}(1+w_{i})\theta_{i}] \underbrace{-k^{2}\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi)}_{k^{2}(\Psi' + \mathcal{H}\Phi)} = 4\pi G a^{2}\delta\rho,
$$

Effective energy density perturbation

where $\frac{1}{2}$ is the density contrast. The density contrast $\frac{1}{2}$ is the density contrast.

 \mathbf{L} time. The energy density and pressure perturbations are denoted by \mathbf{L} Simple universe model: i=m, DE T shear perturbation Ω relation of the anisotropic stress through the relation Ω For DE with single degree of freedom, only one extra parameter:

 ι_{sD}

where H and the conformal Hubble parameter and the prime denotes the prime denotes the derivative with respect to \mathbb{R}^n

and it vanishes if matter is a perfect fluid or consists of a set of scalar fields as in the cases considered in this paper. T , in the cases considered here we have \mathcal{P} , one can obtain the Poisson equations, one can obtain the P

 $0 < \frac{1}{2}$

i . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3) . (3)

k2 (1 + w) , (5)

^l), (4)

However......

$$
\delta'_{i} = -(1+w_{i})[1+\frac{9\mathcal{H}^{2}}{k^{2}}(c_{si}^{2} - c_{ai}^{2})]\theta_{i} - 3\mathcal{H}(c_{si}^{2} - w_{i})\delta_{i} + 3(1+w_{i})\Phi'
$$

$$
\theta'_{i} = \mathcal{H}(3c_{si}^{2} - 1)\theta_{i} + k^{2}(\frac{c_{si}^{2}}{1+w_{i}}\delta_{i} + \Phi)
$$

$$
c_{aD}^2 = w_D - \frac{w'_D}{3\mathcal{H}(1+w_D)}
$$

 $1 + w_D = 0$, $w'_D \neq 0$, is a singularity

1, Forbid crossing -1 a priori, leads to biased results;

2, Models allowing crossing require more degrees of freedom. Constraining multi components DE is cumbersome and model dependent.

Method: divide the parameter space into three regions

Zhao,Xia,Li,Feng,Zhang, PRD(2005)

 $1 + w_D > \epsilon$ *or* $1 + w_D < -\epsilon$, regular, can be considered as single component $-\epsilon < 1 + w_D < \epsilon, \ \delta_D|_{+} = \delta_D|_{-}, \ \theta_D|_{+} = \theta_D|_{-}$

 $\epsilon > 0$ is a small parameter

 c_{sD}^2 is not constrained significantly by data

This approach approximated the results obtained in two-field quintom models accurately for $\epsilon \sim 10^{-5}$

The way to get matching conditions x_i + x_i these metric perturbations transform as

expansion rate H are continues. And for the perturbations, one obtains, one obtains, one obtains, one obtains,

physical. Under the coordinate transformation

Mingzhe Li, Y.Cai, H.Li, R.Brandenberger, X.Zhang, PLB (2011)

In a perturbed universe $w_D = w_D(\eta) + \delta w_D(\eta, x^i)$

 $\delta w_D \rightarrow \widetilde{\delta w}_D = \delta w_D - w'_D$ $\int_D \xi^0$.

We will use the temporal gauge to obtain the matching conditions. In this gauge, the matching hypersurface coincides with $\tilde{\eta} = \text{const.}$ and the equation of this hypersurface

$$
\widetilde{w}_D(\tilde{\eta},\ \tilde{x}^i) \ =\ \text{const.}
$$

implies

$$
\delta w_D\,=\,0\,.
$$

Hence the time shift is

$$
\xi^0\,=\,\frac{\delta w_D}{w_D'}\,\,,
$$

but ξ remains arbitrary. The induced 3-metric of this hypersurface and its extrinsic curvature are expressed as

$$
q_{ij} = a^2 [(1 - 2\tilde{\psi})\delta_{ij} + 2\tilde{E}_{,ij}],
$$

\n
$$
K_{ij} = \frac{q_{ij}}{a} (\mathcal{H} - \mathcal{H}\tilde{A} - \tilde{\psi}') + \frac{1}{a} (\tilde{E}' - \tilde{B})_{,ij},
$$

The matching conditions tell us that the induced metric and the extrinsic curvature should be continuous across the surface, i.e. that is the surface of the surface, i.e. the background, the scale factor and the scale factor and the scale factor

$$
\begin{aligned}\n[\tilde{\psi}]_{\pm} &= [\tilde{E}]_{\pm} = 0\\ \n[\mathcal{H}\tilde{A} + \tilde{\psi}']_{\pm} &= [\tilde{E}' - \tilde{B}]_{\pm} = 0\\ \n[\tilde{\psi}]_{\pm} &\equiv \tilde{\psi}_{+} - \tilde{\psi}_{-} \text{ etc.}\n\end{aligned}
$$

To arbitrary gauge:

where the notation

where the notation

an arbitrary gauge,

an arbitrary gauge,

To arbitrary gauge:
\n
$$
[\psi + \mathcal{H} \frac{\delta w_D}{w'_D}]_{\pm} = 0 ,
$$
\n
$$
[E - \xi]_{\pm} = 0 ,
$$
\n
$$
[\mathcal{H}A + \psi' + (\mathcal{H}' - \mathcal{H}^2) \frac{\delta w_D}{w'_D}]_{\pm} = 0 ,
$$
\n
$$
[E' - B + \frac{\delta w_D}{w'_D}]_{\pm} = 0 .
$$

Specifically, in the Conformal Newtonian gauge used in this paper (B = E = 0 and Φ = A, Ψ = ψ) these conditions To conformal Newtonian gauge:

$$
\begin{aligned}\n[\Psi]_{\pm} &= 0, \\
[\frac{\delta w_D}{w'_D}]_{\pm} &= 0, \\
[\mathcal{H}\Phi + \Psi' + (\mathcal{H}' - \mathcal{H}^2) \frac{\delta w_D}{w'_D}]_{\pm} &= 0.\n\end{aligned}
$$

When D = When Φ = Which are absoluted the absoluted the matter contributions, and dividing the matter Φ

D

$$
\frac{k^2}{a^2}\Phi = -4\pi G\{\rho_D[\delta_D + (1+w_D)\frac{\mathcal{H}}{k^2}\theta_D] + \rho_m[\delta_m + (1+w_m)\frac{\mathcal{H}}{k^2}\theta_m]\} \qquad w_D \simeq -1
$$

$$
[\delta_D]_{\pm} = 0
$$

i
De la componenta de la co
De la componenta de la c w"

]± = 0 . (28)

^k² ^θm]} , (29)

k2 θm) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29) , (29)

k2 θD should be also continuous.

^k² ^θm]} , (29)

k2 θD . (32) θD . (3

^k² ^θ^D . (32)

^D must be non-zero in order to obtain crossing. Thus, the matching condition

IB + 2 . (245 0 . 6 . 6 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3) = 0 . (3)

 \mathbb{R}^2 , \mathbb{R}^2

^k² ^θ^D . (32)

 \mathbb{R}^2 , \mathbb{R}^3 , \mathbb{R}^3

k2 , (31)

k2 , (31)

$$
\frac{\delta w_D}{w'_D} = \frac{1}{w'_D} (\frac{\delta p_D}{\rho_D} - w_D \delta_D) = \frac{c_{sD}^2 - w_D}{w'_D} [\delta_D + \frac{3\mathcal{H}(1 + w_D)}{k^2} \theta_D] + \frac{\theta_D}{k^2}
$$

 $N_{\rm eff}$ we turn to the physical meaning of the second condition in (28). After simple calculations on α

 \blacksquare and (33) coincide with the assumptions (13) used in Ref. [1]. Another way to see that (33) and (33) are \lnot

 $\mathbb{E}[\mathcal{S}^{(1)}]$ and (13) coincide with the assumptions (13) used in Ref. [1]. Another way to see that (33) are $\mathcal{S}^{(1)}$ and

^D must be non-zero in order to obtain crossing. Thus, the matching condition

values we see that if the see that if the matching conditions are satisfied, then all of the matching conditions (28) are satisfied to the matching conditions (28) are satisfied. The matching conditions (28) are satisfied

valid we see that if these matching conditions are satisfied, then all of the matching conditions (28) are satisfied.

|
|Hodi + V + (H" + H2)

k
2 θD - ρmar - ρmar - ρmar - γm (1 + wm)

k2 θα μεταφέρει του μεταφέρει του μεταφέρου του μεταφέρει του μεταφέρει του μεταφέρει του μεταφέρει του μεταφέ
Προϊόνται του μεταφέρει το

 \mathbb{R} = \mathbb{R} = \mathbb{R} perturbations, and dividing the matter contributions into the matter contributions into that of \mathbb{R}

$$
\frac{\delta w_D}{w'_D} = \frac{c_{sD}^2 + 1}{w'_D} \delta_D + \frac{1}{k^2} \theta_D
$$

 $\mathbb{E}[\mathbf{S}^{(1)}]$ coincide with the assumptions (13) used in Ref. [1]. Another way to see that (30) and (33) are f. [1]. Another way to see that (30) and (33) are f. [1]. Another way to see that (30) are f. [1]. Another

valid we see that if the see that if the matching conditions are satisfied, then all of the matching conditions (

energy and that of regular cold matter, the Poisson equation (5) becomes

Because the matching hypersurface is characterized by w^D = −1, one gets the following matching condition for the

energy and that of regular cold matter, the Poisson equation (5) becomes

 $\theta_D]_\pm =$ $\overline{}$ ^D are continuous, and ^w" ^D must be non-zero in order to obtain crossing. Thus, the matching condition $[\theta_D]_{\pm} = 0$ ^D must be non-zero in order to obtain crossing. Thus, the matching condition

 $\mathbb{D}=\{x\in\mathbb{R}^n: x\in\mathbb{R}^n: x\in\mathbb{R$

energy density perturbation of dark energy density perturbation of dark energy of dark energy of dark energy o

 Γ first matching condition in (28) means that the combination Γ (1 μ wD) H μ

a2 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0 Φ = 0

and at the matching hypersurface this becomes

Both \mathbf{B}

Both δ^D and w"

⁼ ¹

 \blacksquare

 \blacksquare

^D are continuous, and ^w"

 $\overline{\mathcal{S}}$

ρ^D

مية.
ويقد المراجع الأسرار المراجع ا

where the subscript m denotes \mathbb{R}^n

energy density perturbation of dark energy

energy density perturbation of dark energy

and at the matching hypersurface this becomes

^D are continuous, and ^w"

The first matching condition in (28) means that the combination in (28) means that the combination $\mathcal{L}(\mathcal{L})$

perturbation we have done the calculations separately for the two cases including and switching off the dark energy perturbations. In Figure 1 we plot our numerical results. One can see the observered between the two cases of two cases of the observer

FIG. 1: Two dimensional constraints on (w_0, w_1) from current observations of CMB + SNIa + BAO. The red solid and black dash lines represent the 2 σ limits for the two cases with and without dark energy perturbations, respectively. The star represents the best fit value.

perturbations are singular and unstable at this point. The quintom model is able to constant of the quintom model is able to constant of the quintom model is able to constant of the cross this boundary naturally, and a sec however it requires more degrees of freedom, and lessons learned when studying the transfer of fluctuations through Parametrized Post-Friedmann (PPF) approach

W.Fang, W.Hu, A.Lewis, arXiv:0808.3125

The same motivation.

c ²*s*⇢

Replace
$$
\delta p = c_s^2 \delta \rho + \frac{3\mathcal{H}(\rho + p)}{k^2} (c_s^2 - c_a^2) \theta
$$

with an assumed relationship between $(\rho_D + p_D)\theta_D$ and $\rho_m\theta_m$ on large scales and a transition scale under which the effective energy density perturbation of dark energy can be neglected. Match the evolution of the metric for scales much larger and much smaller than the transition scale.

Mimic multi fields. Assuming smoothness of DE on small scales and adiabatic initial condition.

One exception: models correspond to imperfect fluids This means the degenerate model has no extra degenerate model \mathbf{r} ⊘ne exception: models correspond to imperfect fluids α and the expectrion only incorrection on the second to imperfect fluids one exception β redefinitions of K(β), the degenerations of β , β and β $L = \frac{1}{2} \int_{0}^{2\pi} \frac{1}{2} \, e^{-\frac{2\pi i}{3} \left(\frac{1}{2} \, e^{-\frac{2\pi i}{3}} \right)} \, e^{-\frac{2\pi i}{3} \left(\frac{1}{2} \, e^{-\frac{2\pi i}{3}} \right)} \, e^{-\frac{2\pi i}{3} \left(\frac{1}{2} \, e^{-\frac{2\pi i}{3}} \right)} \, e^{-\frac{2\pi i}{3} \left(\frac{1}{2} \, e^{-\frac{2\pi i}{3}} \right)} \, e^{-\frac{2\pi i}{3} \left(\frac{1}{2} \, e^{-\frac$ **L EXUAGABLE CONGLUE CONGLUE EN ARRYAGE EN SOURCE EN ABOVE ENCORE ENCORE ENCORE ENCORE EN ABOVE EN A FEDERAL PO**

The energy momentum tensor which sources the gravitational field is obtained through the variation of the action

The box term is equivalent to −2XG quivalent to −2XG quivalent to −2XG quivalent to −2XG quivalent to −2XG qui

Deffayet, Pujolas, Sawicki & Vikman, JCAP (2010)
Now we will invest the data stable can stable from the can stable from the boundary cross the boundary cross the boundary of the boundary cross the boundary of the boundary Now we will investigate whether the dark energy model from this Lagrangian can stably cross the boundary of , (22) Deffayet, Pujolas, Sawicki & Vikman, JCAP (2010)

Kobayashi, Yamaguchi & Yokoyama, PRL (2010) The box term at the right hand side with G α you considered in the context of Galileon theory in the context of Galil Li, Qiu, Cai & Zhang, JCAP (2012) Kamada Kobayashi Yamaguchi & Yokoyama PRD (2011) $T_{\text{min,real}}$, Teologiasm, Tamagachi os Tokoyama, TKD (2011) Kamada, Kobayashi, Yamaguchi &Yokoyama, PRD (2011) V_{c} become its by studying its background evolution and properties of perturbation and perturbations. NUU NUU NUU NU NU (2012) √g δgµ^ν Kobayashi, Yamaguchi &Yokoyama, PRL (2010)

tensor. The equation of motion remains to be a second order differential equation, but there is a curvature-field coupling term appeared in its even though we only consider the minimal coupling to the gravity in the Lagrangian.

+ 2Fφ(∇µX∇^µ^φ + 2X✷φ)+2XFφX∇µX∇^µ^φ + 4X²Fφφ ⁺ ^FX(✷φ∇µX∇^µ^φ − ∇µX∇^µX)=0 . (21)

tensor. The equation of motion remains to be a second order differential equation, but there is a curvature-field coupling term appeared in it even though we only consider the minimal coupling to the gravity in the Lagrangian.

generalization was studied in [35], named as KGB model and in inflation model building [43], named as G-inflation. The box term is equivalent to [−]2XG^φ [−] ^G^X [∇]^µφ∇^νφ∇µ∇ν^φ after integration by parts and dropping a surface term.

 E_1 , $\sqrt{2}a$, $Ca \alpha$ Enang, $3C/H$ (2012) + 2Fφ(∇µX∇µ^φ + 2X✷φ)+2XFφX∇µX∇µ^φ + 4X2Fφφ ⁺ ^FX(✷φ∇µX∇µ^φ − ∇µX∇µX)=0 . (21) $+ 2F(x, \text{Ca} \alpha \text{ }\alpha \text{ }\text{Znang}, \text{3C/FY}$ (2012) $T_{\rm eff}$ \sim $T_{\rm eff}$, \sim $T_{\rm eff}$. (23)

 \mathbb{R}^d redefinitions of \mathbb{R}^d and \mathbb{R}^d , the degenerate Lagrangian may be generated as \mathbb{R}^d

 \mathbb{R}^n redefinitions of \mathbb{R}^n and \mathbb{R}^n and \mathbb{R}^n and \mathbb{R}^n and \mathbb{R}^n as generate \mathbb{R}^n as a generate \mathbb{R}^n and \mathbb{R}^n and \mathbb{R}^n are generated by \mathbb{R}^n and \mathbb{R}^n a

invariance, L can only be

for the degenerate model it is

 $\mathcal{L} = K(\phi,~X) + F(\phi,~X) \nabla^{\mu} \phi \nabla^{\nu} \phi \nabla_{\mu} \nabla_{\nu} \phi = K + F \nabla_{\mu} X \nabla^{\mu} \phi~.$ $\mathcal{L} = \mathbf{R}(\varphi, \mathbf{A}) + \mathbf{R}(\varphi, \mathbf{A}) \mathbf{v}$ by $\varphi \mathbf{v}$ properties of perturbations. $\mathcal{C} = \mathcal{I}^{\mathcal{I}}(t - \mathbf{V})$ which sources the gravitational field is obtained the action of $\mathcal{L} = \mathbf{\Lambda}(\varphi, \mathbf{\Lambda}) +$ The energy momentum tensor which sources the gravitational field is obtained through the variation of the action with respect to the metric tensor, $\mathcal{L} = K(\phi, X) + F(\phi, X) \nabla^{\mu} \phi \nabla^{\nu} \phi \nabla_{\nu} \nabla_{\nu} \phi = K + F \nabla_{\nu} X \nabla^{\mu} \phi$ around the homogeneous background,

Now we will investigate whether the dark energy model from this Lagrangian can stably cross the boundary of

$$
\text{EOM} \quad K_{\phi} - 2X K_{X\phi} - K_{X}\Box\phi - K_{XX}\nabla_{\mu}X\nabla^{\mu}\phi + F[(\Box\phi)^{2} - \nabla_{\mu}\nabla_{\nu}\phi\nabla^{\mu}\nabla^{\nu}\phi + R_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi] \n+ 2F_{\phi}(\nabla_{\mu}X\nabla^{\mu}\phi + 2X\Box\phi) + 2XF_{\phi}X\nabla_{\mu}X\nabla^{\mu}\phi + 4X^{2}F_{\phi\phi} + F_{X}(\Box\phi\nabla_{\mu}X\nabla^{\mu}\phi - \nabla_{\mu}X\nabla^{\mu}X) = 0 \nT^{\mu\nu} = -(K + F\nabla_{\rho}X\nabla^{\rho}\phi)g^{\mu\nu} + (K_{X} - 2XF_{\phi} - F\Box\phi)\nabla^{\mu}\phi\nabla^{\nu}\phi + F(\nabla^{\mu}X\nabla^{\nu}\phi + \nabla^{\nu}X\nabla^{\mu}\phi) \nT^{\mu\nu} = -pg^{\mu\nu} + (\rho + p)u^{\mu}u^{\nu} + (2X)^{3/2}F(a^{\mu}u^{\nu} + a^{\nu}u^{\mu}) \np = K + F\nabla_{\mu}X\nabla^{\mu}\phi , \n\rho = -K + F\nabla_{\mu}X\nabla^{\mu}\phi + 2X(K_{X} - 2XF_{\phi} - F\Box\phi)
$$

 T \sim T

normalized as up the four acceleration which is the four acceleration which is orthogonal to the velocity, i.e., and

where by analogy with k-inflation \mathcal{A} or k-inflation \mathcal{A} with \mathcal{A} with \mathcal{A} with \mathcal{A}

This energy momentum tensor (26) does not have the form of perfect fluid due to the last two terms depending on the

p = K + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F Q + F ρ = − κ το αναφική του αναφική του αναφική του αναφική του ανα

3/2F(a) + avuun + avuu

√

√

2X which is a completely

2X which is

$$
u^{\mu} = \nabla^{\mu} \phi / \sqrt{2X}, \ a^{\mu} = u^{\rho} \nabla_{\rho} u^{\mu}
$$

where by analogy with the four velocity \mathcal{R} or the four velocity \mathcal{R} with the four velocity \mathcal{R} with \mathcal{R}

 $T_{\rm eff}$ = $\mu_{\rm eff}$ + $\mu_{\rm eff}$

$$
\phi(t,\vec{x}) = \phi(t) + \pi(t,\vec{x})
$$

$$
S^{(2)}(\pi) = \frac{1}{2} \int d^3x dt a^3 (A\dot{\pi}^2 - \frac{B}{a^2} \partial_i \pi \partial_i \pi + C\pi^2)
$$

 \mathcal{A} see from the action that the sound speed speed

S =

 \mathbb{R}^n we can see from the sound speed speed

different sound speed squared,

The classical stability requires called

personal

 \blacksquare

^C ⁼ ^d

different sound speed squared,

!

d⁴x

$$
A = K_X + 2XK_{XX} - 6H\dot{\phi}(F + XF_X) - 8XF_{\phi} - 4X^2F_{X\phi} ,
$$

\n
$$
B = K_X - 2(\ddot{\phi} + 2H\dot{\phi})F - 4XF_{\phi} - 2X\ddot{\phi}F_X ,
$$

\n
$$
C = \frac{d}{dt}(6HXF_{\phi} + 2X\dot{\phi}F_{\phi\phi} - \dot{\phi}K_{X\phi}) + 3H(6HXF_{\phi} + 2X\dot{\phi}F_{\phi\phi} - \dot{\phi}K_{X\phi}) + 2X\ddot{\phi}F_{\phi\phi} + K_{\phi\phi}
$$

If the universe is dominated by the scalar field, as discussed in the KGB model \mathcal{A}_i or the G-inflation model \mathcal{A}_i as discussed in the G-inflation model \mathcal{A}_i as discussed in the G-inflation model \mathcal{A}_i \mathbf{r} full treatment of the gravity-p coupled system based on the $(1+\epsilon)$ method will give a slightly give

sound speed depends on the gravity theory, here the gravity theory is Einstein's general relativity. For dark energy studied in this paper we will consider the sound speed in Eq. (36) to express the propagating velocity of the

s of the absence of \sim 0.0 Furthermore, the absence of the absence of ghost mode corresponds to the absence of

[√]g(^K ⁺ ^F [∇]µX∇^µφ) (33)

. <u>(36)</u>

. (36) The Contract of the Contract of

, **(37)**

p = 1/8 mg

^p = 1/8πG. This

, and \mathcal{A} , and \mathcal{A}

$$
c_s^2 = \frac{B}{A} = \frac{K_X - 2(\ddot{\phi} + 2H\dot{\phi})F - 4XF_{\phi} - 2X\ddot{\phi}F_X}{K_X + 2XK_{XX} - 6H\dot{\phi}(F + XF_X) - 8XF_{\phi} - 4X^2F_{X\phi}}
$$

If the universe is dominated by the scalar field, as discussed in the KGB model \mathbb{R}^3

full treatment of the gravity-based on the gravity-p coupled system based on the \mathcal{C} -Misner \mathcal{C} and \mathcal{C}

where both numerator and denominator and denominator and denominator are modified by the Planck mass \mathbb{R}^n

sound speed depends on the gravity theory, here the gravity theory is Einstein's general relativity. For dark en-

where both numerator and denominator and denominator and denominator are modified by the Planck mass \mathcal{P}

INITIAL CONDITIONS FOR DE PERTURBATIONS

 \mathbb{R} the evolutions of \mathbb{R}

3(1 + ^wα) [−] ^Φ ,

where $\frac{1}{2}$ is the density contrast, $\frac{1}{2}$ is the density contrast, $\frac{1}{2}$

Gauge invariant perturbations

$$
\zeta_{\alpha} = \frac{\delta_{\alpha}}{3(1+w_{\alpha})} - \Phi,
$$

$$
\Delta_{\alpha} = \frac{\rho_{\alpha}\delta_{\alpha}}{3} + \frac{\mathcal{H}}{k^2}(\rho_{\alpha} + p_{\alpha})\theta_{\alpha}
$$

ia/(ρα + pa) is the corresponding momentum density per-

/a with the prime denoting the derivative with

 α , (6) α

³H)Φ] . (5)

with the metric perturbations. In this paper, we use the following gauge-independent variables for each species

fully described by one relativistic potential μ is the matter sector, the perturbations are expressed by the perturbed by the perturbe energy-momentum tensor which is gauge dependent. However, for the discussions of perturbations on large scales it is more convenient to use gauge invariant variables constructed by combining the energy-momentum perturbations

with the metric perturbations. In this paper, we use the following gauge-independent variables for each specie

where we have implicitly assumed that the shear perturbations can be neglected and the metric perturbations are neglected and the metric perturbations are neglected and the metric perturbations are neglected and the metric \mathbb{F}_2 described by one relativistic potential \mathbb{F}_2 . In the perturbations are expressed by the perturbed by th energy-momentum tensor which is gauge dependent. However, for the discussions of perturbations on large scales it is more convenient to use gauge invariant variables constructed by combining the energy-momentum perturbations

Γ_{quation} respect to conformal time. Za is a comoving curvature perturbation, and as we will see later in this paper ∆α may be will see later and as we wil the equations governing the evolutions government the evolutions of \mathcal{L} H₂ − Equa $\overline{2}$ Equations

^ζ^α ⁼ ^δ^α

 $= 5$ δρα $= 5$

 $\frac{1}{2}$ turbation, and the conformal Hubble parameter is defined by $\frac{1}{2}$

 \mathbb{R}^2

 $\frac{1}{2}$

α/ρ!

Equations
\n
$$
\zeta'_{\alpha} + 3\mathcal{H}(c_{s\alpha}^2 - c_{a\alpha}^2) \frac{\Delta_{\alpha}}{\rho_{\alpha} + p_{\alpha}} + \frac{k^2}{3\mathcal{H}} \left(\frac{\Delta_{\alpha}}{\rho_{\alpha} + p_{\alpha}} - \zeta_{\alpha} \right) = \frac{k^2}{3\mathcal{H}} \Phi,
$$
\n
$$
\Delta'_{\alpha} + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}}) \Delta_{\alpha} - (\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}}) (\rho_{\alpha} + p_{\alpha}) \zeta_{\alpha} = (\rho_{\alpha} + p_{\alpha}) [\Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} + \frac{k^2}{3\mathcal{H}}) \Phi]
$$
\n
$$
\frac{k^2}{a^2} \Phi = -12\pi G \sum_{\alpha} \Delta_{\alpha}
$$

respect to conformal time. α is a comoving curvature perturbation, and as we will see later in this paper α called an "effective" density perturbation. The conservation of the energy-momentum tensor at the linear order

 $\overline{}$

α

On large scales $k << H$ the Newtonian gravity we can see that ∆^α may be called an effective density perturbation. On super horizon scales On large scales $k << H$ $\zeta'_\alpha + 3\mathcal{H}(c^2_{s\alpha} - c^2_{a\alpha})\frac{\Delta_\alpha}{\alpha + \Delta_\alpha}$ $\rho_{\alpha} + p_{\alpha}$ $= 0 \; ,$ $\Delta'_\alpha + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_\alpha - (\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})(\rho_\alpha + p_\alpha)\zeta_\alpha = (\rho_\alpha + p_\alpha)[\Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Phi]\;,$ $\sum \Delta_\alpha = 0 \ .$ α $\Delta_\alpha=0$.

the horizon during inflation. In the post-inflation epoch, these primordial perturbations re-enter the horizon and

need to consider the equations (7), (8) and (9). There are two types of solutions of these equations, called adiabatic and isocurvature (or entropy) modes. For adiabatic perturbation all comoving curvature perturbations α

r \sim 0. Thus, for adiabatic perturbation, the picture is simplex is simplex is simplex in \mathbb{R}^n

which can be obtained from the perturbed Einstein equations. Comparing this equation with the Poisson equation

Conserved \zeta initial conserved to specify the initial conditions of perturbation equations, we need to specify the initial conditions of the initial conditions. Usually the initial conditions of the initial conditions o are set at the time deep inside the radiation dominated era when the scales corresponding to observations today were

 $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{n} \frac{1}{j} \sum$

(1) barotropic fluids, e.g., cdm, radiation $c_{s\alpha}^2 = c_{a\alpha}^2 = p_{\alpha}'/\rho_{\alpha}'$

 \mathbb{F}_q is conserved on a perfect fluid \mathbb{F}_q is conserved on large scales. The perfect is conserved on large scales.

(1) barotropic fluids, e.g., cdm, radiation
$$
c_{s\alpha}^2 = c_{a\alpha}^2 = p'_{\alpha}/\rho'_{\alpha}
$$

(ρα + pa)ζα = ζρα + pa

interact with matter to component universe θ and structure field inflation. $\sum \Lambda_{\alpha} = \Lambda = 0$ evolutions of the perturbations on super-horizon super-horizon scales $\frac{1}{\alpha}$ (2) single component universe, e.g., single field inflation $\sum \Delta_{\alpha} = \Delta = 0$

same as that of radiation $\mathcal{L}_\mathcal{S}$ radiation $\mathcal{L}_\mathcal{S}$ comoving curvature perturbation is also equal to $\mathcal{L}_\mathcal{S}$

 $\mathcal{L}^{\mathcal{L}}$ are frozen while they are outside the horizon. With Eq. (9), the sum of Eq. (8) over all species gives gives

 α

3

 $\frac{1}{2}$ atic perturbation

same as that of radiation $\mathcal{L}_\mathcal{A}$ radiation $\mathcal{L}_\mathcal{A}$ comoving curvature perturbation is also equal to $\mathcal{L}_\mathcal{A}$

 \overline{a}

= 0 , (7)

is occursation of the isocurvature perturbation of species \mathcal{P}

 Δ = 0 . (9) Δ , (9) Δ

^r = 0. Thus, for adiabatic perturbation, the picture is simple: the primordial perturbations

 $\overline{}$

from quantum vacuum fluctuations and become classical perturbations when their corresponding length scales leave

 $\mathbf{F} = \mathbf{F} \cdot \mathbf{F$

Integration of this equation gives

is occurs to increase a isocurvature per

and Eqs. (7) and (8) become

same as that of radiation \mathcal{L}

 $\mathcal{L} = \mathcal{L} \mathcal{L}$ are outside the horizon. With Eq. (8) over all species gives gives gives gives gives gives gives gives gives

 \mathbb{R}

a2 − <mark>20</mark>

where C is a constant term on the first term on the expanding universe and can be neglected. $H_{\rm eff}$ if one of the comoving curvature perturbations α is not equal to α

 \overline{c}

a2

Adiabatic perturbation $\zeta_{\alpha} = \zeta_r = \text{constant}$., for all α Δ dichatic perturbation Δ are the person all comodes. **Same as the total comoving comoving comoving comoving comoving comoving comoving curvature perturbation** is also equal to ζ_{α} = ζ_{r} = constant. whation $\zeta = \zeta$ = constant for all α $\zeta_{\alpha} - \zeta_r - \text{constant}$, for all α \overline{a} ant., for all α ζ $\frac{1}{2}$ batic perturbation $\zeta_{\alpha} = \zeta_r = \text{constant}$, for all α $\zeta_{\alpha} = \zeta_r = \text{constant}$. δr . (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (

 $\zeta = \frac{1}{\sqrt{2\pi}}$

need to consider the equations (8), (8) and (9). There are two types of solutions of these equations, called a and isocurvature (or entropy) modes. For adiabatic perturbation all comoving curvature perturbations α

is occursation to increase the isocurvature mode. The isocurvature perturbation of species \mathcal{L}

which is constant since $\mathcal{L}^{\mathcal{A}}$ is constant since $\mathcal{L}^{\mathcal{A}}$

 $H = \frac{1}{2}$

interact with matter to cause the CMB anisotropies and structure formation. Hence it is important to inspect t

evolutions of the perturbations on super-horizon scales until the scales re-enter the horizon. For this purpose we only the scales re-

where C is a constant term on the first term on the right hand side decays in the expanding universe and can be neglected.

 \mathbb{H} if one of the comoving curvature perturbations \mathbb{H} is not equal to \mathbb{H} the density perturbation have an interval to \mathbb{H}

need to consider the equations (7), (8) and (9). There are two types of solutions of these equations of the equations, called adiabatic s

 $\rho + p$ \sum α $(\rho_{\alpha} + p_{\alpha})\zeta_{\alpha} = \zeta_r$ (it are decaying mode $\rho + p \sim \rho$ is simple in the picture is simple. $\mathcal{L}_{\mathcal{A}}$ are frozen while they are outside the horizon. With Eq. (9), the sum of Eq. (8) over all species gives g $\alpha = \zeta_r$ decaying mode $\alpha + p_{\alpha}$) $\zeta_{\alpha} = \zeta_r$ $\frac{1}{4}$ $\mathcal{L}(\mathcal{L})$, $\mathcal{L}(\mathcal{L})$, $\mathcal{L}(\mathcal{L})$ where $\frac{1}{\epsilon}$ is a constant on the expanding universe and can be neglected. $\zeta = \frac{1}{\alpha + n} \sum_{\alpha} (\rho_{\alpha} + p_{\alpha}) \zeta_{\alpha} = \zeta_r$ S_{α}) $\zeta_{\alpha} = \zeta_{\alpha}$ decayin $\frac{1}{2}$ \overline{a} \rm{de} . (13) In principle, if the universe contains N components, there should be at most N −1 isocurvature density perturbations. $\zeta = \frac{1}{\rho + n} \sum_{\alpha} (\rho_{\alpha} + p_{\alpha}) \zeta_{\alpha} = \zeta_r$ $\mathsf{P} \perp \mathsf{P}$ relation (11) relatio perturbation is still value in this case \mathcal{L} \mathcal{L} on \mathcal{L} on \mathcal{L} is one species a has isocurvature perturbation, \mathcal{L} decaying mode

.
Γ **1** - 1 - 1 - 1

ζ are frozen while they are outside the horizon. With Eq. (9), the sum of Eq. (8) over all species gives

H , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (11) , (1

 \sim

!

where C is a constant term on the first term on the right hand side decays in the expanding universe and can be

 $H_{\rm eff}$ if one of the comoving curvature perturbations α is not equal to α is not equal to α

^r = 0. Thus, for adiabatic perturbation, the picture is simple: the primordial perturbations

$$
(\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})\zeta = \Phi' + (2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Phi \longrightarrow \Phi^{\text{adi}} = C\frac{\mathcal{H}}{a^2} - \zeta_r(1 - \frac{\mathcal{H}}{a^2}\int \frac{ada}{\mathcal{H}(a)})
$$

 \blacksquare Isocurvature (entropi Integration of this equation gives Is isocurvature (entropic) perturbation $\alpha = 3(\alpha + 2)$

3(ρ + p)

Isocurvature (entropic) perturbation
\n
$$
S_{\alpha} \equiv 3(\zeta_{\alpha} - \zeta_{r}) = \frac{\delta_{\alpha}}{1 + w_{\alpha}} - \frac{3}{4}\delta_{r}
$$

 $\overline{}$

 \pm (H \sim \pm \sim \pm

= 0 , (7)

(ρα + pa)ζα = ζετ , (10)ζα = ζετ ,

H , (11)

(ρ^α + pα)ζ^α = ζ^r , (10)

S^α . (14)

, (15), (15

 $\mathcal{A} = \mathcal{A}$

 \blacksquare

, (15), (15

Sa . (14)

δ^r . (13)

, (15), (15

^ρ ⁺ ^p)ξ^α = 0 . (16)

 $\mathcal{L}(\mathcal{L}) = \mathcal{L}(\mathcal{L})$

^r = 0. Thus, for adiabatic perturbation, the picture is simple: the primordial perturbations

 $\overline{3}$

from quantum vacuum fluctuations and become classical perturbations when their corresponding length scales leave

where C is a constant term on the right hand side decays in the expanding universe and can be neglected. However, if one of the comoving curvature perturbations α is not equal to α

δr . (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (13) - (

 $\frac{1}{\sqrt{2}}$

is occurrent mode. The isocurrent perturbation of species and species and species and species and species and

H + H) + H + H + Page +

^s^α [−] ^c²

ρ + p

 $\mathcal{L} = \mathcal{L} \mathcal{L}$

$$
\zeta = \zeta_r + \frac{\rho_\alpha + p_\alpha}{3(\rho + p)} S_\alpha = \zeta_r + \frac{\xi_\alpha}{\rho + p} \qquad \qquad \xi_\alpha \equiv (\rho_\alpha + p_\alpha) S_\alpha / 3
$$

$$
\xi'_{\alpha} + 3\mathcal{H}(1 + c_{a\alpha}^2)\xi_{\alpha} + 3\mathcal{H}(c_{s\alpha}^2 - c_{a\alpha}^2)\Delta_{\alpha} = 0,
$$

$$
\Delta'_{\alpha} + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_{\alpha} + (\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})(1 - \frac{\rho_{\alpha} + p_{\alpha}}{\rho + p})\xi_{\alpha} = 0
$$

In principle, if the universe contains N components, there should be at most N −1 isocurvature density perturbations. With the presence of isocurvature modes, the equation (11) relating the potential and the total comoving curvature

 $\overline{}$

 α + α + α + α + α

$$
\Phi = \Phi^{\text{adi}} - 4\pi G \frac{\mathcal{H}}{a^2} \int \xi_{\alpha} \frac{a^3}{\mathcal{H}(a)^3} da
$$

where Φ is the contribution of the adiabatic mode given in Eq. (12), and the last term is the contribution from in Eq. (12), and the last term is the contribution from in Eq. (12), and the contribution from in Eq. (12),

the isocurvature perturbation. To get it we have used the equation H. μ 2 = 4π

ξ!

explicitly that both adiabatic and isocurvature perturbations are able to generate the metric perturbation.

$$
\zeta = \zeta_r + \frac{\xi_\alpha}{\rho + p}
$$

explicitly that both adiabatic and isocurvature perturbations are able to generate the metric perturbation.

If \alpha-species is subdominant $(\rho_{\alpha} + p_{\alpha})/(\rho + p) \rightarrow 0$ perturbation and the evolution $(\rho a + \rho a)/(\rho + \rho)$ \mathbb{R}^n is the contribution of the adiabatic mode given in Eq. (12), and the contribution from in Eq. It \alpha-species is subdominant $(\rho_{\alpha} + p_{\alpha})/(\rho + p) \rightarrow 0$

can be solved by integrating Eq. (11) and we obtain the solved by integrating Eq. (11) and we obtain

can be solved by integrating Eq. (11) and we obtain

$$
\Phi \simeq \Phi^{\text{adi}}
$$
\n
$$
\xi'_{\alpha} + 3\mathcal{H}(1 + c_{a\alpha}^{2})\xi_{\alpha} + 3\mathcal{H}(c_{s\alpha}^{2} - c_{a\alpha}^{2})\Delta_{\alpha} = 0
$$
\n
$$
\Delta'_{\alpha} + (4\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}})\Delta_{\alpha} + (\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H})\xi_{\alpha} = 0.
$$

These two equations are the basis for the discussion of dark energy isocurvature perturbations in the radiation and

matter dominated eras in the next section. We can see from Eq. (15) that the contribution of dark energy is operators in the contribution of dark energy is operators in the contribution of dark energy is operators in the c

perturbations relies on the ratio μ p) compared with μ p, where the subscript e represents dark energy. Because μ

 \mathcal{L} is conserved, qualitatively the effect of the isocurvature depends on whether \mathcal{L} grows or decays with \mathcal{L}

time. When the density of dark energy becomes significant at late time, its isocurvature perturbations could m

important contribution to the metric perturbation and we should use the equations (16) to investigate its evolution.

II. DARK ENERGY ISOCURVATURE PERTURBATION ISOCURVATURE PERTURBATION PERTURBATION PER

explicitly that both adiabatic and isocurvature perturbations are able to generate the metric perturbation.

The above equations describe how the isocurvature perturbations evolve on large scales and Eq. (14) or (15) char-

acterizes the contribution of isocurvature perturbations to the total comoving curvature perturbation. The potential

The above equations describe how the isocurvature perturbations evolve on large scales and Eq. (14) or (15) char-

acterizes the contribution of isocurvature perturbations to the total comoving comoving curvature perturbation

These two equations are the basis for the discussion of dark energy isocurvature perturbations in the radiation \mathcal{C}

explicitly that both adiabatic and isocurvature perturbations are able to generate the metric perturbation.

These two equations are the basis for the discussion of dark energy isocurvature perturbations in the radiation and

matter dominated eras in the next section. We can see from Eq. (15) that the contribution of dark energy isocurvature

perturbations relies on the ratio $\mathcal{L}_\mathcal{P}$ and $\mathcal{L}_\mathcal{P}$ and subscript e represents dark energy. Because $\mathcal{L}_\mathcal{P}$

 $\mathcal{L}(\mathcal{L})$ is conserved, qualitatively the isocurvature depends on whether $\mathcal{L}(\mathcal{L})$ grows or decays with $\mathcal{L}(\mathcal{L})$

time. When the density of dark energy becomes significant at late time, its isocurvature perturbations could m
The density of dark energy becomes significant at late time, its isocurvature perturbations could make as a se

important contribution to the metric perturbation and we should use the equations (16) to investigate its evolution.

matter dominated eras in the next section. We can see from Eq. (15) that the contribution of dark energy isocurvature

perturbations relief on the ratio μ p) compared with μ p, where the subscript e represents dark energy. Because

 \tilde{G} r is conserved, qualitatively the effect of the isocurvature depends on whether \tilde{G} grows or decays with \tilde{G}

time. When the density of dark energy becomes significant at late time, its isocurvature perturbations could make as

important contribution to the metric perturbation and we should use the equations (16) to investigate its evolution.

II. DARK ENERGY ISOCURVATURE PERTURBATION ISOCURVATURE PERTURBATION ISOCURVATURE PERTURBATION ISOCURVATURE PER

In this section we will study the dark energy isocurvature perturbations during the radiation and matter dominated

^H(a)³ da , (17)

^H(a)³ da , (17)

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 \mathbb{R}

^H(a)³ da , (17)

 for \sum porturbations The General initial conditions for DE perturbations: termination in the Communities of the Conditions.
Terminations for DE perturbations: AadiAiso **and America** initial conditions for DE perturbations: $\int_{\mathcal{C}_r}$ adiabatic. temperature and initial conditions for DF porturbations. General initial conditions for DE perturbations:

)# ⁼ ²π²

including both adiabatic and isocurvature modes, one usually introduce a vector Xⁱ with two components,

xi ak

k3 Pin (k)

The primordial power spectra Pilon School power spectra Piece School power spectra Piece School primordial which characterize the amplitudes and spectral indices, respectively. We have spectral indices, respectively. One can parameterize the power spectra as ^Pij ⁼ ^Aij (^k abatic and DE isocurvature \blacksquare mixed adiabatic and DE isocurvature metric performance metric perturbations and the isocurvature metric perturbations and therefore metric per therefore metric per therefore metric per the isocurvature. temperature and symbolically we have an interesting we have

s = n22. For simplicity we assume that new ass
The new assume that is not as in the new assume

^j (k"

^s = n¹¹ and niso

"Xi(k)X[∗]

which characterize the amplitudes and spectral indices, respectively. We have α

nadi

General initial conditions for DE perturbations:
\n
$$
\mathcal{X}_i = \begin{cases} \zeta_r & \text{adiabatic} \;, \\ S_D & \text{DE isocurvature} \;. \end{cases}
$$

^l being the transfer function of photons for the initial condition i. There are similar formulas for the CMB EE

nij−1T

 $\mathcal{L}(\mathcal{$

(k)T ^j

27 (27 A)

^s = n¹² = ⁿ11+n²²

describes the correlation between adiabatic and isocurvature perturbations [37], and Aadi

^S^e isocurvature . (28)

As we see from Eq. (17), both adiabatic and isocurvature modes can generate modes can generate metric perturbations and

). (29) The contract of the contract of

, α , α

n11+n22

(k), (35)

kon)nij−1, where Aij and nij and nij are 2−

 $\overline{\mathcal{L}}$

, (30)

^l (32)

^l(k) (33)

^T)iso, (31)

l + 2\$Aadi,iso cos ∆Cristian cos ∆Cristian cos ∆Cristian cos ∆Cristian cos ∆Cristian cos ∆Cristian cos ∆Cristia
Decembre 2011 - Cos ∆Cristian cos ∆Cristian cos ∆Cristian cos ∆Cristian cos → Cristian cos → Cristian cos → C

mordial spectra $\langle \mathcal{X}_i(\mathbf{k}) \mathcal{X}_j^*(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{ij}(k) \delta(\mathbf{k} - \mathbf{k}').$ $\mathcal{P}_{ij} = A_{ij} \left(\frac{k}{k_0}\right)^{n_{ij}-1}$ $A_{ij} = \begin{pmatrix} A_{\rm adi} & \sqrt{A_{\rm adi}} A_{\rm iso} \cos \Delta \end{pmatrix}$ $\sqrt{\sqrt{A_{\rm{adi}}A_{\rm{iso}}\cos{\Delta}}}$ and spectral indices, respectively. The amplitudes and spectral indices, respectively. Also $\sqrt{A_{\text{rel}}(A_{\text{rel}}(A_{\text{rel}})}$ $2\pi^2$ and μ (1) $\nu^*(1)$ $2\pi^2$ σ (k) $\delta(1,-1)$ and σ μ (k) $n \cdot -1$ Primordial spectra $\langle \mathcal{X}_i(\mathbf{k}) \mathcal{X}_j^*(\mathbf{k}') \rangle = \frac{1}{k^3} \mathcal{P}_{ij}(k) \delta(\mathbf{k} - \mathbf{k}')$. $\mathcal{P}_{ij} = A_{ij} \left(\frac{k}{k_0} \right)^{n_{ij} - 1}$ $A_{ij} =$ $\begin{pmatrix} A_{\text{adi}} \\ \sqrt{A_{\text{adi}}A_{\text{iso}}}\cos\Delta & A_{\text{iso}} \end{pmatrix}$, (30) \sim (30) \sim (30) \sim (30) \sim $\frac{1}{4}$ \cos $\gamma_{ij} = A_{ij}(\frac{1}{k_0})$. $\frac{1}{2}$ ra $\langle \mathcal{X}_i(\mathbf{k}) \mathcal{X}_j^*(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{ij}(k) \delta(\mathbf{k})$ $-k'$ \mathcal{D} \mathcal{A} (k) n_{ij} -1 AadiAiso $A_{ij} = \begin{pmatrix} A_{\text{adi}} & \sqrt{A_{\text{adi}}A_{\text{iso}}\cos\Delta} \end{pmatrix}$ α adian α iso cos α describes the correlation between adiabatic and isocurvature perturbations $\frac{37}{27}$ $A_{ij} = \begin{pmatrix} A_{\rm adi} & \sqrt{A_{\rm adi}A_{\rm iso}\cos\Delta} \end{pmatrix}$ $\bigvee A_{\text{adi}} A_{\text{iso}} \cos \Delta$ $2\pi^2$ and $2\pi^2$ and isocurvature modes respectively. The spectral indices are denoted by $\frac{1}{2}$ $\langle X_i(\mathbf{k}) X_j^*(\mathbf{k}') \rangle = \frac{1}{k^3} P_{ij}(k) \delta(\mathbf{k} - \mathbf{k}')$. $\mathcal{P}_{ij} = A_{ij} \left(\frac{k}{k_0} \right)^{n_{ij} - 1}$ \sim μ ₀, A_{adi} A_{adi} $\sqrt{A_{\text{adi}}A_{\text{iso}}\cos\Delta}$ $\langle \mathcal{X}_i(\mathbf{k}) \mathcal{X}_j^*(\mathbf{k}') \rangle = \frac{2\pi}{k^3} \mathcal{P}_{ij}(k) \delta(\mathbf{k} - \mathbf{k}').$ $\mathcal{P}_{ij} = A_{ij} \left(\frac{k}{k_0} \right)^{n_{ij} - 1}$ $A_{ij} = \sqrt{A_{\rm adi} A_{\rm iso}} \cos \Delta$ and $\frac{2\pi^2}{2}$ (1) $\frac{2\pi^2}{2}$ (1) $\frac{8(1-\frac{1}{2})}{2}$ $\sqrt{A_{\text{adi}}A_{\text{iso}}}\cos\Lambda$ ~ 20 μ Primordial spectra

One can parameterize the power spectra as ^Pij ⁼ ^Aij (^k

^s = n22. For simplicity we assume that ncor

describes the correlation between adiabatic and isocurvature perturbations [37], and Aadi

 $\Delta = \frac{A_{\text{adi,iso}}}{\sqrt{A_{\text{adi}}A_{\text{iso}}}}$ correlation betw , (30) , (3 rrelatio $\mathcal{A}_{\rm iso}$ / $\mathcal{A}_{\rm iso}$ $\mathcal{A}_{\rm iso}$ $\mathcal{A}_{\rm iso}$ √
Detween adiabatic and isocurvature $\cos \Delta = \frac{A_{\text{adi,iso}}}{\sqrt{A_{\text{adi,iso}}}}$ correlation between adiabatic and isocurvature and Aiso are the amplitudes of adiabatic and isocurvature modes respectively. The spectral indices are denoted by $\frac{A_{\text{adi,iso}}}{A_{\text{adi}}A_{\text{iso}}}$ correlation between adiabatic and isocurvature and Aiso are the amplitudes of adiabatic and isocurvature modes respectively. The spectral indices are denoted by $\sqrt{\frac{A_{\text{adj}}A_{\text{iso}}\cos\Delta}{\Delta}}$ A_{iso} $\cos \Delta = \frac{A_{\text{adi,iso}}}{\sqrt{2\pi}}$ correlation bety $\sqrt{A_{\rm adi}A_{\rm iso}}$ correlation between adiabatic and isocurvature temperature and the model control called the symbolical lying $\mathcal{L}_\mathcal{A}$ $\cos \Lambda = \frac{A_{\text{adi,iso}}}{\text{correlation between adiabatic and isocumvature}}$ $\overline{}$ $v \cdot \text{rad}(1 \cdot \text{H} \text{so})$ $\cos \Delta = \frac{A_{\text{adi,iso}}}{\sqrt{A_{\text{tot}}A_{\text{tot}}}}$ correlation between adiabatic and isocurvature relation between adiabatic and isocurvat correlation between adiabatic and isocurvature

CMB spectrum
$$
\frac{\delta T}{T} = (\frac{\delta T}{T})_{\text{adi}} + (\frac{\delta T}{T})_{\text{iso}},
$$

$$
C_l = A_{\text{adi}} \hat{C}_l^{\text{adi}} + A_{\text{iso}} \hat{C}_l^{\text{iso}} + 2\sqrt{A_{\text{adi}} A_{\text{iso}}} \cos \Delta \hat{C}_l^{\text{adi,iso}} \quad \hat{C}_l^{ij} = \frac{4\pi}{2l+1} \int d\ln k (\frac{k}{k_0})^{n_{ij}-1} \Theta_l^i(k) \Theta_l^j(k)
$$
Matter power spectrum

Matter power spectrum and B polarization spectra and temperature-polarization spectra and temperature-polarization spectrum T E.

where

 $\overline{}$

and BB polarization spectra and temperature-polarization spectrum T E.

The isocurvature perturbations also affects the matter power spectrum P(k) as follows, and matter power spectrum

where cos

AadiAiso

where cos ∆ = [√]

where cos ∆ = [√]

^s = n¹¹ and niso

^s = n¹¹ and niso

 \sim

with Θi

where $\mathcal{L} = \mathcal{L}$

temperature anisotropies. Symbolically we have

Aadi,iso

Aadi,iso

$$
P(k) = A_{\text{adi}} \hat{P}^{\text{adi}}(k) + A_{\text{iso}} \hat{P}^{\text{iso}}(k) + 2\sqrt{A_{\text{adi}} A_{\text{iso}}} \cos \Delta \hat{P}^{\text{adi,iso}}(k)
$$

$$
\hat{P}^{ij}(k) = \left(\frac{k}{k_0}\right)^{n_{ij} - 1} T^i(k) T^j(k),
$$

P(k) = AadiPˆadi(k) + AisoPˆiso(k)+2\$AadiAiso cos ∆Pˆadi,iso(k) , (34)

k0

Cl - Aadin - A
Aadin - Aadin - Aadin

(k) being the transfer functions of matter perturbation for initial condition i.

l being the transfer function of photons for the initial condition i. The function is the condition of the CMB EEEE

l + AisoCòisoche ann an aisoche ann an an aisoche ann an aisoche ann an aisoche ann an aisoche ann an aisoche
Le

k0)

In order to show the effects of the isocurvature perturbations on CMB and LSS observations, we plot in Fig.1 the TT

^Pˆij (k)=(^k

SUMMARY (I)

- DE perturbations have important effects and usually increase the parameter degeneracies
- Singularity w_D=-1 needs to be handled carefully
- Current data are not sensitive to the sound speed and initial perturbations of DE
- Models corresponds to imperfect fluid need more studies

ANISOTROPIC ROTATION ANGLE INDUCED BY DE COUPLING

"Quintessence and the rest of the world", S. Carroll, PRL (1998)

Quintessence $\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi)$

Nearly massless $m_{\phi} \sim \sqrt{V_{\phi\phi}} < H_0 \sim 10^{-33} \text{eV}$

Hypothetical couplings (besides the gravity) to SM particles: 1, direct coupling

$$
c\frac{\phi}{M}\mathcal{L}(\bar{\psi}\psi, F_{\rho\sigma}F^{\rho\sigma}, G_{\rho\sigma}G^{\rho\sigma},)
$$

A. Long range force, violates equivalence principle, constrained to $c \leq$ $10^{-4}(M/M_{pl});$

B. Instability under quantum corrections, $\delta m_{\phi} \simeq \frac{\Lambda^2}{4\pi M} \sim 10^{-7} \text{eV} >> m_{\phi}$, $M \sim M_{pl}$, $\Lambda \sim \Lambda_{ew}$.

2, derivative coupling, pseudo-Goldstone originated from *U*(1) symmetry breaking $\overline{}$ t_{0} $F(x) = \frac{1}{2} \int_{0}^{x} F(x) \sin(x) dx$ \blacksquare breaking-invariant, but the action integral \blacksquare \blacksquare

$$
\frac{c}{M}\partial_{\mu}\phi\mathcal{O}^{\mu}(\psi, F_{\rho\sigma}, G_{\rho\sigma}, \ldots)
$$

A. shift symmetry $\phi \rightarrow \phi + const.$, guarantees the flatness of the potential; B. propagates spin-dependent force, short range, much weaker constraint from astrophysics $M \geq 10^{10}$ Gev, PDG . W applying the modified theory to CMB, the CMB, the Chern-Simons term is expected to be significant only in the era \sim A. shift symmetry $\varphi \to \varphi + const.$, guarantees the flatness of the potential;

Derivative coupling to photons

$$
\frac{c}{M} \partial_{\mu} \phi A_{\nu} \widetilde{F}^{\mu\nu} \rightarrow -\frac{c}{2M} \phi F_{\mu\nu} \widetilde{F}^{\mu\nu}
$$
\nRotation of polarization

\nRotation angle
$$
\chi = \frac{c}{M} \Delta \phi, \ \Delta \phi = \phi(\eta, \ \vec{x})|_{source} - \phi(\eta, \ \vec{x})|_{receiver}
$$
\nChange of Stokes parameters
$$
(\tilde{Q} \pm i\tilde{U}) = \exp(\pm i2\chi)(Q \pm iU)
$$

where we have used tilde to denote the rotated parameter. More details and the rotated parameter of the rotated \mathbb{R}^2

 $\mathcal{L}(\mathcal{$

where $\frac{1}{2}$ is details can be found in Ref. [13]. More de Applied to CMB, source=last scattering surface

$$
T(\hat{\boldsymbol{n}}) = \sum_{lm} a_{T,lm} Y_{lm}(\hat{\boldsymbol{n}})
$$

$$
(Q \pm iU)(\hat{\boldsymbol{n}}) = \sum_{lm} a_{\pm 2,lm} \pm 2Y_{lm}(\hat{\boldsymbol{n}}).
$$

 E/B decomposition $\frac{1}{2}$ $a_{Elm} \pm i a_{Blm} = - \int d\Omega_{+2} Y_{lm}^*(\hat{\bf n}) (\Omega \pm iU)(\hat{\bf n})$ ization patterns of operation patterns of operation particle particle particle particle particle \mathcal{I} $(Q \pm iU)(\hat{\bf{n}}) = -\sum (a_{E,lm} \pm i a_{B,lm}) \pm i2 Y_{lm}(\hat{\bf{n}})$ *lm* $a_{E,lm} \pm i a_{B,lm} = -$ Z $d\Omega \pm 2Y^*_{lm}(\hat{\bf n}) (Q \pm iU)(\hat{\bf n})$

Power spectra $\langle a_{X',l'm'}^* a_{X,lm} \rangle = C_l^{X'X} \delta_{l'l} \delta_{m'm}$ $X, X' = T, E, B$

with the assumption of statistical isotropy. In the equation above, X" and X denote the temperature T and the t

ς.
(Q, iU) = experimental production of its control to its control of its control of its control of the internet

Parameter and B modes of the polarization field, respectively. The statistical properties of the CMB modes of the statistical properties of the CMB modes of the SMB modes of the SMB modes of the CMB modes of the CMB modes

Simons coupling, CT B

background and fluctuation, and flu

$$
\tilde{a}_{E,lm} \pm i \tilde{a}_{B,lm} = - \int d\Omega \pm 2Y_{lm}^*(\hat{\mathbf{n}})(\tilde{Q} \pm i\tilde{U})(\hat{\mathbf{n}})
$$

$$
= - \int d\Omega \pm 2Y_{lm}^*(\hat{\mathbf{n}}) e^{\pm 2i\chi(\hat{\mathbf{n}})} (Q \pm iU)(\hat{\mathbf{n}})
$$

cases. So we also have a power spectrum for the rotation angle. As usual, we see also have an age of the rotation angle into its int

Isotropic rotation angle $\chi(\hat{\mathbf{n}})=\bar{\chi}=$ *c* $\frac{\text{d}}{M}[\phi(\eta_*)-\phi(\eta_0)]$

$$
\tilde{a}_{E,lm} \pm i \tilde{a}_{B,lm} = e^{\pm 2i\bar{\chi}} (a_{E,lm} \pm i a_{B,lm})
$$

$$
\begin{aligned}\n\tilde{C}_l^{TT} &= C_l^{TT}, \\
\tilde{C}_l^{TE} &= C_l^{TE} \cos(2\bar{\chi}), \\
\tilde{C}_l^{TB} &= C_l^{TE} \sin(2\bar{\chi}), \\
\tilde{C}_l^{EE} &= C_l^{EE} \cos^2(2\bar{\chi}) + C_l^{BB} \sin^2(2\bar{\chi}), \\
\tilde{C}_l^{BB} &= C_l^{EE} \sin^2(2\bar{\chi}) + C_l^{BB} \cos^2(2\bar{\chi}), \\
\tilde{C}_l^{EB} &= \frac{1}{2} \sin(4\bar{\chi}) (C_l^{EE} - C_l^{BB})\n\end{aligned}
$$

Cosmic Birefringence, CPT Violation

Feng,Li,Xia,Chen,Zhang, PRL (2006)

Li & Zhang, PRD (2008) $\mathbb{E} \left[\mathcal{L}_{\mathcal{A}} \right]$ and $\mathcal{L}_{\mathcal{A}}$ for Gaussian theories, the statistical properties of the $\mathcal{L}_{\mathcal{A}}$

The $\mathcal{L}_\mathcal{R}$ is no effect on the temperature field. The rotated polarization field (3) can be decomposed by decomposition field (3) can be decomposed by decomposition field (3) can be decomposed by decomposition fie

which depends on the three dimensional power spectrum of the spectrum of the scattering surface, and the last surface, α

ization patterns of opposite parity. The power spectra are defined as

l = 0.0

background and fluctuation, and fluctuation, and fluctuation, and fluctuation, and

decomposed by the spherical harmonics,

$$
\chi(\hat{\mathbf{n}}) = \frac{c}{M} [\phi(\eta_*, \vec{x}_*) - \phi(\eta_0)] = \frac{c}{M} [\phi(\eta_*, (\eta_0 - \eta_*)\hat{\mathbf{n}}) - \phi(\eta_0)] = \bar{\chi} + \delta \chi(\hat{\mathbf{n}})
$$

$$
\phi(\eta_*, \Delta \eta \hat{\mathbf{n}}) = \phi(\eta_*) + \delta \phi(\eta_*, \hat{\mathbf{n}})
$$

cases. So we also have a power spectrum for the rotation angle. As usual, we separate the rotation angle into i

^X! ,l!m!aX,lm" ⁼ ^C^X!

 $\overline{}$

DE perturbation at last scattering surface

3

l de la segunda de

induced anisotropic rotation angle $\ddot{\text{in}}$ the same way except the spectra should be denoted by tildes. We assume the field $\ddot{\text{in}}$ denoted $\ddot{\text{in}}$ denoted $\ddot{\text{in}}$ denoted $\ddot{\text{in}}$ induced all so also also a Gaussian random variable, as we have done for studies of cosmic scalar fields in other fields in $\$ $indu$ anisotropic rotation angle

$$
\delta \chi(\hat{\mathbf{n}}) = \frac{c}{M} [\phi(\eta_*, \Delta \eta \hat{\mathbf{n}}) - \phi(\eta_*)] = \frac{c}{M} \delta \phi(\eta_*, \hat{\mathbf{n}})
$$

With assumed statistical isotropy of blam, the angular power spectrum of the rotation angle is defined as \mathbb{R}^n

Probing anisotropic rotation angle is helpful to investigate the dynamics of the background part of the background part of the sky and the perturbation of the perturbation \mathbb{R}^n $\frac{1}{2}$ anisotropic rotation angle is helpful to investigate the dynamics of

$$
\delta \chi(\hat{n}) = \sum_{lm} b_{lm} Y_{lm}(\hat{n}) \qquad \langle b^*_{l'm'} b_{lm} \rangle = C_l^{\chi} \delta_{l'l} \delta_{m'm}
$$

$$
C_l^{\chi} = \frac{4\pi c^2}{M^2} \int d\ln k \mathcal{P}_{\phi}(k, \eta_*) j_l^2(k\Delta \eta) \qquad \text{Angular spectrum of rotation angle}
$$

!a[∗]

New distortions due to anisotropic rotation

$$
\begin{array}{rcl}\n\text{(up to second order)} & (\tilde{Q} \pm i\tilde{Q}) & = & e^{\pm 2i(\tilde{\chi}+\delta\chi)}(Q\pm iU) \\
\tilde{C}_{l}^{TE} & = & C_{l}^{TE}\cos\left(2\bar{\chi}\right)(1-2\langle\delta\chi^{2}\rangle) \;, \\
\tilde{C}_{l}^{FB} & = & C_{l}^{TE}\sin\left(2\bar{\chi}\right)(1-2\langle\delta\chi^{2}\rangle) \;, \\
\tilde{C}_{l}^{FE} & = & [C_{l}^{EE}\cos^{2}\left(2\bar{\chi}\right)+C_{l}^{BB}\sin^{2}\left(2\bar{\chi}\right)](1-4\langle\delta\chi^{2}\rangle) \\
& & + & \sum_{l_{1}l_{2}} \left(\begin{array}{cc} l & l_{1} & l_{2} \\ 2 & -2 & 0 \end{array}\right)^{2} \frac{(2l_{1}+1)(2l_{2}+1)}{2\pi} C_{l_{2}}^{X}\{[1+(-1)^{L+1}\cos\left(4\bar{\chi}\right)]C_{l_{1}}^{EE} + [1+(-1)^{L}\cos\left(4\bar{\chi}\right)]C_{l_{1}}^{BB}\} \\
\tilde{C}_{l}^{BB} & = & [C_{l}^{EE}\sin^{2}\left(2\bar{\chi}\right)+C_{l}^{BB}\cos^{2}\left(2\bar{\chi}\right)](1-4\langle\delta\chi^{2}\rangle) \\
& & + & \sum_{l_{1}l_{2}} \left(\begin{array}{cc} l & l_{1} & l_{2} \\ 2 & -2 & 0 \end{array}\right)^{2} \frac{(2l_{1}+1)(2l_{2}+1)}{2\pi} C_{l_{2}}^{X}\{[1+(-1)^{L}\cos\left(4\bar{\chi}\right)]C_{l_{1}}^{EE} + [1+(-1)^{L+1}\cos\left(4\bar{\chi}\right)]C_{l_{1}}^{BB}\} \\
\tilde{C}_{l}^{EB} & = & \frac{1}{2}\sin\left(4\bar{\chi}\right)(C_{l}^{EE}-C_{l}^{BB})(1-4\langle\delta\chi^{2}\rangle) \\
& & + & \sin\left(4\bar{\chi}\right)\sum_{l_{1}l_{2}}
$$

Background evolution breaks Lorentz and CPT, perturbations are stochastic

Power conservation

$$
\sum_{l} (2l+1)(\tilde{C}_{l}^{EE} + \tilde{C}_{l}^{BB})
$$
\n
$$
= \sum_{l} (2l+1)(C_{l}^{EE} + C_{l}^{BB})(1 - 4\langle \delta \chi^{2} \rangle) + \sum_{ll_{1}l_{2}} \left(\begin{array}{cc} l & l_{1} & l_{2} \\ 2 & -2 & 0 \end{array}\right)^{2} \frac{(2l+1)(2l_{1}+1)(2l_{2}+1)}{\pi} C_{l_{2}}^{\chi} (C_{l_{1}}^{EE} + C_{l_{1}}^{BB})
$$
\n
$$
= \sum_{l} (2l+1)(C_{l}^{EE} + C_{l}^{BB})(1 - 4\langle \delta \chi^{2} \rangle) + 4\langle \delta \chi^{2} \rangle \sum_{l_{1}} (2l_{1}+1)(C_{l_{1}}^{EE} + C_{l_{1}}^{BB})
$$
\n
$$
= \sum_{l} (2l+1)(C_{l}^{EE} + C_{l}^{BB})
$$

Consistent with $\tilde{Q}^2 + \tilde{U}^2 = Q^2 + U^2$

First CMB Constraints on Direction-Dependent Cosmological Birefringence from WMAP-7 arXiv:1206.5546

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A Chern-Simons coupling of a new scalar field to electromagnetism may give rise to cosmological birefringence, a rotation of the linear polarization of electromagnetic waves as they propagate over cosmological distances. Prior work has sought this rotation, assuming the rotation angle to be uniform across the sky, by looking for the parity-violating TB and EB correlations a uniform rotation produces in the CMB temperature/polarization. However, if the scalar field that gives rise to cosmological birefringence has spatial fluctuations, then the rotation angle may vary across the sky. Here we search for direction-dependent cosmological birefringence in the WMAP-7 data. We report the first CMB constraint on the rotation-angle power spectrum $C_L^{\alpha\alpha}$ for multipoles between $L = 0$ and $L = 512$. We also obtain a 68% confidence-level upper limit of $\sqrt{C_2^{\alpha\alpha}/(4\pi)} \lesssim 1^{\circ}$ on the quadrupole of a scale-invariant rotation-angle power spectrum.

arXiv:1206.5546v2 [astro-ph.CO] 7 Oct 2012

and Two-point ■ Two-point correlations

They are related to the power spectra as

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 T are related to the power spectra as \mathcal{L}

⁼ !

 \mathbb{R}^n is the Wigner small d function and cos \mathbb{R}^n

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l + Cooperation

^l = 2π

 \mathcal{F}_max and (13),

 $\tilde{\xi}_{+} \equiv \langle (\tilde{Q}+i\tilde{U})^{*}(\hat{\boldsymbol{n}})(\tilde{Q}+i\tilde{U})(\hat{\boldsymbol{n'}})\rangle$ $\tilde{\xi}_- \equiv \langle (\tilde{Q}+i\tilde{U})(\hat{\bm{n}})(\hat{Q}+i\tilde{U})(\hat{\bm{n'}}) \rangle$ $\tilde{\xi}_X \equiv \langle T(\hat{\bm n}) (\tilde{Q} + i \tilde{U}) (\hat{\bm n'}) \rangle \; .$)).

displacements. In the spherical coordinate system with the origin was set to the position of the observer, we need

only evaluate the following three rotated correlation functions for the polarization field by taking nˆ along the z−axis Two-point correlations function functions for the polarization field by taking n° along the polarization for the polarization function for the polarization for the $\frac{1}{2}$ along the $\frac{1}{2}$ along the $\frac{1}{2}$ along t

)" WMAP9+QUaD+BICEP

$$
\tilde{\xi}_{+}(\beta) = \sum_{lm,l'm'} \langle (\tilde{a}_{E,lm}^{*} - i\tilde{a}_{B,lm}^{*})(\tilde{a}_{E,l'm'} + i\tilde{a}_{B,l'm'})\rangle_{2} Y_{lm}^{*}(\hat{n})_{2} Y_{l'm'}(\hat{n'})
$$
\n
$$
= \sum_{lm} (\tilde{C}_{l}^{EE} + \tilde{C}_{l}^{BB})_{2} Y_{lm}^{*}(\hat{n})_{2} Y_{lm}(\hat{n'})
$$
\n
$$
= \sum_{l} \frac{2l+1}{4\pi} (\tilde{C}_{l}^{EE} + \tilde{C}_{l}^{BB}) d_{22}^{l}(\beta) ,
$$

Taylor expansion. The correlation function only depends on the separation of the two points and is invariant under displacements. In the spherical coordinate system with the origin with the origin with the observer, we need to

$$
\tilde{\xi}_{-}(\beta) = \sum_{l} \frac{2l+1}{4\pi} (\tilde{C}_{l}^{EE} - \tilde{C}_{l}^{BB} + 2i \tilde{C}_{l}^{EB}) d_{-22}^{l}(\beta) ,
$$
\n
$$
\tilde{\xi}_{X}(\beta) = -\sum_{l} \frac{2l+1}{4\pi} (\tilde{C}_{l}^{TE} + i \tilde{C}_{l}^{TB}) d_{02}^{l}(\beta) ,
$$

<u>" 1</u>

 $\frac{1}{\sqrt{2}}$

²²(β)d cos β

$$
\tilde{\xi}_{+}(\beta) = \langle \exp\left[2i(\chi(\hat{\boldsymbol{n}'}) - \chi(\hat{\boldsymbol{n}}))\right](Q + iU)^{*}(\hat{\boldsymbol{n}})(Q + iU)(\hat{\boldsymbol{n}'})\rangle
$$
\n
$$
= \langle \exp\left[2i(\chi(\hat{\boldsymbol{n}'}) - \chi(\hat{\boldsymbol{n}}))\right]\rangle \xi_{+}(\beta)
$$
\n
$$
= \exp\left[-2\langle[\delta\chi(\hat{\boldsymbol{n}'}) - \delta\chi(\hat{\boldsymbol{n}})]^{2}\rangle\right] \xi_{+}(\beta)
$$
\n
$$
= \exp\left[-4C^{\chi}(0) + 4C^{\chi}(\beta)\right] \xi_{+}(\beta)
$$
\n
$$
= \exp\left[-4C^{\chi}(0) + 4C^{\chi}(\beta)\right] \sum_{l} \frac{2l+1}{4\pi} (C_{l}^{EE} + C_{l}^{BB}) d_{22}^{l}(\beta)
$$

l!

the MCMC package to search for or put the MCMC package to search for or put the constraints on α

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2l

Before that we note that we have seen as the

e e cos(4 que e cos)e−4C

 \blacksquare

C˜EE

l − C∑

C

2l+1 4π

^l + C˜BB

C

E

^l − C˜BB

l = sin(4 ⊂γ)e−4Cχ(0).
.

Furthermore, according to the Eq. (3), the rotated correlation function is related to the unrotated correlation

 $=$ $\frac{1}{2}$ $\frac{1}{2$

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| _{βγ}(π) − δχ(π)]

 $=$ $\frac{1}{2}$ $\frac{1}{2}$

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) − δχ(nˆ)]²#]ξ+(β)

2l + 1 4π

(CEE

^l + CBB

^l)d^l

 ω and ω the small effect of the correlation between the unrotated polarization angle and the unrotated polarization and the unrotated polarization and the unrotated polarization angle and the unrotated polarization f Gaussian variable $\langle e^{ix} \rangle = e^{-\langle x \rangle/2}$ $=\sum_{l=1}^{\infty} \frac{2l+1}{c_l^{\chi}} P_l(\cos \beta)$ Variance $C^{\chi}(0) = \sum_{l=1}^{\infty} \frac{2l+1}{4\pi}$ for a Gaussian variable x (here it is so mean. The notation CX β) represents the two point correlation CX represents the two point correlation β $\tilde{C}_l^{EE} + \tilde{C}_l^{BB} = e^{-4C^{\chi}(0)} \sum$ $\tilde{C}EE \quad \tilde{C}BB = \cos(4\bar{x})e^{-4C^{\chi}(0)}$ Ωl $\overline{}$ $\tilde{C}^{EE}_l-\tilde{C}^{BB}_l=\cos(4\bar{\chi})e^{-4C^\chi(0)}\sum \frac{2l'+1}{2}(C^{EE}_{l'}-C^{BB}_{l'})\int^1 \, d^{l'}_{-22}(\beta)d^{l}_{-22}(\beta)e^{-4C^\chi(\beta)}d\cos\beta$ $\tilde{C}_l^{EB} = \sin(4\bar{\chi})e^{-4C^{\chi}(0)}\sum \frac{2l'+1}{4}(C_{l'}^{EE} - C_{l'}^{BB})\int_{}^{1} d_{-22}^{l'}(\beta)d_{-22}^{l}(\beta)e^{-\beta}$ $\tilde{C}^{TE}_l = \mathrm{co}$ $\sqrt{2}$ $\overline{\chi}$) $e^{-2C^{\chi}(0)}\sum_{\alpha} \frac{2C^{\alpha}}{2}$ $+1$ $\tilde{C}_l^{TE} = \cos(2\bar{\chi})e^{-2C^{\chi}(0)}\sum \frac{2l'+1}{2}C_{l'}^{TE}\int_{}^{1}d_{02}^{l'}(\beta)d_{20}^{l}(\beta)d\cos\beta = C_{l}^{TE}\cos(2\bar{\chi})e^{-2\theta}$ $\tilde{C}_l^{TB} = \sin(2\bar{\chi})e^{-2C^{\chi}(0)}\sum$ " ¹ l Pl(cos β) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , (19) , $C^{x}(p) = \langle \delta \chi(n) \delta \chi(n') \rangle = \sum_{l} \frac{1}{4\pi} C_l^{x} P_l(\cos p)$ variance $C^{x}(0) = \sum_{l}$ l' $2l' + 1$ $\frac{1}{2} (C_{l'}^{EE} + C_{l'}^{BB})$ \int_1^1 −1 $\tilde{C}_{l}^{EE}+\tilde{C}_{l}^{BB}=e^{-4C^{\chi}(0)}\sum\frac{2l'+1}{2}(C_{l'}^{EE}+C_{l'}^{BB})\int^{\tau}d_{22}^{l'}(\beta)d_{22}^{l}(\beta)e^{4C^{\chi}(\beta)}d\cos\beta.$ $\ddot{\mathbf{r}}$ \overline{R} $C^{EE}_{l'}-C^{BB}_{l'}\big)\int_{-1}^{1}d^l_{-}$ −1 $\tilde{C}^{EB}_l=\sin(4\bar{\chi})e^{-4C^\chi(0)}\sum\frac{2l'+1}{4}(C^{EE}_{l'}-C^{BB}_{l'})\int^1 \, d^{l'}_{-22}(\beta)d^l_{-22}(\beta)e^{-4C^\chi(\beta)}d\cos\beta$ $\sum_{}^{}$ $\frac{2\ell+1}{2}C_l^{TE}\int$ \overline{a} $\tilde{C}_l^{TE} = \cos(2\bar{\chi})e^{-2C^{\chi}(0)}\sum \frac{2l'+1}{2}C_{l'}^{TE}\int_{}^{1} d_{02}^{l'}(\beta)d_{20}^{l}(\beta)d\cos\beta = C_{l}^{TE}\cos(\beta)$ ^l is its variance. Substitute Eq.(17) into the first equation of (16), we have 2 (CEE) \mathbf{v} $\frac{2l}{4\pi}$. (20) l' $2l' + 1$ $\frac{1}{2} (C_{l'}^{EE} - C_{l'}^{BB})$ \int_1^1 −1 $\frac{d^2\Theta}{d^2\omega^2} (C^{EE}_{l'}-C^{BB}_{l'}) \int_{-1}^1 d_{-22}^{l'}(\beta) d_{-22}^{l}(\beta) e^{-4C^\chi(\beta)} d\cos\beta$ l' $2l' + 1$ $\frac{1}{4} (C_{l'}^{EE} - C_{l'}^{BB})$ \int_1^1 −1 $\tilde{C}_l^{EB} = \sin(4\bar{\chi})e^{-4C^{\chi}(0)}\sum_{\nu}\frac{2l^{\prime}+1}{4}(C_{l^{\prime}}^{EE}-C_{l^{\prime}}^{BB})\int_{-1}^{1}d_{-22}^{l^{\prime}}(\beta)d_{-22}^{l}(\beta)e^{-4C^{\chi}(\beta)}d\cos\beta$ l' $2l' + 1$ $\frac{1}{2} C_{l'}^{TE}$ \int_1^1 −1 $d_{02}^{l'}(\beta)d_{20}^{l}(\beta)d\cos\beta = C_{l}^{TE}\cos(2\bar{\chi})e^{-2C^{\chi}(0)}$ l' $2l' + 1$ $\frac{1}{2} C_{l'}^{TE}$ \int_1^1 −1 $\tilde{C}_l^{TB}=\sin(2\bar{\chi})e^{-2C^\chi(0)}\sum \frac{2l'+1}{2}C_{l'}^{TE}\int^1 d_{02}^{l'}(\beta)d_{20}^l(\beta)d\cos\beta=C_l^{TE}\sin(2\bar{\chi})e^{-2C^\chi(0)}\;.$ $\text{Gaussian variable} \quad \langle e^{ix} \rangle = e^{-\langle x^2 \rangle/2}$ $C^{\chi}(\beta) = \langle \delta \chi(\hat{n}) \delta \chi(\hat{n'}) \rangle = \sum \frac{2l+1}{4\pi} C^{\chi}_l P_l(\cos \beta)$ Variance $C^{\chi}(0) = \sum_l \frac{2l+1}{4\pi} C^{\chi}_l$ function of the perturbed rotation angle, i.e., $C_l + C_l = e$ $\sum_{l'} \frac{1}{2} (C_{l'} + C_{l'})$ E_1 $^{-1}$ $\int_{R_1}^{1}$, $\int_{R_2}^{1}$, (2) $\int_{R_1}^{1}$, (2) $\int_{R_2}^{R_2}$, (2) $\int_{R_3}^{R_4}$ (3) $\int_{R_4}^{R_5}$ ^l + C˜BB $\tilde{C}_l^{TE} = \cos(2\bar{\chi})e^{-2C^{\chi}(0)}$ $\begin{array}{c} \begin{array}{c} \begin{array}{c} \text{2} \\ \text{1} \end{array} \end{array}$ d_{0}^{l} $(\beta) d_{20}^{\circ}(\beta) d \cos \beta = C_l^{\alpha/2} \cos(2\chi) e^{-2\zeta(\zeta)}$ Straightforwardly one may obtain that for a Gaussian variable \mathbb{R}^n with zero mean. The notation \mathbb{R}^n represents the two point correlation \mathbb{R}^n l $2l + 1$ 4π $C_l^{\chi} P_l(\cos \beta)$ Variance $C^{\chi}(0) = \sum_l$ $\tilde{C}_{l}^{EE} + \tilde{C}_{l}^{BB} = e^{-4C^{\chi}(0)} \sum \frac{2l' + 1}{\chi_{l'}^{EE} + C_{l'}^{BB}} \int_{-a}^{b} d_{22}^{l'}(\beta) d_{33}^{B}$ \tilde{C}_l^1 \overline{R} \overline{Z} $-\tilde{C}_l^{BB} = \cos(4\bar{\chi})e^{-4}$ " 1
" 1 $\sum \frac{2l'+1}{2}(C_{l'}^{EE}-C_{l'}^{BB})\int_{}^{1}d_{-22}^{l'}(\beta)d_{-22}^{l}(\beta)e^{-4C^{2}}$ \mathbb{R} is a straightforward on \mathbb{R} $\tilde{C}^{TE}_l =$ $\sqrt{2}$ $\mathrm{s}(2\bar\chi) e^{-2C^\chi(0)} \sum_{\alpha} 2 \bar\chi$ l' - \mathbf{z} $\frac{1}{\alpha} C^{TE}_{l'}\int_{}^1 d^{l'}_{02}(\beta)d^l_{20}(\beta)d\cos\beta$ l <mark>a</mark>sin(4 ⊂χ)e−4C $E^B = \sin(2\bar{\chi})e^{-2C\chi}$ $\overline{10}$ −1 $\sum_{\alpha} \frac{2l'+1}{2} C^{TE}_{l'} \int_0^1 d^{l'}_{02}(\beta) d^l_{20}(\beta)$ $2l+1$ Variance $C^{\chi}(0) = \sum_{l} \frac{2l+1}{4\pi} C^{\chi}_{l}$ $\begin{array}{c} \n \mathcal{X} \\
 l\n \end{array}$ ^l + C˜BB $\mathcal{L}_2(\beta) d_{-22}(\beta) e^{-i \phi/\beta} d \cos \beta$ \blacksquare C˜EB ^l = sin(4 ¯χ)e[−]4Cχ(0)!

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Now we can use these formulae to compute the rotated power spectra and use the observational data together with

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$$
\tilde{C}_l^{EE}=\tilde{C}_{l,0}^{EE}+\Delta \tilde{C}_l^{EE}\ ,\quad \tilde{C}_l^{BB}=\tilde{C}_{l,0}^{BB}+\Delta \tilde{C}_l^{BB}\ ,\quad \tilde{C}_l^{EB}=\tilde{C}_{l,0}^{EB}+\Delta \tilde{C}_l^{EB}
$$

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Now we can use these formulae to compute the rotated power spectra and use the observational data together with

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²²(β)

22 (β)d cos β , (24) (β)d cos β , (24)

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Now we can use these formulae to compute the rotated power spectra and use the observational data together with

02(β)d^l

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l sin(2 ⊂χ)e−2Cχ(0) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21) . (21)

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^l , (22)

C˜T B

l = sin(2 ⊂2C)e−2C

together with C

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^l and ^C˜T B

Now we can use these formulae to compute the rotated power spectra and use the observational data together with

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Before that we note that we note

the MCMC package to search for or put the constraints on $\mathcal{A}_\mathcal{A}$

l = sin(2 −2C −2C)

l!

22 l!

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[∆]˜ξ+(β) = ^e−4Cχ(0)(e4Cχ(β) [−] 1)!

^l ⁼ ^C˜EE

 \mathbb{R}^n

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C˜EE

^l may be decomposed into the sums,

 $\frac{1}{\sqrt{2}}$ the MCMC package to search for or put the constraints on $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

2l

2 CT ESC 2011 l!

dl !

 $N_{\rm eff}$ we can use these formulae to compute the rotated power spectra and use the observational data together with $N_{\rm eff}$

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$$
\tilde{C}_{l,0}^{EE} = [C_l^{EE}\cos^2(2\bar{\chi}) + C_l^{BB}\sin^2(2\bar{\chi})]e^{-4C^{\chi}(0)}
$$
\n
$$
\tilde{C}_{l,0}^{BB} = [C_l^{EE}\sin^2(2\bar{\chi}) + C_l^{BB}\cos^2(2\bar{\chi})]e^{-4C^{\chi}(0)}
$$
\n
$$
\tilde{C}_{l,0}^{EB} = \frac{1}{2}\sin(4\bar{\chi})(C_l^{EE} - C_l^{BB})e^{-4C^{\chi}(0)}
$$
\n
$$
\tilde{C}_l^{TE} = C_l^{TE}\cos(2\bar{\chi})e^{-2C^{\chi}(0)}
$$
\nOnly depend on variance which dominates the distortion dominates the distortion

SUMMARY (II)

- DE has natural derivative couplings to SM particles
- Chern-Simons coupling induces anisotropic rotation angle which distorts CMB polarization spectra similar to weak lensing effect
- Current data showed no evidence for the non-zero spectra of rotation angle
- It predicts non-Gaussianities in CMB polarization field

THANKS!