Kerr/CFT correspondence From Thermodynamics

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- The earlier works with Jiang Long, Jia-ju Zhang, and Bo Ning.
- Some earlier works by A. Strominger et.al., A. Catro et.al., F. Larsen, M. Cvetic, et.al..

Outline

- Review of Kerr/CFT correspondence
- Inner horizon thermodynamics
- Holographic pictures from thermodynamics
- Conclusion and discussions

Black hole entropy

- "Black holes: harmonic oscillators in 21th century physics!" (A. Strominger)

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- Holographic principle in quantum gravity;
- One central issue: how to understand the entropy microscopically?
- One of the greatest achievements in string theory: for a class of 5D extremal charged BH, there exists microscopic counting; Strominger and Vafa (1996)
- It relies on string technology: D-branes configuration;
  1. The near horizon geometry includes a AdS factor;
  2. A CFT dual: Cardy's formula ...;

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- The physical degree of freedom resides only at asymptotic boundary;
- In retrospect, this provides another way to understand AdS/CFT, without resorting to string theory;
- It has led to some interesting results in the past few years:
  1. Warped AdS/CFT correspondence;
  2. Kerr/CFT correspondence;

Kerr black holes

A Kerr black hole is characterized by the mass M and angular momentum $J = aM$. It could be described by the metric of the following form

$$ds^2 = -\frac{\Delta}{\hat{\rho}^2}(d\hat{t} - a\sin^2\theta d\hat{\phi})^2 + \frac{\sin^2\theta}{\hat{\rho}^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t}\right)^2 + \frac{\hat{\rho}^2}{\Delta}d\hat{r}^2 + \hat{\rho}^2 d\theta^2,$$

with

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2, \qquad \hat{\rho}^2 = \hat{r}^2 + a^2\cos^2\theta,$$

where we have used the unit $G = \hbar = c = 1$.

- Two horizons: $r_\pm = M \pm \sqrt{M^2 - a^2}$;
- The Hawking temperature, the angular velocity of the horizon and the entropy of the Kerr black hole are

$$T_H = \frac{r_+ - r_-}{8\pi M r_+}, \qquad \Omega_H = \frac{a}{2Mr_+}, \qquad S_{BH} = 2\pi M r_+.$$

Kerr/CFT correspondence

- Conjecture: A Kerr black hole could be holographically described by a 2D CFT with

$$c_L = c_R = 12J, \qquad T_L = \frac{(r_+ + r_-)^2}{8\pi J}, \quad T_R = \frac{r_+^2 - r_-^2}{8\pi J};$$

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$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R);$$

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  2. Superradiant scattering and the low-frequency scattering with the hidden conformal symmetry: the amplitudes are in agreement with the CFT predictions.
- Feature: The set up of Kerr/CFT has nothing to do with string theory;

ASG to set up Kerr/CFT I

- Central charges: read from Asymptotic Symmetry Group(ASG) analysis of NHEK geometry;M.Guica et.al. 0809.4266
- NHEK geometry: near-horizon geometry of an extremal Kerr black hole (NHEK)J.M. Bardeen and G.T. Horowitz (1999)

$$ds^2 = 2J\Gamma\left(-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2(d\phi + rdt)^2\right),$$

where $\Gamma(\theta) = \frac{1+\cos^2\theta}{2}, \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$.

- For fixed θ, it is a warped AdS$_3$, as a $U(1)$ bundle on AdS$_2$;
- $SL(2,R)_R \times U(1)_L$ isometry group;

Asymptotic Symmetry Group(ASG)

- Given a set of consistent boundary condition $\{h_{\mu\nu}\}$;
- The associated ASG is defined as

$$\text{ASG} = \frac{\text{Allowed Symmetry Transformation}}{\text{Trivial Symmetry Transformation}}$$

- "Allowed": the asymp. Killing symmetry $\hat{\chi}$ such that $\mathcal{L}_{\hat{\chi}} g_{\mu\nu} \sim h_{\mu\nu}$;
- "Trivial": some of $\hat{\chi}$ lead to vanishing global charges;
- Moreover, consistency requires $\hat{\chi}$ not diverge at the boundary;

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- In practice, it is often hard to find a "good" boundary condition
  1. Too strong: eliminate any interesting excitations;
  2. Too weak: the generators of ASG are ill-defined;
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- In AdS_3 case, we have the Brown-Henneaux boundary condition;
- In Kerr/CFT, the boundary conditions of NHEK geometry are different, first proposed in 0809.4266;
- Via BBC formalism or Stretched horizon formalism, the ASG could be worked out;

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- The correspondence was inspired by the study of the asymptotic symmetry group of NHEK;
- It is different from Carlip's proposal of holographic CFT descriptions of black holes.
- Under a certain set of boundary conditions, the $U(1)_L$ get enhanced into a Virasoro algebra with central charge $c_L = 12J$;

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- For the extreme case, the dual temperature could be read from the Frolov-Thorne vacuum;
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- Perfect match of the macroscopic entropy of the black hole with the microscopic (CFT) entropy computed by the Cardy formula.
- This has been generalized to many other cases: Kerr BH in higher dimensions, Kerr-Newmann-AdS-dS, RN,...;
- It is only true for extreme rotating or charged black holes;

Kerr/CFT for Nonextreme BH

- Strictly speaking, only the left-moving central charge of NHEK could be read;
- Actually, the dual CFT is not chiral. Its right-moving sectors describe the deviation from the extreme
  1. $c_R = 12J$ with a different set of B.C.;Y.Matsuo et.al. (2009)
  2. c_R from AdS_2 quantum gravity;A. Castro et.al (2009)
- Open issue: no B.C. gives c_L, c_R simultaneously;Y.Matsuo and T.Nishioka 1010.4549,BC et.al. 1105.2878

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- Nonextreme Kerr/CFT: the central charges are still $c_L = c_R = 12J$ even far away from extremality;
- There is no derivation on these central charges;
- For the generic non-extreme case, they are read from the hidden conformal symmetry(HCS) in the low frequency scattering of the probe;A.Castro et.al. 1004.0996
- Let's take 4D Kerr-Newman BH as a prototype;

Kerr-Newman black hole BC and Jiang Long, 1004.5039

For the Kerr-Newman black hole with mass M, angular momentum $J = aM$ and electric charge Q, its metric takes the following form

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{1}{\rho^2}\sin^2\theta\left(adt - (r^2+a^2)d\phi\right)^2,$$

where

$$\begin{aligned} \Delta &= (r^2+a^2) - 2Mr + Q^2, \\ \rho^2 &= r^2 + a^2\cos^2\theta. \end{aligned}$$

The gauge field is $A = -\frac{Qr}{\rho^2}(dt - a\sin^2\theta d\phi)$.

- Horizons: $r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}$
- Entropy at outer horizon: $S_+ = \frac{A_+}{4} = \pi[r_+(r_+ + r_-) - Q^2]$
- Three hairs: (M, J, Q);

Charged scalar scattering

- Let us consider the complex scalar field with mass μ and charge e scattering with the Kerr-Newman black hole;
- The Klein-Gordon equation is

$$(\nabla_\mu + ieA_\mu)(\nabla^\mu + ieA^\mu)\Phi - \mu^2\Phi = 0.$$

- The ansatz $\Phi = e^{-i\omega t + im\phi}\mathcal{R}(r)\mathcal{S}(\theta)$;

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- The angular part is of the form

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\mathcal{S}\right) + \left(\Lambda_{lm} - a^2(\omega^2 - \mu^2)\sin^2\theta - \frac{m^2}{\sin^2\theta}\right)\mathcal{S} = 0.$$

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- The radial part of the wave function is of the form

$$\partial_r(\Delta\partial_r\mathcal{R}) + V_R\mathcal{R} = 0$$

with

$$\begin{aligned} V_R &= -\Lambda_{lm} + 2am\omega + \frac{H^2}{\Delta} - \mu^2(r^2 + a^2), \\ H &= \omega(r^2 + a^2) - eQr - am. \end{aligned}$$

Low-frequency limit

- In the low frequency limit,

$$\omega M << 1, \tag{1.1}$$

  the ω^2 term in the angular equation could be neglected.
- Note that the low frequency limit (1.1) is very different from the near-extreme case, where only the frequencies near the superradiant bound were studied;

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- To simplify our discussion, we focus on the massless scalar.
- The angular equation is just the Laplacian on the 2-sphere with the separation constants taking values $\Lambda_{lm} = l(l+1)$.

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- In the "Near" region,

$$r\omega << 1, \tag{1.2}$$

  the radial equation could be simplified even more, and more importantly, could be written in terms of $SL(2,R)$ quadratic Casimir (for neutral scalar);

Conformal coordinates for non-extremal BH

- Let's introduce the conformal coordinates

$$
\begin{aligned}
\omega^+ &= \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R\phi+2n_Rt}, \\
\omega^- &= \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L\phi+2n_Lt}, \\
y &= \sqrt{\frac{r_+-r_-}{r-r_-}}e^{\pi(T_L+T_R)\phi+(n_L+n_R)t},
\end{aligned}
$$

- Define locally the vector fields

$$
\begin{aligned}
H_1 &= i\partial_+ \\
H_0 &= i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right) \\
H_{-1} &= i(\omega^{+2}\partial_+ + \omega^+ y\partial_y - y^2\partial_-)
\end{aligned}
$$

which obey the $SL(2,R)$ Lie algebra: $[H_0, H_{\pm1}] = \mp iH_{\pm1}$;

- Similarly we can define another set of vector fields $(\tilde{H}_0, \tilde{H}_{\pm1})$ with $+ \leftrightarrow -$;

Casimir

- The quadratic Casimir is

$$\mathcal{H}^2 = \tilde{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1)$$
$$= \frac{1}{4}(y^2 \partial_y^2 - y\partial_y) + y^2 \partial_+ \partial_- .$$

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$$= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-.$$

- The key point: the neutral scalar Laplacian is just the $SL(2,R)$ Casimir

$$\tilde{\mathcal{H}}^2\mathcal{R}(r) = \mathcal{H}^2\mathcal{R}(r) = l(l+1)\mathcal{R}(r), \quad (1.3)$$

with the following identification:

$$n_R = 0, \qquad n_L = -\frac{1}{4M}$$
$$T_R = \frac{r_+ - r_-}{4\pi a}, \qquad T_L = \frac{(r_+ + r_-) - Q^2/M}{4\pi a}, \quad (1.4)$$

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- The vector fields are only defined locally;
- The $SL(2,R) \times SL(2,R)$ symmetry is spontaneously broken down to $U(1)_L \times U(1)_R$ subgroup by the periodic identification

$$\phi \sim \phi + 2\pi.$$

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- This provides an effective way to determine (T_L, T_R) in CFT;
- Moreover the scattering amplitude is in perfect match with CFT predictions;
- For extremal BHs, a new set of conf. coordinates; BC et.al. 1007.4269

J-picture

- As 4D Kerr-Newman is quite similar to 4D Kerr, it has a holographic description via a 2D CFT with

$$c_L = c_R = 12J, \quad T_L = \frac{(r_+ + r_-)^2 - 2Q^2}{8\pi J}, \quad T_R = \frac{r_+^2 - r_-^2}{8\pi J};$$

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- In discussing HCS, the probe scalar carries only angular quantum momentum, but no charge;
- There is a singular $J \to 0$ limit; T. Hartman et.al. 0811.4393
- Such a limit for Kerr BH is singular, and raised a question: what's the holographic dual of Schwartzschild BH?
- For Kerr-Newman, we have another holographic description which make sense even in the $J \to 0$ limit;

Q-picture

- To get so-called Q-picture, one needs to uplift the 4D metric with gauge field to 5D metic

$$ds^2 = ds_4^2 + (\ell_5 d\chi + A)^2,$$

where $\chi \sim \chi + 2\pi$.

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- The charged probe without angular momentum scattering off the BH and tells us

$$T_L^Q = \frac{(r_+ + r_-)^2 - 2Q^2}{4\pi Q^3/\ell_5}, \quad T_R^Q = \frac{r_+^2 - r_-^2}{4\pi Q^3/\ell_5},$$

from HCS in the low frequency scattering;

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- More generally, we have more novel holographic pictures interpolating between J- and Q-pictures.
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- They are generated by an $SL(2,Z)$ transformations, which originate from the modular group of the torus in the uplifted metric;BC and J.J. Zhang 1106.4148
- Physically, these novel pictures come from different U(1) directions in getting the NHEK geometry, and also from the probes carrying both angular momentum and electric charge;

BC and J.J. Zhang 1107.0543

Main insights from recent study

- Our study in the past year find that there are unexpected relations between holographic pictures and black hole inner horizon dynamics;

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- Let's go on...

Inner Cauchy horizon

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- It has been shown by M. Ansorg and J. Hennig (2008,2009) that such inner Cauchy horizon always exists provided that J and Q do not vanish simultaneously, by using the Backlund transformation;
- Moreover they showed that

$$A^+A^- = (8\pi J)^2 + (4\pi Q^2)^2. \tag{2.1}$$

- The area product is mass-independent.

Entropy product

- There has been a long-ignored fact for 4D or 5D multi-charged black hole in string theory: F.Larsen, M. Cvetic 1997

$$S_+ = 2\pi(\sqrt{N_L} + \sqrt{N_R}), \quad S_- = 2\pi(\sqrt{N_L} - \sqrt{N_R}).$$

- Then the entropy product

$$S_+ S_- = 4\pi^2 (N_L - N_R)$$

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- As a result, the entropy product $S_+ S_-$ must be mass-independent;
- This is also true for the Kerr, Kerr-Newman and other black holes which have well-established holographic pictures;
- We may take it as a criterion to see if a black hole has a holographic picture, in the Einstein(-Maxwell) gravity;
- We should take the inner horizon thermodynamics seriously.

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- While for intensive quantities, when $r_+ \leftrightarrow r_-$,

$$T_- = -T_+|_{r_+ \leftrightarrow r_-}, \qquad S_- = S_+|_{r_+ \leftrightarrow r_-}, \qquad \Omega^i_- = \Omega^i_+|_{r_+ \leftrightarrow r_-},$$

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$$T_- = -T_+|_{r_+ \leftrightarrow r_-}, \quad S_- = S_+|_{r_+ \leftrightarrow r_-}, \quad \Omega^i_- = \Omega^i_+|_{r_+ \leftrightarrow r_-},$$

- For 4D Kerr-Newman, the first law of thermodynamics for the outer horizon:

$$\mathrm{d}M = T_+ \mathrm{d}S_+ + \Omega_+ \mathrm{d}J + \Phi_+ \mathrm{d}Q.$$

- And the first law of thermodynamics for the inner horizon:

$$\mathrm{d}M = -T_- \mathrm{d}S_- + \Omega_- \mathrm{d}J + \Phi_- \mathrm{d}Q.$$

- Under reasonable assumption, we have shown that the first law of thermodynamics for the outer horizon always indicates the one for the inner horizon. BC et.al. 1206.2015

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- Strictly speaking, the inner horizon "thermodynamics" is not the thermodynamics law in the standard sense;
- Nevertheless, it could be taken as the response of the black hole under the perturbation;
- In other words, under a perturbation, the response of the black hole is characterized in the thermodynamics laws at both outer and inner horizon;
- Interestingly, the laws encode the universal information, including the central charges and temperatures of dual CFT;

Remarks

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- It breaks down for Myers-Perry black holes in $d \geq 6$, Kerr-AdS(-dS) black holes in $d \geq 4$ and RN-AdS(-dS);

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- It breaks down for Myers-Perry black holes in $d \geq 6$, Kerr-AdS(-dS) black holes in $d \geq 4$ and RN-AdS(-dS);
- 3D case is a little bit subtle, we will comment on it later;

First implications: $c_L = c_R$

- With $\beta_\pm = 1/T_\pm$, we may define the left- and right-moving sectors by

$$\beta_{R,L} = \beta_+ \pm \beta_-, \qquad S_{R,L} = \frac{1}{2}(S_+ \mp S_-).$$

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where Ω could be angular velocity or the electric potential.

- The mass-independence of entropy product indicates that

$$\frac{S_L}{S_R} = \frac{T_L}{T_R}.$$

- Therefore we have

$$c_L = c_R,$$

if we believe the Cardy formula for left-moving and right-moving sectors independently.

4D Kerr-Newman

The first laws of the outer and inner horizons imply the first laws for the right- and left-moving sectors

$$\frac{1}{2}\mathrm{d}M = T_R\mathrm{d}S_R + \Omega_R\mathrm{d}J + \Phi_R\mathrm{d}Q$$
$$= T_L\mathrm{d}S_L + \Omega_L\mathrm{d}J + \Phi_L\mathrm{d}Q,$$

with

$$T_R = \frac{r_+ - r_-}{4\pi[(r_+ + r_-)^2 - 2Q^2]}, \quad T_L = \frac{1}{4\pi(r_+ + r_-)},$$
$$S_R = \frac{\pi}{2}(r_+^2 - r_-^2), \quad S_L = \frac{\pi}{2}[(r_+ + r_-)^2 - 2Q^2],$$
$$\Omega_R = \frac{\sqrt{r_+ r_- - Q^2}}{(r_+ + r_-)^2 - 2Q^2}, \quad \Omega_L = 0,$$
$$\Phi_R = \frac{Q(r_+ + r_-)}{2[(r_+ + r_-)^2 - 2Q^2]}, \quad \Phi_L = \frac{Q}{2(r_+ + r_-)}.$$

J-picture from thermodynamics

- Consider the perturbation with only angular momentum;
- By setting $\mathrm{d}Q = 0$, we get

$$\mathrm{d}J = \frac{T_L}{\Omega_R - \Omega_L}\mathrm{d}S_L - \frac{T_R}{\Omega_R - \Omega_L}\mathrm{d}S_R.$$

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- We identify the left- and right-moving temperatures of dual CFT as

$$T^J_{L,R} = \frac{T_{L,R}}{\Omega_R - \Omega_L},$$

which are exactly the ones obtained from HCS.

- In other words, we have

$$\mathrm{d}J = T^J_L \mathrm{d}S_L - T^J_R \mathrm{d}S_R$$

in CFT, whose meaning is not clear to us.

- Using the Cardy formula $S_{R,L} = \frac{\pi^2}{3} c_{R,L} T^J_{R,L}$, we read the central charges of the dual CFT in the J-picture: $c_{L,R} = 12J$.

J-picture from thermodynamics II

- The key point in the above treatment is to obtain the microscopic dimensionless temperature $T^J_{L,R}$;
- The temperatures $T_{R,L}$ are obtained geometrically and should be of dimension $[T_{R,L}] = \mathcal{L}^{-1}$;

J-picture from thermodynamics II

- The key point in the above treatment is to obtain the microscopic dimensionless temperature $T^J_{L,R}$;
- The temperatures $T_{R,L}$ are obtained geometrically and should be of dimension $[T_{R,L}] = \mathcal{L}^{-1}$;
- One may consider the dual CFT is put in a period box of radius R_J

$$R_J = \frac{1}{\Omega_R - \Omega_L} = \frac{(r_+ + r_-)[(r_+ + r_-)^2 - 2Q^2]}{2J}.$$

such that

$$T^J_{L,R} = \frac{T_{L,R}}{\Omega_R - \Omega_L}.$$

J-picture from thermodynamics III

- Let us consider the thermodynamics law more carefully,

$$\frac{1}{2}\mathrm{d}M = T_R \mathrm{d}S_R + \Omega_R \mathrm{d}J, \ \frac{1}{2}\mathrm{d}M = T_L \mathrm{d}S_L + \Omega_L \mathrm{d}J.$$

- Considering a perturbation with $\mathrm{d}M = \omega, \mathrm{d}J = k$, then we have

$$T^J_R dS_R = R_J \left(\frac{1}{2}\omega - \Omega_R k\right), \quad T^J_L dS_L = R_J \left(\frac{1}{2}\omega - \Omega_L k\right).$$

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- Considering a perturbation with $\mathrm{d}M = \omega, \mathrm{d}J = k$, then we have

$$T_R^J dS_R = R_J \left(\frac{1}{2}\omega - \Omega_R k \right), \quad T_L^J dS_L = R_J \left(\frac{1}{2}\omega - \Omega_L k \right).$$

- In the CFT side, we suppose that

$$T_R^J dS_R = \omega_R^J - q_R^J \mu_R^J, \quad T_L^J dS_L = \omega_L^J - q_L^J \mu_L^J,$$

with $\omega_{R,L}^J$, $q_{R,L}^J$ and $\mu_{R,L}^J$ as the frequencies, the charges, and the chemical potentials of the dual perturbation.

- We may have the identification

$$\omega_{R,L}^J = \frac{R_J}{2}\omega, \ q_{R,L}^J = k, \ \mu_{R,L}^J = R_J \Omega_{R,L}^J$$

Absorption cross section

Explicitly we get the results

$$\omega^J_{R,L} = \frac{R_J}{2}\omega = \frac{T_-^2 - T_+^2}{2T_-T_+(\Omega^J_- - \Omega^J_+)}\omega,$$
$$q^J_{R,L} = k,$$
$$\mu^J_R = R_J\Omega^J_R = \frac{(T_- - T_+)(T_-\Omega^J_+ + T_+\Omega^J_-)}{2T_-T_+(\Omega^J_- - \Omega^J_+)},$$
$$\mu^J_L = R_J\Omega^J_L = \frac{(T_- + T_+)(T_-\Omega^J_+ - T_+\Omega^J_-)}{2T_-T_+(\Omega^J_- - \Omega^J_+)}. \quad (3.1)$$

They are exactly the quantities appearing in absorption cross section

$$\sigma \propto \sinh\left(\frac{\omega^J_L - q^J_L\mu^J_L}{2T^J_L} + \frac{\omega^J_R - q^J_R\mu^J_R}{2T^J_R}\right)$$
$$\times \left|\Gamma\left(h^J_L + i\frac{\omega^J_L - q^J_L\mu^J_L}{2\pi T^J_L}\right)\right|^2 \left|\Gamma\left(h^J_R + i\frac{\omega^J_R - q^J_R\mu^J_R}{2\pi T^J_R}\right)\right|^2, \quad (3.2)$$

which was obtained by considering the low-frequency scattering of a neutral scalar. BC et.al. 1001.3208,1004.5039

Q-picture from thermodynamics

- To read the Q-picture, we need to set $dJ = 0$ in the thermodynamics, then we have

$$\ell_5 \mathrm{d}Q = \frac{\ell_5 T_L}{\Phi_R - \Phi_L} \mathrm{d}S_L - \frac{\ell_5 T_R}{\Phi_R - \Phi_L} \mathrm{d}S_R.$$

- Here we include a factor ℓ_5, which is the scale of uplifted fifth dimension of the Kerr-Newman black hole, to make the left side of the equation have the dimension of horizon area, and thus make the microscopic temperatures at the right side dimensionless.

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- Here we include a factor ℓ_5, which is the scale of uplifted fifth dimension of the Kerr-Newman black hole, to make the left side of the equation have the dimension of horizon area, and thus make the microscopic temperatures at the right side dimensionless.
- In this way, we may read the temperatures and the central charges in the so-called Q-picture

$$T^Q_{L,R} = \frac{\ell_5 T_{L,R}}{\Phi_R - \Phi_L}, \quad c^Q_L = c^Q_R = 6Q^3/\ell_5,$$

which are in exact match with the ones found before: C.M.Chen et.al. 1001.2883

$$T^Q_L = \frac{(r_+ + r_-)^2 - 2Q^2}{4\pi Q^3/\ell_5}, \quad T^Q_R = \frac{r_+^2 - r_-^2}{4\pi Q^3/\ell_5}, \quad c^Q_L = c^Q_R = 6Q^3/\ell_5.$$

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$$dN = T_L dS_L - T_R dS_R. \tag{3.3}$$

- The physical reason is that the thermodynamics of black hole shows how the black hole respond with respect to the perturbation. In the case of Kerr-Newman, the perturbation should carry integer units of electric charge;
- This quantization condition is actually in accord with the quantization of momentum in the extra dimension of uplifted metric and the study in HCS;
- In 4D Kerr-Newman case, we have $\ell_5 = 1/e$;

General pictures from thermodynamics

- More generally, we may consider the perturbation with $(dJ, dQ) = dN(a, b)$ with a pair of coprime integer (a, b), then the first laws change to

$$\frac{1}{2}dM = T_R dS_R + \Omega_R^N dN$$
$$= T_L dS_L + \Omega_L^N dN,$$

from which we may read

$$T_{R,L}^{(a,b)} = \frac{1}{a/T_{R,L}^J + b/T_{R,L}^Q},$$
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- These results are consistent with the ones obtained from conventional ways. BC and J.J. Zhang 1106.4148,1107.0543

Central charges

- Let us define the entropy product function

$$\mathcal{F} \equiv \frac{S_+ S_-}{4\pi^2},$$

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- With respect to every conserved charge N_i , there could be a dual CFT picture with the central charge BC et.al. 1212.1960, 1301.0429

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$$c^i_{R,L} = 6\frac{\partial \mathcal{F}}{\partial N_i}.$$

- From the elementary pictures with respect to N_i, we may generate more pictures via $SL(n, Z)$ transformations;

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$$dM = T^+ dS^+ + \Omega_i^+ dN_i, \qquad dM = -T^- dS^- + \Omega_i^- dN_i$$

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- Quantization condition for elementary picture

$$dN_i = T_{iL} dS_L - T_{iR} dS_R$$

How to understand this relation in CFT?

Conclusion and discussions

- With respect to different kinds of perturbations which may change the conserved charges of the black hole, angular momenta or U(1) charges, the response of the black hole is encoded in the thermodynamics laws;

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- We showed how to do this in various situation: elementary pictures and general pictures;
- The effectiveness and robustness of this method have been checked in all well-known black holes with holographic descriptions;
- It could be used to study the holographic pictures of 5D black ring, in which HCS fails; BC et.al. 1208.4413

Discussions

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- In the Einstein gravity, mass-independence of entropy product is the necessary, but not sufficient condition for a black hole to have HCS; BC et.al. 1301.0429
- The results are in agreement with the general study on the black hole monodromy;A. Castro et.al. 1303.0759

Duality

- The general pictures could be generated by acting $SL(n, Z)$ on the elementary pictures;
- This $SL(n, Z)$ could originate from the modular group of torus, but could also be from the eletromagnetic duality group; BC et.al. 1212.1960
- In a sense, it is very much like the duality group in string theory;
- Certainly, for the black holes in string theory constructed by branes, this relation is more transparent;

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- In 3D, things are a little subtler;
- In 3D TMG, the diffeomorphism is anomalous. As a result, the mass-independence of entropy product is broken, even though there exist holographic descriptions for BTZ and warped BHs;
- It is interesting to see that even in this case, the thermodynamics method still give the consistent pictures;
- In fact, the method is effective for 3D BHs in TMG and NMG, and is expected to be potent in more general case; BC et.al. 1302.6643

Concluding remarks

Thermodynamics of the black hole has been well-known for almost forty years, but it still brings us new surprise. It reflects not only the holographic nature of quantum gravity, but also encodes in itself the information of the holographic pictures and the symmetry among them. It certainly deserves more intense investigations from microscopical point of view. We expect that string theory may shed light on this issue.