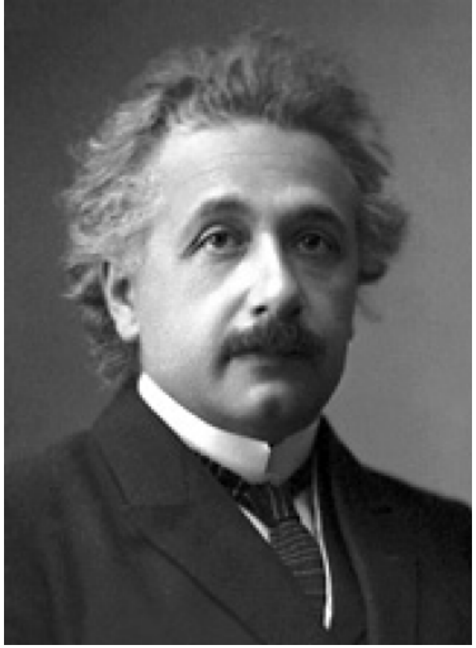
Possible Evidece for Special Relativity with de Sitter Symmetry

Mu-Lin Yan

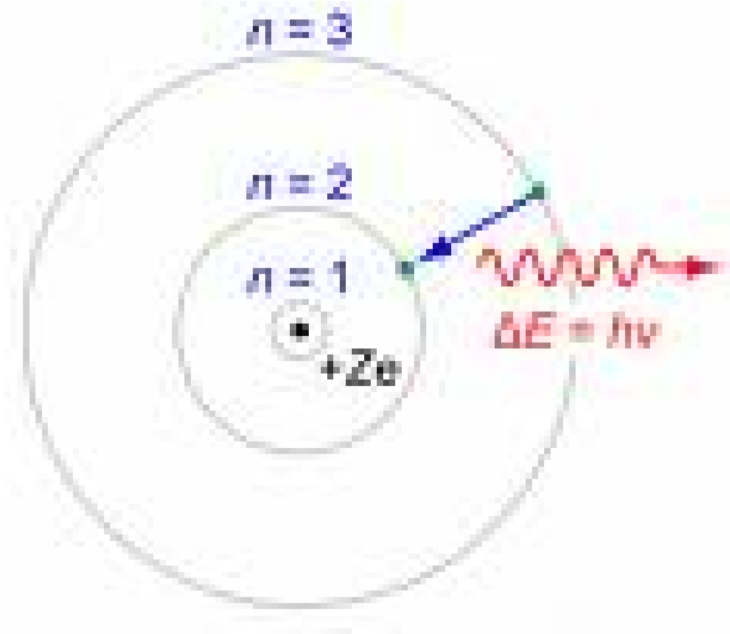
ICTS, USTC

2010 Nov



A.Einstein (1879-1955)

Part I
Fine-structure constant



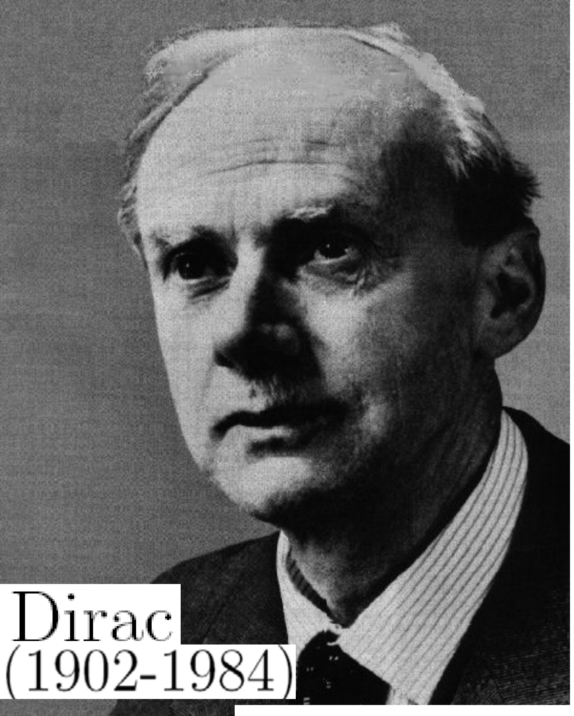
A. Sommerfeld (1868-1951)

N.Bohr (1885-1962)

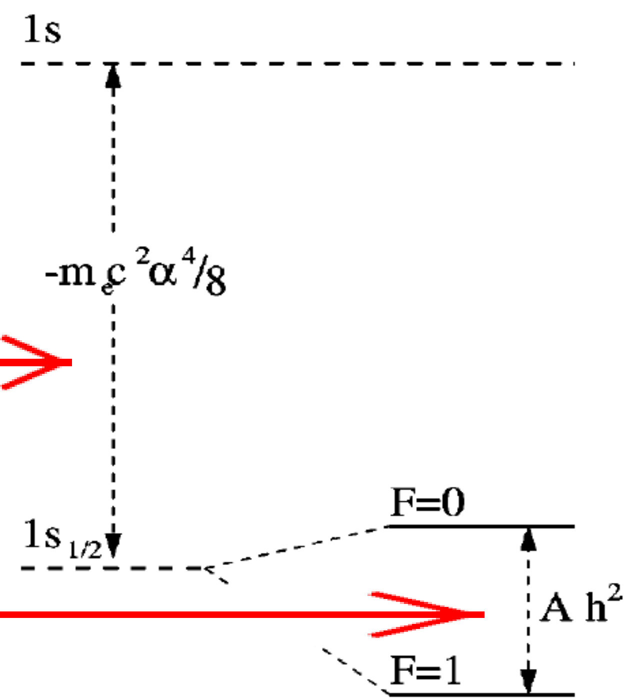
$$\alpha = \frac{e^2}{\hbar c} = (137.035999084(51))^{-1}$$

(A. Sommerfeld, 1916)

α reflects effects of Special Relativity in QM



Dirac
(1902-1984)



Fine structu

Hyperfine strcture

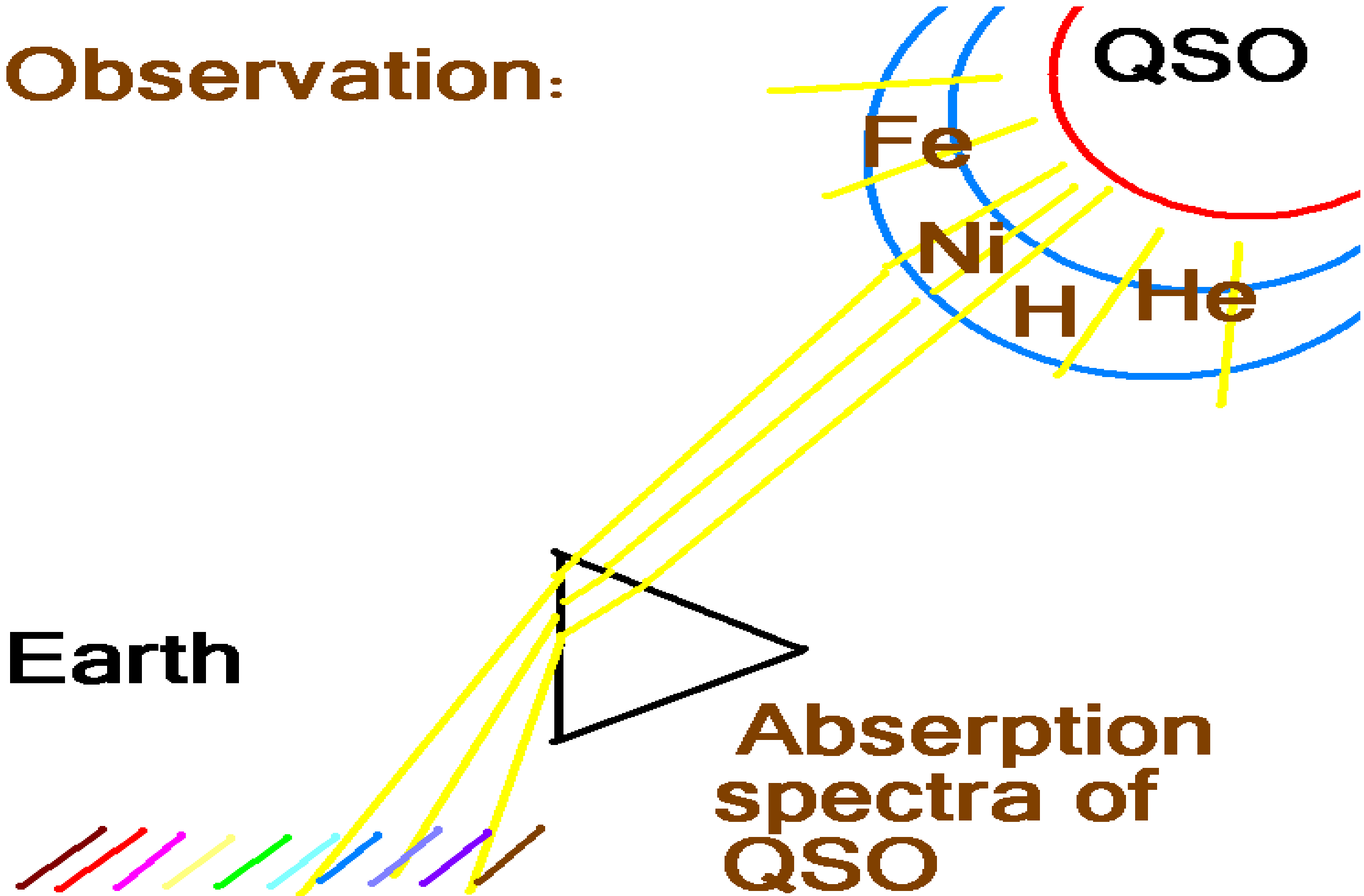
Hydrogen atom, $n = 1$

Dirac hypothesized : (1937)

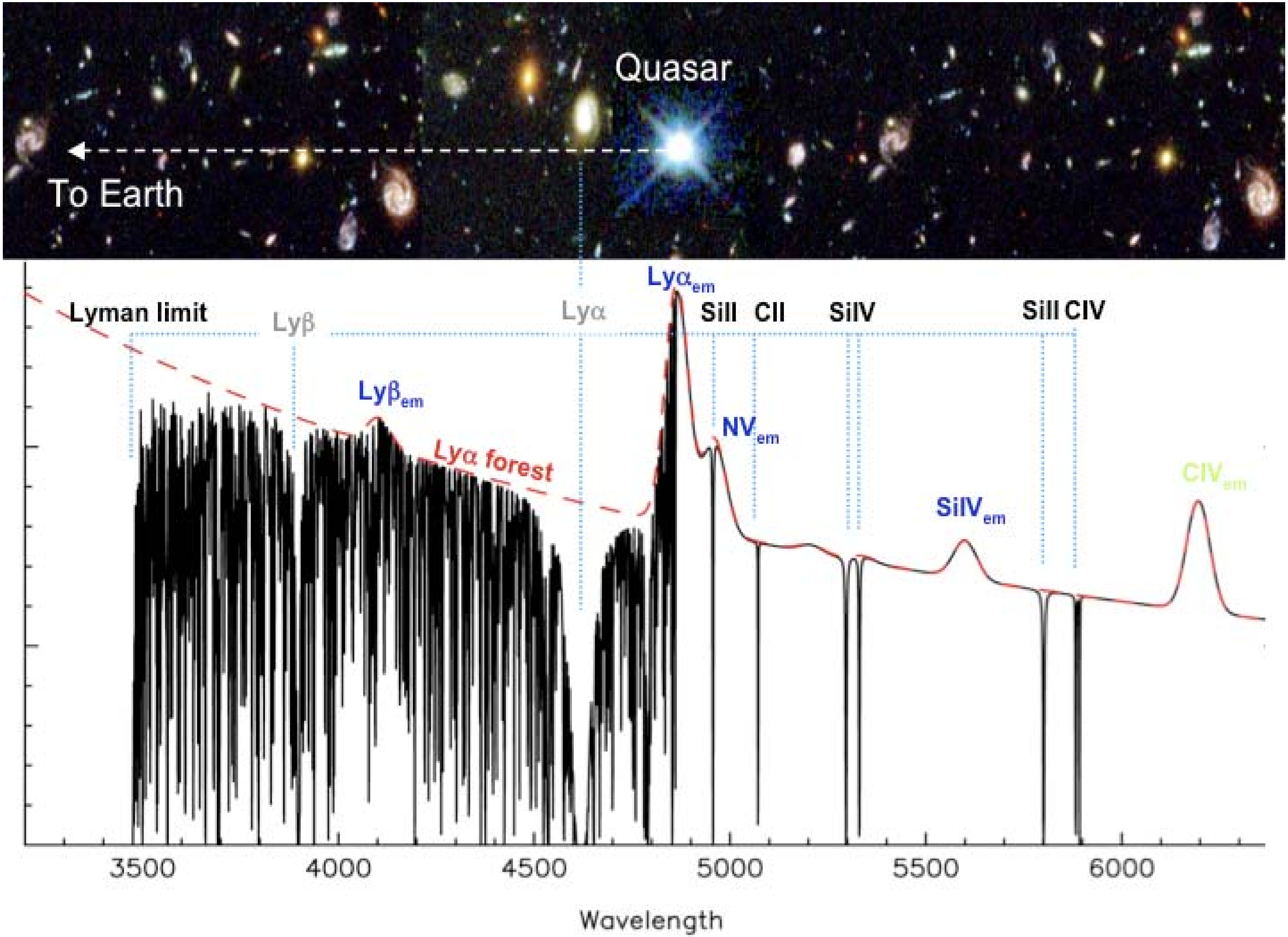
G, α may vary with time !?

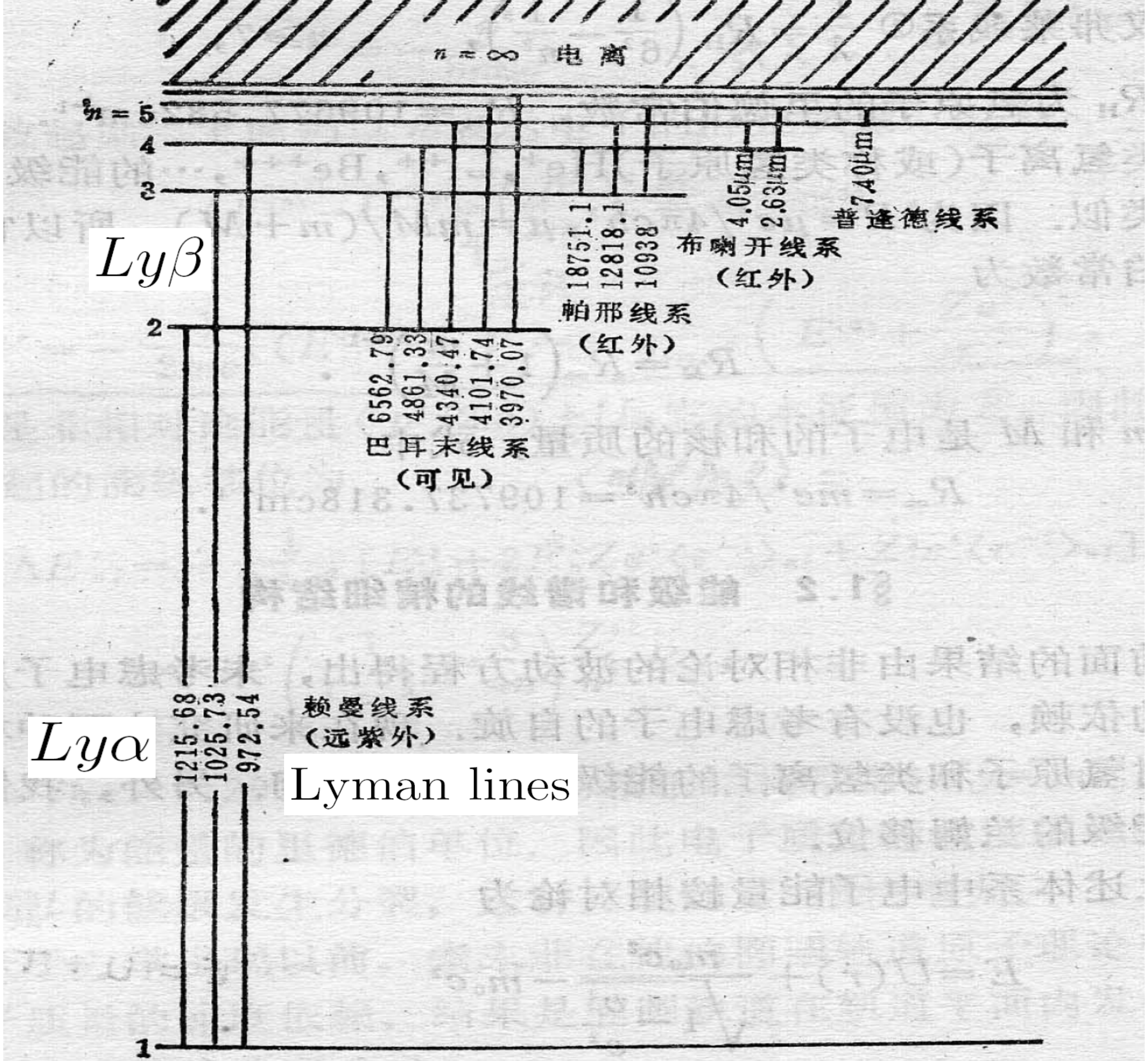
•Does Alpha vary really and truly?

to measure Alpha in several G years ago, we observe absorption spectra of QSO:



W. M. Keck Telescope 2008


Quasar
To Earth
Lyα_em
Lyman limit
Lyβ
Lyα
SiII
CII
SiIV
SiII
CIV
Lyβ_em
NV_em
Lyα forest
CIV_em
SiIV_em
3500
4000
4500
5000
5500
6000
Wavelength



Spectra of Hydrogen atom

- **Absorption spectra are measurble on the Earth:**

$$\omega^{i}_{aQSO}, \quad with \;\; i = Ly\alpha, \;\; Ly\beta \, , \; \mathrm{etc}$$

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- **Hubble, Bohr, Dirac:**

$$\omega^i_{aQSO} = \omega^i_{aQSO}(z, \alpha_{aQSO}) = A(z)\omega^i_{lab}(\alpha_{aQSO})$$

where

$$\begin{aligned} \omega^i_{lab}(\alpha_{aQSO}) &= \omega^i_{lab}(\alpha_{lab}) + \frac{d\omega^i_{lab}(\alpha_{lab})}{d\alpha_{lab}}\Delta\alpha \\ &\equiv \omega^i_{lab}(\alpha_{lab}) + \frac{1}{2}g^i(\alpha_{lab})\Delta\alpha, \quad with \;\; \Delta\alpha = \alpha_{aQSO} - \alpha_{lab} \end{aligned}$$

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- **A measurable quantity on the Earth:**

$$B^{QSO}_{ij} \equiv \frac{\omega^i_{aQSO}}{\omega^j_{aQSO}} = \frac{\omega^i_{lab}(\alpha_{aQSO})}{\omega^j_{lab}(\alpha_{aQSO})} = \frac{\omega^i_{lab}(\alpha_{lab}) + \frac{1}{2}g^i(\alpha_{lab})\Delta\alpha}{\omega^j_{lab}(\alpha_{lab}) + \frac{1}{2}g^j(\alpha_{lab})\Delta\alpha}$$

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Then, α-variation $\Delta\alpha$ measured on the Earth

$$\Delta\alpha = \frac{2(B^{QSO}_{ij} - B^{lab}_{ij})\omega^i_{lab}(\alpha_{lab})\omega^j_{lab}(\alpha_{lab})}{g^i(\alpha_{lab})\omega^i_{lab}(\alpha_{lab}) - g^j(\alpha_{lab})\omega^j_{lab}(\alpha_{lab})} \quad with \;\; B^{lab}_{ij} \equiv \frac{\omega^i_{lab}(\alpha_{lab})}{\omega^j_{lab}(\alpha_{lab})}$$

•**Results (1999-2007):** **Murphy, Webb, etc. PRL**

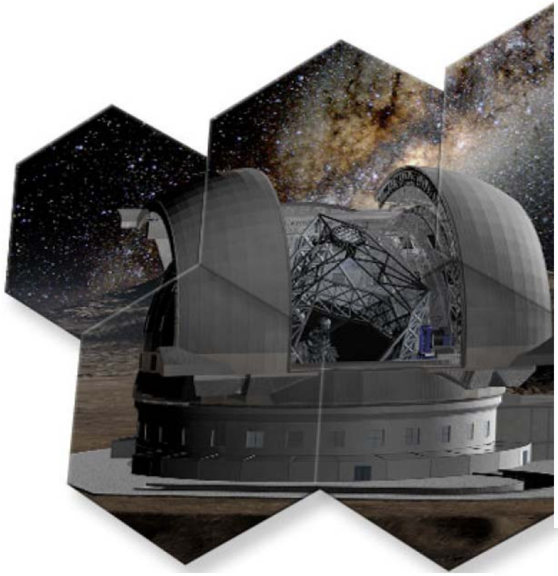
Studied 143 quasar absorption systems.

$0.2 < z_{abs} < 4.2.$

$$\frac{\Delta\alpha}{\alpha} = (-0.57 \pm 0.11) \times 10^{-5}$$

The 134 data points are assigned three epochs: **(Dent, *et al*, PRD (2008))**

average of redshift $\langle z \rangle$	$(\Delta\alpha/\alpha)_{expt}$	epoch t
0.65	$(-0.29 \pm 0.31) \times 10^{-5}$	6.04Gyr
1.47	$(-0.58 \pm 0.13) \times 10^{-5}$	9.29Gyr
2.84	$(-0.87 \pm 0.37) \times 10^{-5}$	11.39Gyr



E-ELT

Name: European Extremely Large Telescope (E-ELT)

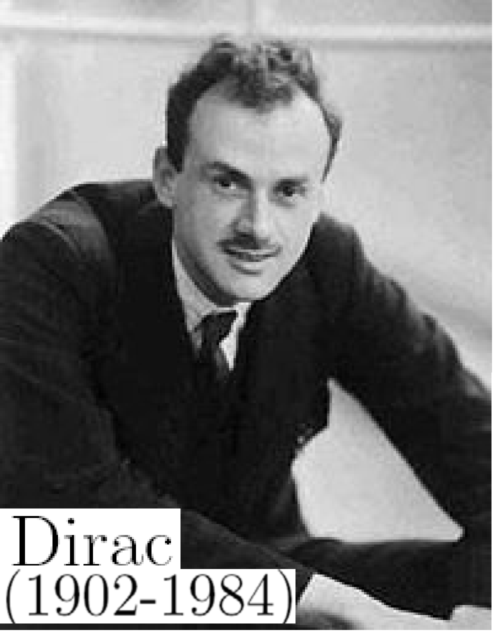
Type: optical to mid-infrared telescope

Aperture: 42 m

Start of operations: 2018 (planned)

Science goals:

detection and characterisation of exoplanets, fundamental physics (e.g., variations of fundamental constants across cosmic history), first objects in the Universe and evolution history of galaxies, black holes (from solar-mass to supermassive), and the nature and distribution of the dark matter and dark energy which dominate the Universe



Part II

Extended Special Relativity (de Sitter SR)

ANNALS OF MATHEMATICS
Vol. 36, No. 3, July, 1935

THE ELECTRON WAVE EQUATION IN DE-SITTER SPACE

BY P. A. M. DIRAC

(Received April 16, 1935)

The equations of atomic physics are usually formulated in terms of the space-time of the special theory of relativity.

a group. It is of interest to examine the effect of the various transformations on the physical equations and so to establish a connexion between physics and the mathematical theory of groups.

Nearly all of the more general spaces have only trivial groups of operations which carry the spaces over into themselves, so they spoil the connexion between physics and group theory. There is one exception, however, namely the de-Sitter space (with no local gravitational fields). This space is associated with a very interesting group, and so the study of the equations of atomic physics in this space is of special interest, from a mathematical point of view.

--Dirac, 1935

陆启铿(1927-)

K.H. Look (Q.K.Lu),

Why the Minkowski metric must be used ?, (1970)

第 23 卷 第 4 期 物 理 学 报 Vol. 23, No. 4
1974 年 7 月 ACTA PHYSICA SINICA July, 1974

典型时空中的运动效应和宇观红移现象*

陆 启 铿 （中国科学院数学研究所）
邹 振 隆 （中国科学院北京天文台）
郭 汉 英 （中国科学院原子能研究所）

1, de Sitter Special Relativity as theory beyond Einstein's SR:

- It's a natural extension of Einstein-SR;
- It's a SR in spacetime with constant curvature 1/R;
- Under Beltrami metric in dS space,enertial law for free paticle holds;(Lu,Zou,Guo)
- dS-transformation to preserve Beltrami metric exists; (Lu,Zou,Guo, 1974)
- Langrangian-Hamiltian for dS-SR exists

  (Yan, Xiao, Huang, Li, 2007).

2,Lagrangian for dS-SR ($\mathcal{SR}_{cR}$)

$$L_{cR} = -m_0 c \frac{\sqrt{B_{\mu\nu}(x) dx^\mu dx^\nu}}{dt} = -m_0 c \sqrt{B_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu}$$

where Beltrami metric:

$$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{1}{R^2 \sigma(x)^2} \eta_{\mu\lambda} \eta_{\nu\rho} x^\lambda x^\rho, \quad \text{with} \quad \sigma(x) \equiv 1 - \frac{1}{R^2} \eta_{\mu\nu} x^\mu x^\nu$$

Solution of Lagrangian eq $\quad \frac{\partial L_{cR}}{\partial x^i} = \frac{d}{dt} \frac{\partial L_{cR}}{\partial \dot{x}^i}$

is:

$$\ddot{x}^j = 0, \qquad \dot{x}^j = \text{constant}.$$

Indicates inertial law holds!

3,Canonic momenta and canonic energy (Hamiltonian)

$$\pi_i = \frac{\partial L_{cR}}{\partial \dot{x}^i} = -m_0 \sigma(x) \Gamma B_{i\mu} \dot{x}^\mu$$

$$H_{cR} = \sum_{i=1}^{3} \frac{\partial L_{cR}}{\partial \dot{x}^i} \dot{x}^i - L_{cR} = m_0 c \sigma(x) \Gamma B_{0\mu} \dot{x}^\mu$$

where

$$\Gamma^{-1} = \sigma(x) \frac{ds}{cdt} = \frac{1}{R} \sqrt{(R^2 - \eta_{ij} x^i x^j)(1 + \frac{\eta_{ij} \dot{x}^i \dot{x}^j}{c^2}) + 2t \eta_{ij} x^i \dot{x}^j - \eta_{ij} \dot{x}^i \dot{x}^j t^2 + \frac{(\eta_{ij} x^i \dot{x}^j)^2}{c^2}}.$$

$$\dot{\Gamma}|_{\ddot{x}^i = 0} = 0$$

Canonic eqs:

$$\dot{x}^i = \frac{\partial H_{cR}}{\partial \pi_i} = \{H_{cR}, x^i\}_{PB}$$

$$\dot{\pi}_i = -\frac{\partial H_{cR}}{\partial x^i} = \{H_{cR}, \pi_i\}_{PB}$$

Comparing with Einstien SR:

	$\mathcal{SR}_c$	$\mathcal{SR}_{cR}$
space-time metric	$\eta_{\mu\nu}$	$B_{\mu\nu}(x)$, (Eq.(3))
Lagrangian	$L_c = -m_0 c^2 \gamma^{-1}$	$L_{cR} = -m_0 c^2 \sigma^{-1} \Gamma^{-1}$
equation of motion	$v^i = \dot{x}^i$ =constant, (or $\dot{\gamma} = 0$)	$v^i = \dot{x}^i$ =constant, (or $\dot{\Gamma} = 0$)
canonic momenta	$\pi_i = -m_0 \gamma \eta_{i\mu} \dot{x}^\mu$	$\pi_i = -m_0 \sigma \Gamma B_{i\mu} \dot{x}^\mu$
Hamiltonian	$H_c = m_0 c \gamma \eta_{0\mu} \dot{x}^\mu$	$H_{cR} = m_0 c \sigma \Gamma B_{0\mu} \dot{x}^\mu$

4,de Sitter trans to preserve $B_{\mu\nu}(x)$

$$t \rightarrow \tilde{t} = \frac{\sqrt{\sigma(a)}}{c\sigma(a,x)}\gamma\left[ct - \beta x^1 - a^0 + \beta a^1 + \frac{a^0 - \beta a^1}{R^2}\frac{a^0 ct - a^1 x^1 - (a^0)^2 + (a^1)^2}{\sigma(a) + \sqrt{\sigma(a)}}\right]$$

$$x^1 \rightarrow \tilde{x}^1 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\gamma\left[x^1 - \beta ct + \beta a^0 - a^1 + \frac{a^1 - \beta a^0}{R^2}\frac{a^0 ct - a^1 x^1 - (a^0)^2 + (a^1)^2}{\sigma(a) + \sqrt{\sigma(a)}}\right]$$

$$x^2 \rightarrow \tilde{x}^2 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^2$$

$$x^3 \rightarrow \tilde{x}^3 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^3$$

Noether charges:

$\text{Noether charges for Lorentz boost}: \quad K^{i}_{cR} = m_0 \Gamma c (x^i - t\dot{x}^i)$

$\text{Charges for space} - \text{transitions (momenta)}: \quad P^{i}_{cR} = m_0 \Gamma \dot{x}^i,$

$\text{Charge for time} - \text{transition (energy)}: \quad E_{cR} = m_0 c^2 \Gamma$

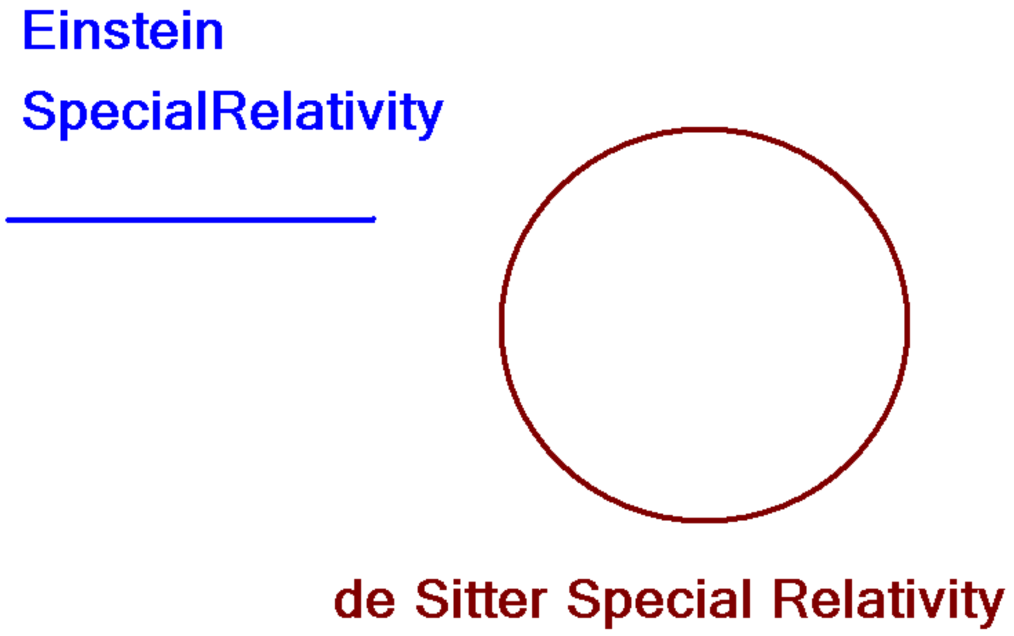
$\text{Charges for rotations in space (angularmomenta)}: \quad L^{i}_{cR} = \epsilon^{i}_{jk} x^j P^{k}_{cR}$

$$p^{\mu}_{cR} \equiv \{p^{0}_{cR},\ p^{i}_{cR}\} = -\frac{1}{\sigma(x)} B^{\mu\nu} \pi_{\nu}$$

$$E^{2}_{cR} = m_0^2 c^4 + c^2 \mathbf{p}^{2}_{cR} + \frac{c^2}{R^2} (\mathbf{L}^{2}_{cR} - \mathbf{K}^{2}_{cR})$$

5,Conclusion: From theory view, E-SR=dS-SR

- **Both of them satisy Copernican priciple:**



- **Which one is true for Nature? Answer: Experiments!**

6, dS-Special Relativistic Quantum mechanics

- **Quantizetion:** $\{x, \pi\}_{PB} \Rightarrow \frac{1}{i\hbar}[x, \hat{\pi}]$
- **Consider Wyle ordering of** (πx)**, and**

$$[x^i,\ \pi_j] = i\hbar\delta^i_j, \qquad [\pi_i,\ \pi_j] = 0, \qquad [x_i,\ x_j] = 0,$$

the solution is

$$\boxed{\pi_\mu = -i\hbar B^{-\frac{1}{4}}\partial_\mu B^{\frac{1}{4}}}$$

where

$$B = det(B_{\mu\nu})$$

- **Classical Dispersion relation**

$$B^{\mu\nu}\pi_\mu\pi_\nu = m_0^2c^2 \Longrightarrow B^{-\frac{1}{4}}\pi_\mu B^{\frac{1}{4}}B^{\mu\nu}B^{\frac{1}{4}}\pi_\nu B^{-\frac{1}{4}} = m_0^2c^2$$

then for spinless particle QM-equation:

$$\frac{1}{\sqrt{B}}\partial_\mu(B^{\mu\nu}\sqrt{B}\partial_\nu)\phi + \frac{m_0^2c^2}{\hbar^2}\phi = 0$$

For spin-1/2 particle, Dirac eq:

$$\left(ie_a^\mu\gamma^a D_\mu - \frac{m_0c}{\hbar}\right)\psi = 0$$

$$D_\mu = \partial_\mu - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab}, \quad e_\mu^a e_\nu^b\eta_{ab} = B_{\mu\nu},$$

$$\omega_\mu^{ab} = \frac{1}{2}(e^{a\rho}\partial_\mu e_\rho^b - e^{b\rho}\partial_\mu e_\rho^a) - \frac{1}{2}\Gamma_{\lambda\mu}^\rho(e^{a\lambda}e_\rho^b - e^{b\lambda}e_\rho^a)$$

Part III

HYDROGEN ATOM IN EARTH-QSO REFERENCE FRAME

1, Earth-QSO light cone

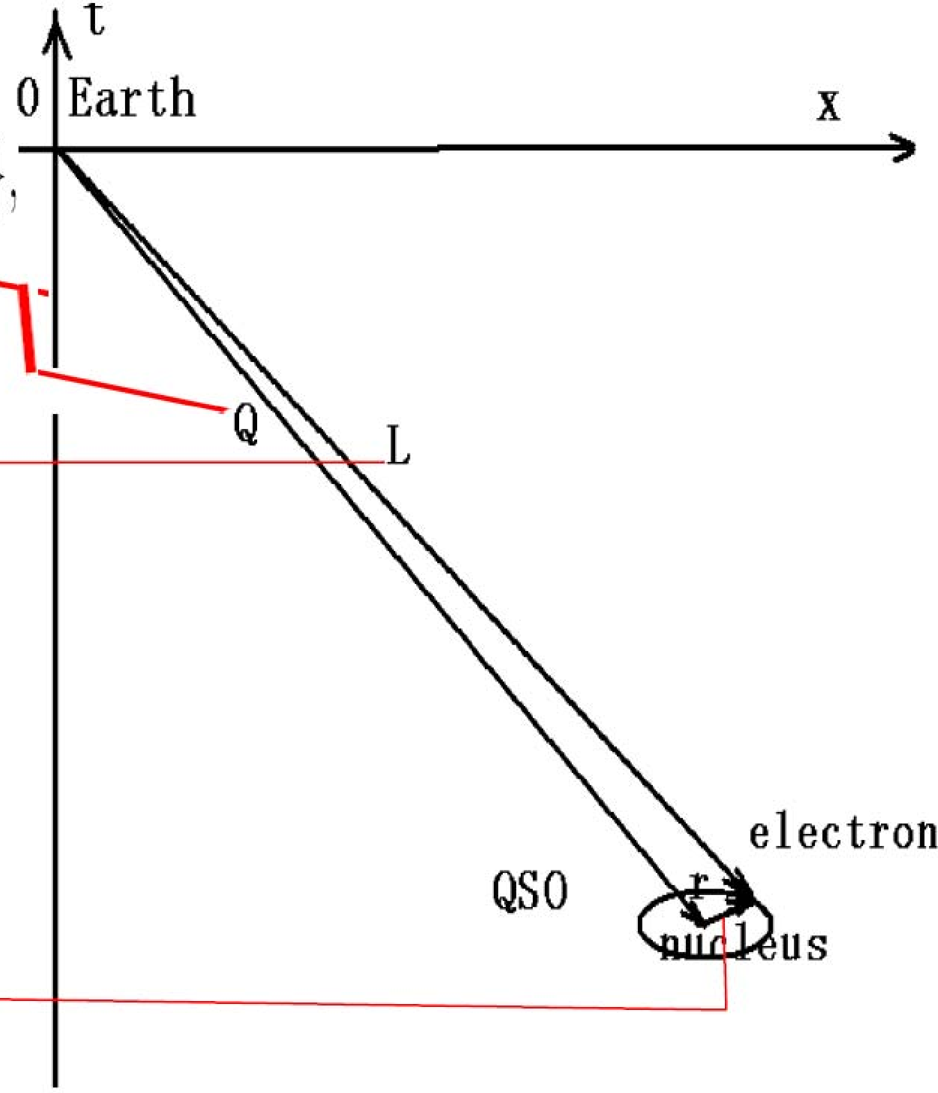
$$B_{\mu\nu}(Q)Q^\mu Q^\nu = \eta_{\mu\nu}Q^\mu Q^\nu = 0$$

$$Q = \{Q^0 \equiv c\,t,\ Q^1 = c\,t,\ Q^2 = 0,\ Q^3 = 0\},$$

$$L = \{L^0 \equiv c\,t,\ L^1,\ L^2,\ L^3\},$$

$$x^i = L^i - Q^i.$$

$$r \equiv \sqrt{-\eta_{ij}x^i x^j}$$



Geometry:

$$B_{\mu\nu}(Q) = \eta_{\mu\nu} + \frac{1}{R^2}\eta_{\mu\lambda}Q^\lambda \eta_{\nu\rho}Q^\rho, \quad B_{ij}(Q) = \eta_{ij} + \frac{c^2t^2}{R^2}\delta_{i1}\delta_{j1}.$$

$$e_a^\mu \; = \; \left(1 - \frac{\eta_{cd}L^c L^d}{2R^2}\right)\eta_a^\mu - \frac{\eta_{ab}L^b L^\mu}{2R^2} + \mathcal{O}(1/R^4),$$

$$\omega_\mu^{ab} \; = \; \frac{1}{2R^2}(\eta_\mu^a L^b - \eta_\mu^b L^a) + \mathcal{O}(1/R^4)$$

2, Dirac eq for hydrogen atom at Earth-QSO frame

$$(ie_a^\mu \gamma^a \mathcal{D}_\mu^L - \frac{\mu c}{\hbar})\psi = 0,$$

where

$$\mu = m_e \left(1 - \frac{m_e}{m_p}\right)$$

$$\mathcal{D}_\mu^L = \frac{\partial}{\partial L^\mu} - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab} - \delta_{\mu 0} ie/(c\hbar)\phi_B(x)$$

Coulomb potential:

$$-B^{ij}(Q)\partial_i\partial_j\phi_B(x)=\left(\nabla^2+\frac{c^2t^2}{R^2}\frac{\partial^2}{\partial(x^1)^2}\right)\phi_B(x)=-4\pi\rho_B(x)=\frac{-4\pi e}{\sqrt{-det(B_{ij}(Q))}}\delta^{(3)}(\mathbf{x})$$

Solution:

$$\phi_B = \frac{e}{r_B}: \qquad r_B = \sqrt{(\tilde{x}^1)^2+(x^2)^2+(x^3)^2}$$

$$\tilde{x}^1 = (1-c^2t^2/(2R^2))x^1$$

By $e_a^\mu = \left(1-\frac{\eta_{cd}L^cL^d}{2R^2}\right)\eta_a^\mu - \frac{\eta_{ab}L^bL^\mu}{2R^2}$

$$\hbar c\beta\left[i\left(1-\frac{\eta_{ab}L^aL^b}{2R^2}\right)\gamma^\mu\mathcal{D}_\mu^L - \frac{i}{2R^2}\eta_{ab}L^a\gamma^bL^\mu\mathcal{D}_\mu^L - \frac{\mu c}{\hbar}\right]\psi = 0$$

Therefore

$$\left(1+\frac{c^2t^2}{2R^2}\right)E\psi = \left[-i\hbar c\vec{\alpha}\cdot\nabla_B + \mu c^2\beta - \left(1+\frac{c^2t^2}{2R^2}\right)\frac{e^2}{r_B}\right]\psi.$$

3, Spectra Energy: Due to dS,

$$p^0 = \frac{E}{c} = i\hbar\left[\frac{1}{c}\partial_t - \frac{ct}{R^2}x^\nu\partial^L_\nu + \frac{5ct}{2R^2}\right]$$

$$E\psi \simeq i\hbar\left(1-\frac{c^2t^2}{R^2}\right)\partial_t\psi$$

$$E\psi = \left[-i\hbar c\left(1-\frac{\lambda c^2t^2}{2R^2}\right)\vec{\alpha}\cdot\nabla_B + \left(1-\frac{\lambda c^2t^2}{2R^2}\right)\mu c^2\beta - \frac{e^2}{r_B}\right]\psi.$$

$$\frac{1}{R^2} \Rightarrow \frac{\lambda}{R^2} \quad with \quad \lambda \equiv \pm 1$$

$\lambda = +1$: de Sitter group: SO(4,1)

$\lambda = -1$: Anti-de Sitter group: SO(3,2)

3’ Adiabatic approximation in QM

Generally, to a $H(x,t)$, we may express it as $H(x,t) = H_0(x) + H'(x,t)$. Suppose two eigenstates $|s\rangle$ and $|m\rangle$ of $H_0(x)$ do not generate, i.e., $\Delta E \equiv \hbar(\omega_m - \omega_s) = \hbar\omega_{ms} \neq 0$. The validness of for adiabatic approximation relies on the fact that the variation of the potential $H'(x,t)$ in the the Bohr time-period $(\Delta T_{ms}^{(Bohr)})\dot{H}'(x,t)_{ms} = (2\pi/\omega_{ms})\dot{H}'(x,t)_{ms}$ is much less than $\hbar\omega_{ms}$. That makes the quantum transition from state $|s\rangle$ to state $|m\rangle$ almost impossible. Thus, the non-adiabatic effect corrections are small enough (or tiny) , and the adiabatic approximations are proper [18].

4, Adiabatic approximation solution to dS-SR-Dirac spectra equation

via $E \Rightarrow i\hbar\partial_t$: above Dirac spectra becomes

$$
\begin{aligned}
i\hbar\partial_t\psi &= H(t)\psi = [H_0(r,e) + H'(t)]\psi, \\
H_0(r,e) &= -i\hbar c\vec{\alpha}\cdot\nabla_B + \mu c^2\beta - \frac{e^2}{r_B} \quad (see\ eq.(50)) \\
H'(t) &= -\left(\frac{\lambda c^2 t^2}{2R^2}\right) H_0(r,0).
\end{aligned}
$$

Then **Adiabatic app wave function**

$$
\psi(t) \simeq \psi_s(\mathbf{r},\hbar_t,\mu_t,\alpha_t)e^{-i\frac{E_s}{\hbar}t}
$$

$$
+\sum_{m\neq s}\frac{\dot{H}'(t)_{ms}}{i\hbar\omega_{ms}^2}\left(e^{i\omega_{ms}t}-1\right)\psi_m(\mathbf{r},\hbar_t,\mu_t,\alpha_t)e^{\left(-i\int_0^t\frac{E_m(\theta)}{\hbar}d\theta\right)}
$$

where
$$\hbar_t = \left(1 - \frac{\lambda c^2 t^2}{2R^2}\right)\hbar,$$
$$\mu_t = \left(1 - \frac{\lambda c^2 t^2}{2R^2}\right)\mu,$$
$$e_t = e.$$

$$\boxed{\alpha_t \equiv \frac{e_t^2}{\hbar_t c} = \left(1 + \frac{\lambda c^2 t^2}{2R^2}\right)\alpha}$$

with $\alpha = \dfrac{e^2}{\hbar c}$

Is adiabatic proper? To examine $\psi(t)$:

$$\dot{H}'(t)_{ms}|_{(m\neq s)} = \langle m|\dot{H}'(t)|s\rangle|_{(m\neq s)} = \frac{2c^2 t}{3R^2}\langle n_m|\frac{e^2}{r}|n_s\rangle e^{-i(\omega_s-\omega_m)t},$$

To estimate $|\dot{H}'(t)_{ms}/\hbar\omega_{ms}^2|$ for $s=1,\ m=2$ and $t \sim 10^9\text{Yr},\ R \sim 10^{12}\text{ly}.$

By Borh wave funs:

$$\left|\frac{\dot{H}'(t)_{21}}{\hbar\omega_{21}^2}\right| \simeq \frac{512\sqrt{2}}{729}\frac{ct}{R^2}\frac{a}{\alpha} \simeq 0.9\times 10^{-40} << \left(\frac{c^2t^2}{R^2} \sim \mathcal{O}(10^{-5})\right) << 1$$

Hence adiabatic app is hightly accurate!

$$\psi(t) \simeq \psi_s(\mathbf{r}, \hbar_t, \mu_t, \alpha_t) e^{-i\frac{E_s}{\hbar}t}$$

$$s = \{n,\ K,\ \mathbf{j}^2,\ j_z\}$$

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha_t - \alpha}{\alpha} = \frac{\lambda c^2 t^2}{2R^2},$$

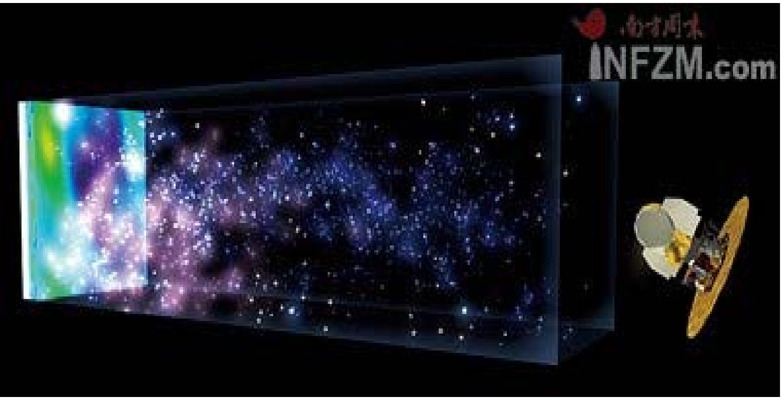
$$\frac{\Delta m_e}{m_e} \equiv \frac{(m_e)_t - m_e}{m_e} = -\frac{\lambda c^2 t^2}{2R^2}.$$

$$\hbar_t = \left(1 - \frac{\lambda c^2 t^2}{2R^2}\right) \hbar,$$

Or, Compton wave lengt $a_c = \hbar/(m_e c)$
and Bohr radius $a = \hbar^2/(m_e e^2) = a_c/\alpha,$

$$\frac{\Delta a_c}{a_c} \equiv \frac{(a_c)_t - a_c}{a_c} = 0,$$

$$\frac{\Delta a}{a} \equiv \frac{a_t - a}{a} = -\frac{\lambda c^2 t^2}{2R^2}$$



♣ t-z relation:

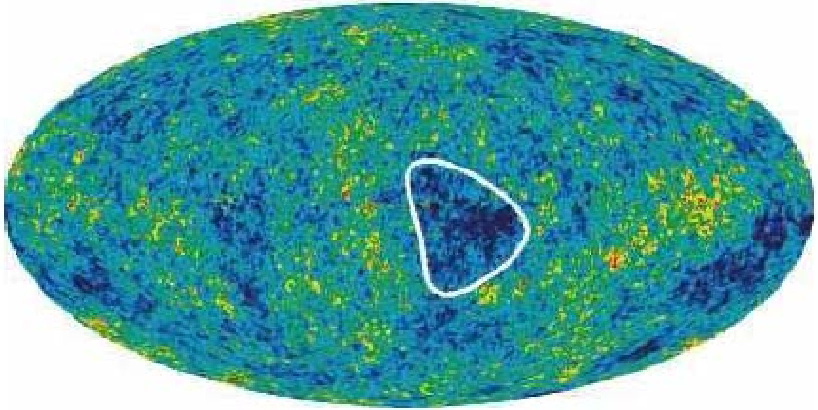
Λ CDM model:

$$t = \int_0^z \frac{dz'}{H(z')(1+z')},$$

$$H(z') = H_0\sqrt{\Omega_{m0}(1+z')^3 + 1 - \Omega_{m0}},$$

$$H_0 = 100\ h \simeq 100 \times 0.705 km \cdot s^{-1}/Mpc,$$

$$\Omega_{m0} \simeq 0.274.$$



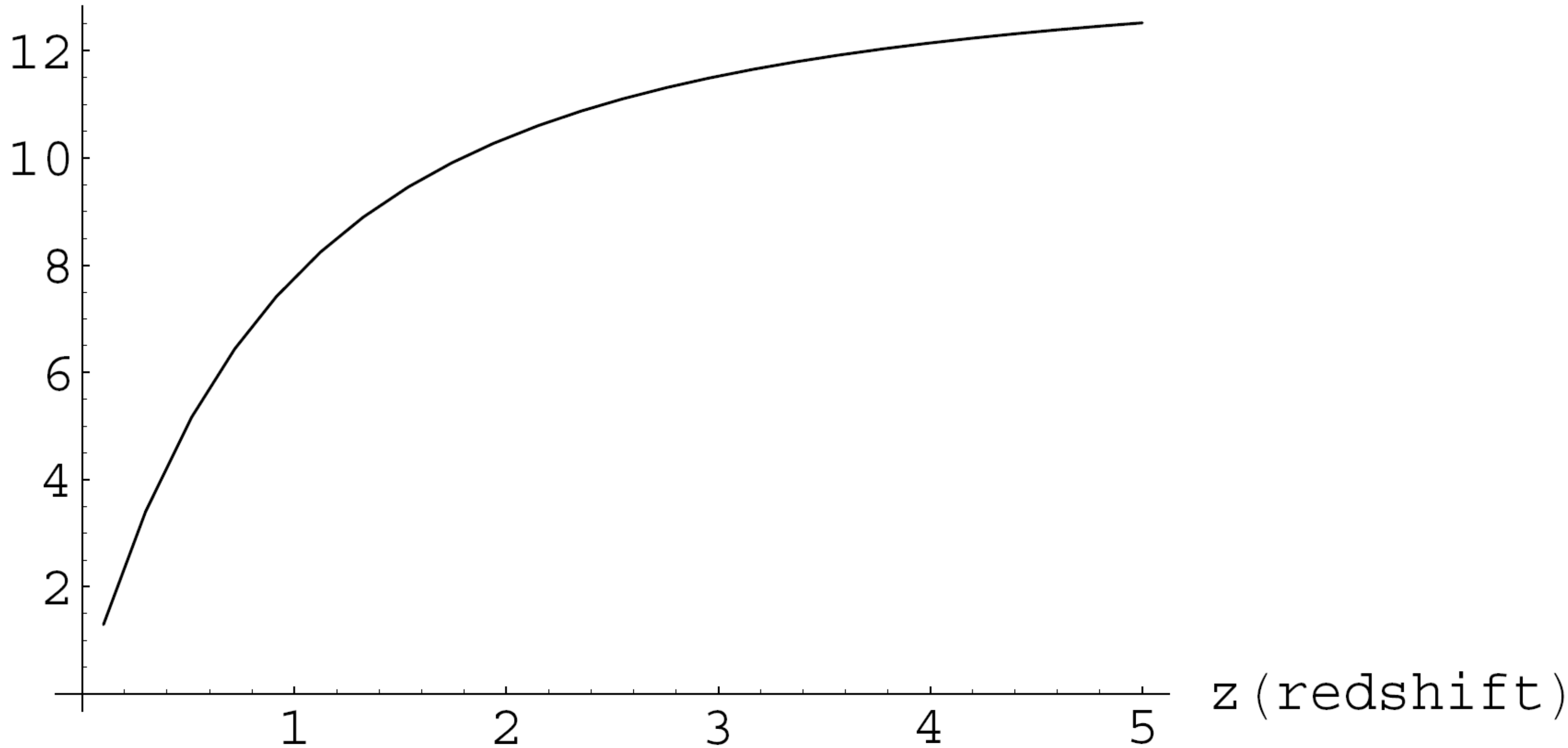


FIG. 2: The $t - z$ relation in ΛCDM model (eq.(98)).

5,Experiments:Murphy et al: 2004
Dent et al: 2008

- **Since**

$$\frac{\Delta\alpha}{\alpha} = (-0.57 \pm 0.11) \times 10^{-5}.$$

- **We have**

$$\lambda = -1$$

the space-time symmetry for $\mathcal{SR}_{cR}$ is anti-de Sitter-$SO(3,2)$

TABLE II: Time variations of $\Delta\alpha/\alpha$: The first two columns are quoted from [38]. Eq. (82) with $R \simeq 2.73 \times 10^{12} ly$, and the ΛCDM model's $t - z$ relation (98) are used.

average of redshift $\langle z \rangle$	$(\Delta\alpha/\alpha)_{expt}$	epoch t	theory prediction of (82)
0.65	$(-0.29 \pm 0.31) \times 10^{-5}$	6.04Gyr	-0.24×10^{-5}
1.47	$(-0.58 \pm 0.13) \times 10^{-5}$	9.29Gyr	-0.58×10^{-5}
2.84	$(-0.87 \pm 0.37) \times 10^{-5}$	11.39Gyr	-0.87×10^{-5}

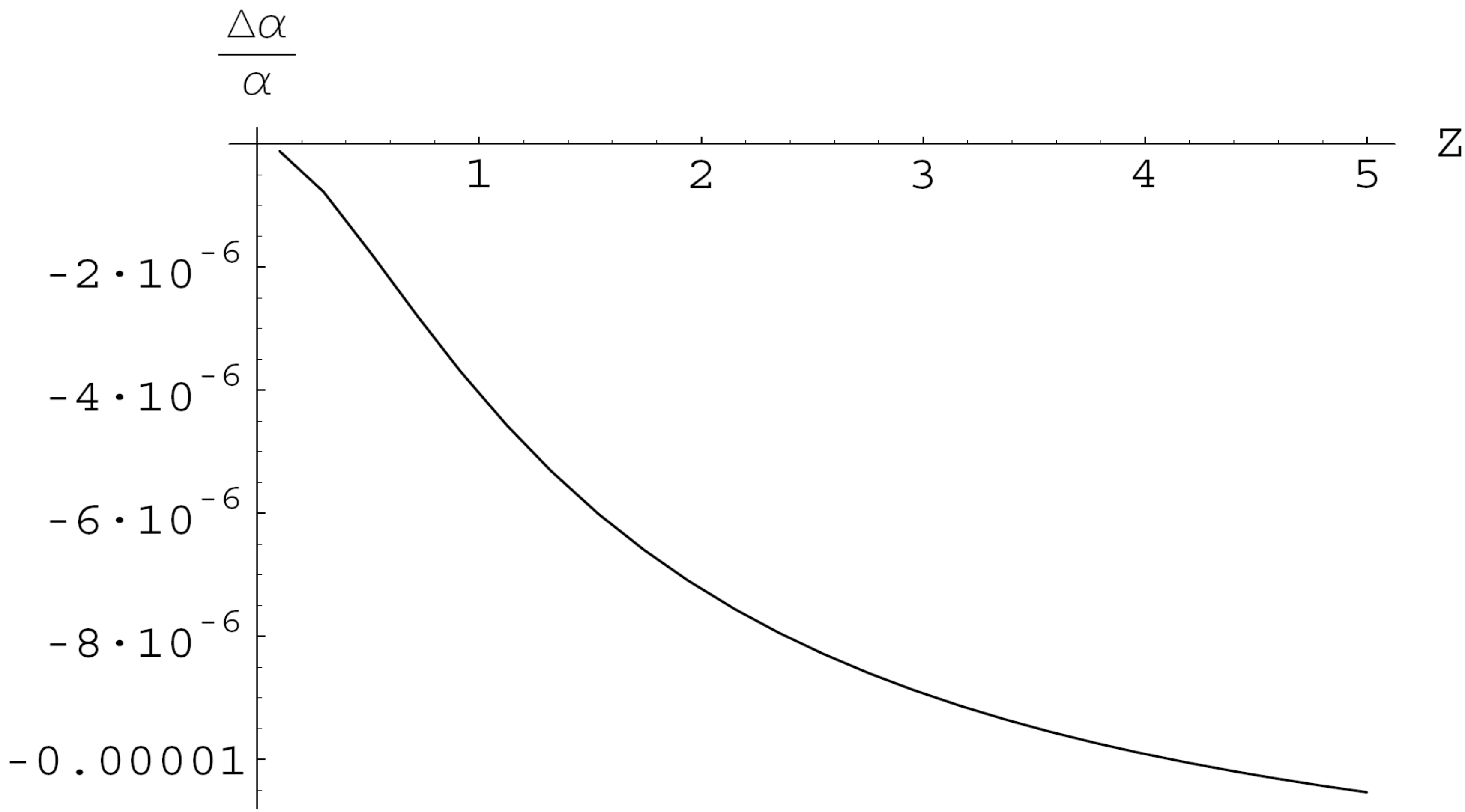
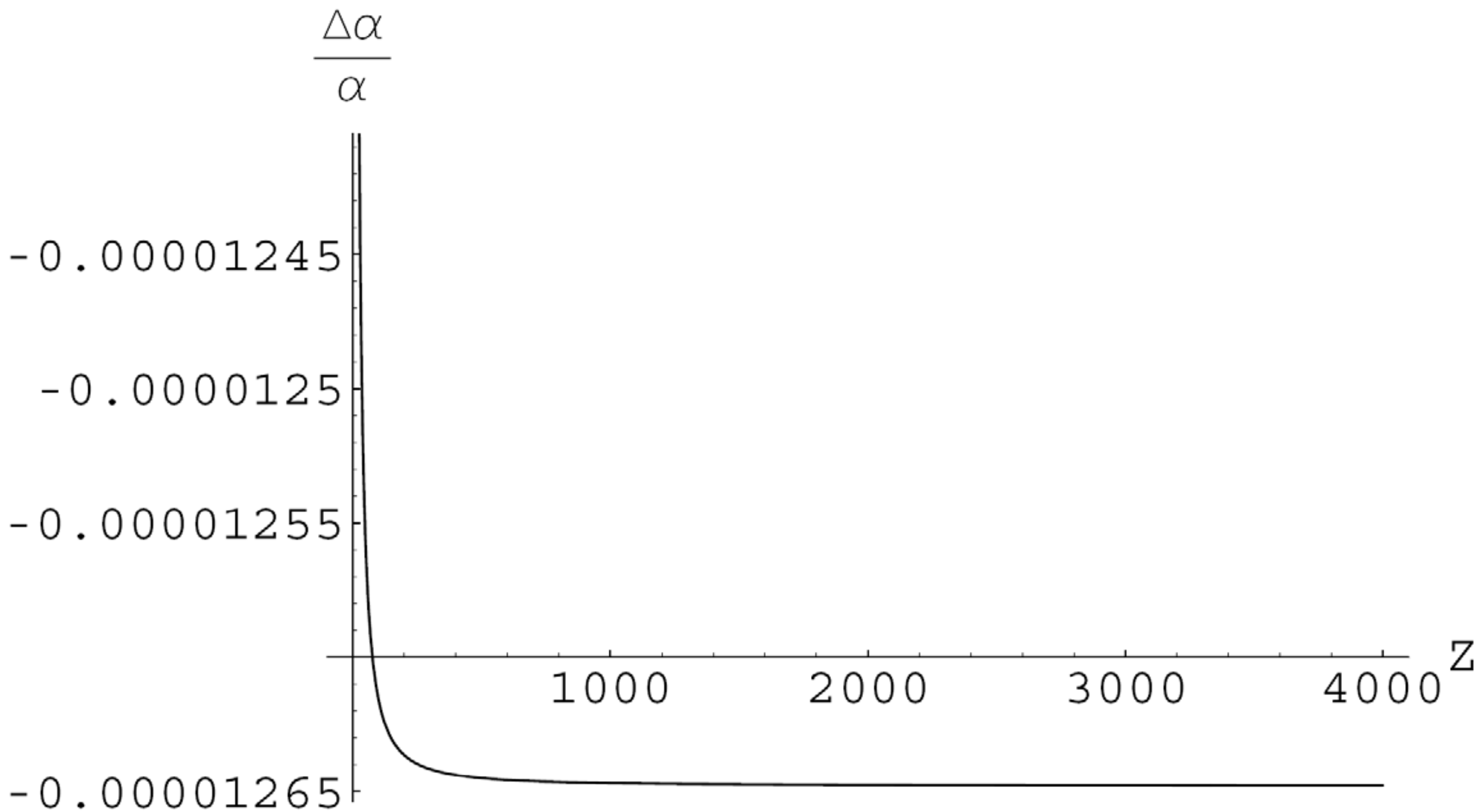


FIG. 3: The $\Delta\alpha/\alpha$ as function of the red shift z.



When $z \geq 10^3$, $\frac{\Delta\alpha}{\alpha}(z)$ is almost independent of z, i.e., α-variation ceases in that very hight z region.

lower bound of $\frac{\Delta\alpha}{\alpha}(z)$ is about $\sim -1.3 \times 10^{-5}$.

This result coincides with BSBM model [28–30]

•Correction to dark energy:

- 俞允强：《热大爆炸宇宙学》pp135:

无论如何，这里出现了一个物理学中前所未有的奇怪境地．如果引力场方程中应包含宇宙项，那么宇宙常数 λ 就和牛顿引力常数 G 一样，是一个基本物理常数．它的值只能从实验来定出，而不可能由理论推断．上面的分析使我们看到，实验只能定出等效宇宙常数 λ_{eff} 或等效真空能 ρ_{eff}，它是 λ 和 ρ_{vac} 的联合效果．这意味着宇宙常数 λ 本身既不能由理论定出，也不能由实测定出．这真是物理学的尴尬．

幸好真空能密度 ρ_{vac} 不是基本物理常数．它作为派生的常数，原则上可以由量子场论算出．这样总算有一条出路：用实验测出等效常数 ρ_{eff}（即 λ_{eff}），再从理论上算出 ρ_{vac}，宇宙常数 λ 就能由（9.5）（或（9.3））定出．

爱因斯坦倾向于认为 $\lambda=0$．如果他是对的，那么测出的 ρ_{eff} 和算出的 ρ_{vac} 应该相等，否则就是他作出了错误的判断．后人终究是能把这问题澄清的．可是宇宙常数问题的研究竟然是如此的戏剧性：理论与实测对比的结果，出现的是很大的困惑．

let's discuss the physics meaning of the universal parameter λ/R^2 in Beltromi metric $B_{\mu\nu} = \eta_{\mu\nu}/\sigma(x) + \eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho \lambda/(R^2\sigma(x)^2)$

- Full Einstein equation:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} - \Lambda_E g_{\mu\nu} = -\frac{8\pi G}{c^4}(T_{\mu\nu} - \rho_{vac}g_{\mu\nu})$$

- Beltromi metric satspys

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}B_{\mu\nu}\mathcal{R} - \Lambda_E B_{\mu\nu} = 0.$$

then we have

$$\Lambda_E = \frac{3\lambda}{R^2}$$

- **Cosmology constant & and corrections:**

$$\Lambda_{eff} = \frac{8\pi G}{c^4}\rho_{vac} + \Lambda_E \equiv \Lambda_{(dark\ energy)} + \Lambda_E$$

using $\lambda = -1$ and $R \simeq 2.73 \times 10^{12} ly$

$$\Lambda_E \simeq -0.45 \times 10^{-60} cm^{-2}.$$

Λ_{eff} can be estimated by the data in [25]

$$\Lambda_{eff} \simeq \frac{3H_0^2}{c^2}(1 - \Omega_{m0}) \simeq 0.138 \times 10^{-55} cm^{-2}$$

Therefore

$$\Lambda_{(dark\ energy)} = \Lambda_{eff} - \Lambda_E \simeq \Lambda_{eff}(1 + \mathcal{O}(10^{-5}))$$

which is a little bit larger than the usual estimations: $\Lambda_{(dark\ energy)} = \Lambda_{eff}$

6, Conclusion

- **dS-Dirac eq of hydrogen can be solved in adiabatic approach;**
- **Cosmic effects on atom levels exist;**
- **Time-variations of fine construction const and electron mass are revealed;**
- **Prediction agree with data;**
- **At cosimic scale, de-SR seems more reliable than Einstein-SR;**
- **More exts/observ are required.**

Hydrogen Atom in de Sitter Special Relativity and Time Variation of Fine-Structure Constant

Mu-Lin Yan*

Abstract

This paper is for solving the de Sitter-Dirac equation for Hydrogen atom. The aim is to explore cosmology effects in atom physics. It is found out that the fine-structure constant changes adiabatically along with cosmic time. Thus, the work provides an example that phenomena of time-variation of fundamental constants can be understood as a well known special relativistic quantum mechanics effect instead of some unknown mysterious scalar field effects.

Thank You !