Hidden Analytic Relations for Two-Loop Higgs Amplitudes in QCD

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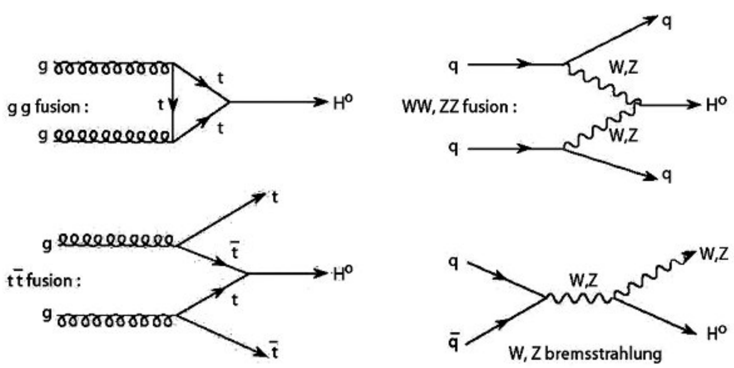
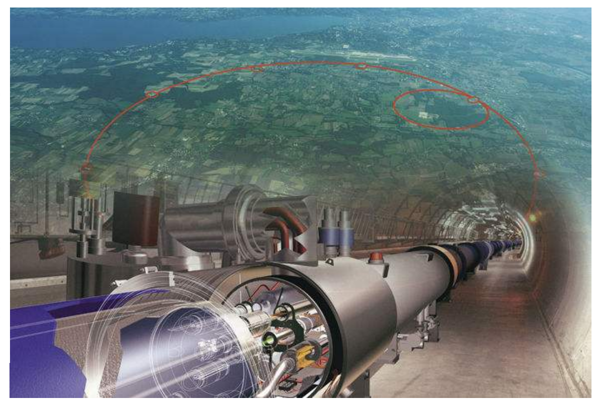
Content

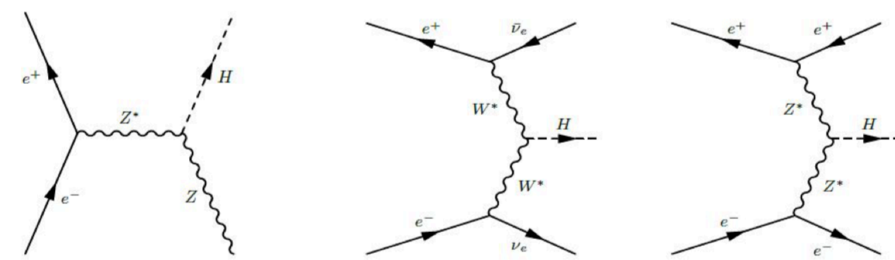
- **Motivations**
- Computations
- The hidden relation
- Summary and outlook

Scattering Amplitudes in Standard Model (SM)

Model → Scattering Amplitude ← Experiment Data







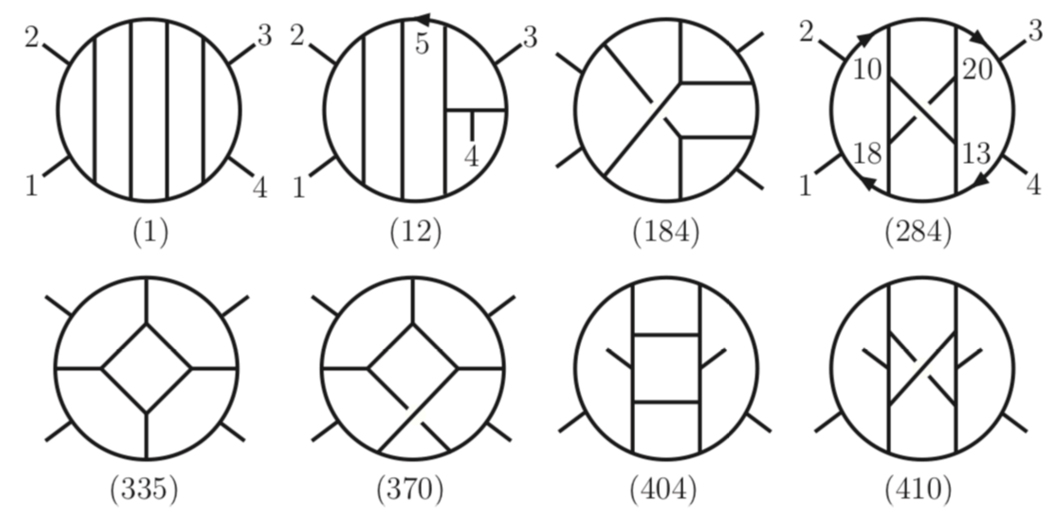
Higgs production in LHC and CEPC

LHC : Perfect agreement with SM.

Future CEPC : higher accuracy, less background noise. Requires even high loop computation of scattering amplitudes.

New Methods in Scattering Amplitudes

- New methods developed during the computation of scattering amplitudes in supersymmetric field thoeries: spinor helicity formalism, on shell unitarity cut …



- N=4 SYM integrand was computed to 5 loops. [Bern et al 2012]

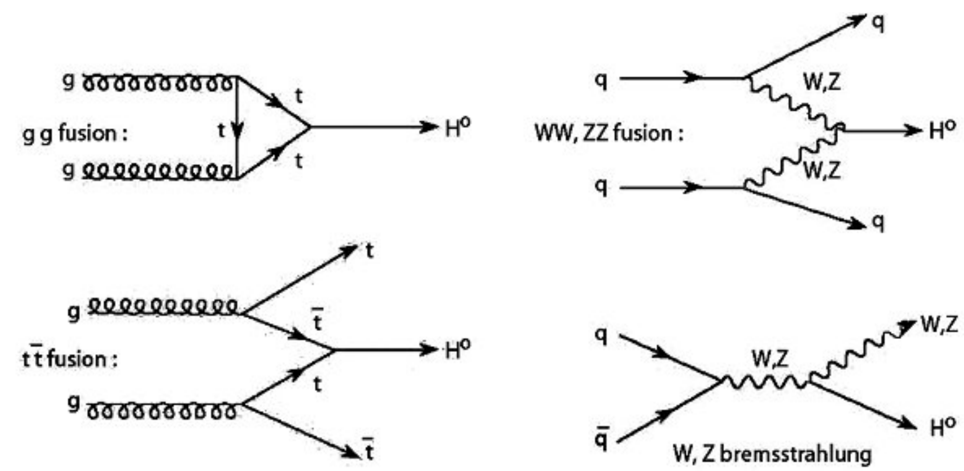
- N=8 supergravity is free of UV divergence to at least 5 loops. [Bern et al 2018]

- Apply these new methods to the computation of loop amplitude in Standard Model.

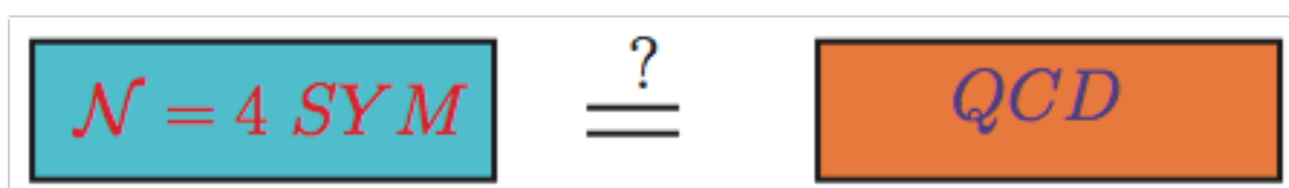
Higgs to 3 parton amplitude and the Maximal Transcendental Principle

Study the properties of Higgs boson is one of the major tasks of LHC and future CEPC.

In LHC, Higgs can be produced by the fusion of gluons and quarks. Higgs to 3 parton amplitudes are very important to understand the LHC data.



The maximally transcendental principle: the maximal transcendental part of N=4 SYM and QCD amplitudes are the same.



An "easy" N=4 computation gives information for the "difficult" QCD amplitude!

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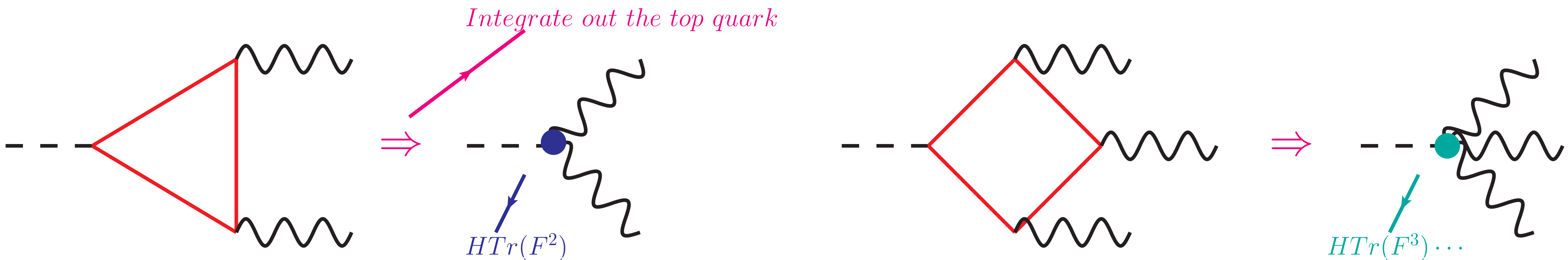
Higgs effective field theory (HEFT)



- When the transverse momentum of Higgs particle is less than the top quark mass, the HEFT [*Wilczek 1977*] is good approximation.

$$\mathcal{L}_{\text{eff}} = \hat{C}_0 O_0 + \frac{1}{m_{\text{t}}^2} \sum_{i=1}^{4} \hat{C}_i O_i + \mathcal{O}\left(\frac{1}{m_{\text{t}}^4}\right)$$

$$O_0 = H\text{Tr}(F^2)$$

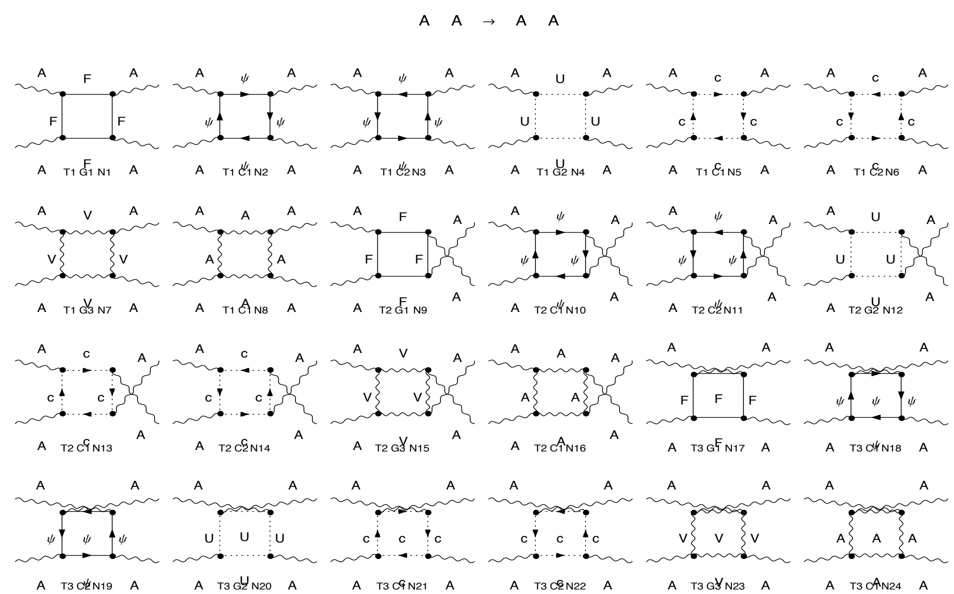
$$O_1 = H\text{Tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)\,, \quad O_2 = H\text{Tr}(D_\rho F_{\mu\nu} D^\rho F^{\mu\nu})\,,$$

$$O_3 = H\text{Tr}(D^\rho F_{\rho\mu} D_\sigma F^{\sigma\mu})\,, O_4 = H\text{Tr}(F_{\mu\rho} D^\rho D_\sigma F^{\sigma\mu})\,.$$

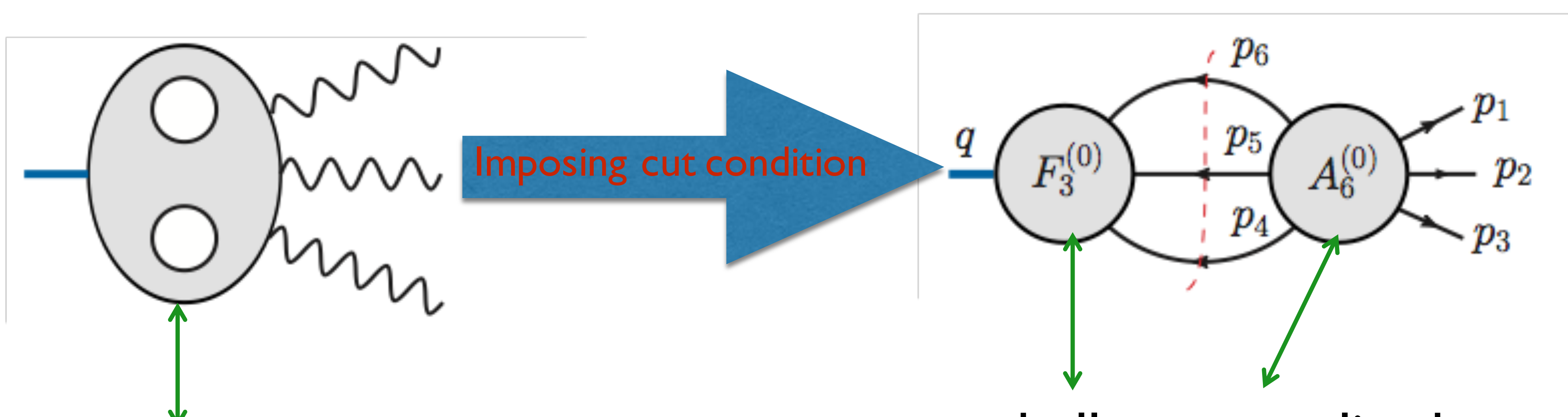
- The leading (dimension 4) contribution of Higgs to 3 parton amplitude was computed in [*Gehrmann, Jaquier, Glover, Koukoutsakis 2011*].

- The contributions of dimension 6 operators. [QJ, Yang 2018] and [QJ, Yang 2019].

Feynman Diagram *vs* Unitarity Cut



Feynman diagrams: a broken vase



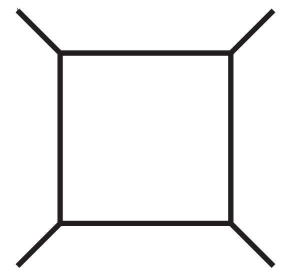
loop amplitude

two on shell tree amplitudes

D-dimensional unitarity cut

- 4-d spinor helicity formalism fails to capture the rational term in non-SUSY theories.

$$A^{----} = (2-2\epsilon)\mu^4$$



$$= \frac{4}{3} + \mathcal{O}(\epsilon)$$

- D-dimensional unitarity uses the D-dimensional tree amplitude.

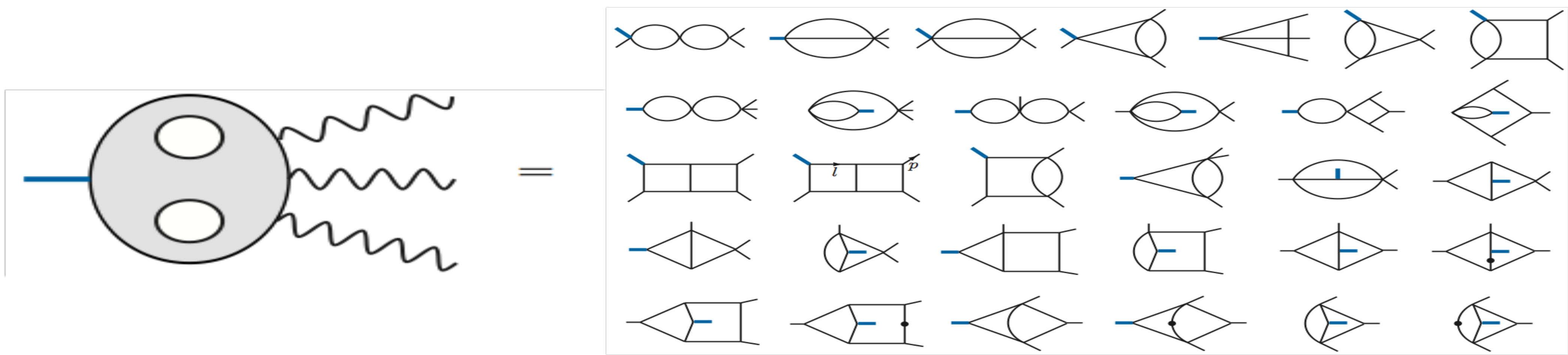
$$\begin{aligned} A(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4) = &- (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) + (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4)) \\ &- \frac{2}{s}\Big[2t(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) + (\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_1)(\epsilon_3 \cdot \epsilon_4) - (\epsilon_1 \cdot p_2)(\epsilon_2 \cdot p_3)(\epsilon_3 \cdot \epsilon_4)\Big] \\ &+ 22 \text{ more terms} \end{aligned}$$

- The polarization vector summing rule:

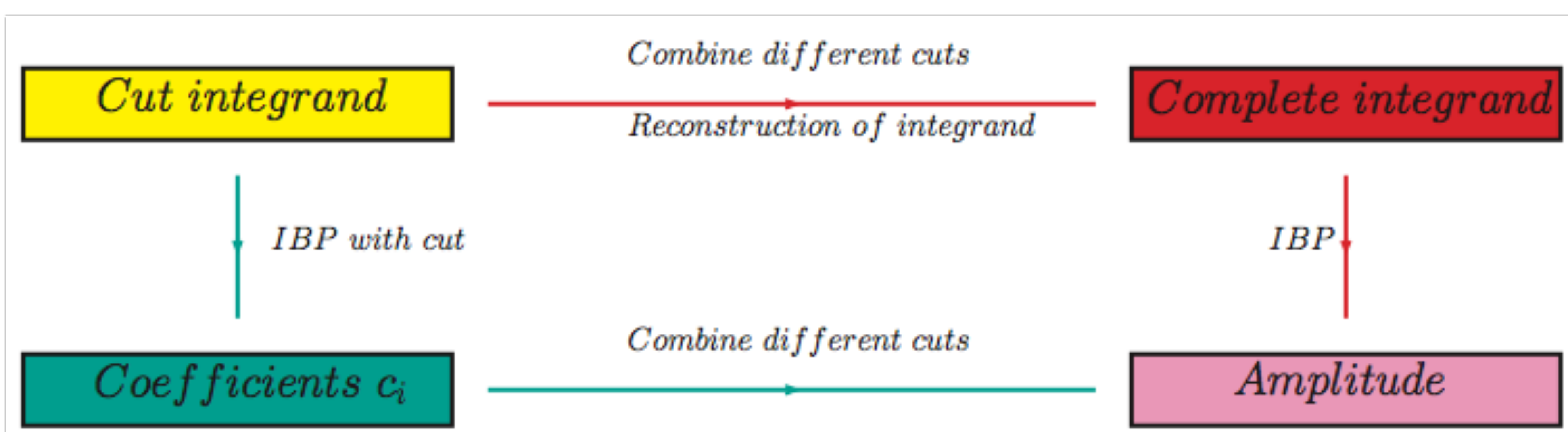
$$\sum_{\text{helicities}} \varepsilon_i^\mu \varepsilon_i^\nu = \eta^{\mu\nu} - \frac{q^\mu p_i^\nu + q^\nu p_i^\mu}{q \cdot p_i}$$

New Strategy of IBP

Loop integrand can be reduced using integration by parts (IBP) relations.



The IBP with cut strategy enhanced the efficiency by a factor of 10.



Simplify the multi-polylog functions using Symbol

$$
\begin{aligned}
A_\alpha^{(2)} = \Big[\frac{1}{2}\Big(& - G(1-z,-z,1-z,0,y) - G(1-z,-z,0,1-z,y) + G(1-z,1-z,0,0,y) \\
& + G(1-z,0,-z,1-z,y) + G(1-z,0,1-z,0,y) - G(1-z,0,1,0,y) \\
& + G(1-z,0,0,1-z,y) + H(1,1,0,0,z) + H(1,0,1,0,z) + H(1,0,0,1,z) \\
& + H(1,0,0,z)G(1-z,y) - H(1,0,z)G(-z,0,y) + H(1,0,z)G(1-z,-z,y) \\
& - G(1,0,y)H(1,0,z) + H(1,z)G(1-z,-z,0,y) + H(1,z)G(1-z,0,-z,y) \\
& - H(1,z)G(1-z,0,0,y) - G(0,-z,1-z,y) + G(0,-z,y)H(1,0,z) \\
& - G(0,1-z,-z,1-z,y) + G(0,1-z,1-z,0,y) + G(0,1-z,0,1-z,y) \\
& + G(0,1-z,-z,y)H(1,z) - G(0,-z,1-z,y) + G(0,1-z,1-z,y)H(0,z) \\
& - G(0,1-z,1,0,y) - G(0,1-z,0,y)H(1,z) - G(0,1-z,0,y)H(0,z) \\
& - H(0,z)G(1-z,0,1-z,y) - H(0,z)G(1-z,0,0,y) - G(0,y)H(1,1,0,z) \\
& - G(0,y)H(1,0,1,z) + G(0,y)H(1,0,0,z) - G(0,1,1-z,0,y) + H(0,1,1,0,z) \\
& - H(0,1,1,z)G(0,y) - G(0,1,0,1-z,y) + H(0,1,0,1,z) \\
& + H(0,1,0,z)G(1-z,y) + H(0,1,0,z)G(0,y) + G(0,1,0,y)H(1,z) \\
& - G(0,1,0,y)H(0,z) - H(0,1,z)G(-z,0,y) - H(0,1,z)G(1,0,y) \\
& + H(0,1,z)G(0,-z,y) + G(0,0,1-z,1-z,y) - G(0,0,1-z,y)H(1,z) \\
& - G(0,0,1-z,y)H(0,z) + H(0,0,1,1,z) + H(0,0,1,z)G(0,y) \\
& + H(0,0,z)G(1-z,1-z,y) + H(0,0,z)G(1-z,0,y) + H(0,0,z)G(0,1-z,y) \\
& + G(0,0,y)H(1,1,z) - G(0,0,y)H(1,0,z) - G(0,0,y)H(0,1,z) \\
& + G(0,0,y)H(0,0,z) + H(0,z)G(-z,1-z,0,y) + H(0,z)G(-z,0,1-z,y) \\
& - H(0,z)G(1-z,-z,1-z,y) - H(0,z)G(1-z,1-z,0,y) \\
& - H(0,z)G(1-z,1,0,y) + H(0,z)G(1,1-z,0,y) + H(0,z)G(1,0,1-z,y)\Big) \\
& + \Big(- G(-z,1-z,1-z,0,y) - G(-z,1-z,0,1-z,y) \\
& - G(-z,0,1-z,1-z,y) - G(1-z,1-z,-z,1-z,y) + G(1-z,1-z,1,0,y) \\
& + G(1-z,1,1-z,0,y) + G(1-z,1,0,1-z,y) - G(1-z,1,0,0,y) \\
& - G(1-z,0,-z,1-z,y) - G(1,1-z,0,0,y) - H(1,1,0,z)G(-z,y) \\
& - H(1,1,z)G(-z,0,y) - G(1,0,1-z,0,y) + H(1,0,1,z)G(-z,y) \\
& - G(1,0,0,1-z,y) + G(1,0,0,y)H(1,z) - H(1,0,z) + H(1,0,z)G(-z,1-z,y) \\
& - H(1,0,z)G(1-z,1-z,y) + H(1,0,z)G(1-z,0,y) + H(1,z)G(-z,1-z,0,y) \\
& + H(1,z)G(-z,0,1-z,y) + H(1,z)G(1-z,1-z,-z,y) \\
& - H(1,z)G(1-z,1,0,y) - G(0,-z,1-z,1-z,y) + G(0,-z,1-z,0,y) \\
& + G(0,-z,0,1-z,y) + G(0,-z,1-z,y) - G(0,-z,y)H(1,1,z) \\
& + G(0,-z,1-z,y)H(0,z) - G(0,-z,0,y)H(1,z) + G(0,-z,1-z,y)H(1,z) \\
& + H(0,1,1,z)G(-z,y) - H(0,1,z)G(-z,1-z,y) + H(0,1,z)G(1-z,1-z,y) \\
& + H(0,1,z)G(1-z,0,y) + G(0,0,-z,1-z,y) - G(0,0,-z,y)H(1,z) \\
& - G(0,0,1,0,y) - H(0,z)G(-z,1-z,1-z,y) + H(0,z)G(1-z,0,0,y) \\
& - H(0,z)G(1,0,0,y)\Big) \\
& + \frac{3}{2}\Big(- G(0,-z,1-z,y)H(0,z) - H(0,1,z)G(1-z,-z,y) \\
& - H(0,0,1,z)G(1-z,y)\Big) \\
& + 2\Big(- G(-z,-z,-z,1-z,y) + G(-z,-z,1-z,1-z,y) \\
& + G(-z,1-z,-z,1-z,y) + G(1-z,-z,-z,1-z,y) - G(1,1,1,0,y) \\
& + G(1,1,0,0,y) + H(1,1,0,z)G(1-z,y) + H(1,1,z)G(-z,-z,y) + G(1,0,1,0,y) \\
& + H(1,z)G(-z,-z,-z,y) - H(1,z)G(-z,-z,1-z,y) \\
& - H(1,z)G(-z,1-z,-z,y) - H(1,z)G(1-z,-z,-z,y) + G(0,1,1,0,y) \\
& + H(0,1,z)G(-z,-z,y) + H(0,0,1,z)G(-z,y)\Big) \\
& + \frac{11}{24}\Big(- G(0,1-z,y)H(0,z) + H(0,1,z)G(0,y) - H(0,z)G(1-z,0,y) \\
& + G(0,y)H(1,0,z)\Big) \\
& + \frac{11}{6}\Big(- G(-z,-z,1-z,y) + G(1-z,1-z,0,y) + G(1-z,0,1-z,y) \\
& + G(1-z,0,0,y) - G(1,1,0,y) + H(1,0,0,z) - H(1,0,z)G(1-z,y) \\
& + H(1,z)G(-z,-z,y) - H(1,z)G(1-z,0,y) + G(0,1-z,1-z,y) \\
& - G(0,1-z,y)H(1,z) + G(0,1-z,0,y) - H(0,1,1,z) + H(0,1,z)G(-z,y) \\
& + G(0,0,1-z,y) + H(0,0,z)G(1-z,y) + H(0,0,z)G(0,y) - G(0,0,y)H(1,z) \\
& + G(0,0,y)H(0,z) + H(0,z)G(1-z,1-z,y) + G(0,y)H(1,1,z)\Big) \\
& + \frac{11}{4}\Big(- H(1,0,1,z) + H(0,1,z)G(1-z,y) + H(0,1,0,z)\Big) \\
& + \frac{11}{3}\Big(- G(1,0,0,y) - G(-z,1-z,1-z,y) - H(1,1,z)G(-z,y) \\
& + H(1,z)G(-z,1-z,y)\Big)
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{55}{12}\Big(- G(1-z,-z,1-z,y) + H(1,z)G(1-z,-z,y) - G(0,1,0,y)\Big) \\
& + \frac{\pi^2}{3}\Big(G(1,1,y) - G(0,1,y) + H(1,1,z) - G(1,0,y) + H(0,0,z) + G(0,0,y)\Big) \\
& + \Big(\frac{\zeta_3}{4} - \frac{33\pi^2}{8}\Big)\Big(- G(1-z,y) + H(1,z) - H(0,z) - G(0,y)\Big) \\
& + \Big(- \frac{121}{48} + \frac{\pi^2}{3}\Big)\Big(- G(1-z,1-z,y) - H(1,1,z) + H(1,z)G(1-z,y) \\
& - H(0,0,z) - G(0,0,y)\Big) \\
& + \Big(\frac{49}{48} - \frac{\pi^2}{8}\Big)\Big(- G(1-z,0,y) - G(0,1-z,y) - H(0,z)G(1-z,y) \\
& + G(0,y)H(1,z) - G(0,y)H(0,z)\Big) \\
& + \Big(\frac{67}{18} + \frac{\pi^2}{12}\Big)\Big(G(-z,1-z,y) + G(1,0,y) - H(1,z)G(-z,y)\Big) \\
& - \Big(\frac{245}{144} - \frac{\pi^2}{8}\Big)H(1,0,z) - \Big(\frac{389}{144} - \frac{\pi^2}{8}\Big)H(0,1,z) \\
& - \Big(\frac{13}{8} + \frac{451\pi^2}{96}\Big)G(1-z,y) + \Big(\frac{13}{8} + \frac{1265\pi^2}{288}\Big)H(1,z) \\
& + \Big(\frac{13}{8} + \frac{1133\pi^2}{288}\Big)\Big(- H(0,z) - G(0,y)\Big) + \frac{11\pi^2 G(1,y)}{36} \\
& - \frac{1}{36}\Big(\frac{5029\pi^2}{24} - 72\zeta_4 + \frac{99\zeta_3}{4} + \frac{3\pi^4}{16} - \frac{1321}{6}\Big)\Big] \\
& + \frac{1}{6}\Big((y+z)(1-y) - z^2\Big)\Big[+ G(1,0,y) + G(-z,1-z,y) - H(1,z)G(-z,y) \\
& + G(0,y)H(1,z) - G(0,y)H(0,z) \\
& + \frac{1}{2}\Big(- G(1-z,0,y) - H(1,0,z) - G(0,1-z,y) - H(0,1,z) \\
& - H(0,z)G(1-z,y) - G(0,y)H(1,z) + G(0,y)H(0,z)\Big) \\
& - \frac{41}{12}\Big(G(1-z,y) - H(1,z)\Big) + \frac{19}{12}H(0,z) - \frac{3\pi^2}{2} + \frac{247}{18}\Big] \\
& + \Big(\frac{25z}{12}(- 1 + \frac{1}{1-y-z}) - \frac{15z^2}{4(1-y-z)} - \frac{yz}{6} + \frac{5z^2}{6}(1 + \frac{2z}{1-y-z} \\
& + \frac{1}{(1-y-z)^2}(1 - 2z + z^2))\Big)\Big[G(0,y)H(0,z) + H(1,0,z) - G(1,0,y)\Big] \\
& + \Big(\frac{25y}{12z} - \frac{9y}{4} + \frac{yz}{6} - \frac{15y^2}{4z} + y^2 + \frac{5y^2}{6}(\frac{2y}{z} + \frac{1}{z^2}(1 - 2y + y^2))\Big) \times \\
& \Big[G(1-z,0,y) - G(1,0,y) - G(-z,1-z,y) + H(1,z)G(-z,y) + G(0,1-z,y) \\
& + H(0,1,z) - G(0,y)H(1,z)\Big] \\
& + \Big(\frac{25z}{12y} - \frac{9z}{4} + \frac{yz}{6} - \frac{15z^2}{4y} + z^2 + \frac{5z^2}{6}(\frac{2z}{y} + \frac{1}{y^2}(1 - 2z + z^2))\Big) \times \\
& \Big[H(0,z)G(1-z,y) - G(-z,1-z,y) + H(1,z)G(-z,y)\Big] \\
& + \frac{1}{36}\Big(63 - 93(y+z) + 4yz + \frac{30z}{y}(1 - 2z + z^2) + \frac{30y}{z}(1 - 2y + y^2) \\
& + 30(y^2 + z^2)\Big)\Big[G(1-z,y) - H(1,z)\Big] \\
& - \frac{1}{36}\Big(- 63z + 60z^2 - 30z^2(1-z)(\frac{1}{y} + \frac{1}{1-y-z}) + 26y(1-y-z)\Big)H(0,z) \\
& - \frac{1}{36}\Big(\frac{93z(1-z)}{2} - \frac{145y}{2} + \frac{27yz}{2} + \frac{79y^2}{2} + \frac{30z}{1-y-z}(- 1 + 2z - z^2) \\
& - \frac{30y^2(1-y)}{z}\Big)G(0,y) \\
& + \frac{\pi^2}{2}\Big(\frac{25z}{36(1-y-z)} - \frac{2z}{9} + \frac{5z^2}{18(1-y-z)}(2z - \frac{9}{2} + \frac{1}{1-y-z}(1 - 2z + z^2)) \\
& - \frac{7z^2}{36} - \frac{19yz}{36} + \frac{17y(1-y)}{36}\Big) \\
& + i\pi\Big[\frac{55}{24}\Big(- H(0,z)G(1-z,y) - H(0,z)G(0,y) - H(0,1,z) \\
& - 2H(1,z)G(-z,y) + H(1,z)G(0,y) - H(1,0,z) - G(1-z,0,y) \\
& + 2G(-z,1-z,y) - G(0,1-z,y) + 2G(1,0,y) + \frac{11}{6}(- H(0,z) + H(1,z) \\
& - G(1-z,y) - G(0,y)) + \frac{1}{3}(y(1-y-z) + z(1-z))\Big) - \frac{77\pi^2}{288} + \frac{3\zeta_3}{4} + \frac{185}{24}\Big]
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{L2;4} = & -2\left[J_4\left(-\frac{uv}{w}\right) + J_4\left(-\frac{vw}{u}\right) + J_4\left(-\frac{wu}{v}\right)\right] \\
& - 8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1 - \frac{1}{u_i}\right) + \frac{\log^4 u_i}{4!}\right] - 2\left[\sum_{i=1}^{3}\mathrm{Li}_2\left(1 - \frac{1}{u_i}\right)\right]^2 \\
& + \frac{1}{2}\left[\sum_{i=1}^{3}\log^2 u_i\right]^2 + 2(J_2^2 - \zeta_2 J_2) - \frac{\log^4(uvw)}{4!} \\
& - \zeta_3 \log(uvw) - \frac{123}{8}\zeta_4,
\end{aligned}
$$

$$
\begin{aligned}
J_4(x) = & \ \mathrm{Li}_4(x) - \log(-x)\mathrm{Li}_3(x) + \frac{\log^2(-x)}{2!}\mathrm{Li}_2(x) \\
& - \frac{\log^3(-x)}{3!}\mathrm{Li}_1(x) - \frac{\log^4(-x)}{48},
\end{aligned}
$$

$$
J_2 = \sum_{i=1}^{3}\left(\mathrm{Li}_2(1 - u_i) + \frac{1}{2}\log(u_i)\log(u_{i+1})\right),
$$

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Transcendental Degree

$$R^{(2)}_{\mathrm{L2;4}} = -2\left[J_4\left(-\frac{uv}{w}\right) + J_4\left(-\frac{vw}{u}\right) + J_4\left(-\frac{wu}{v}\right)\right]$$
$$-8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1-\frac{1}{u_i}\right) + \frac{\log^4 u_i}{4!}\right] - 2\left[\sum_{i=1}^{3}\mathrm{Li}_2\left(1-\frac{1}{u_i}\right)\right]^2$$
$$+\frac{1}{2}\left[\sum_{i=1}^{3}\log^2 u_i\right]^2 + 2(J_2^2 - \zeta_2 J_2) - \frac{\log^4(uvw)}{4!}$$
$$-\zeta_3\log(uvw) - \frac{123}{8}\zeta_4,$$

$$J_4(x) = \mathrm{Li}_4(x) - \log(-x)\mathrm{Li}_3(x) + \frac{\log^2(-x)}{2!}\mathrm{Li}_2(x) - \frac{\log^3(-x)}{3!}\mathrm{Li}_1(x) - \frac{\log^4(-x)}{48},$$
$$J_2 = \sum_{i=1}^{3}\left(\mathrm{Li}_2(1-u_i) + \frac{1}{2}\log(u_i)\log(u_{i+1})\right),$$

- deg=0: Algebraic number, $2-\sqrt{3}$; rational functions, $\frac{uv}{w^2}$
- deg=1: $\pi,\quad \log(1-u-v),\quad \frac{1}{\epsilon}$
- deg=k: $\zeta_k,\quad Li_k(\frac{u}{v})$

$D = 4 - 2\epsilon$

This definition allows us to discuss the transcendental degree of UV or IR divergent quantities

$$deg\left(\frac{\pi^2\zeta_3\log u}{\epsilon^2}\right) = 8$$

The maximally transcendental principle

$$\mathcal{N}=4\ SYM \stackrel{?}{=} QCD$$

- The MT part of N=4 and QCD amplitude are the same?
- The MT part of amplitude is universal in all gauge theories?
- Anomalous dimensions of twist-two operators [*Kotikov, Lipatov, Onishchenko, Velizhanin 2004*]
- The dimension 4 contribution of Higgs to 3 gluon ($H \to 3g$) amplitude in HEFT [*Brandhuber, Travaglini, Yang 2012*].

The MTP of the IR divergence

IR renormalization factor

- IR divergence can be determined by cusp and jet anomalous dimensions.

$$\ln Z \sim \alpha\left(\frac{\gamma_1^{cusp}}{4\epsilon^2}+\frac{\gamma_1^{q,g}}{2\epsilon}\right)+\alpha^2\left(\frac{\gamma_2^{cusp}}{16\epsilon^2}+\frac{\gamma_2^{q,g}}{4\epsilon}\right)+\alpha^3\left(\frac{\gamma_3^{cusp}}{36\epsilon^2}+\frac{\gamma_3^{q,g}}{6\epsilon}\right)$$

- The MT part of cusp anomalous dimension is theory-independent up to 3 loops.

$$\gamma_1^{cusp} = 4\,,$$

$$\gamma_2^{cusp} = \left(\frac{268}{9}-\frac{4\pi^2}{3}\right)C_A-\frac{80}{9}T_F n_f\,,$$

$$\gamma_3^{cusp} = C_A^2\left(\frac{490}{3}-\frac{536\pi^2}{27}+\frac{44\pi^4}{45}+\frac{88}{3}\zeta_3\right)+C_A T_F n_f\left(-\frac{1672}{27}+\frac{160\pi^2}{27}-\frac{224}{3}\zeta_3\right)$$
$$+C_F T_F n_f\left(-\frac{220}{3}+64\zeta_3\right)-\frac{64}{27}T_F^2 n_f^2\,.$$

The MTP of the IR divergence

- The MT part of jet anomalous dimension only depends on the group representation of the particle up to 3 loops.

$$\gamma_1^q = -3C_F\,,$$
$$\gamma_2^q = C_F^2\left(-\frac{3}{2}+2\pi^2-24\zeta_3\right)+C_FC_A\left(-\frac{961}{54}-\frac{11\pi^2}{6}+26\zeta_3\right)+C_FT_Fn_f\left(\frac{130}{27}+\frac{2\pi^2}{3}\right),$$
$$\gamma_3^q = C_F^3\left(-\frac{29}{2}-3\pi^2-\frac{8\pi^4}{5}-68\zeta_3+\frac{16\pi^2}{3}\zeta_3+240\zeta_5\right)$$
$$+C_F^2C_A\left(-\frac{151}{4}+\frac{205\pi^2}{9}+\frac{247\pi^4}{135}-\frac{844}{3}\zeta_3-\frac{8\pi^2}{3}\zeta_3-120\zeta_5\right)$$
$$+C_FC_A^2\left(-\frac{139345}{2916}-\frac{7163\pi^2}{486}-\frac{83\pi^4}{90}+\frac{3526}{9}\zeta_3-\frac{44\pi^2}{9}\zeta_3-136\zeta_5\right)$$
$$+C_F^2T_Fn_f\left(\frac{2953}{27}-\frac{26\pi^2}{9}-\frac{28\pi^4}{27}+\frac{512}{9}\zeta_3\right)$$
$$+C_FC_AT_Fn_f\left(-\frac{17318}{729}+\frac{2594\pi^2}{243}+\frac{22\pi^4}{45}-\frac{1928}{27}\zeta_3\right)$$
$$+C_FT_F^2n_f^2\left(\frac{9668}{729}-\frac{40\pi^2}{27}-\frac{32}{27}\zeta_3\right).$$

$$\gamma_1^g = -\frac{11}{3}C_A+\frac{4}{3}T_Fn_f\,,$$
$$\gamma_2^g = C_A^2\left(-\frac{692}{27}+\frac{11\pi^2}{18}+2\zeta_3\right)+C_AT_Fn_f\left(\frac{256}{27}-\frac{2\pi^2}{9}\right)+4C_FT_Fn_f\,,$$
$$\gamma_3^g = C_A^3\left(-\frac{97186}{729}+\frac{6109\pi^2}{486}-\frac{319\pi^4}{270}+\frac{122}{3}\zeta_3-\frac{20\pi^2}{9}\zeta_3-16\zeta_5\right)$$
$$+C_A^2T_Fn_f\left(\frac{30715}{729}-\frac{1198\pi^2}{243}+\frac{82\pi^4}{135}+\frac{712}{27}\zeta_3\right)$$
$$+C_AC_FT_Fn_f\left(\frac{2434}{27}-\frac{2\pi^2}{3}-\frac{8\pi^4}{45}-\frac{304}{9}\zeta_3\right)-2C_F^2T_Fn_f$$
$$+C_AT_F^2n_f^2\left(-\frac{538}{729}+\frac{40\pi^2}{81}-\frac{224}{27}\zeta_3\right)-\frac{44}{9}C_FT_F^2n_f^2\,.$$

- MTP of IR divergence: $$\gamma^q\Big|_{MT}\ \xrightarrow{C_F\to C_A}\ \gamma^g\Big|_{MT}$$

Hidden relation in $H \to q\bar{q}g$ amplitude

- The dimension 4 contribution of $H \to q\bar{q}g$ and $H \to 3g$ amplitudes are given in [*Gehrmann et al 2011*].

$$\Omega^{(2),finite} = \left(N^2 A_\Omega^{(2)} + N^0 B_\Omega^{(2)} + \frac{1}{N^2} C_\Omega^{(2)} + \frac{N_F}{N} D_\Omega^{(2)} + N N_F E_\Omega^{(2)} + N_F^2 F_\Omega^{(2)} \right)$$

- We rewrite the expressions using quadratic Casmir:

$$C_A = N, \; C_F = \frac{N^2 - 1}{2N}$$

$H \to q\bar{q}g$ amplitude reduces to $H \to 3g$ amplitude by replacing quadratic Casmir from Fundamental to Adjoint. [QJ, Yang 2019]

$$H \to q\bar{q}g \xrightarrow{C_F \to C_A} H \to 3g$$

Verify MTP for dimension-6 operators

$$
\begin{aligned}
A_{QCD;4} = & -\frac{3}{2}\mathrm{Li}_4(u) + \frac{3}{4}\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\mathrm{Li}_3\left(-\frac{u}{v}\right) \\
& + \frac{\log^2(u)}{32}\left[\log^2(u) + \log^2(v) + \log^2(w) - 4\log(v)\log(w)\right] \\
& + \frac{\zeta_2}{8}\left[5\log^2(u) - 2\log(v)\log(w)\right] - \frac{1}{4}\zeta_4 - \frac{1}{2}\zeta_3\log(-q^2) + \mathrm{perms}(u,v,w)\,.
\end{aligned}
$$

$$\mathcal{N}=4\ SYM \overset{\checkmark}{=} QCD$$

$H \to 3g$: **Verified** [QJ, Yang 2018]

The same relation as the dimension-4 operator was verified. [QJ, Yang 2019]

$$H \to q\bar{q}g \xrightarrow{\ C_F \to C_A\ } H \to 3g$$

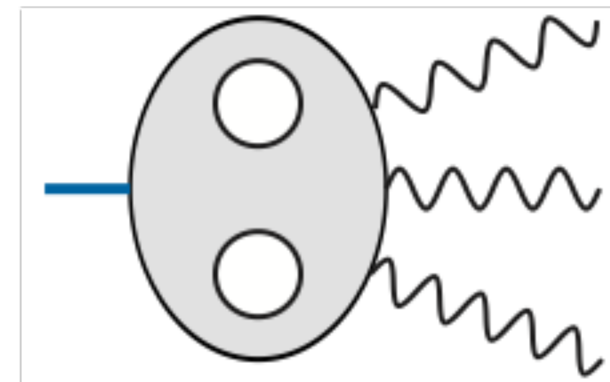
The Universality of MTP

- Changed the effective interaction between Higgs and partons, including $H\bar{\psi}\psi,\ HF^3,\ HFD^2F$
- Replaced quark by scalar fields and computed $H \to \phi\bar{\phi}g$ amplitudes.
- MTP is universal. It does not depend on the spin and is insensitive to the details of interaction.

Content

- Motivations
- Computations
- The hidden relation
- **Summary and outlook**

Summary and outlook



- Analytical Higgs to 3 parton amplitude at two loops.
- Extend MTP to $H \to q\bar{q}g$ amplitude.
- The origin of MTP?
- Any relation between MT part of N=4 SYM and QCD gluon amplitudes?

Thank you!