Soft-collinear effective theory at subleading power

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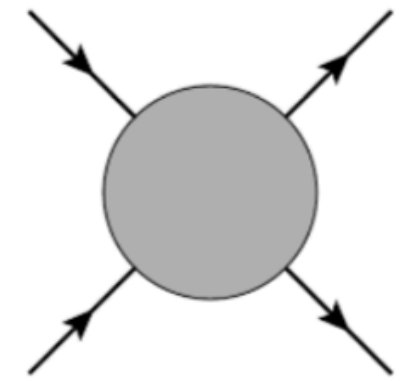
USTC
Sep.7th, 2019

LO power of strong coupling

$$\hat{\sigma}_{a,b\to X} = \alpha_s^{n}\left[\sigma_0 + \alpha_s\sigma_1 + \alpha_s^2\sigma_2 + \alpha_s^3\sigma_3 + \mathcal{O}(\alpha_s^4)\right]$$

LO	NLO	NNLO	N^{3}LO
	$\mathcal{O}(0.1)$	$\mathcal{O}(0.01)$	

- LO:



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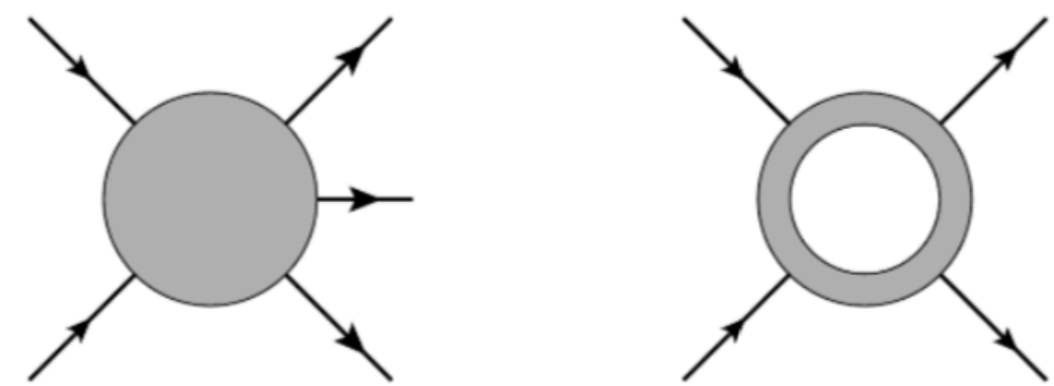
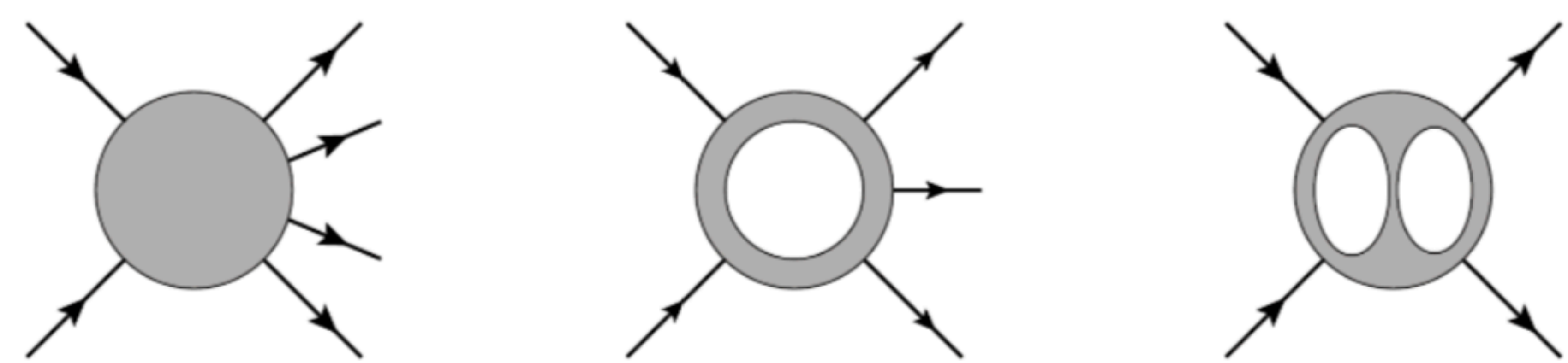


- NNLO:



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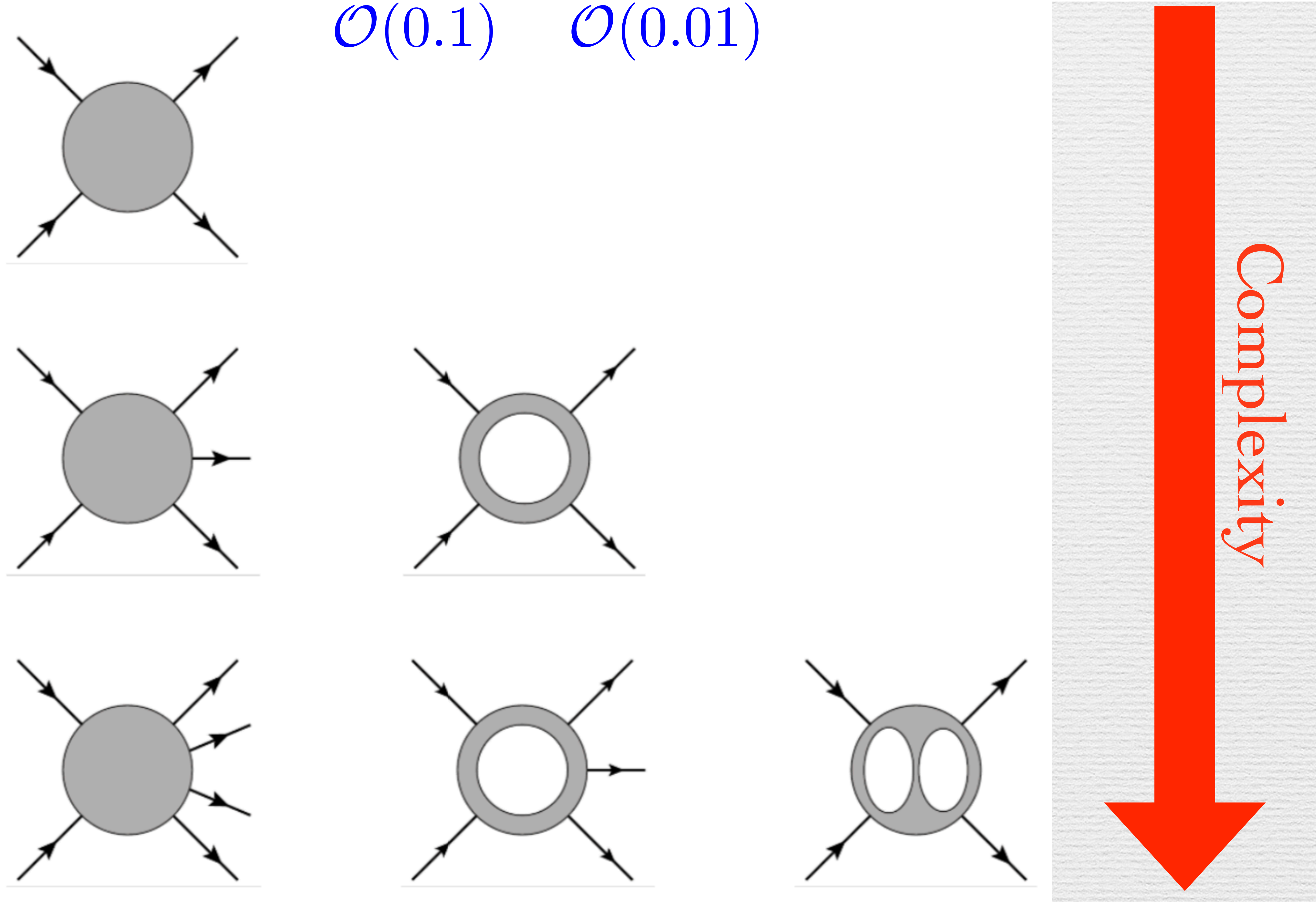

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$$\frac{M_H^2}{s}$$

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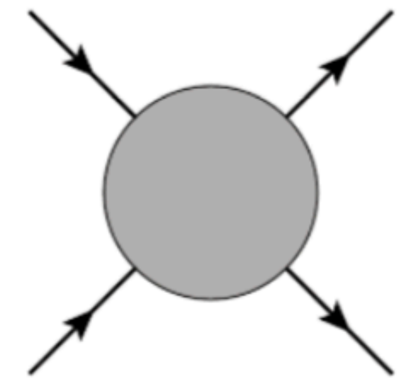
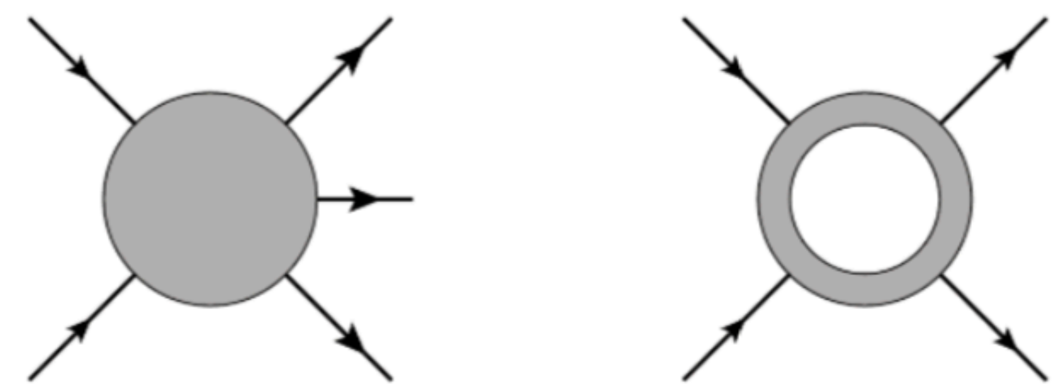


- NLO:



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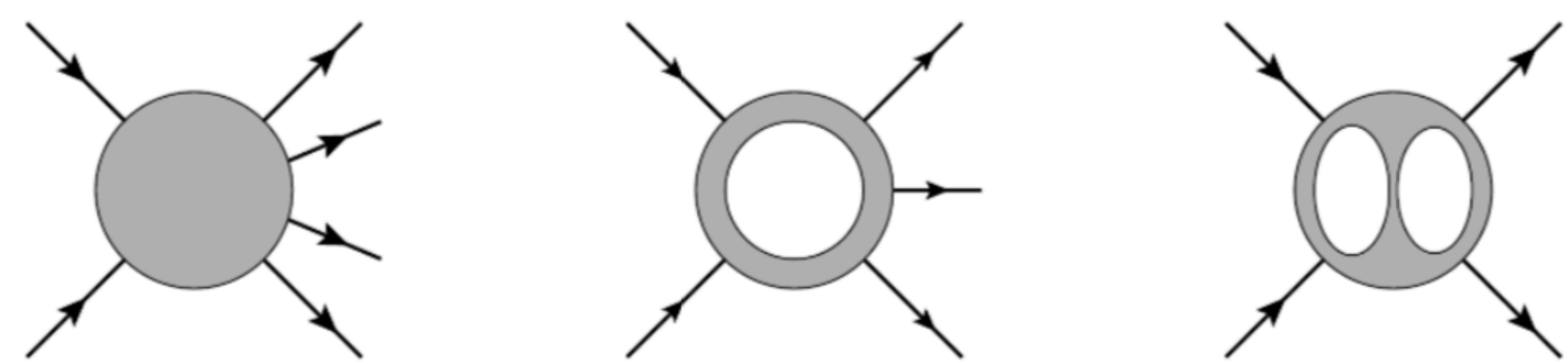

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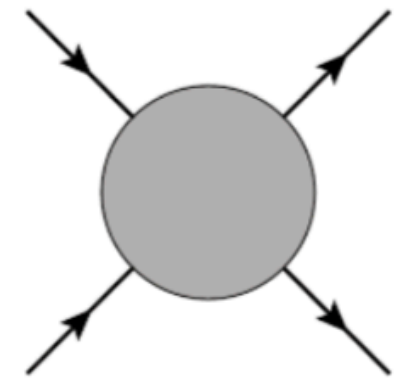
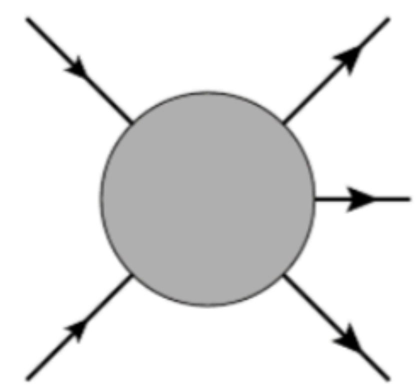
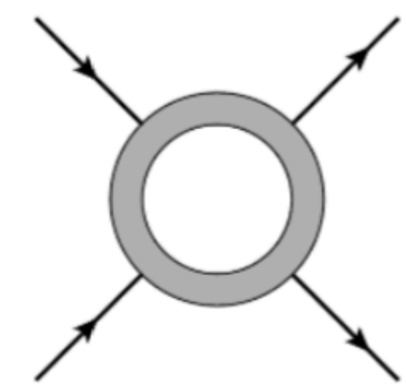


$$\frac{M_H^2}{s}$$

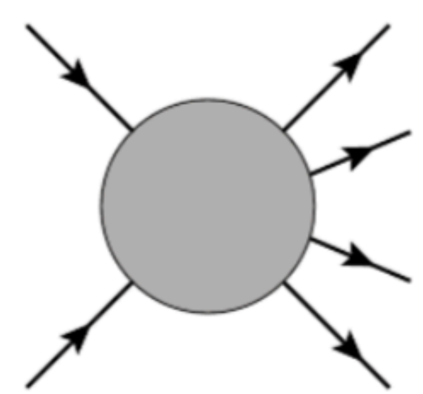
- NLO:

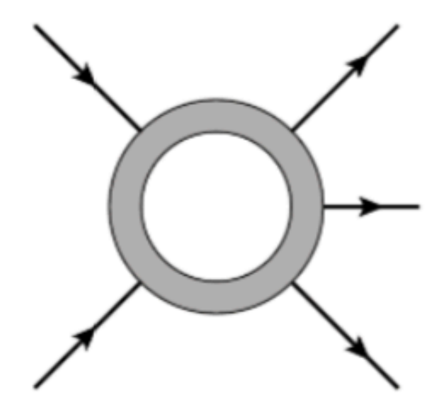


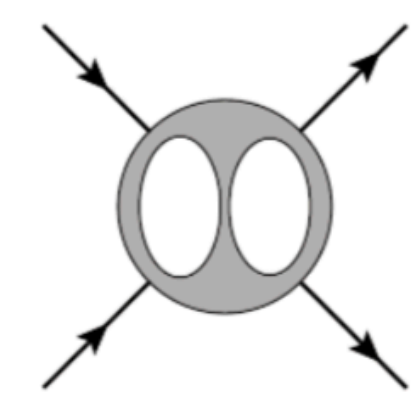


$$\ln^2\left(1-\frac{M_H^2}{s}\right)$$

- NNLO:







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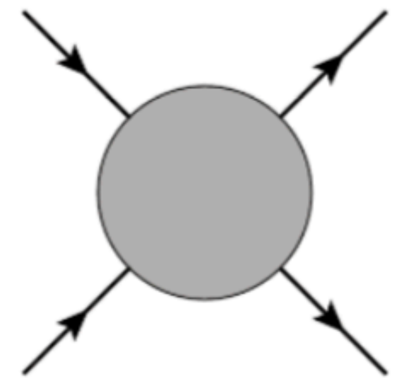

$$\frac{M_H^2}{s}$$

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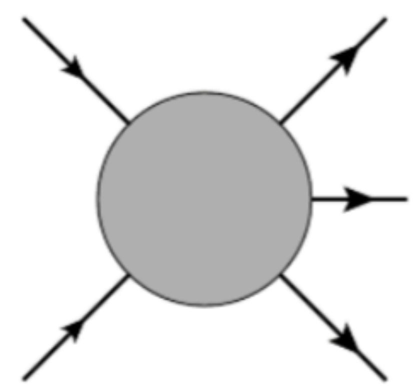

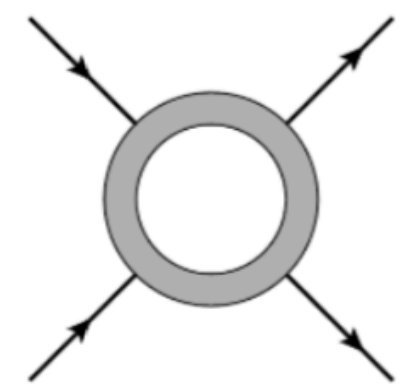

$$\ln^2\left(1 - \frac{M_H^2}{s}\right)$$

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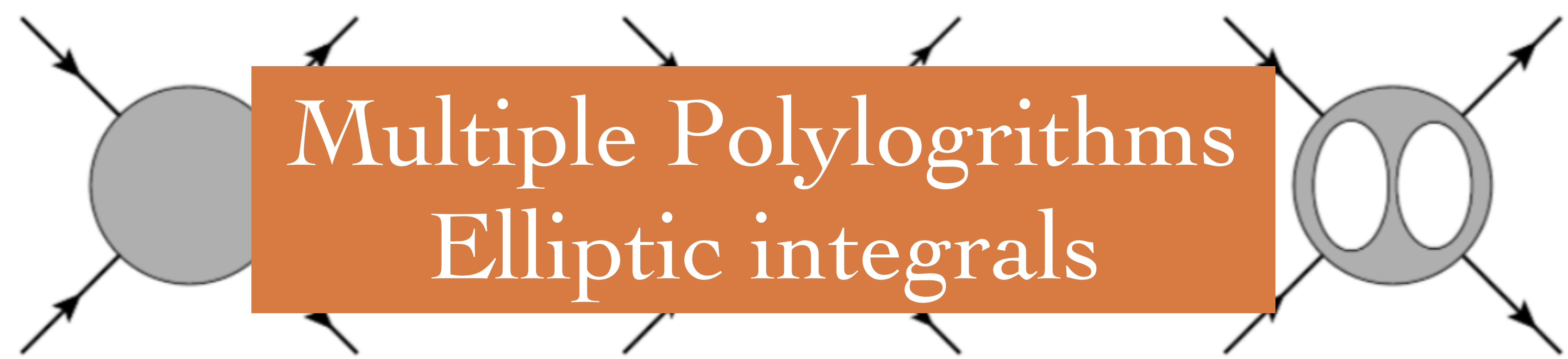

Multiple Polylogrithms
Elliptic integrals

Complexity

If we can not obtain the full result, can we first get an approximation near some limits?

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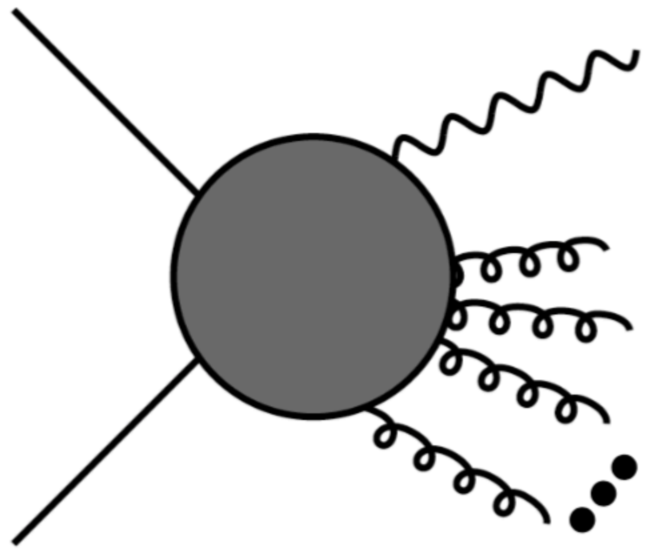
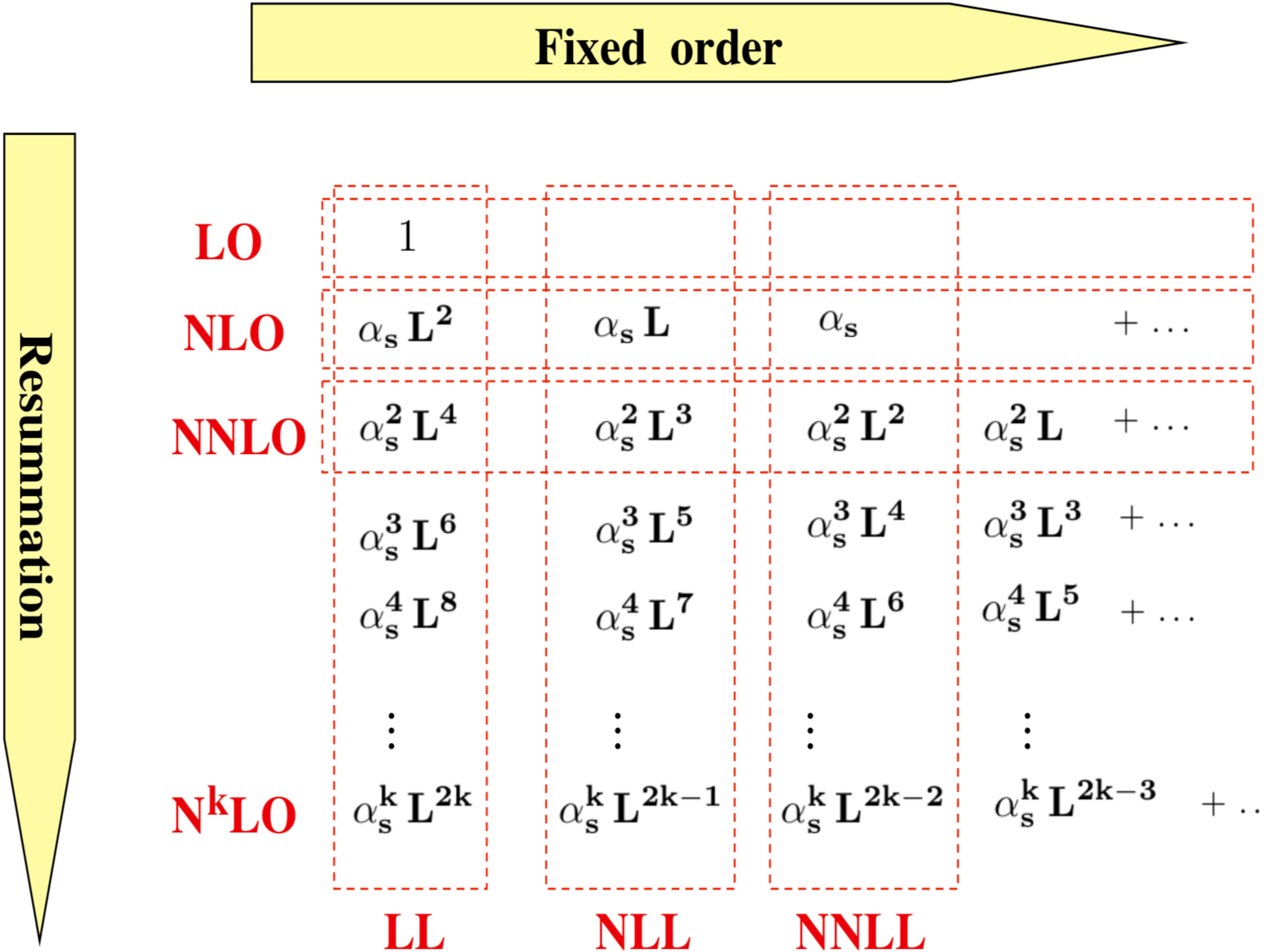
Threshold limit $\quad z = \frac{Q^2}{\hat{s}} \to 1$



$$\frac{d\hat{\sigma}^{(k)}_{q\bar{q}}}{dQ^2} \propto \alpha_s^k \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$


Fixed order
Resummation
LO
NLO
NNLO
N^{k}LO
1
$\alpha_s L^2$ $\alpha_s L$ α_s $+ \dots$
$\alpha_s^2 L^4$ $\alpha_s^2 L^3$ $\alpha_s^2 L^2$ $\alpha_s^2 L$ $+ \dots$
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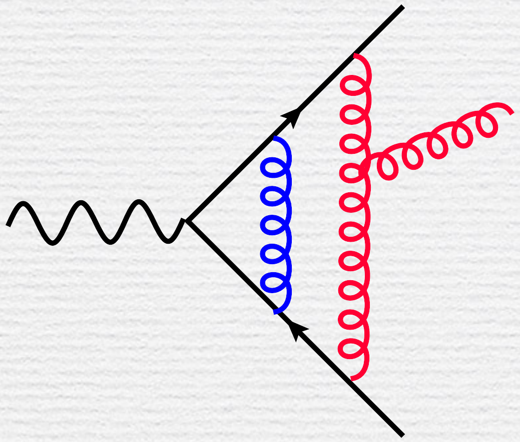
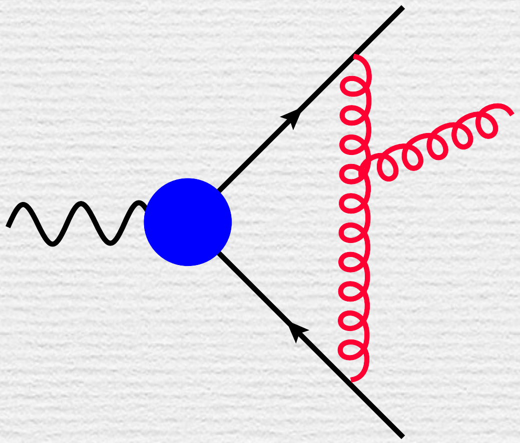
$\alpha_s^k L^{2k}$ $\alpha_s^k L^{2k-1}$ $\alpha_s^k L^{2k-2}$ $\alpha_s^k L^{2k-3}$ $+ \dots$
LL
NLL
NNLL

QCD



10^{-28} s vs. 10^{-25} s

EFT



Drell-Yan process (pp>Z)

NNLO cross-section: Hamberg, van Neerven, Matsuura 1991

Near threshold $z=Q^2/s \to 1$

$$\hat{\sigma}_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \ldots \right]$$

Leading power

Next-to-leading power

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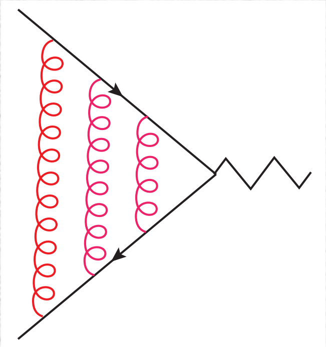
Factorization and Resummation

Soft-collinear effective theory

Leading power logs



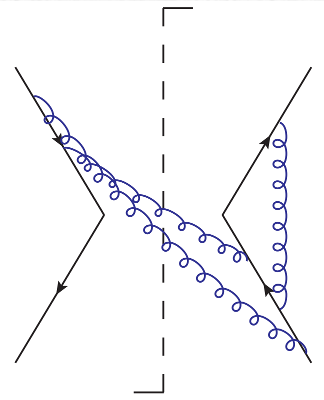
$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, R. N. Lee, '09

T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli and C. Studerus, '10

Wilson lines

$$S_{\mathrm{DY}}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

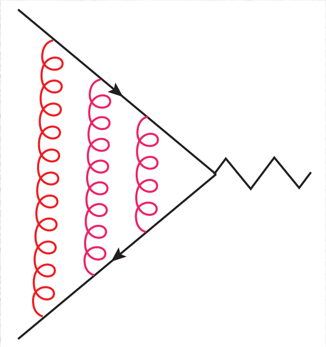


C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog and B. Mistlberger, '13

Y. Li, A. von Manteuffel, R. M. Schabinger and H. X. Zhu, '13

$$\frac{d}{d\ln\mu} H(Q^2,\mu^2) = \left[2\Gamma_{\mathrm{cusp}} \ln\frac{Q^2}{\mu^2} + 2\gamma^V\right] H(Q^2,\mu^2)$$

Leading power logs



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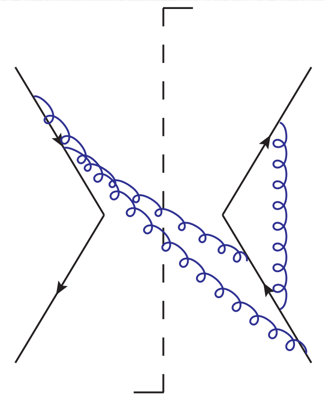

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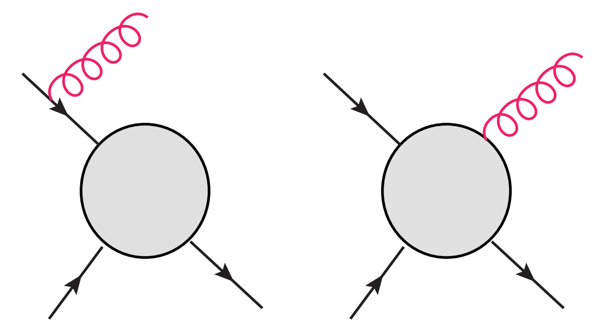
In contrast, much less is understood at NLP.

Structure of NLP logs

- the method of region approach, Bonocore et al 2014, Anastasiou et al 2014, Bahjat-Abbas et al 2018
- diagrammatic factorization techniques, Bonocore et al 2015, Bonocore et al 2016, Del Duca et al:2017



Low-Burnett-Kroll theorem 1958,1968

$$-g_s \sum_{i=1}^{N} \mathbf{T}_i \left(\frac{p_i \cdot \epsilon(k)}{p_i \cdot k} + \frac{\epsilon_\mu(k) k_\nu J_i^{\mu\nu}}{p_i \cdot k} \right) A_0(\{p_i\})$$

$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu}$$

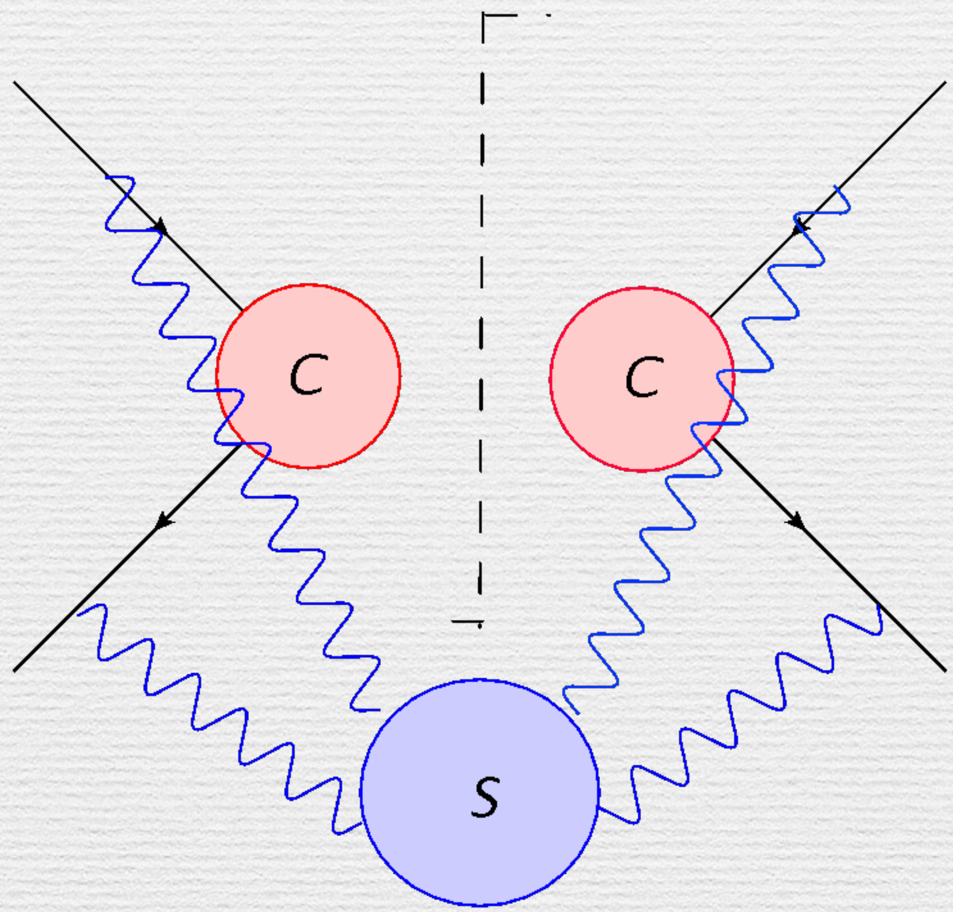
LP factorization

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

$\mu_h \sim Q$

$\mu_s \sim Q(1-z)$

$\mu_c \sim Q\sqrt{1-z}$



$(n_+p, n_-p, p_\perp)$

$Q(1,1,1)$

$Q(1,\lambda^2,\lambda)$

$Q(\lambda^2,\lambda^2,\lambda^2)$

$\lambda = \sqrt{1-z}$

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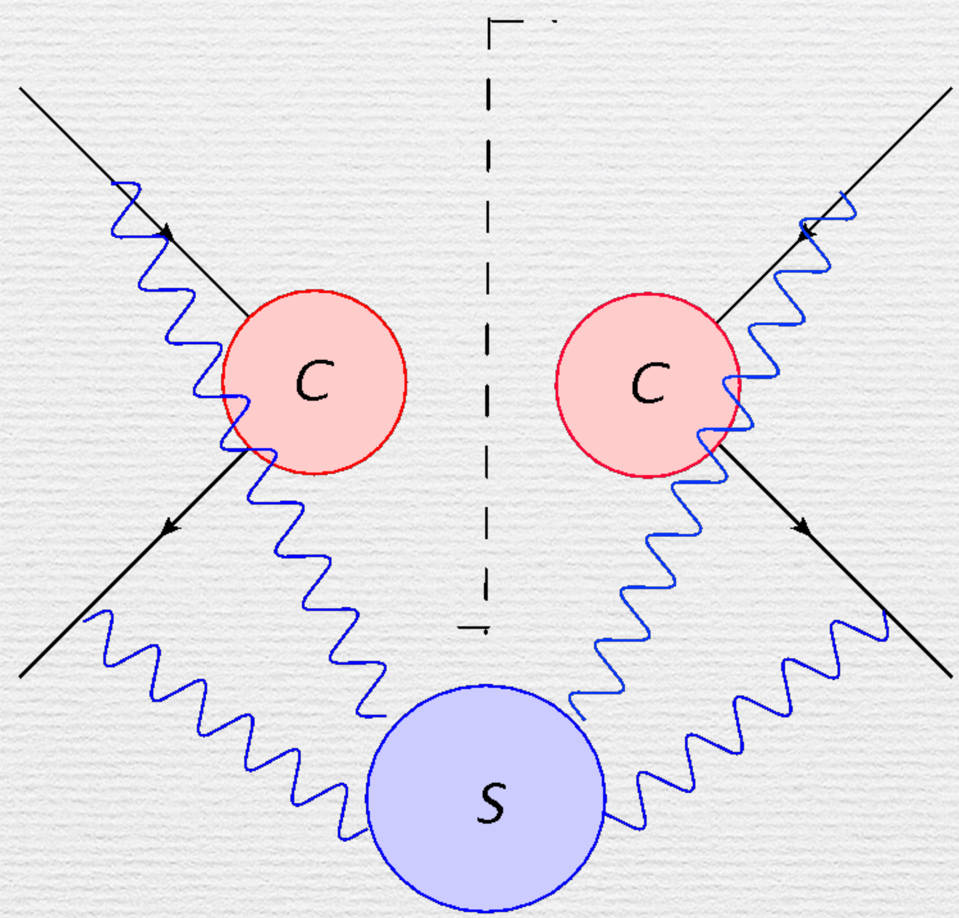

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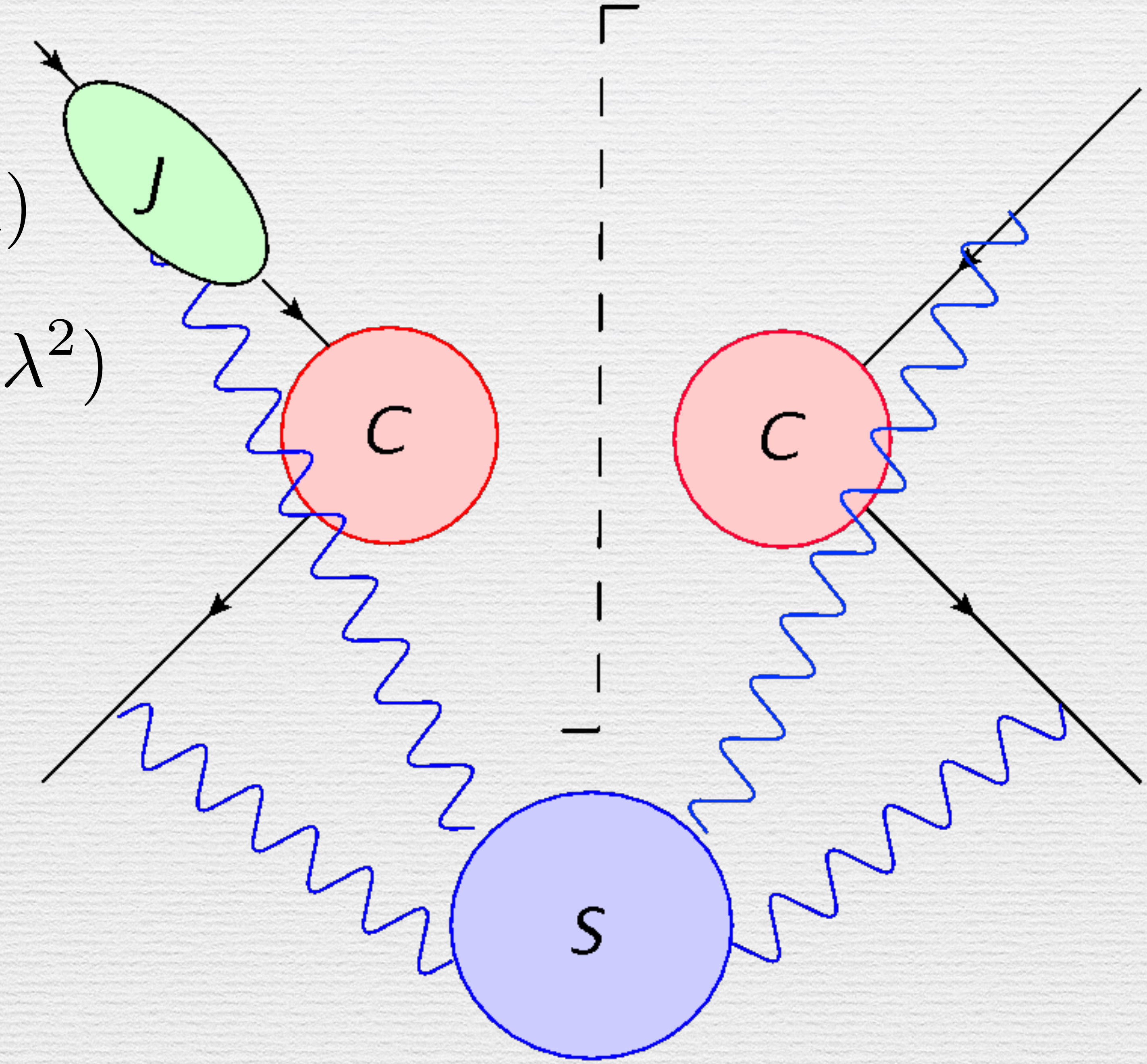
$\lambda = \sqrt{1-z}$

Tricky point: no collinear function at LP

NLP factorization

$Q(1, \lambda^2, \lambda)$

$Q(\lambda^2, \lambda^2, \lambda^2)$



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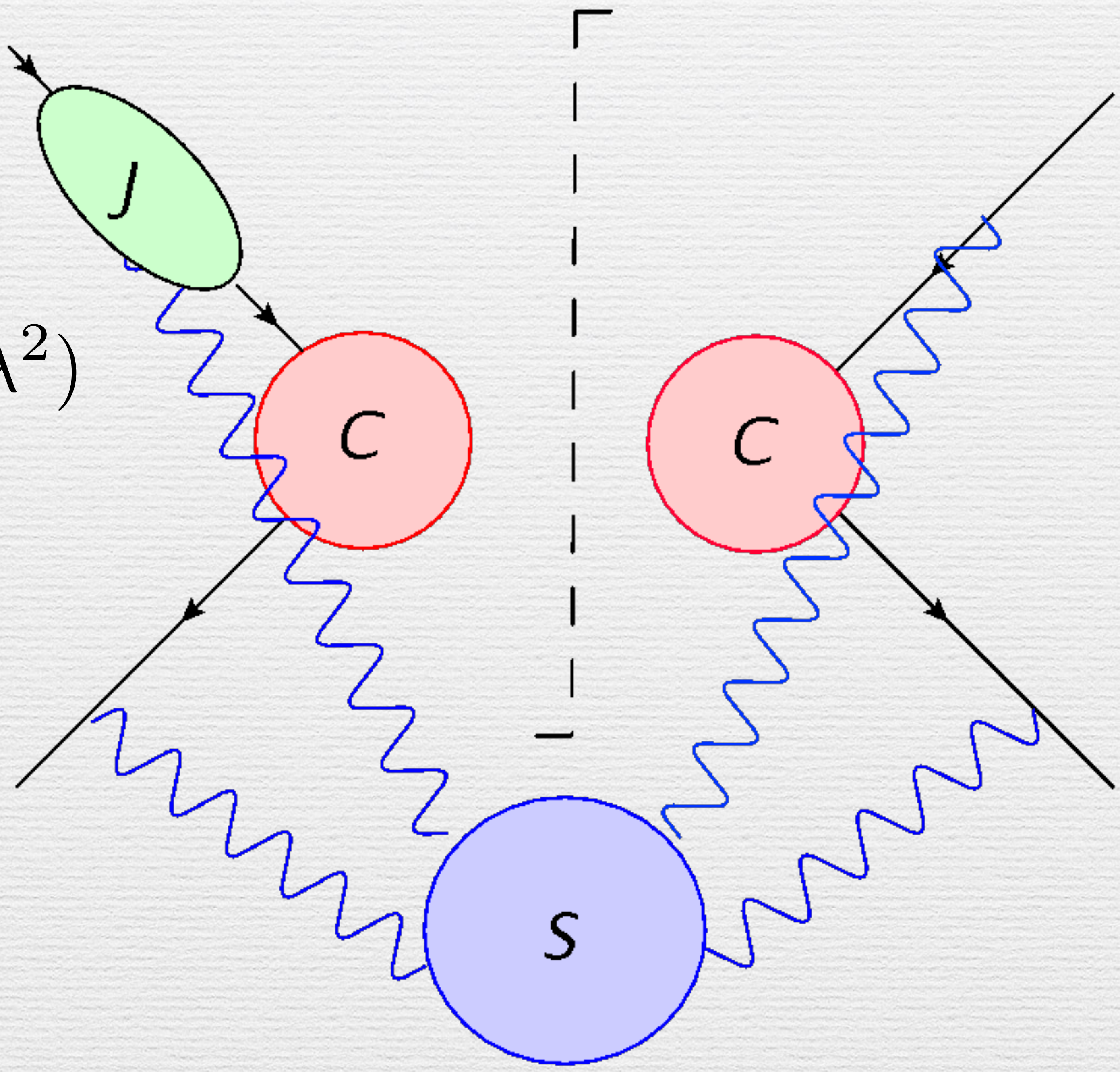

NLP factorization

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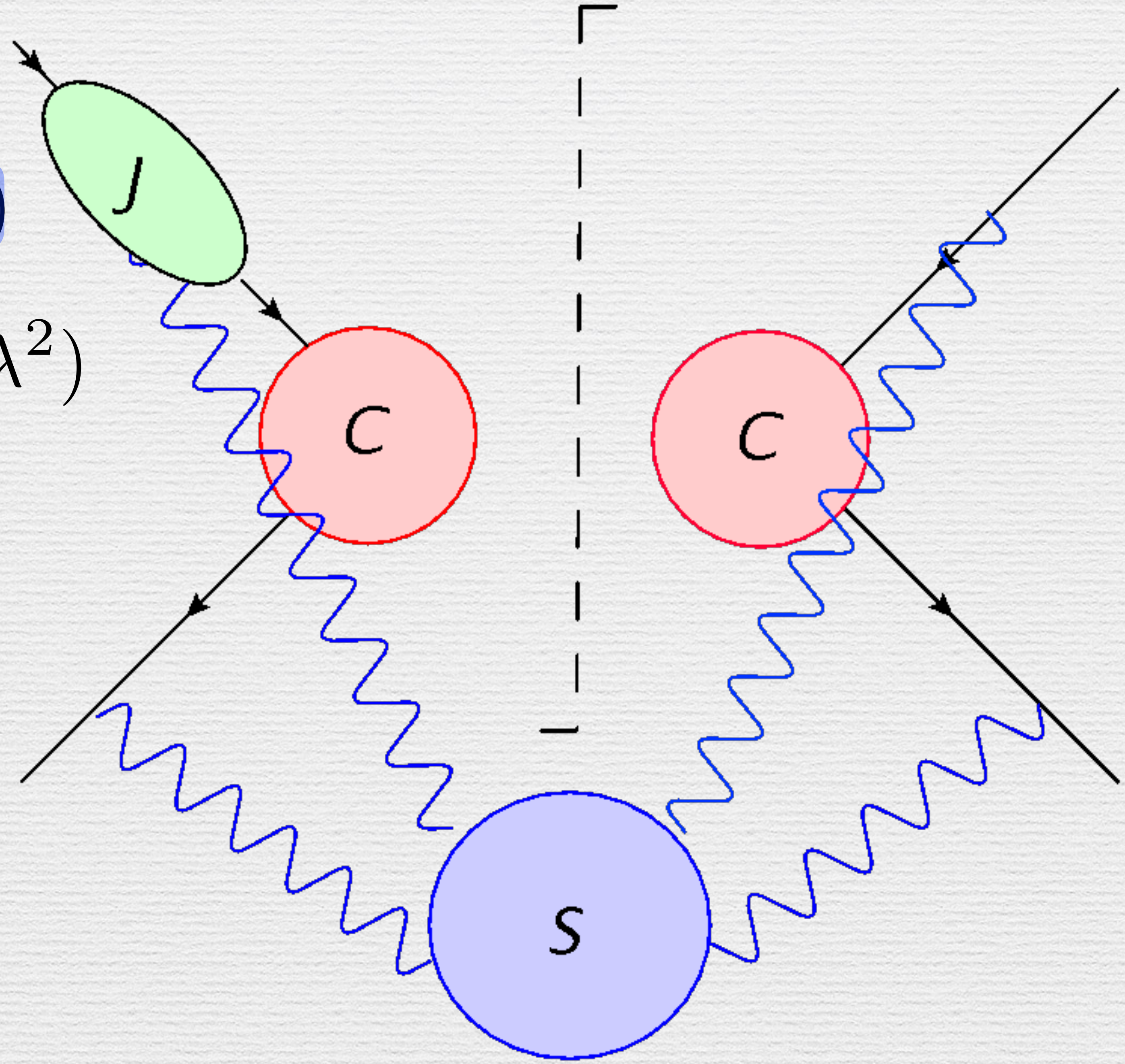
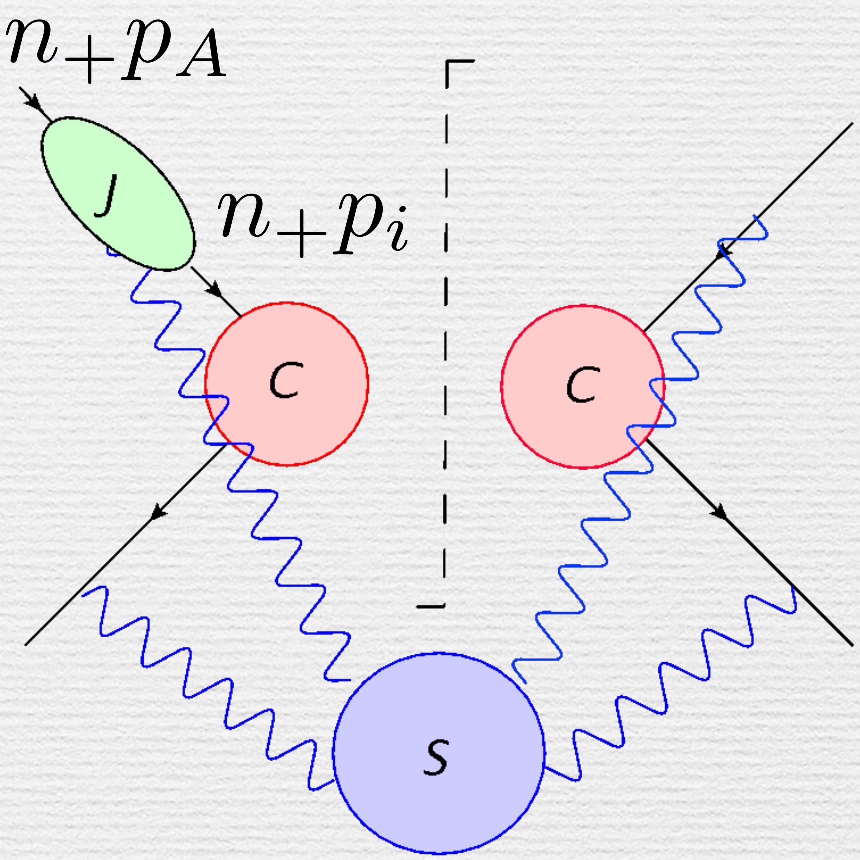


NLP factorization

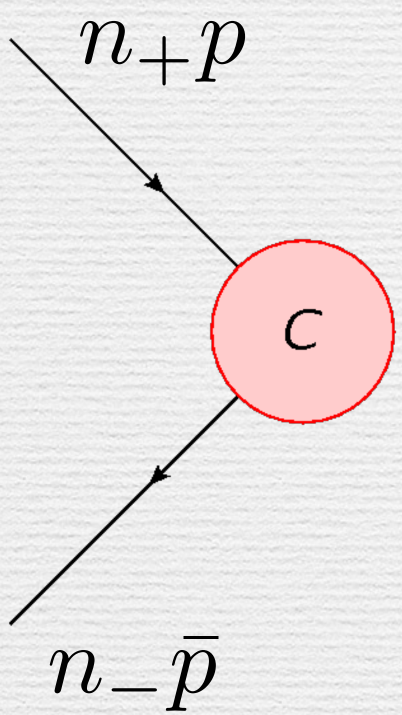
$$\hat{\sigma}(z) = \sum_i \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i' \, D(-\hat{s};\omega_i,\bar{\omega}_i) D^*(-\hat{s};\omega_i',\bar{\omega}_i')$$
$$\times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2+\vec{q}^{\,2}}} \frac{1}{2\pi}$$
$$\int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x} \, \widetilde{S}(x;\omega_i,\bar{\omega}_i,\omega_i',\bar{\omega}_i')$$



$$D(-\hat{s};\omega_i,\bar{\omega}_i) = \int d(n_+p_i) d(n_-\bar{p}_i) \, C(n_+p_i, n_-\bar{p}_i)$$
$$\times J(n_+p_i, x_a n_+ p_A;\omega_i) \, \bar{J}(n_-\bar{p}_i, -x_b n_- p_B;\bar{\omega}_i)$$

$$n_+p_i, n_-\bar{p}_i \sim O(1)$$
$$\omega_i, \bar{\omega}_i \sim O(\lambda^2)$$

Hard function



QCD

Soft-collinear effective theory

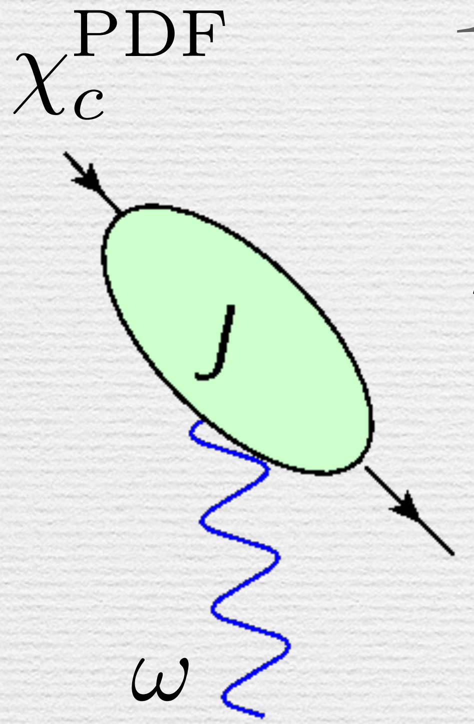
$$\bar{\psi}\gamma_\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\, J_\mu^{A0}(t,\bar{t}\,)$$

$$C^{A0}(n_+p, n_-\bar{p}) = \int dt\, d\bar{t}\, e^{-itn_+p - i\bar{t}n_-\bar{p}}\, \widetilde{C}^{A0}(t,\bar{t}\,)$$

$$J_\mu^{A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\mu}\chi_c(tn_+)$$

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

NLP jet function



$$i\int d^4z\, e^{i\omega(n_+z)/2}\,\mathbf{T}\left[\chi_{c,\alpha a}(tn_+)\bar\chi_{c,d}(z)\frac{\not n_+}{2}\chi_{c,e}(z)\right]$$

$$= 2\pi\int du\,\widetilde J_{\alpha\beta,abde}(t,u;\omega)\,\chi^{\rm PDF}_{c,\beta b}(un_+)$$

NLP quark-gluon interaction: Beneke et al 2002

$$\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar\chi_c x_\perp^\mu x_\perp^\nu\left[i\partial_\nu in_-\partial\mathcal{B}^+_\mu\right]\frac{\not n_+}{2}\chi_c \qquad \mathcal{B}^\mu_\pm = Y^\dagger_\pm\left[iD^\mu_s Y_\pm\right]$$

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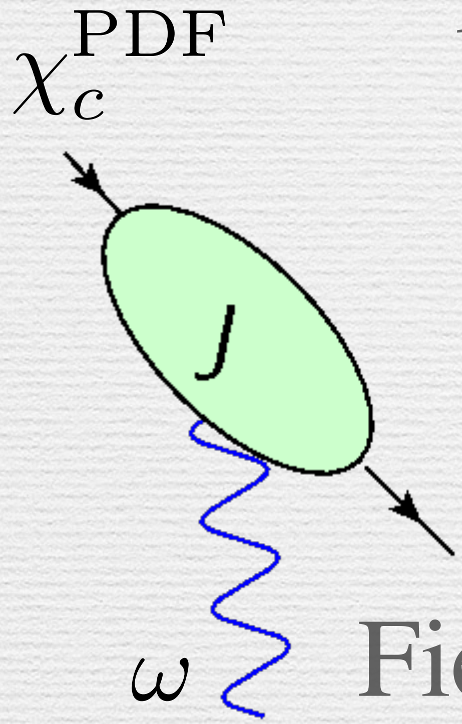

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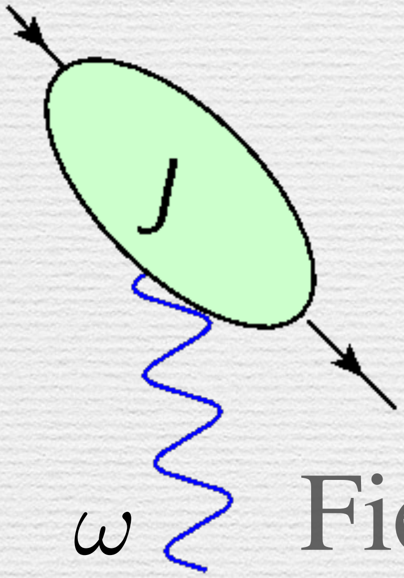
Field definition of radiative jet function

del Duca 1990,
Bonocore et al '15,'16
Moult et al '19

NLP quark-gluon interaction: Beneke et al 2002

$$\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar{\chi}_c x_\perp^\mu x_\perp^\nu\left[i\partial_\nu i n_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c \qquad \mathcal{B}^\mu_\pm = Y^\dagger_\pm\left[iD^\mu_s Y_\pm\right]$$

NLP jet function



$\chi_c^{\rm PDF}$ χ_c ω

$$i\int d^4z\, e^{i\omega(n_+z)/2}\,\mathbf{T}\left[\chi_{c,\alpha a}(tn_+)\bar\chi_{c,d}(z)\frac{\not n_+}{2}\chi_{c,e}(z)\right]$$

$$= 2\pi\int du\,\widetilde J_{\alpha\beta,abde}(t,u;\omega)\,\chi^{\rm PDF}_{c,\beta b}(un_+)$$

Field definition of radiative jet function

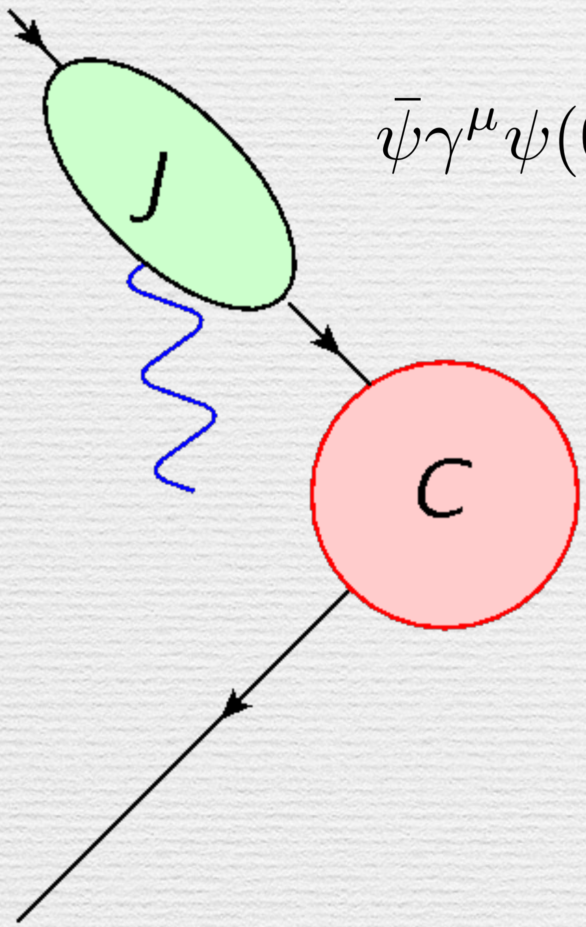
del Duca 1990,
Bonocore et al '15,'16
Moult et al '19

NLP quark-gluon interaction: Beneke et al 2002

$$\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar\chi_c x^\mu_\perp x^\nu_\perp\left[i\partial_\nu in_-\partial\mathcal{B}^+_\mu\right]\frac{\not n_+}{2}\chi_c \qquad \mathcal{B}^\mu_\pm = Y^\dagger_\pm\left[iD^\mu_s Y_\pm\right]$$

LO: $$J^{\mu\rho}_{2\xi;\alpha\beta,abde}(n_+p,n_+p';\omega) = -\frac{g^{\mu\rho}_\perp}{n_+p}\delta(n_+p-n_+p')\delta_{\alpha\beta}\delta_{ad}\delta_{eb}$$

NLP factorization

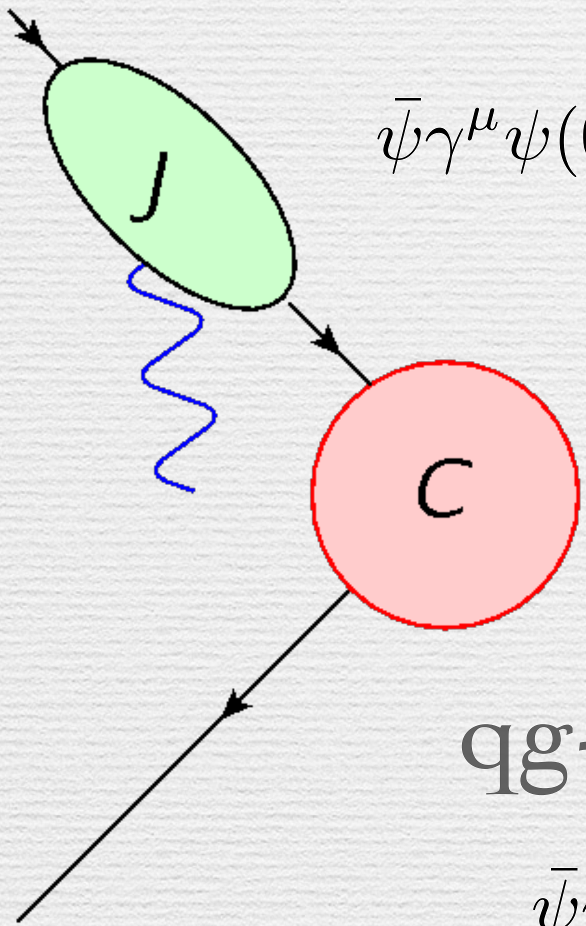


$O(1)$ $\quad$ $O(\lambda^2)$

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,) \left[J^\mu_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^\mu + \ \bar{c}\text{-term} \right]$$

$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^\mu = i \int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(2)}_{2\xi}(x) \right]$$

NLP factorization



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$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(2)}_{2\xi}(x)\right]$$

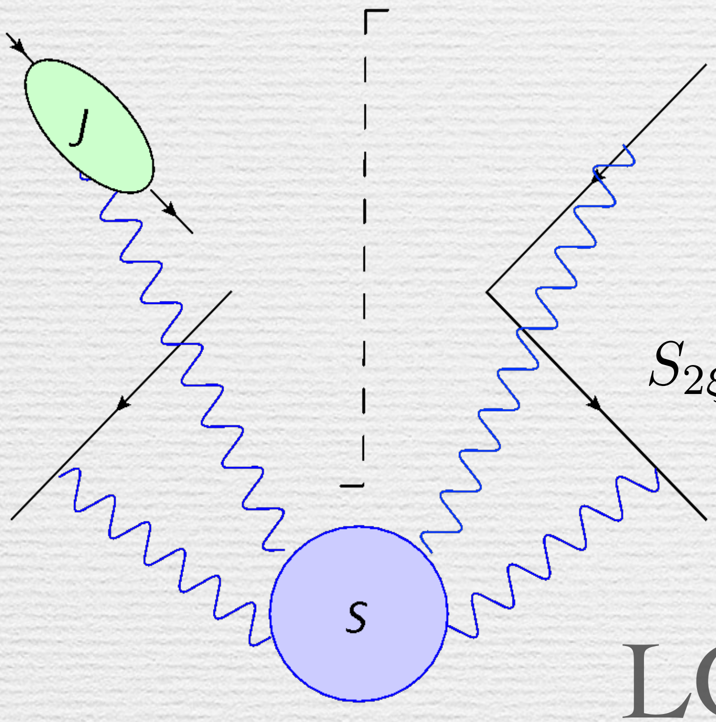
qg-channel:

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}) \left[\left(J^{T1}_{A0,\xi q}(t,\bar{t})\right)^\mu + \bar{c}\text{-term}\right]$$

$$\left(J^{T1}_{A0,\xi q}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(1)}_{\xi q}(x)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q}_+ \not{A}_{c\perp}\chi_c + \text{h.c.}$$

Soft function at NLP



$$\mathcal{B}_{\pm}^{\mu} = Y_{\pm}^{\dagger}\left[iD_s^{\mu} Y_{\pm}\right]$$

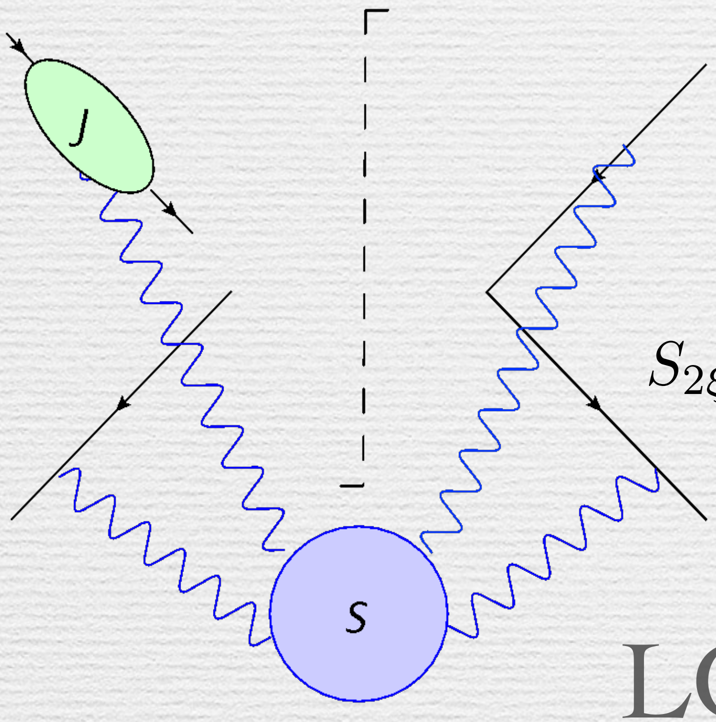
$$\widetilde{\mathcal{S}}_{2\xi}\,(x, z_-) = \bar{\mathbf{T}}\left[Y_+^{\dagger}(x) Y_-(x)\right] \mathbf{T}\left[Y_-^{\dagger}(0) Y_+(0) \frac{i\partial_{\perp}^{\nu}}{in_-\partial} \mathcal{B}_{\perp\nu}^{+}(z_-)\right]$$

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+ z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+ z)/2}\, \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

LO:

$$S_{2\xi}(\Omega, \omega) = \frac{\alpha_s C_F}{2\pi}\left\{\theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon} + \ln\frac{\Omega^2}{\mu^2}\right) + \left[\frac{1}{\omega}\right]_+ \theta(\omega)\theta(\Omega - \omega)\right\}$$

Soft function at NLP



$$\mathcal{B}^{\mu}_{\pm} = Y^{\dagger}_{\pm}\left[iD^{\mu}_{s} Y_{\pm}\right]$$

$$\widetilde{\mathcal{S}}_{2\xi}\left(x, z_-\right) = \bar{\mathbf{T}}\left[Y^{\dagger}_{+}(x) Y_{-}(x)\right] \mathbf{T}\left[Y^{\dagger}_{-}(0) Y_{+}(0) \frac{i\partial^{\nu}_{\perp}}{in_-\partial} \mathcal{B}^{+}_{\perp\nu}(z_-)\right]$$

$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}\, \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

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$$S_{2\xi}(\Omega,\omega) = \frac{\alpha_s C_F}{2\pi}\left\{\theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon} + \ln\frac{\Omega^2}{\mu^2}\right) + \left[\frac{1}{\omega}\right]_+ \theta(\omega)\theta(\Omega-\omega)\right\}$$

A puzzle: divergence at LO

RG condition

$$S_{2\xi}(\Omega,\omega) = \frac{\alpha_s C_F}{2\pi}\left\{\theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon} + \ln\frac{\Omega^2}{\mu^2}\right) + \left[\frac{1}{\omega}\right]_+ \theta(\omega)\theta(\Omega-\omega)\right\}$$

$$S_{2\xi}(\Omega,\omega)_{|\text{ren}} = \int d\Omega' \int d\omega'\, Z_{2\xi,2\xi}(\Omega,\omega;\Omega',\omega')\, S_{2\xi}(\Omega',\omega')_{|\text{bare}} + \int d\Omega'\, Z_{2\xi,x_0}(\Omega,\omega;\Omega')\, S_{x_0}(\Omega')_{|\text{bare}}$$

$$Z_{2\xi,2\xi}(\Omega,\omega;\Omega,\omega') = \delta(\Omega-\Omega')\delta(\omega-\omega') + \mathcal{O}(\alpha_s)\,,$$

$$Z_{2\xi,x_0}(\Omega,\omega;\Omega') = \frac{\alpha_s C_F}{2\pi}\frac{1}{\epsilon}\delta(\Omega-\Omega')\delta(\omega) + \mathcal{O}(\alpha_s^2)$$

The mixing term subtracts the divergent part of the first term on the right-hand side, resulting in a finite, renormalized soft function

Auxiliary soft function

$$S_{x_0}(\Omega) = \theta(\Omega)$$ at LO

We propose

$$S_{x_0}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{-2i}{x^0 - i\varepsilon}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \right] | 0 \rangle$$

"Theta-soft function" in NLP thrust distribution,

Moult, Stewart, Vita, Zhu '18

Auxiliary soft function

$$S_{x_0}(\Omega) = \theta(\Omega) \quad \text{at LO}$$

We propose

$$S_{x_0}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{-2i}{x^0 - i\varepsilon}\, \frac{1}{N_c} \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\right]|0\rangle$$

"Theta-soft function" in NLP thrust distribution,

Moult, Stewart,Vita, Zhu '18

We check this form by requiring the poles cancel at two-loop level

Check

$$S_{2\xi}^{(2)} + Z_{2\xi\, x_0}^{(1)} S_{x_0}^{(1)} + Z_{2\xi\, x_0}^{(2)} S_{x_0}^{(0)} + Z_{2\xi\, 2\xi}^{(1)} S_{2\xi}^{(1)} = \text{finite}$$

Under assumption that the off-diag has only subleading pole

$$S_{2\xi}^{(2)} - \frac{1}{4} Z_{2\xi\, x_0}^{(1)} \left(3 Z_{2\xi\, 2\xi}^{(1)} + Z_{x_0\, x_0}^{(1)} \right) S_{x_0}^{(0)} = \mathcal{O}\left(\frac{1}{\epsilon^2} \right)$$

Check

$$S^{(2)}_{2\xi} + Z^{(1)}_{2\xi\, x_0} S^{(1)}_{x_0} + Z^{(2)}_{2\xi\, x_0} S^{(0)}_{x_0} + Z^{(1)}_{2\xi\, 2\xi} S^{(1)}_{2\xi} = \text{finite}$$

Under assumption that the off-diag has only subleading pole

$$S^{(2)}_{2\xi} - \frac{1}{4} Z^{(1)}_{2\xi\, x_0} \left(3 Z^{(1)}_{2\xi\, 2\xi} + Z^{(1)}_{x_0\, x_0} \right) S^{(0)}_{x_0} = \mathcal{O}\left(\frac{1}{\epsilon^2} \right)$$

↓ Same as LP soft fun.

Check

$$S^{(2)}_{2\xi} + Z^{(1)}_{2\xi\, x_0} S^{(1)}_{x_0} + Z^{(2)}_{2\xi\, x_0} S^{(0)}_{x_0} + Z^{(1)}_{2\xi\, 2\xi} S^{(1)}_{2\xi} = \text{finite}$$

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↓ Known from 1-loop ↓ Same as LP soft fun.

Check

$$S^{(2)}_{2\xi} + Z^{(1)}_{2\xi\, x_0} S^{(1)}_{x_0} + Z^{(2)}_{2\xi\, x_0} S^{(0)}_{x_0} + Z^{(1)}_{2\xi\, 2\xi} S^{(1)}_{2\xi} = \text{finite}$$

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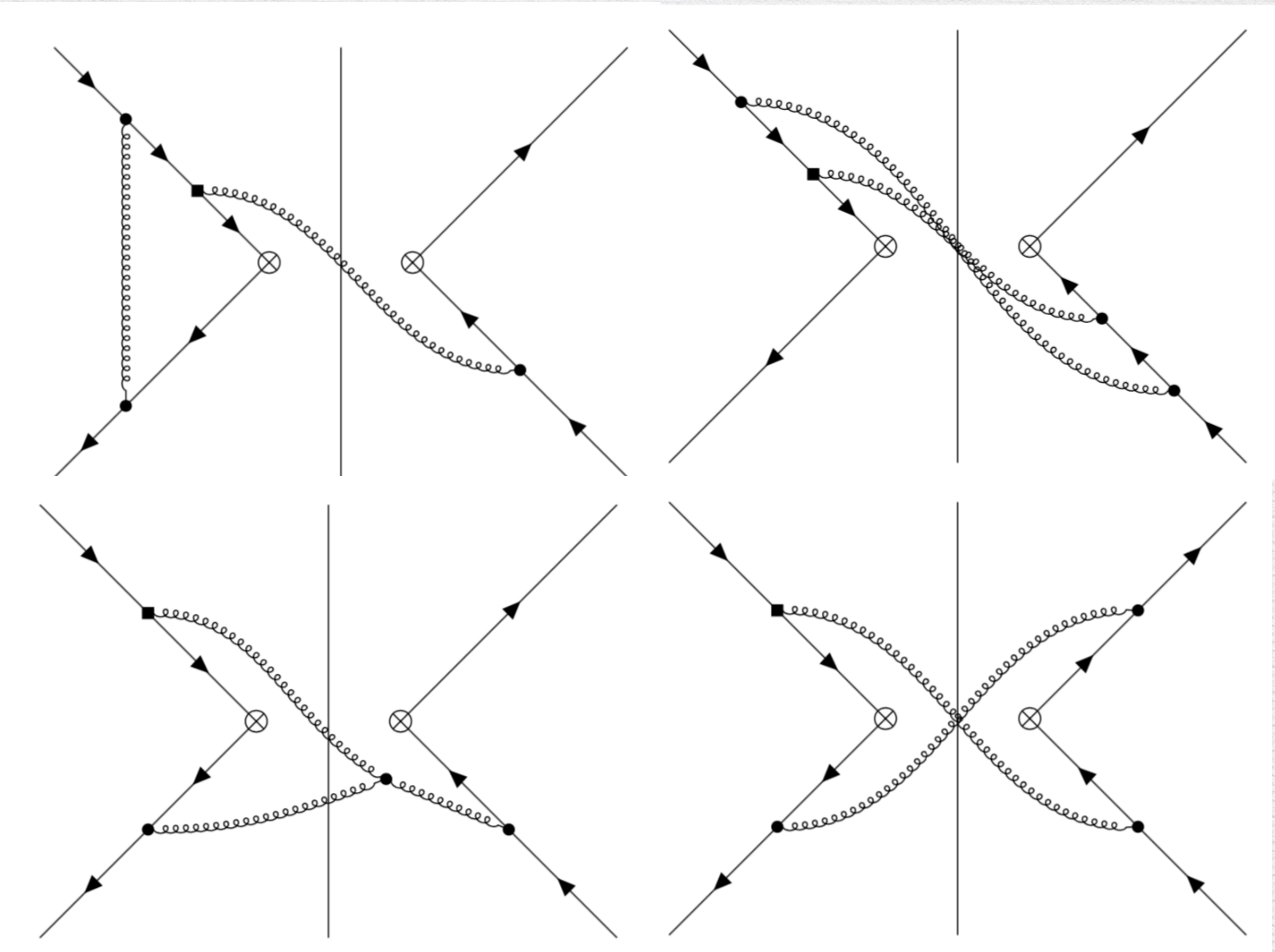
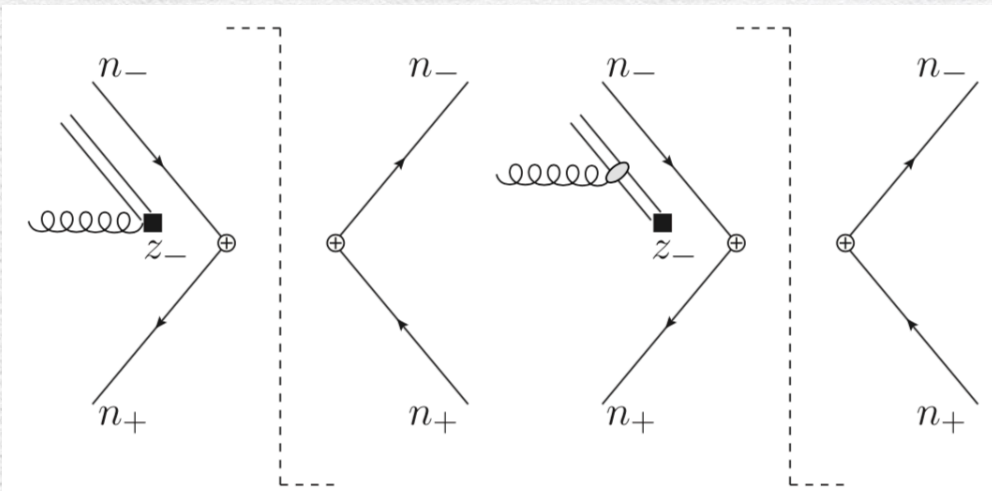
↓ Same as LP soft fun.

$$S_{2\xi}^{(2)}$$

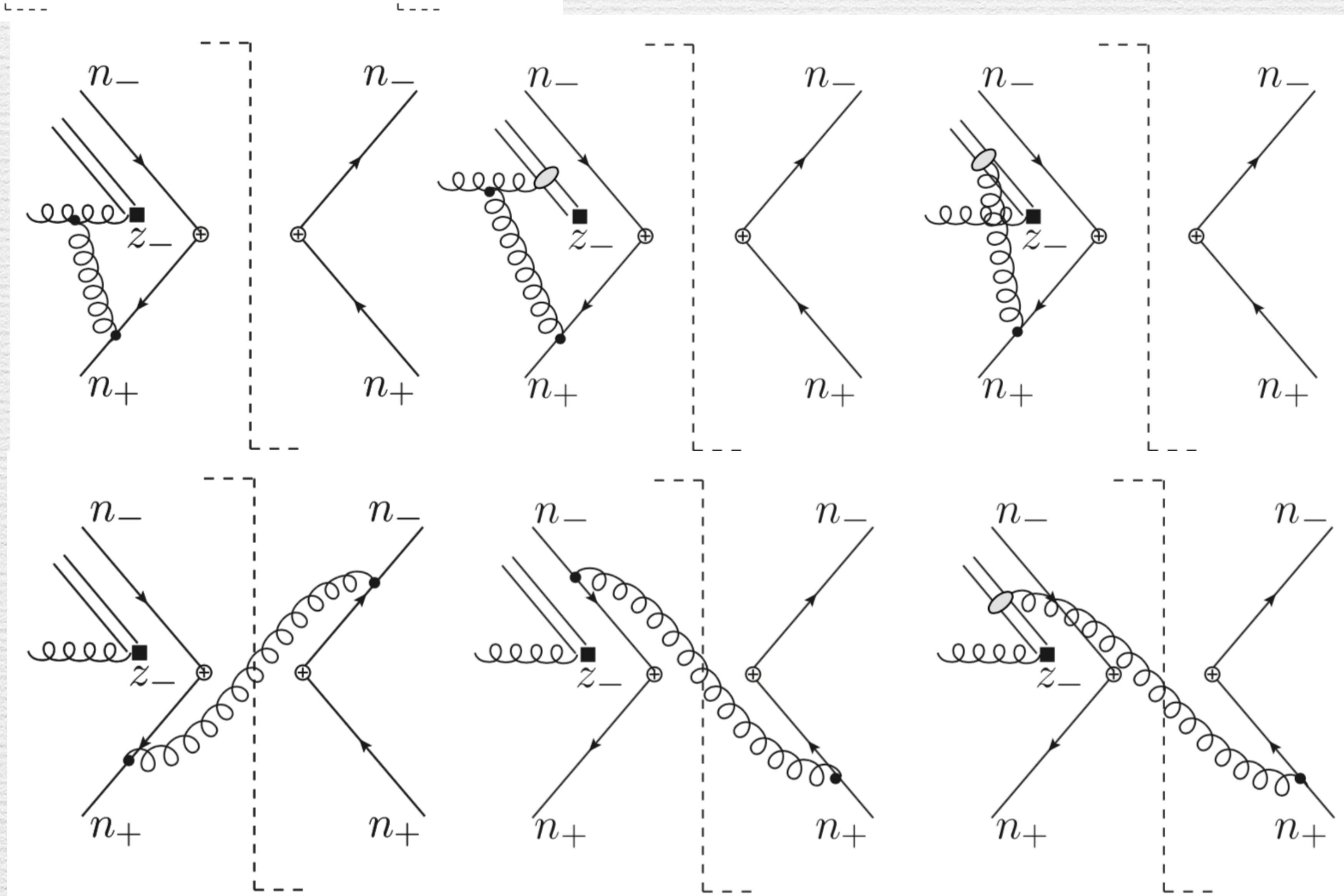


$$Z^{(1)}_{2\xi\ 2\xi}$$



$$\bar{\mathbf{T}}\left[Y_+^\dagger(x)Y_-(x)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)\right]$$

$$\mathcal{B}^\mu_\pm = Y_\pm^\dagger\left[iD_s^\mu Y_\pm\right]$$



RG eq. of soft fun.

$$\frac{d}{d\ln\mu}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x_0}(\Omega) \end{pmatrix} = \frac{\alpha_s}{\pi}\begin{pmatrix} 4C_F\ln\dfrac{\mu}{\mu_s} & -C_F\delta(\omega) \\ 0 & 4C_F\ln\dfrac{\mu}{\mu_s} \end{pmatrix}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x_0}(\Omega) \end{pmatrix}$$

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$$S_{2\xi}^{\mathrm{LL}}(\Omega,\omega,\mu) = \frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\exp\left[-4S^{\mathrm{LL}}(\mu_s,\mu)\right]\theta(\Omega)\delta(\omega)$$

$$\alpha_s \ln\frac{\mu}{\mu_s} \qquad\qquad \alpha_s \ln^2\frac{\mu}{\mu_s}$$

RG eq. of soft fun.

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Similarly, for the hard function

$$H(Q^2,\mu) = \exp\left[\underbrace{4S(\mu_h,\mu)}_{\alpha_s\ln^2\frac{\mu}{\mu_h}}\right]$$

Kinematic corrections

In the partonic c.o.m frame, the energy of the soft hadronic final state is expanded as

$$[x_1 p_1 + x_2 p_2 - q]^0 = p^0_{X_s} = \sqrt{\hat{s}} - \sqrt{Q^2 + \vec{q}^{\,2}} = \frac{\Omega_*}{2} - \frac{\vec{q}^{\,2}}{2Q} + O(\lambda^6)$$

$$\Omega_* = 2Q\frac{1-\sqrt{z}}{\sqrt{z}} = Q(1-z) + \frac{3}{4}Q(1-z)^2 + O(\lambda^6)$$

The soft function expands

$$S_{\rm DY}(Q(1-z)) + \frac{1}{Q}S_{K1}(Q(1-z)) + \frac{1}{Q}S_{K2}(Q(1-z)) + \mathcal{O}(\lambda^4)$$

$$S_{K1}(\Omega) = \frac{\partial}{\partial\Omega}\partial^2_{\vec{x}}S_0\left(\Omega,\vec{x}\right)_{|\vec{x}=0}\,,$$

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$$\Delta_{ab}(z) = \frac{\hat{\sigma}_{ab}(z)}{z}$$

→ $S_{K3}(\Omega) = \Omega\, S_0(\Omega, \vec{x})_{|\vec{x}=0}$

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$$\sum_{i=1}^{3} S_{Ki}(\Omega) = 2\,\frac{\alpha_s C_F}{\pi}$$

Kinematic corrections

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No kine.cor.

Final results

$$\mu_h \sim Q$$
$$\mu_s \sim Q(1-z)$$
$$\mu_c \sim Q\sqrt{1-z}$$

$$\Delta^{\text{LL}}_{\text{NLP}}(z) = -\exp\left[4S^{\text{LL}}(\mu_h,\mu_c) - 4S^{\text{LL}}(\mu_s,\mu_c)\right] \times \frac{8C_F}{\beta_0}\ln\frac{\alpha_s(\mu_c)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

Why we evolve the hard/soft function to the jet scale?

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Why we evolve the hard/soft function to the jet scale?

We use the LO jet function.

Recover the general scale dependence by the AP splitting kernels

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Why we evolve the hard/soft function to the jet scale?

We use the LO jet function.

Recover the general scale dependence by the AP splitting kernels

$$\frac{d}{d\ln\mu}\hat{\sigma}_{ab}(z,\mu) = -\sum_c \int_z^1 dx\left(P_{ca}(x)\hat{\sigma}_{cb}\left(\frac{z}{x},\mu\right) + P_{cb}(x)\hat{\sigma}_{ac}\left(\frac{z}{x},\mu\right)\right)$$

$$P_{ab}^{\rm LP}(x) = \left(2\Gamma_{\rm cusp}(\alpha_s)\,\frac{1}{[1-x]_+} + 2\gamma^{\phi}(\alpha_s)\delta(1-x)\right)\delta_{ab}$$
$$P_{ab}^{\rm NLP} = \gamma_{ab}^{\rm NLP}(\alpha_s)$$

Final results

$$\frac{d}{d\ln\mu}\Delta_{\rm NLP}(z,\mu)$$
$$= -4\left[\Gamma_{\rm cusp}(\alpha_s)\left(\underline{\ln(1-z)} - \gamma_E - \psi\left(1+\frac{d}{d\ln(1-z)}\right)\right) + \gamma^{\phi}(\alpha_s)\right]\Delta_{\rm NLP}(z,\mu)$$
$$+ K(z,\mu)$$

$$K(z,\mu) = \underline{-2\gamma_{qq}^{\rm NLP}(\alpha_s)\int_z^1 dy\,\Delta_{\rm LP}(y,\mu)} - 4\,\Gamma_{\rm cusp}(\alpha_s)(1-z)\Delta_{\rm LP}(z,\mu)$$

LL

$$\Delta_{\rm NLP}^{\rm LL}(z,\mu) = \exp\left[4S^{\rm LL}(\mu_h,\mu) - 4S^{\rm LL}(\mu_s,\mu)\right] \times \frac{-8C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

Expansion

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \exp\left[-2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right]\exp\left[+2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s}\right]$$
$$\times\,(-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

$$L_\mu = \ln\mu/Q$$

$$\begin{aligned}
\Delta^{\rm LL}_{\rm NLP}(z,\mu) =& -\theta(1-z)\Bigg\{4C_F\frac{\alpha_s}{\pi}\Big[\ln(1-z)-L_\mu\Big] \\
&+8C_F^2\left(\frac{\alpha_s}{\pi}\right)^2\Big[\ln^3(1-z)-3L_\mu\ln^2(1-z)+2L_\mu^2\ln(1-z)\Big] \\
&+8C_F^3\left(\frac{\alpha_s}{\pi}\right)^3\Big[\ln^5(1-z)-5L_\mu\ln^4(1-z)+8L_\mu^2\ln^3(1-z)-4L_\mu^3\ln^2(1-z)\Big] \\
&+\frac{16}{3}C_F^4\left(\frac{\alpha_s}{\pi}\right)^4\Big[\ln^7(1-z)-7L_\mu\ln^6(1-z)+18L_\mu^2\ln^5(1-z)-20L_\mu^3\ln^4(1-z) \\
&\qquad +8L_\mu^4\ln^3(1-z)\Big] \\
&+\frac{8}{3}C_F^5\left(\frac{\alpha_s}{\pi}\right)^5\Big[\ln^9(1-z)-9L_\mu\ln^8(1-z)+32L_\mu^2\ln^7(1-z)-56L_\mu^3\ln^6(1-z) \\
&\qquad +48L_\mu^4\ln^5(1-z)-16L_\mu^5\ln^4(1-z)\Big]\Bigg\}+\mathcal{O}(\alpha_s^6\times(\log)^{11})
\end{aligned}$$

Expansion

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \exp\left[-2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right]\exp\left[+2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s}\right]$$
$$\times\,(-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

$$\begin{aligned}\Delta^{\rm LL}_{\rm NLP}(z,\mu) = &-\theta(1-z)\Big\{4C_F\frac{\alpha_s}{\pi}\big[\ln(1-z)-L_\mu\big]\\
&+8C_F^2\left(\frac{\alpha_s}{\pi}\right)^2\big[\ln^3(1-z)-3L_\mu\ln^2(1-z)+2L_\mu^2\ln(1-z)\big]\\
&+8C_F^3\left(\frac{\alpha_s}{\pi}\right)^3\big[\ln^5(1-z)-5L_\mu\ln^4(1-z)+8L_\mu^2\ln^3(1-z)-4L_\mu^3\ln^2(1-z)\big]\\
&+\frac{16}{3}C_F^4\left(\frac{\alpha_s}{\pi}\right)^4\big[\ln^7(1-z)-7L_\mu\ln^6(1-z)+18L_\mu^2\ln^5(1-z)-20L_\mu^3\ln^4(1-z)\\
&\qquad+8L_\mu^4\ln^3(1-z)\big]\\
&+\frac{8}{3}C_F^5\left(\frac{\alpha_s}{\pi}\right)^5\big[\ln^9(1-z)-9L_\mu\ln^8(1-z)+32L_\mu^2\ln^7(1-z)-56L_\mu^3\ln^6(1-z)\\
&\qquad+48L_\mu^4\ln^5(1-z)-16L_\mu^5\ln^4(1-z)\big]\Big\}+\mathcal{O}(\alpha_s^6\times(\log)^{11})\end{aligned}$$

Hamberg, van Neervan, Matsuura, 1991

$L_\mu = \ln\mu/Q$

Expansion

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \exp\left[-2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right]\exp\left[+2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s}\right]$$
$$\times(-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = -\,\theta(1-z)\left\{4C_F\frac{\alpha_s}{\pi}\left[\ln(1-z)-L_\mu\right]\right.$$
$$+\,8C_F^2\left(\frac{\alpha_s}{\pi}\right)^2\left[\ln^3(1-z)-3L_\mu\ln^2(1-z)+2L_\mu^2\ln(1-z)\right]$$
$$+\,8C_F^3\left(\frac{\alpha_s}{\pi}\right)^3\left[\ln^5(1-z)-5L_\mu\ln^4(1-z)+8L_\mu^2\ln^3(1-z)-4L_\mu^3\ln^2(1-z)\right]$$
$$+\,\frac{16}{3}C_F^4\left(\frac{\alpha_s}{\pi}\right)^4\left[\ln^7(1-z)-7L_\mu\ln^6(1-z)+18L_\mu^2\ln^5(1-z)-20L_\mu^3\ln^4(1-z)\right.$$
$$\left.+\,8L_\mu^4\ln^3(1-z)\right]$$
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Hamberg, van Neervan, Matsuura, 1991

$L_\mu = \ln\mu/Q$

Florian, Mazzitelli, Moch, Vogt, '14

Kramer, Laenen, Spira, '96, Kidonakis '07

Summary and outlook

Threshold logarithms appear in a double scale process

Leading power resummation has been known for thirty years

The next-to-leading power resummation has only been obtained recently

New structures appear in the soft function

Summary and outlook

Extension of our method to next-to-leading logarithms will reveal the full complexity of the next-to-leading power structure, as indicated in the anomalous dimension of the relevant operators M.Beneke, M.Garny, R.Szafron, JW '17,'18

Summary and outlook

Extension of our method to next-to-leading logarithms will reveal the full complexity of the next-to-leading power structure, as indicated in the anomalous dimension of the relevant operators M.Beneke, M.Garny, R.Szafron, JW '17,'18

Thank you for your attention!

Back up slides

Subleading power operators

$$\mathcal{L}_{\rm SCET} = \sum_{i=1}^{N} \mathcal{L}_i(\psi_i, \psi_s) + \mathcal{L}_s(\psi_s) \qquad (6)$$

The general structure of subleading operators

$$J = \int dt \; C(\{t_{i_k}\}) \, J_s(0) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \dots) \qquad (7)$$

where

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k} n_{i+}) \, , \qquad (8)$$

with gauge-invariant collinear "building blocks"

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}^{\mu}_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^{\mu}_{\perp i} W_i] & \text{collinear gluon} \end{cases} \qquad (9)$$

LP:

$$J_i^{A0}(t_i) = \psi_i(t_i n_{i+})\,. \tag{10}$$

NLP $[O(\lambda), O(\lambda^2)]$:

- $i\partial_\perp$ $\quad\rightarrow J^{A1} = i\partial_\perp J^{A0}$
- $in_- D_s \equiv in_-\partial + g_s n_- A_s$ $\quad\rightarrow$ eliminated by E.o.M
- more building blocks $\quad\rightarrow J^{B1} = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+})$
- new building blocks, e.g., $n_-\mathcal{A}$ $\quad\rightarrow$ eliminated by E.o.M
- pure soft sector J_s, e.g., $q \sim O(\lambda^3)$, $F_s^{\mu\nu} \sim O(\lambda^4)$, not needed at NLP
- time-ordered product operators

$$J_i^{T1}(t_i) = i\int d^4x\, \mathbf{T}\left\{J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x)\right\} \tag{11}$$

Anomalous dimensions

With the definition $\mathbf{\Gamma} \equiv -\left(\frac{d}{d\ln\mu}\mathbf{Z}\right)\mathbf{Z}^{-1}$,

$$\Gamma_{PQ}(x,y) =$$

$$\delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{k,l}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right]$$

$$+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}(y)\,, \tag{12}$$

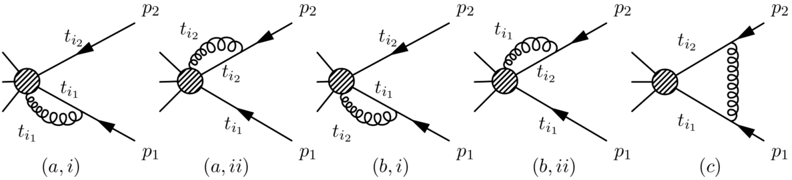
In the calculation, we have used offshellness to regularize the IR poles, and found that they cancel between the soft and collinear contributions.

$$\mathbf{\Gamma} = \begin{pmatrix} \Gamma_{PQ} & \Gamma_{PT(Q')} \\ \Gamma_{T(P')Q} & \Gamma_{T(P')T(Q')} \end{pmatrix} = \begin{pmatrix} \Gamma_{PQ} & 0 \\ \Gamma_{T(P')Q} & \Gamma_{P'Q'} \end{pmatrix} \tag{13}$$

$$\gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s}{\pi} \quad \text{and} \quad \gamma_{i_k}(\alpha_s) = \begin{cases} -\dfrac{3\alpha_s C_F}{4\pi} & ({\rm q}) \\ 0 & ({\rm g}) \end{cases} \tag{14}$$

$$\gamma^i = \begin{pmatrix} \gamma^i_{PQ} & 0 \\ 0 & \gamma^i_{P'Q'} \end{pmatrix}, \quad \gamma^{ij} = \begin{pmatrix} 0 & 0 \\ \gamma^{ij}_{T(P')Q} & 0 \end{pmatrix}. \tag{15}$$

Collinear anomalous dimensions, B1 to B1 with F=2



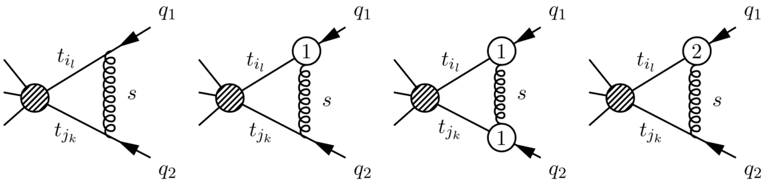
$$
\gamma^{i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y) = \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{\delta_{\alpha\gamma}\delta_{\beta\delta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+
- \theta(x-y)\frac{1-\frac{\bar{x}}{2}}{\bar{y}} - \theta(y-x)\frac{1-\frac{x}{2}}{y}\Bigg)
- \frac{1}{4}\left(\sigma_\perp^{\nu\mu}\right)_{\alpha\gamma}\left(\sigma_{\perp\nu\mu}\right)_{\beta\delta}\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right)\Bigg\}. \quad (16)
$$

B1 to B1 with F=1

$$
\begin{aligned}
\gamma^{i}_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_\beta}(x,y) \;=\; & \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+ \\
& - \frac{\theta(x-y)}{\bar{y}}\left(1+\frac{\bar{x}(\bar{x}+\bar{y})}{2x}\right) - \frac{\theta(y-x)}{2y}(\bar{x}+\bar{y})\Bigg) \\
& + \frac{1}{4}\left([\gamma_\perp^\mu,\gamma_\perp^\nu]\right)_{\alpha\beta}(x+y)\bar{x}\left(\frac{\theta(x-y)}{\bar{y}x}+\frac{\theta(y-x)}{y\bar{x}}\right)\Bigg\} \\
& - \frac{\alpha_s(\mathbf{C_F}+\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2})}{4\pi}\Bigg\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}+\bar{y})+\frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right) \\
& + \frac{1}{2}\left([\gamma_\perp^\mu,\gamma_\perp^\nu]\right)_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}-y-1)+\frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right)\Bigg\} \\
& + \frac{\alpha_s\mathbf{C_F}}{4\pi}\,\bar{x}\,(\gamma_\perp^\mu\gamma_\perp^\nu)_{\alpha\beta}\,, \qquad (17)
\end{aligned}
$$

consistent with previous results [hep-ph/0404217, hep-ph/0508250] and recent work [1806.01278].

Soft anomalous dimensions

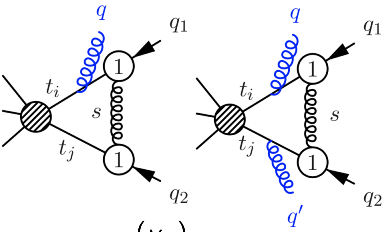


$$\gamma^{ij}_{\left(J^{T1}_{\chi,\xi}\right)_i \left(J^{T1}_{\chi,\xi}\right)_j, \left(J^{A1}_{\partial^\mu \chi}\right)_i \left(J^{A1}_{\partial^\nu \chi}\right)_j} = \frac{2\alpha_s}{\pi} \mathbf{T}_i \cdot \mathbf{T}_j \, G^{\mu\nu}_{ij} \, , \qquad (20)$$

$$G^{\mu\nu}_{ij} \equiv \left(g^{\mu\nu} - \frac{n^\nu_{i-} n^\mu_{j-}}{n_{i-} n_{j-}} \right) \frac{1}{(n_{i-} n_{j-}) P_i P_j} \, . \qquad (21)$$

- The single insertions with $\mathcal{L}^{(1)}$ never contribute to the one-loop anomalous dimension matrix to $\mathcal{O}(\lambda^2)$.
- The soft one-loop diagrams within a single collinear direction do not contribute to the anomalous dimension at any power of λ.

Two $\mathcal{L}^{(1)}$ insertions in two directions



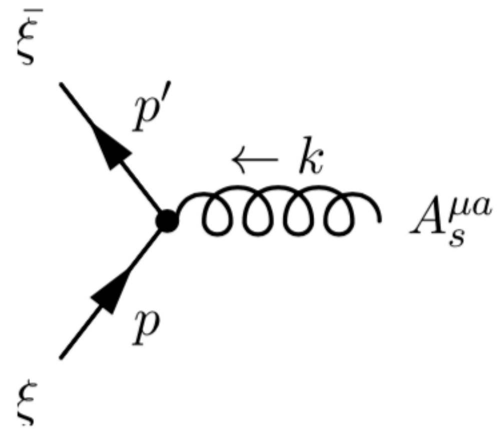
$$\gamma^{ij}_{\left(J^{T1}_{\chi_\alpha,\xi}\right)_i \left(J^{T1}_{\chi_\beta,\xi}\right)_j, \left(J^{B1}_{\mathcal{A}^\mu_b \chi_\gamma}\right)_i \left(J^{A1}_{\partial^\nu \chi_\delta}\right)_j}(y_{i_1})$$

$$= -\frac{\alpha_s}{\pi}\, \mathbf{T}^b_i\, (\mathbf{T}_i \cdot \mathbf{T}_j) G^{\lambda\nu}_{ij} \left(\gamma_{\lambda\perp i}\gamma^\mu_{\perp i} + \frac{\gamma^\mu_{\perp i}\gamma_{\lambda\perp i}}{\bar{y}} \right)_{\alpha\gamma} \delta_{\beta\delta}\,, \tag{23}$$

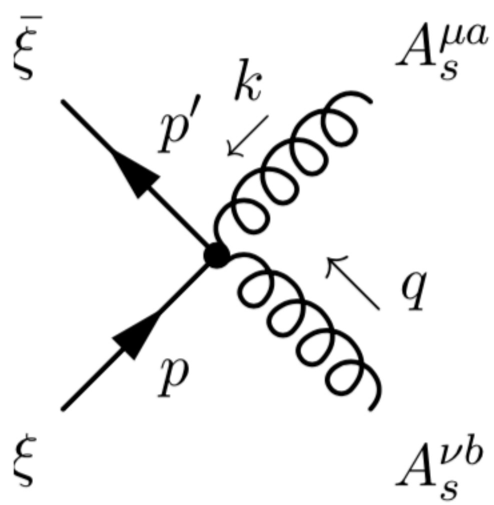
$$\gamma^{ij}_{\left(J^{T1}_{\chi_\alpha,\xi}\right)_i \left(J^{T1}_{\chi_\beta,\xi}\right)_j, \left(J^{B1}_{\mathcal{A}^\mu_b \chi_\gamma}\right)_i \left(J^{B1}_{\mathcal{A}^\nu_c \chi_\delta}\right)_j}(y_{i_1}, y_{j_1})$$

$$= \frac{\alpha_s}{2\pi}\, \mathbf{T}^b_i \mathbf{T}^c_j (\mathbf{T}_i \cdot \mathbf{T}_j)\, G^{ij}_{\lambda\kappa} \left(\gamma^\lambda_{\perp i}\gamma^\mu_{\perp i} + \frac{\gamma^\mu_{\perp i}\gamma^\lambda_{\perp i}}{\bar{y}_{i_1}} \right)_{\alpha\gamma} \left(\gamma^\kappa_{\perp j}\gamma^\nu_{\perp j} + \frac{\gamma^\nu_{\perp j}\gamma^\kappa_{\perp j}}{\bar{y}_{j_1}} \right)_{\beta\delta} \tag{24}$$

Feynman rules in SCET up to next-to-next-to leadng power

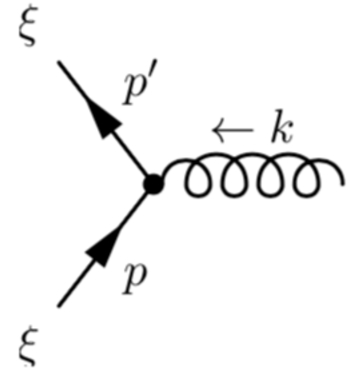


$$
ig_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \frac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}
$$

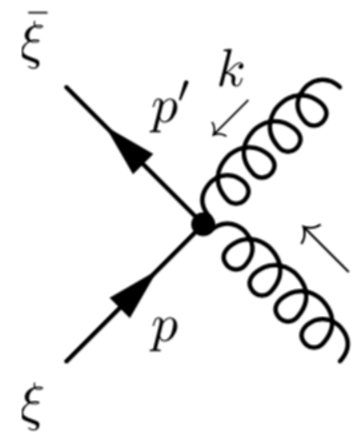


$$
ig_s^2 [t^a, t^b] \begin{cases} 0 & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\sigma (g_{\rho\mu} g_{\sigma\nu} - g_{\rho\nu} g_{\sigma\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\sigma}(k+q,p,p') \frac{\not{n}_+}{2} (g_{\rho\mu} g_{\sigma\nu} - g_{\rho\nu} g_{\sigma\mu}) & \\ + \frac{1}{2} \frac{\not{n}_+}{2} X_\perp^\rho X_\perp^\sigma n_-^\lambda \Big[g_{\rho\mu} (q_\sigma g_{\lambda\nu} - q_\lambda g_{\sigma\nu}) & \\ - g_{\rho\nu} (k_\sigma g_{\lambda\mu} - k_\lambda g_{\sigma\mu}) \Big] & \mathcal{O}(\lambda^2) \end{cases}
$$

$$
S^{\rho\nu}(k,p,p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu + (k X_\perp) X_\perp^\rho n_-^\nu + X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]
$$



$$A_c^{\mu a} \quad i g_s t^a \begin{cases} C_\mu(p',p)\,\frac{\not{n}_+}{2} & \mathcal{O}(\lambda^0) \\ 0 & \mathcal{O}(\lambda) \\ 0 & \mathcal{O}(\lambda^2) \end{cases}$$



$$A_c^{\mu a} \qquad A_c^{\nu b}$$

$$i g_s^2 \begin{cases} C^{ab}_{\mu\nu}(p',p,k,q)\,\frac{\not{n}_+}{2} & \mathcal{O}(\lambda^0) \\ 0 & \mathcal{O}(\lambda) \\ 0 & \mathcal{O}(\lambda^2) \end{cases}$$

$$C^\mu(p',p) \equiv n_-^\mu + \frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\mu + \gamma_\perp^\mu \frac{\not{p}_\perp}{n_+p} - \frac{\not{p}'_\perp}{n_+p'} n_+^\mu \frac{\not{p}_\perp}{n_+p}\,,$$

$$C^{ab}_{\mu\nu}(p',p,k,q) \equiv \Gamma_\mu(p')\frac{t^a t^b}{n_+(p+q)}\Gamma_\nu(p) + \Gamma_\nu(p')\frac{t^b t^a}{n_+(p+k)}\Gamma_\mu(p)\,,$$

$$\Gamma^\mu(p) \equiv \gamma_\perp^\mu - \frac{\not{p}_\perp}{n_+p} n_+^\mu\,.$$

More can be found in JHEP 1811 (2018) 112