Higher-order corrections for heavy quarkonium production and decay from non-relativistic QCD

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Outline of the talk

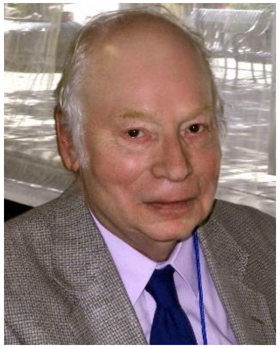
- Brief review of NRQCD factorization to quarkonium production/decay
- NLO QCD and relativistic corrections to $J/\Psi \rightarrow 3\gamma$ and confront BESIII data
- NNLO QCD correction to $\gamma\gamma^* \rightarrow \eta_c$ form factor and confront BaBar data
- NNLO QCD correction to $\eta_c \rightarrow$ light hadrons and $\mathrm{Br}[\eta_c \rightarrow \gamma\gamma]$
- NNLO QCD correction to $e^+ e^- \rightarrow J/\Psi + \eta_c$ at B factories
- Search for graviton via $J/\Psi \rightarrow \gamma$ + Graviton (via missing energy)
- Summary

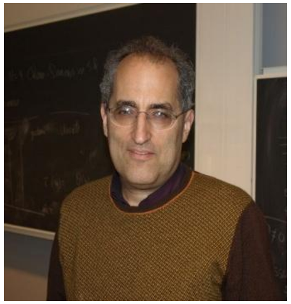
有效场论是物理研究的现代方法论。可用来处理任意存在多个特征标度的系统

- 有效场论的基本哲学：聚焦最相关自由度，高能标动力学被吸收进有效相互作用的短程系数里，无需知道更微观的高能理论即可得到误差可控的预言。精度通过系统性添加高量纲算符而改善

- 有效场论的原则：
  - 保留最相关自由度
  - 对称性作为指导原则
  - 写下对称性允许的所有可能的相互作用
  - 数幂原则（power counting）误差分析
  - 仅在一定能标范围内有效

- 所有物理理论都可被看做有效理论
  - 标准模型，广义相对论，…
  - 手征微扰论， 四费米子弱作用,…



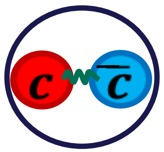
See X.-H. Liu, J. Wang, C. Zhang’s talks in this workshop

量子色动力学的有效场论

- 模型无关
  从第一性原理出发

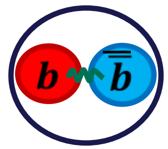
研究重夸克偶素的模型无关的理论工具: NRQCD

重夸克偶素是由重夸克和反重夸克形成的非相对论性的束缚态



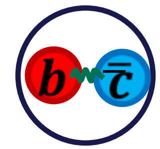
粲偶素

J/ψ, η_c



底偶素

Υ, η_b



B_c , B^*_c

重夸克偶素是QCD中最简单的强子，类似于QED中的正负电子偶素和氢原子

非相对论性 意味着 $v/c \ll 1$,

重夸克偶素有三个分得很开的特征能标：

微扰	非微扰
$M \gg Mv \gg Mv^2$	

重夸克偶素是研究QCD的理想探针, 深化理解微扰和非微扰效应如何interplay

Table 1: Quarkonium energy scales

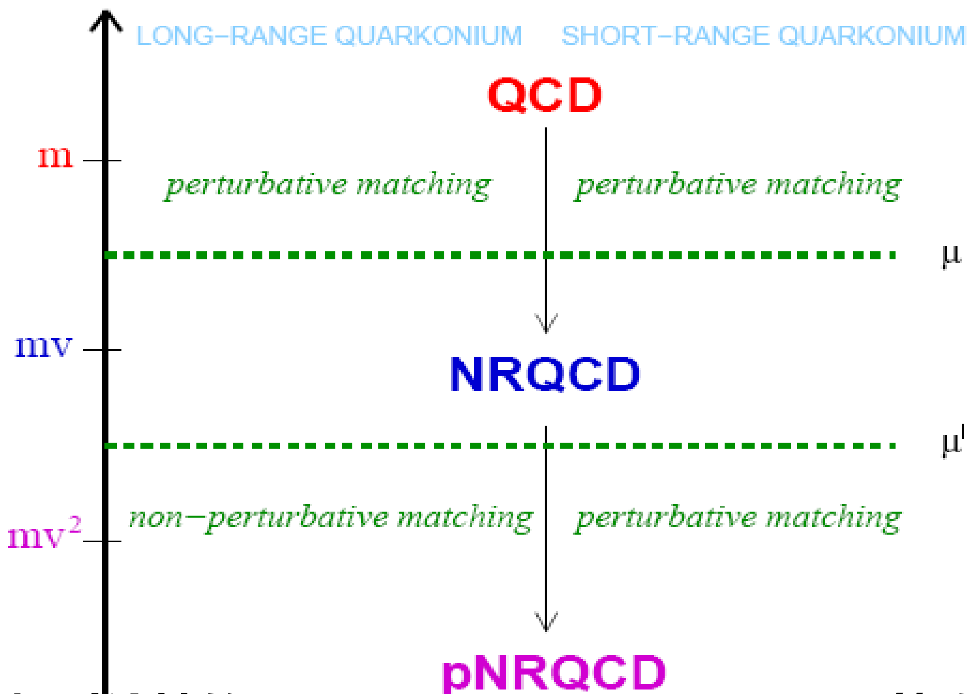
	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
$\alpha_s(M)$	0.35	0.22	0.11
$\alpha_s(Mv)$	0.52	0.35	0.16
$\alpha_s(Mv^2)$	$\gg 1$	$\gg 1$	0.35

Nonrelativistic QCD (NRQCD):

Paradigm of EFT, tailored for describing heavy quarkonium dynamics: exploiting NR nature of quarkonium

Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1995)



NRQCD factorization is viewed as being first principle of QCD

This scale separation is usually referred to as **NRQCD factorization.**

The NRQCD short-dist. coefficients can be computed in perturbation theory, order by order

非相对论性的QCD (non-relativistic QCD,简称NRQCD)，是专门为描述**重夸克偶素而**打造的**QCD有效场论**。它的优点是系统地**分离短程(l<1/M)、微扰**效应和**长程(l>1/M)、非微扰效应**。**NRQCD**是关于**夸克速度v**和**强作用耦合常数** α_s 的**双重展开**。**NRQCD被公认等同于QCD第一性原理!**

构建NRQCD有效拉氏量

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm light} + \overline{\Psi}(i\gamma^\mu D_\mu - m_Q)\Psi,$$

可以通过matching或field redefinition to derive NRQCD effective lagrangian. 在树图阶，最简单的方法是通过所谓的 Foldy-Wouthuysen-Tani (FWT)变换：

$$\Psi \to \exp(-i\gamma\cdot \mathbf{D}/2m_Q)\,\Psi. \qquad \Psi = \begin{pmatrix}\psi \\ \chi\end{pmatrix}.$$

核心物理思想是：在非相对论极限下，重夸克场和反重夸克场完全退耦！

$$\begin{pmatrix}\psi \\ \chi\end{pmatrix}^\dagger \begin{pmatrix} -m_Q + iD_0 + \mathbf{D}^2/2m_Q & 0 \\ 0 & m_Q + iD_0 - \mathbf{D}^2/2m_Q \end{pmatrix} \begin{pmatrix}\psi \\ \chi\end{pmatrix}.$$

非对角元 cancels order by order in 1/mQ；iterative procedure

NRQCD Lagrangian (characterized by velocity (v/c) expansion)

Gauge invariance, U(1) phase inv., rotation symmetry, C, P, T as guiding principle

$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\rm light} = -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}\, i\not{D} q,$$

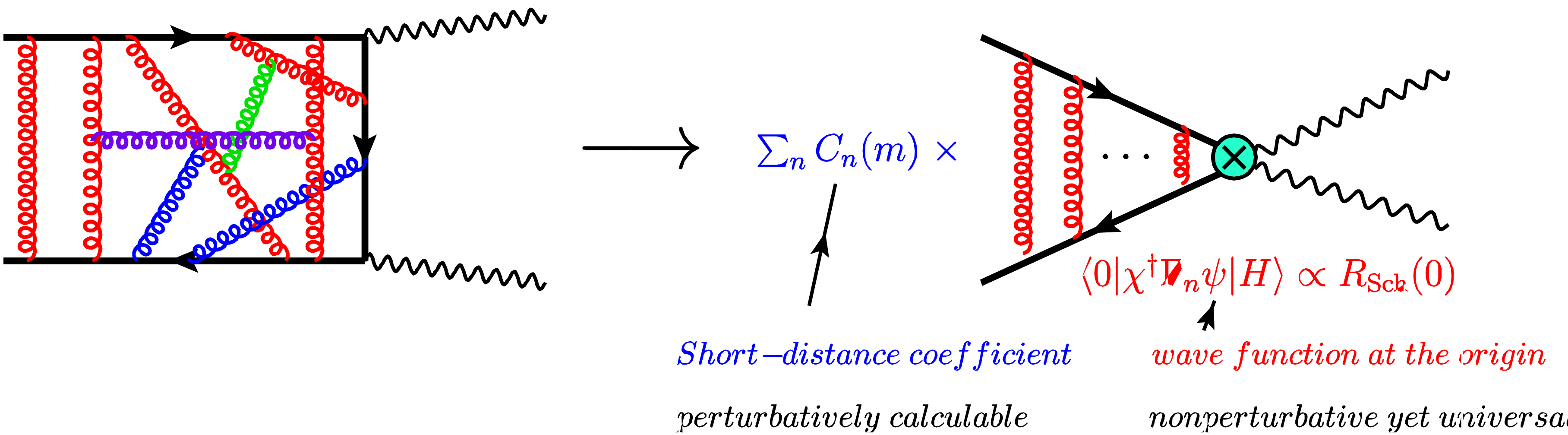
$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

$$\begin{aligned}\delta\mathcal{L}_{\rm bilinear} = & \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\ & + \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\ & + \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\ & + \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),\end{aligned}$$

Very similar to HQET, but with different power counting

NRQCD因子化的物理图像

Quarkonium is a QCD bound state involving several distinct scales



Separate the short-distance effect and long-distance dynamics

Asymptotic freedom: $\alpha_s(m)$<<1, one can invoke perturbation theory

NRQCD is the mainstream tool in studying quarkonium (see Brambilla et al. EPJC 2011 for a review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes:

Charmonia: $v^2/c^2 \sim 0.3$ not truly non-relativistic to some extent

Bottomonia: $v^2/c^2 \sim 0.1$ a better "non-relativistic" system

Exemplified by

$e^+e^- \to J/\psi + \eta_c$ at B factories **(exclusive charmonium production)**

Unpolarized/polarized J/ψ production at hadron colliders (**inclusive**)

Very active field in recent years (Chao's group, Kniehl's group, Wang's group, Bodwin's group, Qiu's group …) marked by a plenty of PRLs

The strategy of determining the **NRQCD** short-distance coefficients (**NRQCD** SDCs)

In principle, NRQCD short-distance coefficients can be computed via the standard perturbative matching procedure:

Computing simultaneously amplitudes in both perturbative QCD and NRQCD, then solve the equations to determine the NRQCD SDCs.

Threshold phenomenon is signaled by four relevant modes: **hard ($k^\mu \sim m$)**, **potential ($k^0 \sim mv^2, |k| \sim mv$)**, **soft ($k^\mu \sim mv$)**, **ultrasoft ($k^\mu \sim mv^2$).**

Elucidated by the **Strategy of region** by Beneke & Smirnov 1997

分区域展开

The **NRQCD** SDCs is associated with the contribution from hard region

Practically, one often directly extract the hard-region contribution in an arbitrary multi-loop diagrams. We then lose track of IR threshold symptom such as Coulomb singularity

Strategy of region (one-loop threshold vertex diagram as a heuristic example)

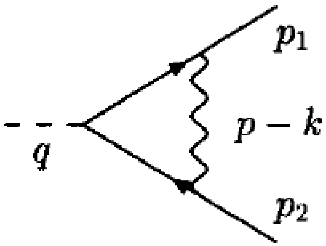
Beneke and Smirnov, NPB, 1998

$\gamma^* \to \bar{Q}(p_1)Q(p_2)$

$p_1^2 = p_2^2 = m^2$

$y \equiv m^2 - \frac{q^2}{4} = p^2 \ll q^2.$

$\hat{y} = y/q^2$



$q = p_1 + p_2$

$\beta = (1 - 4m^2/q^2)^{1/2} = \sqrt{-4\hat{y}}.$

$$I_1 \equiv \int \frac{[dk]}{(k^2 + q\cdot k - y)(k^2 - q\cdot k - y)(k-p)^2},$$

The exact result:

$$= e^{\epsilon\gamma_E}\, y^{-1-\epsilon}\, \frac{1}{2}\, \Gamma(\epsilon)\, {}_2F_1\left(\frac{1}{2}, 1+\epsilon; \frac{3}{2}; -\frac{1}{4\hat{y}}\right)$$

$$= e^{\epsilon\gamma_E}\left(\frac{4}{q^2}\right)^{1+\epsilon}\left\{\frac{(4\hat{y})^{-\epsilon}}{\sqrt{\hat{y}}}\,\frac{\sqrt{\pi}\,\Gamma(\epsilon+1/2)}{8\epsilon}\right.$$

Potential region

The expanded result

$$\left. - \frac{\Gamma(\epsilon)}{2(1+2\epsilon)} \sum_{n=0}^{\infty} \frac{\Gamma(1+\epsilon+n)}{\Gamma(1+\epsilon)}\, \frac{1+2\epsilon}{1+2\epsilon+2n}\, \frac{(-4\hat{y})^n}{n!}\right\}.$$

Hard region

Strategy of region (one-loop threshold vertex diagram as a heuristic example)

Beneke and Smirnov, NPB, 1998

large/hard (h): $k_0 \sim q$, $\boldsymbol{k} \sim q$,

$$I_1^h = \int \frac{[dk]}{k^2(k^2 + q\cdot k)(k^2 - q\cdot k)} = e^{\epsilon\gamma_E}\left(\frac{4}{q^2}\right)^{1+\epsilon}\left(-\frac{1}{2}\right)\frac{\Gamma(\epsilon)}{1+2\epsilon}.$$

potential (p): $k_0 \sim y/q$, $\boldsymbol{k} \sim \sqrt{y}$,

soft (s): $k_0 \sim \sqrt{y}$, $\boldsymbol{k} \sim \sqrt{y}$,

or *ultrasoft* (us): $k_0 \sim y/q$, $\boldsymbol{k} \sim y/q$.

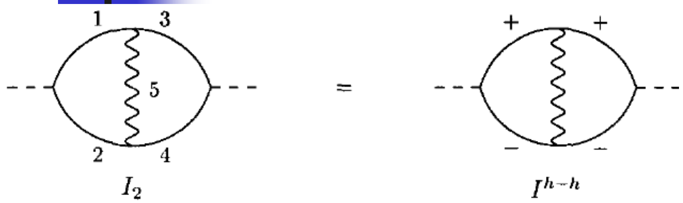
$$I_1^p = \frac{(-1)}{q} e^{\epsilon\gamma_E} \int \frac{d^{d-1}\boldsymbol{k}}{\pi^{d/2-1}} \frac{1}{(\boldsymbol{k}^2 + y)(\boldsymbol{k} - \boldsymbol{p})^2} = e^{\epsilon\gamma_E} \frac{y^{-\epsilon}}{\sqrt{q^2 y}} \frac{\sqrt{\pi}\,\Gamma(\epsilon + 1/2)}{2\epsilon}.$$

Notorious Coulomb divergence

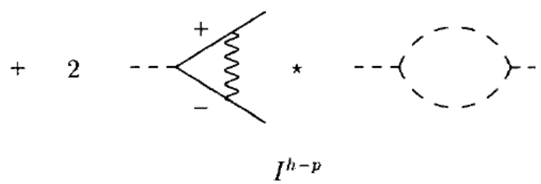
Soft/ultrasoft regions yield scaleless integral, thus make vanishing results

Upon summing I_h and I_p , one reproduces the original Integrals, order by order in threshold expansion

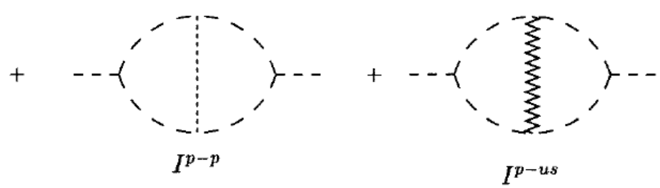
Strategy of region for two-loop threshold diagrams



$$I_2 \equiv \int \frac{[dk][dl]}{(k^2+q\cdot k-y)(k^2-q\cdot k-y)(l^2+q\cdot l-y)(l^2-q\cdot l-y)(k-l)^2}$$



$$q^2 I_2^{\text{h-h}} = \pi^2\left(\frac{1}{\epsilon} - 2\ln q^2 + 6\ln 2\right) + 21\zeta(3) - 4(8+3\pi^2)\,\hat{y} + \mathcal{O}(\hat{y}^2).$$



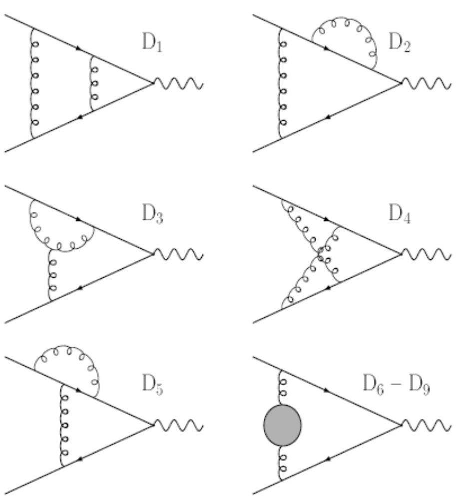
When both loop momenta get hard

For the sake of extracting the hard-matching coefficients, it suffices to compute only the hard regions corresponding to $I_{h\text{-}h}$ -- this brings forth enormous simplification in practice. E.g., we never need worry about the contamination due to unphysical Coulomb divergence.

Example: Two-loop matching of heavy quark vector current

$$\langle\psi(p)|\bar{Q}\gamma^\mu Q|0\rangle = \Lambda^{\mu i}(p)\Bigg[C_0\left(\alpha_s,\frac{m_Q}{\mu}\right)\langle\psi|\psi^\dagger\sigma_i\chi|0\rangle(\mu) + \frac{C_1\left(\alpha_s,\frac{m_Q}{\mu}\right)}{6m_Q^2}\langle\psi|\psi^\dagger\vec{D}^2\sigma_i\chi|0\rangle(\mu)+\mathcal{O}(v^4)\Bigg],$$

$$C_0\left(\alpha_s,\frac{m_Q}{\mu}\right) = 1 - \frac{2C_F\,\alpha_s(m_Q)}{\pi} + c_2(m_Q/\mu)\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots,$$

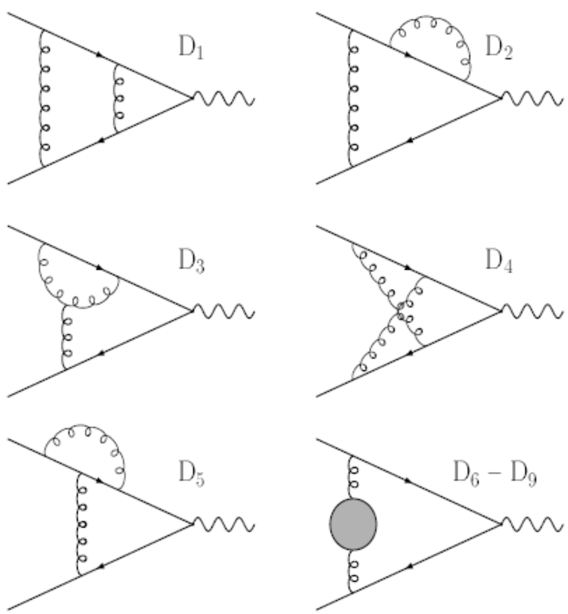


Czarnecki and Melkinov 97
Beneke, Smirnov and Signer 97

Matching equation:

$$Z_{2,\mathrm{QCD}}\,\Gamma_{\mathrm{QCD}} = C_0\,Z_{2,\mathrm{NRQCD}}\,Z_J^{-1}\,\Gamma_{\mathrm{NRQCD}} + \mathcal{O}(v^2),$$

Example: Two-loop matching of heavy quark vector current



	Colour factor	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$	finite
D_1	C_F^2	$\frac{9}{32}$	$-\frac{27}{64}-\frac{5\pi^2}{24}$	$-\frac{81}{128}-\frac{133\pi^2}{96}-\frac{5\pi^2\ln 2}{12}-\frac{35\zeta(3)}{8}$
D_2	C_F^2	$-\frac{3}{16}$	$-\frac{43}{32}$	$\frac{733}{192}+\frac{971\pi^2}{576}$
D_3	$C_F C_A$	$\frac{15}{32}$	$-\frac{5}{64}-\frac{\pi^2}{16}$	$\frac{715}{384}-\frac{319\pi^2}{576}-\frac{\pi^2\ln 2}{8}-\frac{21\zeta(3)}{16}$
D_4	$C_F\left(C_A-2C_F\right)$	0	$\frac{3}{16}-\frac{\pi^2}{16}$	$-\frac{39}{32}-\frac{251\pi^2}{1152}-\frac{3\pi^2\ln 2}{8}-\frac{31\zeta(3)}{16}$
D_5	$C_F\left(C_A-2C_F\right)$	$-\frac{9}{32}$	$-\frac{19}{64}$	$\frac{761}{384}+\frac{1157\pi^2}{1152}+\frac{\pi^2\ln 2}{6}-\frac{3\zeta(3)}{4}$
D_6	$C_F T_F n_f$	$-\frac{1}{8}$	$\frac{5}{48}$	$-\frac{355}{288}-\frac{5\pi^2}{48}$
D_7	$C_F C_A$	$\frac{19}{128}$	$-\frac{53}{768}$	$\frac{6787}{4608}+\frac{95\pi^2}{768}$
D_8	$C_F C_A$	$\frac{1}{128}$	$\frac{1}{768}$	$\frac{361}{4608}+\frac{5\pi^2}{768}$
D_9	$C_F T_F$	$-\frac{1}{4}$	$\frac{13}{48}$	$-\frac{145}{96}+\frac{5\pi^2}{72}$
Sum	C_F^2	$\frac{21}{32}$	$-\frac{99}{64}-\frac{\pi^2}{12}$	$\frac{637}{384}-\frac{733\pi^2}{576}+\zeta(3)$
	$C_F C_A$	$\frac{11}{32}$	$-\frac{49}{192}-\frac{\pi^2}{8}$	$\frac{4811}{1152}+\frac{209\pi^2}{576}-\frac{\pi^2\ln 2}{3}-4\zeta(3)$
	$C_F T_F n_f$	$-\frac{1}{8}$	$\frac{5}{48}$	$-\frac{355}{288}-\frac{5\pi^2}{48}$
	$C_F T_F$	$-\frac{1}{4}$	$\frac{13}{48}$	$-\frac{145}{96}+\frac{5}{72}$

$$c_2(m_Q/\mu) = C_F^2\, c_{2,A} + C_F C_A\, c_{2,NA} + C_F T_F\, n_f\, c_{2,L} + C_F T_F\, c_{2,H},$$

$$c_{2,A} = \pi^2\left[\frac{1}{6}\ln\left(\frac{m_Q^2}{\mu^2}\right) - \frac{79}{36} + \ln 2\right] + \frac{23}{8} - \frac{\zeta(3)}{2},$$

$$c_{2,NA} = \pi^2\left[\frac{1}{4}\ln\left(\frac{m_Q^2}{\mu^2}\right) + \frac{89}{144} - \frac{5}{6}\ln 2\right] - \frac{151}{72} - \frac{13\zeta(3)}{4},$$

$$c_{2,L} = \frac{11}{18},$$

$$c_{2,H} = -\frac{2\pi^2}{9} + \frac{22}{9}.$$

Anomalous dimension of NRQCD current:

$$\gamma_J = \frac{d\ln Z_J}{d\ln\mu} = -C_F\left(2C_F + 3C_A\right)\frac{\pi^2}{6}\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3).$$

The ubiquitous symptom of NRQCD factorization: often plagued with huge QCD radiative correction

Most of the NRQCD successes based on the NLO QCD predictions.

However, the NLO QCD corrections are often large:

$e^+e^- \to J/\psi + \eta_c$	K factor: $1.8 \sim 2.1$	Zhang *et.al.*
$e^+e^- \to J/\psi + J/\psi$	K factor: $-0.31 \sim 0.25$	Gong *et.al.*
$p + p \to J/\psi + X$	K factor: ~ 2	Campbell *et.al.*
$J/\psi \to \gamma\gamma\gamma$	K factor: ≤ 0	Mackenzie *et.al.*

… …

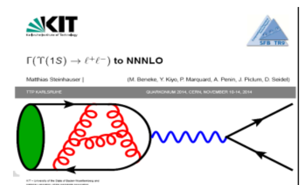
The existing NNLO corrections are rather **few**: all related to S-wave quarkonium **decay**

1. $\Upsilon\,(J/\Psi) \rightarrow e^+ e^-$

NNLO corrections were first computed by two groups in 1997:

Czarnecki and Melkinov; Beneke, Smirnov, and Signer;

N3LO correction available very recently: Steinhausser et al. (2013)



2. $\eta_c \rightarrow \gamma\ \gamma$

NNLO correction was computed by Czarnecki and Melkinov (2001) : (neglecting light-by-light)

3. $B_c \rightarrow l\ \nu$:

NNLO correction computed by Onishchenko, Veretin (2003); Chen and Qiao, (2015)

Perturbative convergence of these decay processes appears to be rather poor

$$\Gamma(J/\psi \to \ell\ell) = \Gamma^{(0)}\left[1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41\,n_f)\left(\frac{\alpha_s}{\pi}\right)^2\right]^2$$

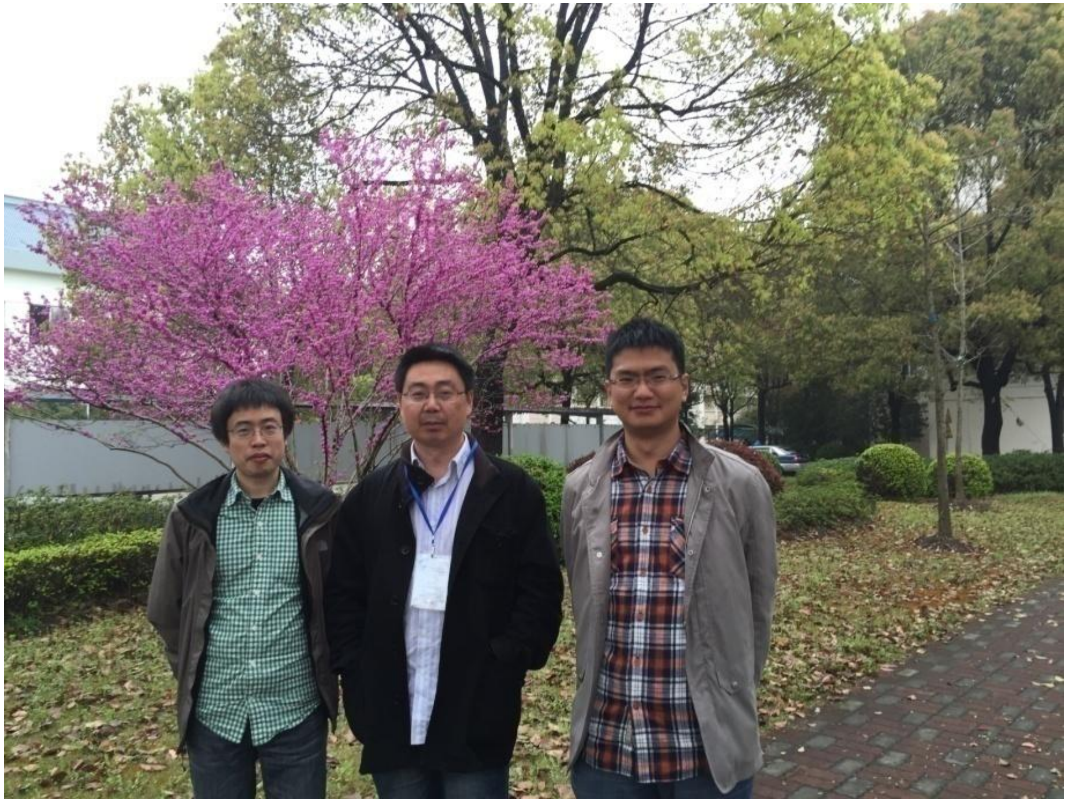
$$+(-2091 + 120.66\,n_f - 0.82\,n_f^2)\left(\frac{\alpha_s}{\pi}\right)^3$$

$$\Gamma(B_c \to \ell\nu) = \Gamma^{(0)}\left[1 - 1.39\frac{\alpha_s}{\pi} - 23.7\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]^2$$

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma^{(0)}\left[1 - 1.69\frac{\alpha_s}{\pi} - 56.52\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]^2$$

So calculating the higher order QCD correction is imperative to test the usefulness of NRQCD factorization!

The long-term collaboration team: Feng, Sang and Jia



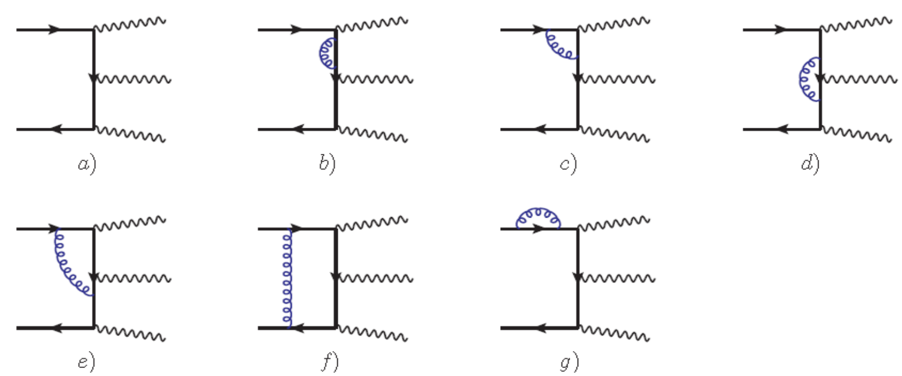
Solving long-standing puzzle concerning rare decay J/ϕ→3γ

Feng, **Jia**, Sang, **PRD (R)**, 2013

对于稀有衰变J/ϕ到3光子的理论预言长期陷入困境，因为NLO辐射修正与相对论修正将衰变分支比的预言拉为负值。

2012年BESIII实验精确测量衰变分支比

首次考虑了单圈及相对论展开的联合修正，发现其贡献一个很大的正修正



$$\Gamma(J/\psi \to 3\gamma) = \frac{8(\pi^2-9)e_c^6\alpha^3}{9m_c^2}\left|\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle\right|^2 \left\{1 - 12.630\,\frac{\alpha_s}{\pi} + \left[\frac{132-19\pi^2}{12(\pi^2-9)} - \left(\frac{16}{9}\ln\frac{\mu_f^2}{m_c^2}+G\right)\frac{\alpha_s}{\pi}\right]\langle v^2\rangle_{J/\psi} + \cdots\right\}$$

$$G = 68.913$$

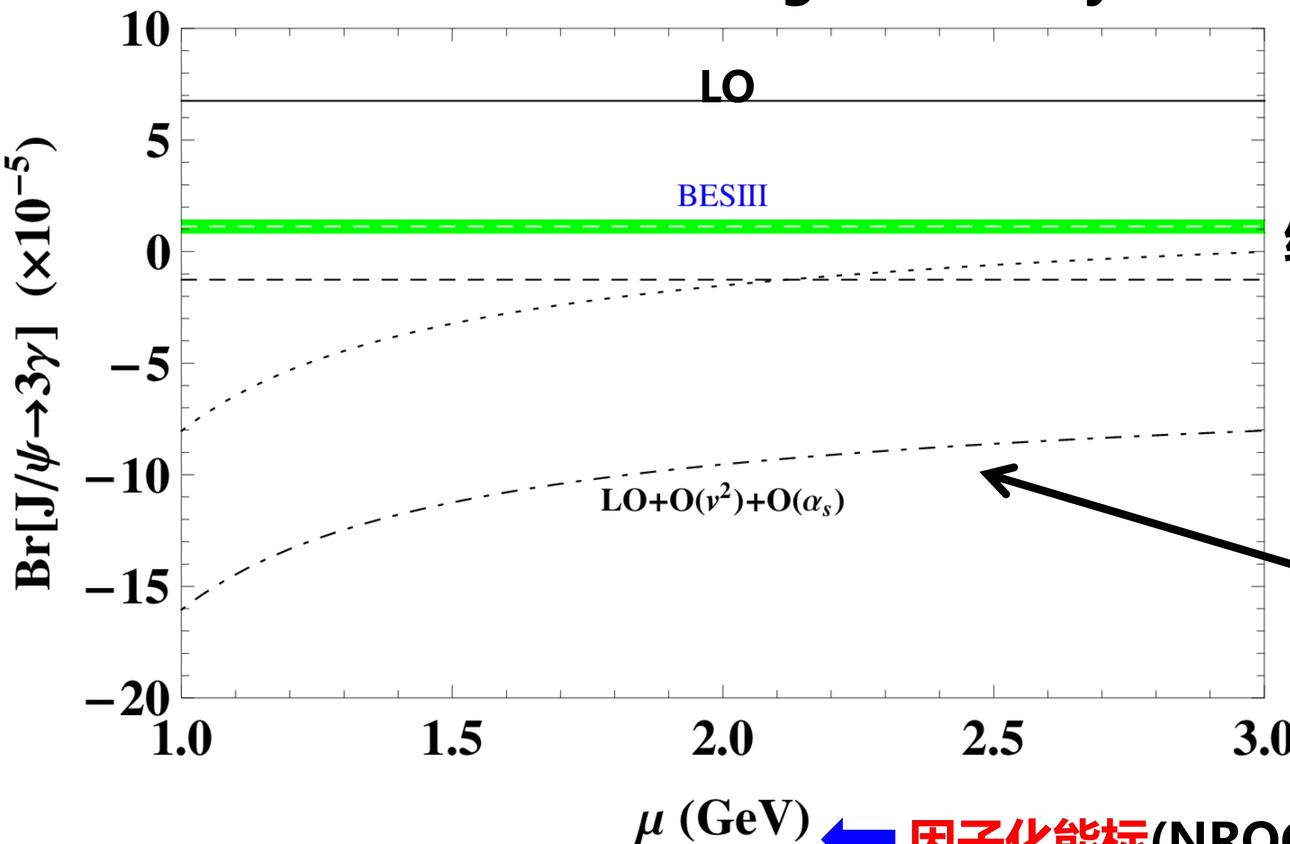
O($\alpha_s v^2$)新修正

NRQCD是关于v和 α_s的双重展开

Solving long-standing puzzle concerning rare decay J/ψ→3γ

Feng, **Jia**, Sang, **PRD (R)**, 2013

Theoretical dilemma：negative decay ratio, hence unphysical



Is NRQCD wrong?

绿带子代表BESIII实验值

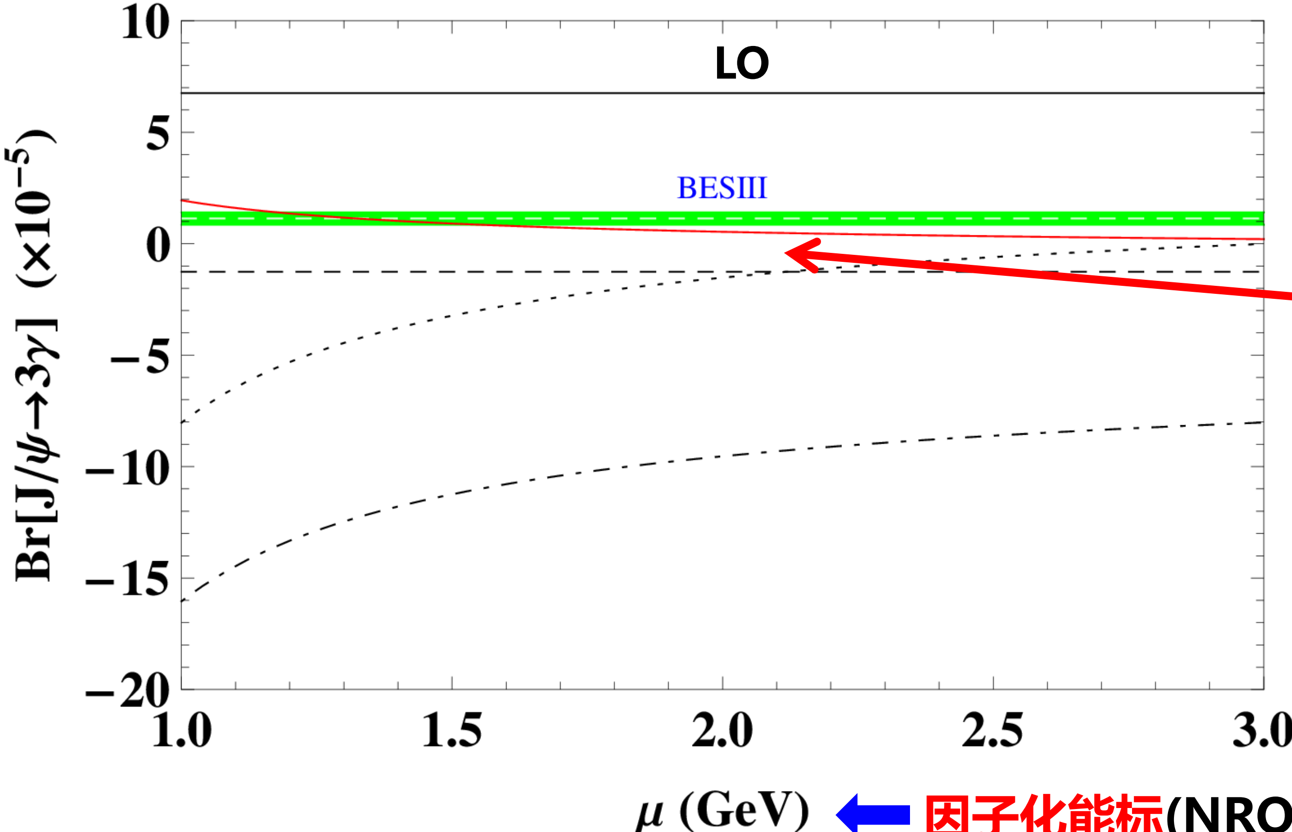
点划线代表**2013年之前 NRQCD最完整预言**

因子化能标(NRQCD紫外截断)

Solving long-standing puzzle concerning rare decay J/ψ→3γ

Feng, **Jia**, Sang, **PRD (R)**, 2013

加上新的α_s及相对论联合修正后，NRQCD预言和BESIII相吻合！



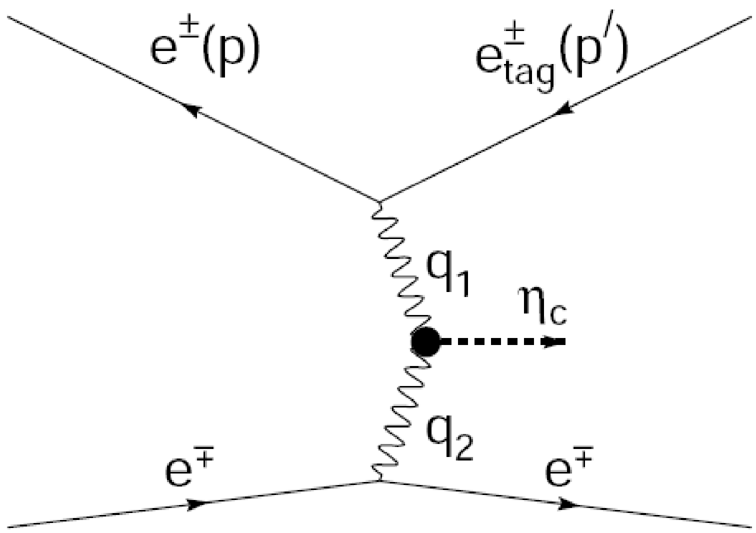
Successfully solve the puzzle

红线代表**加上新的$\alpha_s v^2$联合修正后的预言**

← **因子化能标(NRQCD紫外截断)**

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor Experiment

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**

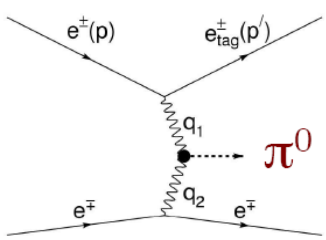


$$q_2^2 \approx 0$$

$$q_1^2 = -Q^2 = (p' - p)^2$$

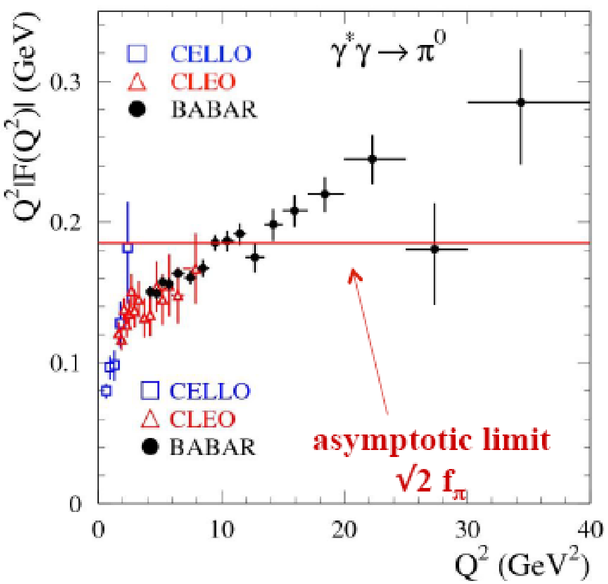
Babar measures the $\gamma\gamma^* \rightarrow \eta_c$ transition form factor in the momentum transfer range from **2 to 50 GeV²**.

Digression: recall the surprise brought by BaBar two-photon experiment on $\gamma\gamma^* \rightarrow \pi^0$



The π^0 Transition Form Factor

Comparison of the result of experiment to the QCD limit

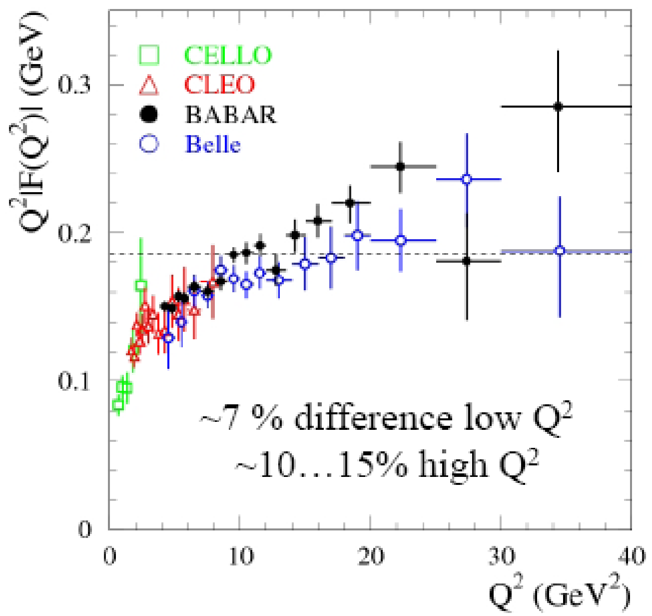


- Experiment:
  In Q^2 range 4-9 GeV^2 CLEO results are consistent with more precise BaBar data
- QCD prediction (Brodsky-Lepage '79):
  at high Q^2 data should reach asymptotic limit (either from below or from above)

  $Q^2 F(Q^2) = \sqrt{2} f_\pi = 0.185$ GeV
  assuming the asymptotic DA

Belle did not confirm BaBar measurement on $\gamma\gamma^* \rightarrow \pi^0$! Situation needs clarification

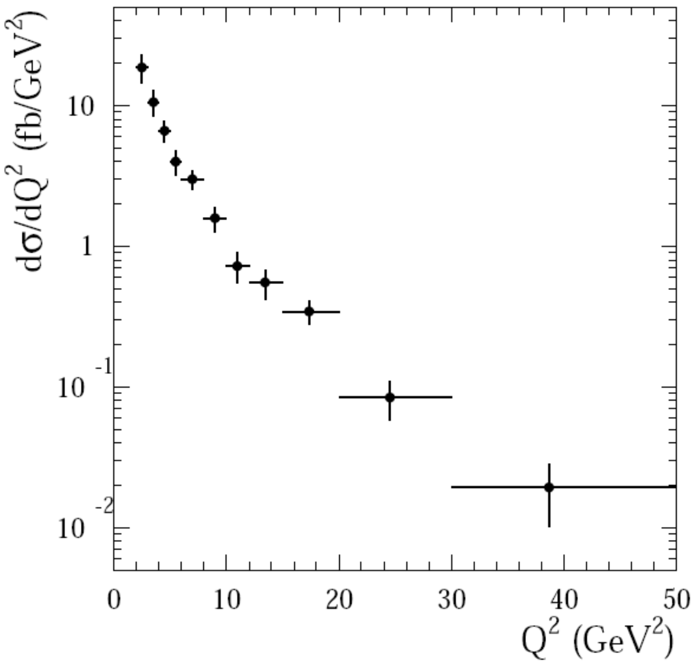
Comparison with BELLE, arXiv:1205.3249



- Difference BABAR – BELLE ~$2\sigma_{syst}$
- BELLE has lower detection efficiency (~factor 2)
- BELLE has higher systematic uncertainties

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor: There also exists BaBar measurements!

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



Q^2 interval (GeV^2)	$\overline{Q^2}$ (GeV^2)	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV^2)	$\lvert F(\overline{Q^2})/F(0)\rvert$
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	0.740 ± 0.085
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	0.680 ± 0.073
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	0.629 ± 0.057
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	0.555 ± 0.056
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	0.563 ± 0.043
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	0.490 ± 0.049
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	0.385 ± 0.048
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	0.395 ± 0.047
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	0.385 ± 0.038
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	0.261 ± 0.041
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	0.204 ± 0.049

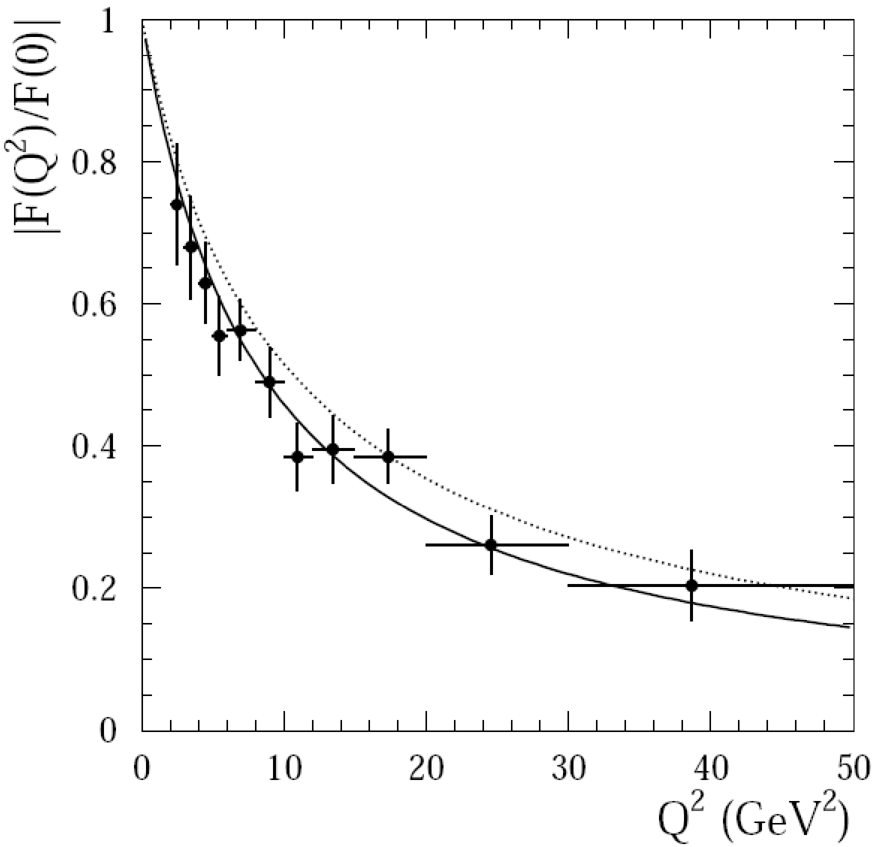
$$F(Q^2): \quad \gamma^*\gamma \rightarrow \eta_c \quad \text{form factor}$$

$$F(0): \quad \eta_c \rightarrow \gamma\gamma \quad \text{form factor}$$

$$\frac{d\sigma(e^+e^- \rightarrow \eta_c e^+e^-)}{dQ^2} \times \mathcal{B}(\eta_c \rightarrow K\bar{K}\pi)$$

Investigation on $\gamma\gamma^* \to \eta_c$ form factor
Experiment

BaBar Collaboration: **Phys. Rev. D81 (2010) 052010**



The solid curve is from a simple monopole fit:

$$|F(Q^2)/F(0)| = \frac{1}{1 + Q^2/\Lambda}$$

with $\Lambda = 8.5 \pm 0.6 \pm 0.7\ \mathrm{GeV}^2$

The dotted curve is from pQCD prediction

Feldmann and Kroll, Phys. Lett. B 413, 410 (1997)

Investigation on $\gamma\gamma^* \to \eta_c$ form factor
Previous investigation

- $k_\perp$ factorization: Feldmann *et.al.*, Cao *and Huang*
- Lattice QCD: Dudek *et.al.*,
- **J/ψ** -pole-dominance: Lees *et.al.*,
- QCD sum rules: Lucha *et.al.*,
- light-front quark model: Geng *et.al.*,
- Dyson-Schwinger approach: Chang, Chen, Ding, Liu, Roberts, 2016

All yield predictions compatible with the data, at least in the small Q^2 range.

So far, so good. Unlike $\gamma\gamma^* \to \pi^0$, there is no open puzzle here

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Motivation

- Model-independent method is always welcome.

  (NRQCD is the first principle approach from QCD)

- In the normalized form factor, nonperturbative NRQCD matrix element cancels out. Therefore, our predictions are free from any freely adjustable parameters!

- Is LO/NLO NRQCD prediction sufficient?

- The momentum transfer is not large enough, we are not bothered by resumming the large collinear logarithms.

The first **NNLO** calculation for (exclusive) quarkonium production process

Feng, Jia, Sang, PRL (2017)

Definition for form factor:

$$\langle \eta_c(p) | J^\mu | \gamma(k,\varepsilon) \rangle = i e^2 \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

NRQCD factorization demands:

Factorization scale

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda) \frac{\langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

Short-distance coefficient (SDC)
We are going to compute it to NNLO

$$\overline{R_{\eta_c}}(\Lambda) \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \psi(\Lambda) | \eta_c \rangle,$$

$$\overline{R_\psi}(\Lambda)\, \epsilon \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \boldsymbol{\sigma} \psi(\Lambda) | \psi(\epsilon) \rangle \,,$$

Investigation on $\gamma\gamma^* \to \eta_c$ form factor
Perturbative series for NRQCD SDCs

Upon general consideration, the SDC can be written as

$$C(Q,m,\mu_R,\mu_\Lambda) = C^{(0)}(Q,m)\Bigg\{1 + C_F\frac{\alpha_s(\mu_R)}{\pi}f^{(1)}(\tau)$$
$$+\frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2+m^2}C_F f^{(1)}(\tau) - \pi^2 C_F\left(C_F+\frac{C_A}{2}\right)\right.$$
$$\left.\times\ln\frac{\mu_\Lambda}{m} + f^{(2)}(\tau)\right] + \mathcal{O}(\alpha_s^3)\Bigg\},$$

RG invariance

IR pole matches **anomalous dimension** of NRQCD pseudo-scalar density

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Theoretical calculation

$$C^{(0)}(Q,m) = \frac{4e_c^2}{Q^2+4m^2}$$

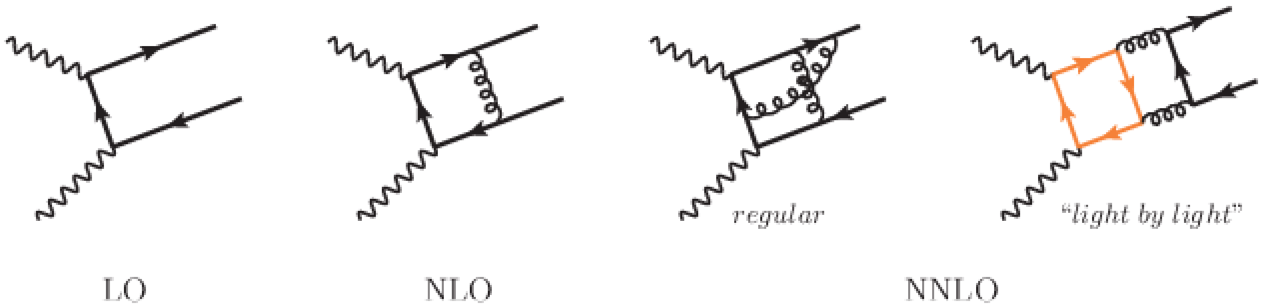
Tree-level SDC

$$f^{(1)}(\tau) = \frac{\pi^2(3-\tau)}{6(4+\tau)} - \frac{20+9\tau}{4(2+\tau)} - \frac{\tau(8+3\tau)}{4(2+\tau)^2}\ln\frac{4+\tau}{2} + 3\sqrt{\frac{\tau}{4+\tau}}\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}$$
$$+\frac{2-\tau}{4+\tau}\left(\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}\right)^2 - \frac{\tau}{2(4+\tau)}\mathrm{Li}_2\left(-\frac{2+\tau}{2}\right),$$

$$\tau \equiv \frac{Q^2}{m^2}$$

NLO QCD correction

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Feynman diagrams



Numer of diagrams

LO	NLO	NNLO (regular)	NNLO ("light by light")
2	8	108	12

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

NNLO corrections

$$f^{(2)}(\tau) = f^{(2)}_{\rm reg}(\tau) + f^{(2)}_{\rm lbl}(\tau).$$

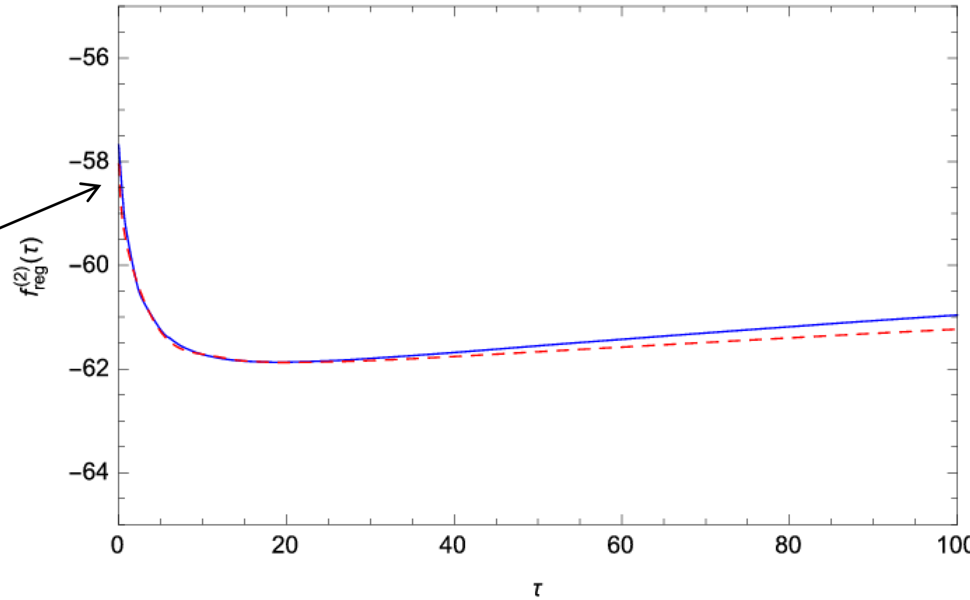
regular

Light-by-light
UV/IR finite

At $\tau \gg 0$, the value of $f^{(2)}_{\rm reg}(\tau)$
is compatible with
asymptotic behavior $\ln^2\tau$
solving ERBL equation by
Yang, NPB 2009

Reproduce
known NNLO
corr. to ηc->γγ

Czarnecki et al.
2001



Investigation on $\gamma\gamma^* \to \eta_c$ form factor
NNLO corrections

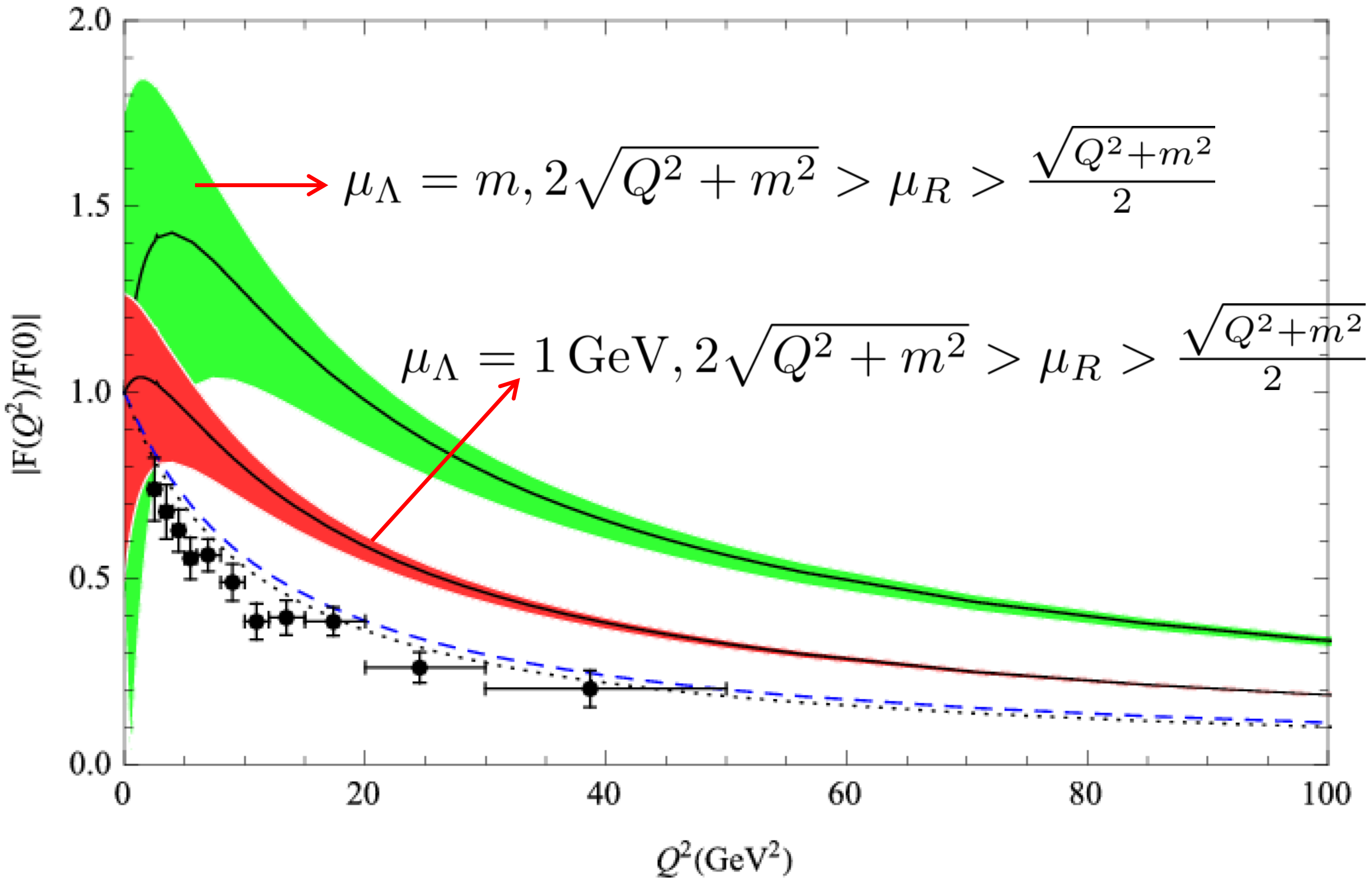
τ	1	5	10	25	50
$f^{(2)}_{\rm reg}$	-59.420(6)	-61.242(6)	-61.721(7)	-61.843(8)	-61.553(8)
$f^{(2)}_{\rm lbl}$	$0.49(1)$ $-0.65(1)i$	$-0.48(1)$ $-0.72(1)i$	$-1.10(1)$ $-0.71(1)i$	$-2.13(1)$ $-0.69(1)i$	$-3.07(1)$ $-0.68(1)i$
$f^{(2)}_{\rm reg}$	-59.636(6)	-61.278(6)	-61.716(7)	-61.864(8)	-61.668(8)
$f^{(2)}_{\rm lbl}$	$0.79(1)$ $-12.45(1)i$	$-5.61(1)$ $-13.55(1)i$	$-9.45(1)$ $-13.83(1)i$	$-15.32(1)$ $-14.03(1)i$	$-20.26(1)$ $-14.10(1)i$

Table 1: $f^{(2)}_{\rm reg}(\tau)$ and $f^{(2)}_{\rm lbl}(\tau)$ at some typical values of τ. The first two rows for η_c and the last two for η_b.

Contribution from light-by-light is not always negligible!

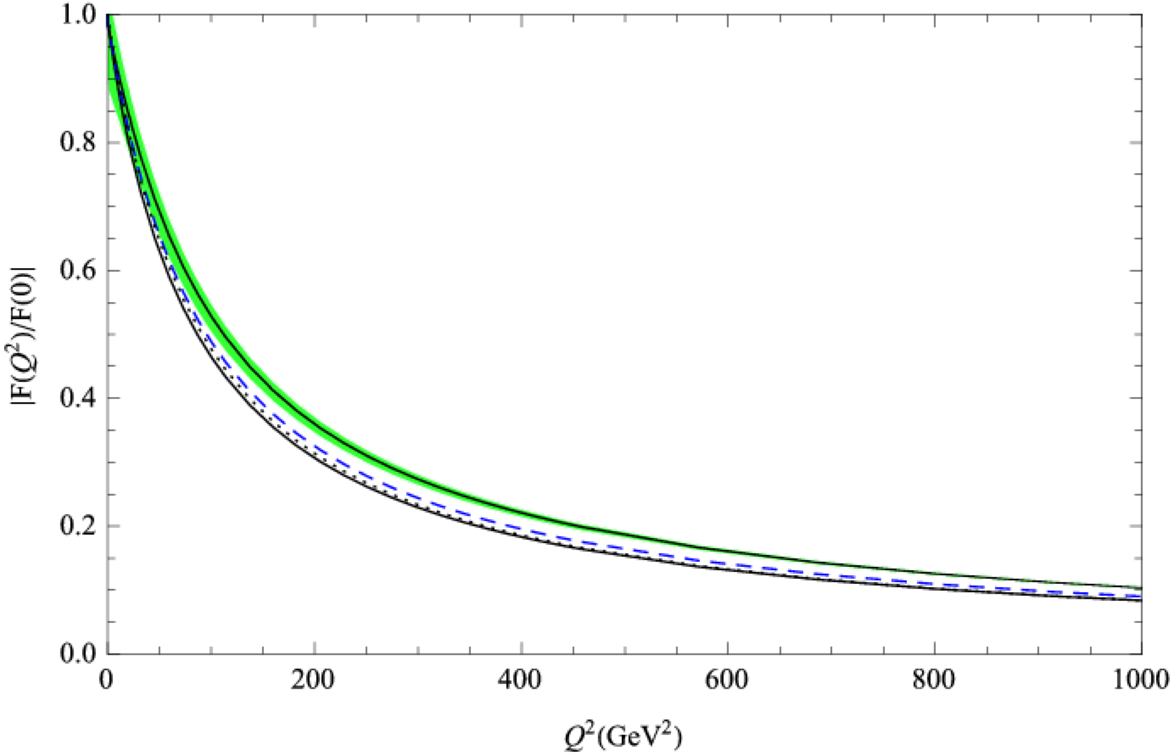
Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Theory vs Experiment

Our Prediction is free of nonperturbative parameters!



$\gamma\gamma^* \rightarrow \eta_c$ **: NNLO predictions seriously fails to describe data!**

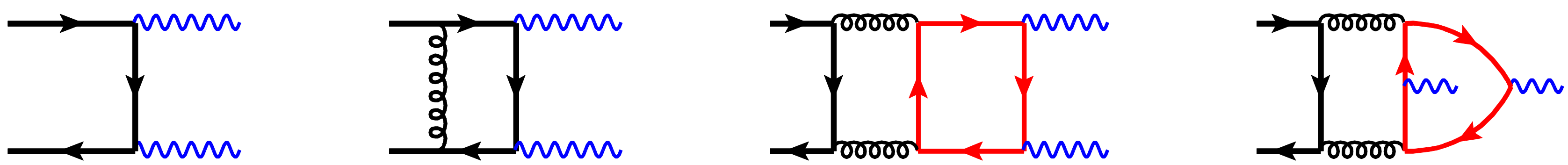
Prediction to $\gamma\gamma^* \rightarrow \eta_b$ form factor



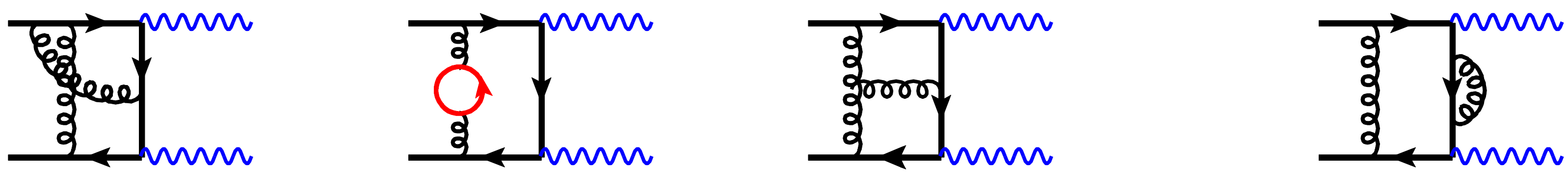
Convergence of perturbation series looks reasonably well.
Await **CEPC/ILC** to test our predictions?

As a by-product, we also have a complete NNLO prediction for $\eta_c \rightarrow 2\gamma$ (including "light-by-light" diagrams)

We can focus on form factor at $Q^2 = 0$:



LO NLO NNLO ("*light by light*")



NNLO (*regular*)

Updated NNLO predictions to $\eta_c \to 2\gamma$

NNLO correction was previously computed by Czarnecki and Melkinov (2001) (neglecting light-by-light);

Here we present a complete/highly precise NNLO predictions

Form factor at $Q^2=0$:

$$F(0) = \frac{e_c^2}{m^{5/2}} \langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle \left\{ 1 + C_F \frac{\alpha_s(\mu_R)}{\pi} \left(\frac{\pi^2}{8} - \frac{5}{2} \right) + \frac{\alpha_s^2}{\pi^2} \left[C_F \left(\frac{\pi^2}{8} - \frac{5}{2} \right) \frac{\beta_0}{4} \ln \frac{\mu_R^2}{m^2} - \pi^2 C_F \left(C_F + \frac{C_A}{2} \right) \ln \frac{\mu_\Lambda}{m} + f_{\rm reg}^{(2)}(0) + f_{\rm lbl}^{(2)}(0) \right] + \mathcal{O}(\alpha_s^3) \right\},$$

NRQCD factorization scale dependence

$$f_{\rm reg}^{(2)}(0) = -21.107\,897\,97(4) C_F^2 - 4.792\,980\,00(3) C_F C_A - \left(\frac{13\pi^2}{144} + \frac{2}{3} \ln 2 + \frac{7}{24} \zeta(3) - \frac{41}{36} \right) C_F T_F n_L + 0.223\,672\,013(2) C_F T_F n_H, \quad (8)$$

$$f_{\rm lbl}^{(2)}(0) = \left(0.731\,284\,59 + i\pi \left(\frac{\pi^2}{9} - \frac{5}{3} \right) \right) C_F T_F \sum_i^{n_L} \frac{e_i^2}{e_Q^2} + (0.646\,965\,57 + 2.073\,575\,56 i) C_F T_F n_H, \quad (9)$$

$$\Gamma(\eta_c \to 2\gamma) = (\pi \alpha^2/4) |F(0)|^2 M_{\eta_c}^3 .$$

A recent paper by Wu, Brodsky et al. (1804.06106) claims that PMC+fixed NNLO can resolve this puzzle.

A solution to the $\gamma\gamma^* \to \eta_c$ puzzle using the Principle of Maximum Conformality

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[2] *Department of Physics, Chongqing University, Chongqing 401331, P.R. China*
[3] *School of Physical Science and Technology, Southwest University, Chongqing 400700, P.R. China and*
[4] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

(Dated: April 18, 2018)

The next-to-next-to-leading order (NNLO) pQCD prediction for the $\gamma\gamma^* \to \eta_c$ form factor was evaluated in 2015 using nonrelativistic QCD (NRQCD). A strong discrepancy between the NRQCD prediction and the BaBar measurements was observed. Until now there has been no solution for this puzzle. In this paper, we present a NNLO analysis by applying the Principle of Maximum Conformality (PMC) to set the renormalization scale. By carefully dealing with the light-by-light diagrams at the NNLO level, the resulting high precision PMC prediction agrees with the BaBar measurements within errors, and the conventional renormalization scale uncertainty is eliminated. The PMC is consistent with all of the requirements of the renormalization group, including scheme-independence. The application of the PMC thus provides a rigorous solution for the $\gamma\gamma^* \to \eta_c$ form factor puzzle, emphasizing the importance of correct renormalization scale-setting. The results also support the applicability of NRQCD to hard exclusive processes involving charmonium.

PACS numbers: 13.66.Bc, 14.40.Pq, 12.38.Bx

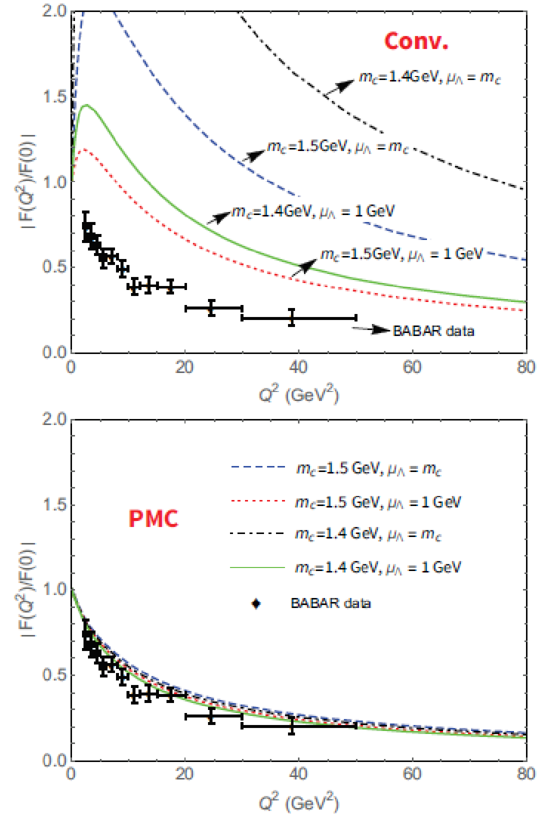


FIG. 4: The NNLO ratio $|F(Q^2)/F(0)|$ versus Q^2 using conventional (Up) and PMC (Down) scale-settings for different values for the quark mass m_c and the factorization scale μ_Λ.

Complete NNLO correction to η_c → light hadrons (first NNLO calculation for inclusive process involving quarkonium)

Feng, Jia, Sang, PRL 119, 252001 (2017)

NLO perturbative corr. 1979/1980

[7] R. Barbieri, E. d'Emilio, G. Curci and E. Remiddi, Nucl. Phys. B **154**, 535 (1979).

[8] K. Hagiwara, C. B. Kim and T. Yoshino, Nucl. Phys. B **177**, 461 (1981).

40 years lapsed from NLO to NNLO;

Another ??? years to transition into NNNLO QCD corrections?

Promising only if **Alpha-Loop** takes over?

NRQCD factorization for η_c → light hadrons – up to relative order-v^4 corrections

Bodwin, Petrelli PRD (2002)

$$\Gamma({}^1S_0\to \mathrm{LH}) = \frac{F_1({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_1({}^1S_0)|{}^1S_0\rangle + \frac{G_1({}^1S_0)}{m^4}\langle {}^1S_0|\mathcal{P}_1({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^3S_1)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^3S_1)|{}^1S_0\rangle + \frac{F_8({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^1P_1)}{m^4}\langle {}^1S_0|\mathcal{O}_8({}^1P_1)|{}^1S_0\rangle + \frac{H_1^1({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^1({}^1S_0)|{}^1S_0\rangle + \frac{H_1^2({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^2({}^1S_0)|{}^1S_0\rangle.$$

$$\mathcal{O}_1({}^1S_0)=\psi^\dagger\chi\chi^\dagger\psi, \tag{2.2a}$$

$$\mathcal{P}_1({}^1S_0)=\frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi+\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\psi\right], \tag{2.2b}$$

$$\mathcal{O}_8({}^3S_1)=\psi^\dagger\boldsymbol{\sigma}T_a\chi\cdot\chi^\dagger\boldsymbol{\sigma}T_a\psi, \tag{2.2c}$$

$$\mathcal{O}_8({}^1S_0)=\psi^\dagger T_a\chi\chi^\dagger T_a\psi, \tag{2.2d}$$

$$\mathcal{O}_8({}^1P_1)=\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\chi\cdot\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\psi, \tag{2.2e}$$

$$\mathcal{Q}_1^1({}^1S_0)=\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi, \tag{2.2f}$$

$$\mathcal{Q}_1^2({}^1S_0)=\frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\psi+\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\chi\chi^\dagger\psi\right], \tag{2.2g}$$

$$\mathcal{Q}_1^3({}^1S_0)=\frac{1}{2}[\psi^\dagger\chi\chi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E}+g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\psi-\psi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E}+g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\chi\chi^\dagger\psi], \tag{2.2h}$$

NRQCD factorization for $\eta_c \rightarrow$ light hadrons – up to relative order-v^4 corrections

Brambilla, Mereghetti, Vairo, 0810.2259

$$\begin{aligned}
\Gamma(^1S_0 \to \text{l.h.}) &= \frac{2\,\mathrm{Im}\, f_1(^1S_0)}{M^2}\langle H(^1S_0)|\mathcal{O}_1(^1S_0)|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, g_1(^1S_0)}{M^4}\langle H(^1S_0)|\mathcal{P}_1(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_8(^3S_1)}{M^2}\langle H(^1S_0)|\mathcal{O}_8(^3S_1)|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, f_8(^1S_0)}{M^2}\langle H(^1S_0)|\mathcal{O}_8(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_8(^1P_1)}{M^4}\langle H(^1S_0)|\mathcal{O}_8(^1P_1)|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, s_{1\text{-}8}(^1S_0, ^3S_1)}{M^4}\langle H(^1S_0)|\mathcal{S}_{1\text{-}8}(^1S_0, ^3S_1)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f'_{8\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{O}'_{8\,\mathrm{cm}}|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, g_{8a\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{P}_{8a\,\mathrm{cm}}|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_{1\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{O}_{1\,\mathrm{cm}}|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, h'_1(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}'_1(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, h''_1(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}''_1(^1S_0)|H(^1S_0)\rangle \\
&+ \frac{2\,\mathrm{Im}\, g_8(^3S_1)}{M^4}\langle H(^1S_0)|\mathcal{P}_8(^3S_1)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, g_8(^1S_0)}{M^4}\langle H(^1S_0)|\mathcal{P}_8(^1S_0)|H(^1S_0)\rangle \\
&+ \frac{2\mathrm{Im}\, g_8(^1P_1)}{M^6}\langle H(^1S_0)|\mathcal{P}_8(^1P_1)|H(^1S_0)\rangle + \frac{2\mathrm{Im}\, h'_8(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}'_8(^1S_0)|H(^1S_0)\rangle \\
&+ \frac{2\mathrm{Im}\, h_8(^1D_2)}{M^6}\langle H(^1S_0)|\mathcal{Q}_8(^1D_2)|H(^1S_0)\rangle + \frac{2\mathrm{Im}\, h_1(^1D_2)}{M^6}\langle H(^1S_0)|\mathcal{Q}_1(^1D_2)|H(^1S_0)\rangle \\
&+ \frac{2\mathrm{Im}\, d_8(^1S_0, ^1P_1)}{M^5}\langle H(^1S_0)|\mathcal{D}_{8\text{-}8}(^1S_0, ^1P_1)|H(^1S_0)\rangle,
\end{aligned}$$

Notice the explosion of number of higher-dimensional operators!

NRQCD factorization for $\eta_c \rightarrow$ light hadrons – Current status of radiative corrections

$$\Gamma(\eta_c \to \mathrm{LH}) = \frac{F_1(^1S_0)}{m^2}\langle \eta_c|\mathcal{O}_1(^1S_0)|\eta_c\rangle + \frac{G_1(^1S_0)}{m^4}\langle \eta_c|\mathcal{P}_1(^1S_0)|\eta_c\rangle + \mathcal{O}(v^3\Gamma),$$

To warrant predictive power, we only retain terms through relative order-v^2

$$F_1(^1S_0) = \frac{\pi\alpha_s^2 C_F}{N_c}\left\{1 + \frac{\alpha_s}{\pi}f_1 + \frac{\alpha_s^2}{\pi^2}f_2 + \cdots\right\}$$

$$G_1(^1S_0) = -\frac{4\pi\alpha_s^2 C_F}{3N_c}\left\{1 + \frac{\alpha_s}{\pi}g_1 + \cdots\right\}.$$

W.Y.Keung, I. Muzinich, 1983

$$f_1 = \frac{\beta_0}{2}\ln\frac{\mu_R^2}{4m^2} + \left(\frac{\pi^2}{4} - 5\right)C_F + \left(\frac{199}{18} - \frac{13\pi^2}{24}\right)C_A - \frac{8}{9}n_L - \frac{2n_H}{3}\ln 2, \quad (3a)$$

Barbieri et al., 1979
Hagiwara et al., 1980

$$g_1 = \frac{\beta_0}{2}\ln\frac{\mu_R^2}{4m^2} - C_F\ln\frac{\mu_\Lambda^2}{m^2} - \left(\frac{49}{12} - \frac{5\pi^2}{16} - 2\ln 2\right)C_F + \left(\frac{479}{36} - \frac{11\pi^2}{16}\right)C_A - \frac{41}{36}n_L - \frac{2n_H}{3}\ln 2. \quad (3b)$$

Guo,Ma,Chao, 2011

Our calculation of short-distance coefficient utilizes **Method of Region (Beneke and Smirnov 1998)** to directly extract the hard region contribution from multi-loop diagrams

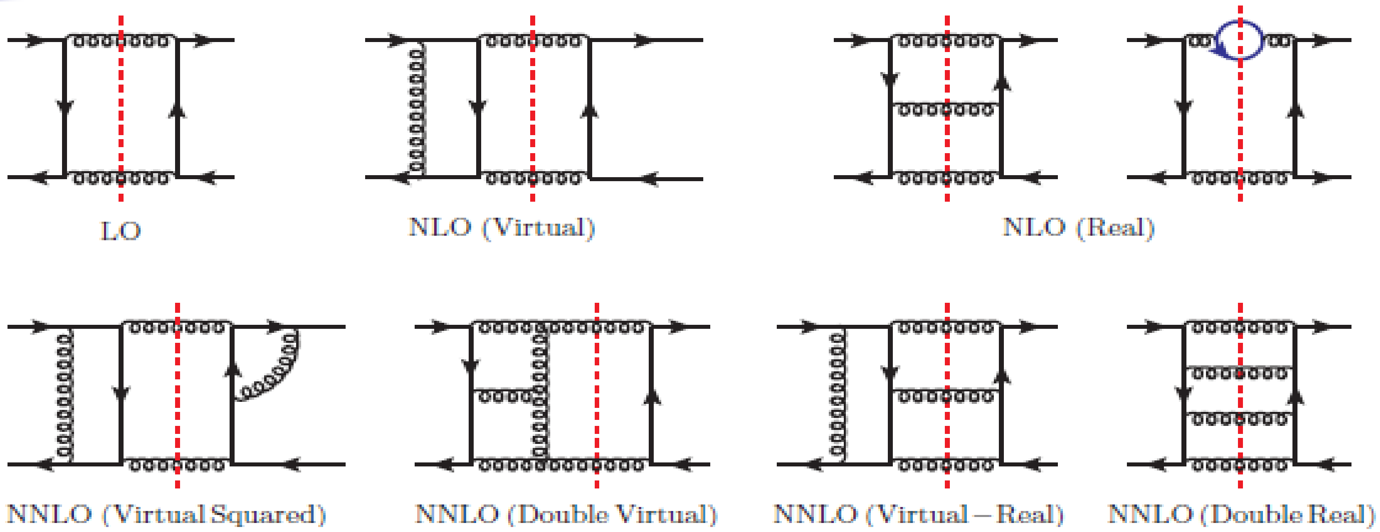


FIG. 1: Representative cut Feynman diagrams responsible for the quark reaction $c\bar{c}(^1S_0^{(1)}) \to c\bar{c}(^1S_0^{(1)})$ through NNLO in α_s. The vertical dashed line denotes the Cutkosky cut.

Roughly 1700 3-loop forward-scattering diagrams, divided into 4 distinct cut topologies; Cutkosky rule is imposed

Employ a well-known trick to deal with phase-space type integrals

Key technique: using IBP to deal with phase-space integral

$$\int \frac{d^D p_i}{(2\pi)^D} 2\pi\, i\, \delta^+(p_i^2) = \int \frac{d^D p_i}{(2\pi)^D} \left(\frac{1}{p_i^2 + i\varepsilon} - \frac{1}{p_i^2 - i\varepsilon} \right).$$

duction. Finally, we end up with 93 MIs for the "Double Virtual" type of diagrams, 89 MIs for the "Virtual-Real" type of diagrams, and 32 MIs for "Double Real" type of diagrams, respectively. To the best of our knowledge, this work represents the first application of the trick (4) in higher-order calculation involving quarkonium.

The nontrivial aspects of the calculation

Encounter some rather time-consuming MIs using sector decomposition method (Fiesta)

Roughly speaking, 10^5 CPU core hour is expensed; Run numerical integration at the GuangZhou Tianhe Supercomputer Center/China Grid.

Explicitly verify the cancellation of IR poles among the 4 types of cut diagrams. Starting from the $1/\varepsilon^4$ poles, observe the exquisite cancelation until $1/\varepsilon$

See Y.-Q. Ma's talk for new algorithm of evaluating MI

Our key results

$$f_2 = \hat{f}_2 + \frac{3\beta_0^2}{16}\ln^2\frac{\mu_R^2}{4m^2} + \left(\frac{\beta_1}{8} + \frac{3}{4}\beta_0\hat{f}_1\right)\ln\frac{\mu_R^2}{4m^2}$$
$$-\pi^2\left(C_F^2 + \frac{C_A C_F}{2}\right)\ln\frac{\mu_\Lambda^2}{m^2}, \quad (5)$$

→ NNLO SDC

Same IR divergence as η_c → 2γ!

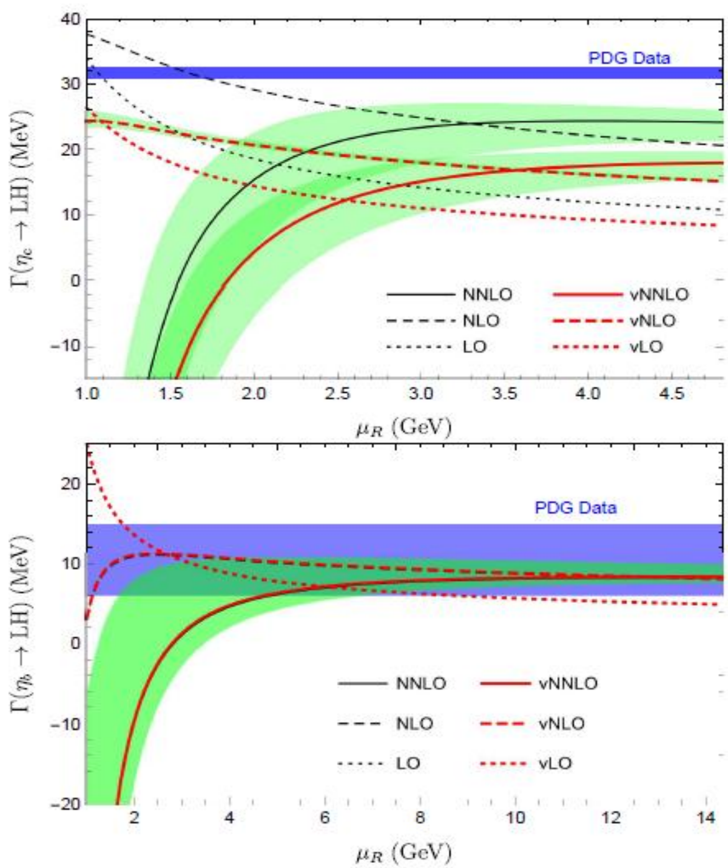
$$\hat{f}_2 = -0.799(13)N_c^2 - 7.4412(5)n_L N_c - 3.6482(2)N_c$$
$$+0.37581(3)n_L^2 + 0.56165(5)n_L + 32.131(5)$$
$$-0.8248(3)\frac{n_L}{N_c} - \frac{0.67105(3)}{N_c} - \frac{9.9475(2)}{N_c^2}. \quad (6)$$

grals. Concretely, $\hat{f}_2 = -50.1(1)$ for η_c hadronic decay, and $-69.5(1)$ for η_b decay. For completeness, here we also enumerate the numerical values of the non-logarithmic parts of f_1 and g_1 in (3): $\hat{f}_1 = 10.62$, $\hat{g}_1 = 16.20$ for η_c hadronic decay; $\hat{f}_1 = 9.73$, $\hat{g}_1 = 15.06$ for η_b decay.

Validate the NRQCD factorization for S-wave onium inclusive decay at NNLO!
We also obtain the following RGE for the leading 4-fermion NRQCD operator:

$$\frac{d\langle\mathcal{O}_1(^1S_0)\rangle_{\eta_c}}{d\ln\mu_\Lambda^2} = \alpha_s^2\left(C_F^2 + \frac{C_A C_F}{2}\right)\langle\mathcal{O}_1(^1S_0)\rangle_{\eta_c}$$
$$-\frac{4}{3}\frac{\alpha_s}{\pi}C_F\frac{\langle\mathcal{P}_1(^1S_0)\rangle_{\eta_c}}{m^2} + \cdots, \quad (7)$$

Phenomenological study: hadronic width



Input parameters:

$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_c} = 0.470\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_c} = \frac{0.430\,\mathrm{GeV}^2}{m_c^2},$$

$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_b} = 3.069\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_b} = -0.009. \qquad (9)$$

PDG values:

$$\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8\ \mathrm{MeV},$$

$$\Gamma_{\mathrm{had}}(\eta_b) = 10^{+5}_{-4}\ \mathrm{MeV}$$

FIG. 2: The predicted hadronic widths of η_c (top) and η_b (bottom) as functions of μ_R, at various level of accuracy in α_s and v expansion. The horizontal blue bands correspond to the measured hadronic widths taken from PDG 2016 [4], with $\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8$ MeV and $\Gamma_{\mathrm{had}}(\eta_b) = 10^{-4}_{+5}$ MeV. The label "LO" represents the NRQCD prediction at the lowest-order α_s and v, and the label "NLO" denotes the "LO" prediction plus the $\mathcal{O}(\alpha_s)$ perturbative correction, while the label "NNLO" signifies the "NLO" prediction plus the $\mathcal{O}(\alpha_s^2)$ perturbative correction. The label "vLO" represents the "LO" prediction together with the tree-level order-v^2 correction, and the label "vNLO" designates the "vLO" prediction supplemented with the relative order-α_s and order-$\alpha_s v^2$ correction, while the label "vNNLO" refers to the "vNLO" prediction further supplemented with the order-α_s^2 correction. The green bands are obtained by varying μ_Λ from 1 GeV to twice heavy quark mass, and the central curve inside the bands are obtained by setting μ_Λ equal to heavy quark mass.

Phenomenological study of Br($\eta_{c,b} \to \gamma\gamma$), Non-Perturbative matrix elements cancel out

For η_c more than 10σ discrepancy !

$$\mathrm{Br}(\eta_c \to \gamma\gamma) = \frac{8\alpha^2}{9\alpha_s^2}\Bigg\{1 - \frac{\alpha_s}{\pi}\left[4.17\ln\frac{\mu_R^2}{4m_c^2} + 14.00\right] + \frac{\alpha_s^2}{\pi^2}\left[4.34\ln^2\frac{\mu_R^2}{4m_c^2} + 22.75\ln\frac{\mu_R^2}{4m_c^2} + 78.8\right] + 2.24\langle v^2\rangle_{\eta_c}\frac{\alpha_s}{\pi}\Bigg\}, \quad (10a)$$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = \frac{\alpha^2}{18\alpha_s^2}\Bigg\{1 - \frac{\alpha_s}{\pi}\left[3.83\ln\frac{\mu_R^2}{4m_b^2} + 13.11\right] + \frac{\alpha_s^2}{\pi^2}\left[3.67\ln^2\frac{\mu_R^2}{4m_b^2} + 20.30\ln\frac{\mu_R^2}{4m_b^2} + 85.5\right] + 1.91\langle v^2\rangle_{\eta_b}\frac{\alpha_s}{\pi}\Bigg\}. \quad (10b)$$

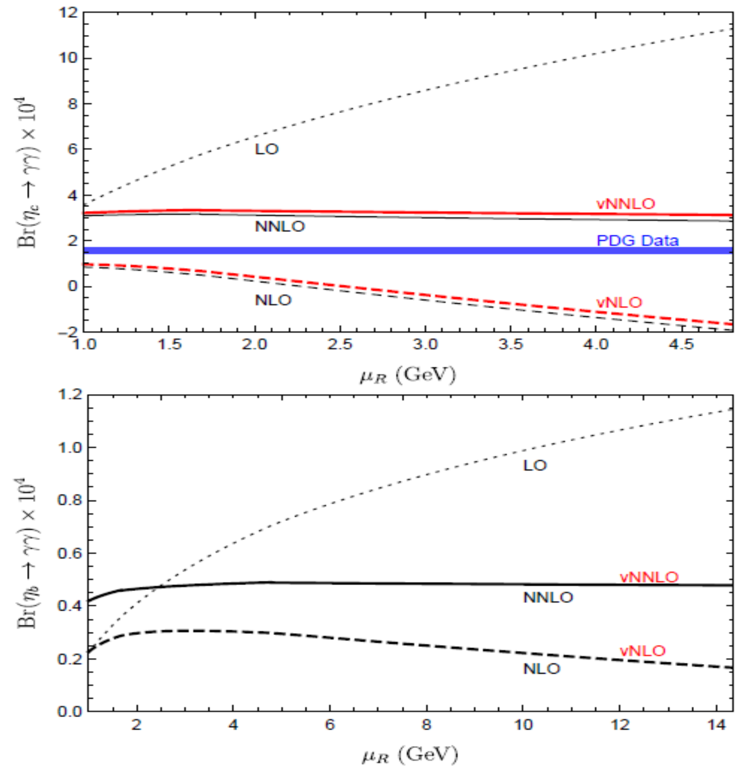


FIG. 3: The predicted branching fractions of $\eta_c \to \gamma\gamma$ (top) and $\eta_b \to \gamma\gamma$ (bottom) as functions of μ_R, at various level of accuracy in α_s and v. The blue band corresponds to the measured branching ratio for $\eta_c \to \gamma\gamma$ taken from PDG 2016 [4], with $\mathrm{Br}(\eta_c \to \gamma\gamma) = (1.59 \pm 0.13) \times 10^{-4}$. The labels characterizing different curves are the same as in Fig. 2.

To date most refined predictior for $\eta_b \to \gamma\gamma$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = (4.8 \pm 0.7) \times 10^{-5},$$

A famous puzzle since 2002: exclusive double charmonium production: $e^+ e^- \rightarrow J/\Psi + \eta_c$ at B factories

(F. Feng, Y. J., W.-L.Sang, arXiv:1901.08447[hep-ph]

Next-to-next-to-leading-order QCD corrections to $e^+e^- \to J/\psi + \eta_c$ at B factories

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(Dated: January 25, 2019)

Within the nonrelativistic QCD (NRQCD) factorization framework, we compute the long-awaited $\mathcal{O}(\alpha_s^2)$ correction for the exclusive double charmonium production process at B factories, *i.e.*, $e^+e^- \to J/\psi + \eta_c$ at $\sqrt{s} = 10.58$ GeV. For the first time, we confirm that NRQCD factorization does hold at next-to-next-to-leading-order (NNLO) for exclusive double charmonium production. It is found that including the NNLO QCD correction greatly reduces the renormalization scale dependence, and also implies the reasonable perturbative convergence behavior for this process. Our state-of-the-art prediction is consistent with the BaBar measurement.

PACS numbers:

A biggest puzzle in Standard Model in the beginning of this century

4. Phenomenology. The production rate initially measured by BELLE is $\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9$ fb [1], later shifted to $\sigma[J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb [44], where $\mathcal{B}_{>n}$ denotes the branching fraction for the η_c into n charged tracks. An indepednent measurement by BABAR in 2005 yields $\sigma[J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb [45].

The LO NRQCD predictions by three groups are smaller
Than Belle measurements by an order of magnitude!

E. Braaten, J. Lee, PRD 2003
K. Y. Liu, Z. G. He, K. T. Chao, PLB 2003 — LO NRQCD factorization
K. Hagiwara, E. Kou, C. F. Qiao, PLB 2003

J. P. Ma, Z. G. Si, PRD 2004 — LO light-cone approach

A crucial progress is the large NLO perturbative correction

Very significant NLO correction comes as a surprise

$$e^+e^- \to J/\psi + \eta_c \qquad \text{K factor: } 1.8 \sim 2.1$$

Y. J. Zhang, Y. J. Gao and K.-T. Chao, PRL 2006
B. Gong, J.-X. Wang, PRD 2008

One may naturally wonder: how about the size of the NNLO QCD corrections? We have to wait for 14 years…

Two-loop, 5 point amplitude is the frontier, especially massive quark!

One influential 2011 review article claims that "The calculation of … is perhaps beyond the current state of the art"

NRQCD factorization formula for exclusive double-charmonium production

$$\langle J/\psi(P_1,\lambda)+\eta_c(P_2)|J^\mu_{\rm EM}|0\rangle = i\,F(s)\,\epsilon^{\mu\nu\rho\sigma}P_{1\nu}P_{2\rho}\varepsilon^*_\sigma(\lambda),$$

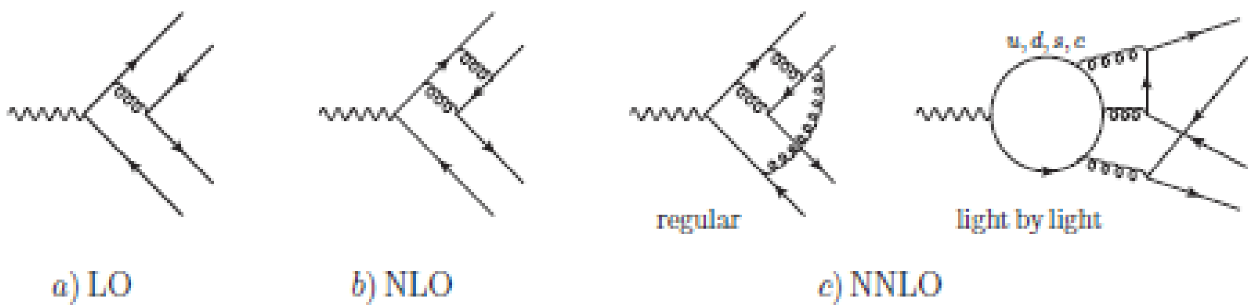
$$F(s) = \sqrt{4M_{J/\psi}M_{\eta_c}}\langle J/\psi|\psi^\dagger\boldsymbol{\sigma}\cdot\epsilon\chi|0\rangle\langle\eta_c|\psi^\dagger\chi|0\rangle \times \left[f+g_{J/\psi}\langle v^2\rangle_{J/\psi}+g_{\eta_c}\langle v^2\rangle_{\eta_c}+\cdots\right],$$

$$\sigma[e^+e^-\to J/\psi+\eta_c] = \frac{4\pi\alpha^2}{3}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)^3|F(s)|^2 = \sigma_0+\sigma_2+\mathcal{O}(\sigma_0 v^4),$$

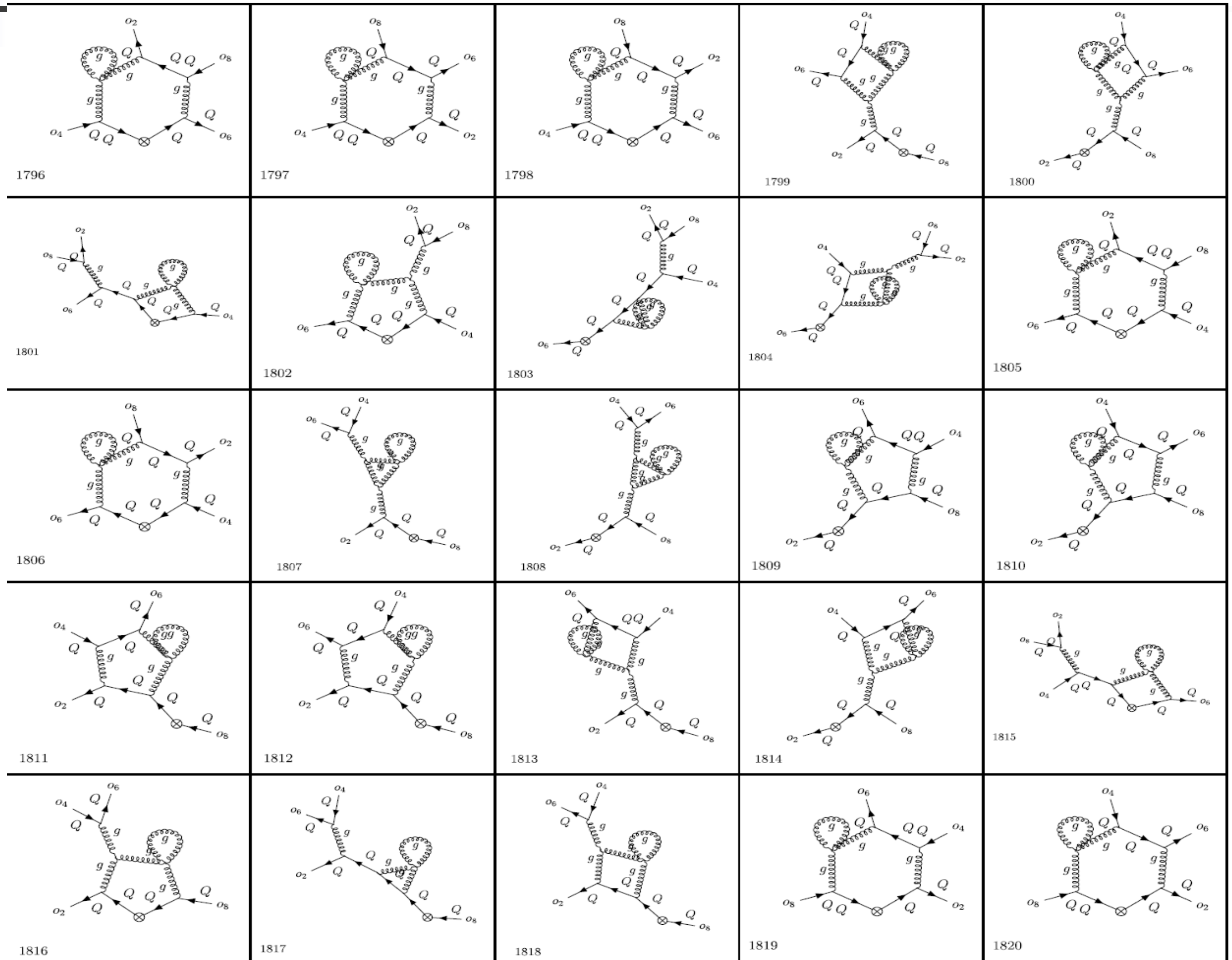
$$f = f^{(0)}+\frac{\alpha_s}{\pi}f^{(1)}+\frac{\alpha_s^2}{\pi^2}f^{(2)}+\cdots,$$

$$g_H = g_H^{(0)}+\frac{\alpha_s}{\pi}g_H^{(1)}+\cdots.$$

$$|f|^2 = |f^{(0)}|^2+\frac{\alpha_s}{\pi}2{\rm Re}(f^{(0)}f^{(1)*}) + \left(\frac{\alpha_s}{\pi}\right)^2\left[2{\rm Re}(f^{(0)}f^{(2)*})+|f^{(1)}|^2\right],$$



About 2000 two-loop diagrams; Cutting-edge NNLO calculation, 1->4 topology



700 master integrals; most complex-valued; Hard efforts in computing them with high precision

$$f^{(2)} = f^{(0)}\left\{\frac{\beta_0^2}{16}\ln^2\frac{s}{4\mu_R^2} - \left(\frac{\beta_1}{16}+\frac{1}{2}\beta_0\hat{f}^{(1)}\right)\ln\frac{s}{4\mu_R^2} + (\gamma_{J/\psi}+\gamma_{\eta_c})\ln\frac{\mu_\Lambda^2}{m^2} + \mathsf{F}(r)\right\},$$

log(muR) dictated By RG invariance

Specific form of single IR pole in hard region

$$\gamma_{J/\psi} = -\frac{\pi^2}{12}C_F\left(2C_F+3C_A\right),$$

$$\gamma_{\eta_c} = -\frac{\pi^2}{4}C_F\left(2C_F+C_A\right).$$

Required by the validity of NRQCD factorization

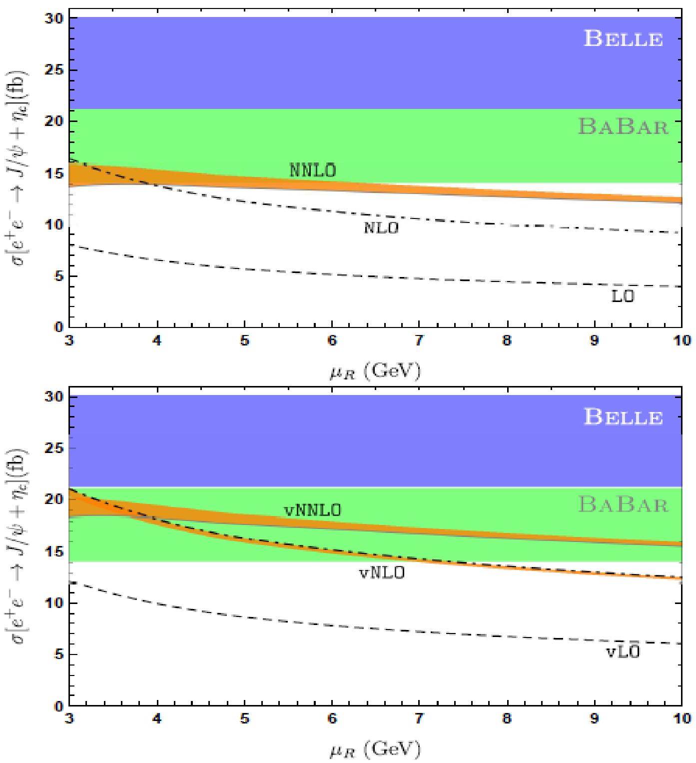
$$\mathrm{Re}\,\mathsf{F}(r=0.0700) = -25\pm 4,$$
$$\mathrm{Re}\,\mathsf{F}(r=0.1009) = -21\pm 5.$$

This is the main result!

Phenomenology: our state-of-the-art predictions

TABLE I: Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ at $\sqrt{s} = 10.58$ GeV. Each column is labeled by the powers of α_s and v, and given in units of fb. We fix $\mu_\Lambda = m$, and consider $\mu_R = 2m$ and $\sqrt{s}/2$. The two upper rows and the two lower rows correspond to $m = 1.4$ GeV and $m = 1.68$ GeV, respectively.

μ_R	LO	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s v^2)$	$\mathcal{O}(\alpha_s^2)$	Total
$2m$	8.48	4.36	8.64	0.34	−3.7(5)	18.1(5)
$\frac{\sqrt{s}}{2}$	5.52	2.84	6.48	1.18	1.6(2)	17.6(2)
$2m$	5.59	1.44	4.71	−0.33	−1.4(4)	10.0(4)
$\frac{\sqrt{s}}{2}$	4.16	1.07	4.08	0.06	0.7(2)	10.1(2)



$$\sigma = \sigma_{\text{LO}} \left[1 + \frac{\sigma^{(v^2)}}{\sigma_{\text{LO}}} + \frac{\sigma^{(\alpha_s)}}{\sigma_{\text{LO}}} + \frac{\sigma^{(\alpha_s v^2)}}{\sigma_{\text{LO}}} + \frac{\sigma^{(\alpha_s^2)}}{\sigma_{\text{LO}}} \right].$$

$$\sigma = 8.48\,\text{fb}\,[1 + 0.51 + 1.02 + 0.04 - 0.44(6)],$$
$$\sigma = 5.52\,\text{fb}\,[1 + 0.51 + 1.17 + 0.21 + 0.28(4)],$$
$$\sigma = 5.59\,\text{fb}\,[1 + 0.26 + 0.84 - 0.06 - 0.25(6)],$$
$$\sigma = 4.16\,\text{fb}\,[1 + 0.26 + 0.98 + 0.01 + 0.16(5)],$$

New NNLO piece!

Conclusion of 1901.08447

- Reducing renormalization scale dependence

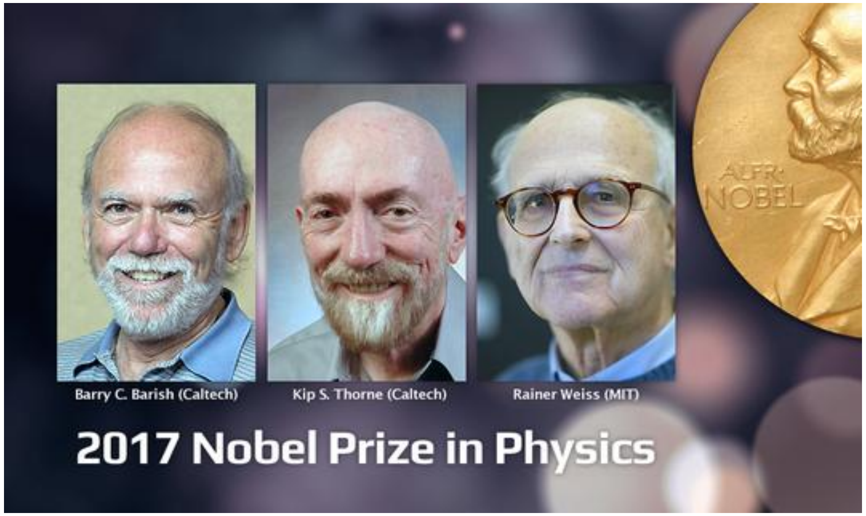
- See decent perturbative convergence behavior

- Agree with BaBar data, yet not Belle

Call for Belle 2 for re-measurement of this channel

Fun: graviton search in quarkonium decay at BESIII experiments

Gravitational wave was finally seen by LIGO in 2015, after 100 years birth of General Relativity by Einstein



Recall, miraculously, both classical EW wave and photo-electric effect were discovered by Hertz in 1887

Unfortunately, searching for quantum graviton looks hopeless

Search for quantum graviton from quarkonium decay at BESIII

Quarkonium decay into photon plus graviton: a golden channel to discriminate General Relativity from Massive Gravity?

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(Dated: November 27, 2017)

Abstract

After the recent historical discovery of gravitational wave, it is curious to speculate upon the detection prospect of the quantum graviton in the terrestrial accelerator-based experiment. We carefully investigate the "golden" channels, $J/\psi(\Upsilon) \to \gamma+$graviton, which can be pursued at BESIII and Belle 2 experiments, by searching for single-photon plus missing energy events. Within the effective field theory (EFT) framework of General Relativity (GR) together with Nonrelativistic QCD (NRQCD), we are capable of making solid predictions for the corresponding decay rates. It is found that these extremely suppressed decays are completely swamped by the Standard Model background events $J/\psi(\Upsilon) \to \gamma + \nu\bar{\nu}$. Meanwhile, we also study these rare decay processes in the context of massive gravity, and find the respective decay rates in the limit of vanishing graviton mass drastically differ from their counterparts in GR. Counterintuitive as the failure of smoothly recovering GR results may look, our finding is reminiscent of the van Dam-Veltman-Zakharov (vDVZ) discontinuity widely known in classical gravity, which can be traced to the finite contribution of the helicity-zero graviton in the massless limit. Nevertheless, at this stage we are not certain about the fate of the discontinuity encountered in this work, whether it is merely a pathology or not. If it could be endowed with some physical significance, the future observation of these rare decay channels, would, in principle, shed important light on the nature of gravitation, whether the graviton is strictly massless, or bears a very small but nonzero mass.

PACS numbers: *04.60.Bc, 14.40.Pq, 14.70.Kv*

General Relativity (GR) should be regarded as the low-energy EFT of quantum gravity (Donoghue 1994)

Einsein-Hilbert action

$$\kappa = \sqrt{32\pi G_N},$$

$$S = S_{\rm grav} + S_{\rm matt} = \int d^4x\sqrt{-g}(\mathcal{L}_{\rm grav} + \mathcal{L}_{\rm SM}).$$

$$\mathcal{L}_{\rm grav} = -\Lambda - \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R^{\mu\nu}R_{\mu\nu} + \cdots ,$$

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - \frac{1}{4}g^{\mu\alpha}g^{\nu\beta}G^a_{\mu\nu}G^a_{\alpha\beta} + \sum_f \bar{q}_f(i\gamma^a e^\mu_a D_\mu - m_f)q_f + \cdots .$$

Weak field expansion: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},$

$$\mathcal{L}_{\rm int} = -\frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu} = \mathcal{L}_{\bar{f}f\mathcal{G}} + \mathcal{L}_{\bar{f}fg\mathcal{G}} + \mathcal{L}_{\bar{f}f\gamma\mathcal{G}} + \mathcal{L}_{gg\mathcal{G}} + \mathcal{L}_{\gamma\gamma\mathcal{G}} + \cdots ,$$

Combining GR+NRQCD to account for quarkonium decay J/Ψ → γ+G

D. Bai, W. Chen, Y.J. arXiv:1711.09058

LO

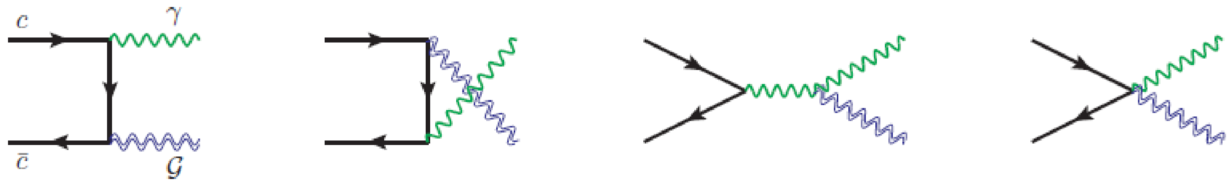

FIG. 1: Four LO Feynman diagrams for $c\bar{c}(^3S_1^{(1)}) \to \gamma + \mathcal{G}$.

Including NLO QCD correction

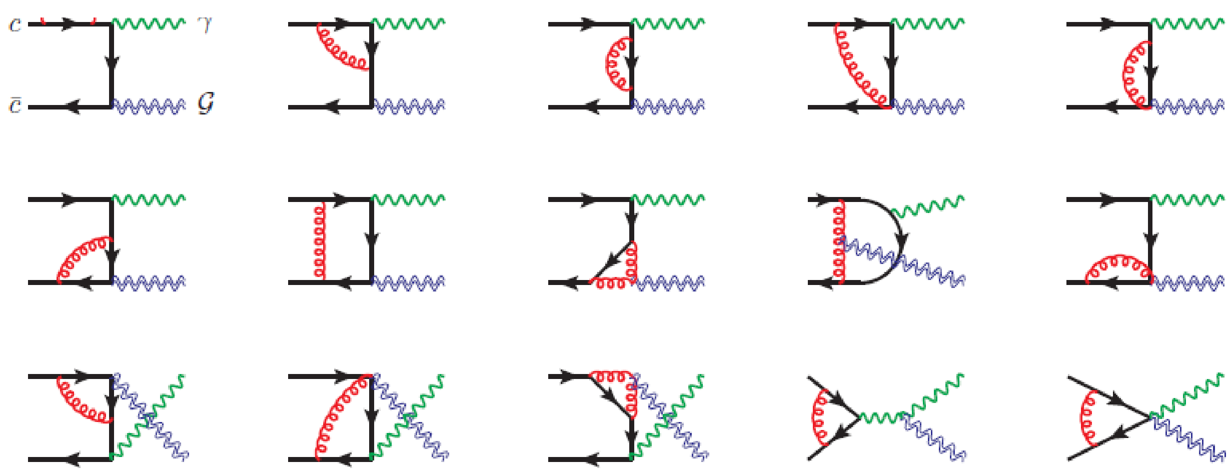

FIG. 2: Representative Feynman diagrams for $c\bar{c}(^3S_1^{(1)}) \to \gamma + \mathcal{G}$ in NLO in α_s.

It is fun that all nature's four forces are united in those diagrams!

Predicted partial widths

Massless graviton (GR): LO prediction accidently vanishes!
Have to proceed to the NLO in α_s and v:

$$\Gamma[J/\psi \to \gamma + \mathcal{G}] = \frac{4e_c^2 \alpha G_N}{27} N_c \left| R_{J/\psi}(0) \right|^2 \left(\langle v^2 \rangle_{J/\psi} + \frac{3C_F \alpha_s}{4\pi}(1 - 4\ln 2) \right)^2 .$$

Massive graviton: nonzero prediction at LO in v at tree level

$$\Gamma[J/\psi \to \gamma + \mathcal{G}] = \frac{2e_c^2 \alpha G_N}{9} N_c \left| R_{J/\psi}(0) \right|^2 .$$

Manifestation of famous vDVZ discontinuity:
Helicity-zero graviton doesn't decouple in the M_G->0 limit

Numerical results

This decay is a golden channel to discriminate whether Graviton mass is strictly zero or not!

$$\mathrm{Br}(J/\psi \to \gamma + \mathcal{G}) = (2 \sim 8) \times 10^{-40}, \qquad \mathrm{GR}$$
$$\mathrm{Br}(J/\psi \to \gamma + \mathcal{G}) = 1.4 \times 10^{-37}. \qquad \mathrm{MG}$$

Not too much suppressed relative to $\mu \to e\,\gamma$, with BR ~ 10^{-34}

$$\mathrm{Br}(\Upsilon(1S) \to \gamma + \mathcal{G}) = (3 \sim 4) \times 10^{-39}, \qquad \mathrm{GR}$$
$$\mathrm{Br}(\Upsilon(1S) \to \gamma + \mathcal{G}) = 4.1 \times 10^{-37}. \qquad \mathrm{MG}$$

Practically speaking, these channels are much rarer than the dominant SM background $J/\Psi \to \gamma\, \nu\, \nu$, with BR ~ 10^{-10}

$$\Gamma[J/\psi \to \gamma\nu\bar{\nu}] = N_\nu \frac{2}{27} e_c^2 \alpha G_F^2 M_{J/\psi}^2 N_c \left|R_{J/\psi}(0)\right|^2,$$

Summary

- **Investigated NNLO QCD corrections to $\gamma\gamma^* \rightarrow \eta_c$, ($\chi_{c0,2} \rightarrow 2\gamma$), $\eta_c \rightarrow$ LH.** Observe significant NNLO corrections. Alarming discrepancy with the existing measurements.

- Perturbative expansion seems to have poor convergence behavior for charmonium

(exception is the double charmonium production at B factory, $e^+ e^- \rightarrow J/\Psi + \eta_c$)

- Perturbative expansion bears much better behavior for bottomonium

Personal (biased) perspectives

NRQCD factorization is from first princple of QCD, has very solid ground!
Unfortunately, maybe Nature is just not so mercy to us …

The charm quark is simply not heavy enough to warrant the trustworthy application of NRQCD factorization to charmonium, just like one cannot fully trust HQET to cope with charmed hadron.

Symptom: m_c is not much greater than Λ_{QCD}, bigger value of α_s at charm mass scale -> damage convergence of perturbative expansion.

But we should still trust NRQCD to be capable of rendering qualitatively correct phenomenology for charmonium. We may need be less ambitious for soliciting precision predictions

Thanks for your attention!