Strong Interaction and Holography

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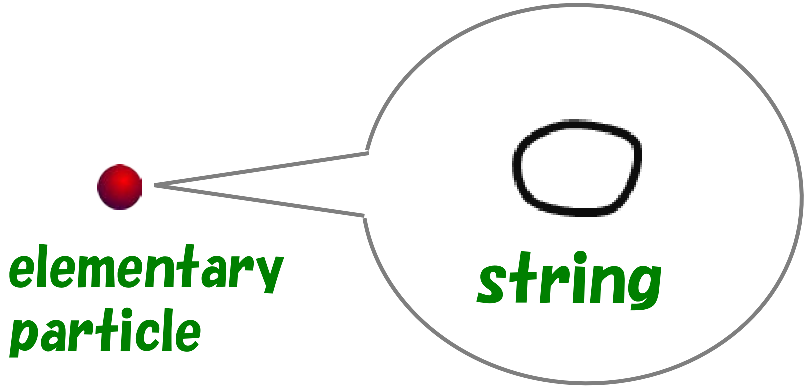
1 Introduction

What is string theory ?



- All kinds of particles may be described by a single string!
- String theory naturally contains gravity
  in a consistent way as a quantum theory!

→ **It could be an ultimate unified theory!**

- This is still a very attractive scenario, but not confirmed.

I'm sorry

No ultimate theory today.

Today's main topic is

Holographic dual

What is holographic dual？

Equivalence between two apparently different theories

Gauge theory (theory of elementary particles) in a flat 4 dim spacetime

＝

String theory in a curved 10 dim spacetime

1 time + 3 space dimensions

equivalent !

- They look completely different.
  But they are claimed to be completely equivalent.
- I am not saying that every gauge theory has a string theory description. But many examples have been found.
- It is still a conjecture without a rigorous proof.
  But there are many many highly non-trivial evidences.

Gauge theory (theory of elementary particles) in a flat 4 dim spacetime

=

String theory in a curved 10 dim spacetime

Because it relates two theories with different spacetime dimensions, it is called "holographic dual"

(※ also called: gauge/string dual, gauge/gravity dual, AdS/CFT correspondence…)

- Why can this be true?
- How can 4 dim theory describe 10 dim theory?

I'll come to these points. Please wait for a while.

- This duality was found in 1997 by Maldacena, and people were shocked.

Maldacena

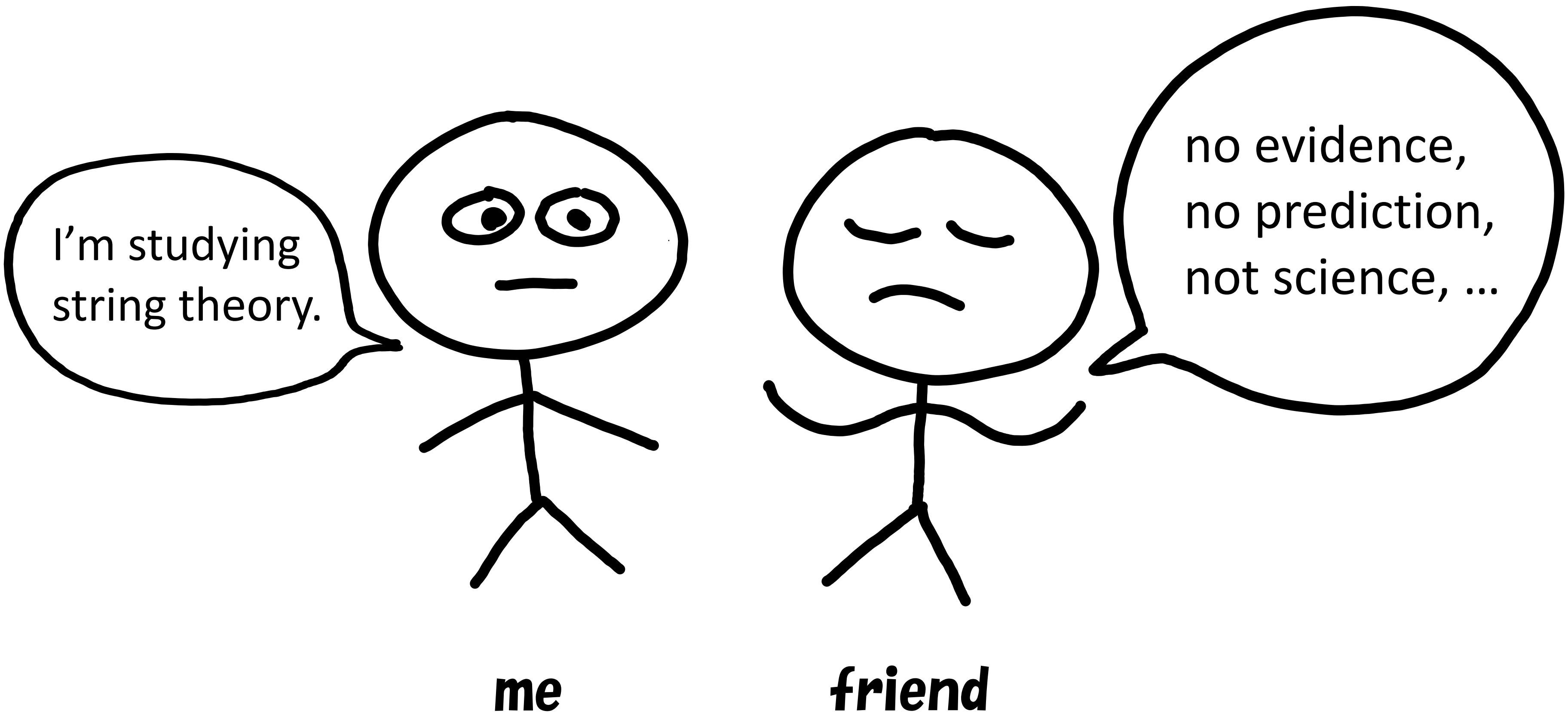
You start with the brane
and the brane is BPS
Then you go near the brane
and the space is AdS
Who knows what it means
I don't I confess
Ehhhh! Maldacena!

By Jeff Harvey (Parody of "Macarena")

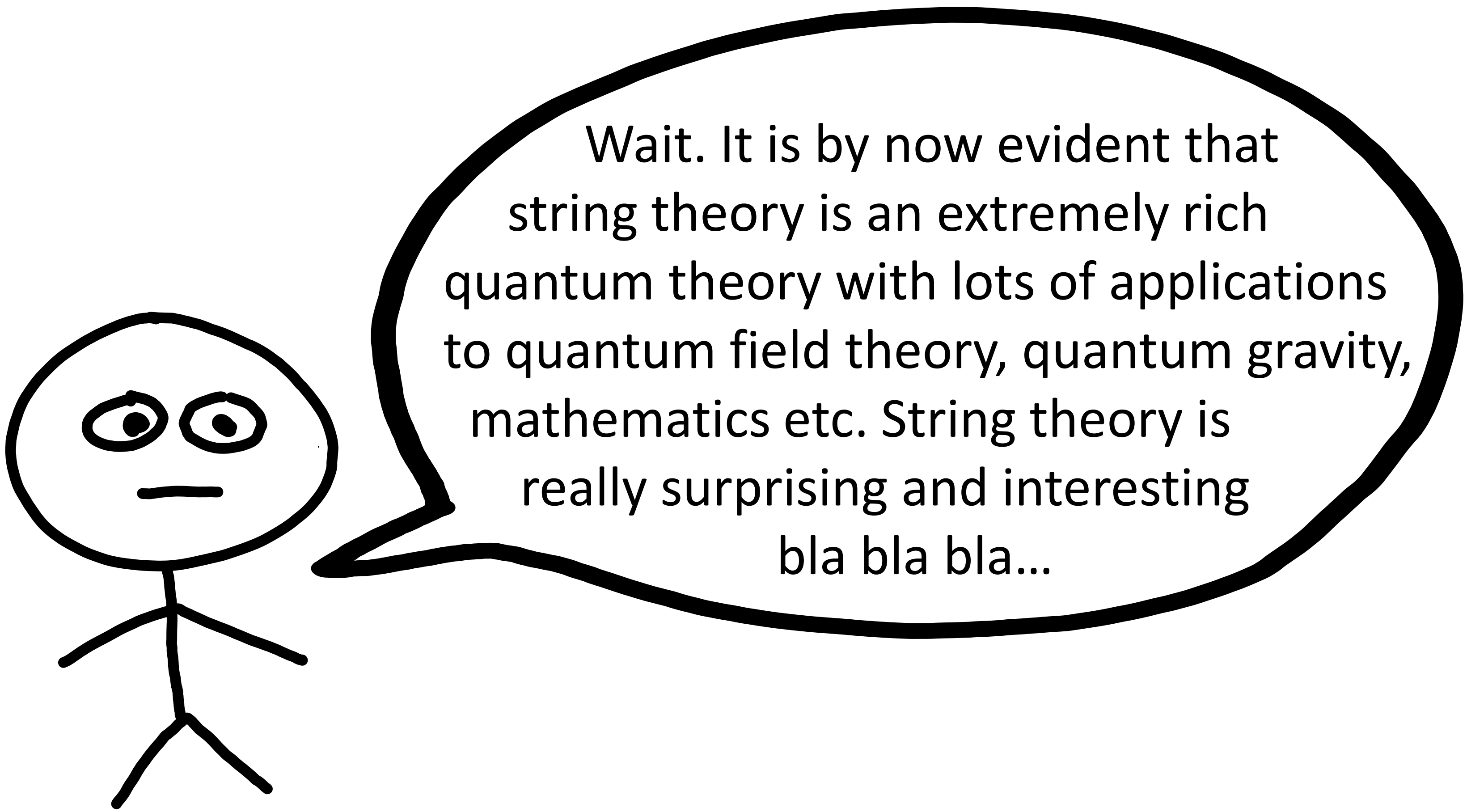
(Physicists dancing "Maldacena" at the conference "Strings '98")

- The paper of this discovery has been cited more than14,000 times in INSPIRE, which is (probably) the most cited paper in physics.

- **By the way, when I say "I'm studying string theory", some people criticize as**

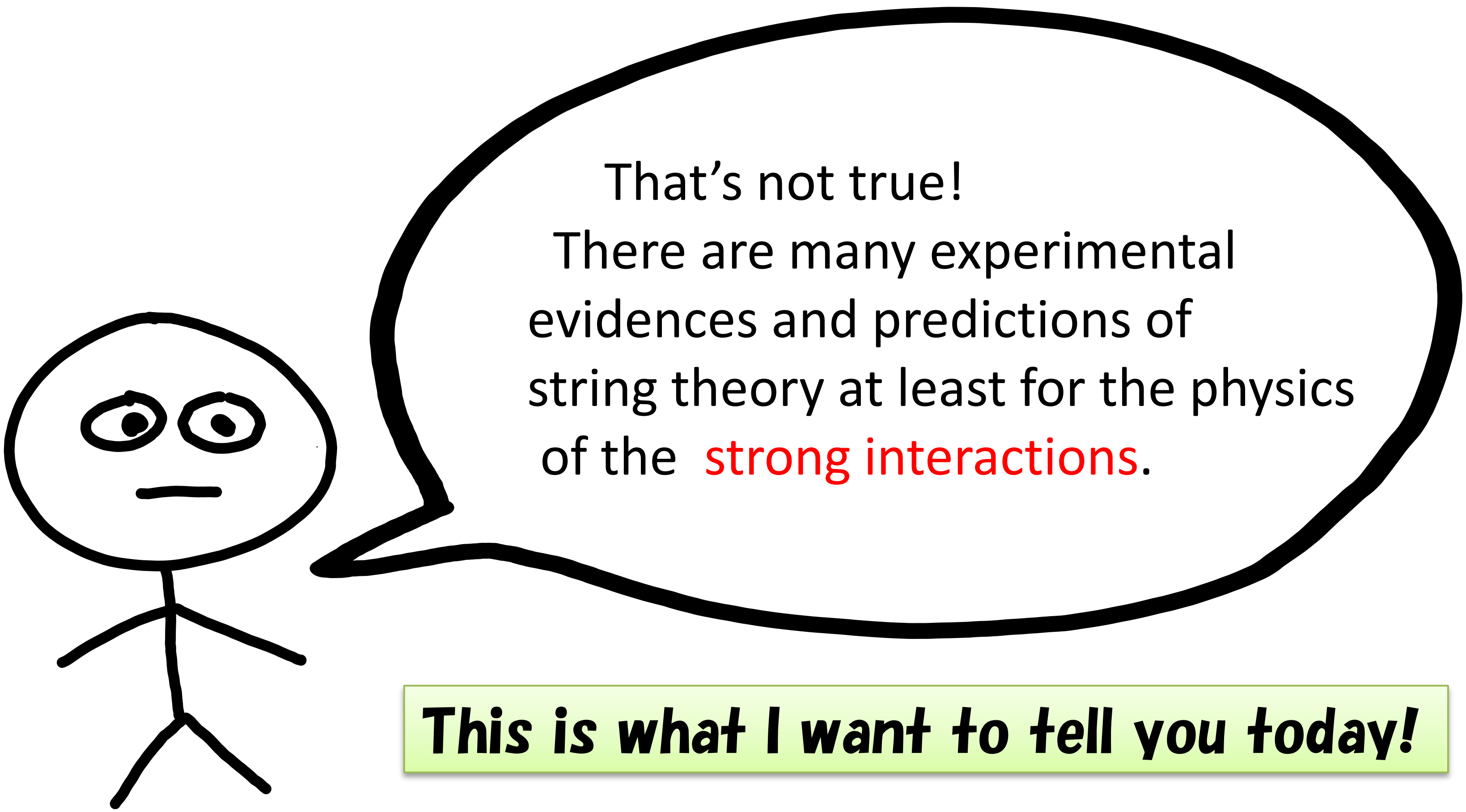


- **Until around 15 years ago, I used to say something like,**



me

- **Now, I can dispute the criticism in an easier way like**

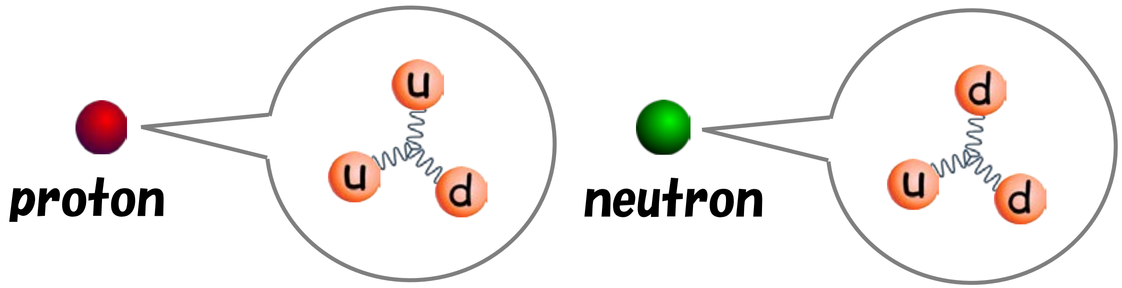


me

This is what I want to tell you today!

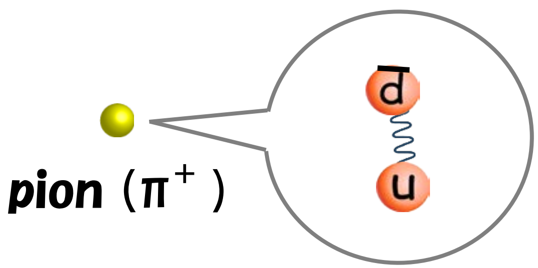
What is strong interaction?

- Protons and neutrons are made of 3 quarks



u : up quark
d : down quark

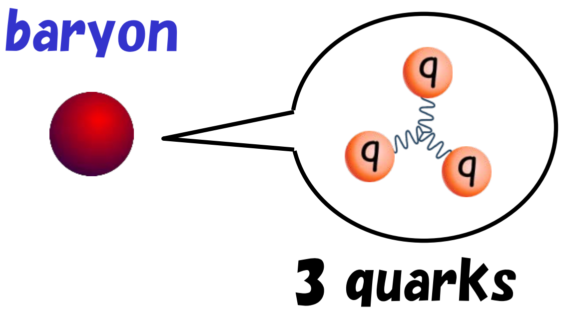
- There are particles composed of a quark-antiquark pair.

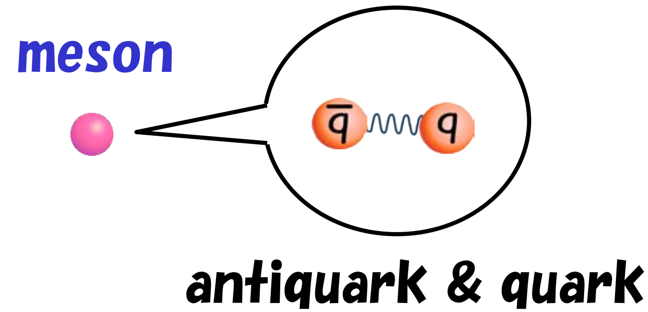


$\bar{d}$: anti-particle of down quark

- These quarks are tightly bound by the force called "strong force" or "strong interaction".

- In general, composite particles bound by the strong interactions are called "hadrons".





hadron
- baryon: p, n, Λ, Σ, Δ etc.
- meson: π, K, η, ρ, ω etc.

There are more than a few hundred kinds of hadrons found in the experiments.

- The fundamental theory of strong interaction was established in the 1970's. It is a gauge theory called

QCD

(Quantum Chromodynamics)

SU(Nc) gauge theory $(N_c = 3)$

A_μ ψ^i $i = 1, 2, \cdots, N_f$

g q (u d s $\cdots$)

(gluon) (quark)

← the particle that mediates strong interaction

This is a very simple and beautiful theory.
But it is notoriously difficult to analyze.

- Solving QCD is one of the most important problems in theoretical physics.

- Can we use "holography" to analyze QCD?

Gauge theory (theory of elementary particles) in a flat 4 dim spacetime	=	String theory in a curved 10 dim spacetime

- Solving QCD is one of the most important problems in theoretical physics.

- Can we use "holography" to analyze QCD?

QCD = **String theory in a curved 10 dim spacetime**

Next, I will explain:

- We can find a string theory description of a gauge theory that is equivalent to QCD at low energies.
- Although the approximations are not very good, we can calculate many quantities that can be measured by experiments.

Plan of Talk

√ 1 Introduction

2 D-brane

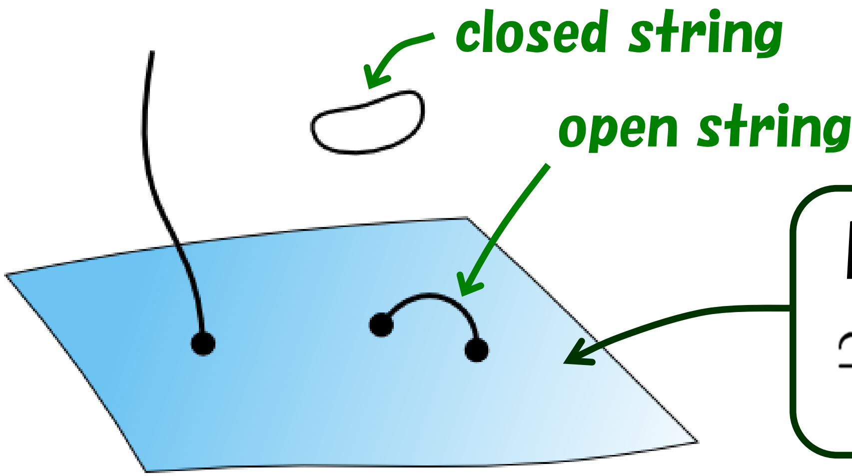
3 Holographic dual and QCD

4 Results

5 Discussion

2 D-brane

What is D-brane?

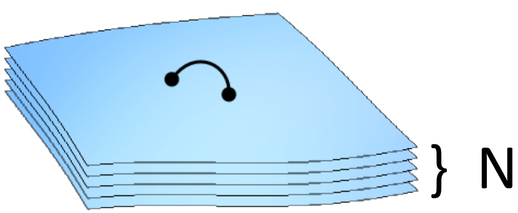


Dp-brane

$\simeq$ (p+1) dim object on which open strings can end.

D-branes played a key role in the 2nd string revolution

- **Open string contains gauge particles.**
  If we have N D-branes, U(N) gauge theory is realized.



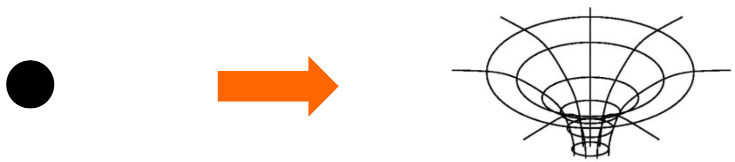
Dp-brane

→ (p+1) dim **U(N) gauge theory**

D-branes in Supergravity

(Low energy effective theory of superstring theory)

- ***General relativity: heavy object → curved spacetime***



Solution of Einstein eq.

Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

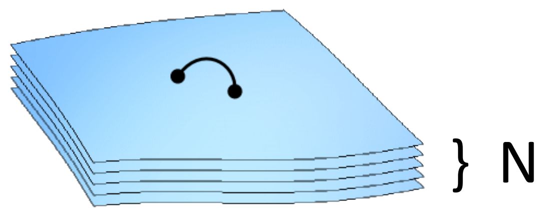
- ***Similarly, D-brane makes the spacetime curved.***

Solution of supergravity EOM

3 Holographic dual and QCD

Basic idea of holographic dual

- 2 ways to describe a D-brane system



U(N) gauge theory ⟷ Sting theory in a curved spacetime

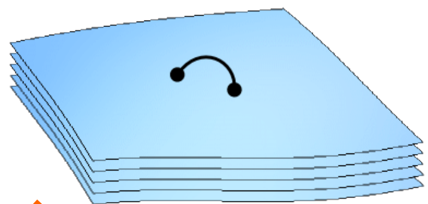
equivalent !

Because they describe the same object, there should be a correspondence.

The most famous example

[Maldacena]

- **Consider D3-branes**



D3-branes

$\mathcal{N} = 4$ Super Yang-Mills theory

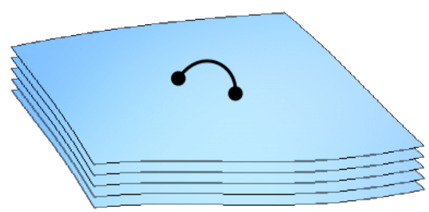
equivalent!

Type IIB superstring theory in $AdS_5 \times S^5$

gauge field × 1
Weyl fermion × 4
scalar field × 6

Yang-Mills theory (= Theory of gluons)

- **Consider N D4-branes wrapped on an S^1 and impose anti-periodic b.c. for the fermion fields.** [Witten]



D4-branes on S^1

Yang-Mills theory ⟷ **String theory in the D4-brane background**

equivalent!

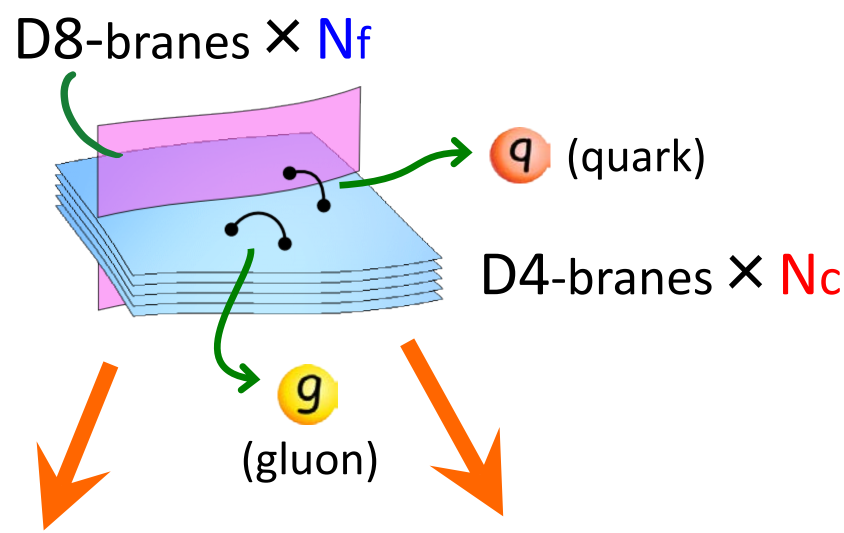
(**more precisely, Yang-Mills theory + massive fields**)

(**Wick rotated black D4-brane solution**)

Holographic QCD

[Sakai-S.S.]

- In order to add quarks, we add D8-branes to the previous system.



SU(N_c) QCD with N_f quarks

(more precisely, QCD+massive fields)

⟷ equivalent!

String theory in the D4-brane background with N_f D8-branes

SU(N_c) QCD
with N_f quarks

↔ equivalent!

String theory in the
D4-brane background
with N_f D8-branes

(more precisely,
QCD + massive fields)

mass M_{KK} (~ 1 GeV),
which corresponds to the cut-off scale

$1/N_c$ expansion ↔ perturbative expansion with respect to the coupling in string theory

$1/\lambda$ expansion ↔ derivative expansion

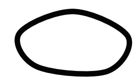
coupling constant ('t Hooft coupling)
at the M_{KK} scale

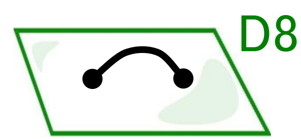
The leading order terms in the $1/N_c$ and $1/\lambda$ expansions
(good approximation at large N_c & strong coupling)
can be easily computed using string theory !

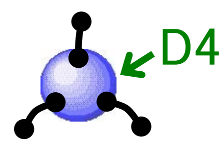
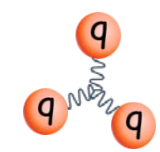
4 Results

Can we find hadrons?

Particles in the system

- closed string  ↔ glueball 

- open string on D8-brane  ↔ meson 

- D4-brane wrapped on S^4  ↔ baryon 

Note: the topology of the background is $\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$

↑ 4 dim sphere

Using this dictionary, we can calculate many quantities in low energy hadron physics!

Meson effective theory

Neglecting the modes that carry momenta in the S^4 directions, the effective theory on D8 becomes a 5 dim U(Nf) gauge theory

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$h(z) = (1+z^2)^{-1/3}$

$(M_{\text{KK}} = 1 \text{ unit})$

Only two parameters: κ and M_{KK}

★ 5 dim gauge theory → 4 dim meson theory

complete sets of functions of z

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z) \qquad A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z)$$

$(\mu = 0 \sim 3)$

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\Rightarrow \quad S_{5\text{dim}}(A) = S_{4\text{dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)$$

★ Surprisingly, this 4 dim meson theory reproduces many old phenomenological models

- Skyrme model ⇒ a model to describe baryons as solitions
- Vector meson dominance ⇒ a model to describe the coupling to photons
- Gell-Mann Sharp Wagner model ⇒ a model of ω meson
- Hidden local symmetry ⇒ a model of ρ meson

★ A rough estimate of the masses and couplings

mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

coupling	our model	experiment
f_π	[92.4 MeV]	92.4 MeV
L_1	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	4.81	5.99
g_ρ	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	4.63 GeV	$2.8 \sim 4.2$ GeV

input ([776], [1], [92.4 MeV])

(※ the numbers are a little old, but more or less the same)

It seems to work! (though not perfectly)

Other mesons

[Imoto-Sakai-S.S.]

Other mesons, including those with higher spins, are obtained from the excited open string modes.

- 1st excited states

→ $a_2(1320),\ b_1(1235),\ \pi(1300),\ a_0(1450),\ \cdots$

- 2nd excited states

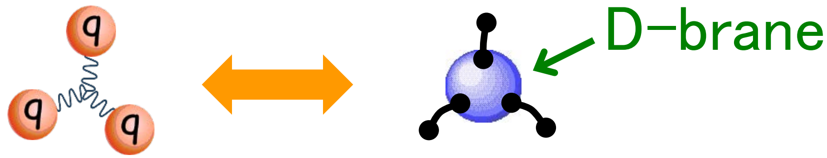
→ $\rho_3(1690),\ \pi_2(1670),\ \cdots$

- 3rd excited states

→ $a_4(2040),\ \cdots$

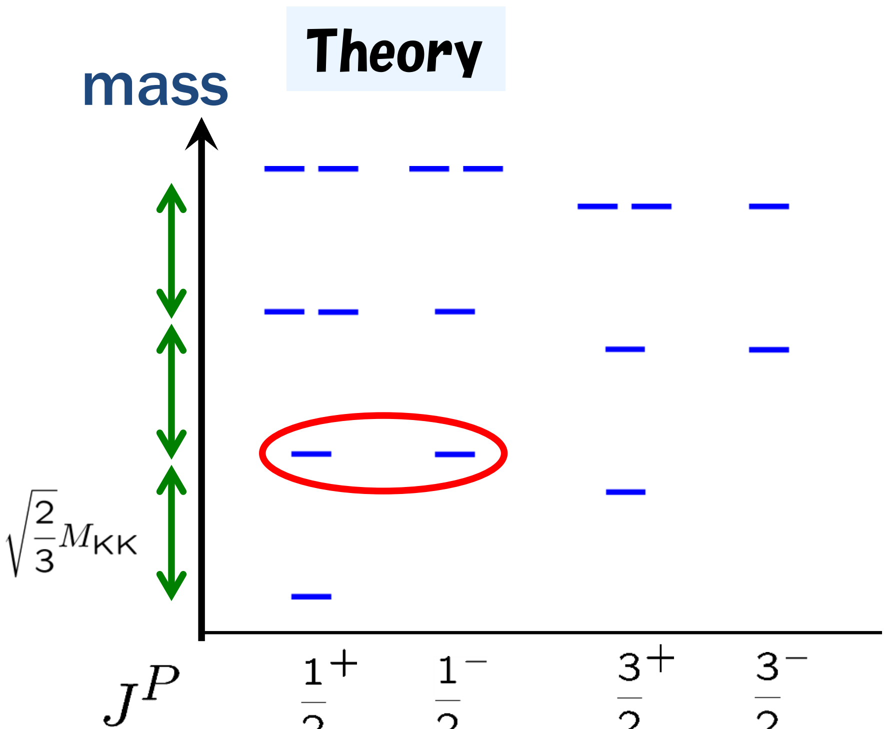
Baryon spectrum

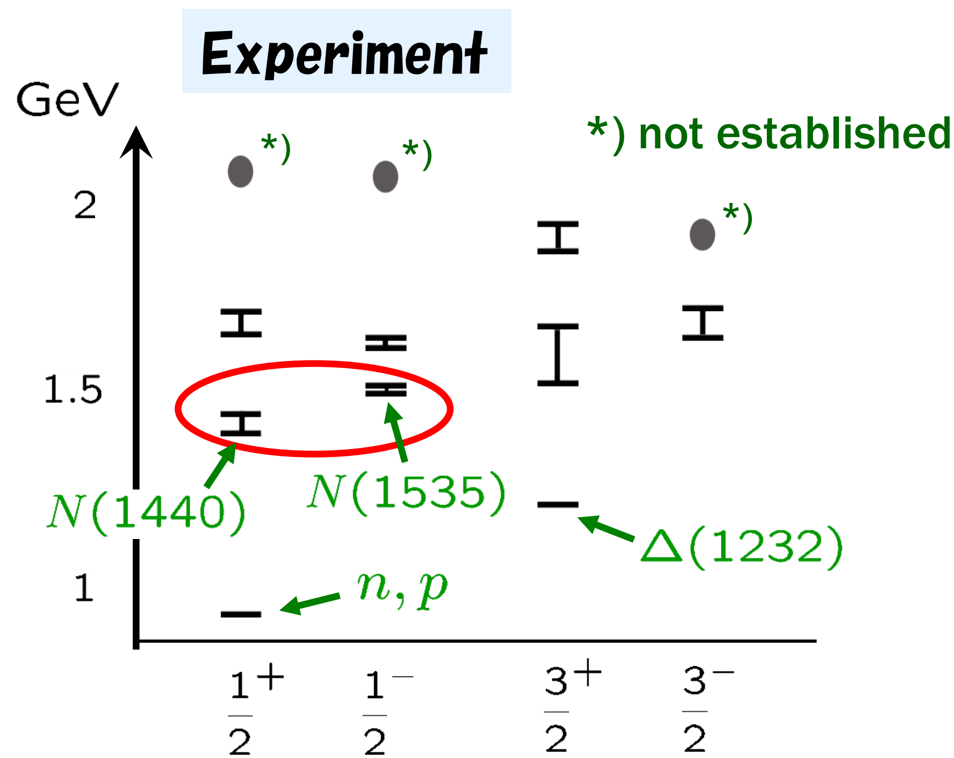
[Hata-Yamato-Sakai-S.S.]



= **A soliton in the 5 dim gauge theory**

Quantizing the solitons, we can analyze baryon spectrum:





Properties of nucleon

[Hashimoto-Sakai-S.S.]

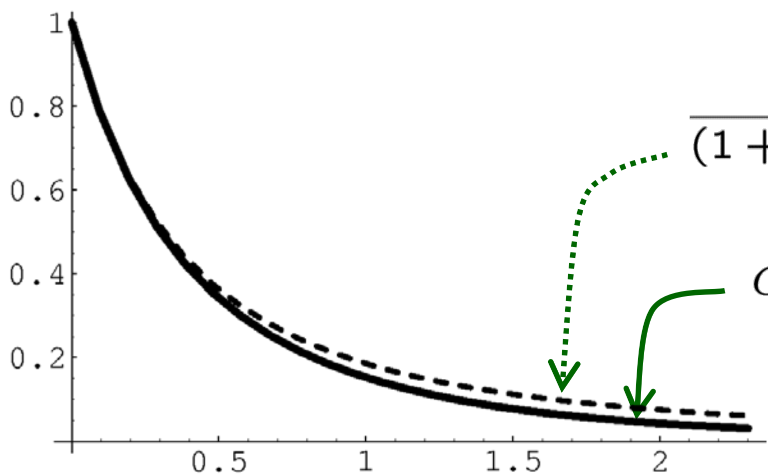
	our result	exp.
$\langle r^2\rangle^{1/2}_{I=0}$	0.74 fm	0.81 fm
$\langle r^2\rangle^{1/2}_{I=1}$	0.74 fm	0.94 fm
$\langle r^2\rangle^{1/2}_{A}$	0.54 fm	0.67 fm
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

[See also,
Hong-Rho-Yee-Yi, Hata-Murata-Yamato, Kim-Zahed]

Electromagnetic form factor

～ charge distribution of a proton

Form factor



(momentum)^2 Q^2 GeV^2

$$\frac{1}{(1+Q^2/\Lambda^2)^2}$$ dipole（≒ experiment）

$$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2}$$ our result

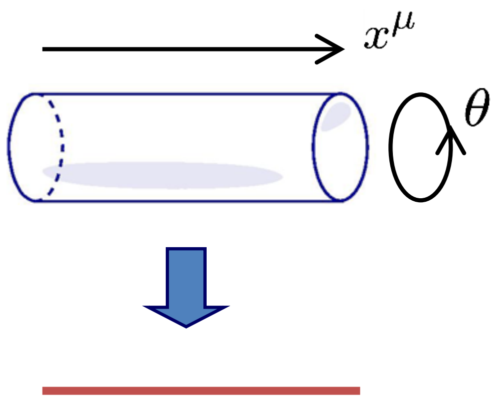
5 Discussion

What is “spacetime dimensions” ?

Why can 4 dim theory be equivalent to higher dim theory

easy example: S^1 compactification

5 dim



4 dim

$$\int d^4x\, d\theta \left(|\partial_\mu \varphi|^2 + |\partial_\theta \varphi|^2\right)$$

$$\varphi(x^\mu, \theta) = \sum_n \varphi^{(n)}(x^\mu)\, e^{in\theta}$$

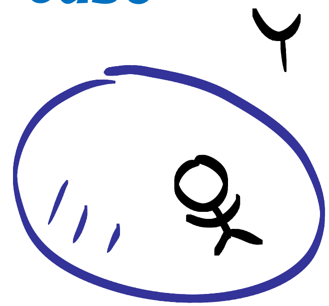
$$\int d^4x \sum_n \left(|\partial_\mu \varphi^{(n)}|^2 + n^2 |\varphi^{(n)}|^2\right)$$

lesson: 4 dim theory with ∞ particles = higher dim theory

It is known that SU(Nc) QCD with Nc → ∞ contains ∞ tower of hadrons. It is not too surprising that it can be described by a higher dim theory.

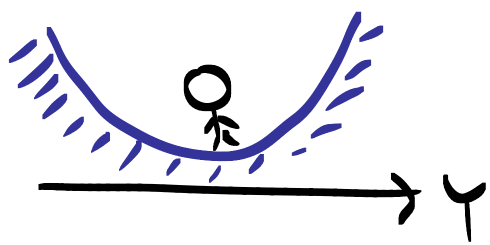
Can we see the extra-dimensions?

- compact case



One can guess the shape of Y by observing Kaluza-Klein modes

- holographic QCD



One can try to see the extra-dim through excited mesons.

- You may think we should be able to see the extra-dim more clearly by putting much higher energy than QCD scale.

  However, in that case, the string theory description becomes highly curved and difficult to handle.

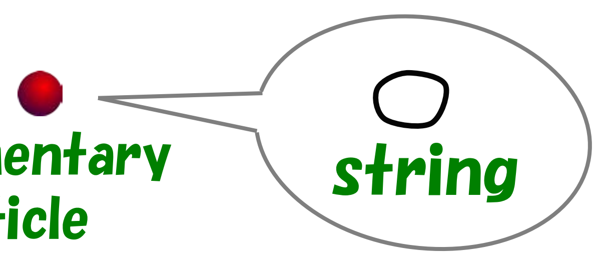
Which one is more fundamental? Elementary particle or string?

Gauge theory (theory of elementary **particles**) in a flat **4 dim** spacetime = **String** theory in a curved **10 dim** spacetime

- If they are completely equivalent, one cannot tell which one is more fundamental.
- In holographic QCD, we observed

Composite particle ⟷ string

This is different from the conventional wisdom:



elementary particle

Summary and outlook

- Holographic dual is very interesting.
- It is fun to play with hadrons using string theory.
- We should be able to improve the accuracy.
  This is one of the important future problems.
- In particular, we want to send the cut-off scale
  to infinity to remove all the artifacts.
  How can we do this?
- Understanding holographic dual better may lead us
  to new understanding of particle physics.

– Thank you ! –