Barrier from Chaos: operator entanglement dynamics from reduced density matrices

arXiv: 1907.09581, H.Wang and T. Zhou

中国科技大学交叉学科理论中心

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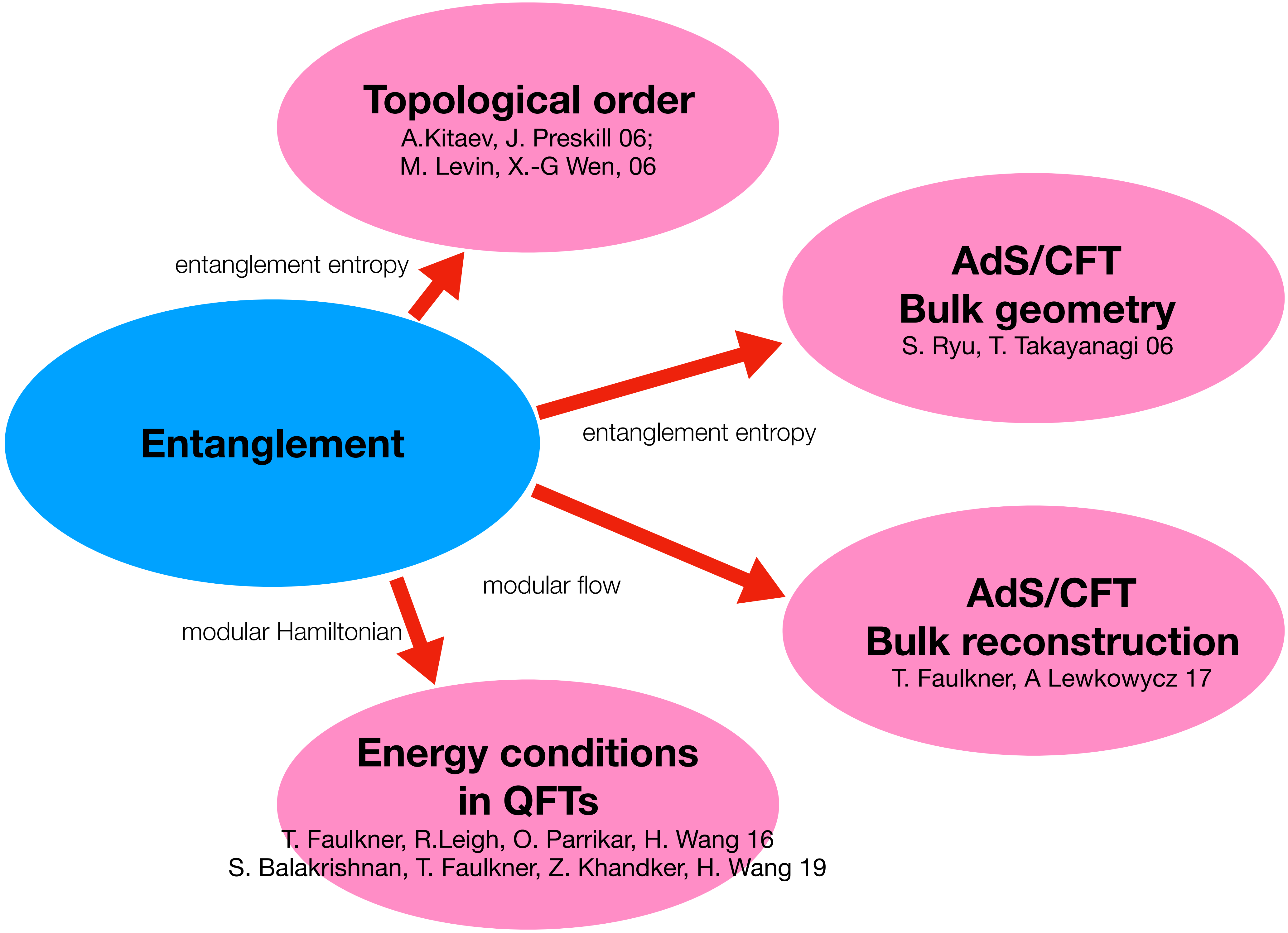
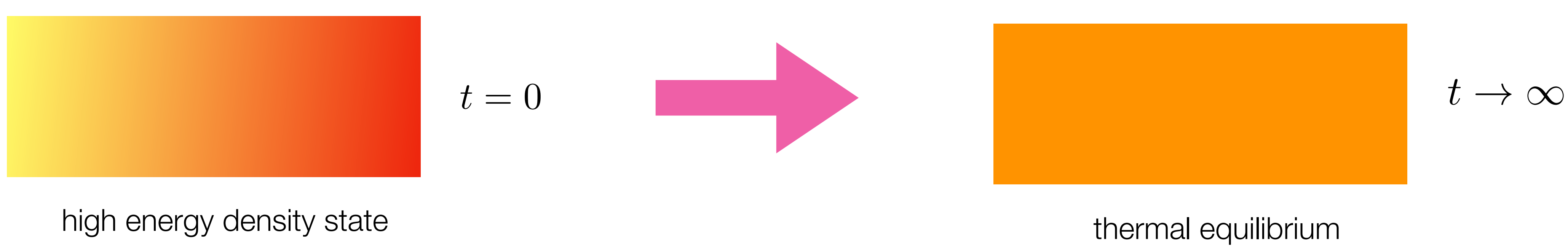
Motivation



Topological order
A.Kitaev, J. Preskill 06;
M. Levin, X.-G Wen, 06
entanglement entropy
AdS/CFT
Bulk geometry
S. Ryu, T. Takayanagi 06
Entanglement
entanglement entropy
modular flow
modular Hamiltonian
AdS/CFT
Bulk reconstruction
T. Faulkner, A Lewkowycz 17
Energy conditions
in QFTs
T. Faulkner, R.Leigh, O. Parrikar, H. Wang 16
S. Balakrishnan, T. Faulkner, Z. Khandker, H. Wang 19

Motivation

- Entanglement entropy in dynamical settings
- e.g. thermalization after quench (global or local)
- Universal behavior: growth —> saturation
- In AdS/CFT — entanglement “tsunami” H. Liu et al (2013)
- For thermo-field-double (TFD) state, probes BH interior T. Hartman et al (2013)



Motivation

- In these analysis, the entanglement is defined w.r.t. *states*
- One can extend definition of entanglement to *operators*
- Construct the corresponding *operator state*
- Entanglement between subsystems in the operator state
- Examples: unitary evolution operator, etc
- We are interested in the *dynamics* of *operator entanglement*

Outline:

- Set up: (reduced density) operator entanglement
- Quenched dynamics for operator entanglement
    1. Two-dimensional rational conformal field theories (CFTs)
    2. Random unitary circuits
    3. Holographic CFTs (AdS/CFT)
- Discussion: a new measure of quantum chaoticity?
- Future directions

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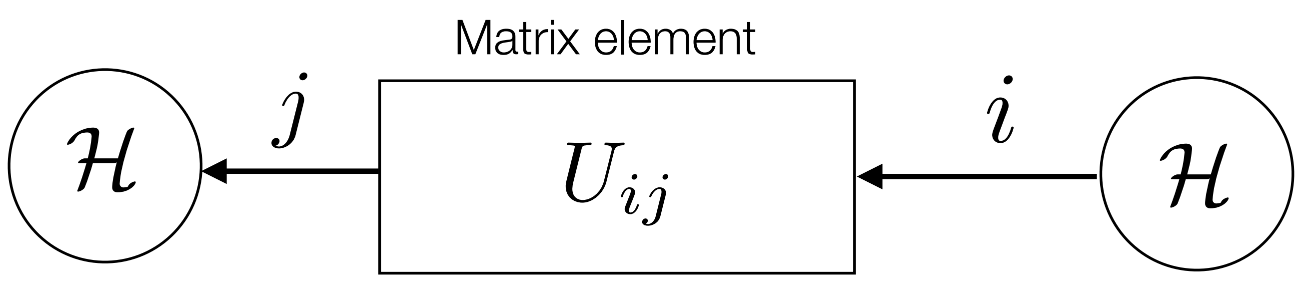
Set up:

Operator $U : \mathcal{H} \to \mathcal{H}$ Matrix element: $U = \sum_{ij} U_{ij} |i\rangle\langle j|$



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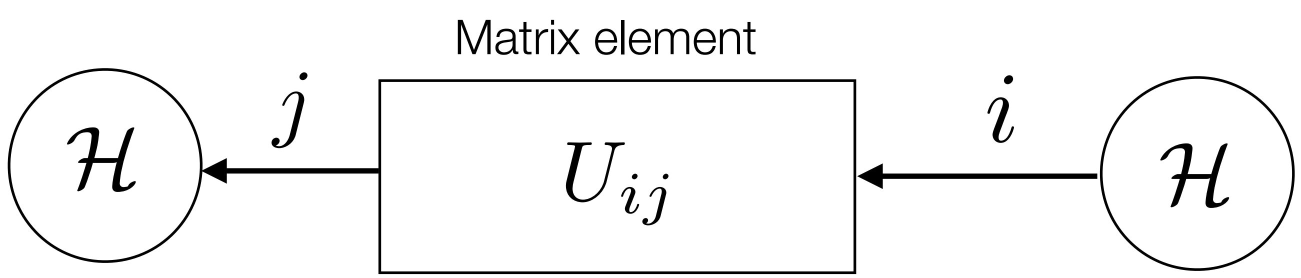
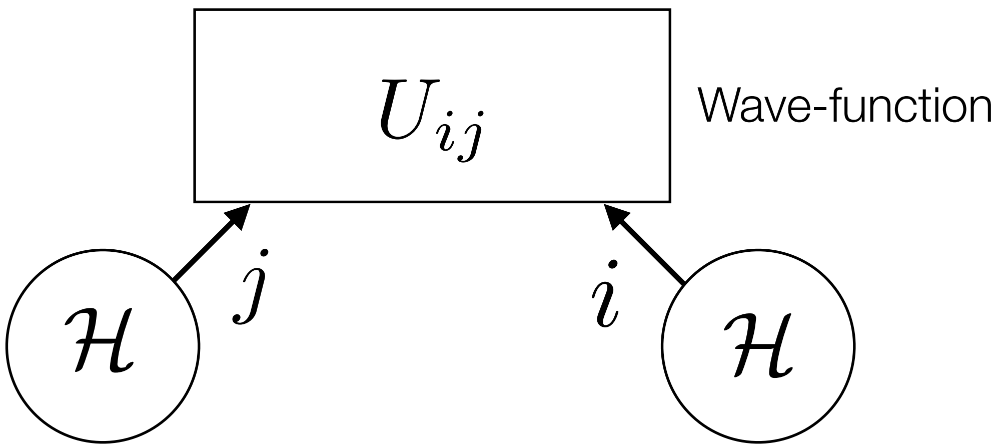


Can be viewed as a state defined on "doubled" Hilbert space $\mathcal{H} \otimes \mathcal{H}$



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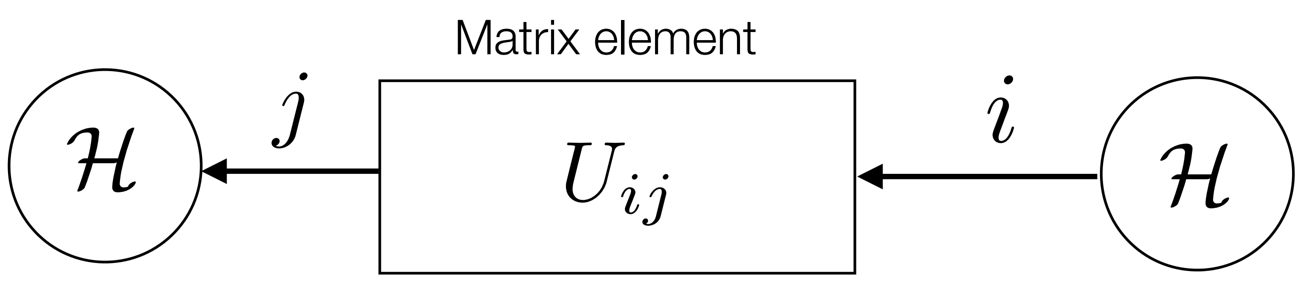

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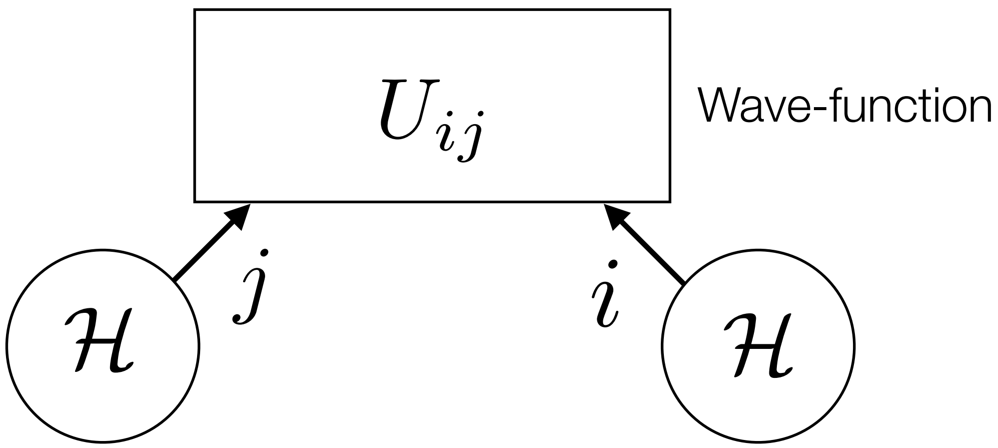
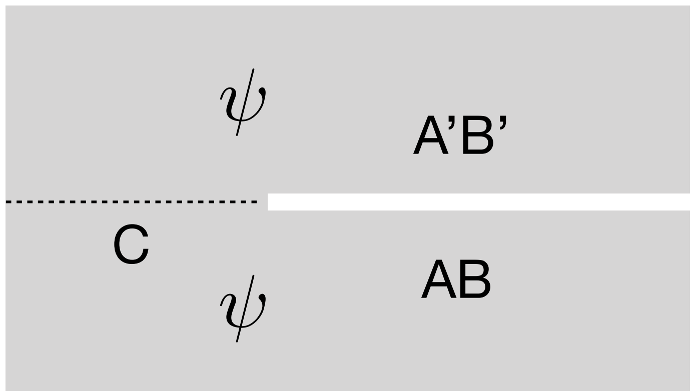


Operator state: $|U\rangle = \frac{1}{\mathrm{tr}\,(UU^\dagger)} \sum_{ij} U_{ij} |i\rangle \otimes |j\rangle$

Set up:

Specifically, take U to be the reduced density matrix

$$U \to \rho_{AB}(t) = \mathrm{tr}_C |\psi(t)\rangle\langle\psi(t)| \qquad : \mathcal{H}_{AB} \to \mathcal{H}_{AB}$$



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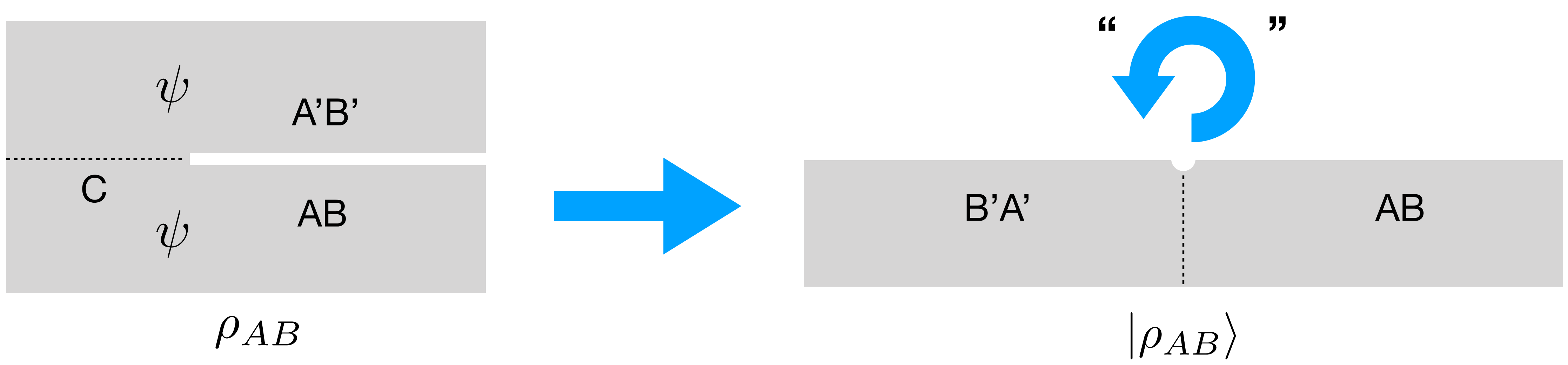
Reduced density operator state: $|\rho_{AB}(t)\rangle \in \mathcal{H}_{AB} \otimes \mathcal{H}_{AB}$

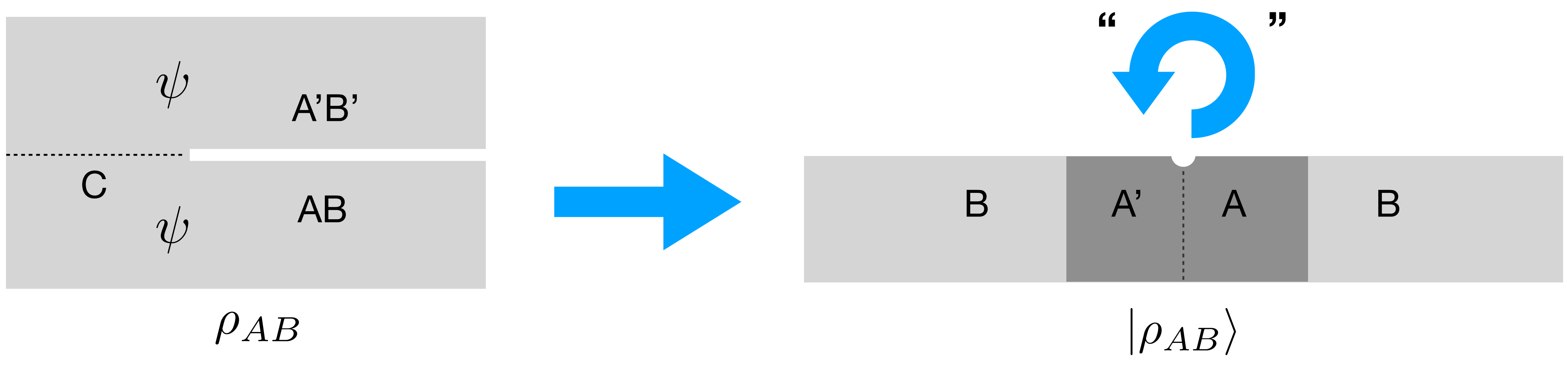
$$|\rho_{AB}(t)\rangle \propto \sum_{ij} (\rho_{AB}(t))_{ij} \, |i\rangle \otimes |j\rangle \qquad \langle\rho_{AB}(t)|\psi_{AB}(t)\rangle = \mathrm{tr}\left(\rho^2_{AB}(t)\right)$$



Set up:

Operator entanglement (renyi entropy) of subsystem $\mathcal{H}_A \otimes \mathcal{H}_A$

$$S_n^{\mathrm{op}}\left(A, \rho_{AB}(t)\right) = \frac{1}{1-n} \log \mathrm{tr}_{\mathcal{H}_A \otimes \mathcal{H}_A} \left[\mathrm{tr}_{\mathcal{H}_B \otimes \mathcal{H}_B} \left(|\rho_{AB}(t)\rangle\langle\rho_{AB}(t)| / \|\rho_{AB}(t)\|^2 \right) \right]^n$$



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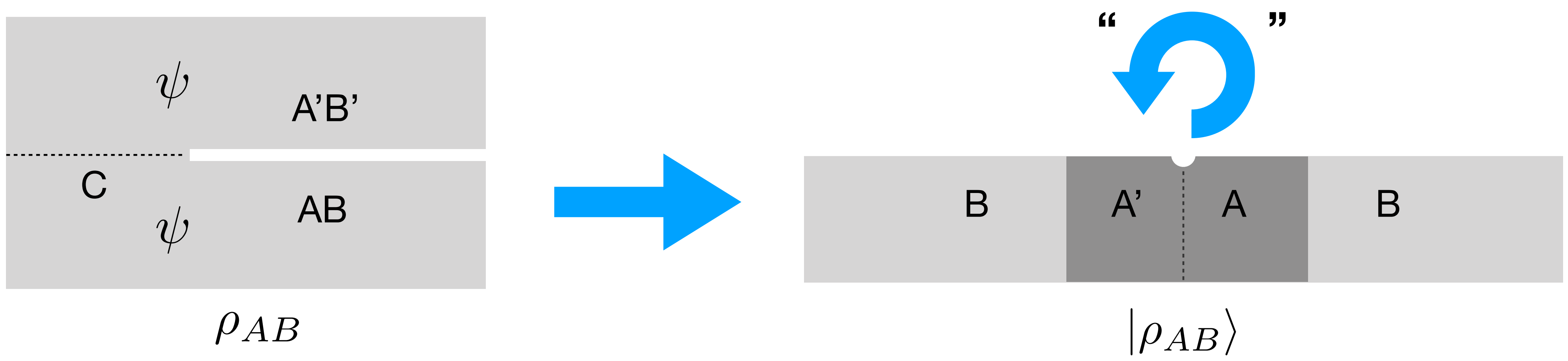
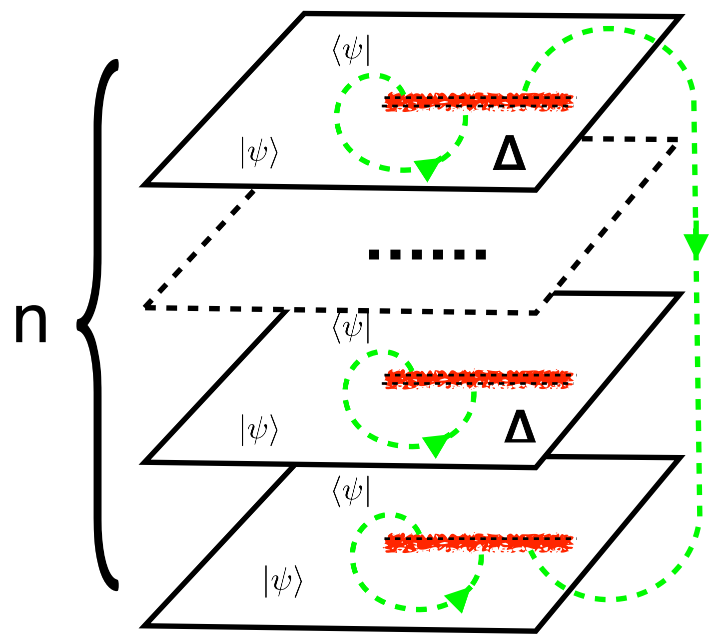


Replica trick (quick recap)

Renyi entropy: $$S_n(A,\psi) = \frac{1}{1-n}\log \mathrm{tr}_{H_A}\left[\mathrm{tr}_{H_{\bar{A}}}|\psi\rangle\langle\psi|\right]^n$$



$$= \mathrm{Tr}\left(\rho_A^\psi\right)^n$$

- Partition function on a branched manifold (replica)
- Can be generalized to computing operator entanglement
- More complicated replica structure

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- Conclude/outlook

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- What happens in the middle? Can the whole process be captured by MPS?
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- Examine this question in 3 classes of models, varying in chaoticity

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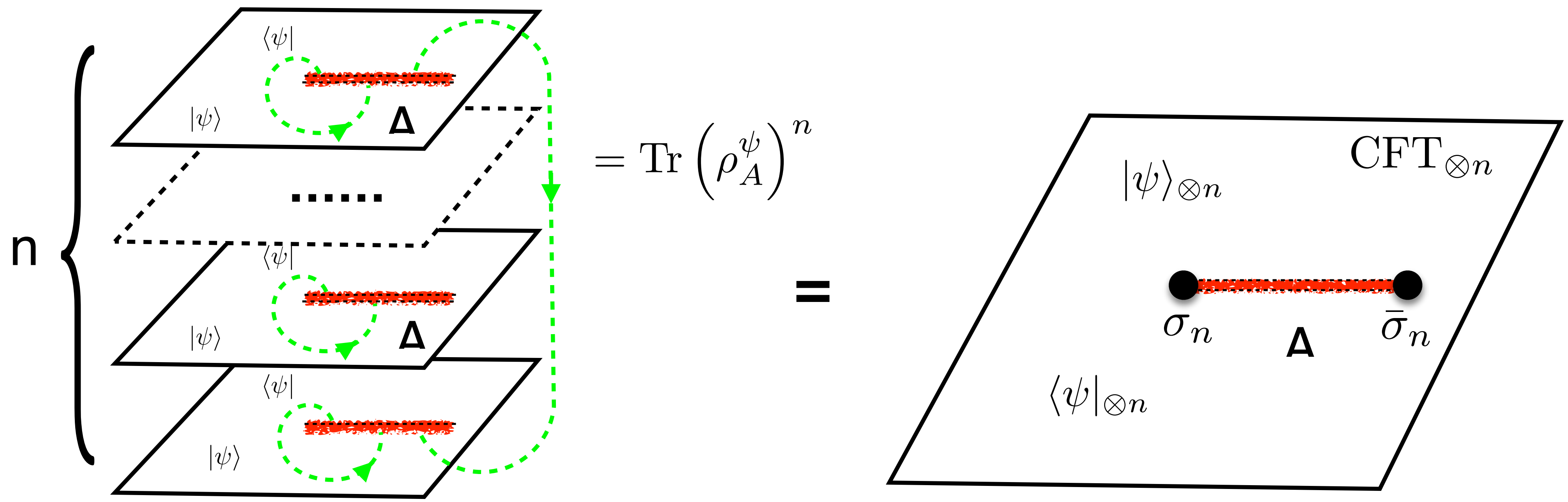
2-dimensional rational CFTs

State Renyi entropy:

- Branched-manifold <—> orbifold CFT: $\mathrm{CFT}_n \equiv \mathrm{CFT}^{\otimes n}/\mathbb{Z}_n$



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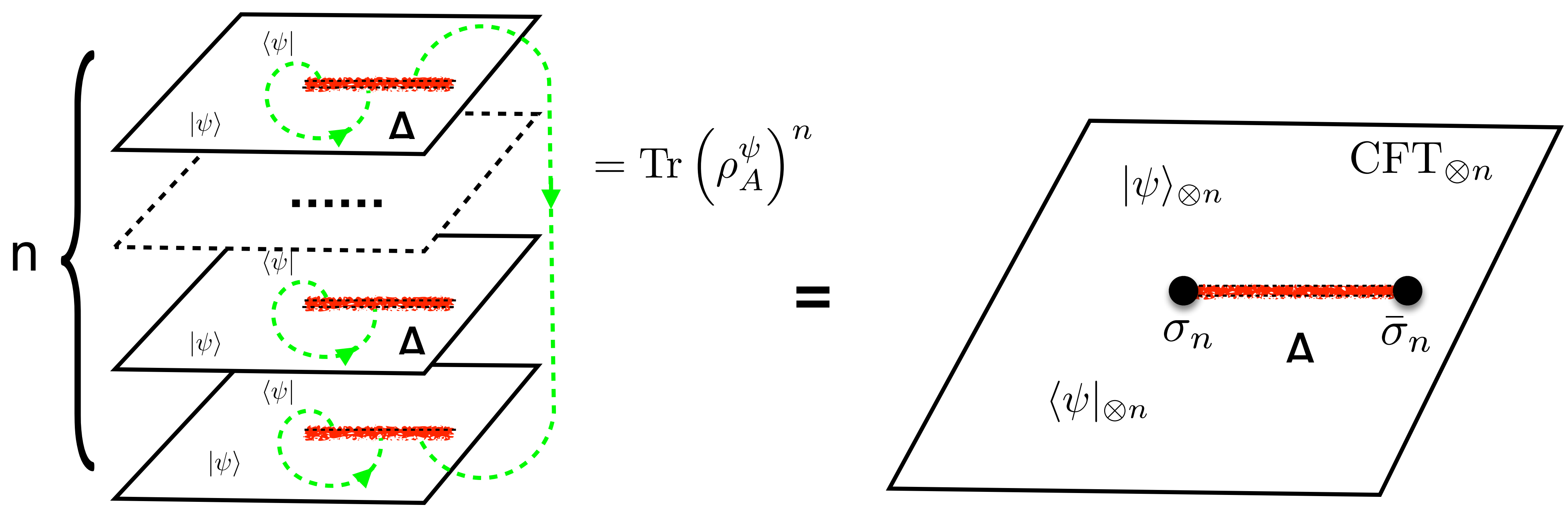
$$S_n(A,\psi) = \langle \psi^{\otimes n} | \sigma_n \bar{\sigma}_n | \psi^{\otimes n} \rangle_{\mathrm{CFT}_n}$$



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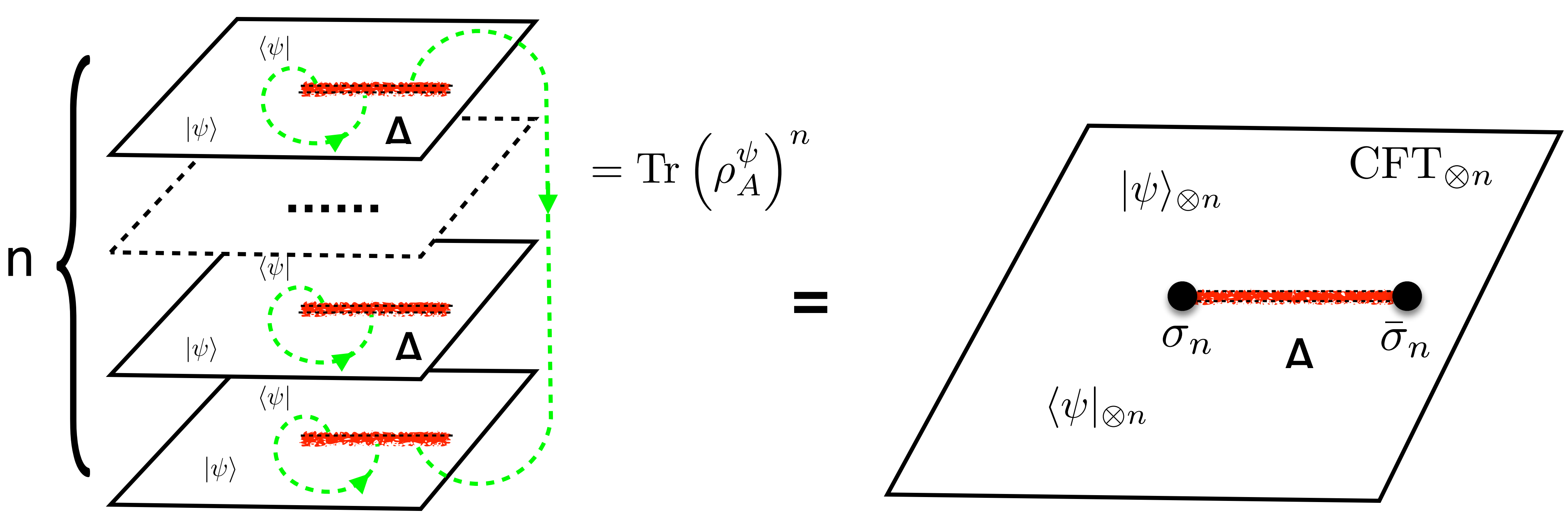
- Definite conformal dimensions $h_n = \frac{nc}{24}\left(1 - \frac{1}{n^2}\right)$



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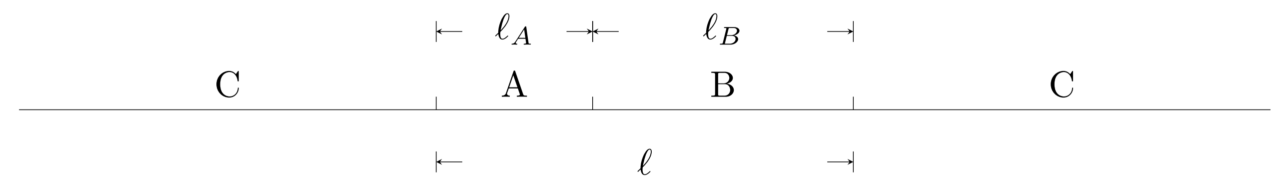

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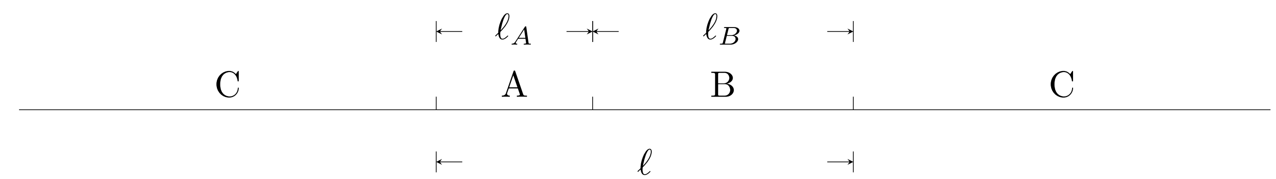

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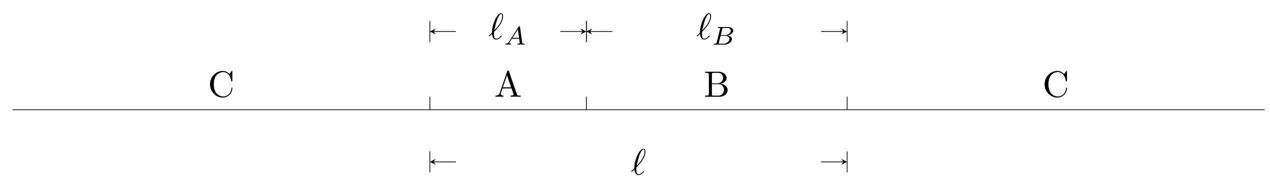

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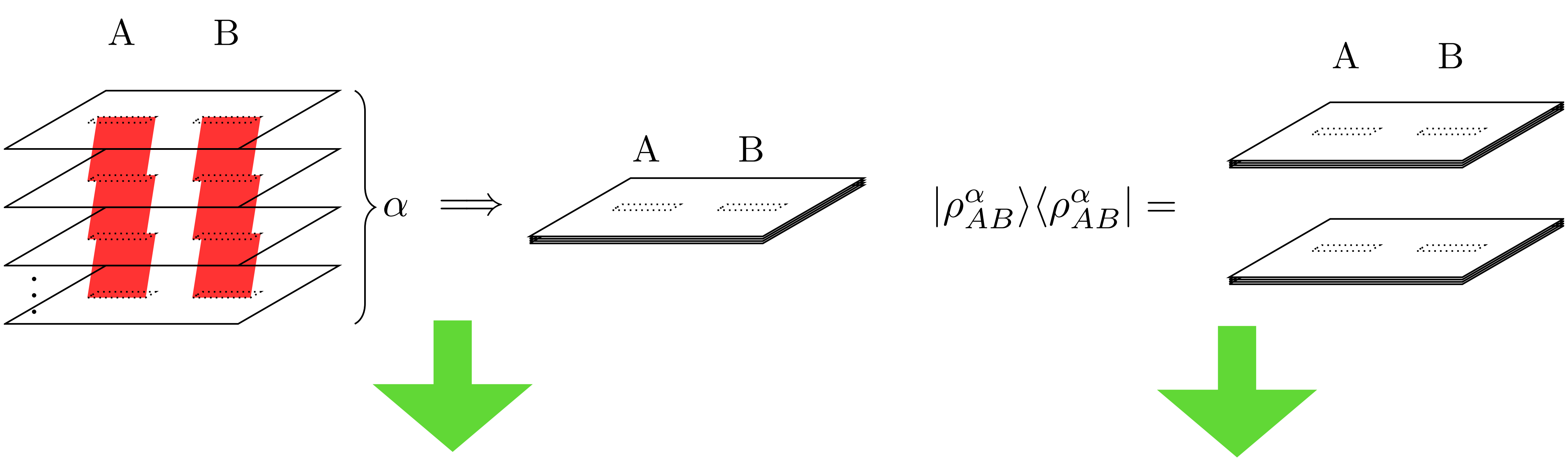

Operator state $|\rho^{\alpha}_{AB}\rangle$

Operator state “density matrix”

2-dimensional rational CFTs

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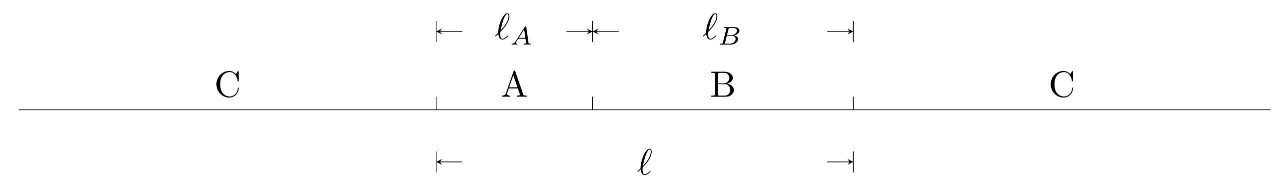

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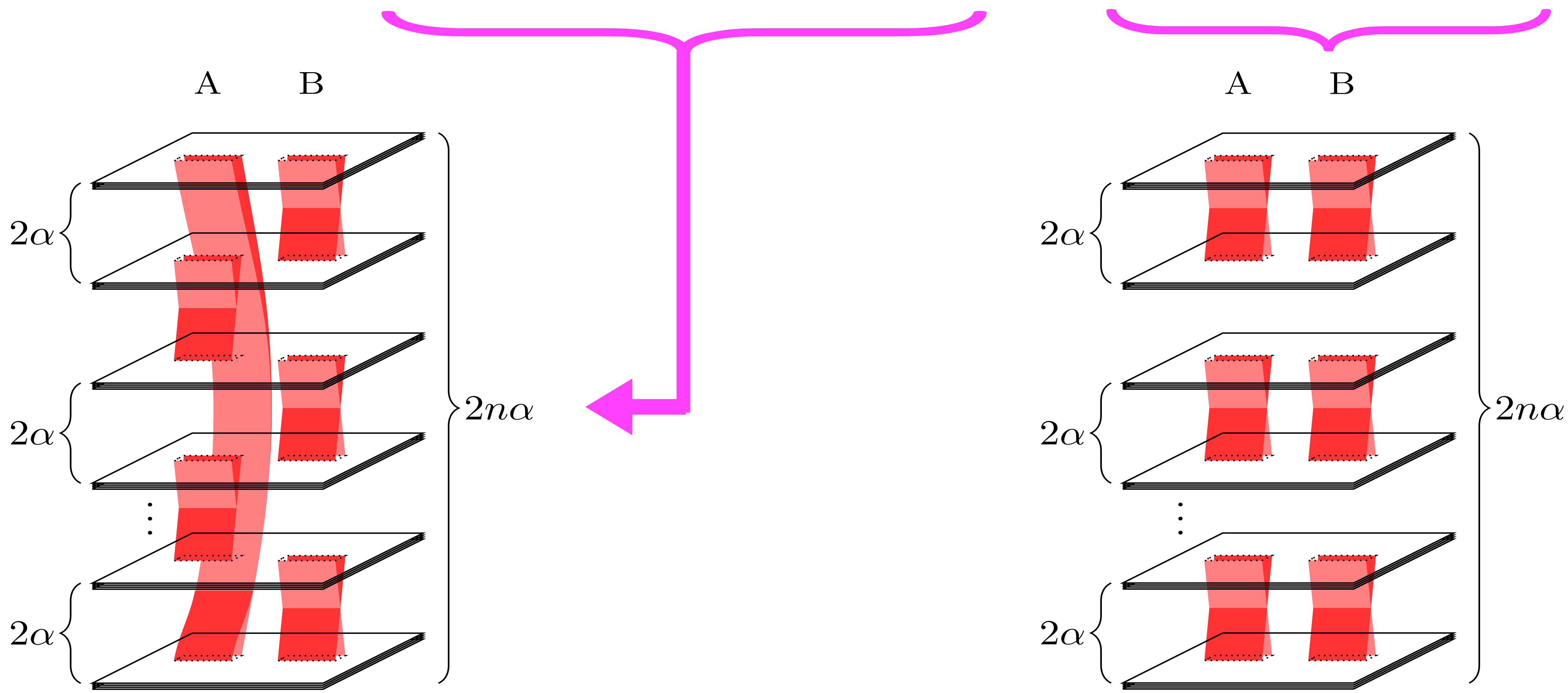

2-dimensional rational CFTs

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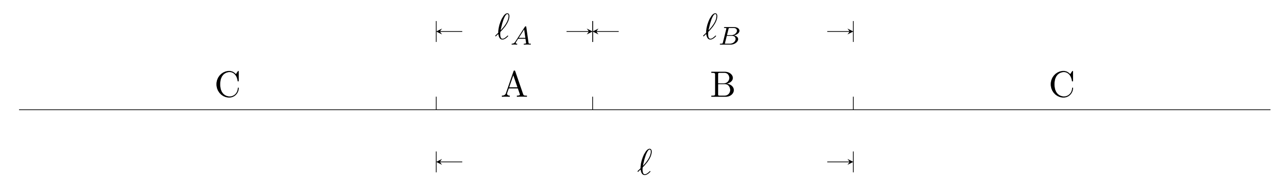

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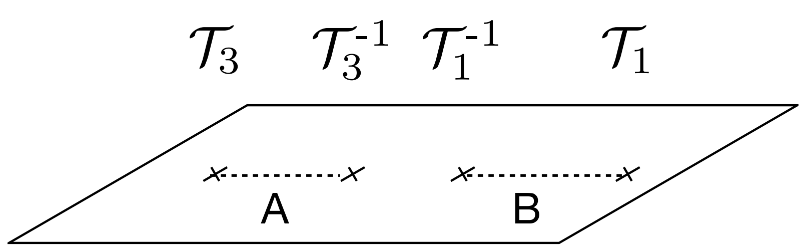
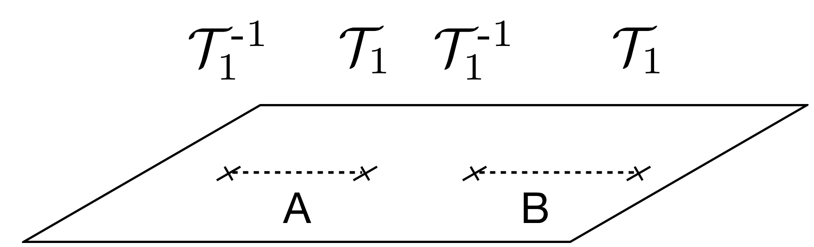
- Replica trick on more complicated branched manifold —> $\mathrm{CFT}_{2n\alpha}$



1st term



2nd term

2-dimensional rational CFTs

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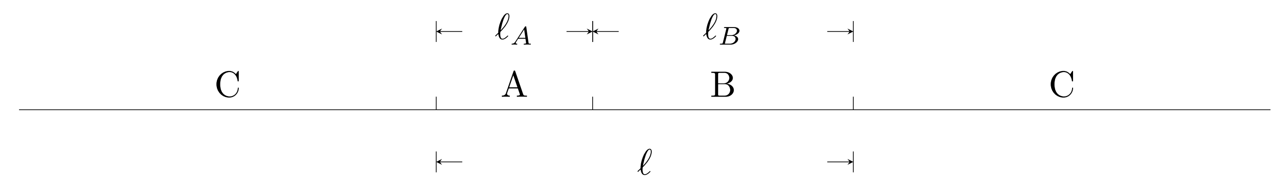

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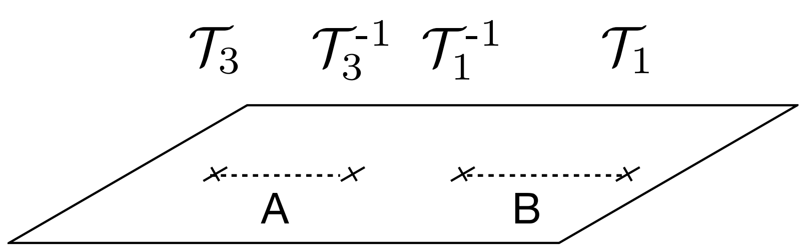

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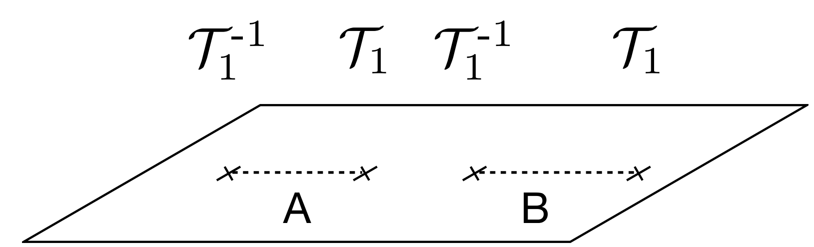


2nd term

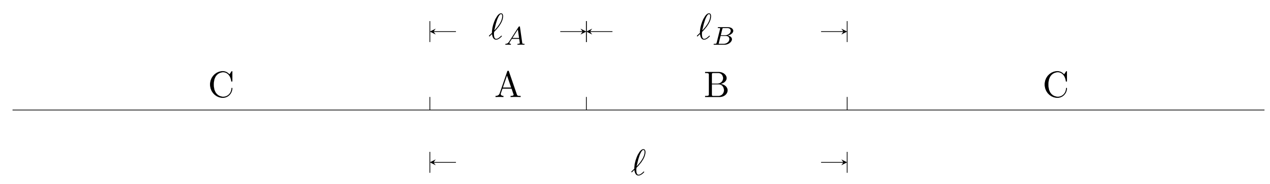
- Different twist operators:

$$\mathcal{T}_3^{-1} = (\alpha+1, ..., 3\alpha)(3\alpha+1, ..., 5\alpha)...\left((2n-1)\alpha+1, ..., 2n\alpha, 1, ..., \alpha\right)$$

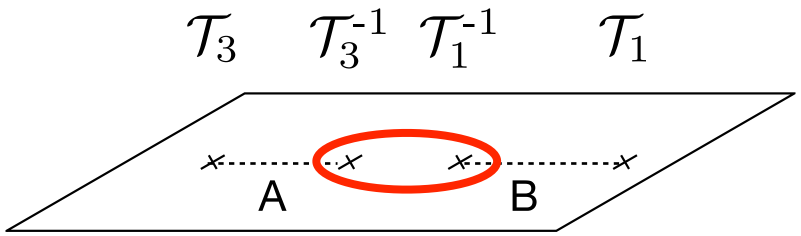
$$\mathcal{T}_1 = (123...2\alpha)(2\alpha+1...4\alpha)...\left((2n-2)\alpha+1, ..., 2n\alpha\right)$$

2-dimensional rational CFTs

Operator Renyi entropy:



- In the limit of adjacent A and B, use the derived OPE:



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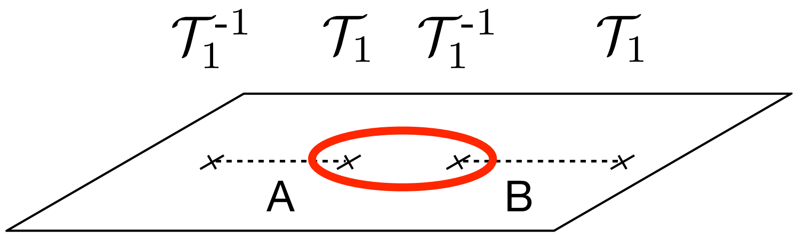

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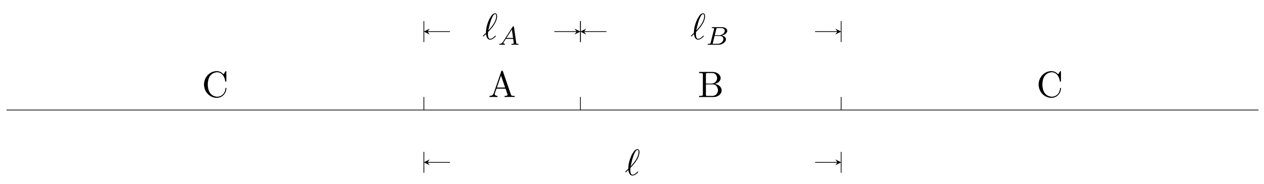

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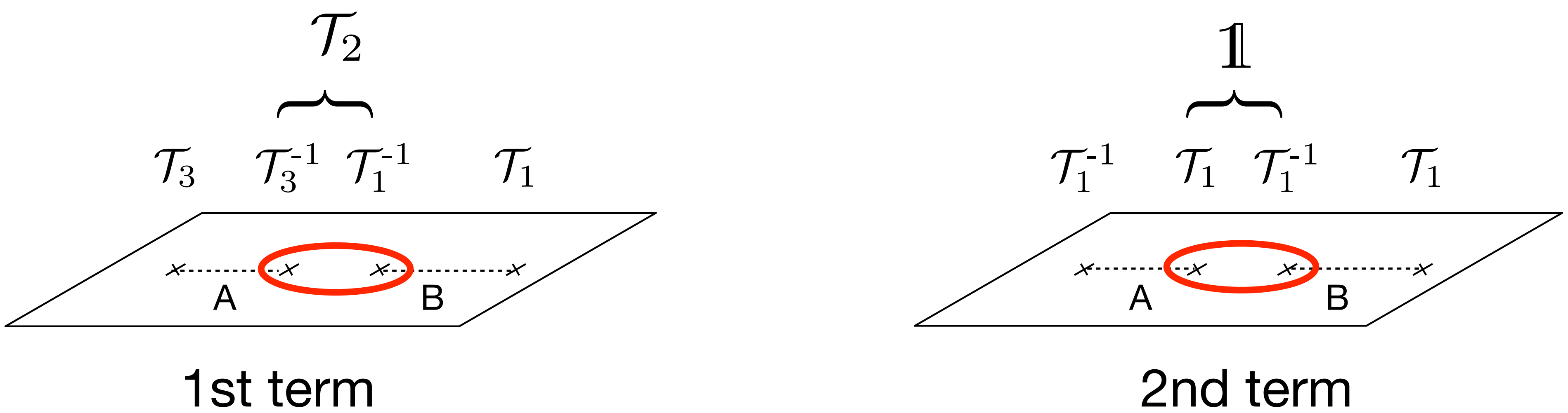
$$\mathcal{T}_3^{-1}\mathcal{T}_1^{-1} \to \mathcal{T}_2, \quad \mathcal{T}_1\mathcal{T}_1^{-1} \to \mathbb{1}$$

$$\mathcal{T}_2 = (\alpha+1, 3\alpha+1, ..., (2n-1)\alpha+1)^{-1}(1, 2\alpha+1, ..., (2n-2)\alpha+1)$$



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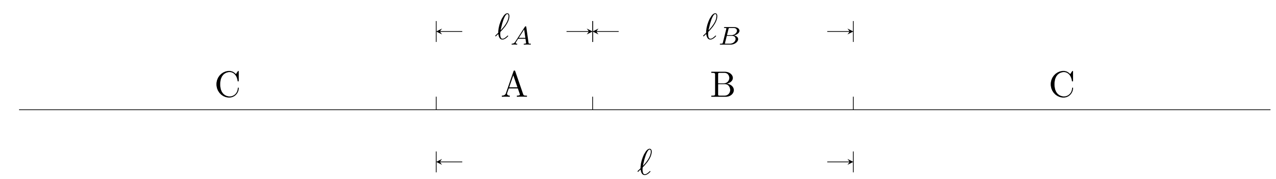

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- Correlation function of different twist operators:

$$S_n^{\text{op}} = \frac{1}{1-n}\ln\left[\frac{\langle\mathcal{T}_1(\ell_B)\mathcal{T}_2(0)\mathcal{T}_3(-\ell_A)\rangle_{\psi(t)}}{\langle\mathcal{T}_1(\ell_B)\mathcal{T}_1^{-1}(-\ell_A)\rangle_{\psi(t)}}\right]$$

$$h_1 = h_3 = \frac{n\alpha c}{12}\left(1-\frac{1}{4\alpha^2}\right)$$

$$h_2 = \frac{nc}{12}\left(1-\frac{1}{n^2}\right)$$

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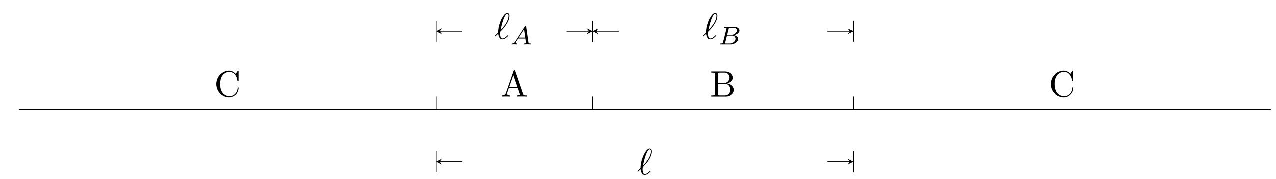


- In the limit of adjacent A and B, use the derived OPE:

$$\mathcal{T}_3^{-1}\mathcal{T}_1^{-1} \to \mathcal{T}_2, \quad \mathcal{T}_1\mathcal{T}_1^{-1} \to \mathbb{1}$$

$$\mathcal{T}_2 = (\alpha+1, 3\alpha+1, ..., (2n-1)\alpha+1)^{-1}\,(1, 2\alpha+1, ..., (2n-2)\alpha+1)$$

- Correlation function of different twist operators:

$$S_n^{\mathrm{op}} = \frac{1}{1-n}\ln\left[\frac{\langle \mathcal{T}_1(\ell_B)\mathcal{T}_2(0)\mathcal{T}_3(-\ell_A)\rangle_{\psi(t)}}{\langle \mathcal{T}_1(\ell_B)\mathcal{T}_1^{-1}(-\ell_A)\rangle_{\psi(t)}}\right]$$

$$h_1 = h_3 = \frac{n\alpha c}{12}\left(1 - \frac{1}{4\alpha^2}\right)$$

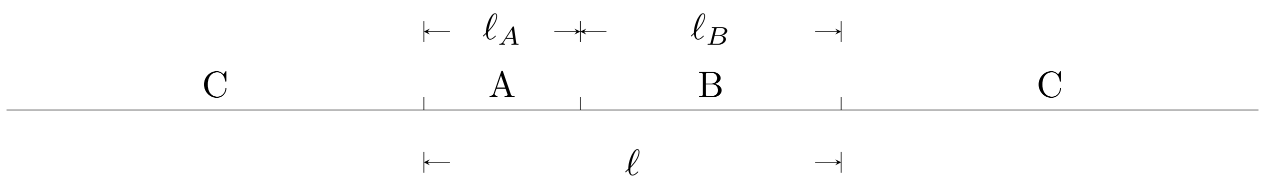
$$h_2 = \frac{nc}{12}\left(1 - \frac{1}{n^2}\right)$$

- To set up quenched initial state, use conformal boundary state:

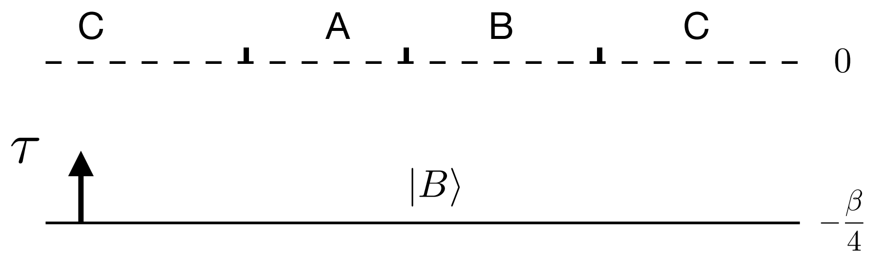
$$|\psi(t=0)\rangle = e^{-\frac{\beta}{4}H}|B\rangle$$

2-dimensional rational CFTs

Operator Renyi entropy:

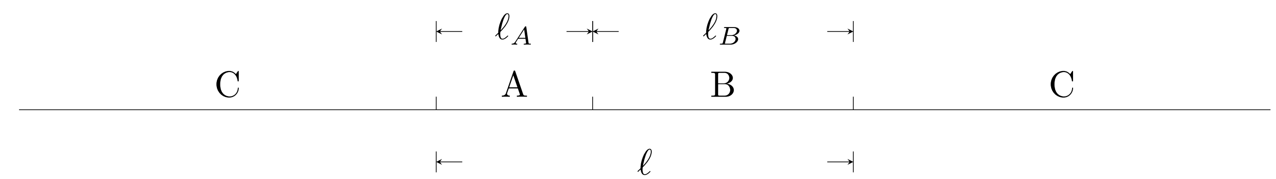


- Euclidean path-integral for $|\psi(t=0)\rangle = e^{-\frac{\beta}{4}H}|B\rangle$



2-dimensional rational CFTs

Operator Renyi entropy:



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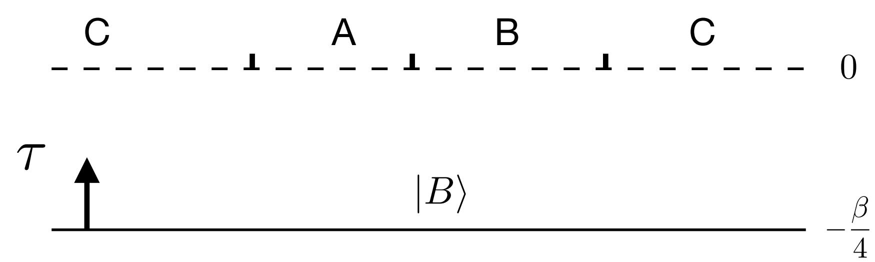
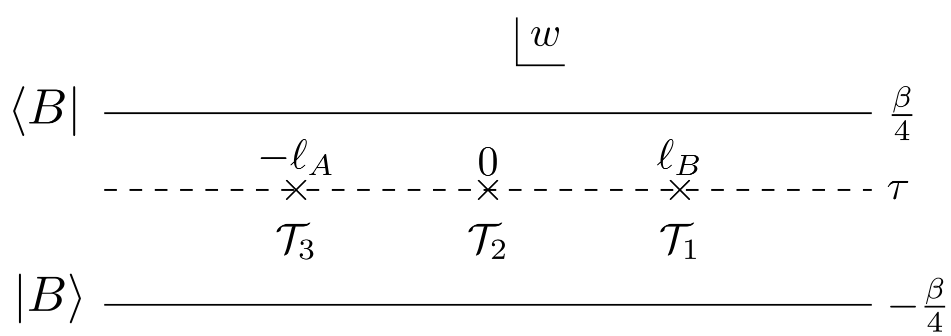
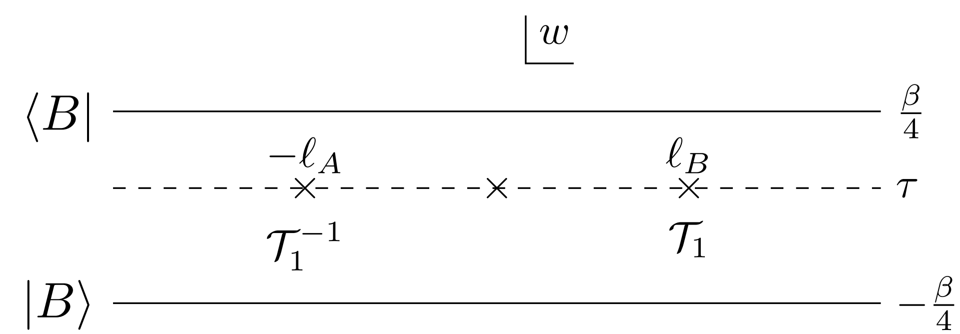


- Wick-rotate $t=-i\tau$, Euclidean correlation function:



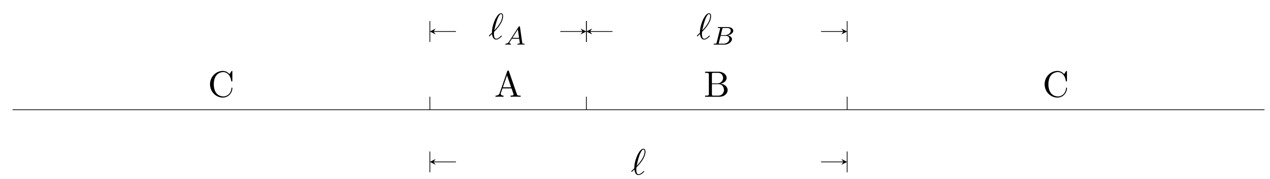
$\langle \mathcal{T}_1(\ell_B)\mathcal{T}_2(0)\mathcal{T}_3(-\ell_A)\rangle_{\text{stripe}}$



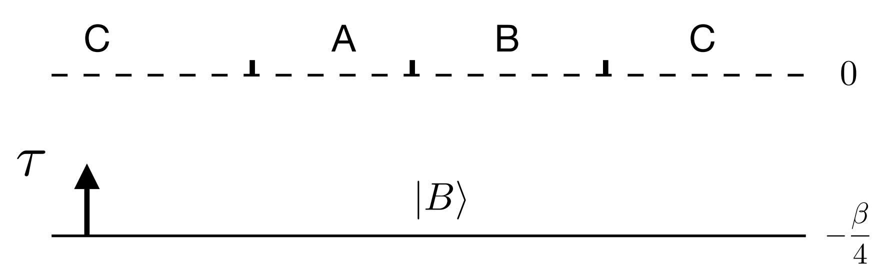
$\langle \mathcal{T}_1(\ell_B)\mathcal{T}_1^{-1}(-\ell_A)\rangle_{\text{stripe}}$

2-dimensional rational CFTs

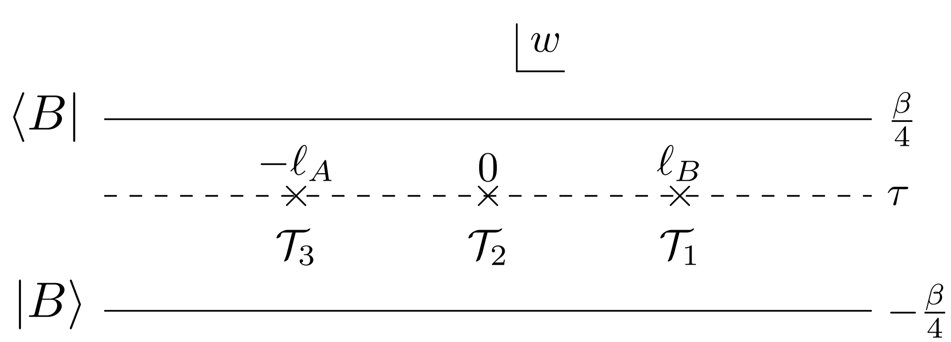
Operator Renyi entropy:



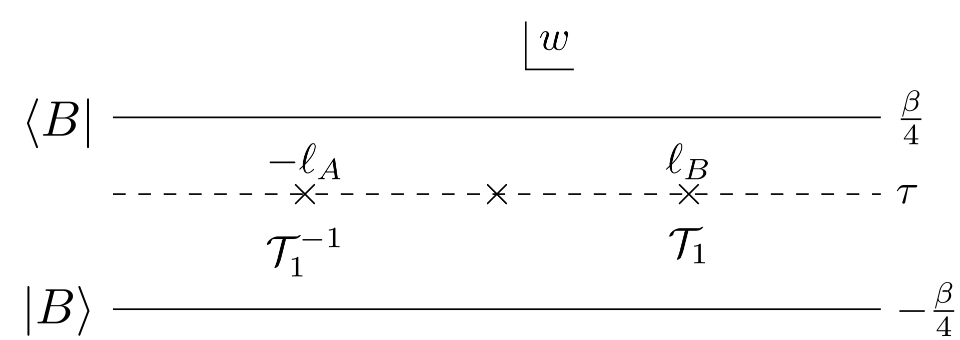
- Euclidean path-integral for $|\psi(t=0)\rangle = e^{-\frac{\beta}{4}H}|B\rangle$



- Wick-rotate $t=-i\tau$, Euclidean correlation function:



$\langle \mathcal{T}_1(\ell_B)\mathcal{T}_2(0)\mathcal{T}_3(-\ell_A)\rangle_{\text{stripe}}$

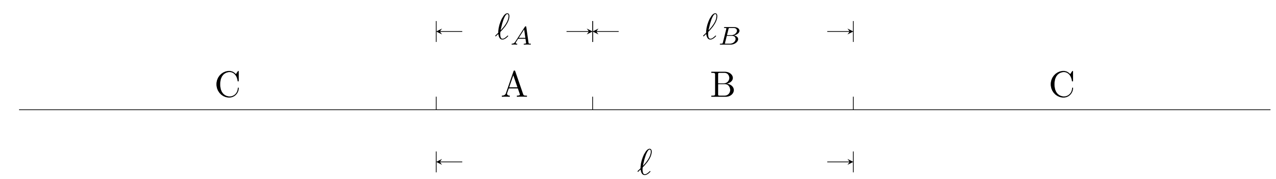


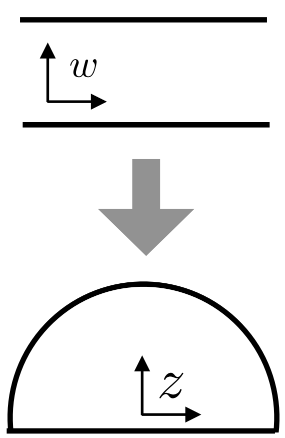
$\langle \mathcal{T}_1(\ell_B)\mathcal{T}_1^{-1}(-\ell_A)\rangle_{\text{stripe}}$

- Conformal map from stripe to upper-half-plane (UHP) by $z = i\exp\left[-\frac{2\pi}{\beta}w\right]$

2-dimensional rational CFTs

Operator Renyi entropy:





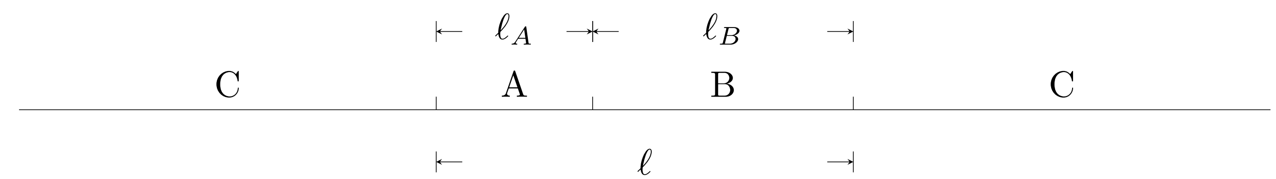
- Via $z = i\exp\left[-\frac{2\pi}{\beta}w\right]$, correlation function in the UHP

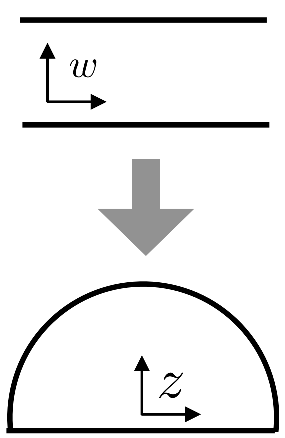
$$S_n^{\mathrm{op}} = \frac{1}{1-n}\ln\left[\left(\frac{\beta}{2\pi}\right)^{2h_2}\frac{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_2(z_2,\bar{z}_2)\mathcal{T}_3(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_1^{-1}(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}\right]$$

$$z_1 = i\exp\left[\frac{2\pi}{\beta}(\ell_B + i\tau)\right],\ z_2 = i\exp\left[\frac{2\pi}{\beta}(i\tau)\right],\ z_3 = i\exp\left[\frac{2\pi}{\beta}(-\ell_A + i\tau)\right]$$

2-dimensional rational CFTs

Operator Renyi entropy:





- Via $z = i \exp\left[-\frac{2\pi}{\beta} w\right]$, correlation function in the UHP

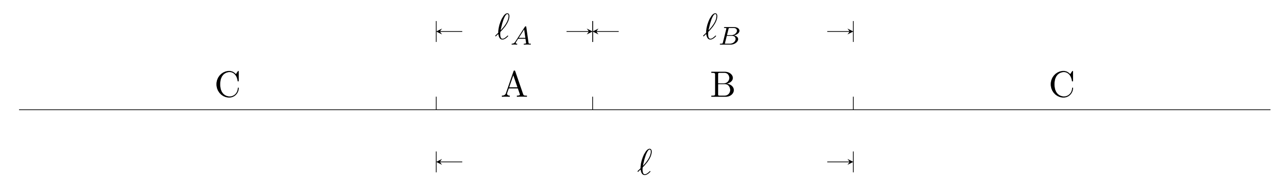
$$S_n^{\mathrm{op}} = \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1, \bar{z}_1) \mathcal{T}_2(z_2, \bar{z}_2) \mathcal{T}_3(z_3, \bar{z}_3) \rangle_{\mathrm{UHP}}}{\langle \mathcal{T}_1(z_1, \bar{z}_1) \mathcal{T}_1^{-1}(z_3, \bar{z}_3) \rangle_{\mathrm{UHP}}} \right]$$

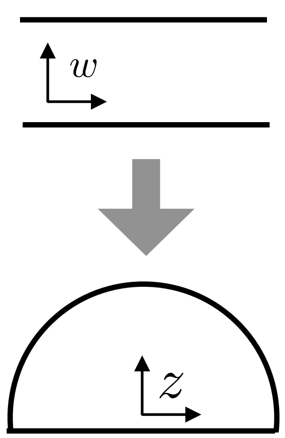
$$z_1 = i \exp\left[\frac{2\pi}{\beta}(\ell_B + i\tau)\right], \; z_2 = i \exp\left[\frac{2\pi}{\beta}(i\tau)\right], \; z_3 = i \exp\left[\frac{2\pi}{\beta}(-\ell_A + i\tau)\right]$$

- Conformal boundary condition on real axis from $|B\rangle$

2-dimensional rational CFTs

Operator Renyi entropy:





- Via $z = i\exp\left[-\frac{2\pi}{\beta}w\right]$, correlation function in the UHP

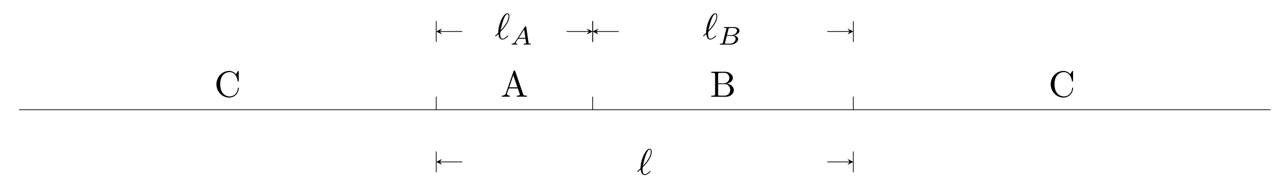
$$S_n^{\mathrm{op}} = \frac{1}{1-n}\ln\left[\left(\frac{\beta}{2\pi}\right)^{2h_2}\frac{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_2(z_2,\bar{z}_2)\mathcal{T}_3(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_1^{-1}(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}\right]$$

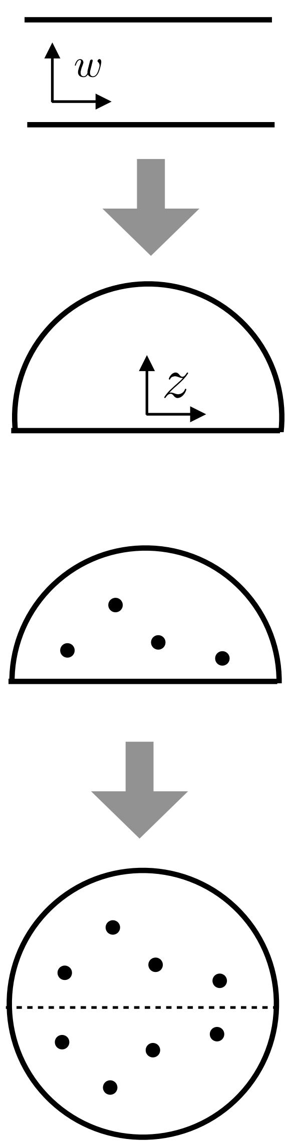
$$z_1 = i\exp\left[\frac{2\pi}{\beta}(\ell_B + i\tau)\right],\ z_2 = i\exp\left[\frac{2\pi}{\beta}(i\tau)\right],\ z_3 = i\exp\left[\frac{2\pi}{\beta}(-\ell_A + i\tau)\right]$$

- Conformal boundary condition on real axis from $|B\rangle$
  - $T = \bar{T}$ on $z = \bar{z}$, can analytically extend $T(z)$ to $\mathbb{C}$ using $T(z) = \bar{T}(\bar{z})$

2-dimensional rational CFTs

Operator Renyi entropy:





- Via $z = i\exp\left[-\frac{2\pi}{\beta}w\right]$, correlation function in the UHP

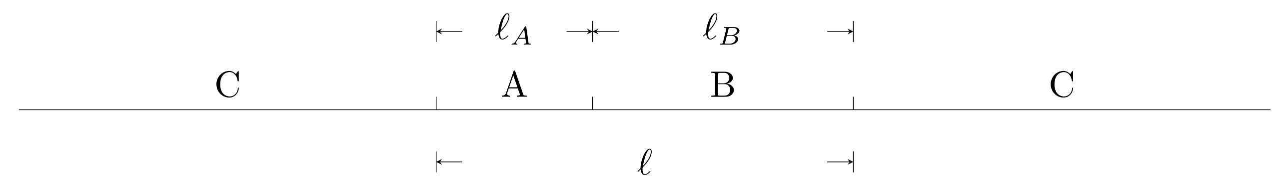
$$S_n^{\rm op} = \frac{1}{1-n}\ln\left[\left(\frac{\beta}{2\pi}\right)^{2h_2}\frac{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_2(z_2,\bar{z}_2)\mathcal{T}_3(z_3,\bar{z}_3)\rangle_{\rm UHP}}{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_1^{-1}(z_3,\bar{z}_3)\rangle_{\rm UHP}}\right]$$

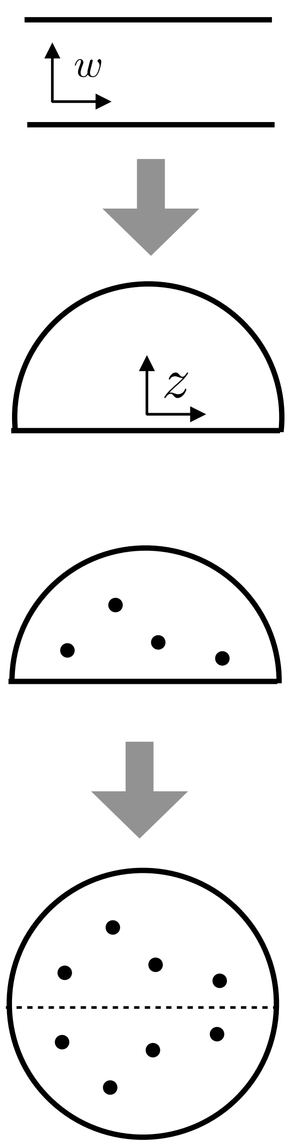
$$z_1 = i\exp\left[\frac{2\pi}{\beta}(\ell_B + i\tau)\right],\ z_2 = i\exp\left[\frac{2\pi}{\beta}(i\tau)\right],\ z_3 = i\exp\left[\frac{2\pi}{\beta}(-\ell_A + i\tau)\right]$$

- Conformal boundary condition on real axis from $|B\rangle$
  - $T = \bar{T}$ on $z = \bar{z}$, can analytically extend $T(z)$ to $\mathbb{C}$ using $T(z) = \bar{T}(\bar{z})$
  - "Method of image": $\langle\ldots\rangle_{\rm UHP}$ same transformation property as a holomorphic correlation function with "images" inserted $\langle\ldots\rangle_{\mathbb{C}}$

2-dimensional rational CFTs

Operator Renyi entropy:





- Via $z = i\exp\left[-\frac{2\pi}{\beta}w\right]$, correlation function in the UHP

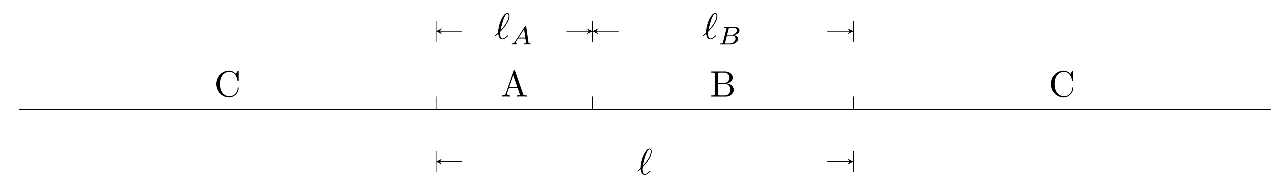
$$S_n^{\mathrm{op}} = \frac{1}{1-n}\ln\left[\left(\frac{\beta}{2\pi}\right)^{2h_2}\frac{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_2(z_2,\bar{z}_2)\mathcal{T}_3(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}{\langle\mathcal{T}_1(z_1,\bar{z}_1)\mathcal{T}_1^{-1}(z_3,\bar{z}_3)\rangle_{\mathrm{UHP}}}\right]$$

$$z_1 = i\exp\left[\frac{2\pi}{\beta}(\ell_B + i\tau)\right],\ z_2 = i\exp\left[\frac{2\pi}{\beta}(i\tau)\right],\ z_3 = i\exp\left[\frac{2\pi}{\beta}(-\ell_A + i\tau)\right]$$

- Conformal boundary condition on real axis from $|B\rangle$
  - $T = \bar{T}$ on $z = \bar{z}$, can analytically extend $T(z)$ to $\mathbb{C}$ using $T(z) = \bar{T}(\bar{z})$
  - "Method of image": $\langle ... \rangle_{\mathrm{UHP}}$ same transformation property as a holomorphic correlation function with "images" inserted $\langle ... \rangle_{\mathbb{C}}$
  - $\langle\mathcal{O}_1(z_1,\bar{z}_1)...\mathcal{O}_n(z_n,\bar{z}_n)\rangle_{\mathrm{UHP}} \sim \langle\mathcal{O}_1(z_1)\bar{\mathcal{O}}_1(\bar{z}_1)...\mathcal{O}_n(z_n)\bar{\mathcal{O}}_n(\bar{z}_n)\rangle_{\mathbb{C}}$

2-dimensional rational CFTs

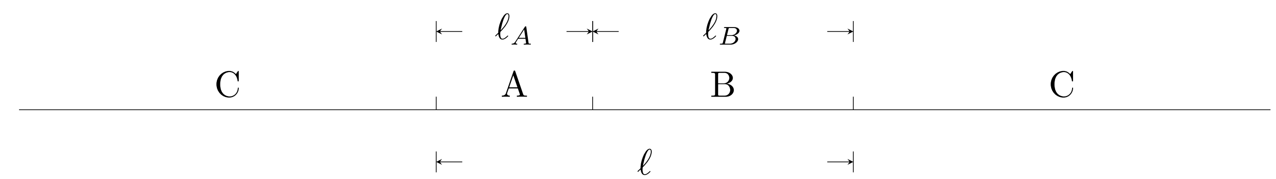
Operator Renyi entropy:



- $$S_n^{\rm op} \sim \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1)\mathcal{T}_2(z_2)\mathcal{T}_3(z_3)\mathcal{T}_1(\bar{z}_1)\mathcal{T}_2(\bar{z}_2)\mathcal{T}_3(\bar{z}_3)\rangle_{\mathbb{C}}}{\langle \mathcal{T}_1(z_1)\mathcal{T}_1^{-1}(z_3)\mathcal{T}_1(\bar{z}_1)\mathcal{T}_1^{-1}(\bar{z}_3)\rangle_{\mathbb{C}}} \right]$$

2-dimensional rational CFTs

Operator Renyi entropy:



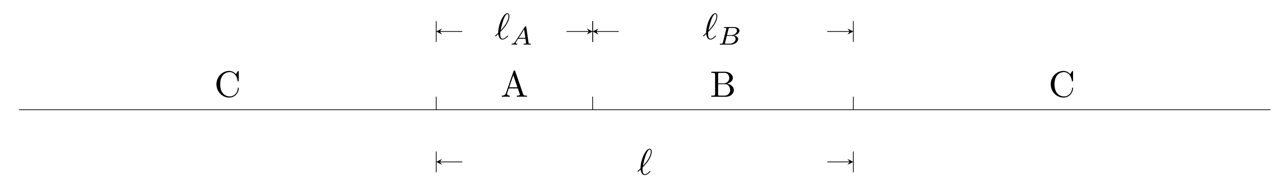
- $$S_n^{\text{op}} \sim \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1) \mathcal{T}_2(z_2) \mathcal{T}_3(z_3) \mathcal{T}_1(\bar{z}_1) \mathcal{T}_2(\bar{z}_2) \mathcal{T}_3(\bar{z}_3) \rangle_{\mathbb{C}}}{\langle \mathcal{T}_1(z_1) \mathcal{T}_1^{-1}(z_3) \mathcal{T}_1(\bar{z}_1) \mathcal{T}_1^{-1}(\bar{z}_3) \rangle_{\mathbb{C}}} \right]$$
- Wick-rotate back to real time $\tau \to it$

$$S_n^{\text{op}} \sim \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1) \mathcal{T}_2(z_2) \mathcal{T}_3(z_3) \mathcal{T}_1(z_4) \mathcal{T}_2(z_5) \mathcal{T}_3(z_6) \rangle_{\mathbb{C}}}{\langle \mathcal{T}_1(z_1) \mathcal{T}_1(z_3) \mathcal{T}_1(z_4) \mathcal{T}_1(z_6) \rangle_{\mathbb{C}}} \right]$$

$$z_1 = i \exp\left[\frac{2\pi}{\beta}(\ell_B - t)\right],\ z_2 = i \exp\left[-\frac{2\pi}{\beta} t\right],\ z_3 = i \exp\left[\frac{2\pi}{\beta}(-\ell_A - t)\right]$$
$$z_4 = -i \exp\left[\frac{2\pi}{\beta}(\ell_B + t)\right],\ z_5 = -i \exp\left[\frac{2\pi}{\beta} t\right],\ z_6 = -i \exp\left[\frac{2\pi}{\beta}(-\ell_A + t)\right]$$

2-dimensional rational CFTs

Operator Renyi entropy:



- $$S_n^{\mathrm{op}} \sim \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1)\mathcal{T}_2(z_2)\mathcal{T}_3(z_3)\mathcal{T}_1(\bar{z}_1)\mathcal{T}_2(\bar{z}_2)\mathcal{T}_3(\bar{z}_3)\rangle_{\mathbb{C}}}{\langle \mathcal{T}_1(z_1)\mathcal{T}_1^{-1}(z_3)\mathcal{T}_1(\bar{z}_1)\mathcal{T}_1^{-1}(\bar{z}_3)\rangle_{\mathbb{C}}} \right]$$
- Wick-rotate back to real time $\tau \to it$

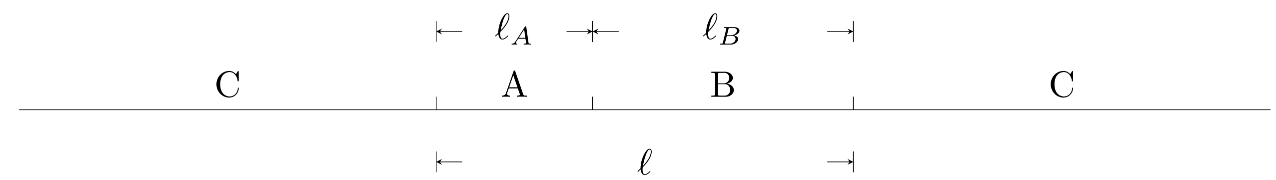
$$S_n^{\mathrm{op}} \sim \frac{1}{1-n} \ln \left[\left(\frac{\beta}{2\pi} \right)^{2h_2} \frac{\langle \mathcal{T}_1(z_1)\mathcal{T}_2(z_2)\mathcal{T}_3(z_3)\mathcal{T}_1(z_4)\mathcal{T}_2(z_5)\mathcal{T}_3(z_6)\rangle_{\mathbb{C}}}{\langle \mathcal{T}_1(z_1)\mathcal{T}_1(z_3)\mathcal{T}_1(z_4)\mathcal{T}_1(z_6)\rangle_{\mathbb{C}}} \right]$$

$$z_1 = i\exp\left[\frac{2\pi}{\beta}(\ell_B - t)\right],\ z_2 = i\exp\left[-\frac{2\pi}{\beta}t\right],\ z_3 = i\exp\left[\frac{2\pi}{\beta}(-\ell_A - t)\right]$$
$$z_4 = -i\exp\left[\frac{2\pi}{\beta}(\ell_B + t)\right],\ z_5 = -i\exp\left[\frac{2\pi}{\beta}t\right],\ z_6 = -i\exp\left[\frac{2\pi}{\beta}(-\ell_A + t)\right]$$

- In the high energy/long time limit $|\ell_B \pm t|, |\ell_A \pm t|, t \gg \beta$, parameters near the boundary of conformal moduli, correlation function —> OPE

2-dimensional rational CFTs

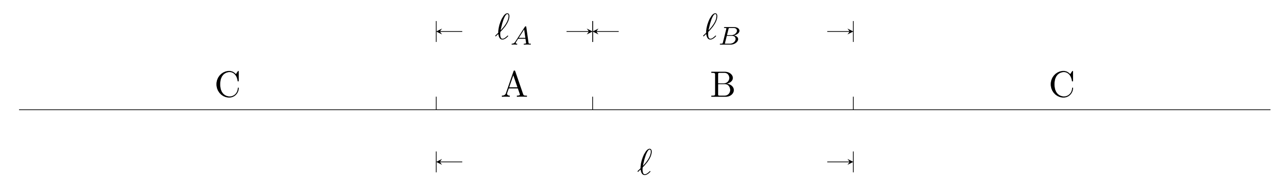
Operator Renyi entropy:



- “Chiral” OPE: $\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \to (z_2 - z_1)^{h_3 - h_1 - h_2}\mathcal{O}_3(z_2) + \ldots$

2-dimensional rational CFTs

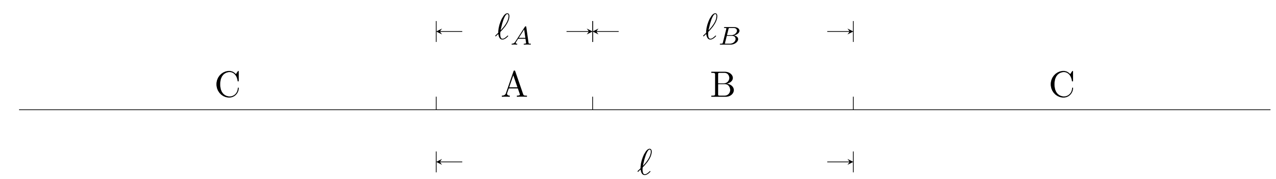
Operator Renyi entropy:



- "Chiral" OPE: $\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \to (z_2 - z_1)^{h_3 - h_1 - h_2}\mathcal{O}_3(z_2) + \ldots$
- Not all chiral OPEs are legitimate —> "True" OPE occurs on stripe/UHP: e.g. $(z_1 \to z_2, \bar{z}_1 \to \bar{z}_2)$ or $(z_1 \leftrightarrow \bar{z}_1)$

2-dimensional rational CFTs

Operator Renyi entropy:



- "Chiral" OPE: $\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \to (z_2 - z_1)^{h_3 - h_1 - h_2}\mathcal{O}_3(z_2) + \dots$
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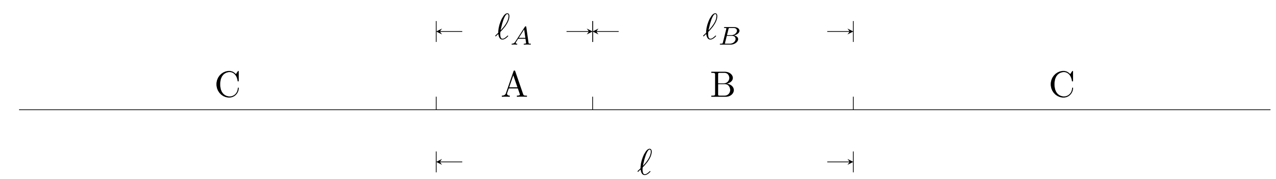
- For rational CFTs, all chiral OPE singularities are present

Satisfying the crossing equation by finite number of blocks (C.Asplund, et al 2015)

2-dimensional rational CFTs

Operator Renyi entropy:



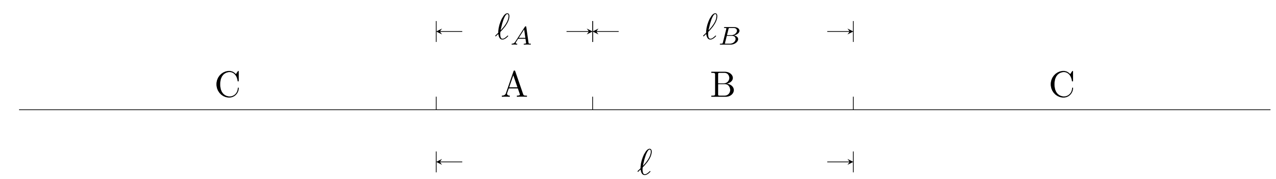
- “Chiral” OPE: $\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \to (z_2 - z_1)^{h_3 - h_1 - h_2}\mathcal{O}_3(z_2) + \ldots$
- Not all chiral OPEs are legitimate —> “True” OPE occurs on stripe/UHP: e.g. $(z_1 \to z_2, \bar{z}_1 \to \bar{z}_2)$ or $(z_1 \leftrightarrow \bar{z}_1)$
- For rational CFTs, all chiral OPE singularities are present

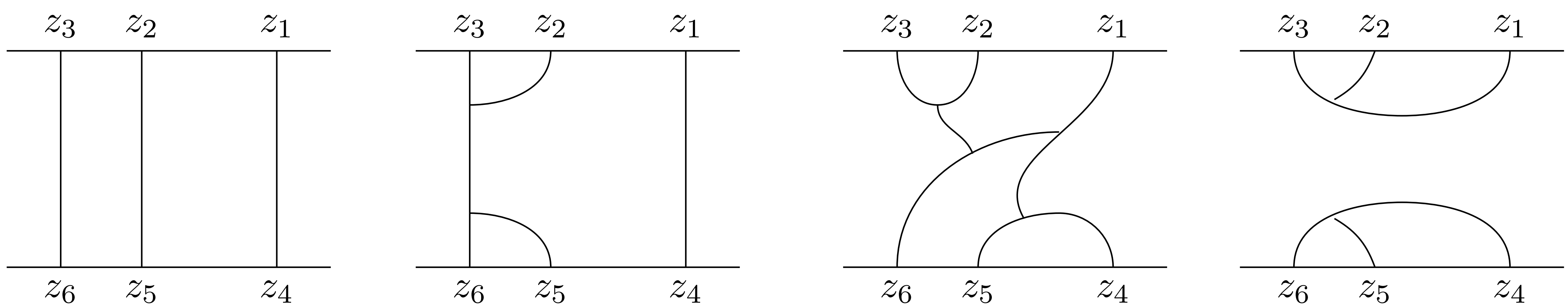
  Satisfying the crossing equation by finite number of blocks (C.Asplund, et al 2015)

- Different chiral OPE channels —> different regimes of the operator entanglement dynamics

2-dimensional rational CFTs

Operator Renyi entropy:





$0 < t < \ell_A/2$ | $\ell_A/2 < t < \ell_B/2$ | $\ell_B/2 < t < \ell/2$ | $t > \ell/2$

$$S_n^{\mathrm{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}t\right]$$

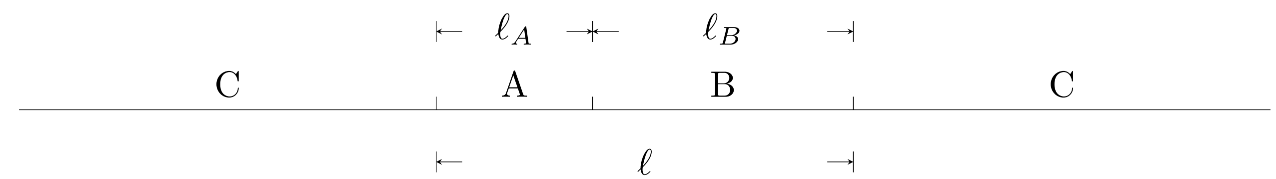
$$S_n^{\mathrm{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}\frac{\ell_A}{2}\right]$$

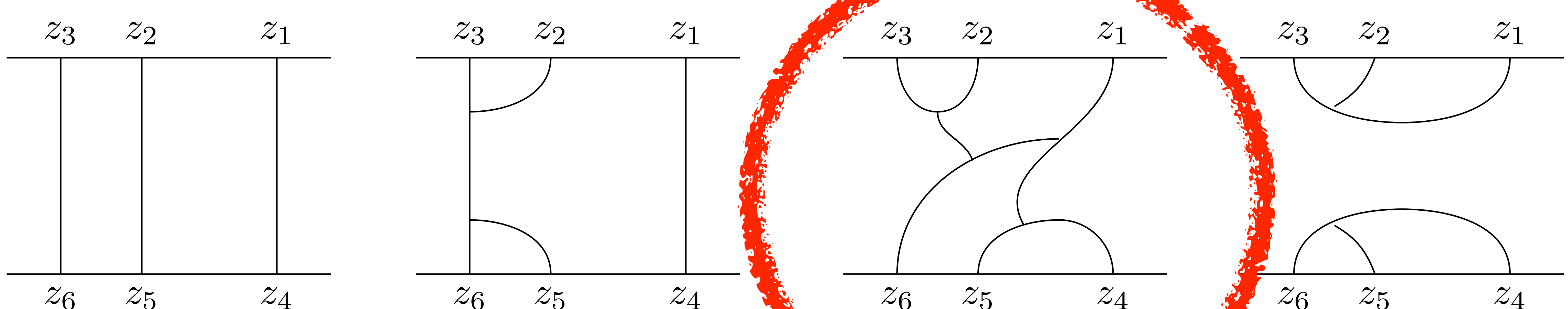
$$S_n^{\mathrm{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}\left(\frac{\ell}{2} - t\right)\right]$$

$$S_n^{\mathrm{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right)\right]$$

2-dimensional rational CFTs

Operator Renyi entropy:





$0 < t < \ell_A/2$

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$t > \ell/2$

$$S_n^{\text{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}t\right]$$

$$S_n^{\text{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}\frac{\ell_A}{2}\right]$$

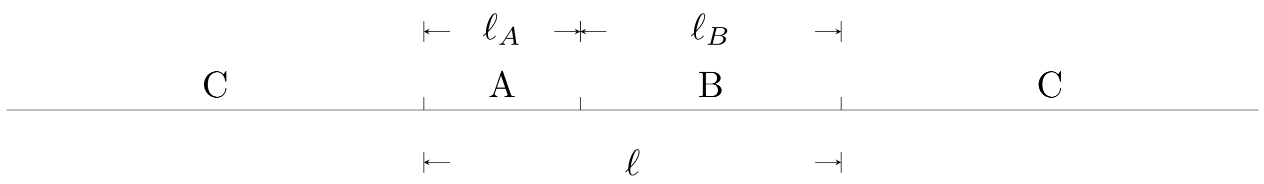
$$S_n^{\text{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi}{\beta}\left(\frac{\ell}{2} - t\right)\right]$$

Rational CFTs

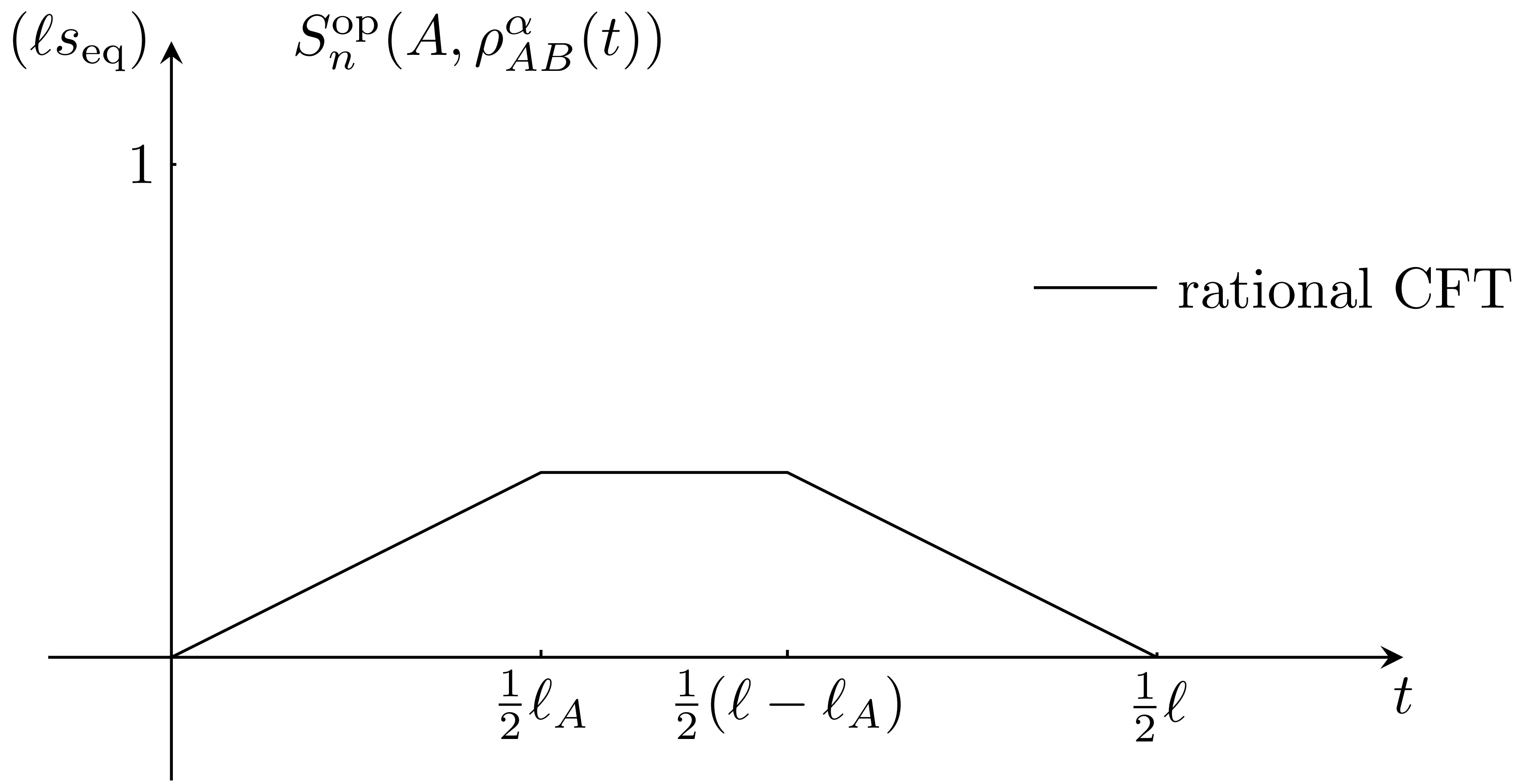
$$S_n^{\text{op}} \approx \frac{2h_2}{n-1}\left[\ln\left(\frac{2\pi}{\beta}\right)\right]$$

2-dimensional rational CFTs

Operator Renyi entropy:



Put together:



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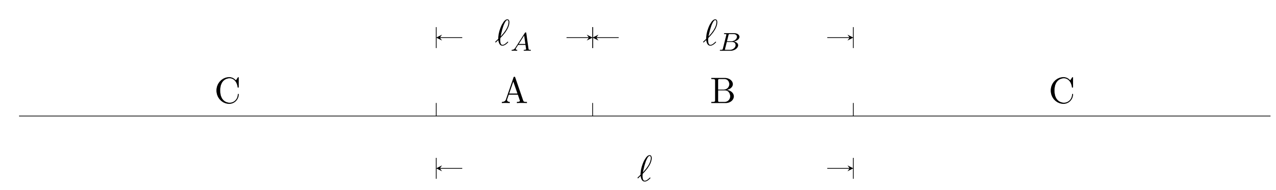

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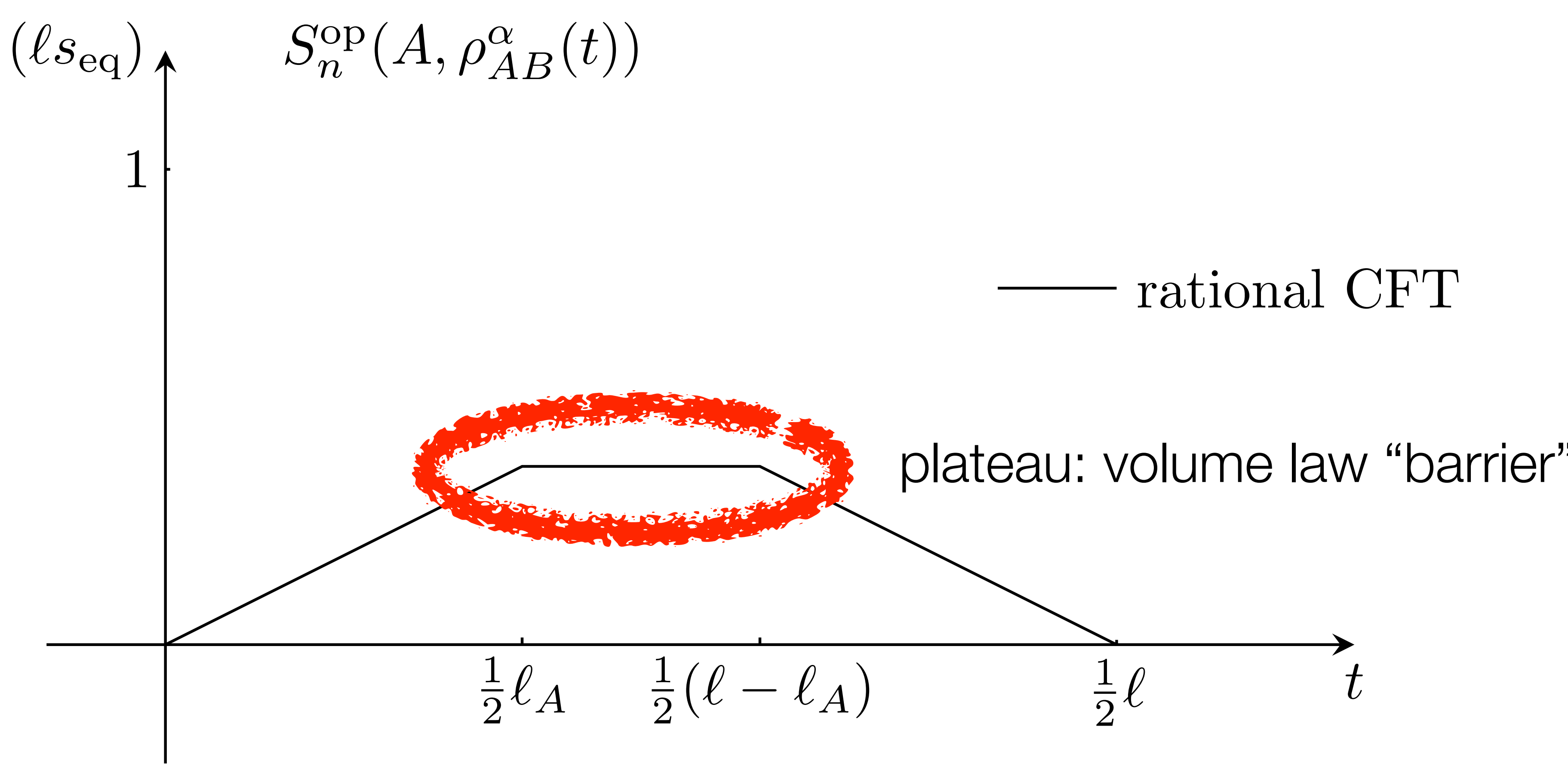


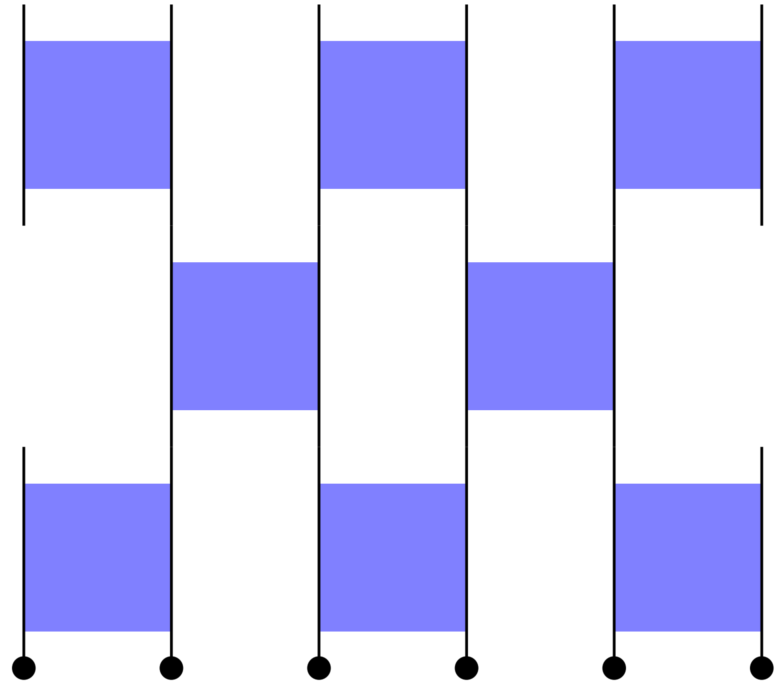
Outline:

- Set up: (reduced density) operator entanglement
- Quenched dynamics for operator entanglement
  1. Two-dimensional rational conformal field theories (CFTs)
  2. Random unitary circuits
  3. Holographic CFTs (AdS/CFT)
- Discussion: a new measure of quantum chaoticity?
- Future directions

Random Unitary Circuits

Random Unitary Circuits

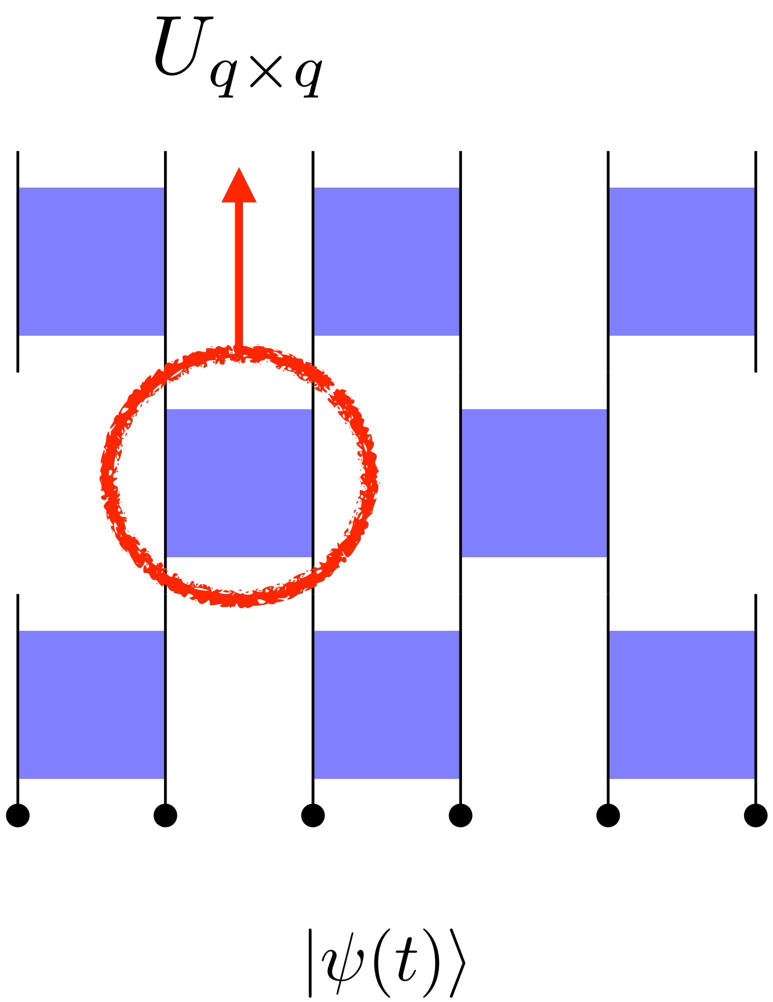
- A tensor network circuit



$|\psi(t)\rangle$

Random Unitary Circuits

- A tensor network circuit
- Each node: a Haar random unitary matrix



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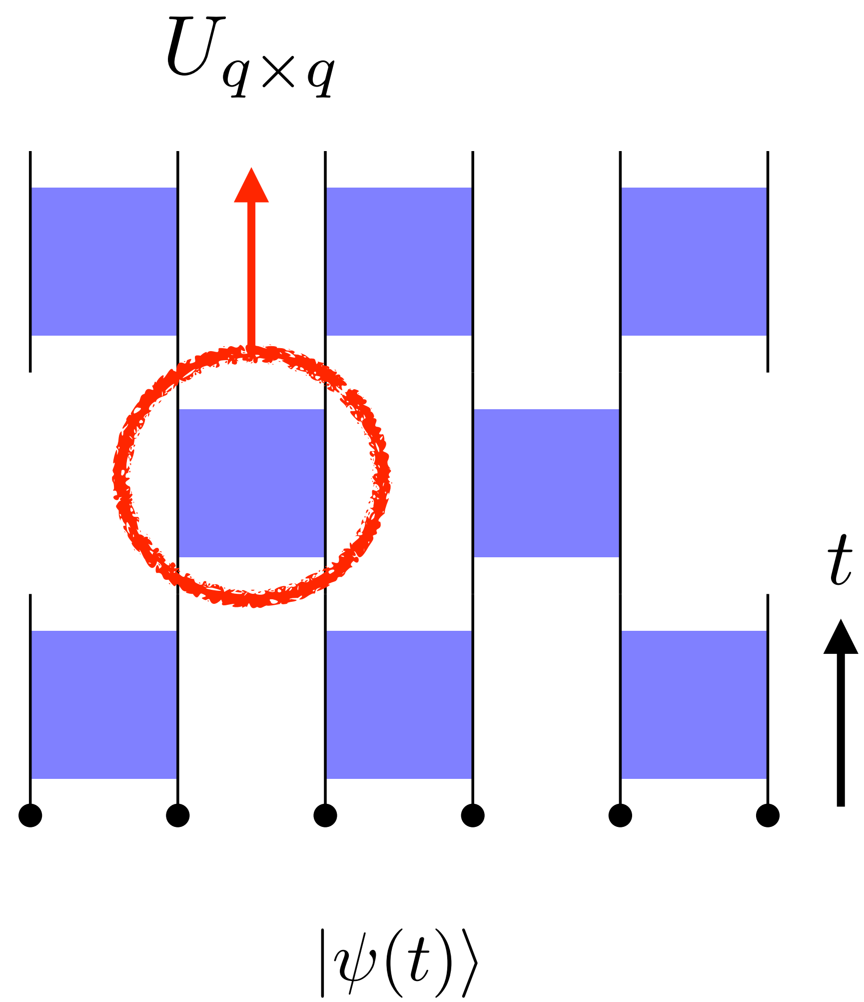

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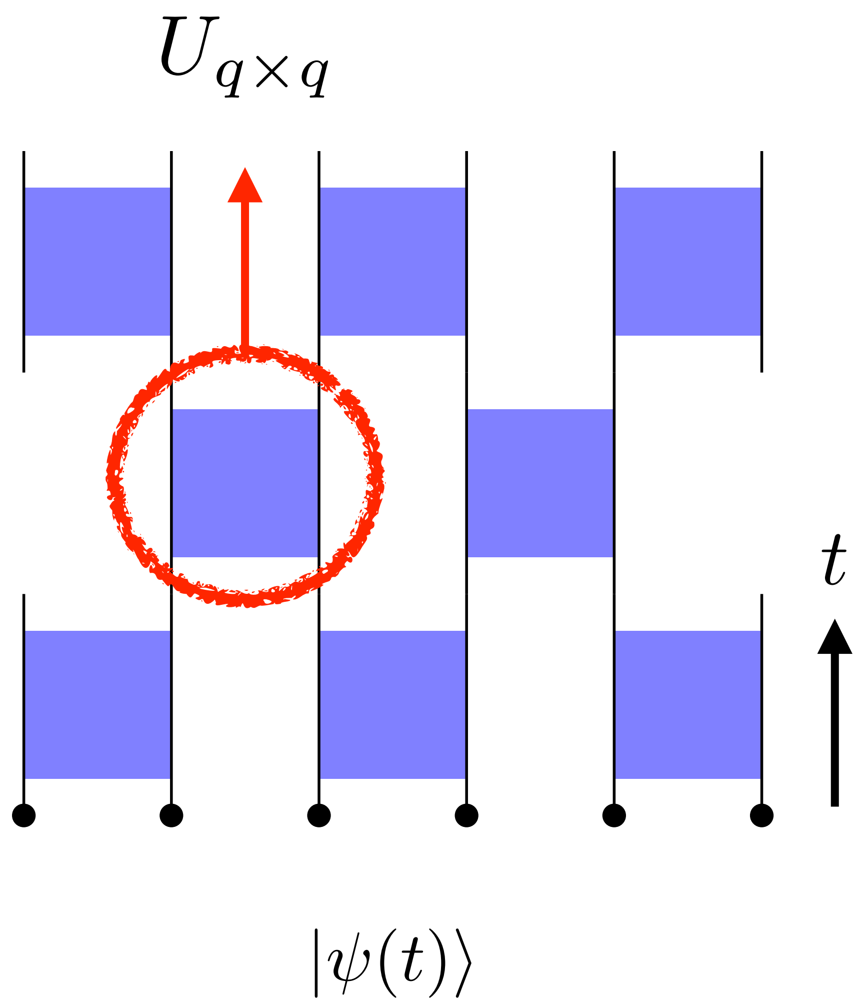

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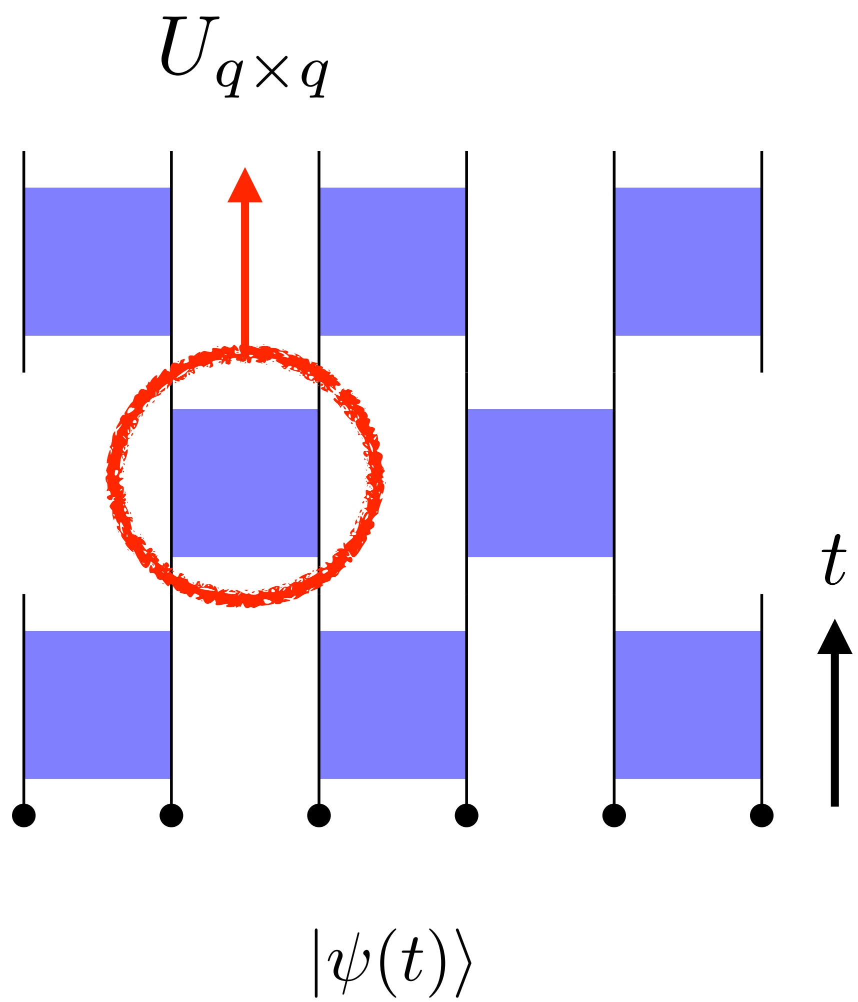

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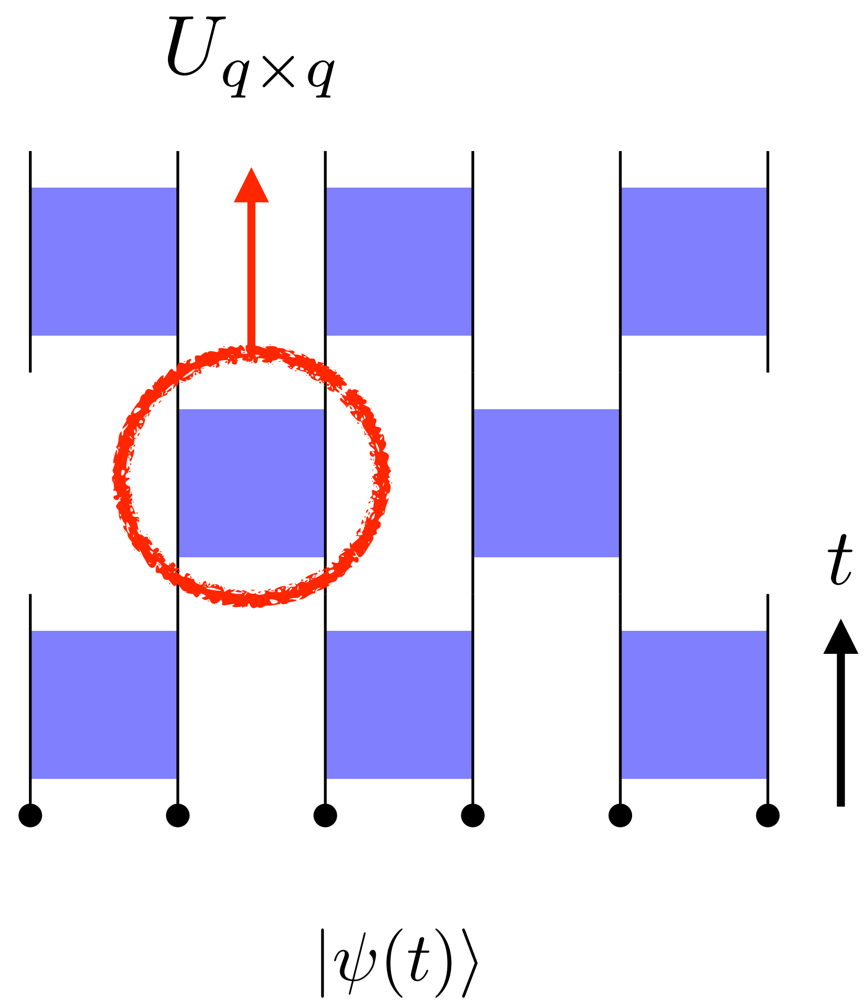
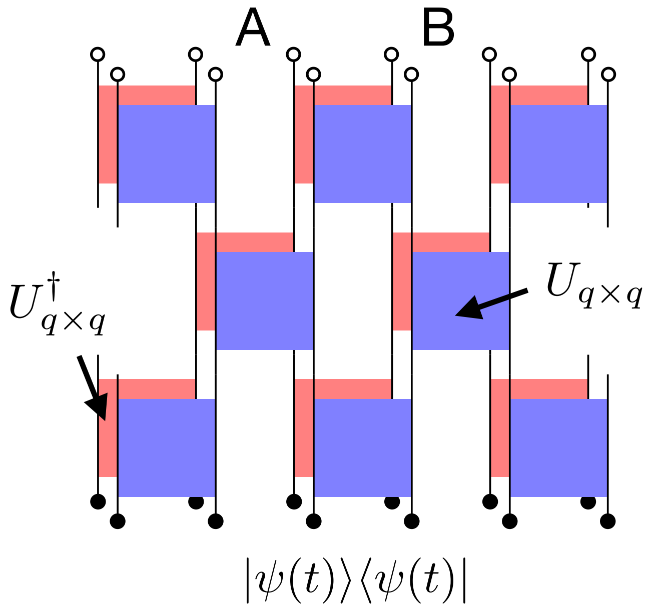
- OTOC, state entanglement dynamics were studied. e.g. A. Nahum, et al; C. Keyserlingk, et al; V. Khemani, et al (2017); T. Zhou, et al (2018)



Random Unitary Circuits

Operator entanglement dynamics $S_n^{\mathrm{op}}\left(A, \rho_{AB}^{\alpha}(t)\right), \alpha=1$



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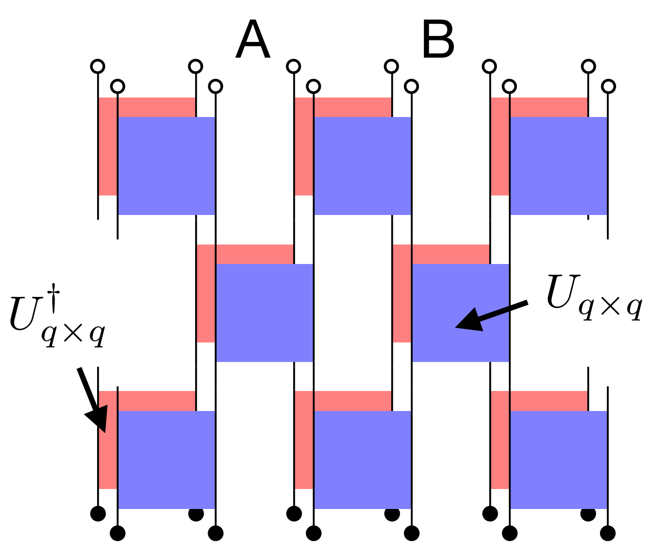
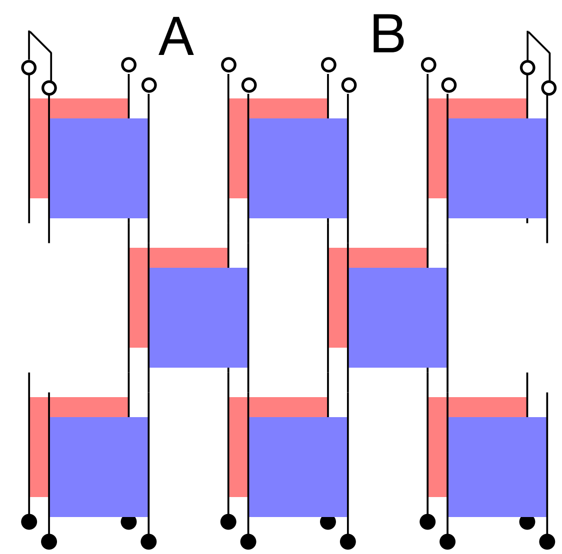


$|\psi(t)\rangle\langle\psi(t)|$



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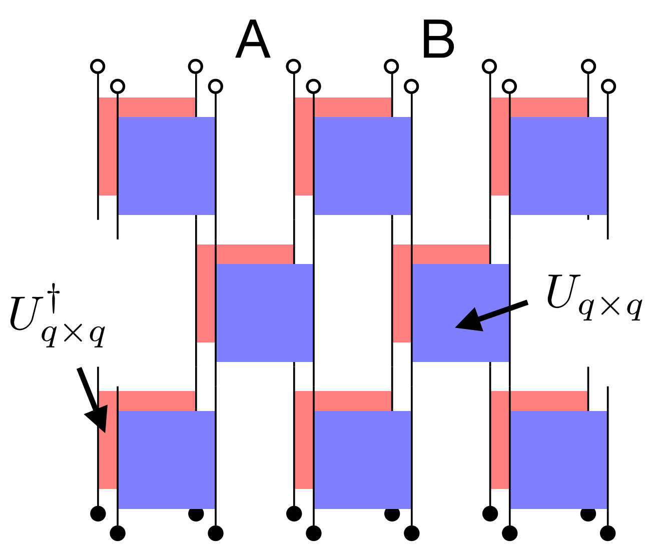

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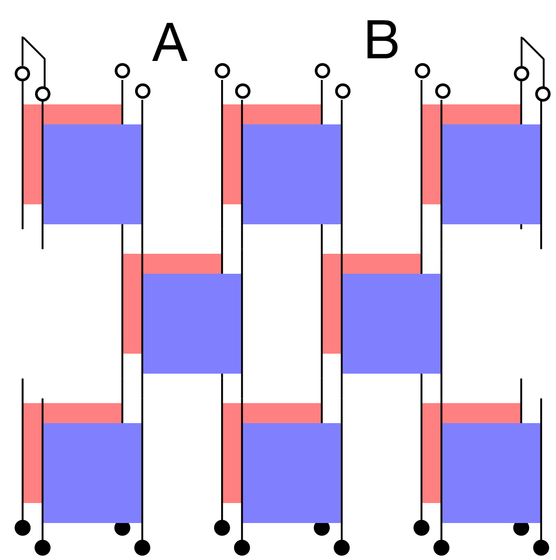

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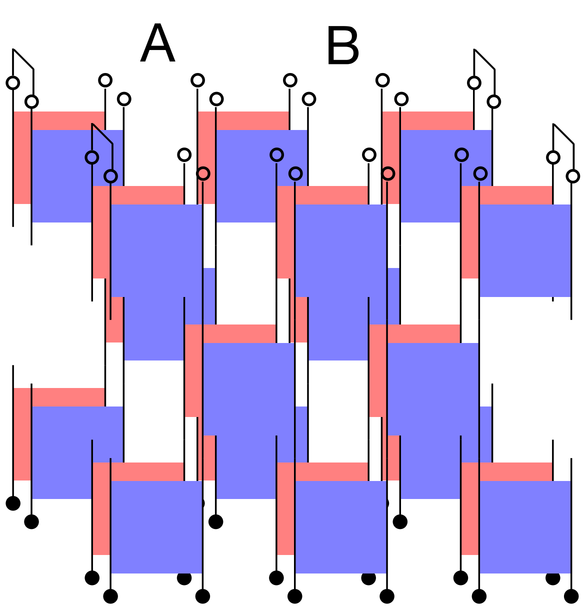

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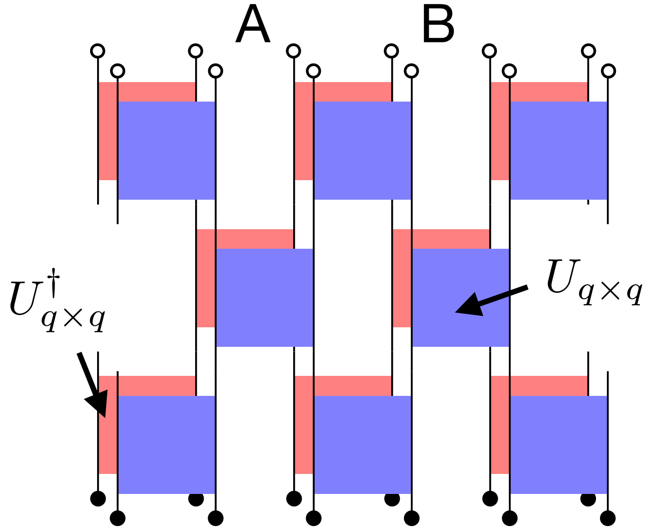

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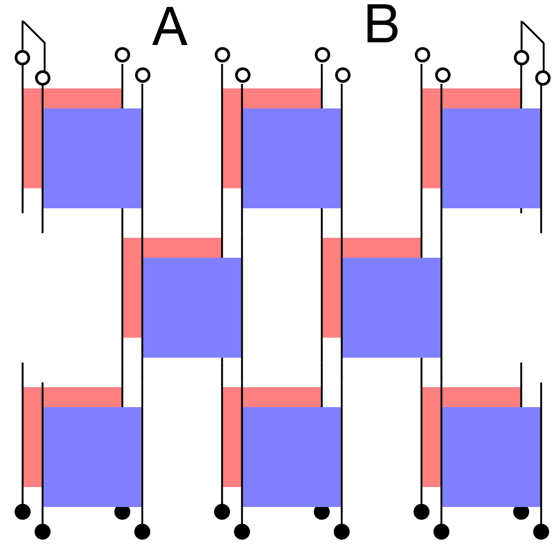

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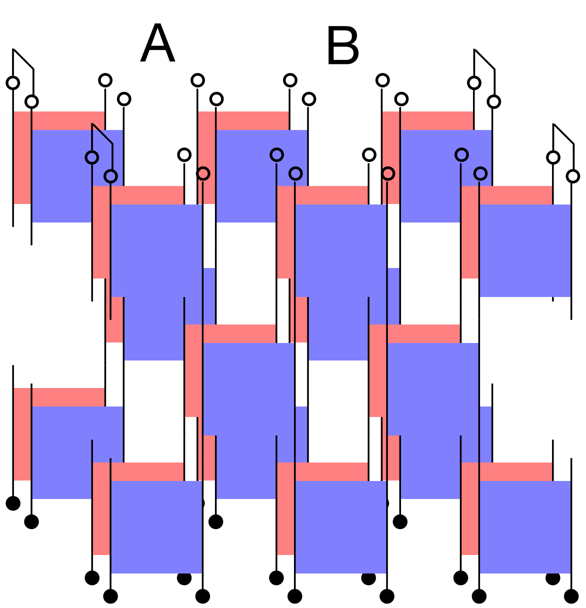

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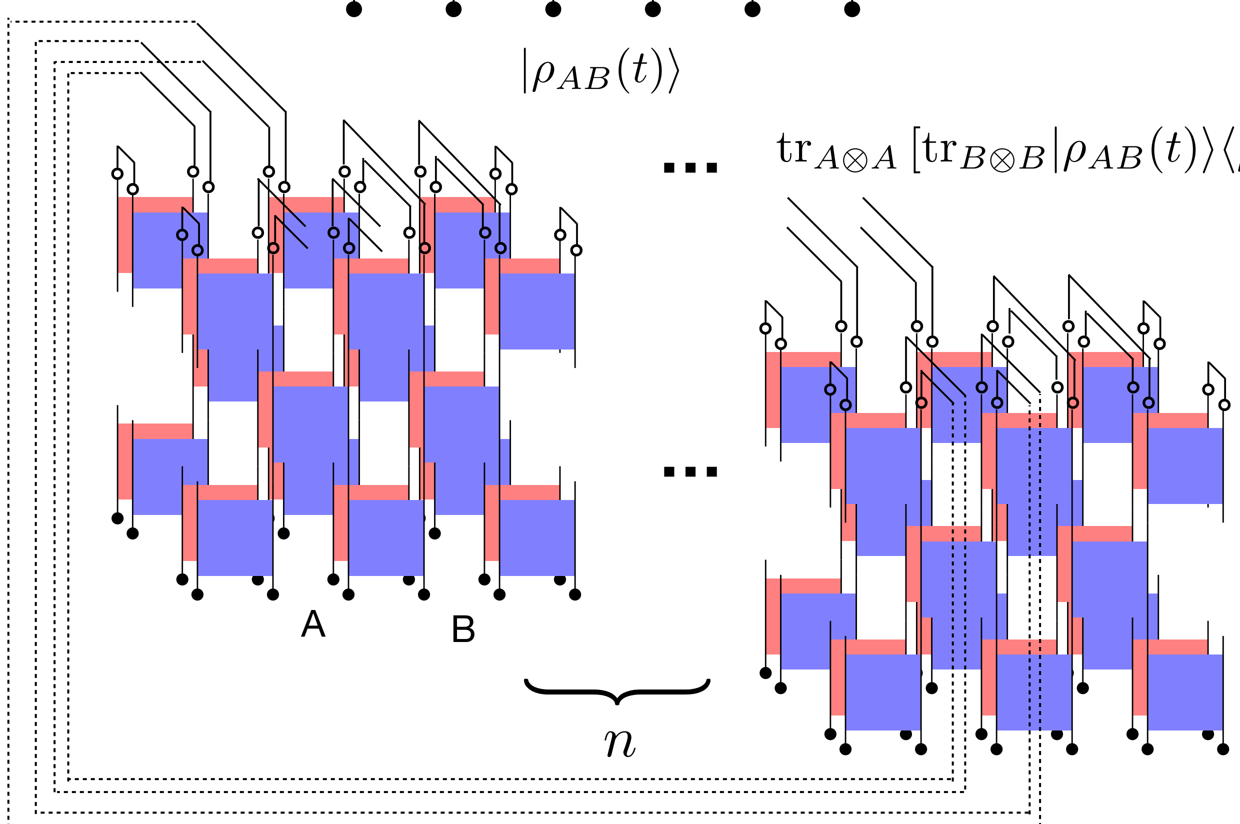

$\mathrm{tr}_{A\otimes A}\left[\mathrm{tr}_{B\otimes B}|\rho_{AB}(t)\rangle\langle\rho_{AB}(t)|\right]^n$

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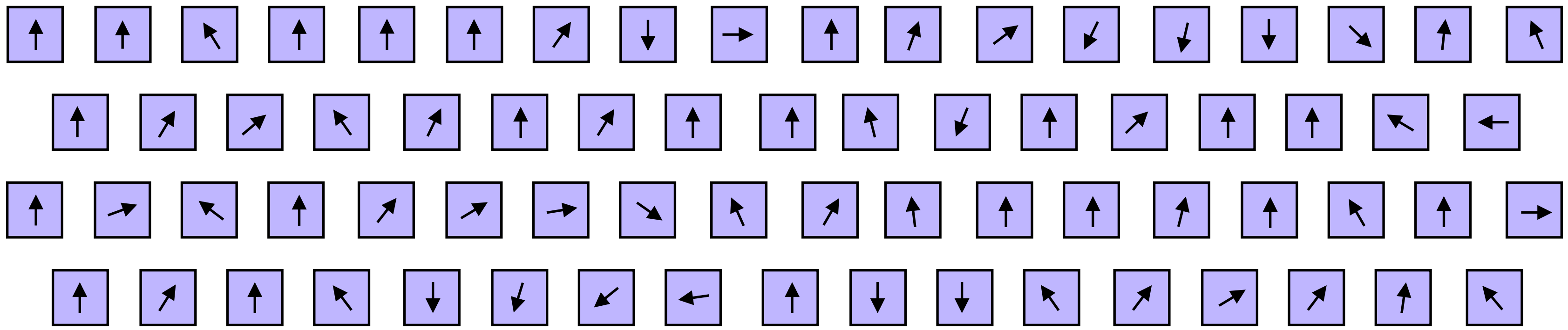
T. Zhou et al (2018)



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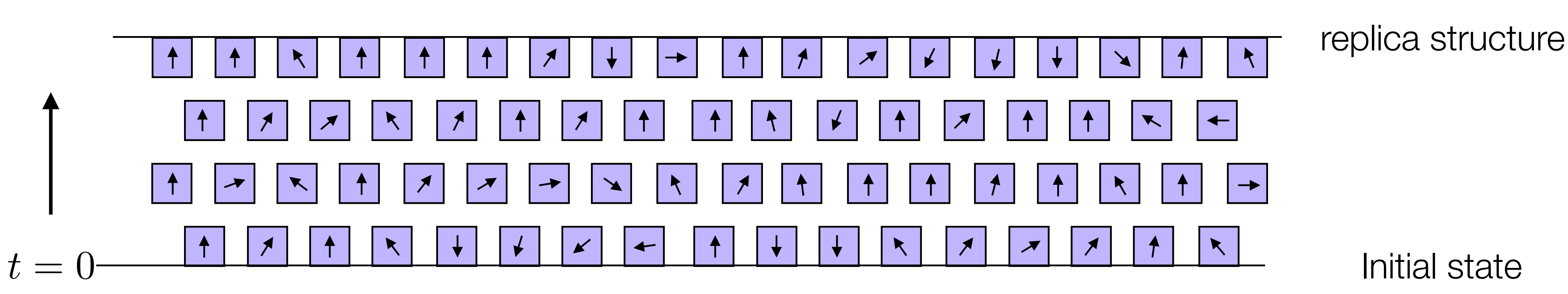

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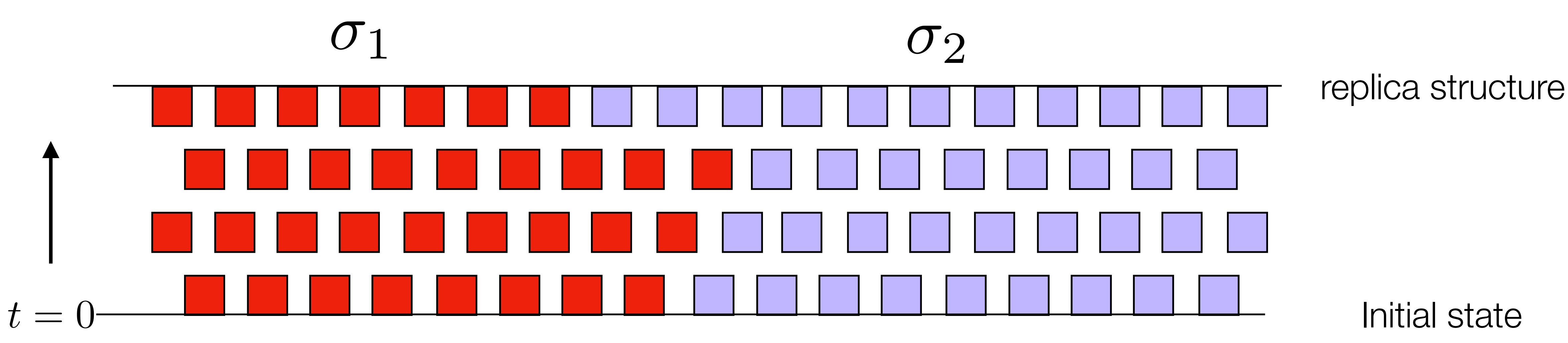

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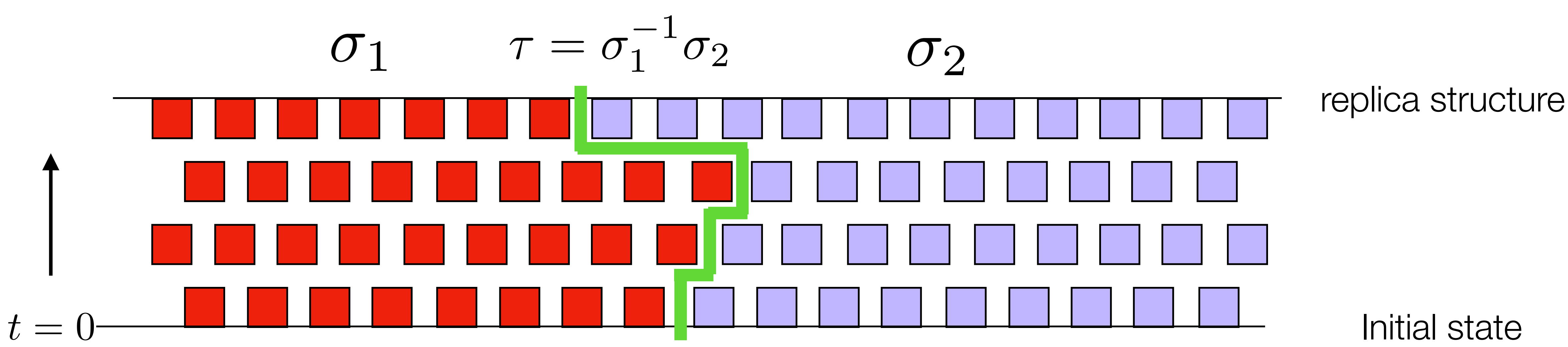

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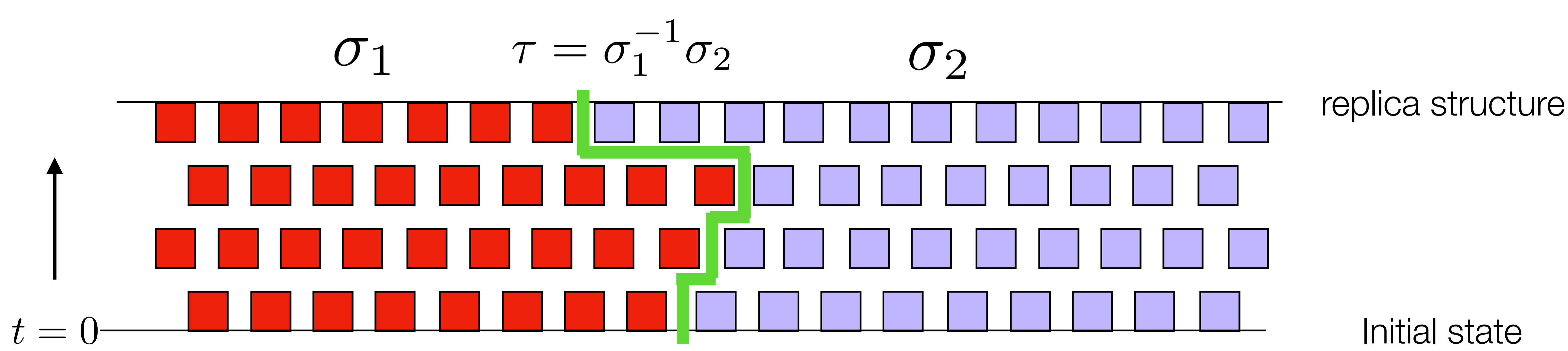
- Free energy of configurations: domain wall picture
- Minimize the free energy of domain walls



Random Unitary Circuits

“Energetics” of domain walls

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- Elementary domain walls: single transpositions

e.g. $(123) = (12)(23)$ —> 2 elementary domain walls

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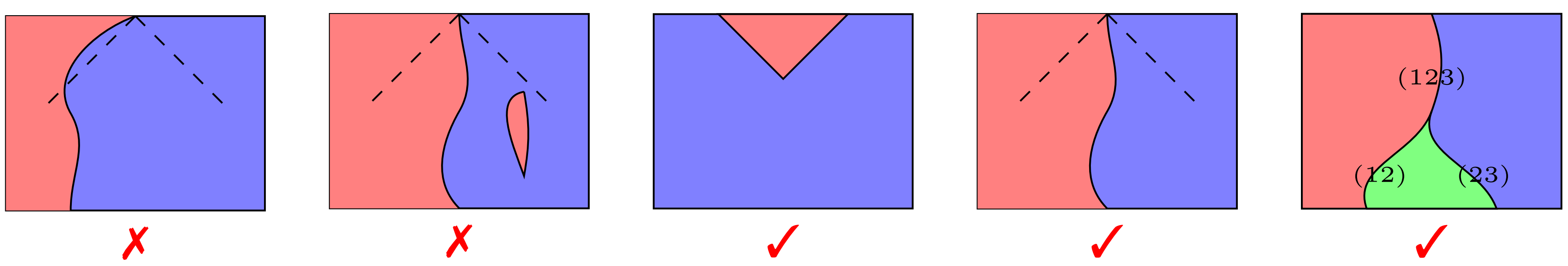

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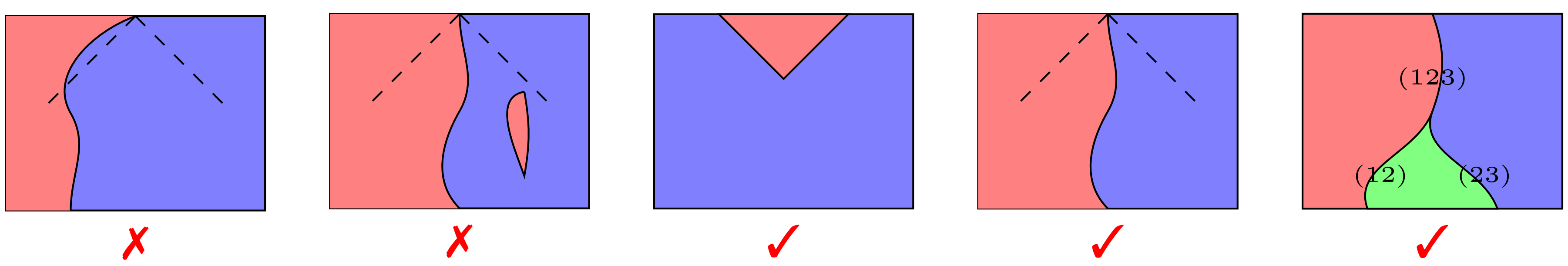

- Each time-step costs free energy $ds = \ln q$ per elementary domain wall

Random Unitary Circuits

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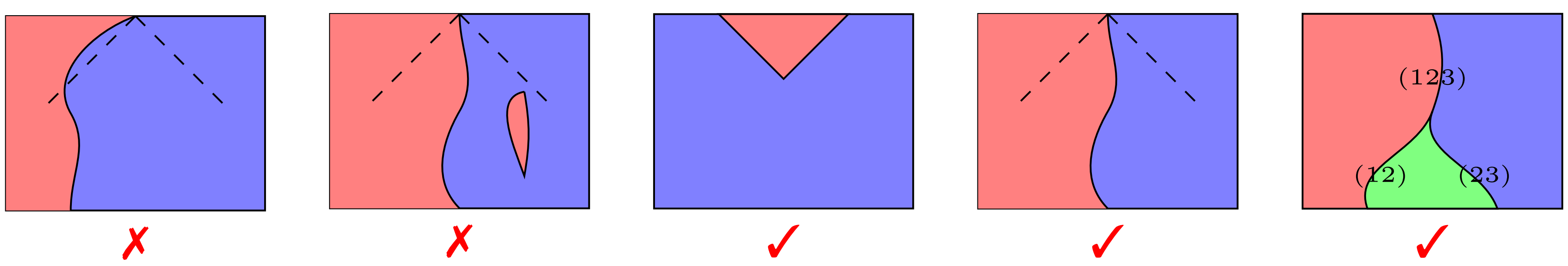

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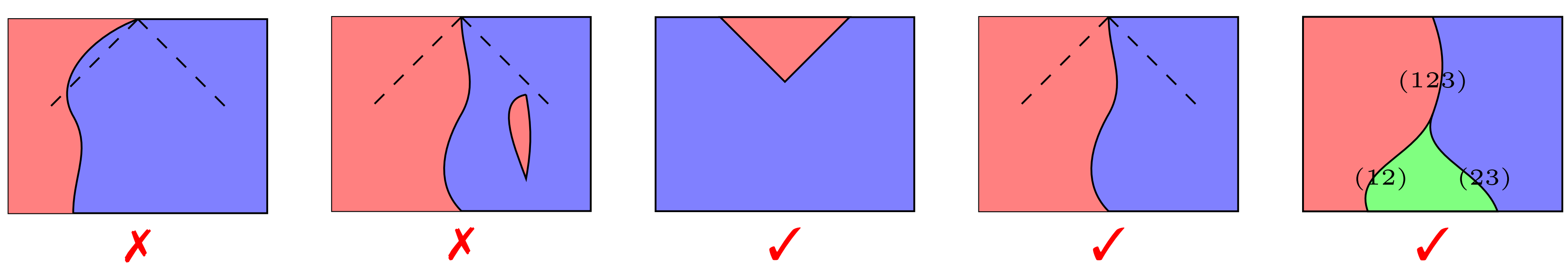

- Each time-step costs free energy $ds = \ln q$ per elementary domain wall
- Domain wall “interactions” suppressed by $\mathcal{O}\left(1/\ln q\right)$
- Large $q \gg 1$ limit: free elementary domain walls (modulo branching and annihilation) taking shortest path

Random Unitary Circuits

Operator entanglement dynamics

$$S_2^{\mathrm{op}}(A, \rho_{AB}) = -\ln \mathrm{tr}_{A\otimes A}\left[\mathrm{tr}_{B\otimes B}|\rho_{AB}\rangle\langle\rho_{AB}|\right]^2 - 2S_2(AB)$$

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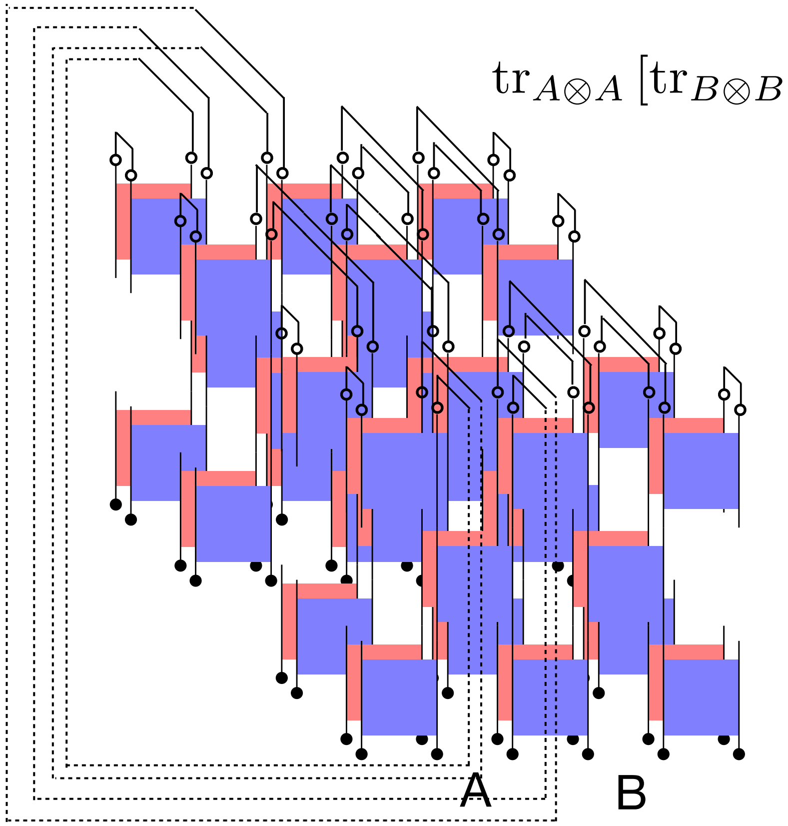

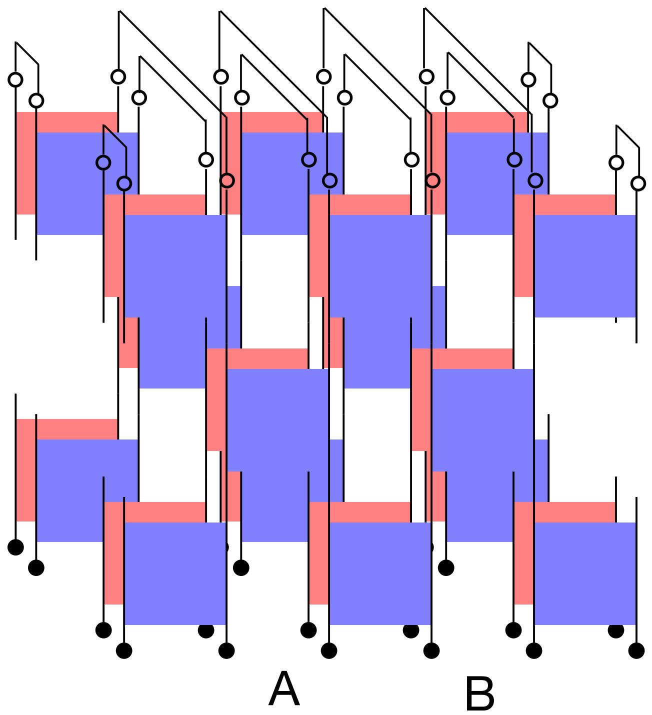

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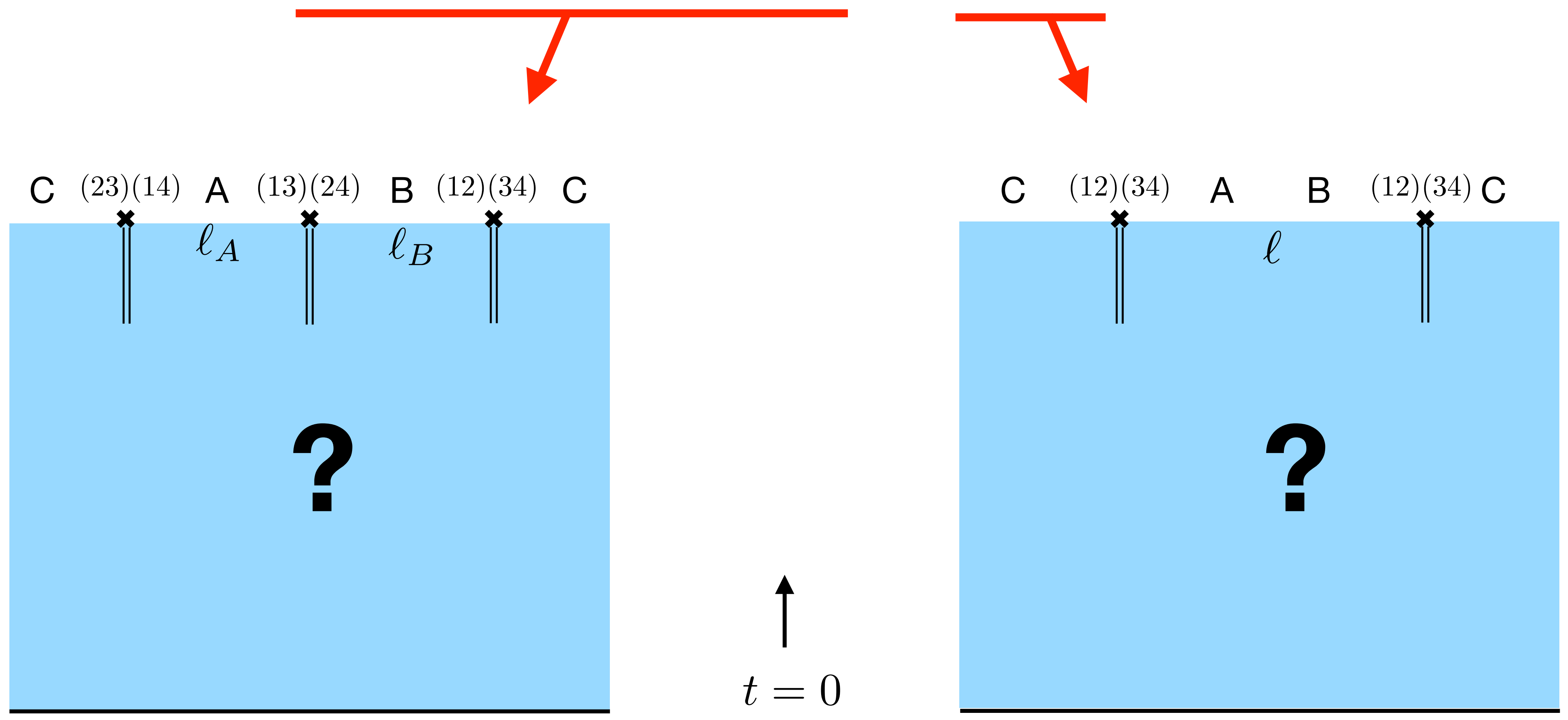

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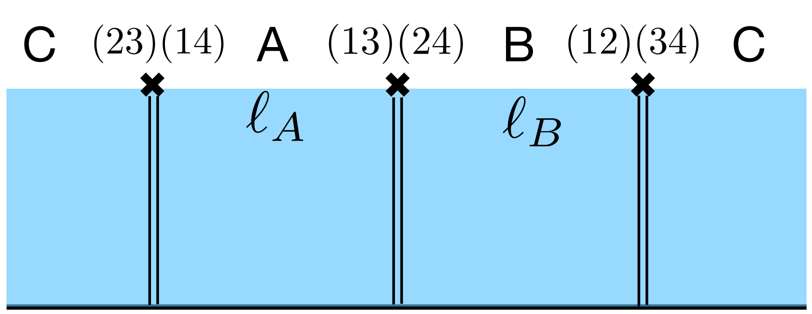
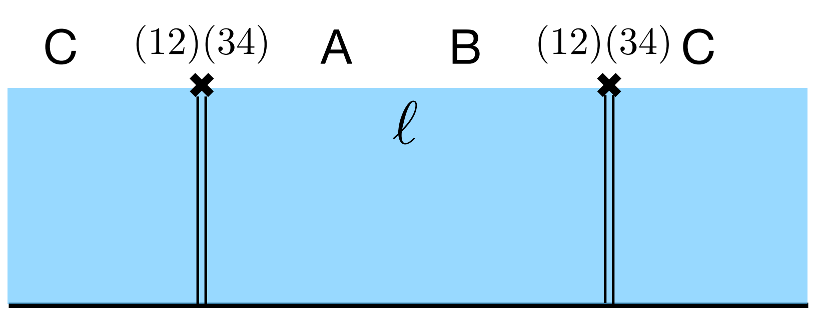
phase 1

$t < \ell_A/2$



$t = 0$



$6 \ln q \; t$

$4 \ln q \; t$

Random Unitary Circuits

Operator entanglement dynamics

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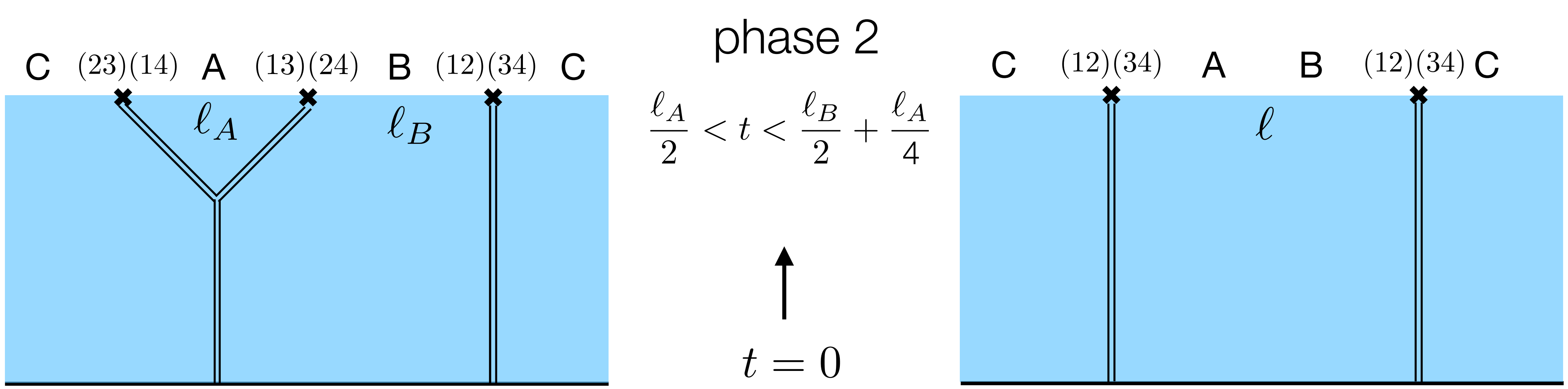

$\ell_A \ln q + 4 \ln q \; t$

$4 \ln q \; t$

Random Unitary Circuits

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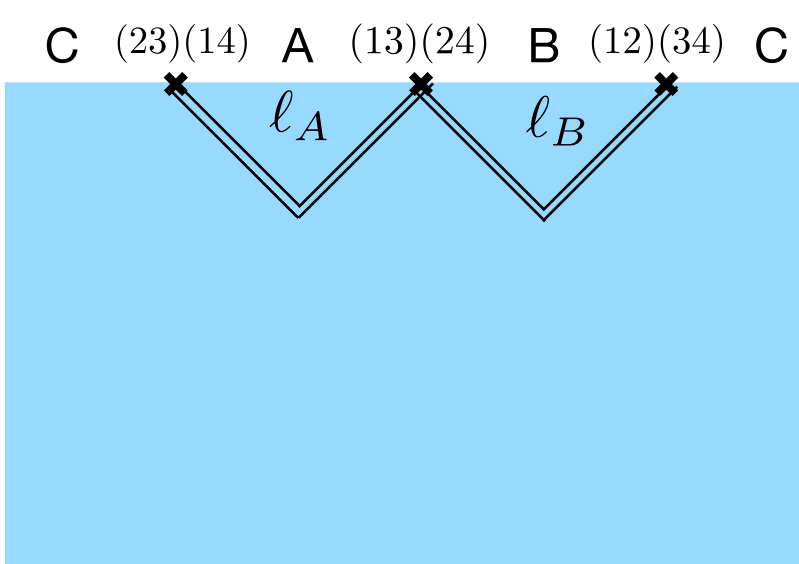
$$S_2^{\mathrm{op}}(A, \rho_{AB}) = \underline{-\ln \mathrm{tr}_{A\otimes A}\left[\mathrm{tr}_{B\otimes B} |\rho_{AB}\rangle\langle\rho_{AB}|\right]^2} - \underline{2S_2(AB)}$$

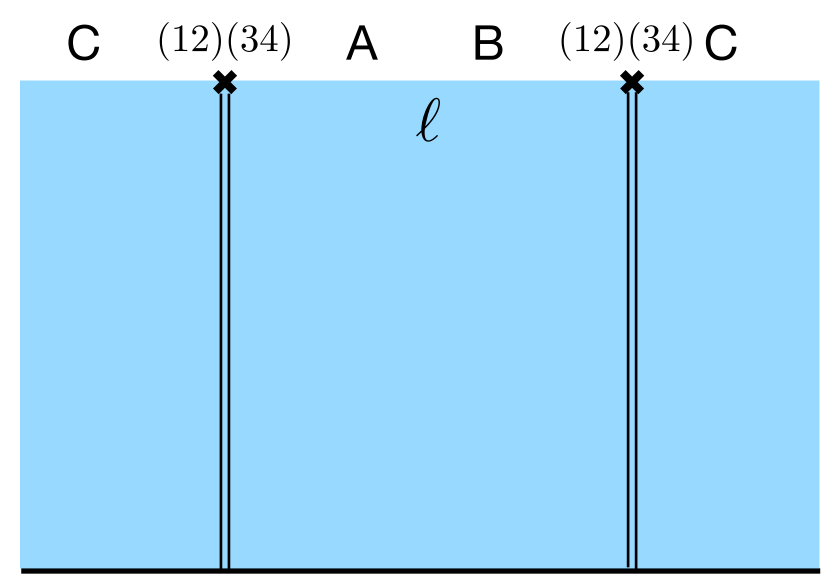


$$2(\ell_A + \ell_B)\ln q$$

phase 3

$$\frac{\ell_B}{2} + \frac{\ell_A}{4} < t < \frac{\ell}{2}$$

$t = 0$



$$4 \ln q \; t$$

Random Unitary Circuits

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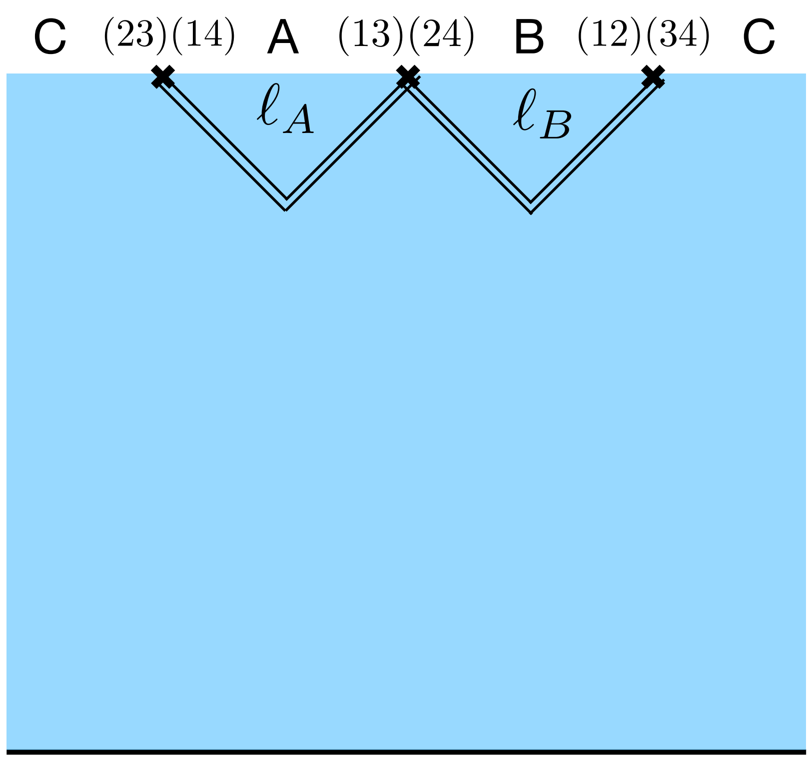
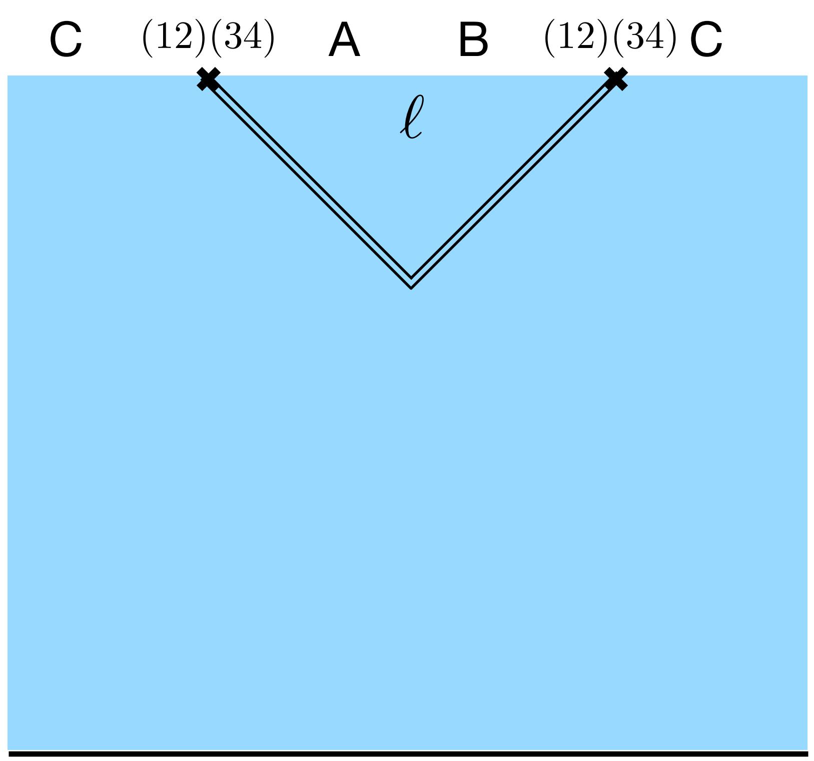


phase 4

$t > \ell/2$

$t = 0$



$2(\ell_A + \ell_B)\ln q$

$2\ell \ln q$

Random Unitary Circuits

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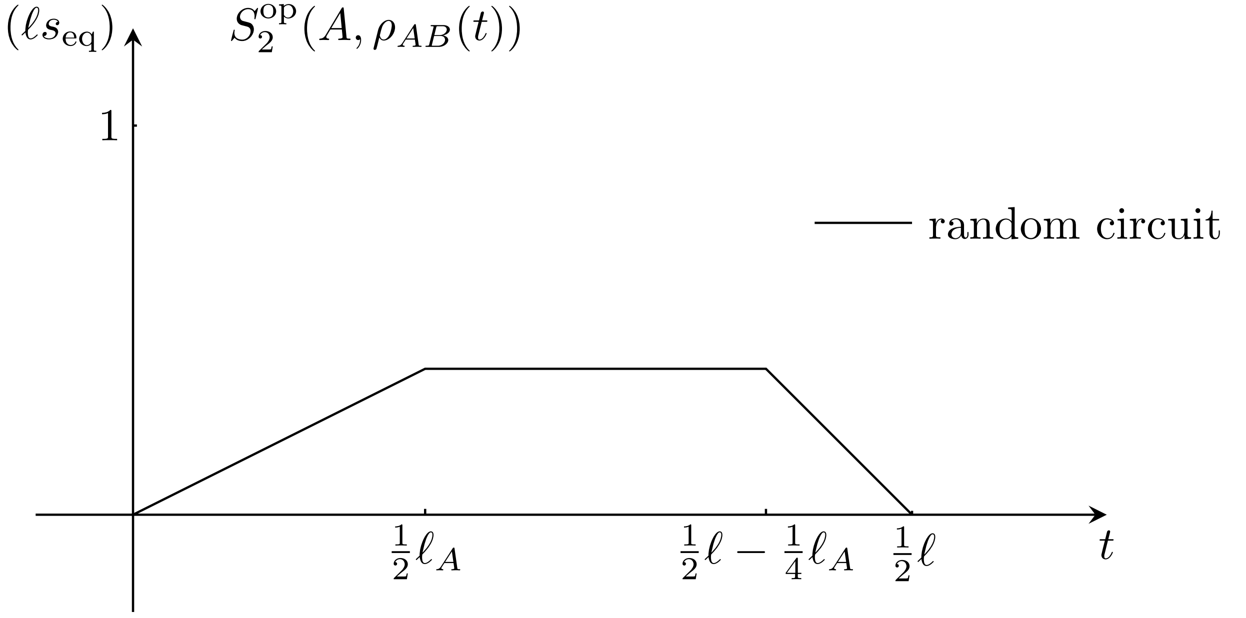
$$S_2^{\mathrm{op}}(A,\rho_{AB}) = -\ln \mathrm{tr}_{A\otimes A}\left[\mathrm{tr}_{B\otimes B}|\rho_{AB}\rangle\langle\rho_{AB}|\right]^2 - 2S_2(AB)$$

Put together: $q \to \infty$

$$S^{\mathrm{op}}_{n=2}(A,\rho_{AB}(t)) = \ln q \begin{cases} 2t & t < \frac{\ell_A}{2} \\ \ell_A & \frac{\ell_A}{2} < t < \frac{\ell_B}{2} + \frac{\ell_A}{4} \\ 2\ell - 4t & \frac{\ell_B}{2} + \frac{\ell_A}{4} < t < \frac{\ell}{2} \\ 0 & \frac{\ell}{2} < t \end{cases}$$



Random Unitary Circuits

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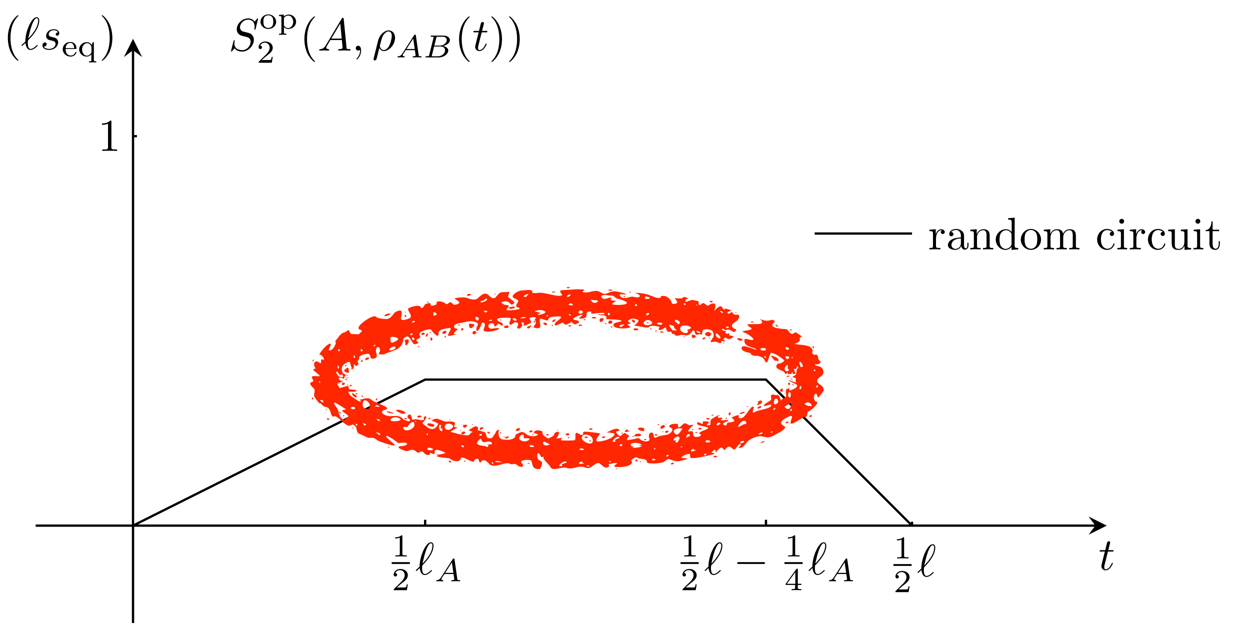

- Longer plateau than rational CFTs: $\left(\frac{\ell_A}{2}, \frac{\ell_B}{2}\right) \subset \left(\frac{\ell_A}{2}, \frac{\ell_B}{2} + \frac{\ell_A}{4}\right)$

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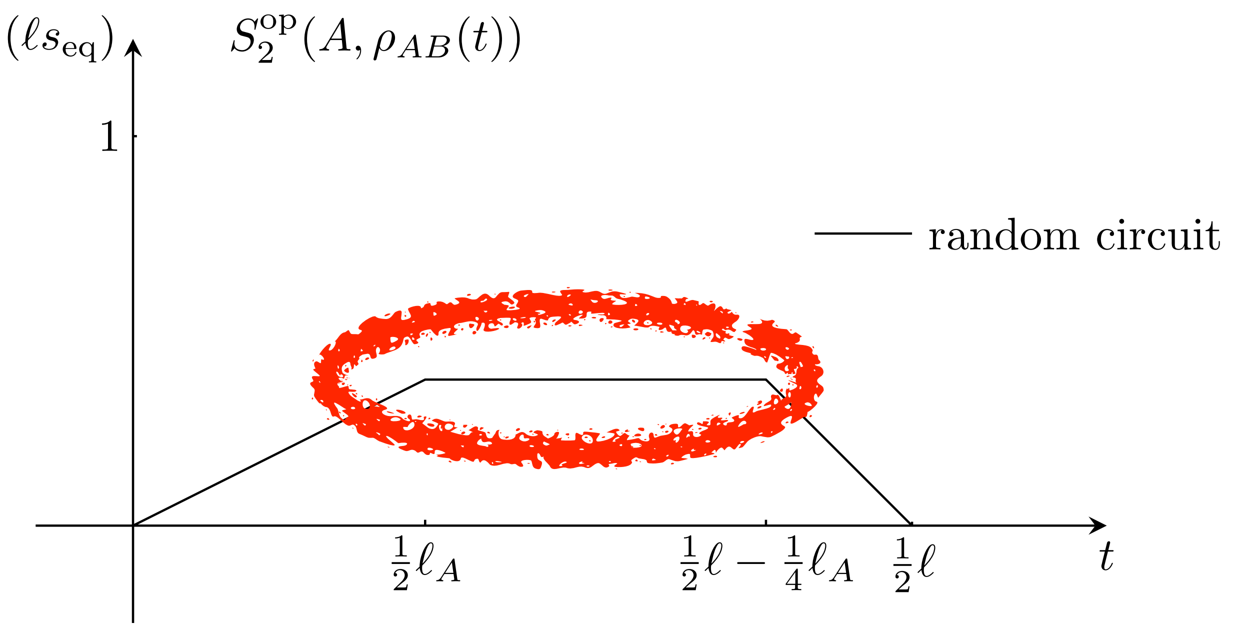

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- Can generalize to general. $n,\alpha\in\mathbb{N}$, same transitions

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- Set up: (reduced density) operator entanglement
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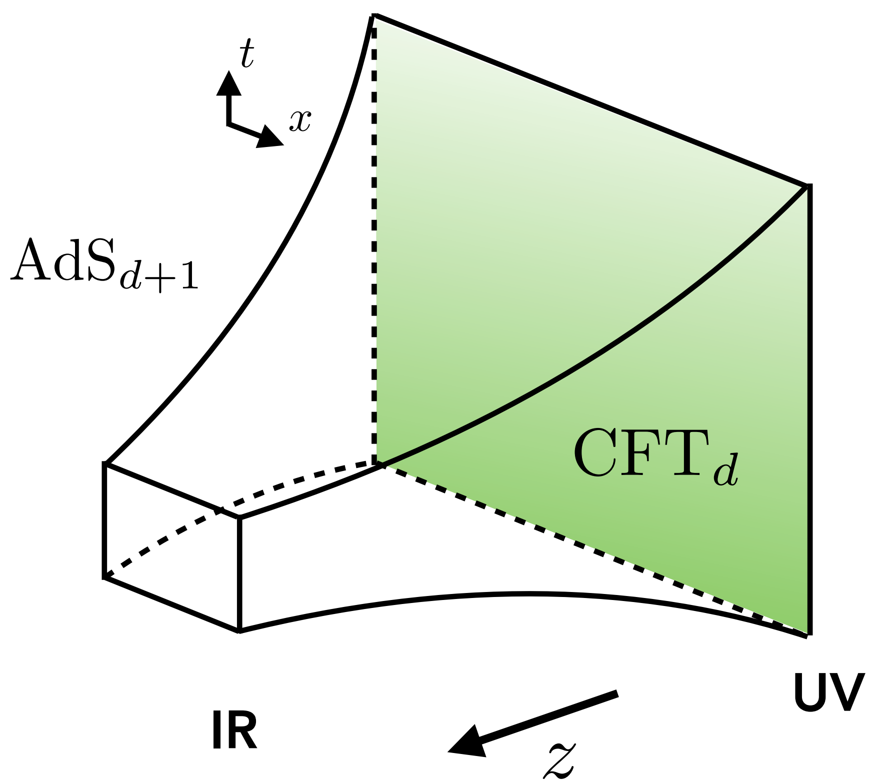
- Future directions

Holographic CFTs

- Dual to weakly coupled gravity (e.g. Einstein gravity) in AdS
- Large central charge, strong coupling (large spectral gap)
- Maximum quantum chaos J. Maldacena, S. Shenker, D. Stanford (2015)
- Universality of a large class of CFTs



Holographic CFTs

Entanglement entropy in AdS/CFT

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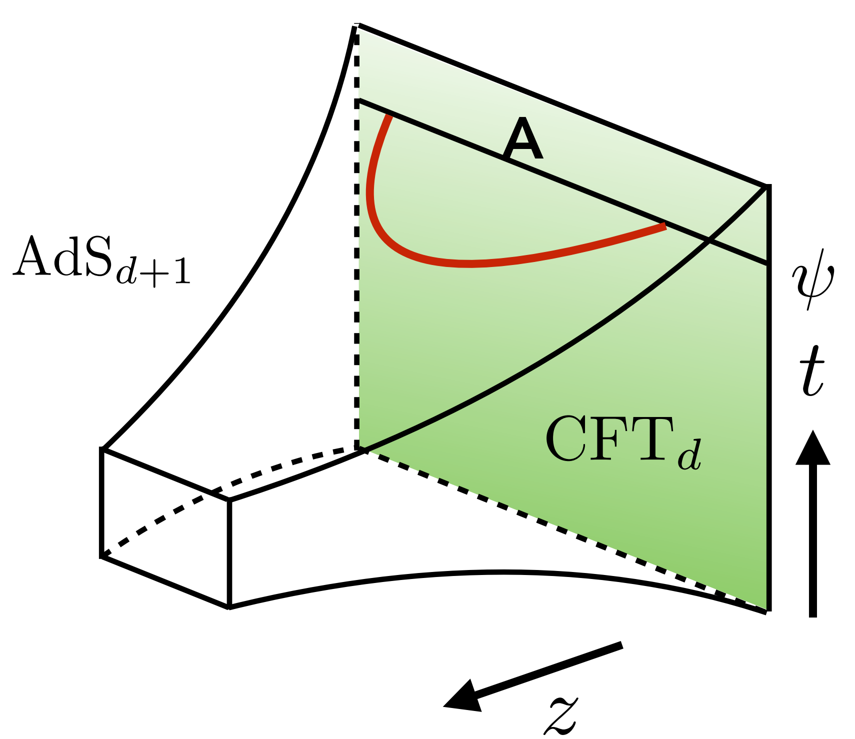
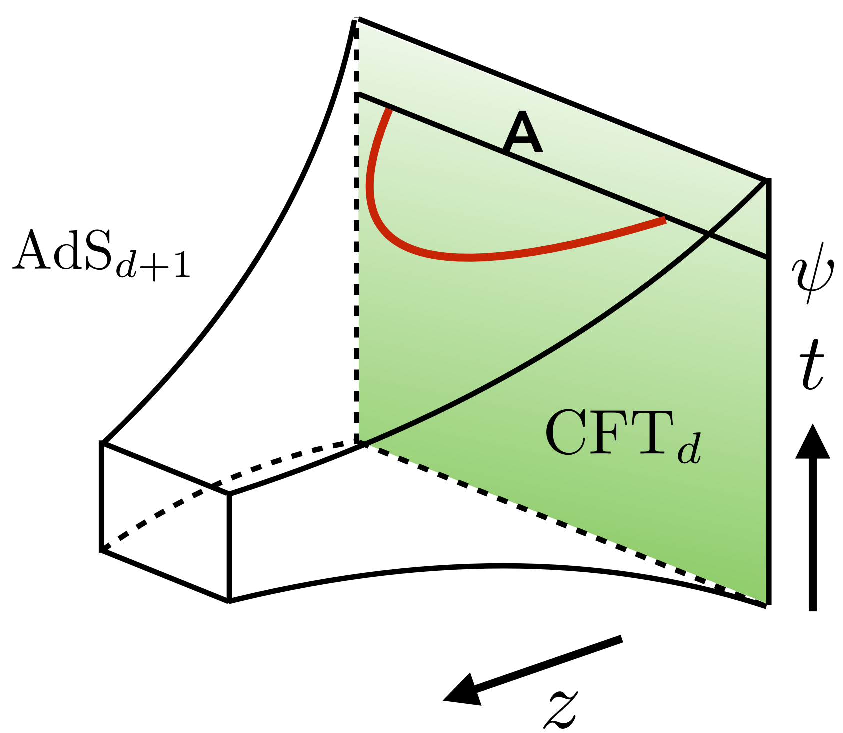
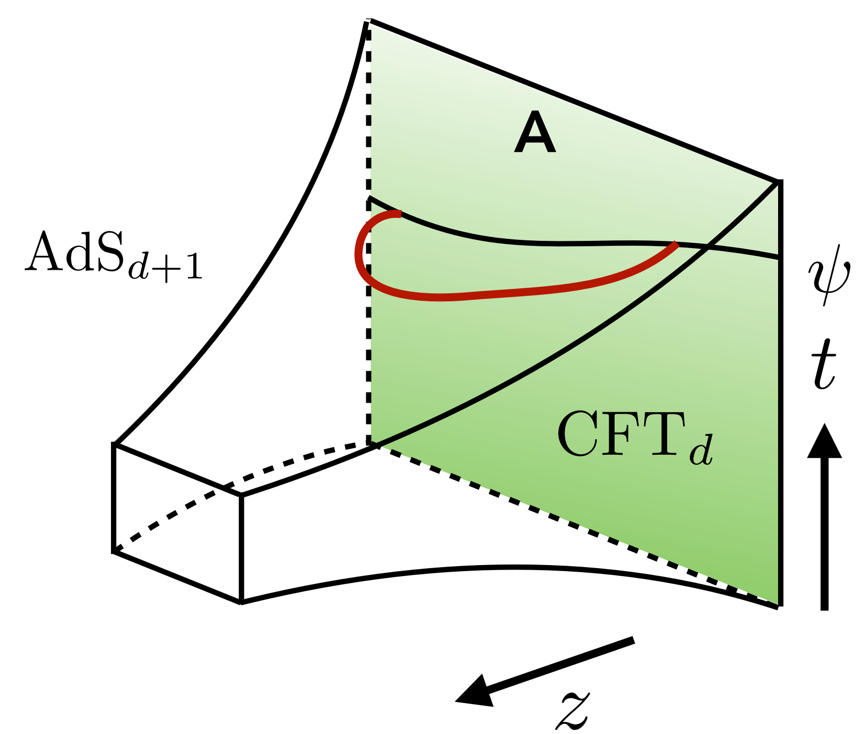


Holographic CFTs

Entanglement entropy in AdS/CFT

- Connection between boundary CFT and bulk geometry
- Ryu-Takayanagi (RT) formula: minimal surface area in static states
- Hubeny-Ragamani-Takayanagi (HRT): extremal surface in dynamical states





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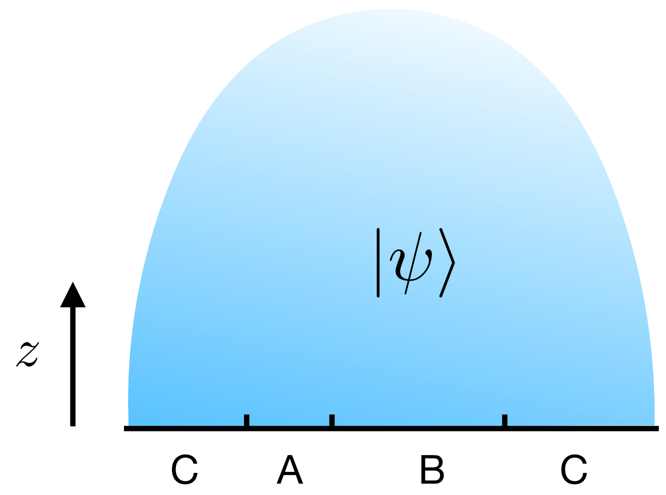

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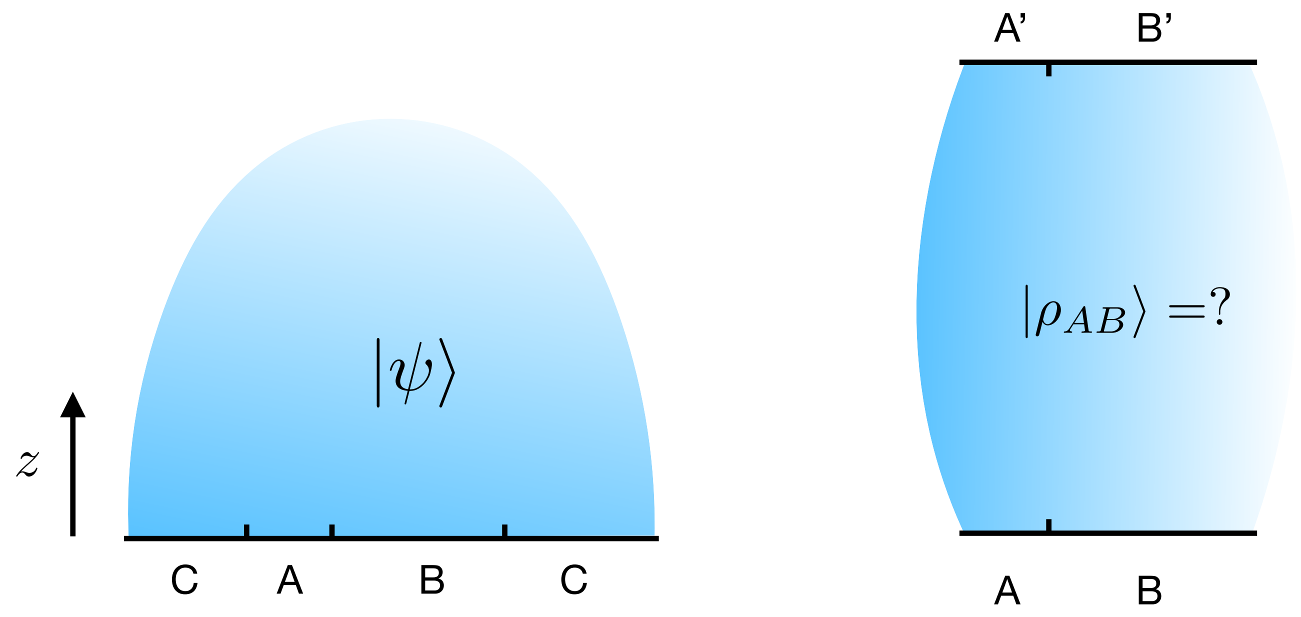

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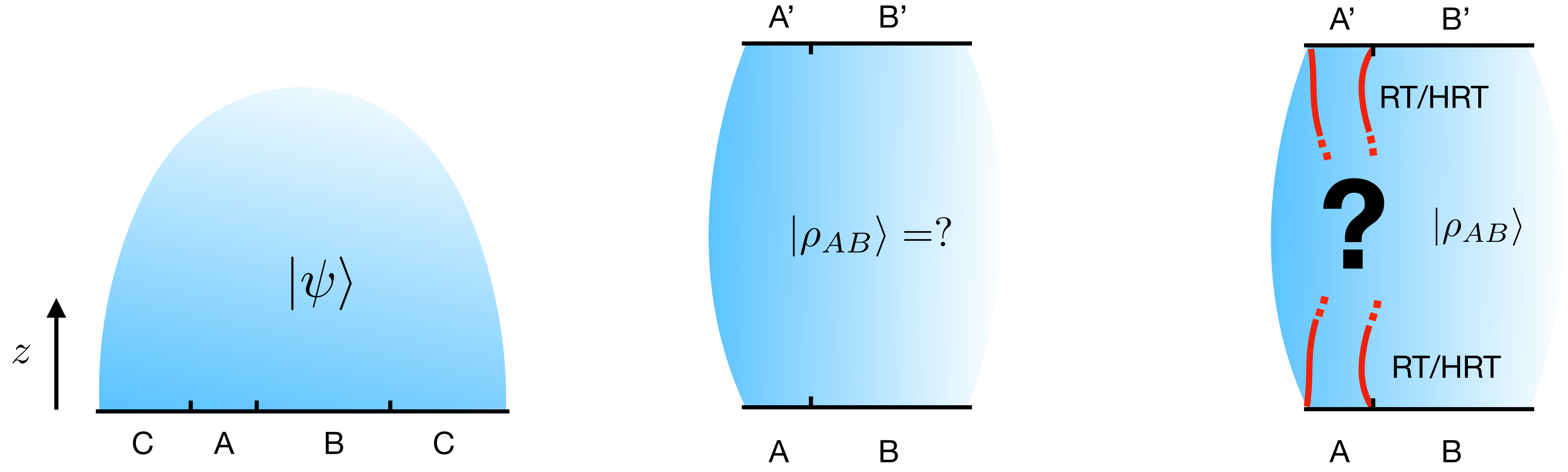

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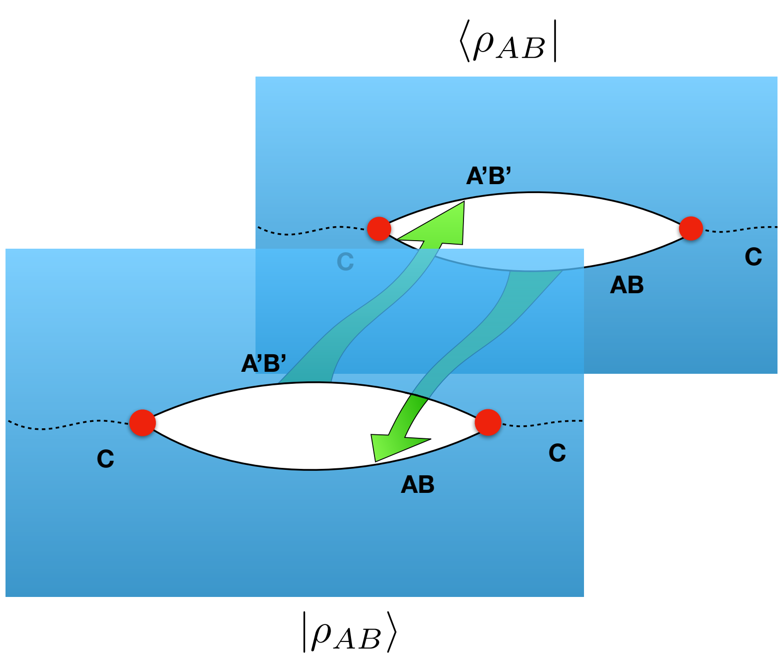

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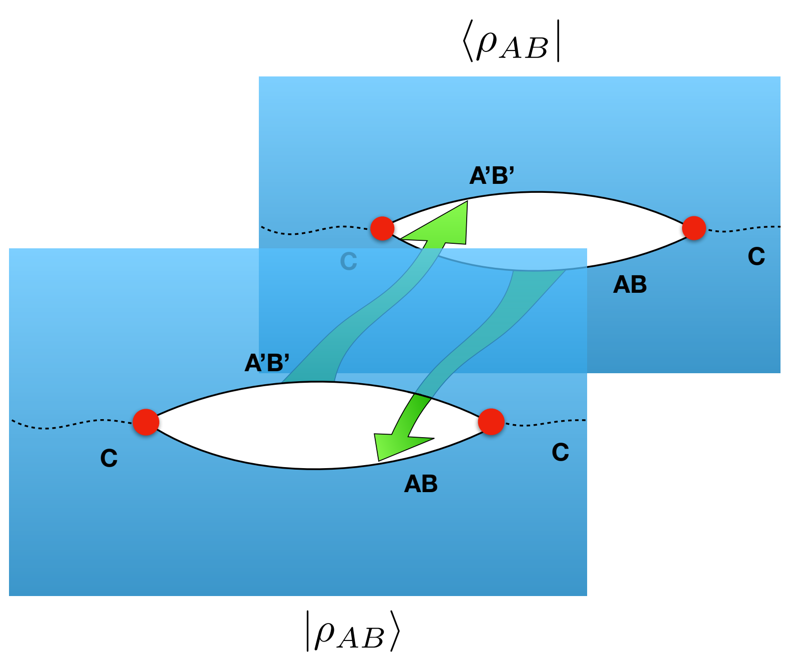

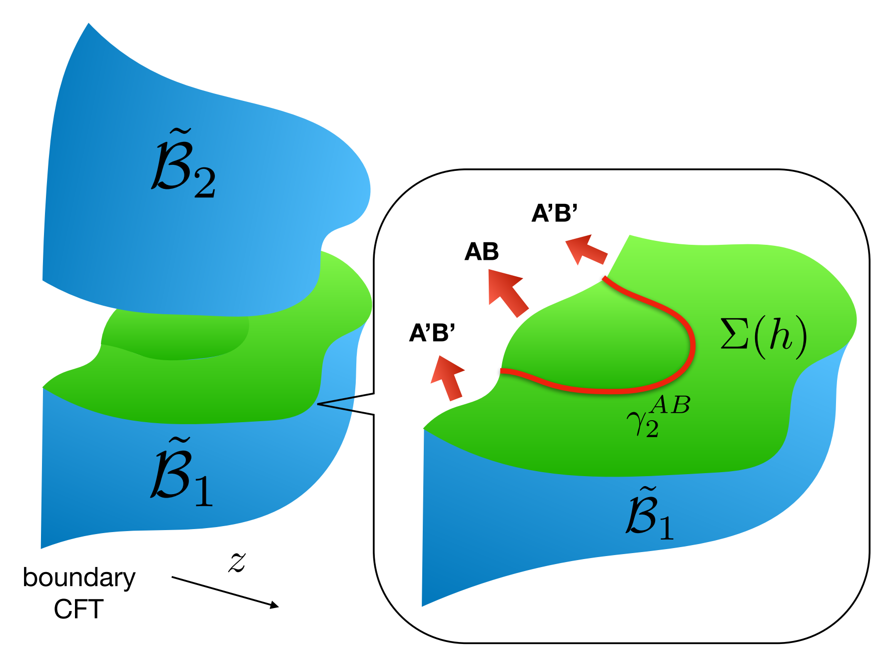

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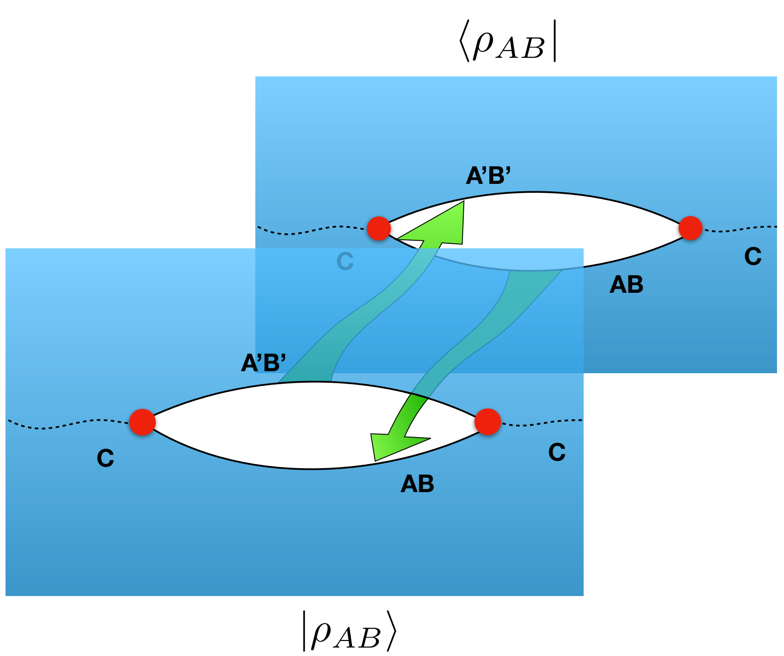

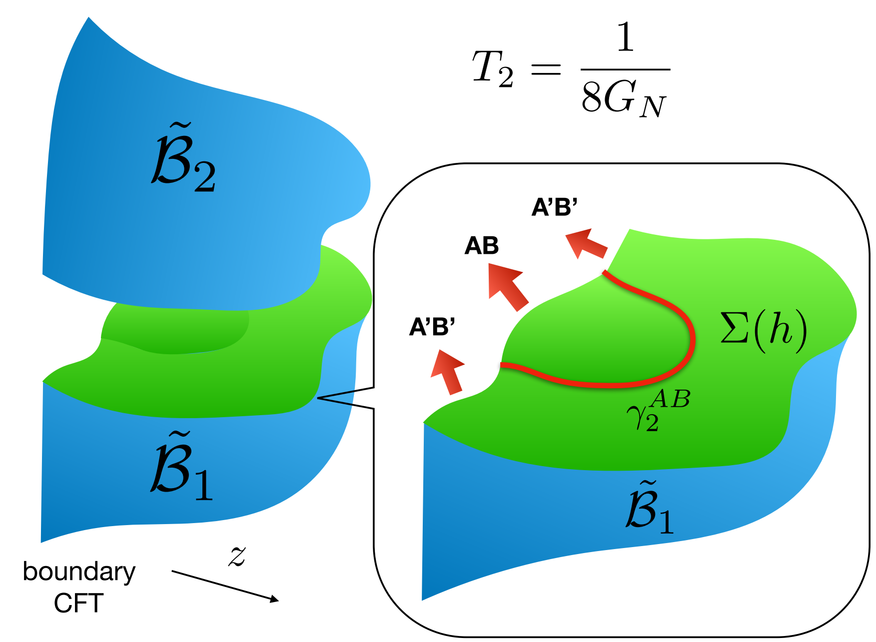

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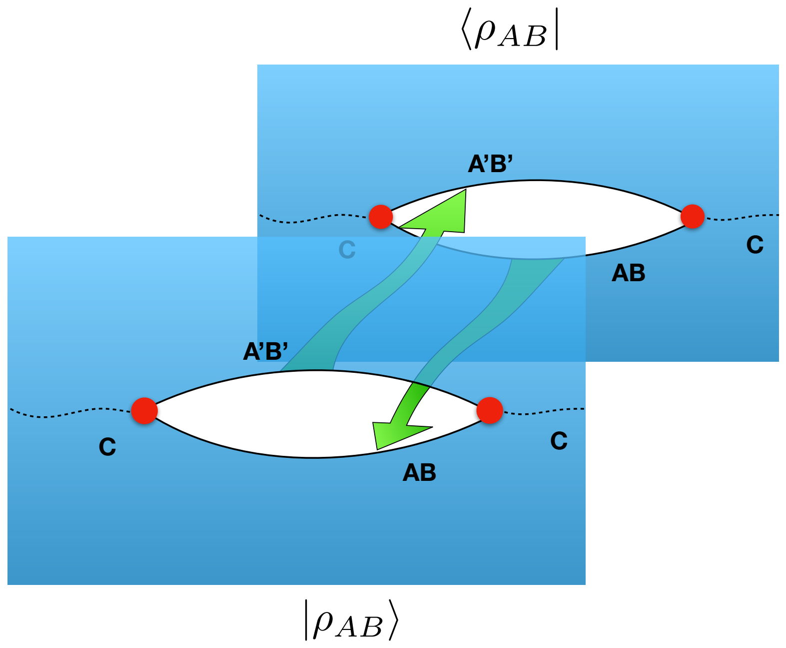

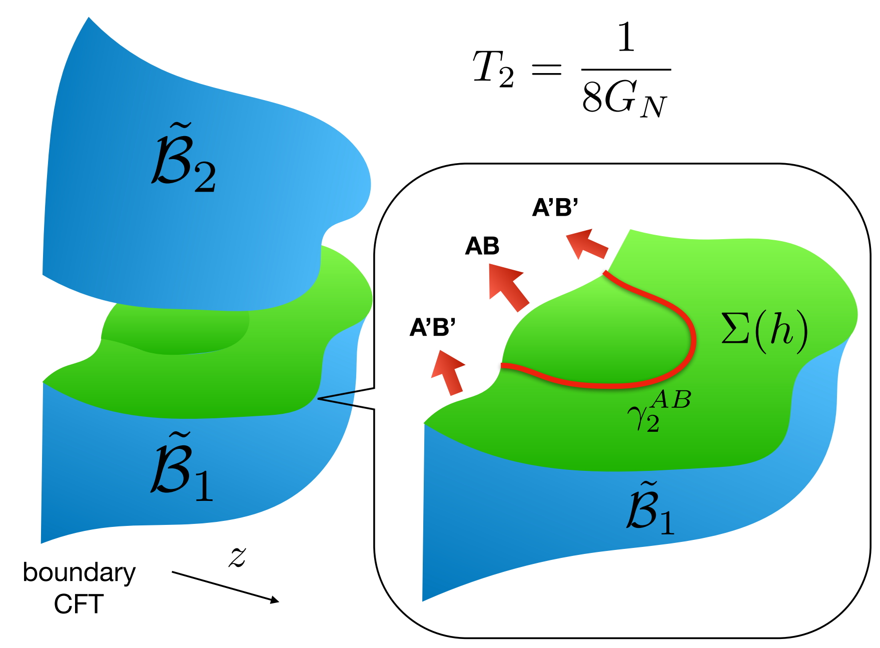

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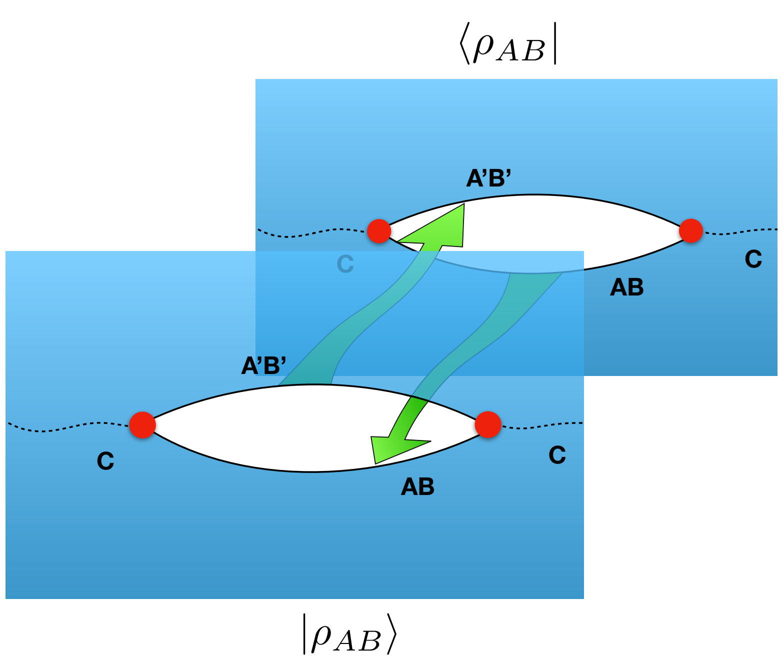

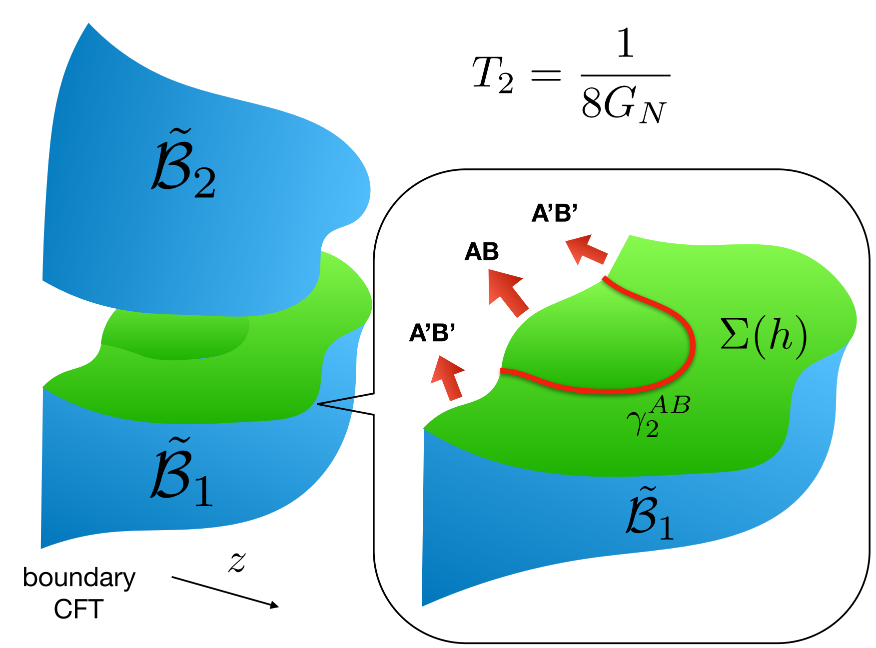

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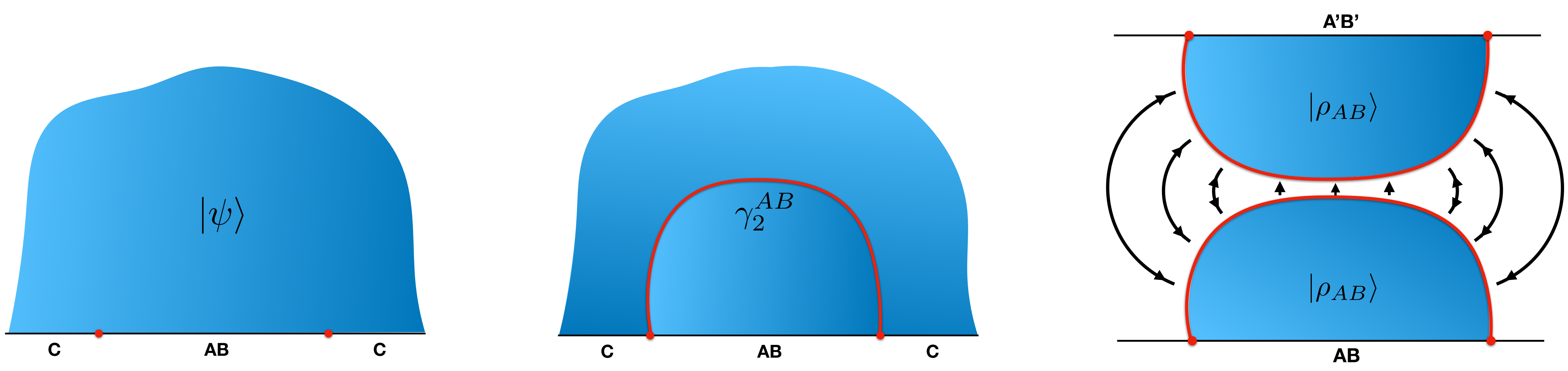
- Key ingredients: back-react + identify across “cosmic brane”



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- Dynamical (e.g. quenched) state: identify across HRT surface

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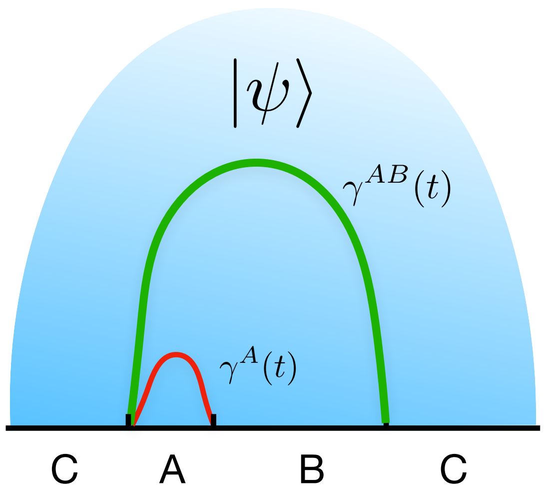

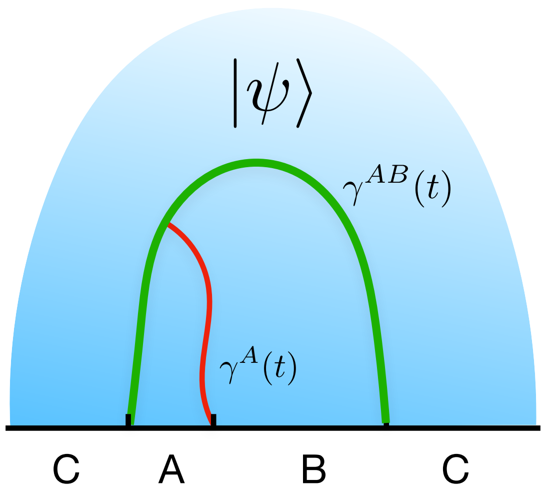

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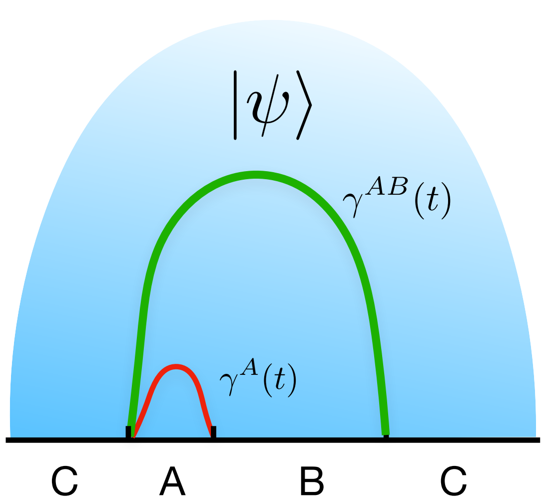

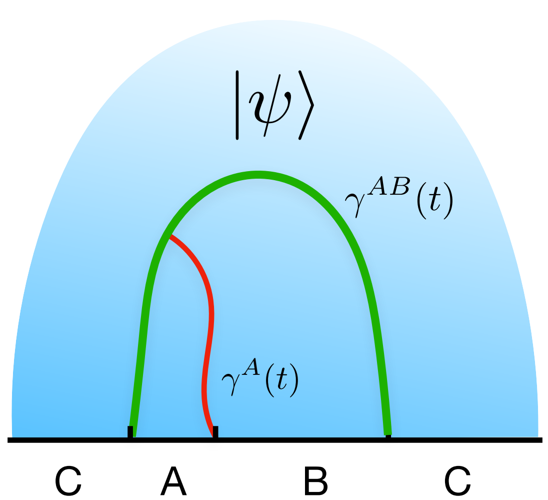

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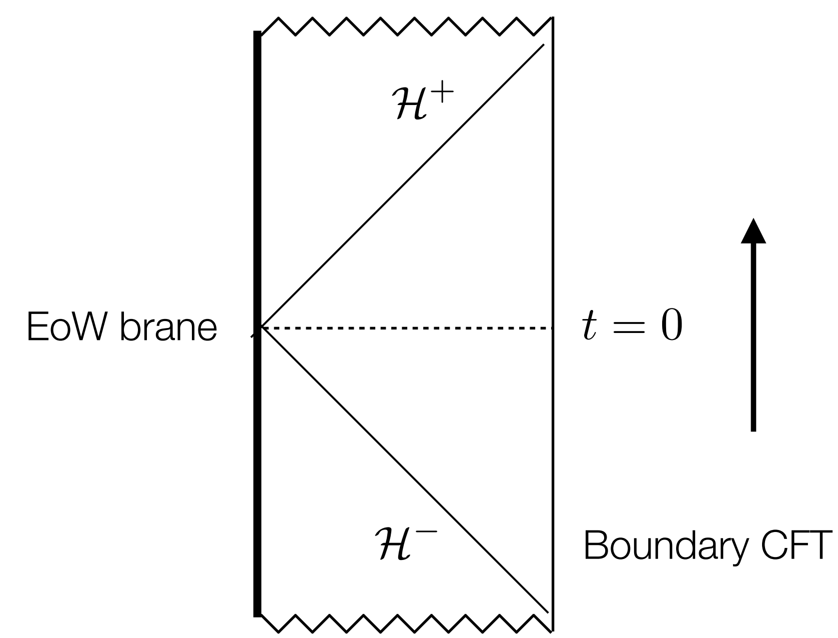

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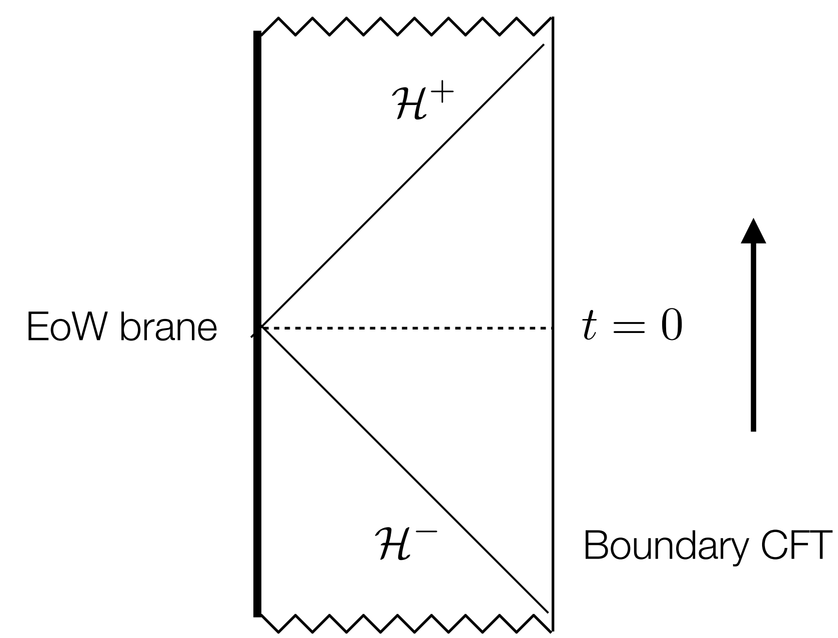

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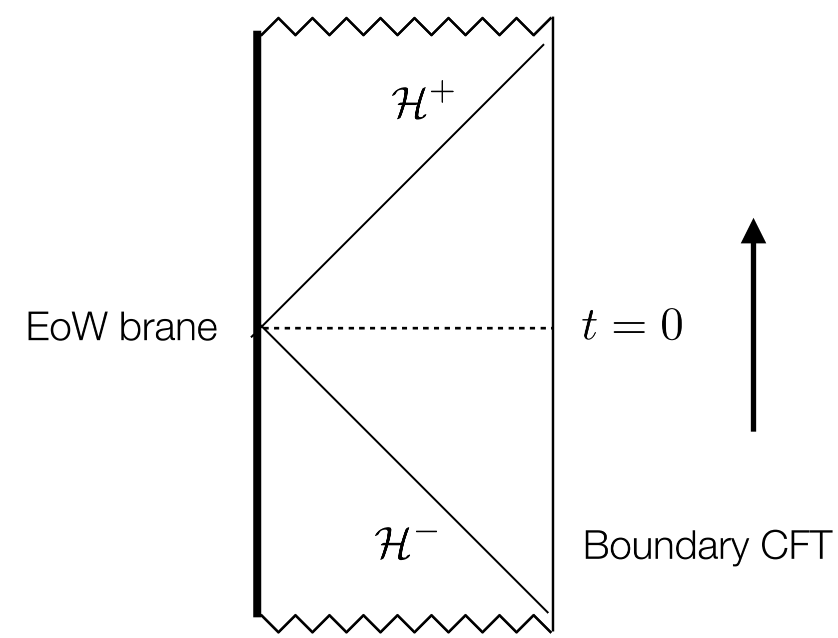

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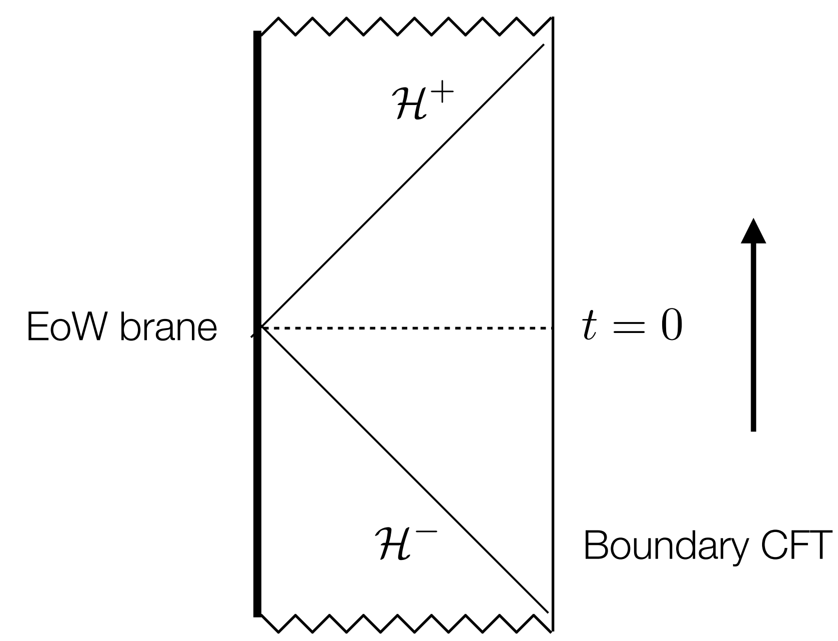

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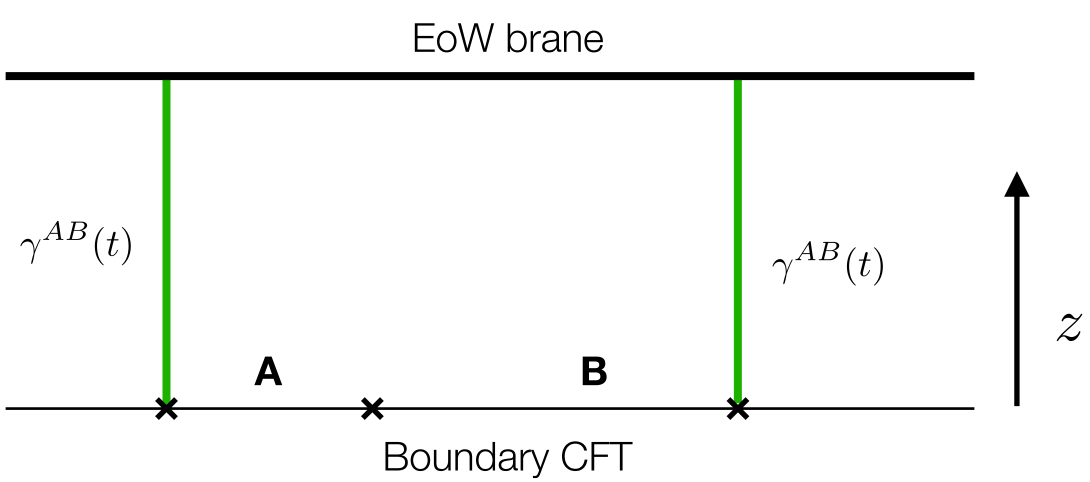

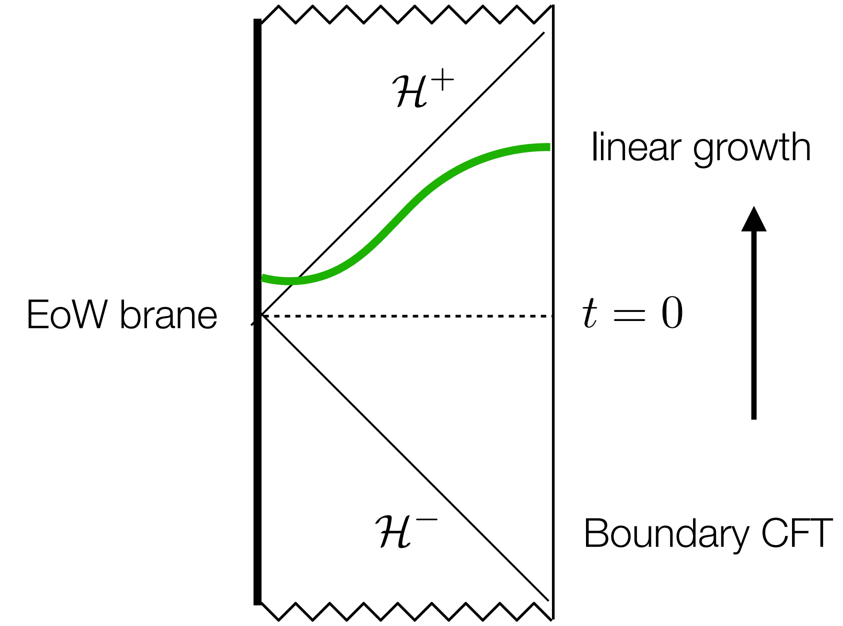

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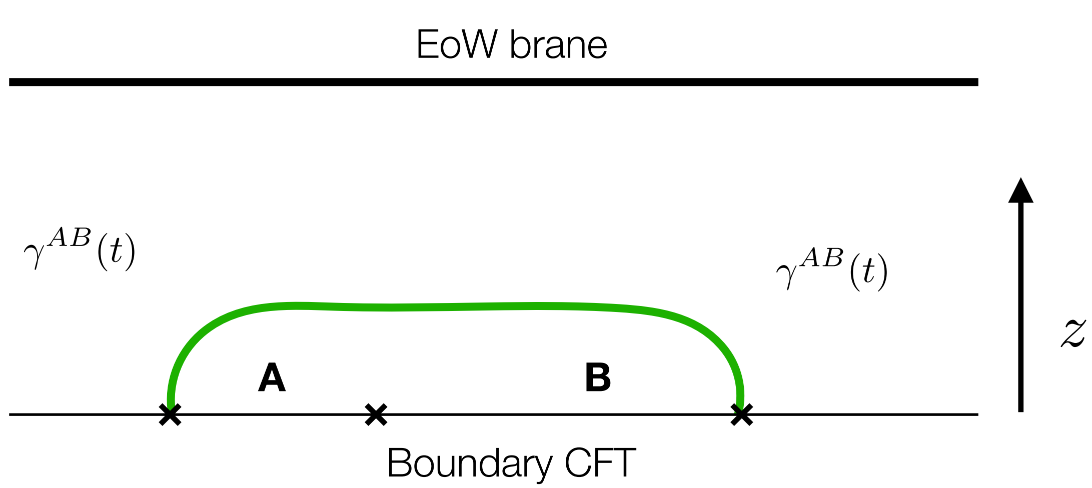

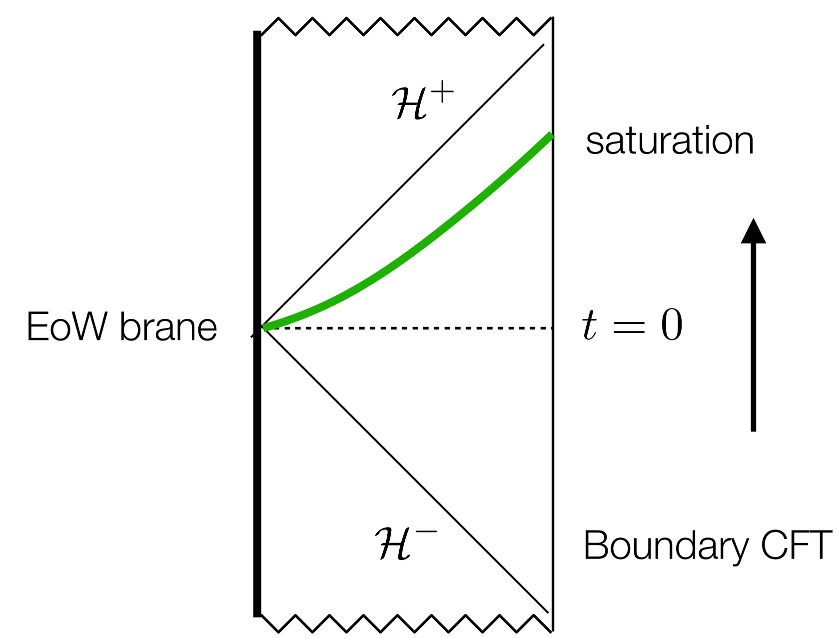

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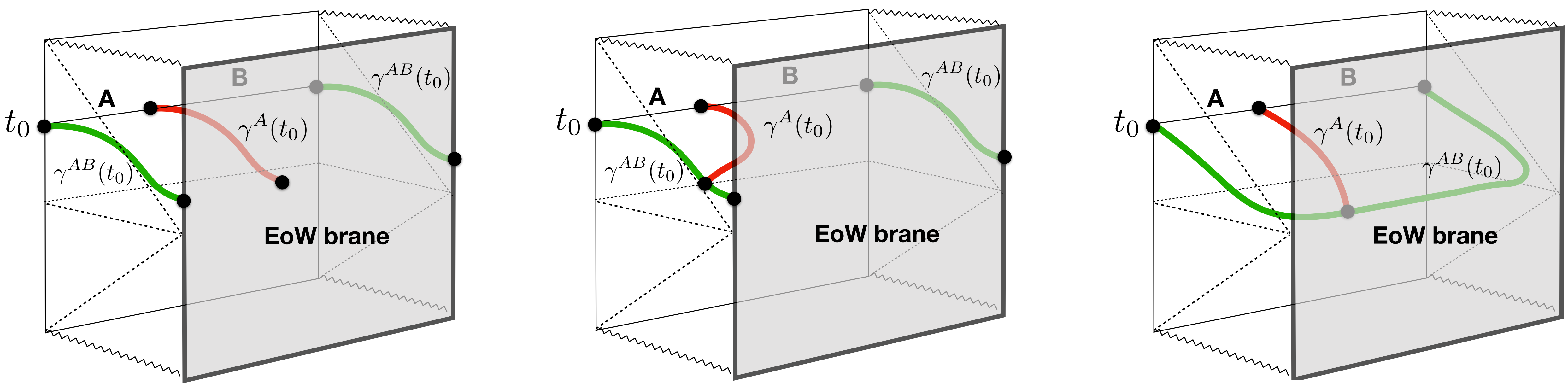
Phases of $\gamma^{A}(t)$:



linear growth $\sim v_d t$ saturation $\sim \ell_A$ short-range $\sim \beta$

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In AdS3/CFT2: $$S^{\mathrm{op}}_{A}(t) = \begin{cases} \frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right) + \frac{2\pi c}{3\beta}t & t < \frac{\ell_A}{2} \\ \frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right) + \frac{\pi c}{3\beta}\ell_A & \frac{\ell_A}{2} < t < \frac{\ell}{2} \\ \frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right) + \mathcal{O}\left(e^{-\frac{2\pi}{\beta}\ell_{A,B}}\right) & t > \frac{\ell}{2} \end{cases}$$

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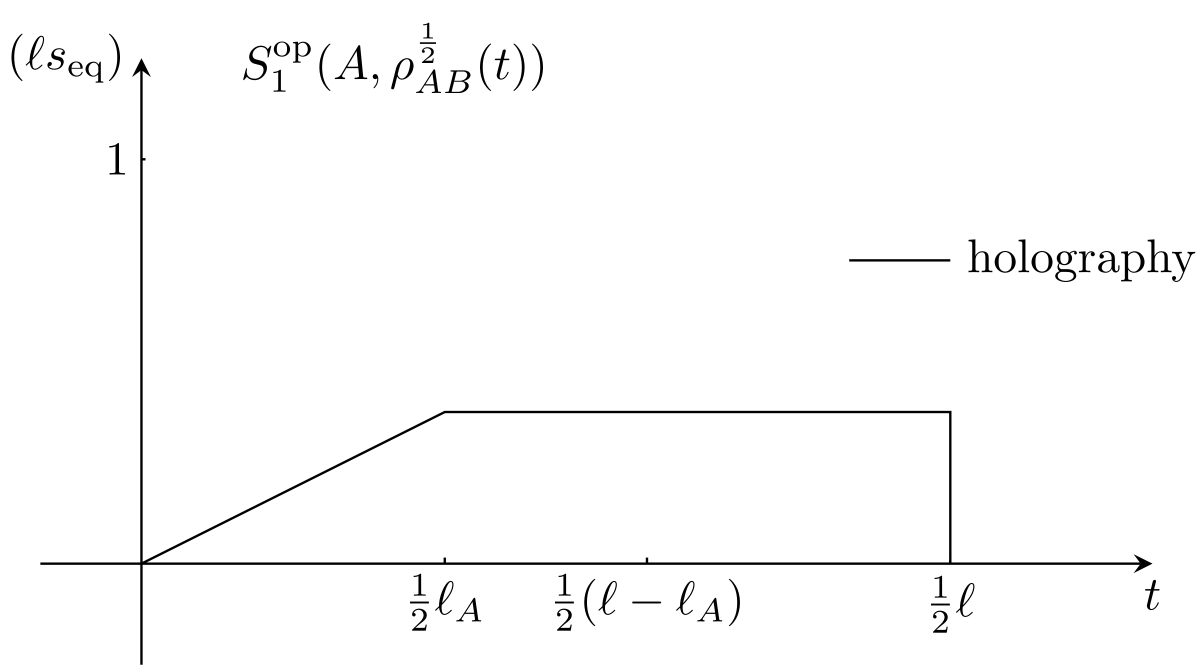

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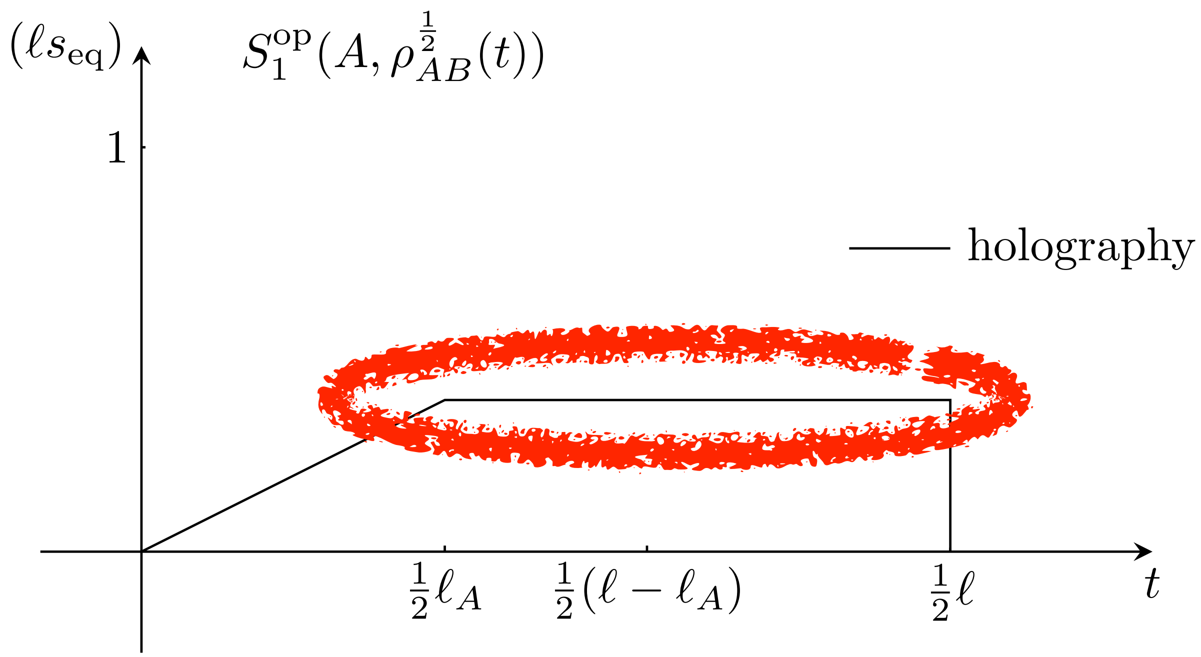

- Longest plateau among models

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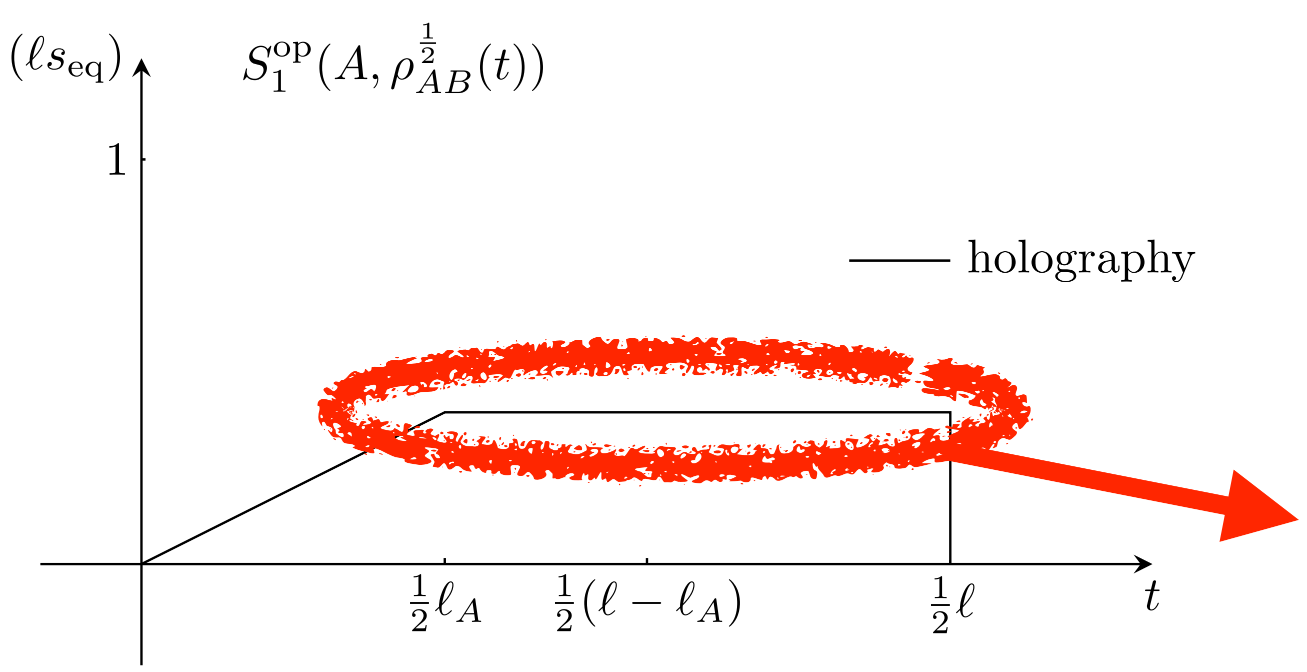
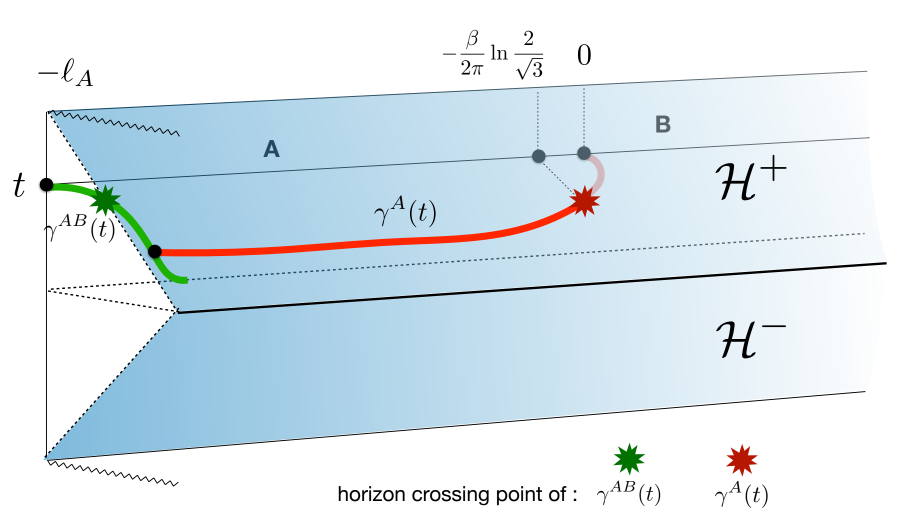
Operator *entanglement* entropy: $S_{\rm ent}^{\rm op}\left(A,\rho_{AB}^{1/2}\right)$ i.e. "reflective entropy" T. Faulkner, et al (2019)

In AdS3/CFT2:

$$S_A^{\rm op}(t)=\begin{cases}\frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right)+\frac{2\pi c}{3\beta}t & t<\frac{\ell_A}{2}\\ \frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right)+\frac{\pi c}{3\beta}\ell_A & \frac{\ell_A}{2}<t<\frac{\ell}{2}\\ \frac{c}{3}\ln\left(\frac{2\pi}{\beta}\right)+\mathcal{O}\left(e^{-\frac{2\pi}{\beta}\ell_{A,B}}\right) & t>\frac{\ell}{2}\end{cases}$$



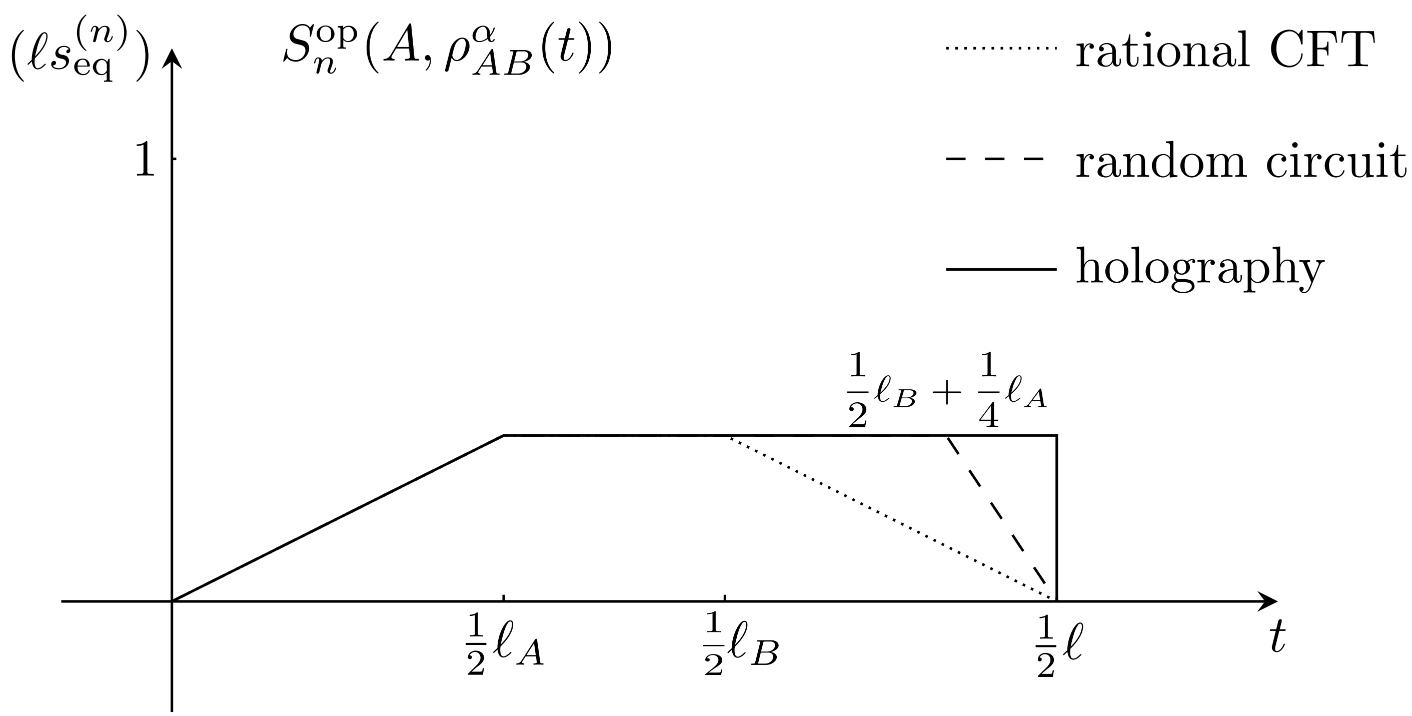
- Longest plateau among models
- Contribution behind horizon



Outline:

- Set up: (reduced density) operator entanglement
- Quenched dynamics for operator entanglement
    1. Two-dimensional rational conformal field theories (CFTs)
    2. Random unitary circuits
    3. Holographic CFTs (AdS/CFT)
- Discussion: a new measure of quantum chaoticity?
- Conclude/outlook

Discussion: ***a new measure of quantum chaoticity?***



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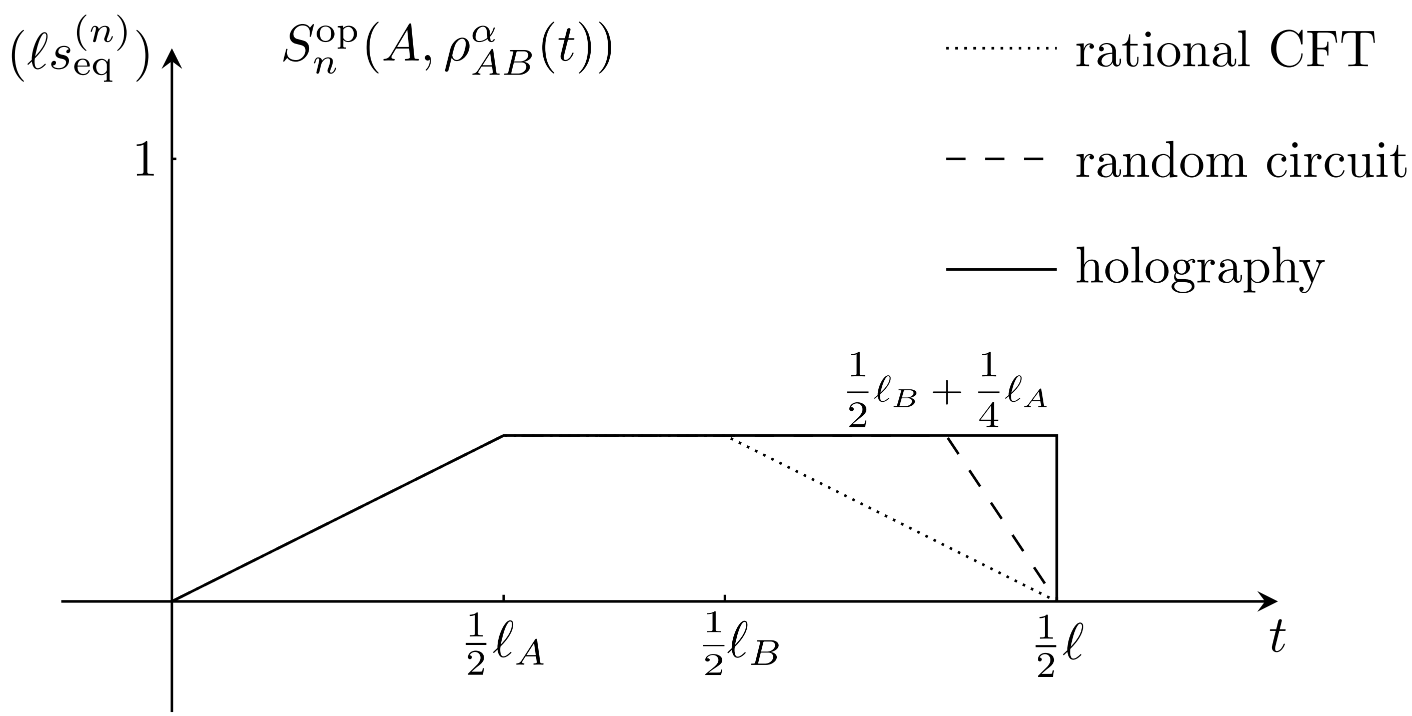

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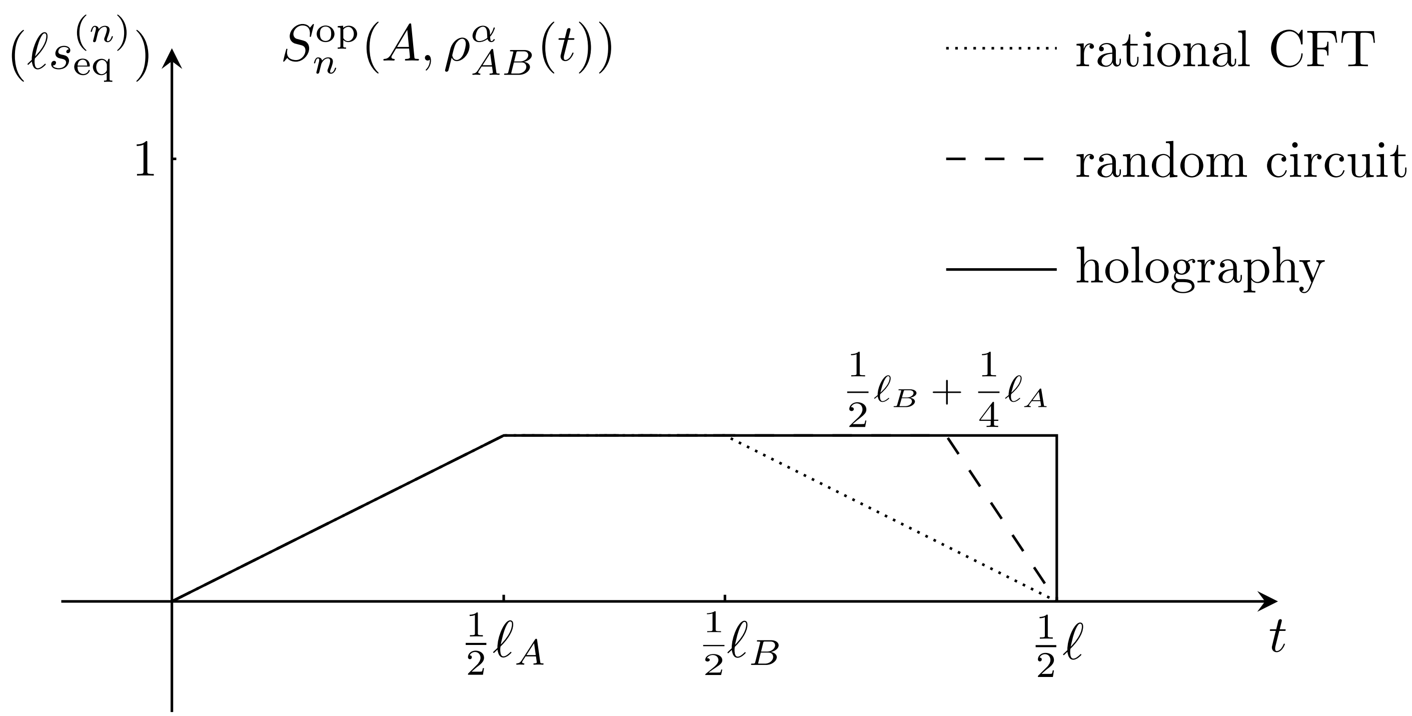

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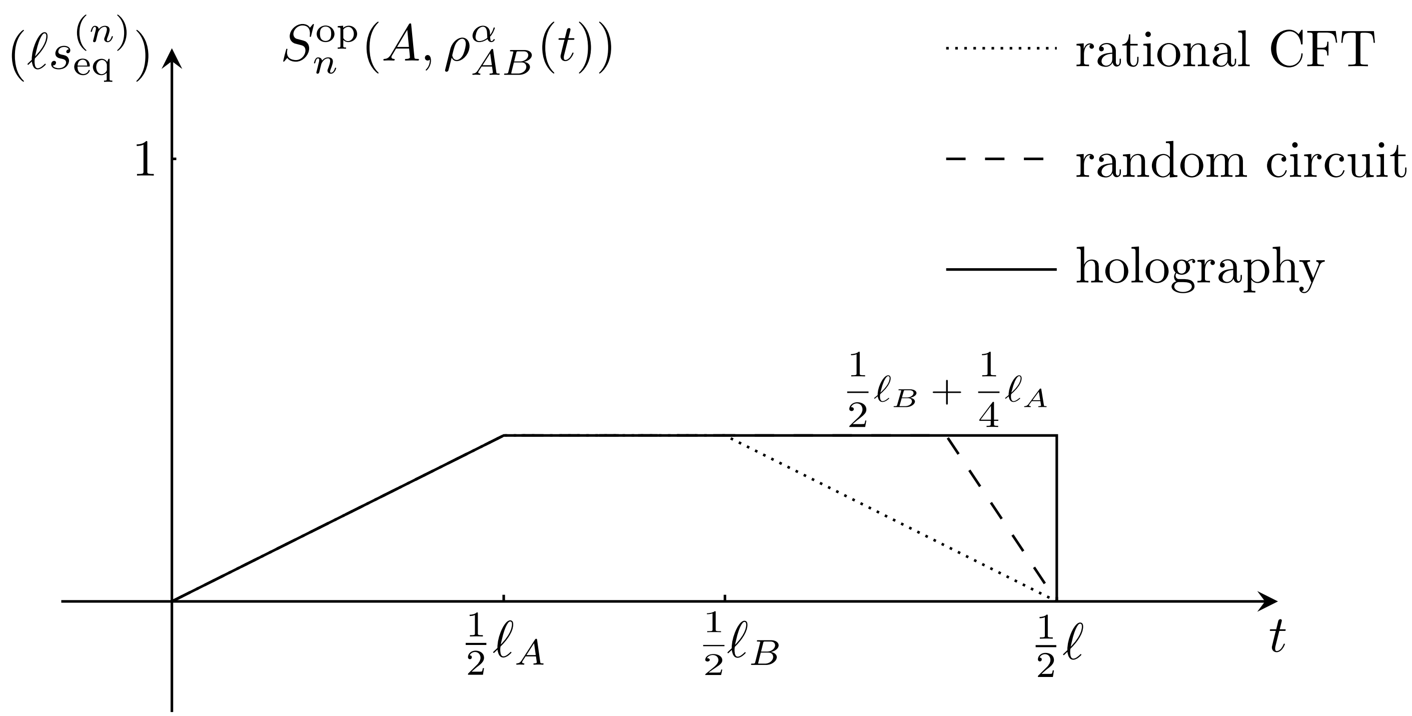

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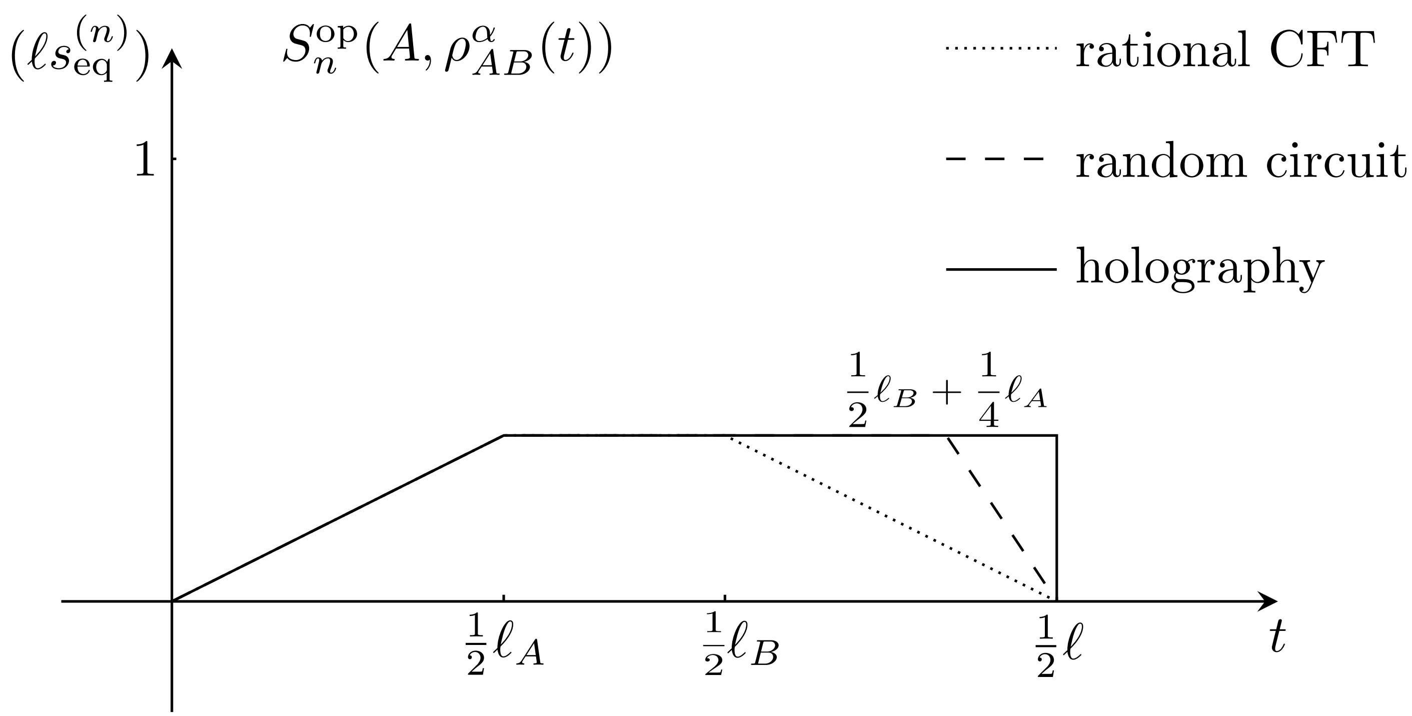

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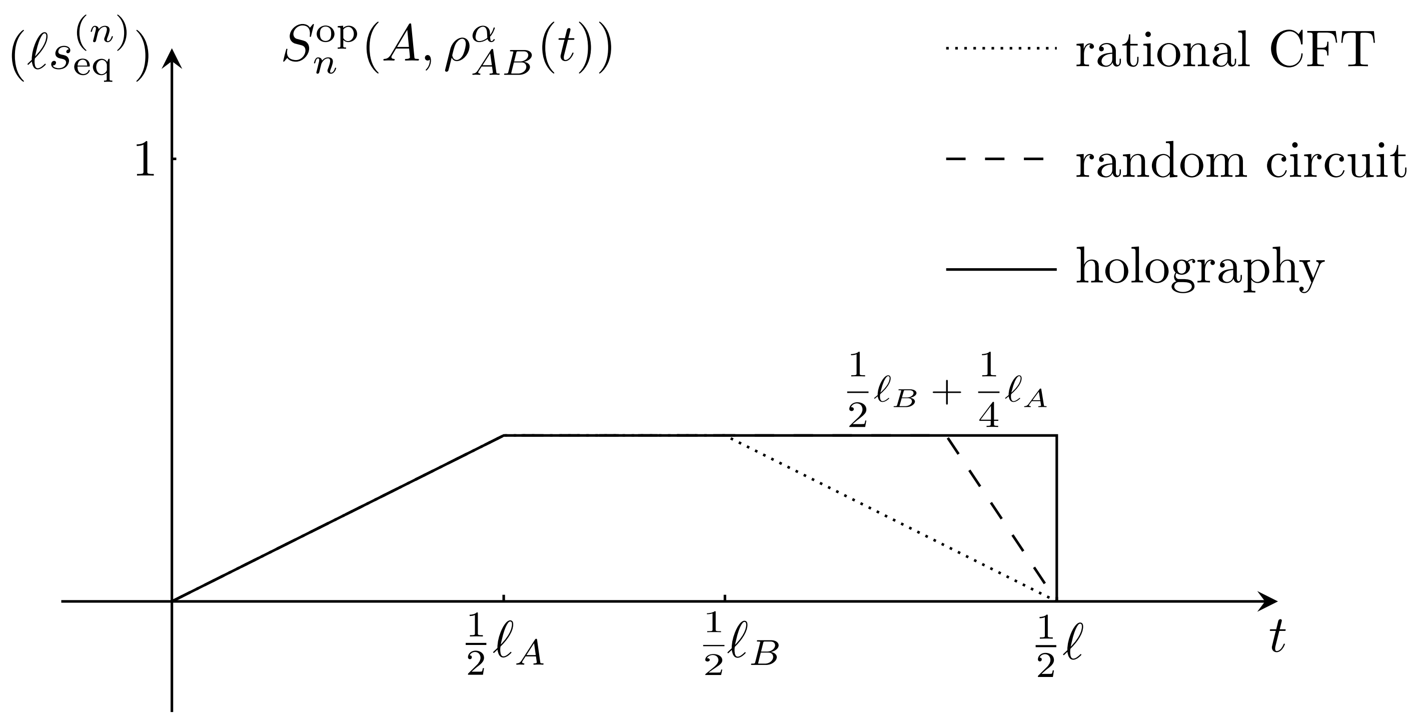

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Thank you!